

# Aspects of Neutrino Oscillation Tomography of the Earth

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**Neutrino Tomography of the Earth is one of the most interesting, important and not yet fully explored field of reserach in today's Neutrino Physics.**

**Two very different methods: neutrino absorption tomography and neutrino oscillation tomography.**

**This talk: comments on Methods, Problems and Goals of Neutrino Oscillation Tomography of the Earth.**

**Can be done with atmospheric, solar and SN neutrinos, in principle.**

**At present, neutrino oscillation tomography of the Earth with atmospheric neutrino seems to be the most feasible (in view of the existing and upcoming adequate experiments).**

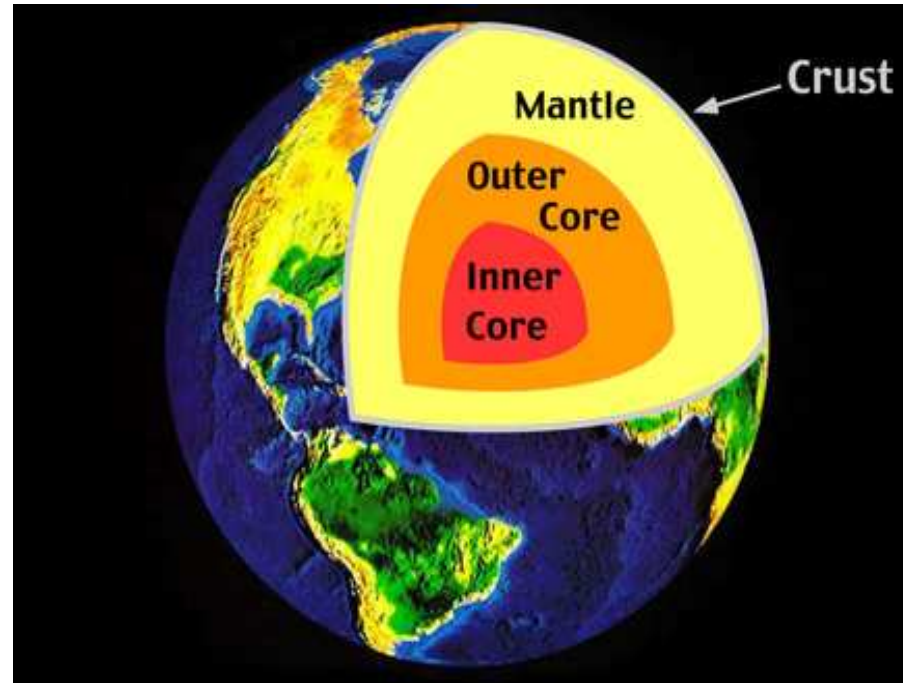
# Articles on using atmospheric neutrinos for neutrino oscillation tomography of the Earth.

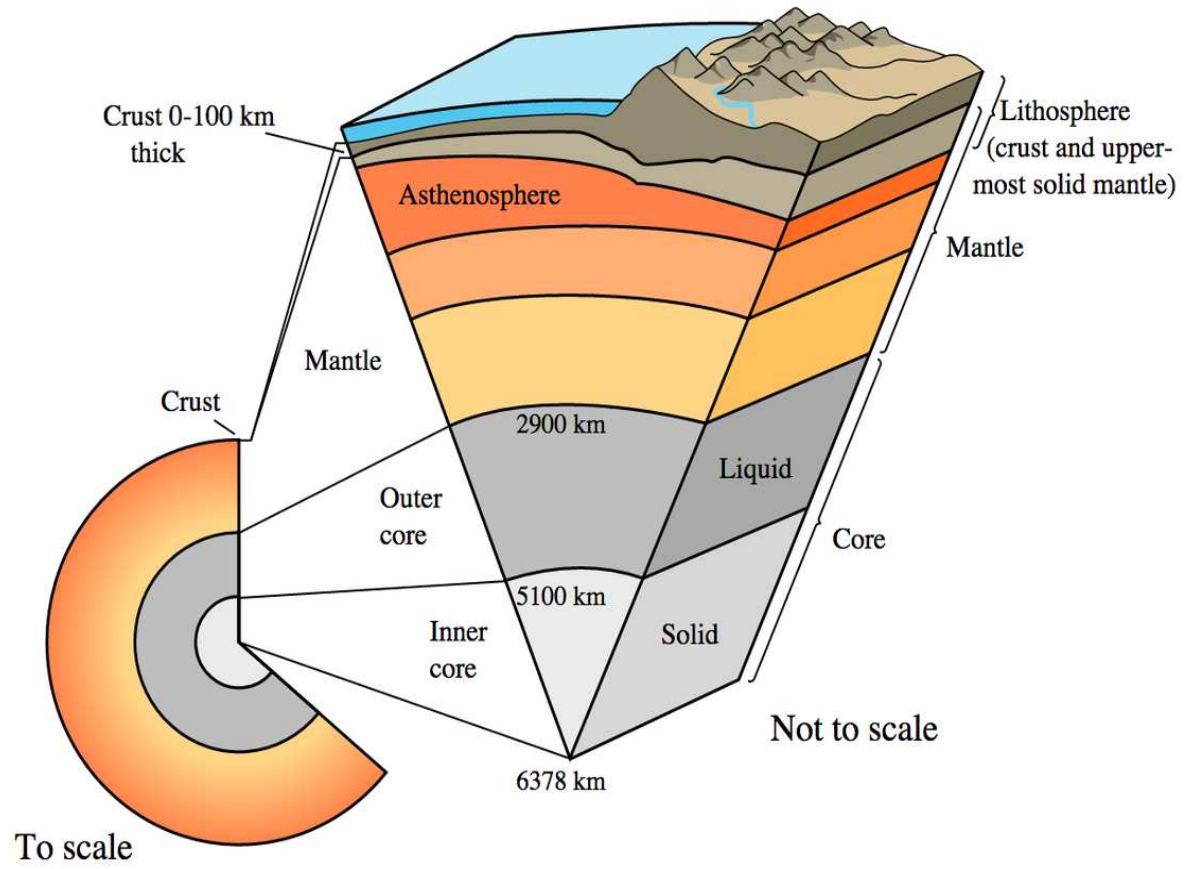
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(The list probably is not complete.)

# The Earth





The method is based on the fact that given the measured  $3 - \nu$  oscillation parameters  $\Delta m_{21}^2$ ,  $|\Delta m_{31}^2|$ ,  $\sin^2 \theta_{12}$ ,  $\sin^2 \theta_{13}$ ,  $\sin^2 \theta_{23}$ ,  
 for  $E_\nu \sim (2 - 10)$  GeV ( $E_\nu \sim 0.2$  GeV, DUNE),  
 $P_{3\nu}(\nu_{\mu(e)} \rightarrow \nu_{e(\mu)})$  or  $P_{3\nu}(\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)})$   
 exhibits a strong dependence on  $\rho_E(r)$  along the  $\nu$  path in the Earth via

$$V_{eff} = \pm \sqrt{2} G_F N_e(r) = \pm \sqrt{2} G_F Y_e \frac{\rho_E(r)}{m_N},$$

$$Y_e = \frac{N_e(r)}{N_p(r) + N_n(r)}$$

**Isotopically symmetric medium:**  $N_e(r) = N_p(r) = N_n(r)$ :  $Y_e = 0.5$ .

$\text{sgn}(\Delta m_{31}^2)$  – not yet determined from data.

$\Delta m_{31}^2 > 0$  (**NO**):  $P_{3\nu}(\nu_{\mu(e)} \rightarrow \nu_{e(\mu)})$  – resonantly enhanced,  $P_{3\nu}(\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)})$  – suppressed.

$\Delta m_{31}^2 < 0$  (**IO**):  $P_{3\nu}(\nu_{\mu(e)} \rightarrow \nu_{e(\mu)})$  – suppressed,  $P_{3\nu}(\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)})$  – resonantly enhanced.

