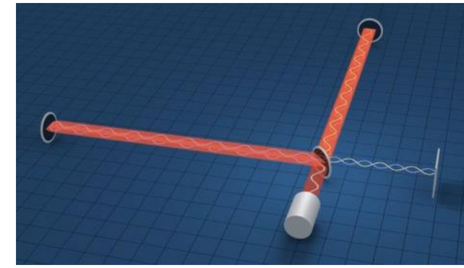
Computational and Statistical Aspects of Neutrino Oscillation Tomography

João Coelho
APC Laboratory

05 July 2023

Some Numbers



Yes, LIGO can measure 10^{-19} m with a 10^{-6} m wavelength. But they have 10^{24} photons per second!

- Oscillation length: L/E ~ 500 km/GeV
- Structures sensitive to oscillation > 100 km
- Volume of the Earth: 10^{12} km^3
- Number of objects with size ~(100 km)^3: **10^6**

Measuring Oscillations with A Million Atmospheric Neutrinos

C. A. Argüelles [1,*] P. Fernández [2,3,†] I. Martínez-Soler [1,‡] and M. Jin (靳淼辰) [1,§]

[1] *Department of Physics & Laboratory for Particle Physics and Cosmology, Harvard University, Cambridge, MA 02138, USA*

[2] *University of Liverpool, Department of Physics, Liverpool, United Kingdom*

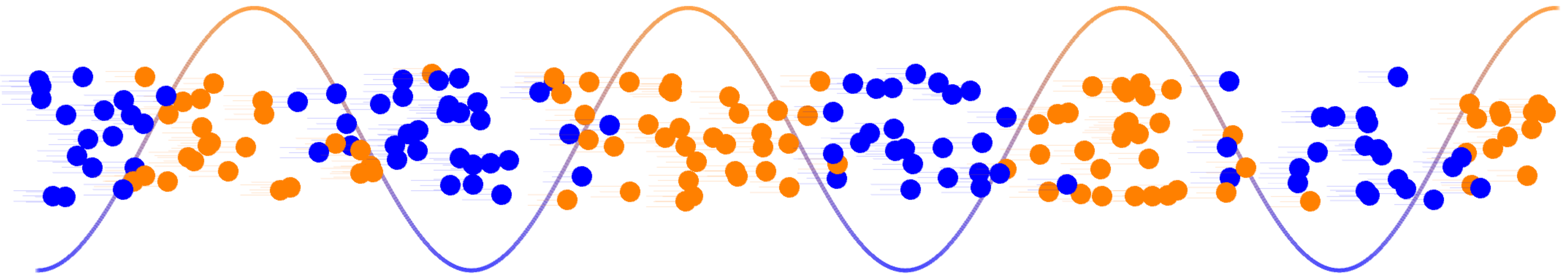
[3] *Donostia International Physics Center DIPC, San Sebastián/Donostia, E-20018, Spain*

