Neutrino Tomography with 3D Earth Models

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Contents

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Simulations with OscProb

README.md

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OscProb

OscProb is a small set of classes aimed at computing exact neutrino oscillation probabilities with a few different models.

OscProb contains a basic framework for computing neutrino oscillation probabilities. It is integrated into ROOT, so that each class can be used as you would any ROOT class.

Available classes are:

- **PremModel:** Used for determining neutrino paths through the earth
- **PMNS_Fast:** Standard 3-flavour oscillations
- **PMNS_Iter:** Standard 3-flavour oscillations (iterative)
- **PMNS_Sterile:** Oscillations with any number of neutrinos
- **PMNS_NSI:** Oscillations with 3 flavours including Non-Standard Interactions
- **PMNS_Deco:** Oscillations with 3 flavours including a simple decoherence model
- **PMNS_LIV:** Oscillations with 3 flavours including Lorentz Invariance Violations
- **PMNS_Decay:** Oscillations with 3 flavours including neutrino decays of the second and third neutrino mass states \nu_2 and \nu_3. [Requires external library Eigen3, see the instructions below.]
- **Absorption:** Computes absorption probabilities for high-energy neutrinos

- https://github.com/joaoabcoelho/OscProb

Original OscProb Schematic

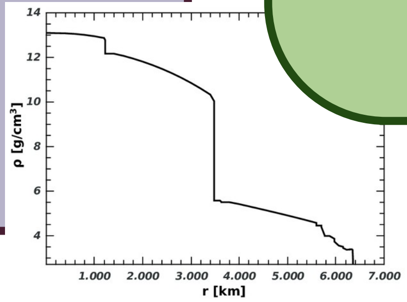
Detector Position (radius)

Class: PremModel

<u>Earth Model Table</u>

- Outer depth
- Density
- Z/A
- "index"

for each layer

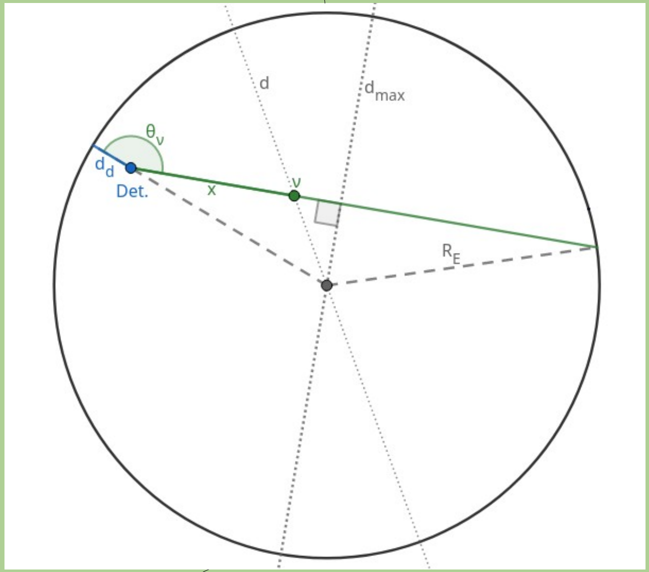


Original OscProb Schematic

Zenith Angle

Detector Position (radius)

Class: PremModel



Earth Model Table

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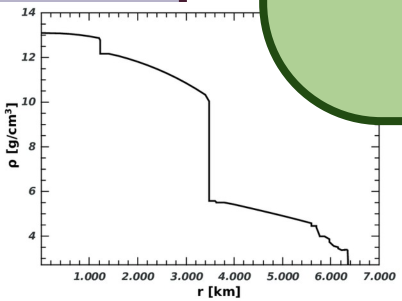


NuPath Object

For each section: distance, layer info

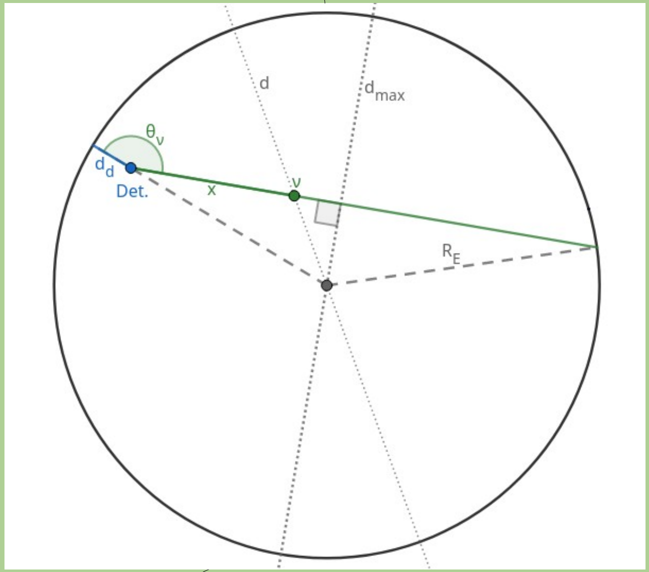
Det

Original OscProb Schematic

Zenith Angle

Detector Position (radius)

Class: PremModel



Oscillation Probability
$P_{\alpha\rightarrow\beta}(E, \cos(\theta))$

PMNS Class

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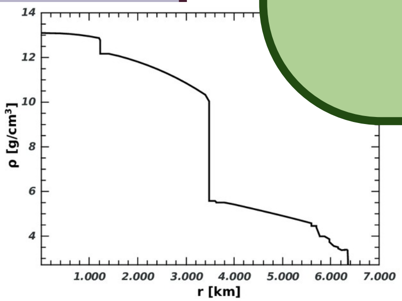
for each layer