For  $E_\nu \sim (2 - 10)$  **GeV**, effects of  $\Delta m_{21}^2$  and  $\delta_{CP}$  – subleading.  
 ( $E_\nu \sim 0.2$  GeV (DUNE),  $\Delta m_{31}^2$  “averaged out” .)

There is a strong dependence of the results on  $\sin^2 \theta_{23}$  ( $\cos^2 \theta_{23}$ ) (not precisely measured yet,  $\sin^2 \theta_{23} = (0.42 - 0.58)$  allowed at  $3\sigma$ )  
 and on the currently unknown **neutrino mass ordering (NO or IO)**.

## Atmospheric $\nu$ experiments (ORCA, IceCube, HK, INO, DUNE, $E_\nu > 1$ GeV)

Subdominant  $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$  and  $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$  oscillations in the Earth.

$$P_{3\nu}(\nu_e \rightarrow \nu_\mu) \cong P_{3\nu}(\nu_\mu \rightarrow \nu_e) \cong s_{23}^2 P_{2\nu}, P_{3\nu}(\nu_e \rightarrow \nu_\tau) \cong c_{23}^2 P_{2\nu},$$
$$P_{3\nu}(\nu_\mu \rightarrow \nu_\mu) \cong 1 - s_{23}^4 P_{2\nu} - 2c_{23}^2 s_{23}^2 \left[ 1 - \text{Re} (e^{-i\kappa} A_{2\nu}(\nu_\tau \rightarrow \nu_\tau)) \right],$$

$P_{2\nu} \equiv P_{2\nu}(\Delta m_{31}^2, \theta_{13}; E, \theta_n; N_e)$ : 2- $\nu$   $\nu_e \rightarrow \nu'_\tau$  oscillations in the Earth,  
 $\nu'_\tau = s_{23} \nu_\mu + c_{23} \nu_\tau$ ;  $\Delta m_{21}^2 \ll |\Delta m_{31(32)}^2|$ ,  $E_\nu \gtrsim 2$  GeV;

$\kappa$  and  $A_{2\nu}(\nu_\tau \rightarrow \nu_\tau) \equiv A_{2\nu}$  are known phase and 2- $\nu$  amplitude.

**NO**:  $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$  **matter enhanced**,  $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$  - **suppressed**

**IO**:  $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$  **matter enhanced**,  $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$  - **suppressed**

**No charge identification (SK, HK, IceCube-PINGU, ANTARES-ORCA); event rate (DIS regime):**  $[2\sigma(\nu_l + N \rightarrow l^- + X) + \sigma(\bar{\nu}_l + N \rightarrow l^+ + X)]/3$

**Charge identification: INO, DUNE; event rate (DIS regime):**

$\sigma(\nu_l + N \rightarrow l^- + X)$ ,  $\sigma(\bar{\nu}_l + N \rightarrow l^+ + X)$

$E_\nu \sim 200$  MeV (DUNE),

$$P_{3\nu}(\nu_e \rightarrow \nu_\mu) \cong P_{3\nu}(\nu_\mu \rightarrow \nu_e) \cong \cos^2 \theta_{23} \tilde{P}_{2\nu},$$

$\tilde{P}_{2\nu} \equiv P_{2\nu}(\Delta m_{21}^2, \theta_{12}; E, \theta_n; N_e)$ : 2- $\nu$   $\nu_e \rightarrow \tilde{\nu}'_\tau$  oscillations in the Earth,  
 $\tilde{\nu}'_\tau = c_{23} \nu_\mu - s_{23} \nu_\tau$ .

**All detectors: strong dependence on  $\sin^2 \theta_{23}$  and on neutrino MO.**



$\sin^2 \theta_{23}$ : T2K, NO $\nu$ A, T2HK, DUNE

Neutrino mass ordering (NO or IO): JUNO, ORCA, PINGU, DUNE, T2HKK.

JUNO will begin physics runs in the first half of 2024.

HK is under construction and is scheduled to start data-taking in 2028.

DUNE is also under construction.

The method is sensitive to changes of  $\rho_E(r)$  over scales  $L \gtrsim L_{\text{osc}}/(2\pi)$ . In the cases of interest  $L_{\text{osc}}/(2\pi) \sim (0.5 - 1.0) \times 10^3$  km.

Thus, very “fine” structures in  $\rho_E(r)$  are difficult (if not impossible) to “resolve.”

At present our knowledge about the interior composition of the Earth and its density structure is based primarily on seismological and geophysical data.

See, e.g., W.F. McDonough, “Treatise on Geochemistry: The Mantle and Core”, vol. 2 (ed. R. W. Carlson, Elsevier-Pergamon, Oxford, 2003), p. 547.  
B.L.N. Kennett, Geophys. J. Int. **132**, 374 (1998);  
G. Masters and D. Gubbins, Phys. Earth Planet. Inter. **140**, 159 (2003).

These data were used to construct the Preliminary Reference Earth Model (PREM) of the density distribution of the Earth: A. M. Dziewonski and D. L. Anderson, “Preliminary reference earth model,” Phys. Earth Planet. Interiors **25** (1981) 297.

In the PREM model,  $\rho_E$  is assumed to be spherically symmetric,  $\rho_E = \rho_E(r)$ ,  $r$  being the distance from the Earth center, and there are two major density structures - the core and the mantle, and a certain number of substructures (shells or layers). The mantle has five shells in the model, while the core is divided into an Inner Core (IC) and Outer Core (OC). There are also the ocean and two crust “thin” layers.

The change of  $\rho_E$  ( $N_e$ ) from the mantle to the core, according to PREM, can well be approximated by a step function, the transition zone being relatively narrow (width  $\sim 100$  km).

# The Earth (PREM)

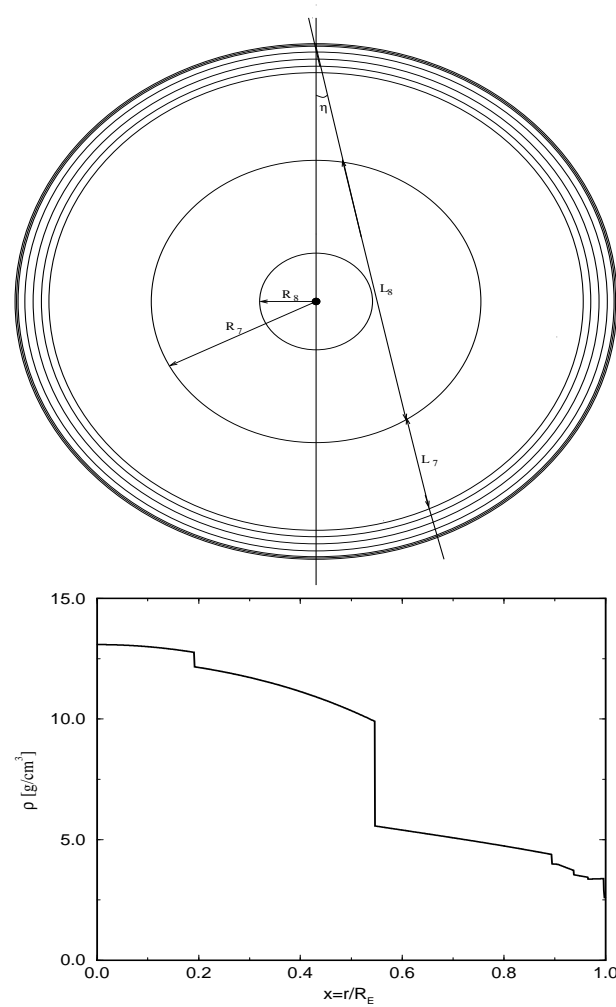


FIG. 1. Density profile of the Earth.

$R_{\oplus} = 6371$  km;  $R_{IC} = 1221.5$  km;  $R_C = 3480$  km;  $D_{\text{man}} = 2891$  km;  
 $\bar{\rho}_{\text{man}} = 4.45$  g/cm<sup>3</sup>;  $\bar{\rho}_C = 10.99$  g/cm<sup>3</sup>;  $\bar{\rho}_{IC}^{16} = 12.89$  g/cm<sup>3</sup>;  $\bar{\rho}_{OC} = 10.90$  g/cm<sup>3</sup>.

