Probing the properties of Core-Mantle Boundary using Atmospheric Neutrino Oscillation

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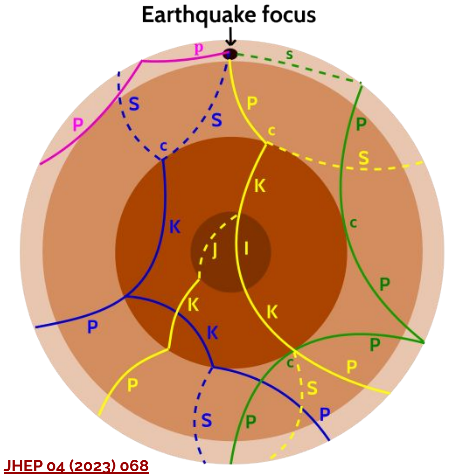
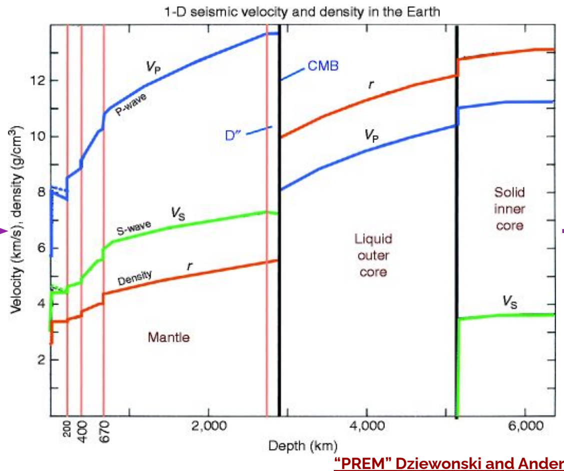
Outline

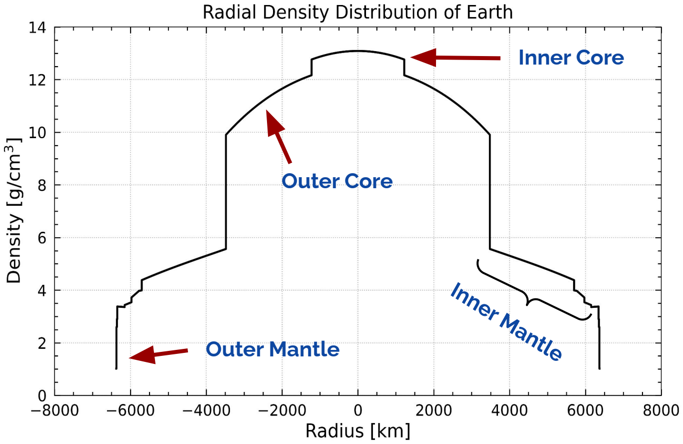
- Information About Earth's interior
- Atmospheric Neutrinos
- Earth's Matter Effects: Key to Probe Internal Structure of Earth
- ICAL at INO
- Neutrinos for Probing the Properties of Core-Mantle Boundary (CMB)
  - Locating CMB
  - Measuring Density Jump at CMB

The Interior of Earth

- Information about the interior of Earth is obtained from indirect probes used in traditional **seismic** and **gravitational** studies → **Preliminary Reference Earth Model (PREM)**



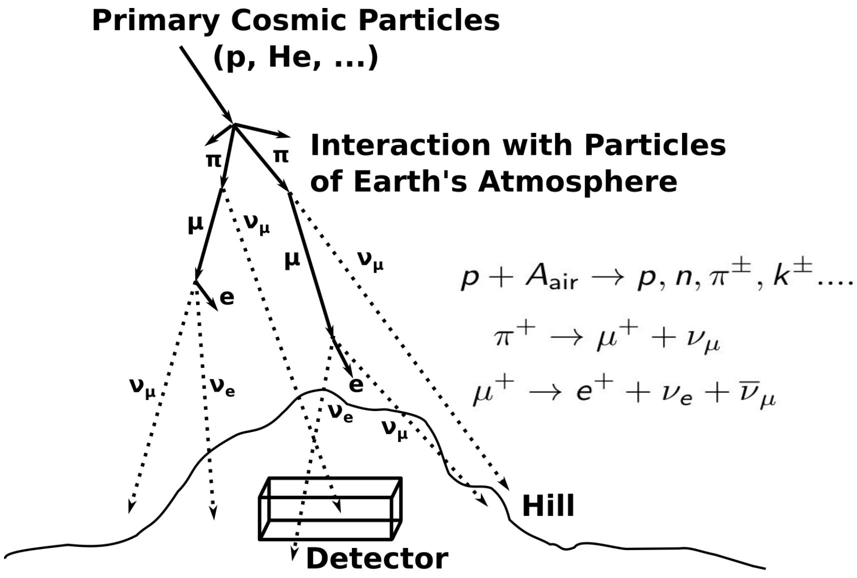


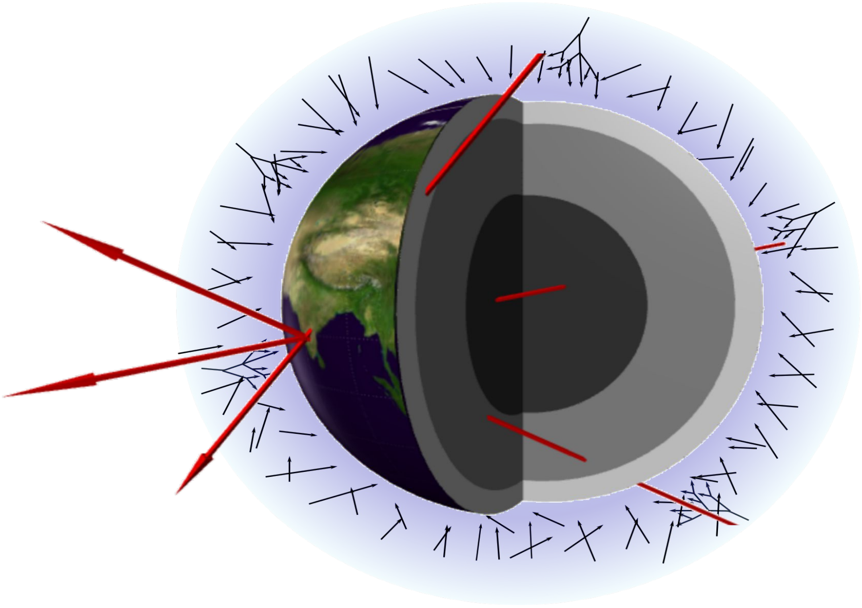


- Broadly classified: two concentric shell - the outer one is mantle, and the inner one with a much higher density is core
- Mantle consists of hot rocks of silicate and core is composed of metals like iron and nickel
- Outer core is expected be liquid (absence of S-waves and decrease in the velocity of P-waves)
- Core-Mantle Boundary (CMB): the largest compositional discontinuity within the Earth at a depth of 2891 km
- The large density contrast across the CMB