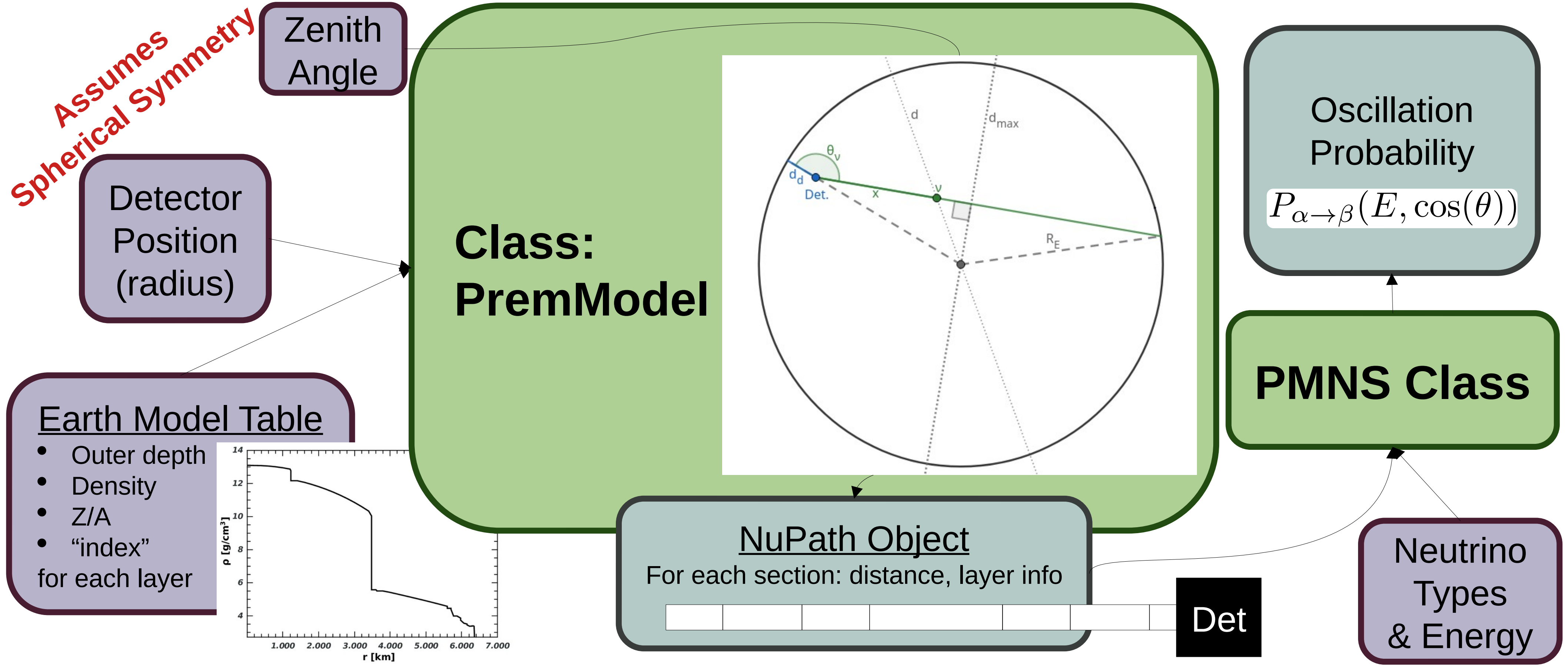
NuPath Object

For each section: distance, layer info

Det

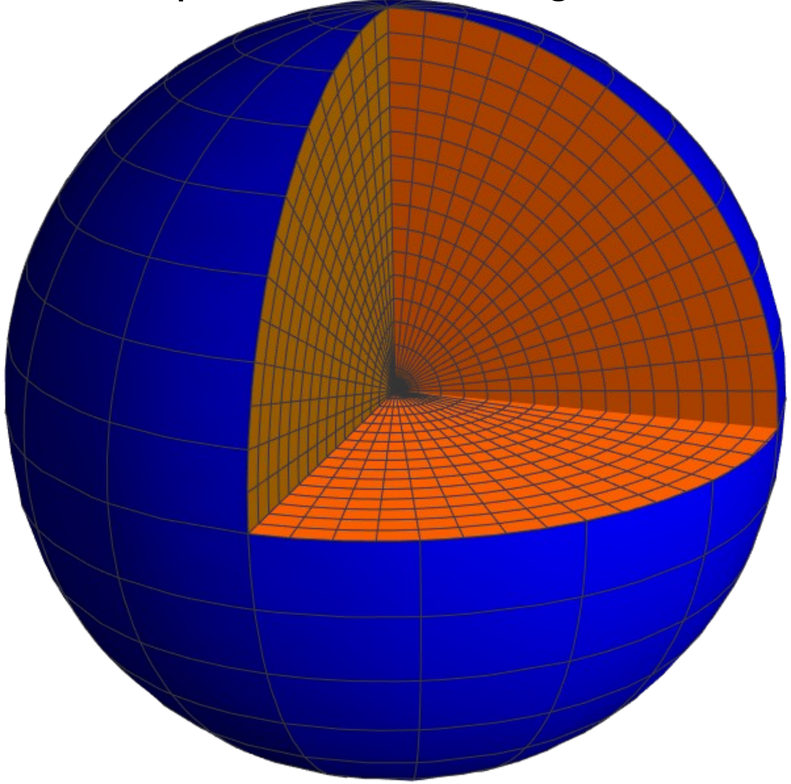
Neutrino Types & Energy

Original OscProb Schematic



Remove Spherical Symmetry

<u>3D Binned Earth Model</u>
Depth, Latitude, Longitude



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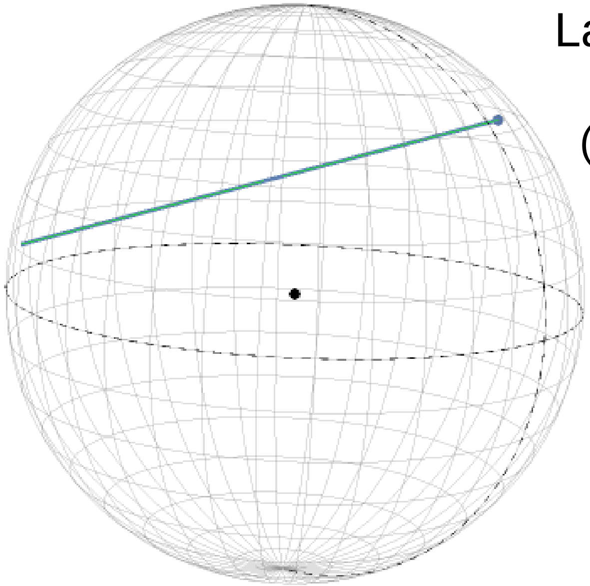
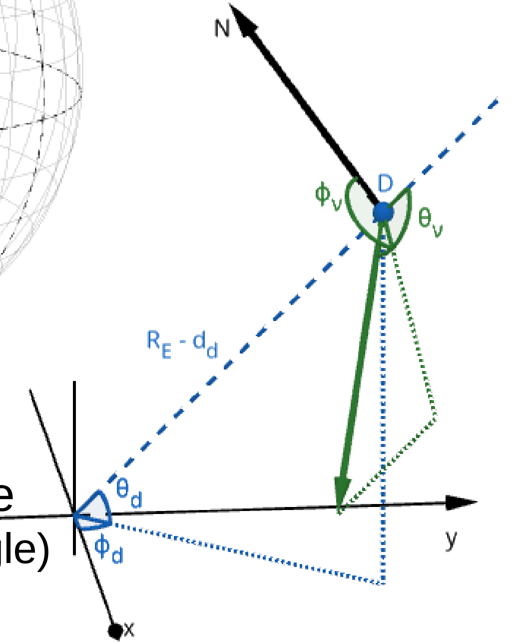
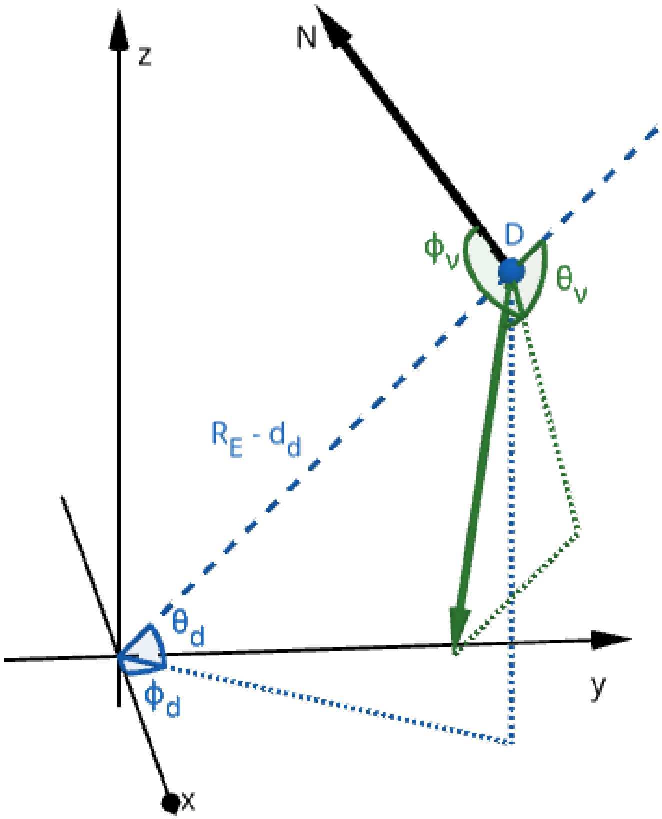