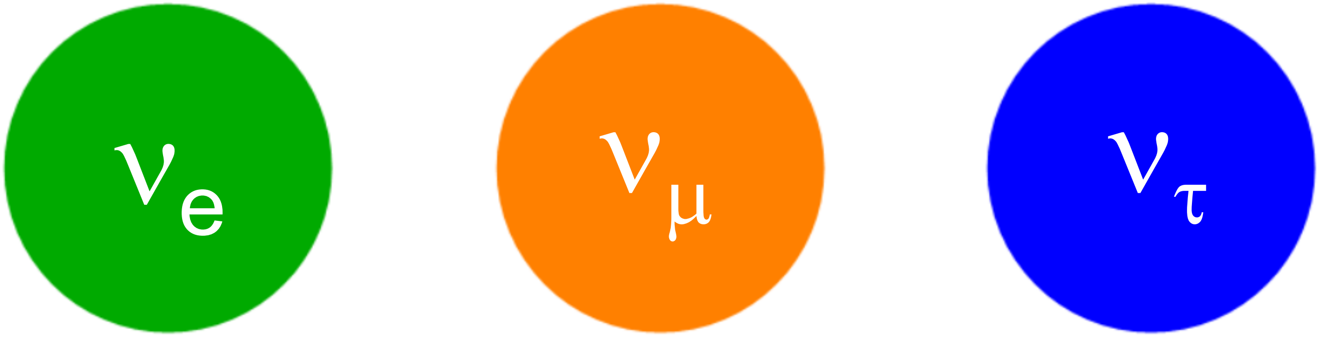
https://arxiv.org/abs/2211.02666

Neutrino Oscillations

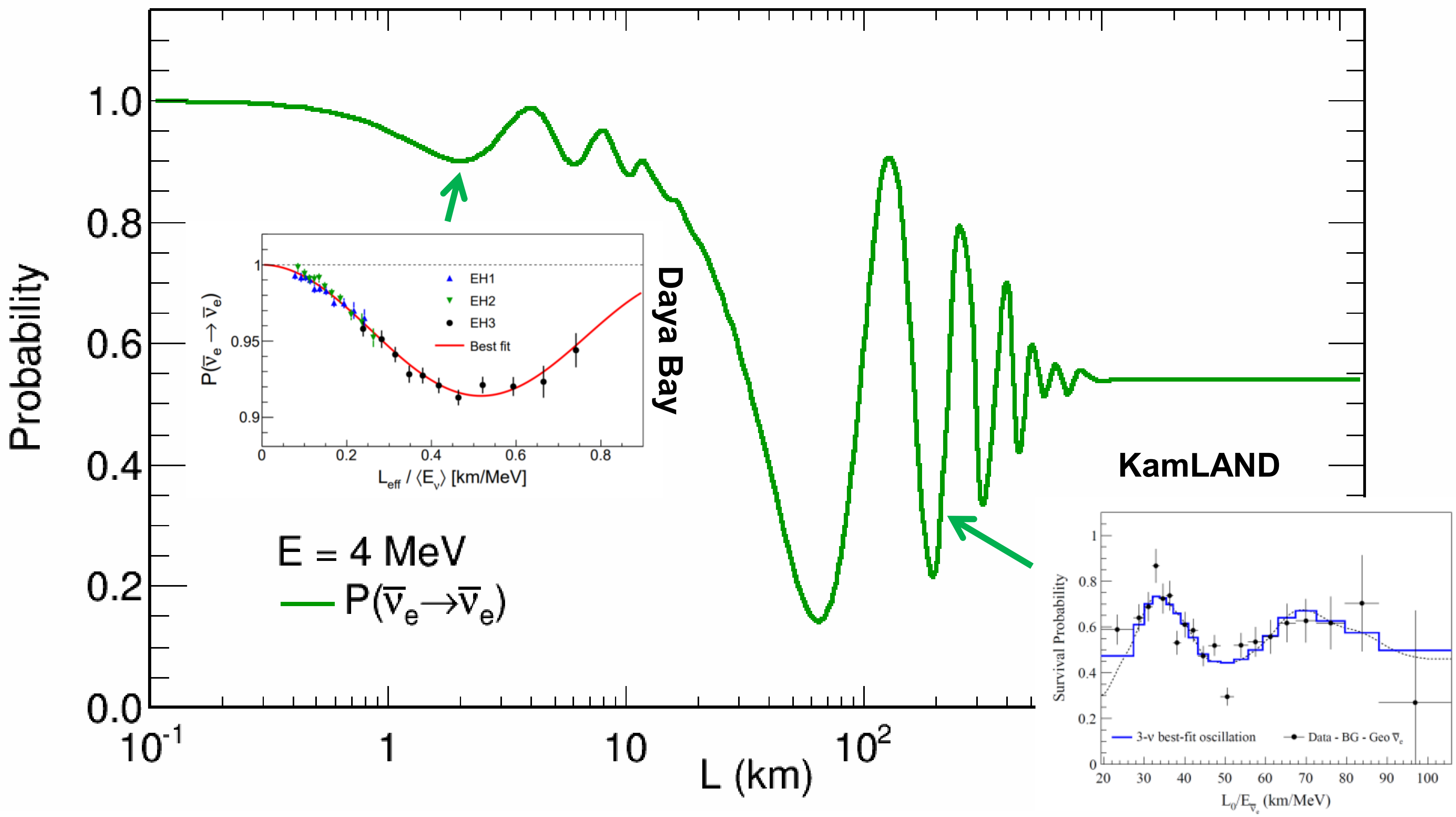
- Neutrinos are created in a superposition of mass states
- Time evolution generates flavour oscillations