Atmospheric Neutrinos

- Complementary information may be obtained using the observations of **neutrinos that pass through Earth**
- Neutrino exploit the **weak interactions** to probe the internal structure of Earth

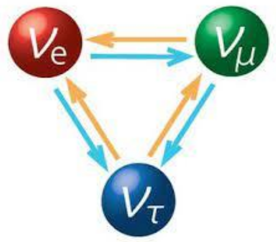




- Flux should be up-down symmetric
- Almost isotropic flux

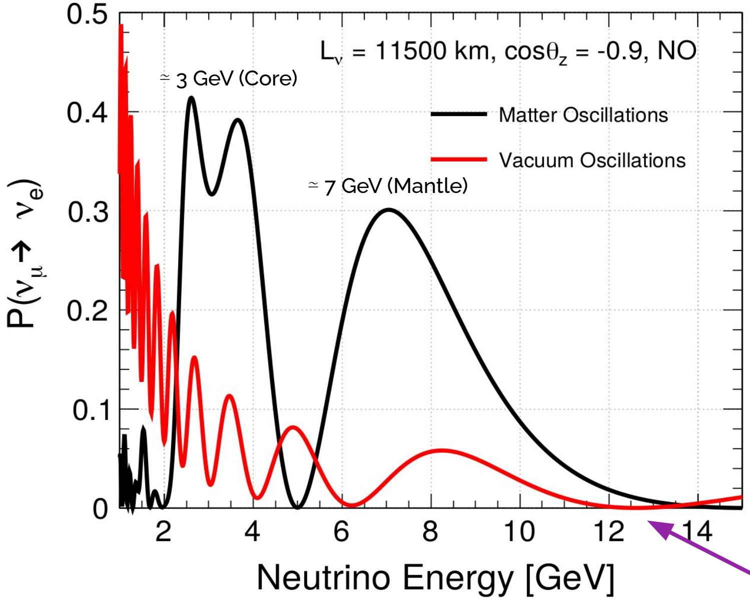
- Baseline: **15 km to 12757 km**
- Energy range: **0.1 GeV to ~10 TeV**

Neutrino Oscillations



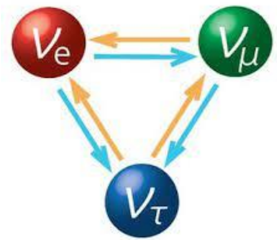
- Neutrino changes its flavor while propagating
- Neutrino oscillation probability get modified significantly in matter

Earth's matter vs. Vacuum



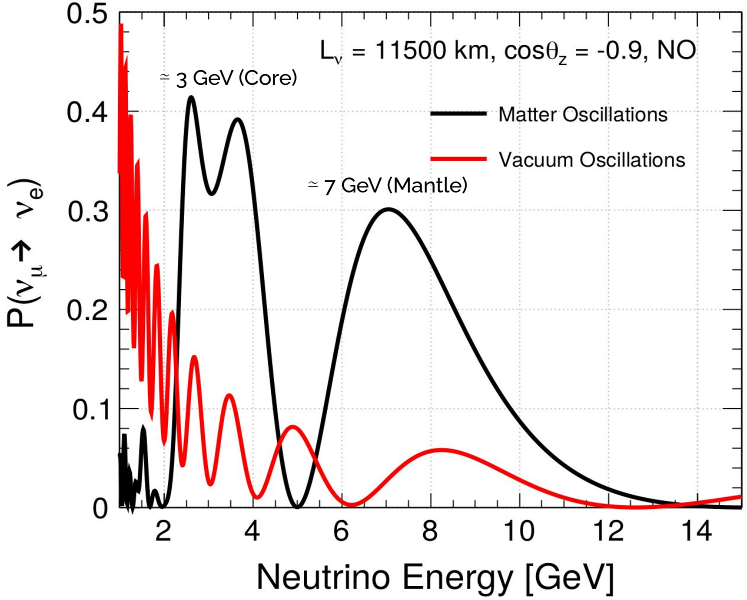
$$E_{\rm res} = \frac{\Delta m^2_{31}\cos 2\theta_{13}}{2\sqrt{2}G_F N_e} \simeq 7\ \text{GeV}\left(\frac{4.5\ \text{g/cm}^3}{\rho}\right)\left(\frac{\Delta m^2_{31}}{2.4\times 10^{-3}\ \text{eV}^2}\right)\cos 2\theta_{13}$$

Neutrino Oscillations

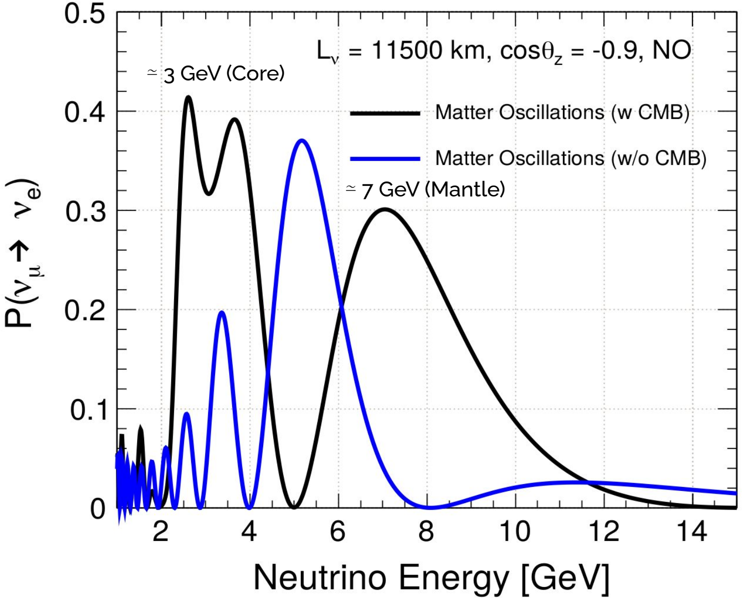


- Neutrino changes its flavor while propagating
- Neutrino oscillation probability get modified significantly in matter
- In absence of high density jump at CMB, the NOLR/PR resonance does not observed

Earth's matter vs. Vacuum



CMB vs. w/o CMB



Matter Effects: Key to Probe Internal Structure of Earth

- The atmospheric neutrinos undergo coherent forward scattering with ambient electrons inside Earth which leads to the modification of neutrino oscillation probabilities