Latitude & Longitude of Detector
(in addition to depth)

Neutrino Trajectory with Azimuthal Angle
(in addition to zenith angle)

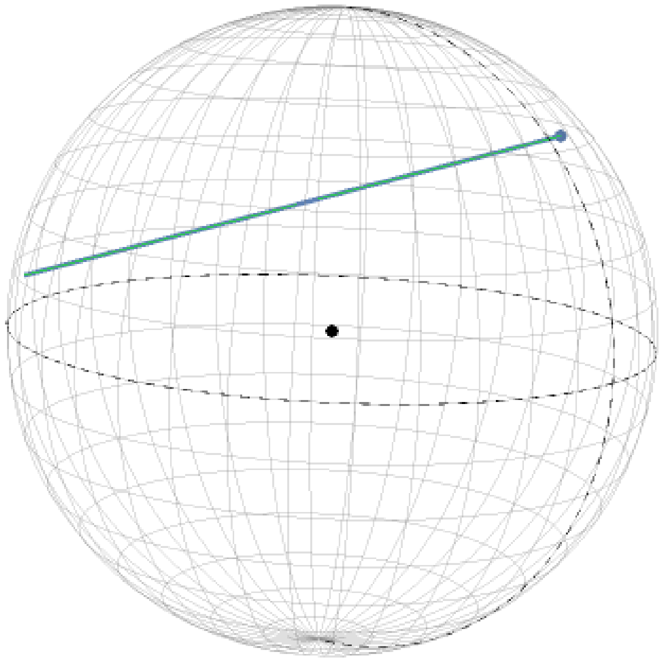


Neutrino Trajectory Equations



- Use geometry to determine coordinates at an arbitary point along the neutrino's trajectory
  - Unit vector from detector along trajectory:
    $$\hat{v} = \cos(\theta_\nu)\hat{r_d} + \sin(\theta_\nu)\left[\cos(\phi_\nu)\hat{\theta_d} - \sin(\phi_\nu)\hat{\phi_d}\right]$$
  - Vector from Earth's center to arbitary point:
    $$\vec{x_\nu} = \vec{r_d} + d_x\hat{v} \quad \text{(distance } d_x \text{ away from detector)}$$
  - Coordinate functions:
    $$r(d_x) = |\vec{x_\nu}| \qquad \sin(\theta(d_x)) = \frac{\vec{x_\nu}\cdot\hat{z}}{|\vec{x_\nu}|} \qquad \tan(\phi(d_x)) = \frac{\vec{x_\nu}\cdot\hat{y}}{\vec{x_\nu}\cdot\hat{x}}$$
- Invert equations (for bin boundary crossings)

Using a 3D Binned Earth



- Find point where neutrino enters Earth

$$d_x(r = R_E) \equiv d_0 \qquad \theta(d_0) \qquad \phi(d_0)$$

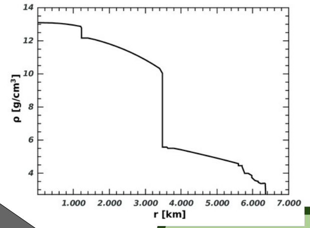
- Determine neutrino path segments
  - Find next bin boundary crossing
  - Record bin information + length

- Neutrino Oscillation Probability

Update for OscProb

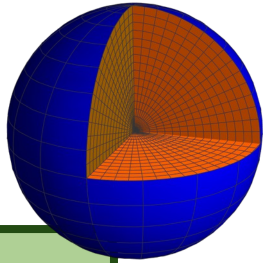
- Added a new class:

PremModel →



EarthModel

PremModel | EarthModelBinned



- New calculation scheme for FillPath in EarthModelBinned:
  - Uses equations found for locations of bin crossings

Initial
θ, ϕ, x

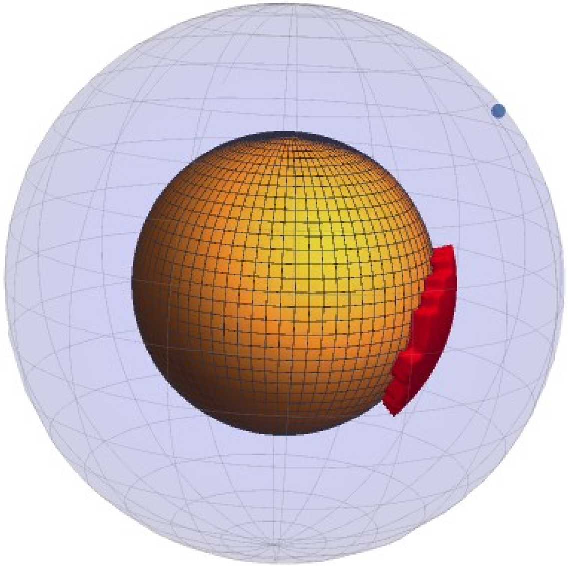
Loop over depth bins:

1) Find x at next bin edge
2) Check for θ/ϕ bin crossings
3) Record remaining segment