Quantum Mechanics

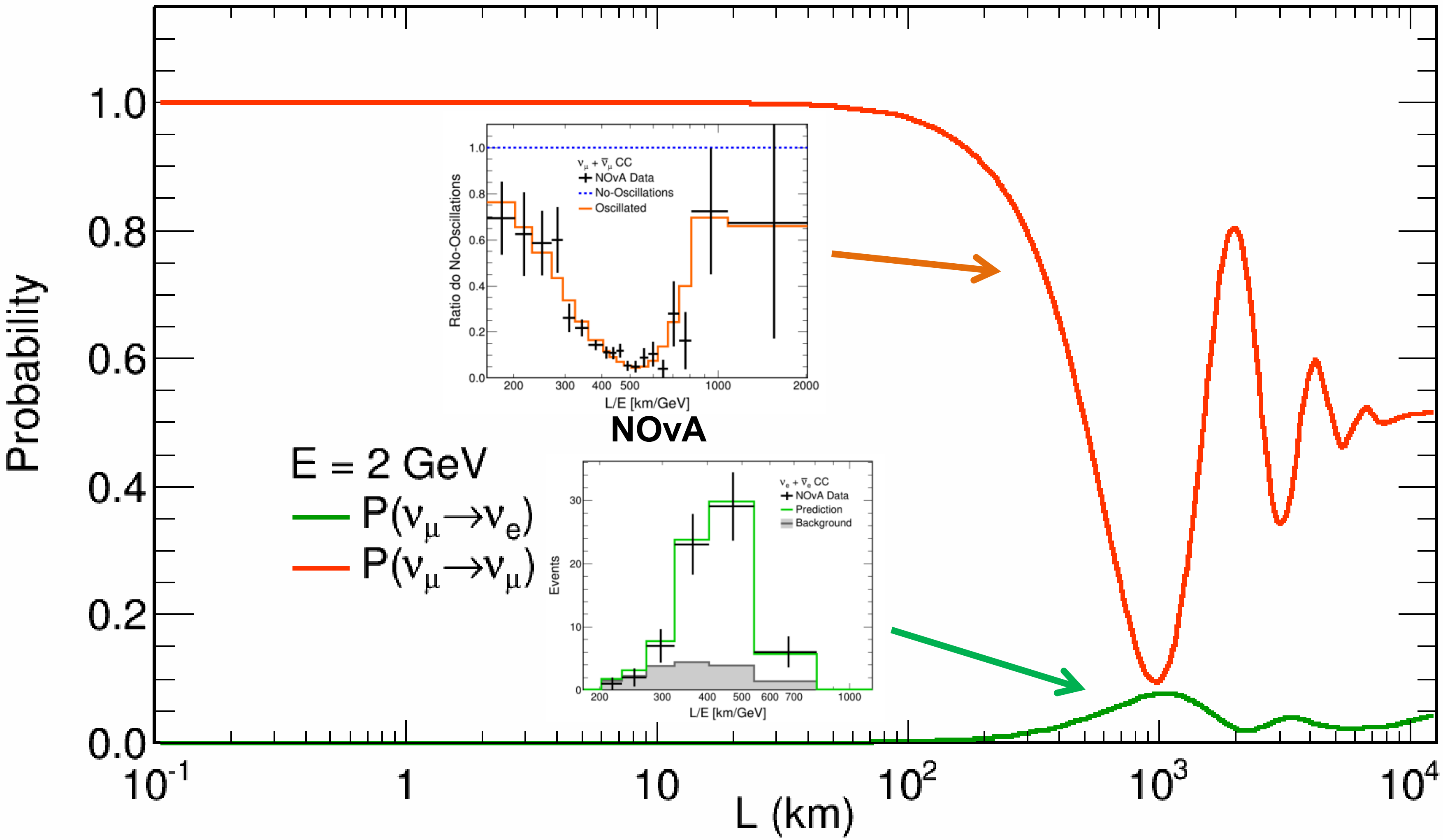