**PREM: good/useful “Reference” model;**  
 $\rho_E^{\text{PREM}}(\mathbf{r}) = \rho_E^{\text{PREM}}(r)$ , in particular. Used in practically all studies of  $\nu$  tomography of the Earth.

**Future: more sophisticated  $\rho_E(\mathbf{r})$  suggested by geophysics/seismology can/should be used.**

**Spherically symmetric  $\rho_E(r)$ : the  $\nu$  trajectory through the Earth is specified by the nadir (zenith) angle  $\theta_n$  (or  $\theta_z$ ).**

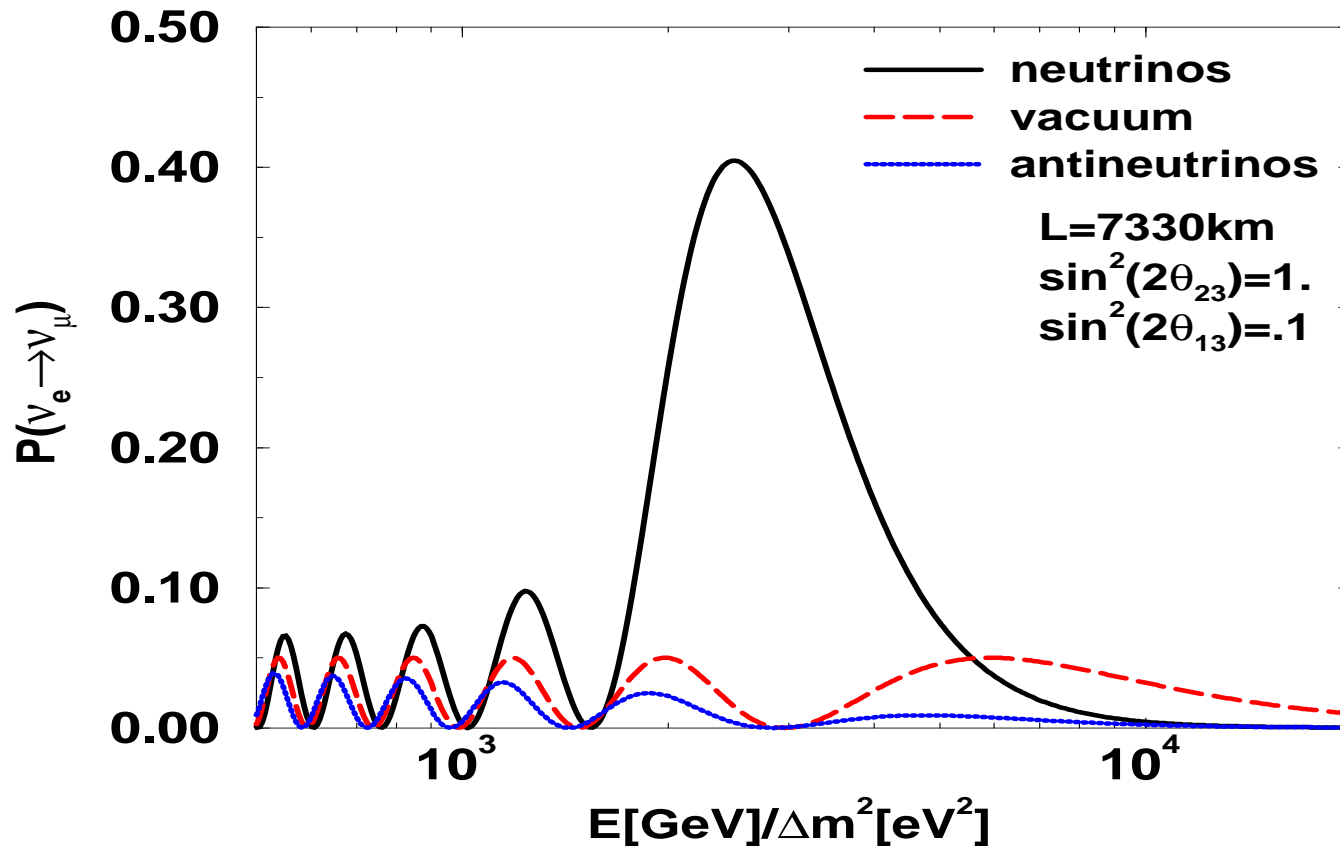
**For  $\theta_n \leq 33.17^\circ$ , or path lengths  $L \geq 10660$  km, neutrinos cross the Earth core.**

**The path length for neutrinos which cross only the Earth mantle is given by  $L = 2R_\oplus \cos \theta_n$ ,  $\theta_n$ .**

**If neutrinos cross the Earth core, the lengths of the paths in the mantle,  $2L^{\text{man}}$ , and in the core,  $L^{\text{core}}$ , are determined by:  $L^{\text{man}} = R_\oplus \cos \theta_n - (R_c^2 - R_\oplus^2 \sin^2 \theta_n)^{\frac{1}{2}}$ ,  $L^{\text{core}} = 2(R_c^2 - R_\oplus^2 \sin^2 \theta_n)^{\frac{1}{2}}$ .**

**$\rho_E(r)$  ( $N_e^E(r)$ ) changes relatively little around the quoted mean values along the trajectories of neutrinos which cross a substantial part of the Earth mantle, or the mantle, the outer core and the inner core.**

# Earth matter effect in $\nu_\mu \rightarrow \nu_e, \bar{\nu}_\mu \rightarrow \bar{\nu}_e$ in the mantle (MSW)



$$\Delta m_{31}^2 = 2.5 \times 10^{-3} \text{ eV}^2, E^{res} = 6.25 \text{ GeV}; P^{3\nu} = \sin^2 \theta_{23} P_m^{2\nu} = 0.5 P_m^{2\nu};$$

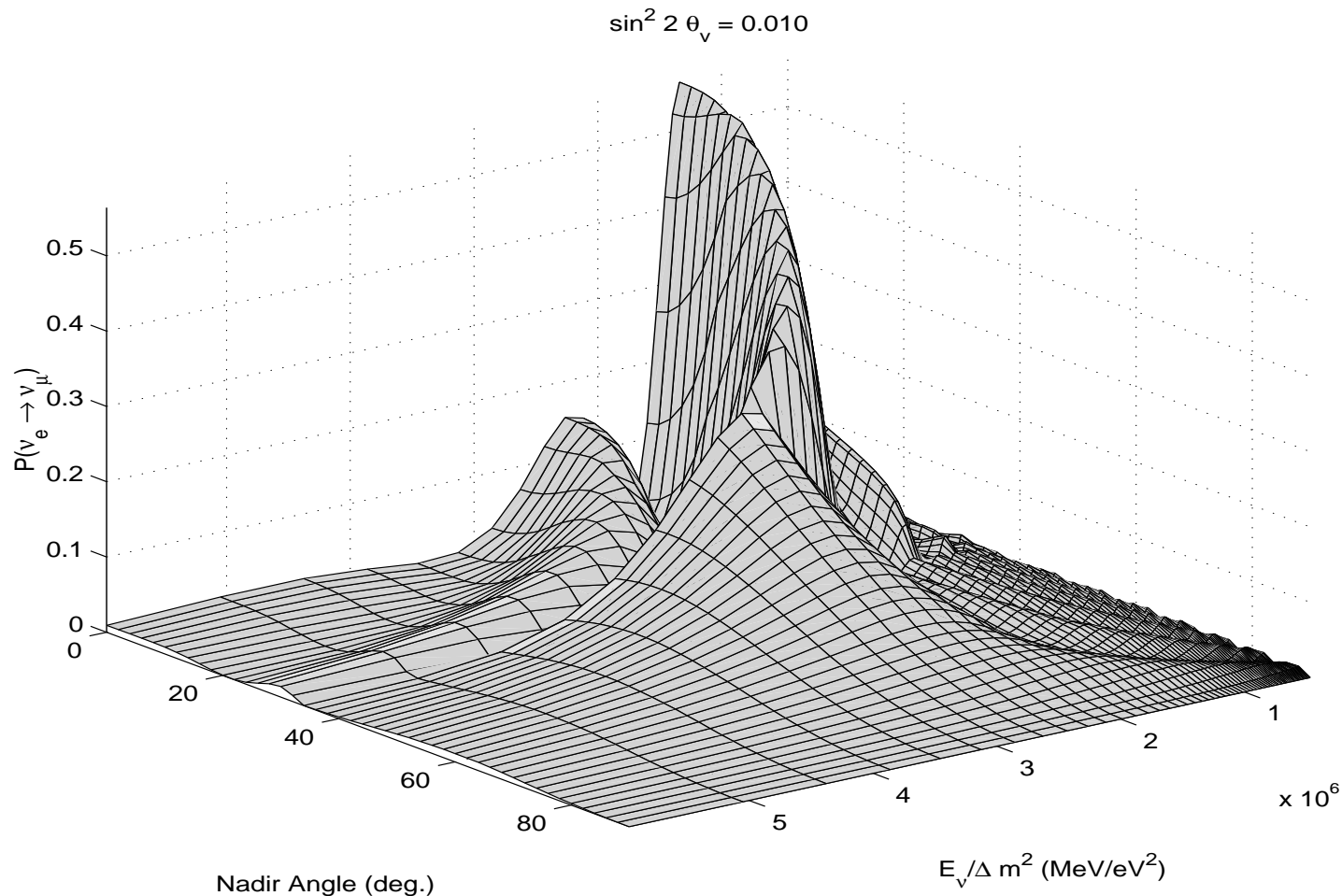
$$N_e^{res} \cong 2.3 \text{ cm}^{-3} N_A; L_m^{res} = L^v / \sin 2\theta_{13} \cong 6250 / 0.32 \text{ km}; 2\pi L / L_m \cong 0.75\pi (\neq \pi).$$