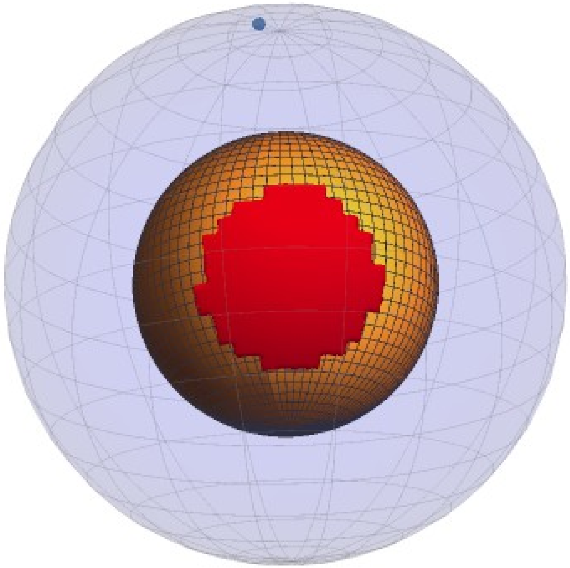
For each crossing (in order):

1) Record segment to crossing
2) Find x at next crossing
   (for same coordinate)

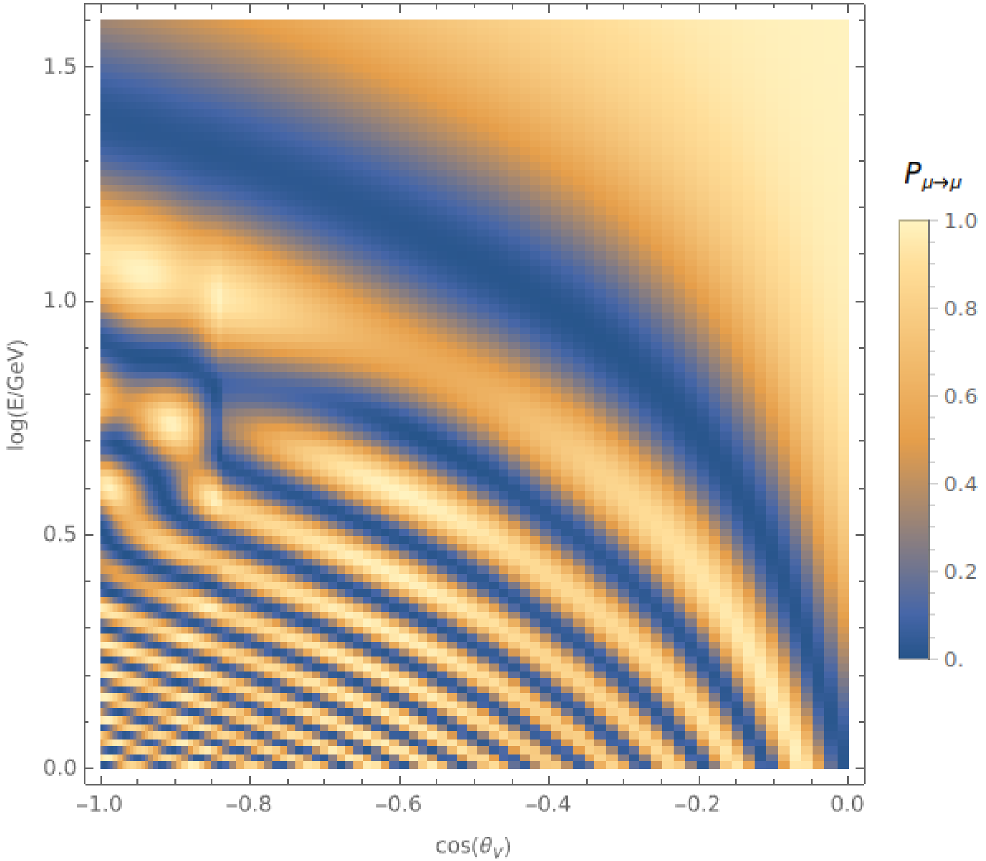
Sample Model

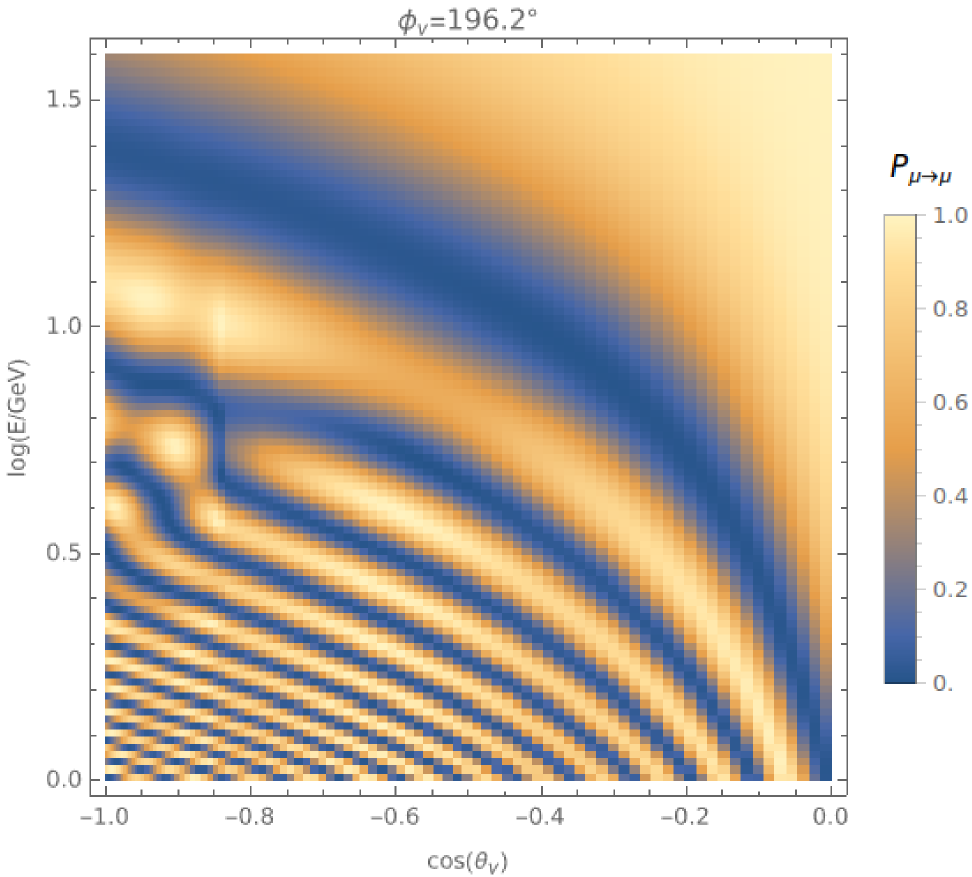


- PREM with modifiable region
  - Size and location of African LLSVP
- Detector at ORCA location