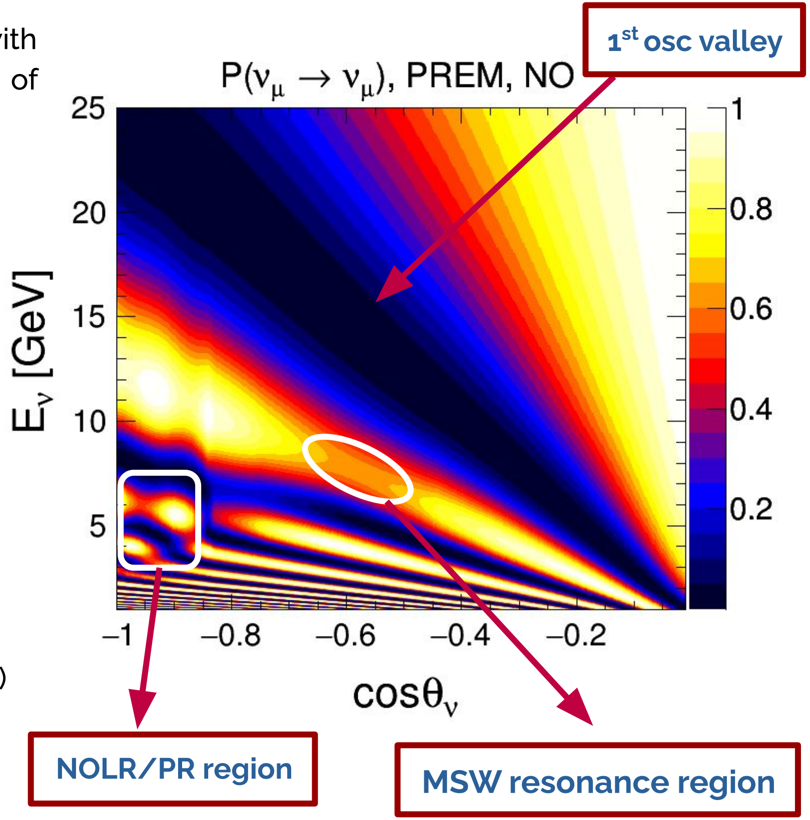
Electron number density

$$V_{CC} = \pm\sqrt{2}G_F N_e \approx \pm 7.6 \times Y_e \times 10^{-14} \left[\frac{\rho}{g/cm^3}\right] \text{eV}$$

$$Y_e = N_e/(N_p + N_n)$$

ρ denotes the matter density
+1 (-1) for neutrino (antineutrino)

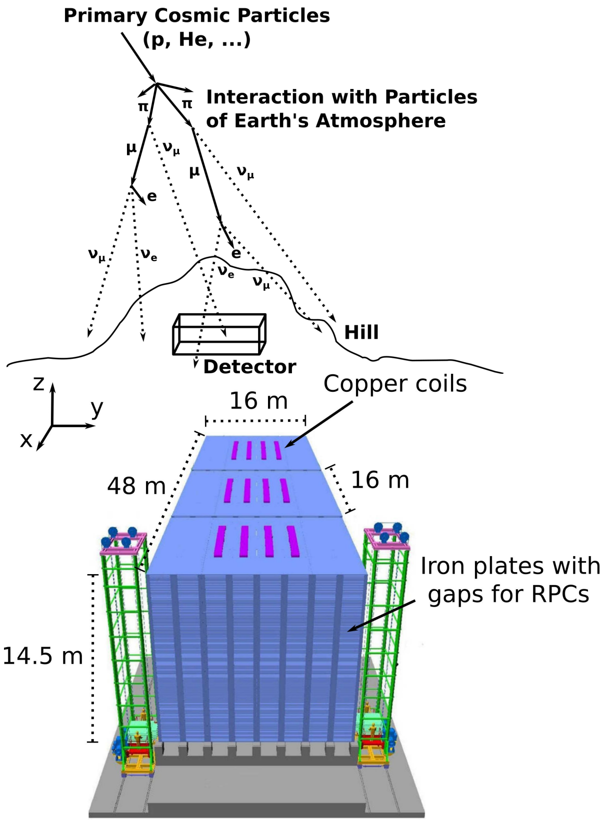
- MSW resonance (L. Wolfenstein, PRD 17 (1978) 2369)
- Neutrino Oscillation Length Resonance (NOLR) (Petcov, PLB 434 (1998) 321) or Parametric Resonance (PR) (Akhmedov, NPB 538 (1999) 25)



Iron Calorimeter Detector (ICAL) at INO

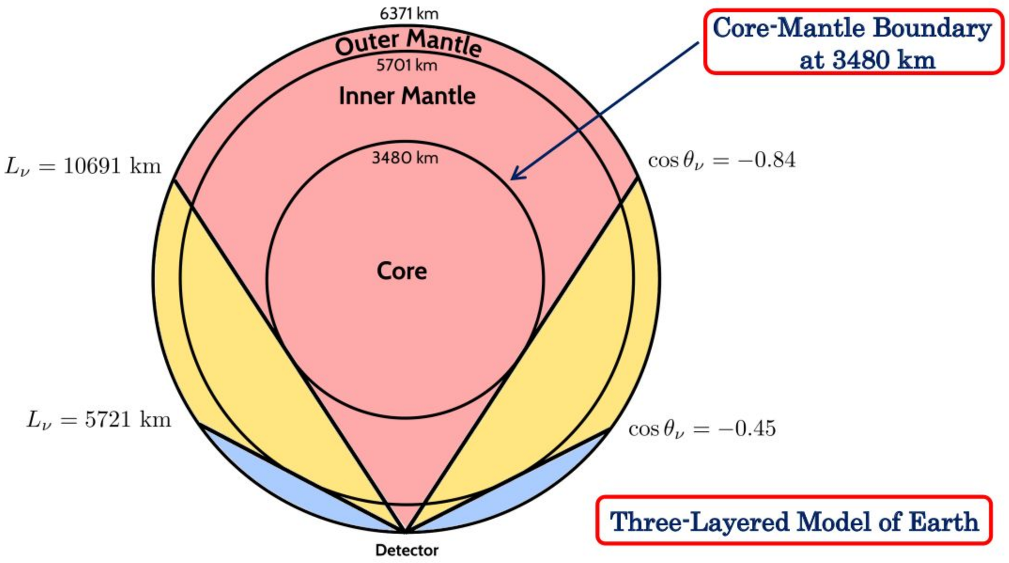
- **ICAL@INO:** 50 kton magnetized iron calorimeter detector at the proposed India-based Neutrino Observatory (INO)
- **Location:** Bodi West Hills, Theni District, Tamil Nadu, India
- **Aim:** To determine neutrino mass ordering and precision measurement of atmospheric neutrino oscillation parameters
- **Source:** Atmospheric neutrinos and antineutrinos in the multi-GeV range of energies over a wide range of baselines
- **Uniqueness:** Charge identification capability helps to distinguish μ^- and μ^+ and hence, ν_μ and $\bar{\nu}_\mu$
- **Muon energy range:** 1 – 25 GeV
- **Muon energy resolution:** ~ 10%
- **Baselines:** 15 – 12000 km
- **Muon zenith angle resolution:** ~ 1°