I. Mocioiu, R. Shrock, 2000

**Sensitivity to  $\bar{\rho}_{man}$  along the  $\nu$  path in the mantle.**

# Resonance-like Amplification of Oscillations of Neutrinos Crossing the Earth Core

# Earth matter effects in $\nu_\mu \rightarrow \nu_e, \bar{\nu}_\mu \rightarrow \bar{\nu}_e$ (NOLR)



S.T.P., 1998;

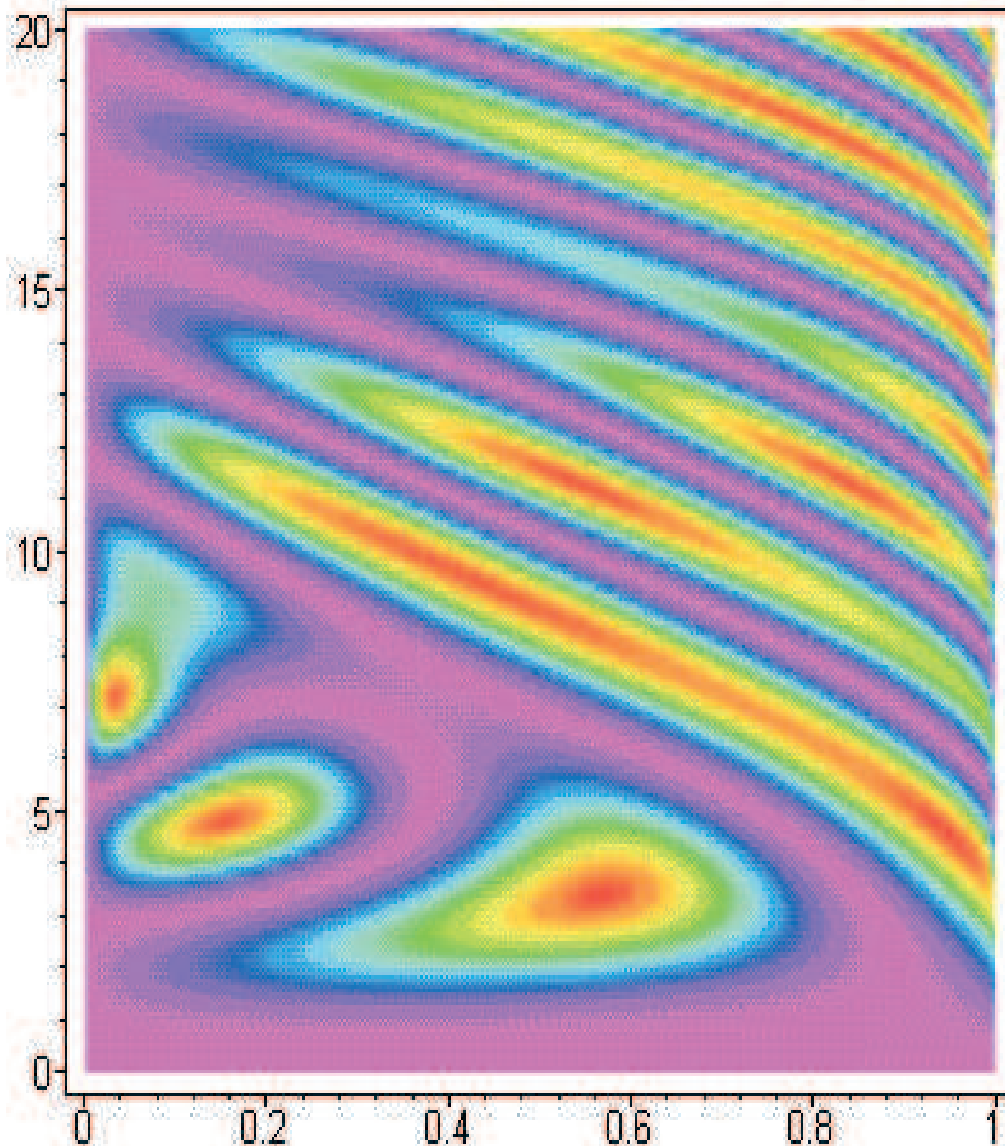
M. Chizhov, M. Maris, S.T.P., 1998; M. Chizhov, S.T.P., 1999

$P(\nu_e \rightarrow \nu_\mu) \equiv P_{2\nu} \equiv (s_{23})^{-2} P_{3\nu}(\nu_{e(\mu)} \rightarrow \nu_{\mu(e)})$ ,  $\theta_\nu \equiv \theta_{13}$ ,  $\Delta m^2 \equiv \Delta m_{\text{atm}}^2$ ;