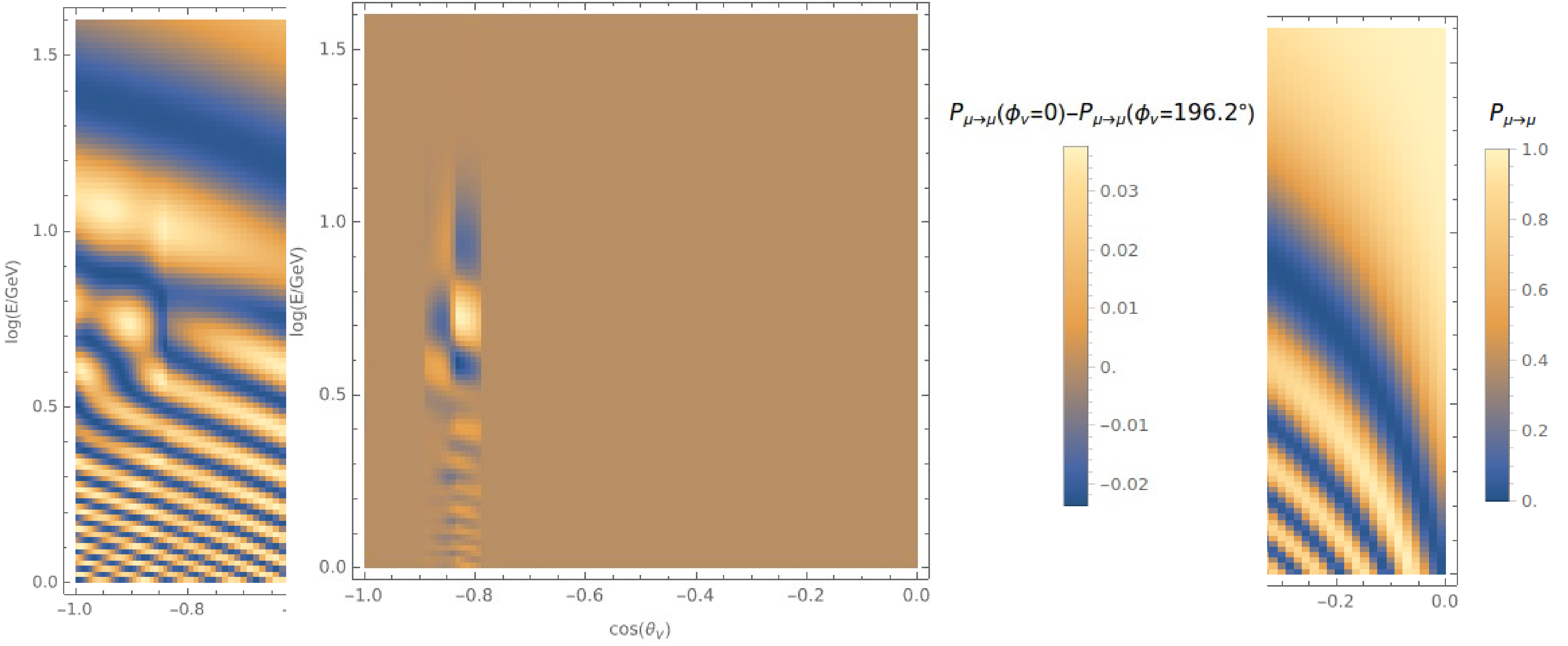


Sample Oscillograms

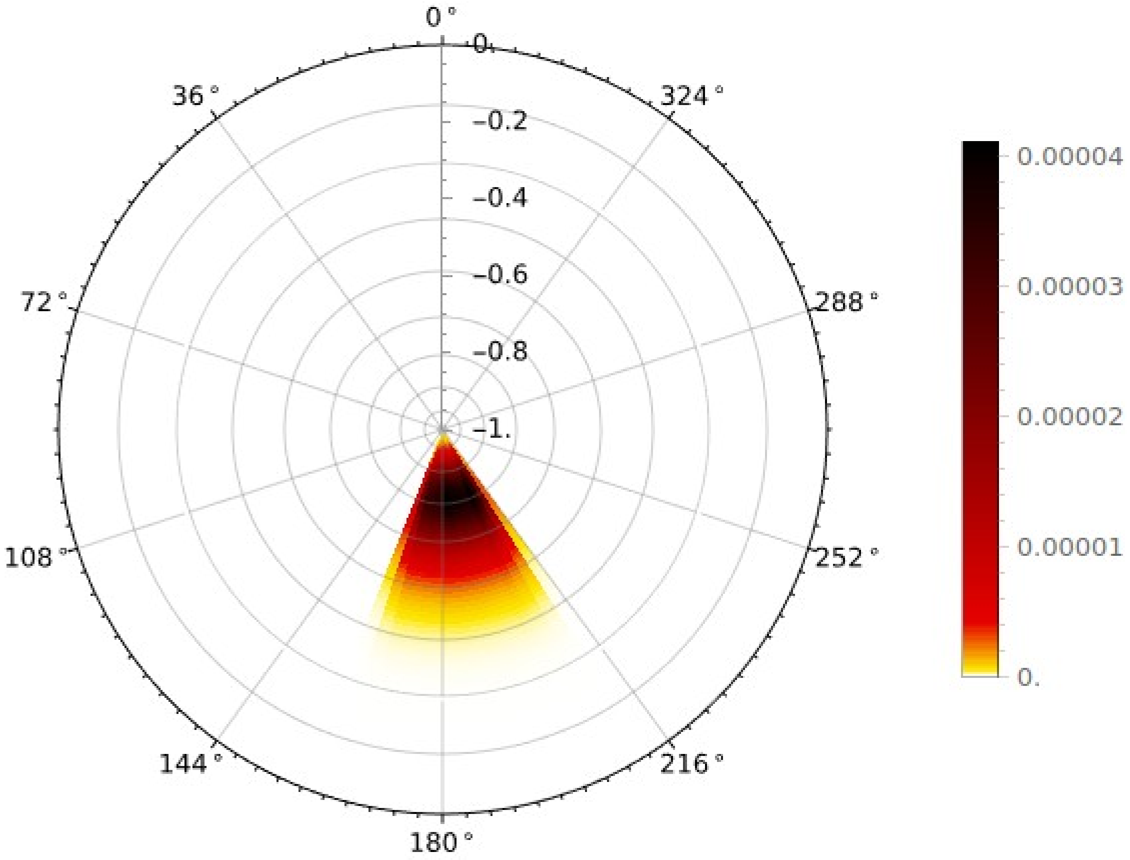


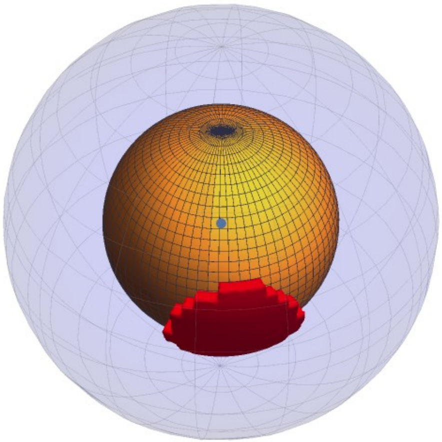


Sample Oscillograms



Sample Chi^2 Plot





- Chi^2 for ORCA
  - On cos(Zenith) vs Azimuthal plane
  - Without Azimuthal smearing

Outlook

- Framework ready for calculating oscillation probabilities
- Work needs to be done on simulating realistic detector effects with measuring the azimuthal angle of the neutrino’s trajectory

Thank you!