Next talk by
Dr. Anil Kumar



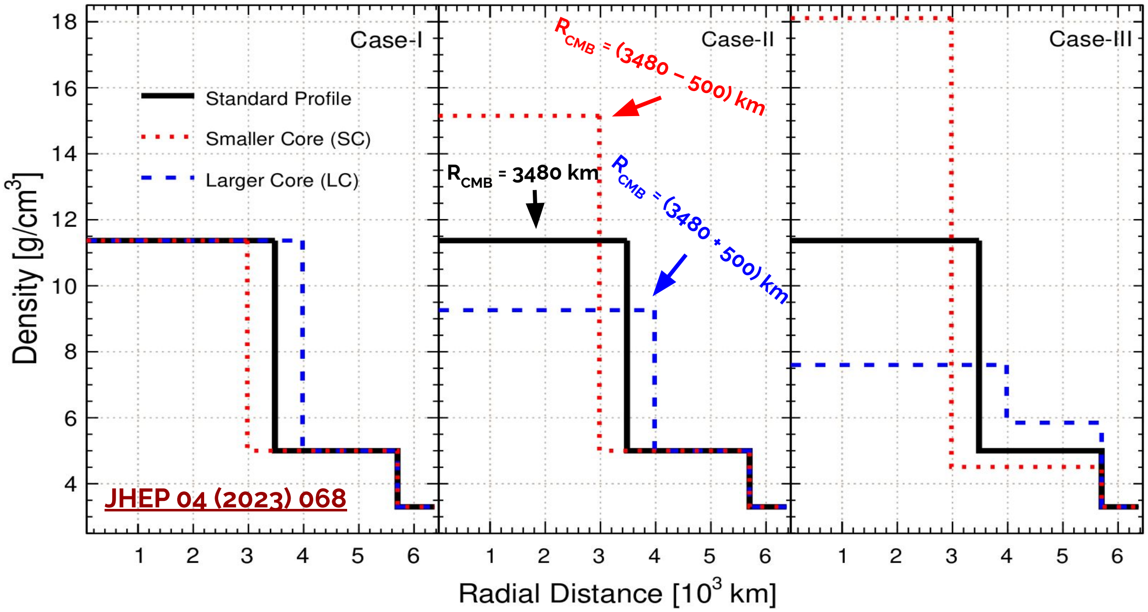
Pramana 88 (2017) 5, 79, arXiv:1505.07380

Locating Core-Mantle Boundary (CMB) Using Atmospheric Neutrinos



Based on: A. K. Upadhyay, A. Kumar, S. K. Agarwalla, and A. Dighe JHEP 04 (2023) 068

A Few Toy Models of Earth with Varying CMB



Case-I: Densities of all layers fixed and $M_\oplus$ is not invariant

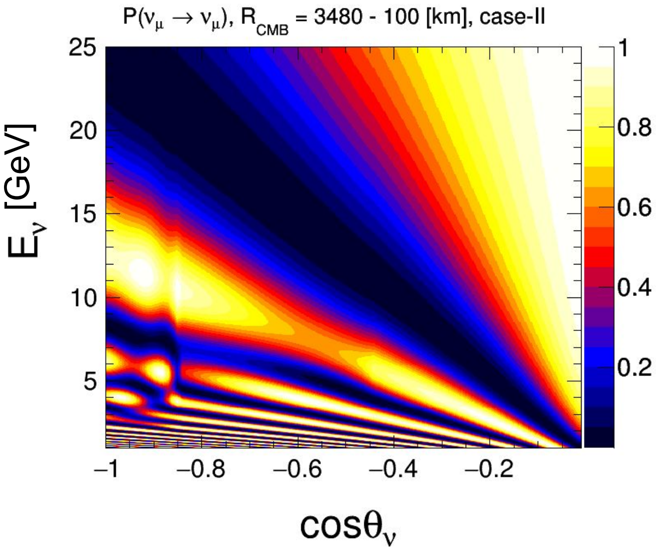
Case-II: Densities of inner & outer mantle fixed. Core density varies to keep $M_\oplus$ invariant

Case-III: Core & inner mantle densities vary to keep their masses fixed. Outer mantle density fixed & $M_\oplus$ invariant

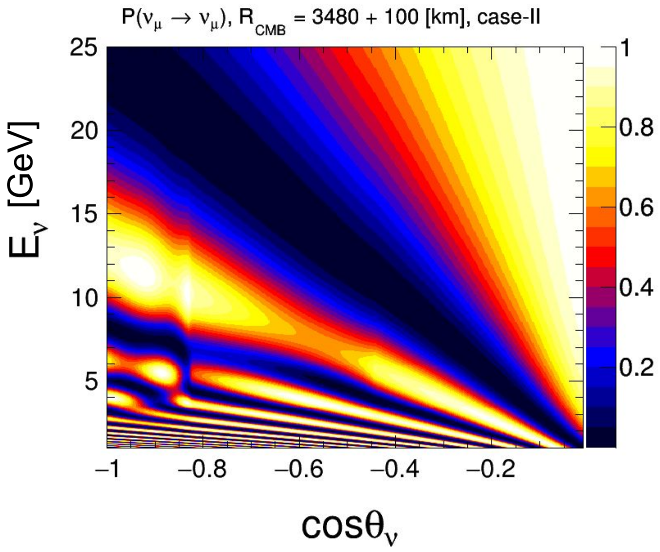
Effect of CMB Variation on Oscillograms

Case-II

Smaller Core (SC)



Larger Core (LC)



- For smaller core, the NOLR/PR shifts to left and patterns shrink
- For larger core, the NOLR/PR shifts to right and patterns broaden
- No modification in MSW region

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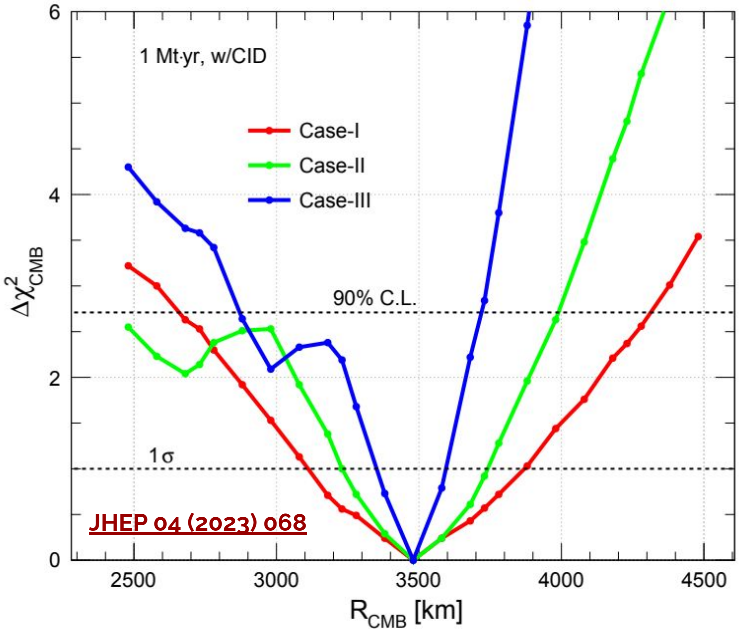
Opposite modification in NOLR/PR region for SC and LC