**Absolute maximum: Neutrino Oscillation Length Resonance (NOLR);**

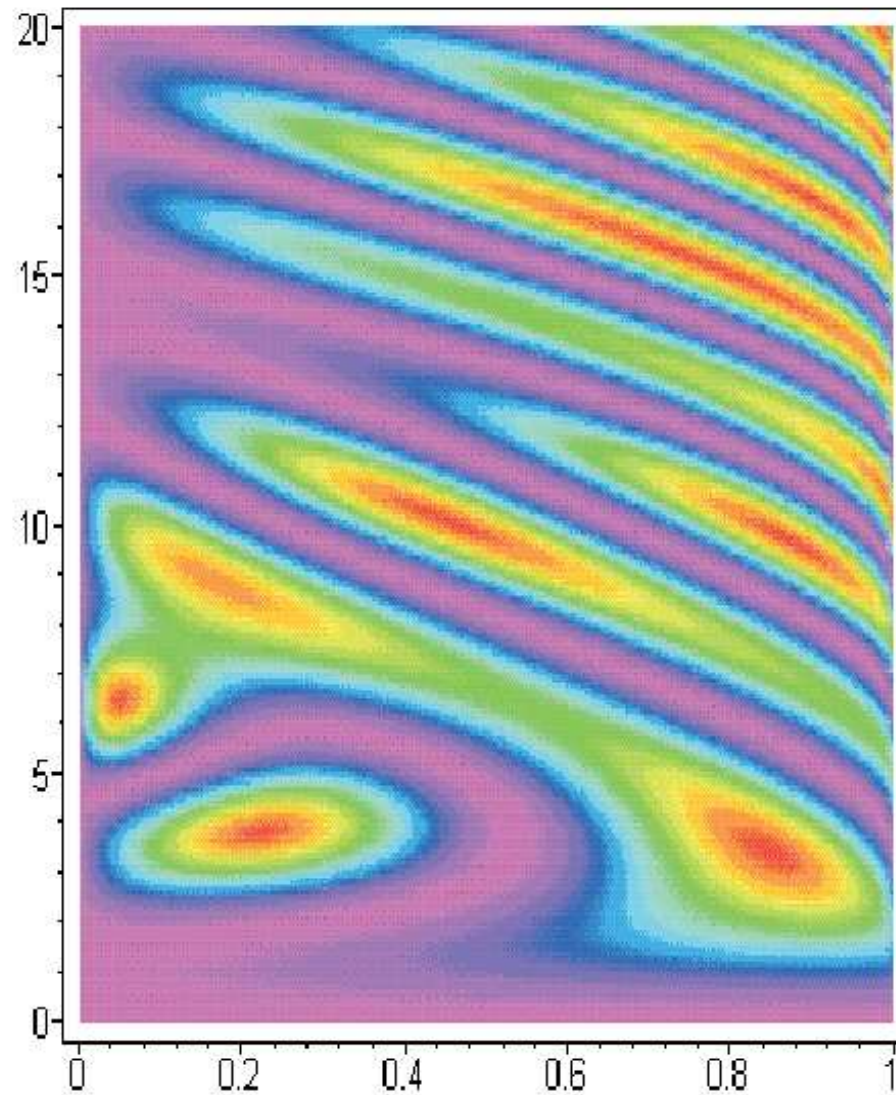
**Local maxima: MSW effect in the Earth mantle or core.**





$(s_{23})^{-2} P_{3\nu}(\nu_{e(\mu)} \rightarrow \nu_{\mu(e)}) \equiv P_{2\nu}$ ; **NOLR: “Dark Red Spots”,  $P_{2\nu} = 1$ ;**  
**Vertical axis:  $\Delta m^2/E$  [ $10^{-7} eV^2/MeV$ ]; horizontal axis:  $\sin^2 2\theta_{13}$ ;  $\theta_n = 0$**   
 M. Chizhov, S.T.P., 1999 (hep-ph/9903399,9903424)

**Sensitivity to  $\bar{\rho}_{\text{man}}$ ,  $\bar{\rho}_{\text{Core}}$ , and to  $(\bar{\rho}_{\text{Core}}/\bar{\rho}_{\text{man}})_{mcb}$  at mantle-core boundary.**



The same for  $\theta_n = 23^\circ$ .

**Vertical axis:**  $\Delta m^2/E$  [ $10^{-7} eV^2/MeV$ ]; **horizontal axis:**  $\sin^2 2\theta_{13}$ .

M. Chizhov, S.T.P., 1999 (hep-ph/9903399,9903424)

- For Earth center crossing  $\nu$ 's ( $\theta_n = 0$ ) and, e.g.  $\sin^2 2\theta_{13} = 0.01$ , **NOLR** occurs at  $E \cong 4$  **GeV** ( $\Delta m^2(atm) = 2.5 \times 10^{-3} \text{ eV}^2$ ).

S.T.P., hep-ph/9805262

- For the Earth core crossing  $\nu$ 's:  $P_{2\nu} = 1$  **due to NOLR** when

$$\tan \Phi^{\text{man}}/2 \equiv \tan \phi' = \pm \sqrt{\frac{-\cos 2\theta''_m}{\cos(2\theta''_m - 4\theta'_m)}},$$

$$\tan \Phi^{\text{core}}/2 \equiv \tan \phi'' = \pm \frac{\cos 2\theta'_m}{\sqrt{-\cos(2\theta''_m) \cos(2\theta''_m - 4\theta'_m)}}$$

$\Phi^{\text{man}}$  ( $\Phi^{\text{core}}$ ) - phase accumulated in the Earth mantle (core),  
 $\theta'_m$  ( $\theta''_m$ ) - the mixing angle in the Earth mantle (core).

$P_{2\nu} = 1$  **due to NOLR** for  $\theta_n = 0$  (Earth center crossing  $\nu$ 's) at, e.g.  $\sin^2 2\theta_{13} = 0.034; 0.154$ ,  $E \cong 3.5; 5.2$  **GeV** ( $\Delta m^2_{31} = 2.5 \times 10^{-3} \text{ eV}^2$ ).

**At the same time for  $E = 3.5$  GeV (5.2 GeV), the probability  $P_{2\nu} \gtrsim 0.5$  for the values of  $\sin^2 2\theta_{13}$  from the interval  $0.02 \lesssim \sin^2 2\theta_{13} \lesssim 0.10$  ( $0.04 \lesssim \sin^2 2\theta_{13} \lesssim 0.26$ ).**

M. Chizhov, S.T.P., Phys. Rev. Lett. 83 (1999) 1096 (hep-ph/9903399); Phys. Rev. Lett. 85 (2000) 3979 (hep-ph/0504247); Phys. Rev. D63 (2001) 073003 (hep-ph/9903424).

The Earth matter effects in  $\nu_{e(\mu)} \rightarrow \nu_{\mu(e)}$ ,  $\bar{\nu}_{e(\mu)} \rightarrow \bar{\nu}_{\mu(e)}$ ,  $\nu_e \rightarrow \nu_\tau$  and  $\bar{\nu}_e \rightarrow \bar{\nu}_\tau$  oscillations are significant for  $E_\nu \sim (2 - 10)$  **GeV**.

**MSW in the Mantle:**  $E_\nu \sim (6 - 7)$  **GeV** ( $|\Delta m_{31}^2| \equiv \Delta m^2(atm) = 2.5 \times 10^{-3}$  eV<sup>2</sup>).

The mantle-core (NOLR) enhancement of  $P_m^{2\nu}$  (or  $\bar{P}_m^{2\nu}$ ):

$P_{2\nu} = 1$  **due to NOLR** for  $\theta_n = 0$  (Earth center crossing  $\nu$ 's) at,  
e.g.  $\sin^2 2\theta_{13} = 0.034; 0.154$ ,  $E \cong 3.5; 5.2$  **GeV** ( $|\Delta m_{31}^2| = 2.5 \times 10^{-3}$  eV<sup>2</sup>).

For  $E = 3.5$  **GeV**, the probability  $P_{2\nu} \gtrsim 0.5$  for the values of  $\sin^2 2\theta_{13}$  from the interval  $0.02 \lesssim \sin^2 2\theta_{13} \lesssim 0.10$ .

The current b.f.v. of  $\sin^2 2\theta_{13} = 0.084$ .