Backup Slides

Equations: Coordinates

- Distance from Earth's center:

$$r = \sqrt{r_d^2 + x^2 + 2 r_d x \cos(\theta_\nu)}$$

- Latitude:

$$\sin(\theta) = \frac{(r_d + x\cos(\theta_\nu))\sin(\theta_d) + x\sin(\theta_\nu)\cos(\phi_\nu)\cos(\theta_d)}{\sqrt{r_d^2 + x^2 + 2 r_d x \cos(\theta_\nu)}}$$

- Longitude:

$$\tan(\phi) = \frac{(r_d + x\cos(\theta_\nu))\cos(\theta_d)\sin(\phi_d) - x\sin(\theta_\nu)(\cos(\phi_\nu)\sin(\theta_d)\sin(\phi_d) + \sin(\phi_\nu)\cos(\phi_d))}{(r_d + x\cos(\theta_\nu))\cos(\theta_d)\cos(\phi_d) - x\sin(\theta_\nu)(\cos(\phi_\nu)\sin(\theta_d)\cos(\phi_d) - \sin(\phi_\nu)\sin(\phi_d))}$$

Equations: Distance from Detector in term of...

- Distance from Earth's center: $$x = -r_d \cos(\theta_\nu) \pm \sqrt{r^2 - r_d^2 \sin^2(\theta_\nu)}$$

$$(+ \text{ for } x > -r_d \cos(\theta_\nu))$$

- Latitude:

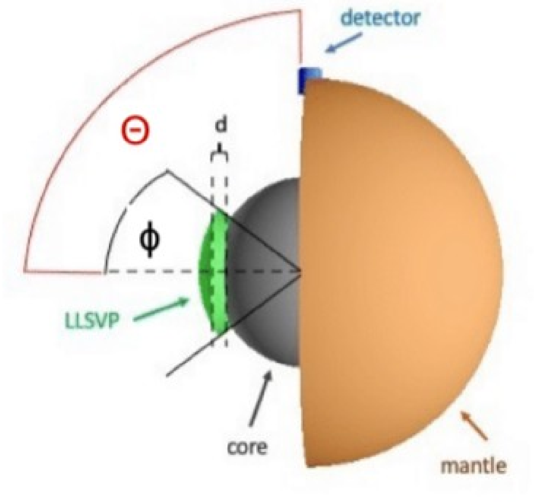
$$\frac{x}{r_d} = \frac{\cos(\theta_\nu)\left(\sin^2(\theta) - \sin^2(\theta_d)\right) - \sin(\theta_\nu)\left[\cos(\phi_\nu)\sin(\theta_d)\cos(\theta_d) \mp \sin(\theta)\sqrt{\cos^2(\theta) - \sin^2(\phi_\nu)\cos^2(\theta_d)}\right]}{\left(\sin(\theta_\nu)\cos(\phi_\nu)\cos(\theta_d) + \cos(\theta_\nu)\sin(\theta_d)\right)^2 - \sin^2(\theta)}$$

$$\left(- \text{ for } \begin{array}{l} x < \dfrac{\cos(\phi_\nu)\cos(\theta_d) r_d}{\sin(\theta_\nu)\sin(\theta_d) - \cos(\theta_\nu)\cos(\phi_\nu)\cos(\theta_d)} \text{ and } \tan(\theta_d) < \cot(\theta_\nu)\cos(\phi_\nu) \\ \text{or} \\ x \geq \dfrac{\cos(\phi_\nu)\cos(\theta_d) r_d}{\sin(\theta_\nu)\sin(\theta_d) - \cos(\theta_\nu)\cos(\phi_\nu)\cos(\theta_d)} \text{ and } \tan(\theta_d) \geq \cot(\theta_\nu)\cos(\phi_\nu) \end{array} \right)$$

- Longitude:

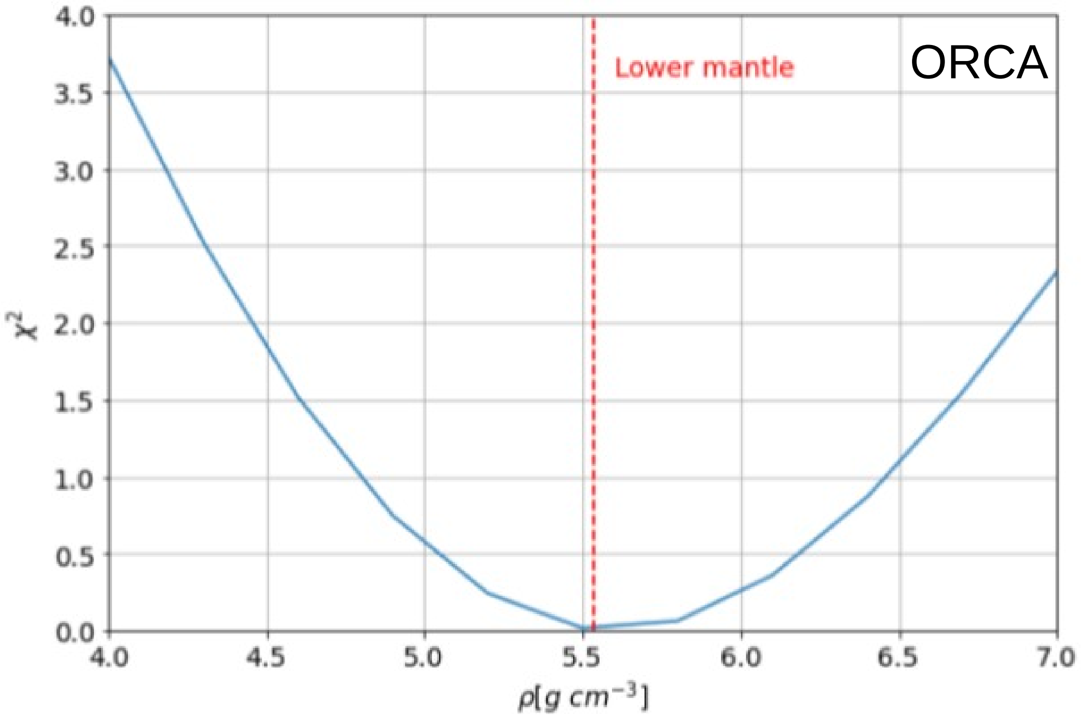
$$x = \frac{\cos(\theta_d)(\tan(\phi_d) - \tan(\phi)) r_d}{[\sin(\theta_\nu)\cos(\phi_\nu)\sin(\theta_d) - \cos(\theta_\nu)\cos(\theta_d)](\tan(\phi_d) - \tan(\phi)) + \sin(\theta_\nu)\sin(\phi_\nu)(1 - \tan(\phi_d)\tan(\phi))}$$