Sensitivity for Locating Core-Mantle Boundary

$$\Delta\chi^2_{\mathrm{CMB}} = \chi^2(\text{modified } R_{\mathrm{CMB}}) - \chi^2(\text{standard } R_{\mathrm{CMB}})$$

- 1 Mt·yr exposure at ICAL
- Marginalization over: (1) systematic uncertainties, (2) Oscillation parameter: (2a) $\sin^2\theta_{23}$: (0.36, 0.66), (2b) $\Delta m^2_{\mathrm{eff}}$: (2.1, 2.6) $\times 10^{-3}$ eV² (2c), mass ordering: (NO, IO)



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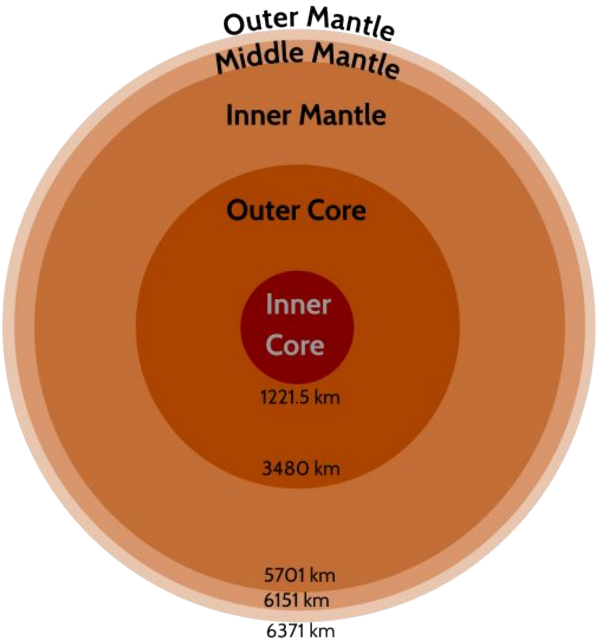
		$\Delta\chi^2_{\mathrm{CMB}}$	
		w CID	w/o CID
Case-I	R_{CMB} = 2980 km	1.53	1.01
	R_{CMB} = 3980 km	1.44	0.95
Case-II	R_{CMB} = 2980 km	2.53	1.66
	R_{CMB} = 3980 km	2.63	1.67
Case-III	R_{CMB} = 2980 km	2.09	1.33
	R_{CMB} = 3980 km	8.07	5.23

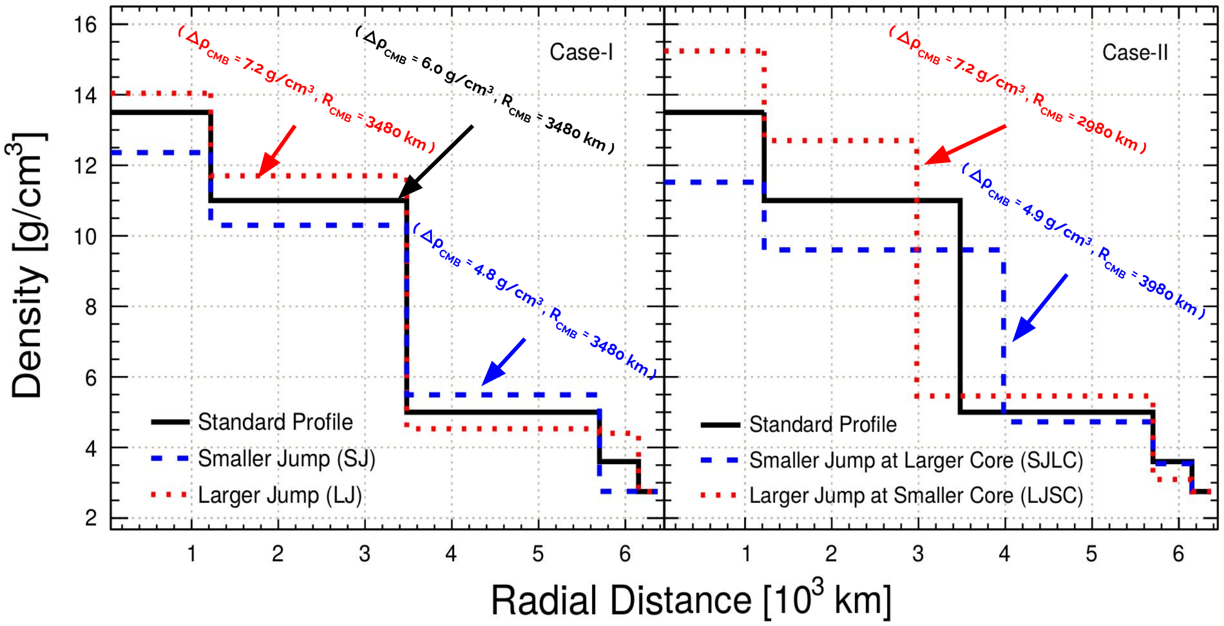
- 1σ precision of about ± 380 km, **± 250 km**, and ± 120 km for Case-I, **Case-II**, and Case-III, respectively.

Measuring Correlation Between Density Jump at CMB and Location of CMB Using Atmospheric Neutrinos



- **Five layers:** Inner Core (IC), Outer Core (OC), Inner Mantle (IM), Middle Mantle (MM), and Outer Mantle (OM)
- **Density Jumps:** IC-OC, CMB, IM-MM, and MM-OM boundary; $\Delta\rho_{IC\text{-}OC}$, $\Delta\rho_{CMB}$, $\Delta\rho_{IM\text{-}MM}$, and $\Delta\rho_{MM\text{-}OM}$

Toy Models of Earth with Modifying $\Delta\rho_{CMB}$ & R_{CMB}



- **Constraints:**
  - Mass of Earth is fixed
  - Moment of inertia of Earth is fixed
  - Hydrodynamic condition ($\rho_{inner\ layer} > \rho_{outer\ layer}$) is satisfied
  - Density of outer mantle is fixed
  - Electron number density ratio: $Y_e = N_e / (N_p + N_n)$ or Z/A :