The effects of Earth matter on the oscillations of atmospheric (and accelerator) neutrinos have not been observed so far; **can be used for performing tomography of the Earth with neutrinos having  $E_\nu \sim (2 - 10)$  GeV.**

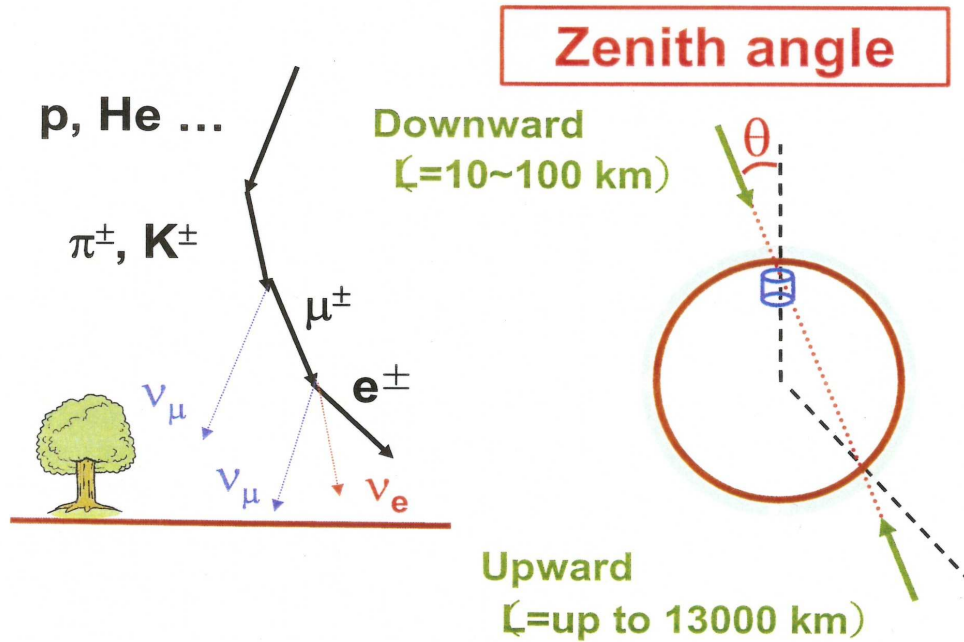
**Atmospheric  $\nu$ s are a perfect tool for performing Earth tomography:**

- i) consist of significant fluxes of  $\nu_\mu$ ,  $\nu_e$ ,  $\bar{\nu}_\mu$  and  $\bar{\nu}_e$ , produced in the interactions of cosmic rays with the Earth atmosphere,
- ii) have a wide range of energies spanning the interval from a few MeV to multi-GeV to multi-TeV,
- iii) being produced isotropically in the upper part of the Earth atmosphere at a height of  $\sim 15$  km, they travel distances from  $\sim 15$  km to 12742 km before reaching detectors located on the Earth surface, crossing the Earth along all possible directions and thus “scanning” the Earth interior.

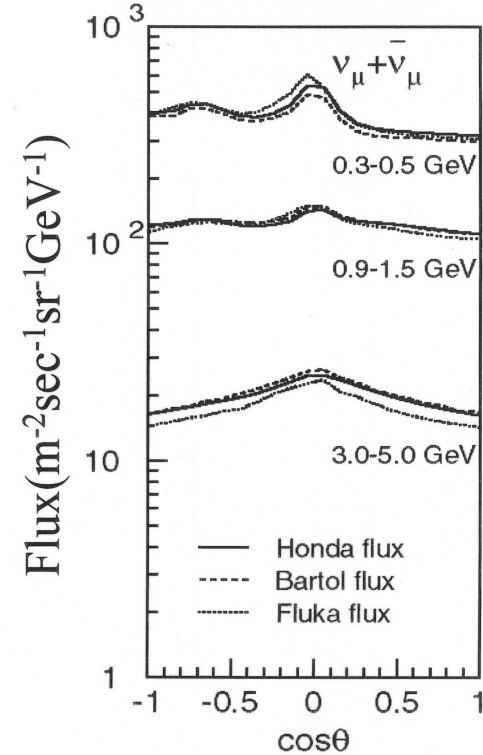
See, e.g., T.K. Gaisser and M. Honda, *Ann. Rev. Nucl. Part. Sci.* **52** (2002) 153  
[arXiv:hep-ph/0203272]

The interaction rates that allow to get information about the Earth density distribution can be obtained in the **currently taking data IceCube experiment and in the future experiments PINGU, ORCA (within KM3Net project), Hyper Kamiokande and DUNE, which are under construction.**

# Atmospheric neutrinos



Zenith angle dist. of Atmospheric  $\nu$  flux



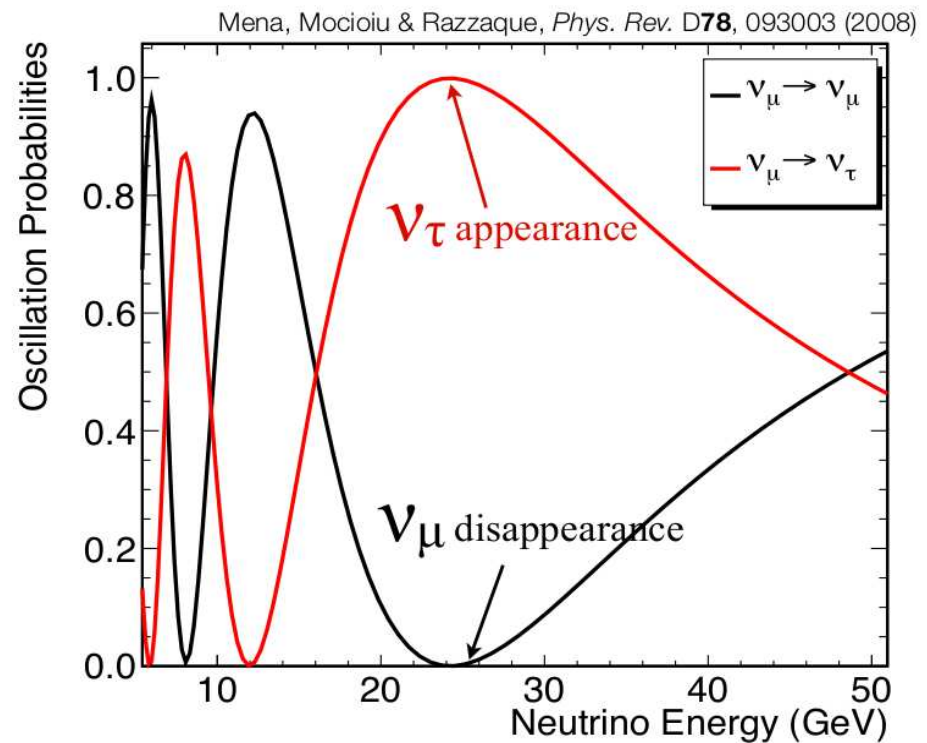
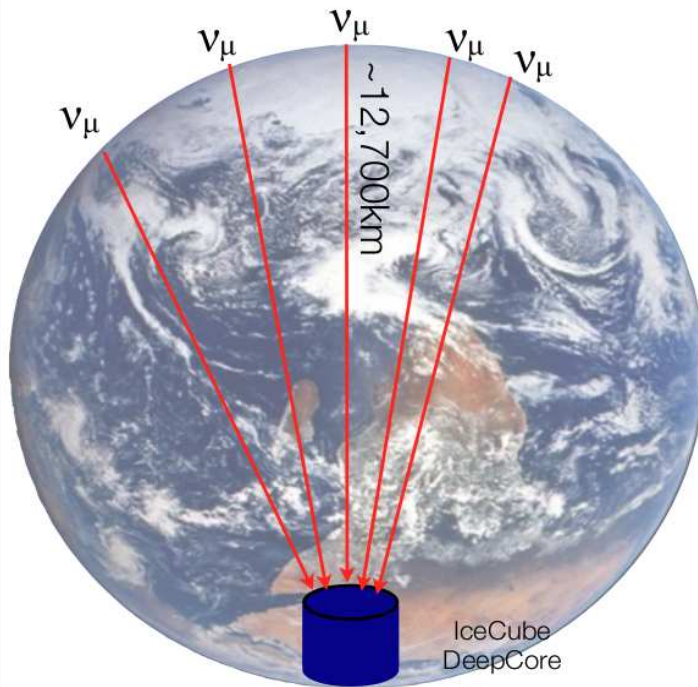
$E_\nu > \text{a few GeV}$   
Up/Down Symmetry



# Neutrino Oscillation Source

- Oscillation
- IceCube-DeepCore Physics
- PINGU
- Beyond

- Northern Hemisphere  $\nu_\mu$  oscillating over one earth radii produces  $\nu_\mu$  ( $\nu_\tau$ ) oscillation minimum(maximum) at  $\sim 25$  GeV
  - Covers all possible terrestrial baselines
  - “Beam” is free and never turns off