Neutrino Oscillation Tomography with LLSVPs



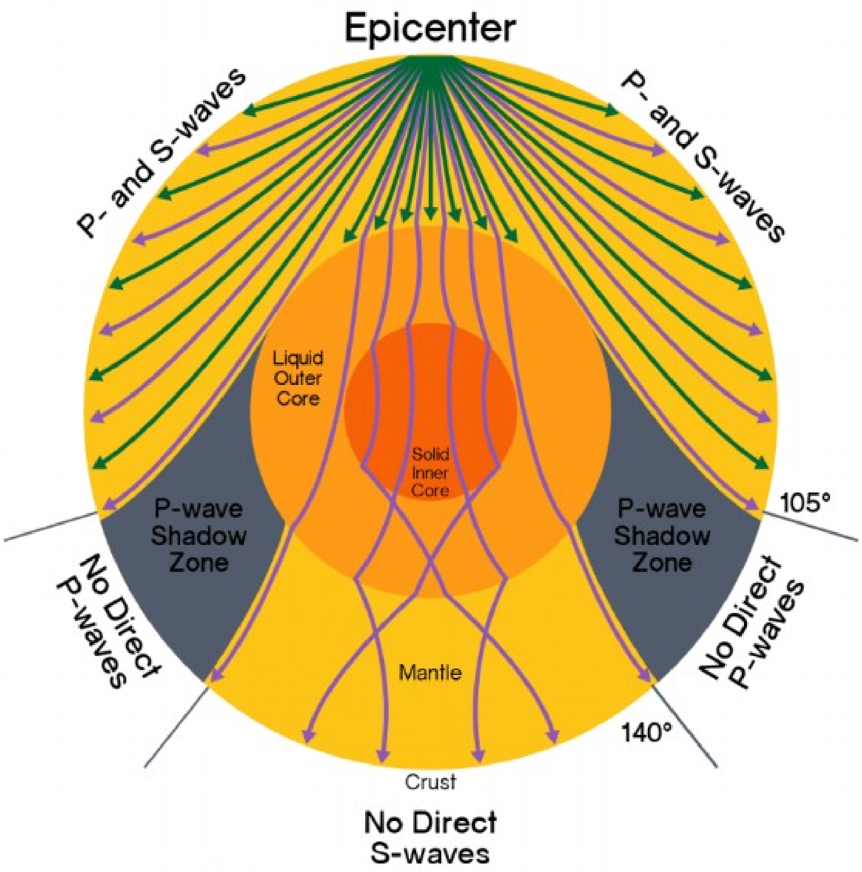
LLSVP model with *ROOT:TGeoManager*

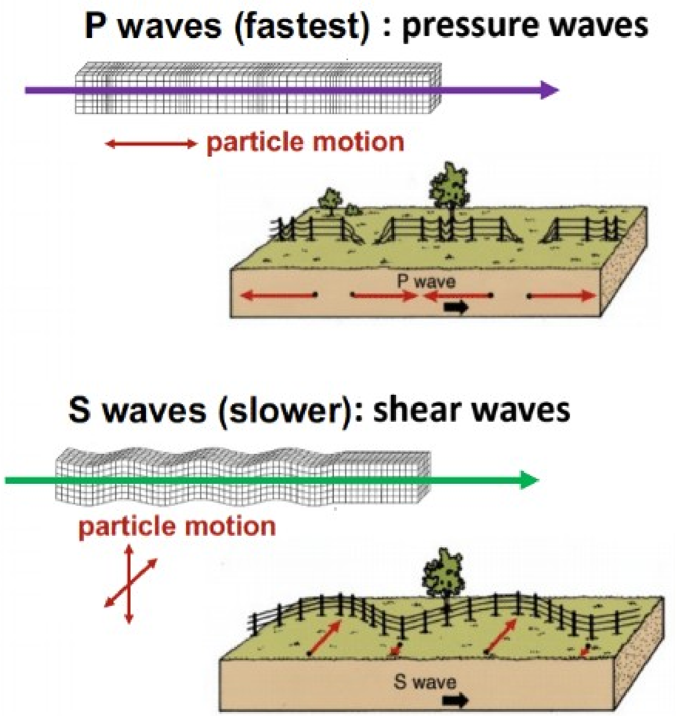
- PREM -> Model with LLSVP
- Constrain density of LLSVP