    Y_e (Inner Core) = 0.466

    Y_e (Outer Core) = 0.466

    Y_e (Mantle) = 0.494

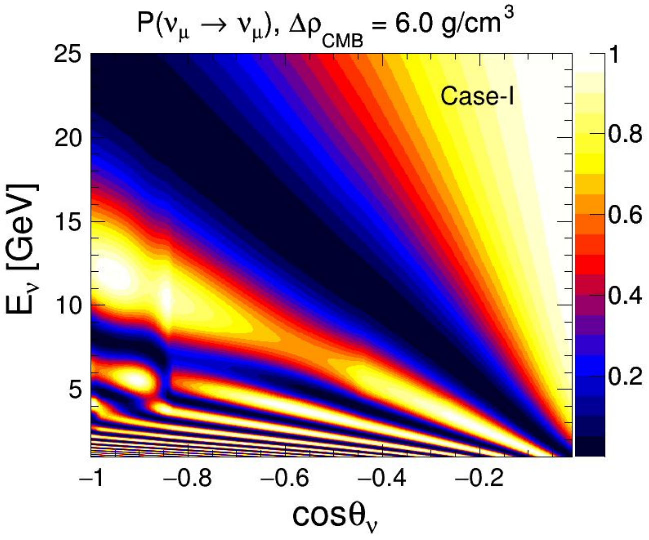
Case-I: Density jump ($\Delta\rho_{CMB}$) modify at the standard R_{CMB} = 3480 km

Case-II: Density jump ($\Delta\rho_{CMB}$) and location of CMB (R_{CMB}) both modify simultaneously

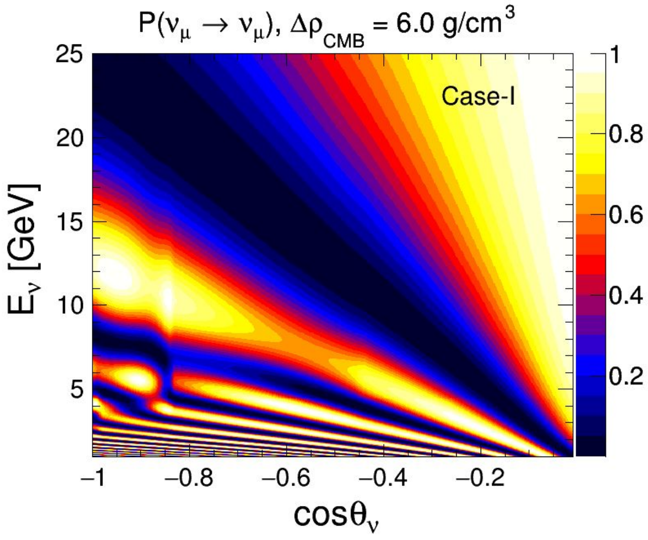
Effect of $\Delta\rho_{CMB}$ Variation on Oscillograms

Case-I

Smaller Jump (SJ)



Larger Jump (LJ)



Observations:

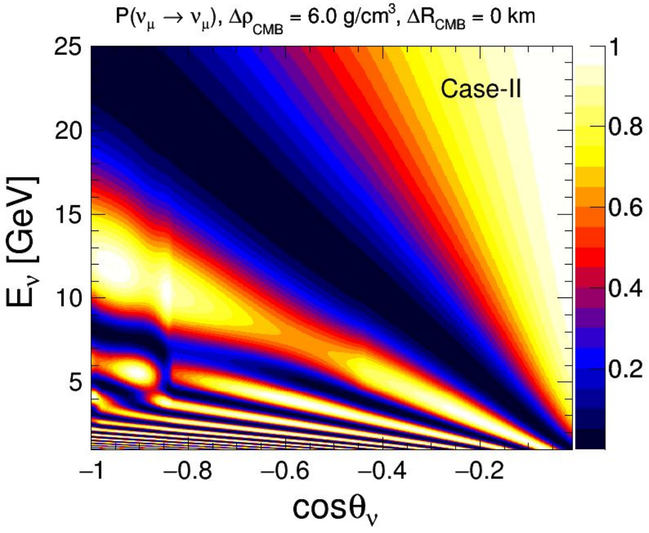
- For SJ:
  - NOLR/PR shift to left
  - MSW patterns shrink to right
- For LJ:
  - NOLR/PR shift to right
  - MSW patterns stretched to left

Opposite modification in MSW and NOLR/PR region for SJ and LJ scenarios

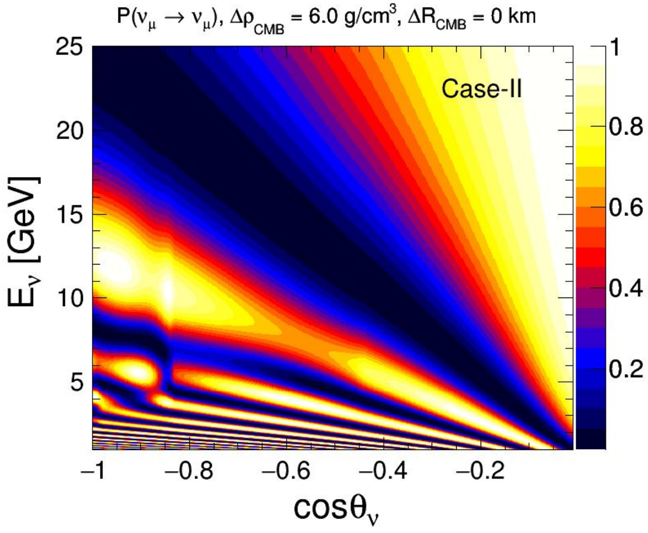
Effect of $\Delta\rho_{CMB}$ & R_{CMB} Variation on Oscillograms

Case-II

LJSC



SJLC



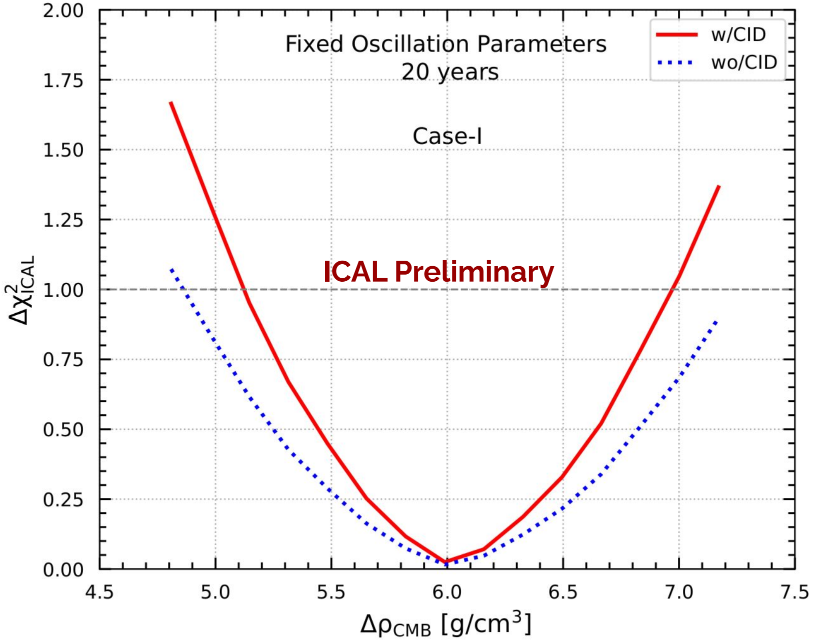
Observations:

- **For LJSC**:
  - NOLR/PR shifts to right and patterns shrink
  - MSW patterns shrink to right
- **For SJLC**:
  - NOLR/PR shifts to left and patterns broaden
  - MSW patterns stretched to left

Opposite modification in MSW and NOLR/PR region for LJSC and SJLC scenarios

Sensitivity for Measuring Density Jump $\Delta\rho_{CMB}$ at Standard R_{CMB}

Case-I

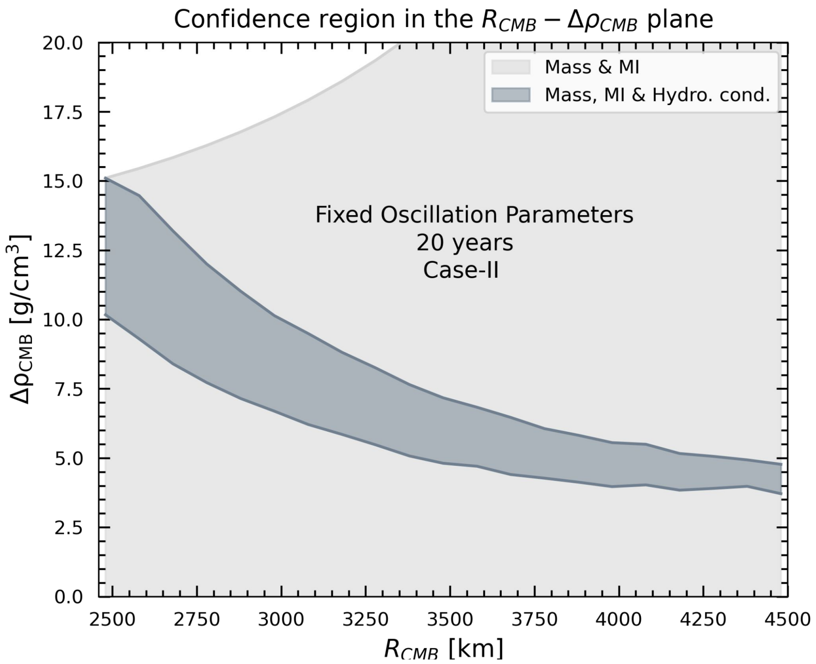


$6.0^{+0.97}_{-0.88}$ g/cm^3

Relative precision of about 15% at the 1σ (w/CID)

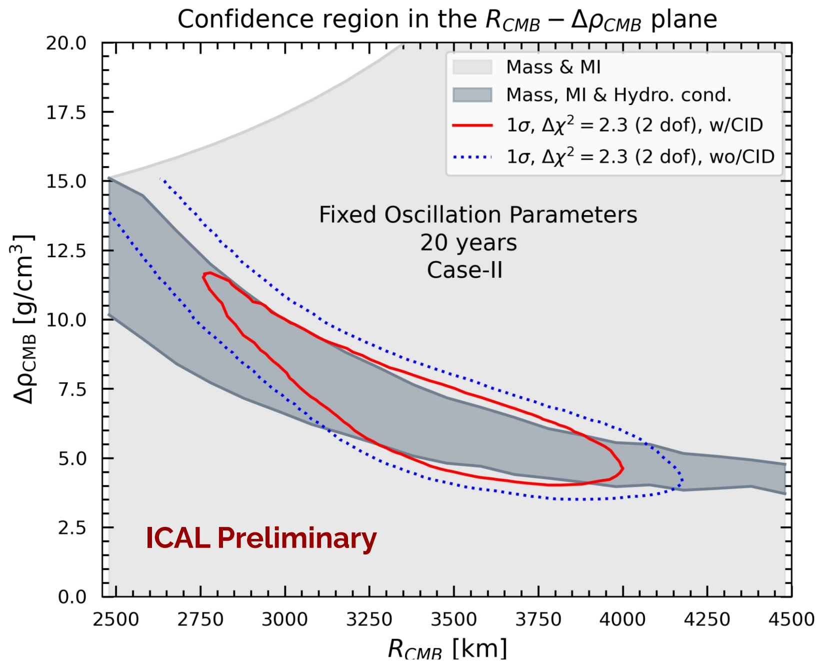
Sensitivity for Measuring Correlation Between $\Delta\rho_{CMB}$ & R_{CMB}