The fluxes of atmospheric  $\nu_{e,\mu}$  of energy  $E$ , which reach the detector after crossing the Earth along a given trajectory specified by the value of  $\theta_n$ ,  $\Phi_{\nu_{e,\mu}}(E, \theta_n)$ , are given by the following expressions in the case of the 3-neutrino oscillations under discussion:

$$\Phi_{\nu_e}(E, \theta_n) \cong \Phi_{\nu_e}^0 (1 + [s_{23}^2 r - 1] P_m^{2\nu}),$$

$$\Phi_{\nu_\mu}(E, \theta_n) \cong \Phi_{\nu_\mu}^0 (1 + s_{23}^4 [(s_{23}^2 r)^{-1} - 1] P_m^{2\nu} - 2c_{23}^2 s_{23}^2 [1 - \text{Re} (e^{-i\kappa} A_m^{2\nu}(\nu_\tau \rightarrow \nu_\tau))]) ,$$

where  $\Phi_{\nu_{e(\mu)}}^0 = \Phi_{\nu_{e(\mu)}}^0(E, \theta_n)$  is the  $\nu_{e(\mu)}$  flux in the absence of neutrino oscillations and

$$r \equiv r(E, \theta_n) \equiv \frac{\Phi_{\nu_\mu}^0(E, \theta_n)}{\Phi_{\nu_e}^0(E, \theta_n)} .$$

$s_{23}^2$ : **b.f.v.** 0.573 (0.575) **NO (IO)**;  $3\sigma$  **CL: (0.415-0.619)**.

$r(E, \theta_n) \cong (2.6 \div 4.5)$  for neutrinos giving the main contribution to the multi-GeV samples,  $E \cong (2 \div 10)$  GeV.

M. Honda, 1995.



Hyper Kamiokande (5SK), IceCube-PINGU, KM3Net-ORCA; DUNE

Iron Magnetised detector: INO

INO: 50 or 100 kt (in India);  $\nu_\mu$  and  $\bar{\nu}_\mu$  induced events detected ( $\mu^+$  and  $\mu^-$ );  
not designed to detect  $\nu_e$  and  $\bar{\nu}_e$  induced events.

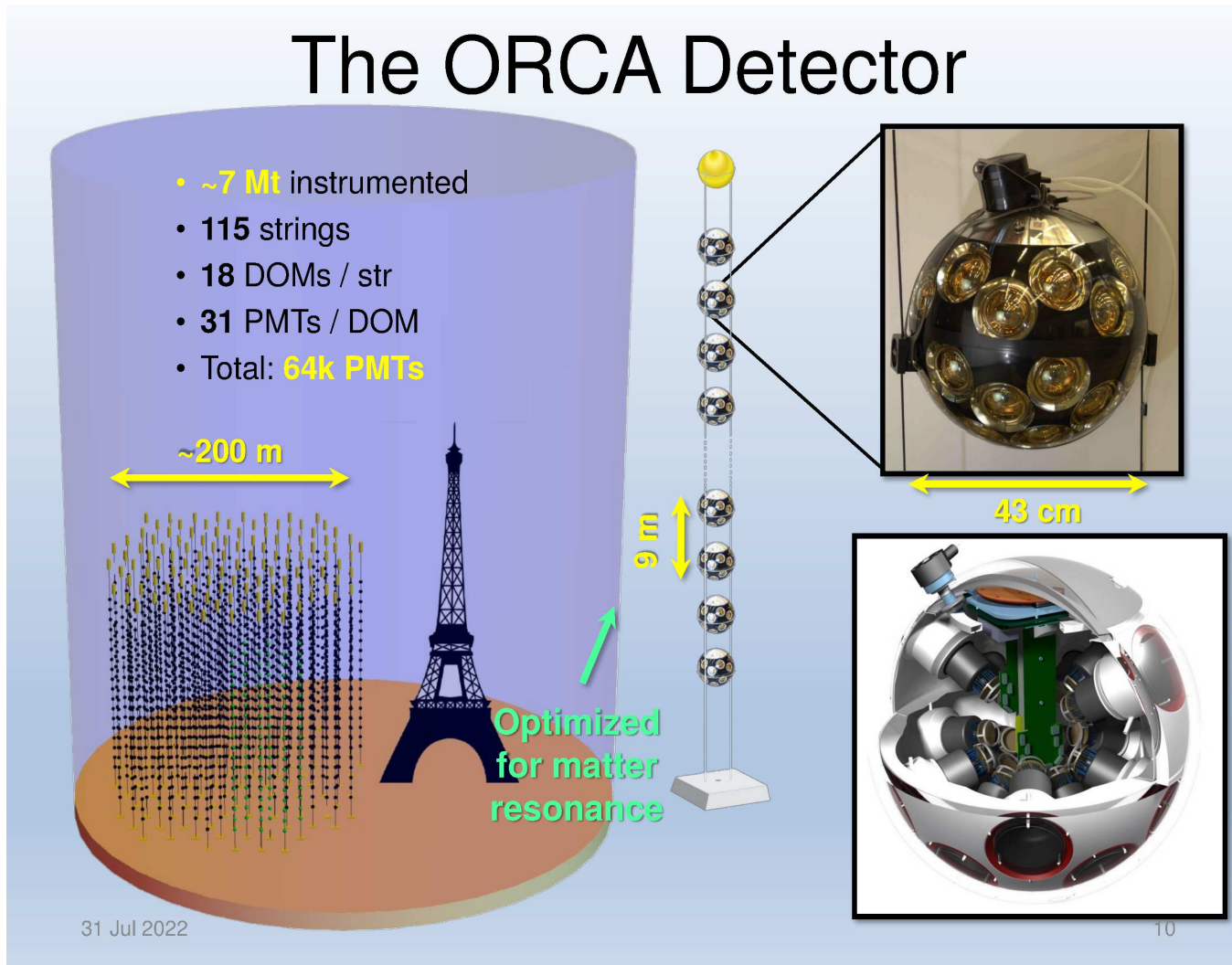
IceCube at the South Pole: PINGU

PINGU: 50SK;  $\nu_\mu$  and  $\bar{\nu}_\mu$  induced events detected ( $\mu^+$  and  $\mu^-$ , no  $\mu$  charge identification); Challenge:  $E_\nu \gtrsim 2$  GeV (?)

KM3Net in Mediteranian sea: ORCA (near Toulon)

Studies of atmospheric  $\nu$  oscillations with DUNE.

# The ORCA Detector



J. Coelho, talk at MMTE 2022

ORCA detector - part of the KM3Net Project. Located in the Mediterranean sea 40 km off the coast of Toulon, France. Letter of Intent: [arXiv:1601.07459](https://arxiv.org/abs/1601.07459). The total instrumented volume of ORCA will be  $\sim 3.7$  Mton of sea water;  $E_\nu \gtrsim 2$  GeV.

The Earth matter effects in the  $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$  ( $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$ ) oscillations depend on

$$V = \begin{pmatrix} + \\ - \end{pmatrix} \sqrt{2} G_F N_e^{(E)}(r)$$

$$N_e^{(E)}(r) = \rho_E(r) Y_e / m_N,$$

$Y_e$  - electron fraction number (or  $Z/A$  factor).

For isotopically symmetric matter  $Y_e = 0.5$ .

Earth core composition models:  $Y_e^c = 0.466 - 0.471$ .

Earth mantle composition models:  $Y_e^{man} = 0.490 - 0.496$ .

In arXiv:2111.13048 (F. Capozzi, STP) we used:  $Y_e^{man} = 0.490$  and  $Y_e^c = 0.467$ .

However, varying  $Y_e^c$  and  $Y_e^{man}$  in the indicated respective intervals has no effect on the results on the  $\bar{\rho}_{man}$ ,  $\bar{\rho}_C$ ,  $\bar{\rho}_{OC}$ ,  $\bar{\rho}_{IC}$  determination (with ORCA detector).