L. Maderer, et al.

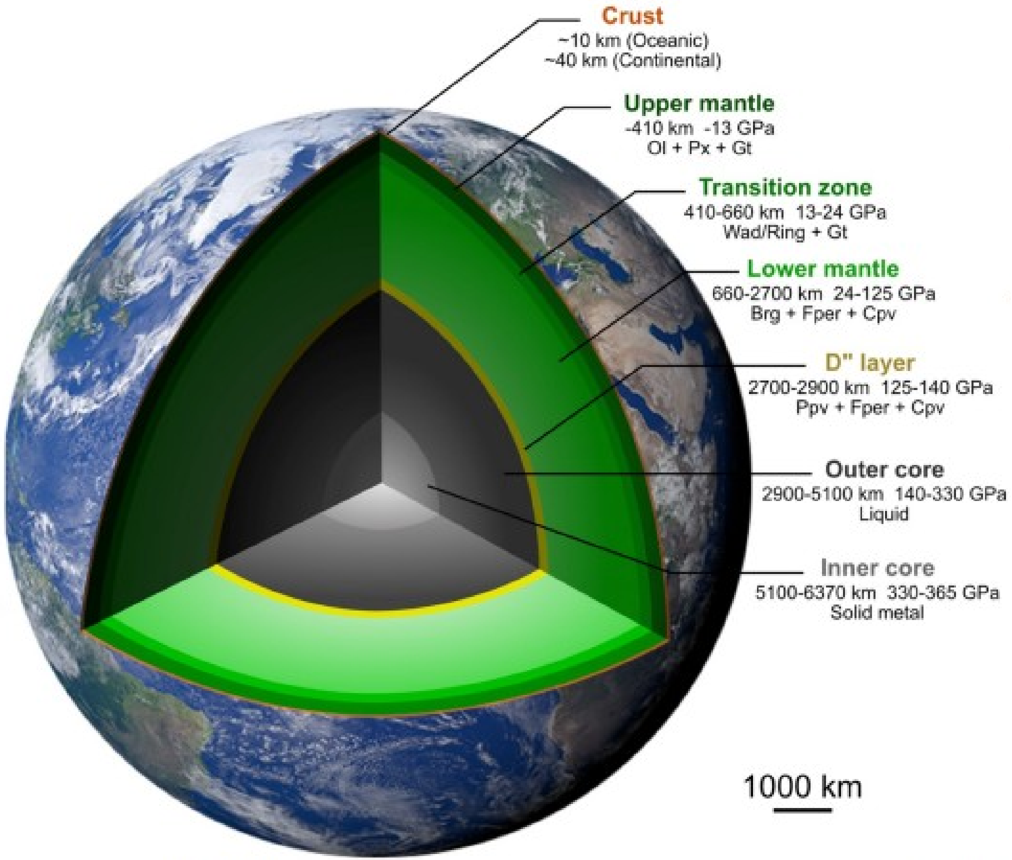
Inside the Earth: Seismology



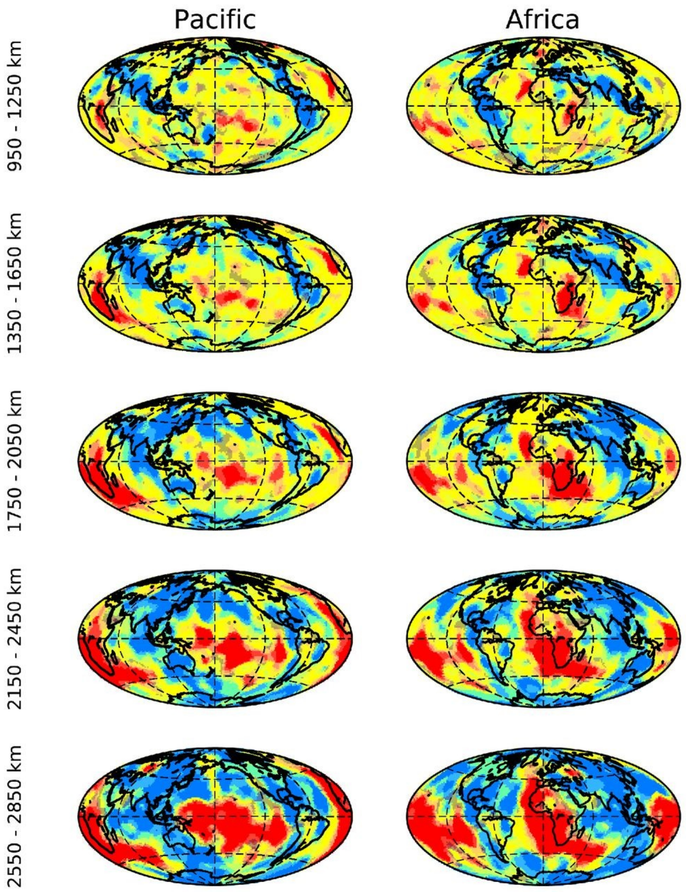


Inside the Earth: Some Questions

- Inner/Outer Core Boundary
  - Where is it?
  - How big is the density change?
- Core Composition
  - Light element percentage?
  - Is there any hydrogen?
- Asymmetries
  - What are LLSVPs?



Large Low-Shear-Velocity Provinces (LLSVPs)

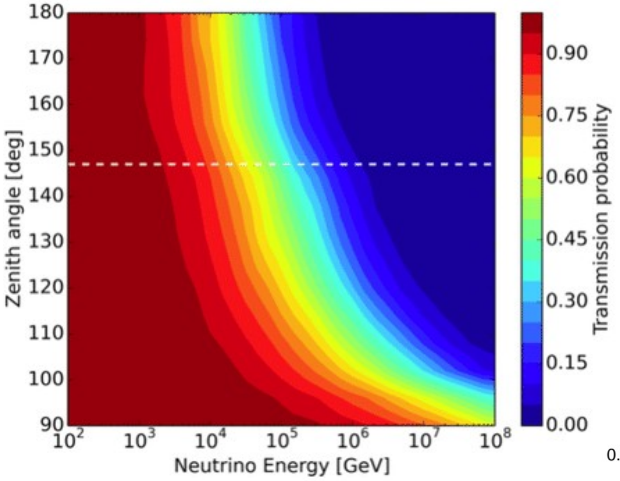


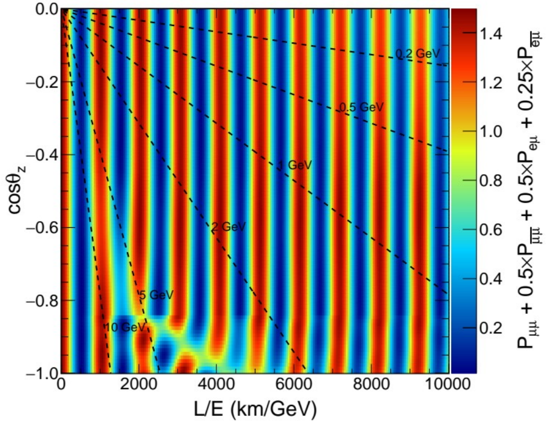
- Large regions in the lower mantle where seismic waves have a lower shear velocity
  - Sharp boundaries
  - Stable
- Makeup
  - Chemical?
  - Thermal?

https://doi.org/10.1093/gji/ggw324 (S. Cottaar, V. Lekic)

Neutrino Tomography

- Neutrinos interact with matter
  - High energy neutrinos have higher cross sections
    - Absorption profiles tell us about density
  - Low energy neutrinos' oscillation patterns are affected by matter
    - Oscillation profiles tell us about density





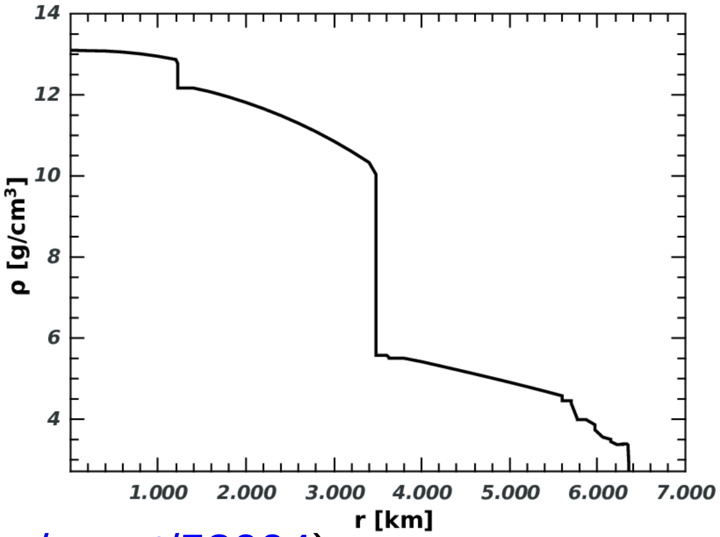
Neutrino Oscillation Tomography

$$H = \frac{1}{2E}\left(U^\dagger \begin{bmatrix} 0 & 0 & 0 \\ 0 & \Delta m_{21}^2 & 0 \\ 0 & 0 & \Delta m_{31}^2 \end{bmatrix} U + 2\sqrt{2} G_F N_e E \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right)$$

- With matter effect, neutrino oscillation is sensitive to electron number density

$$N_e = \frac{N_A}{m_n}\left(\frac{Z}{A}\right)\rho$$

  - Depends on density and composition
- Use atmospheric neutrinos to study Earth



(See slides from MMTE Workshop 2022: https://indico.fnal.gov/event/53004)

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