Case-II



Sensitivity for Measuring Correlation Between $\Delta\rho_{CMB}$ & R_{CMB}

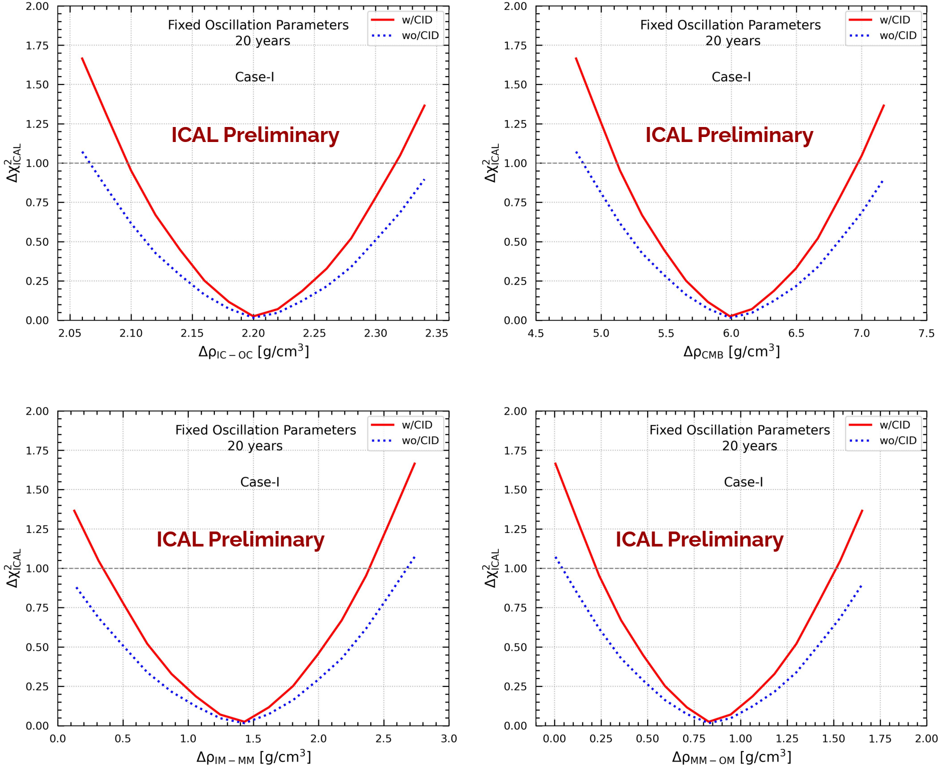
Case-II



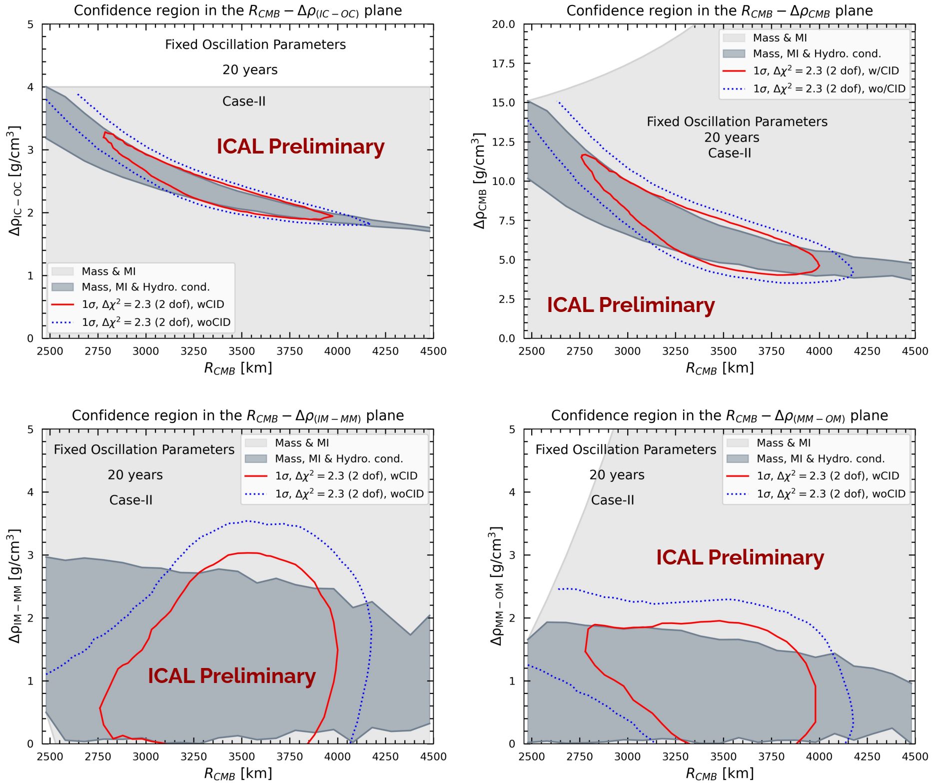
- Relative precision for **locating R_{CMB}** is around **9%** at the 1σ (w/CID) w.r.t standard R_{CMB}
- Relative precision for constraining **density jump at CMB ($\Delta\rho_{CMB}$)** is around **25%** at the 1σ (w/CID) w.r.t standard $\Delta\rho_{CMB}$

Sensitivity for Measuring Correlation Between $\Delta\rho_{IC-OC}$, $\Delta\rho_{CMB}$, $\Delta\rho_{IM-MM}$, $\Delta\rho_{MM-OM}$ & R_{CMB}

Case-I



Case-II



Summary

- In combination with gravitational and seismic studies, neutrino oscillations and absorption based measurements would pave the way for **"Multi-Messenger Tomography of Earth"**
- Atmospheric neutrinos have energies in the multi-GeV range where the Earth matter effects are significant, hence they would serve as probes of the internal structure of Earth
- ICAL can detect 331 μ^- and 146 μ^+ core passing events in 10 years
- ICAL detector with 1000 kt·yr exposure would be able to **locate the CMB** with a precision of about **±250 km** at 1σ confidence level
- ICAL detector with 1000 kt·yr exposure would be able to measure the density jump at inner-outer core boundary (**$\Delta\rho_{IC-OC}$**) and CMB (**$\Delta\rho_{CMB}$**) at the standard R_{CMB} = 3480 km with a relative precision of about **5%** and **15%** at the 1σ C.L, respectively
- ICAL would be able to constraint the allowed region between **$\Delta\rho_{IC-OC}$ - R_{CMB}** around **6%** for **R_{CMB}** and **8%** for **$\Delta\rho_{IC-OC}$** w.r.t. standard $\Delta\rho_{IC-OC}$ and R_{CMB} radius at 1σ level with 1 Mt·yr exposure
- ICAL would be able to constraint the allowed region between **$\Delta\rho_{CMB}$ - R_{CMB}** around **9%** for **R_{CMB}** and **25%** for **$\Delta\rho_{CMB}$** w.r.t. standard $\Delta\rho_{CMB}$ and R_{CMB} radius at 1σ level with 1 Mt·yr exposure

We acknowledge the support from the Department of Atomic Energy (DAE), Department of Science and Technology (DST), and Science and Engineering Research Board (SERB), Govt. of India

Thank you!

Backup

Statistical Analysis

In this analysis, the χ^2 statistics is expected to give median sensitivity of the experiment in the frequentist approach.

$$\chi^2_- = \min_{\xi_l} \sum_{i=1}^{N_{E'^{\text{rec}}_{\text{had}}}} \sum_{j=1}^{N_{E^{\text{rec}}_{\mu}}} \sum_{k=1}^{N_{\cos\theta^{\text{rec}}_{\mu}}} \left[2(N^{\text{theory}}_{ijk} - N^{\text{data}}_{ijk}) - 2N^{\text{data}}_{ijk} \ln\left(\frac{N^{\text{theory}}_{ijk}}{N^{\text{data}}_{ijk}}\right)\right] + \sum_{l=1}^{5} \xi_l^2$$

where,

$$N^{\text{theory}}_{ijk} = N^0_{ijk}\left(1 + \sum_{l=1}^{5} \pi^l_{ijk}\xi_l\right)$$

Similarly, χ^2_+ is defined for μ^+

$$\chi^2_{\text{ICAL}} = \chi^2_- + \chi^2_+$$

The Interior of Earth

Properties	Value (±)	Units
Earth's Mass	$(5.9722 \pm 0.006) \times 10^{24}$	kg
Mean moment of inertia	$(8.01736 \pm 0.00097) \times 10^{37}$	kg m^2
Earth's mean radius	6371.23 (0.01)	km
Core-mantle boundary	3483 ± 5	km
Inner-outer core boundary	1220 ± 10	km

- W.F. McDonough, Elsevier (2003), p. 547
- W. Chen et. al. Journal of Geodesy 89,179–188 (2015)
- B. Luzum et.al. Celestial Mechanics and Dynamical Astronomy 110, 293–304 (2011)