Determining the composition of, e.g., the OC or IC is technically a very different problem with respect to determining  $\bar{\rho}_{man}$ ,  $\bar{\rho}_C$ ,  $\bar{\rho}_{OC}$ ,  $\bar{\rho}_{IC}$ : **does not require varying  $\bar{\rho}_i$  in the study.**

The determination of the radial density distributions in the mantle and core,  $\rho_{man}(r)$  and  $\rho_c(r)$ , from seismological and geophysical data is not direct and suffers from uncertainties.

B. A. Bolt, Q. J. R. Astron. Soc. **32**, 367 (1991).

B.L.N. Kennett, Geophys. J. Int. **132**, 374 (1998);

G. Masters and D. Gubbins, Phys. Earth Planet. Inter. **140**, 159 (2003).

It requires the knowledge, in particular, of the seismic wave speed velocity distribution in the interior of the Earth, which depends on the pressure, temperature, composition and elastic properties of the Earth's interior that are not known with a good/high precision.

Often  $\rho_E(r)$  is determined using an empirical relation between the seismic wave velocities and  $\rho_E(r)$  (one example is the Birch law, which may fail at the higher densities of the core) and the so-called “Adams-Williamson equation” (from 1923). E. Williamson and L.H. Adams, J. Wash. Acad. Sci. **13** (1923) 413.

An approximate and perhaps rather conservative estimate of this uncertainty for  $\rho_{man}(r)$  is  $\sim 5\%$ ; for the core density  $\rho_c(r)$  it is larger and can be significantly larger (Bolt:1991,Kennett:1998,Masters:2003).

**A precise knowledge of  $\rho_E(r)$  and of  $\bar{\rho}_{\text{man}}$ ,  $\bar{\rho}_C$  and  $\bar{\rho}_{\text{IC}}$ , is essential for understanding the physical conditions and fundamental aspects of the structure and properties of the Earth's interior (including the dynamics of mantle and core, the bulk composition of the Earth's three structures, the generation, properties and evolution of the Earth's magnetic field and the gravity field of the Earth) (Bolt:1991, Yoder:1995, McDonough:2003, McDonough:2008zz).**

**The thermal evolution of the Earth's core, in particular, depends critically on the density change across the inner core - outer core boundary (see, e.g., Baffet:1991).**

**An independent determination of  $\rho_E(r)$  and of  $\bar{\rho}_{\text{man}}$ ,  $\bar{\rho}_C$  and  $\bar{\rho}_{\text{IC}}$ , as well as about the composition of the OC, IC, lower mantle, etc. is highly desirable and would be extremely useful.**

**The neutrino tomography of the Earth is a unique alternative method of getting information about the density profile and the composition of the Earth.**

**The neutrino tomography of the Earth – two different sets of goals:**  
**A. information about the densities of the different Earth layers,**  
**B. information about the composition of the different layers.**

**The set A. includes.**

- Getting information about the mantle (upper, lower), core, outer core, inner core densities.
- Getting information about the location and the width of, and the changed of density in, the mantle-core transitions zone.
- Getting information about the OC - IC transition (location, width of the zone, jump of density).

**The set B. includes.**

Getting information about the compositions of the OC, IC and other possible layers (e.g., lower mantle, the composition of the mantle-core transition zone).

Some of the goals (especially those related to the inner core and the composition of the various layers) are extremely challenging but worth the efforts.

**Determine the experimental conditions (statistics, systematics, resolutions, efficiencies of identification of the different types of events, etc.) that could allow to achieve each of the different goals.  
Should be done for each of the detectors that can be used for neutrino tomography of the Earth.**

**In setting the specific goals the collaboration with the colleagues-geophysicists is extremely important.**



## Important Constraints

The total Earth mass constraint:

$$M_{\oplus} = (5.9722 \pm 0.0006) \times 10^{24} \text{ kg}$$

Earth momentum of inertia constraint:

$$I_{\oplus} = (8.01736 \pm 0.00097) \times 10^{37} \text{ kg m}^2.$$

$$I_{\oplus} = 0.330745 M_{\oplus} R_{\oplus}^2.$$

Earth hydrostatic equilibrium (EHE) constraint:

$$\rho_{man} \leq \rho_{OC} \leq \rho_{IC}.$$

**The three constraints should always be implemented in the simulations.**

Typically in the analyses of the sensitivity of a given experiment to the density in a given Earth layer, one changes the **PREM** density in a given layer  $\rho_i(r)$ ,  $i = \text{IC, OC, man}$ , by a factor  $(1 + \kappa_i)$ ,  $\kappa_i$  is  $r$ -independent real constant:

$$\rho_i(r) \rightarrow (1 + \kappa_i)\rho_i(r).$$

In order to satisfy the  $M_{\oplus}$  constraint, one has to correspondingly change the density in another Earth layer.

In the case of  $\rho_{OC}$  ( $\rho_{man}$ ) variation compensated by a change of  $\rho_{man}$  ( $\rho_{OC}$ ), for example, the  $M_{\oplus} + \text{EHE constraints}$  imply approximately:  
 $(-49.6\%) \lesssim \Delta\rho_{OC} \lesssim 18.3\%$  ( $(-8.3\%) \lesssim \Delta\rho_{man} \lesssim 22.8\%$ ).

**The sensitivity to a given  $\rho_i(r)$  depends strongly on, e.g., whether the  $M_{\oplus}$  constraint is implemented or not, and – if it is implemented– on how it is implemented.**

From the studies performed so far it seems that using the neutrino oscillation tomography method it is very difficult to get information about  $\rho_{IC}(r)$ .

**When implementing the  $M_{\oplus}$  constraint, maximal sensitivity to, e.g.,  $\rho_{OC}$  ( $\rho_{man}$ ) is achieved when the variation of  $\rho_{OC}$  ( $\rho_{man}$ ) is compensated by change of  $\rho_{man}$  ( $\rho_{OC}$ )**

**The results are very different - the sensitivity is much worse - if the  $M_{\oplus}$  constraint is not implemented.**

**The sensitivity is maximal for the largest allowed value of  $\sin^2 \theta_{23}$  ( $\cos^2 \theta_{23}$ ) in the case of the effects in the interval  $E_{\nu} \sim (2 - 10)$  GeV ( $E_{\nu} \sim (100 - 300)$  MeV).**

**Getting information about  $\rho_{IC}(r)$  and about the the OC-IC transition ( $\rho_{IC}(r)/\rho_{OC}(r)$  at the transition) – remarkably challenging problem.**

If we implement both the  $M_{\oplus}$  and  $I_{\oplus}$  constraints, the variation of  $\rho_i(r)$  in a given layer leading to variations of both  $M_{\oplus}^i$  and  $I_{\oplus}^i$  of the layer, is impossible to compensate by a corresponding change of  $\rho_j(r)$  of just one layer  $j$  (leading to changes of  $M_{\oplus}^j$  and  $I_{\oplus}^j$  in this layer). In order to have both the  $M_{\oplus}$  and  $I_{\oplus}$  constraints satisfied, one has to implement corresponding changes of the densities of at least two other layers  $\rho_j(r)$  and  $\rho_k(r)$ ,  $j \neq k \neq i$ .

**Example:** suppose we study the sensitivity of a given detector to  $\rho_{OC}$  by  $\rho_{OC}(r) \rightarrow (1 + \kappa_{OC})\rho_{OC}(r)$  and varying  $\kappa_{OC}$ . The  $M_{\oplus}$  and  $I_{\oplus}$  constraints can be simultaneously satisfied only if one compensates the effect of variation of  $\rho_{OC}$  on  $M_{\oplus}$  and  $I_{\oplus}$  by compensating it by corresponding changes of densities of at least two other layers, say  $\rho_{man}$  and  $\rho_{IC}$ .

## Conclusion.

The neutrino tomography of the Earth is a promising powerful alternative method of obtaining valuable information about the Earth interior. It is at initial stage of development. The neutrino tomography of the Earth can improve, or even push the boundaries of, our understanding of the Earth interior, which can have far-reaching fundamental implications. The physics potential of the neutrino tomography of the Earth should be fully explored.

For the success of this Endeavour the collaboration with the colleagues-geophysicists is crucial.