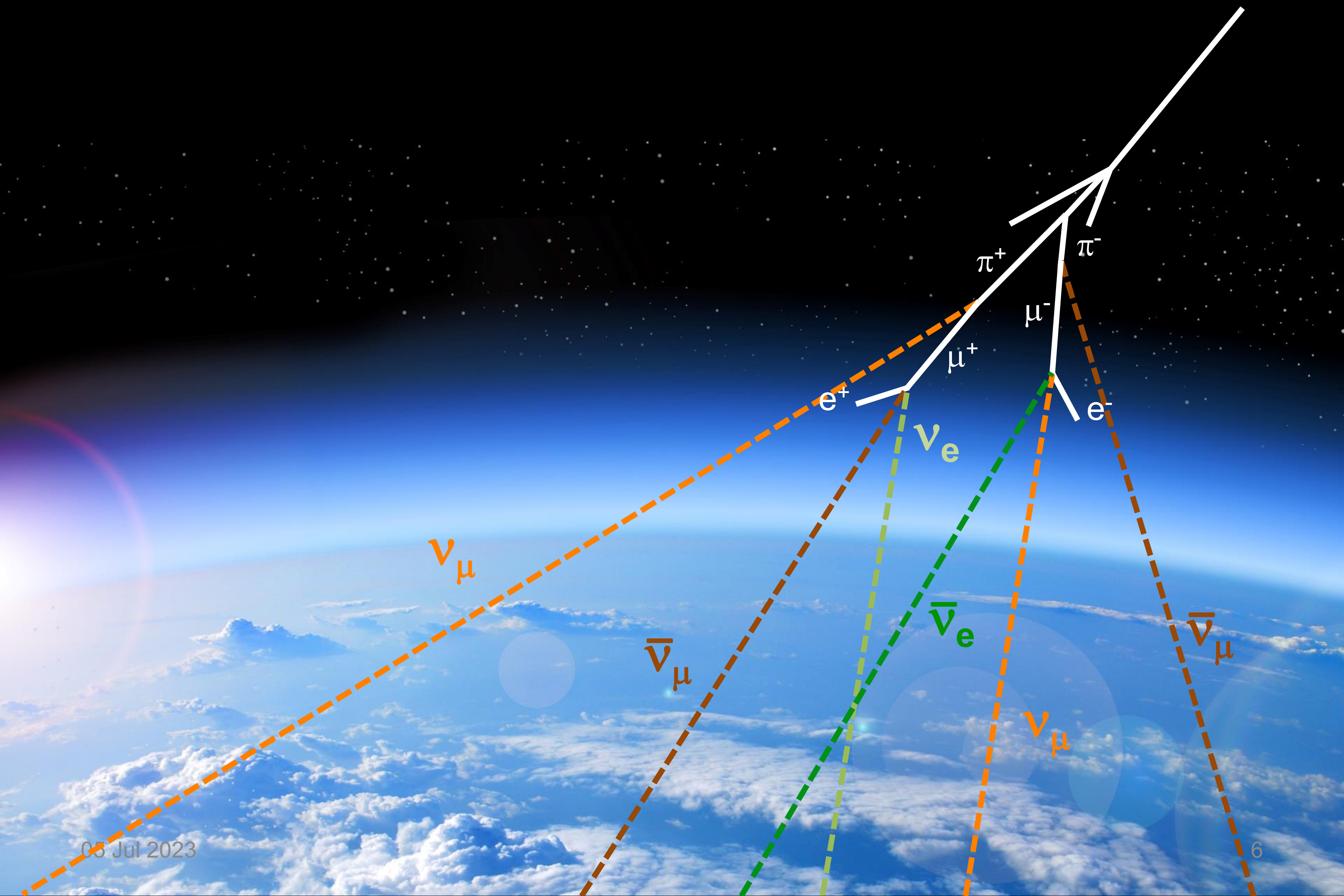
The Data: Reactor Neutrinos



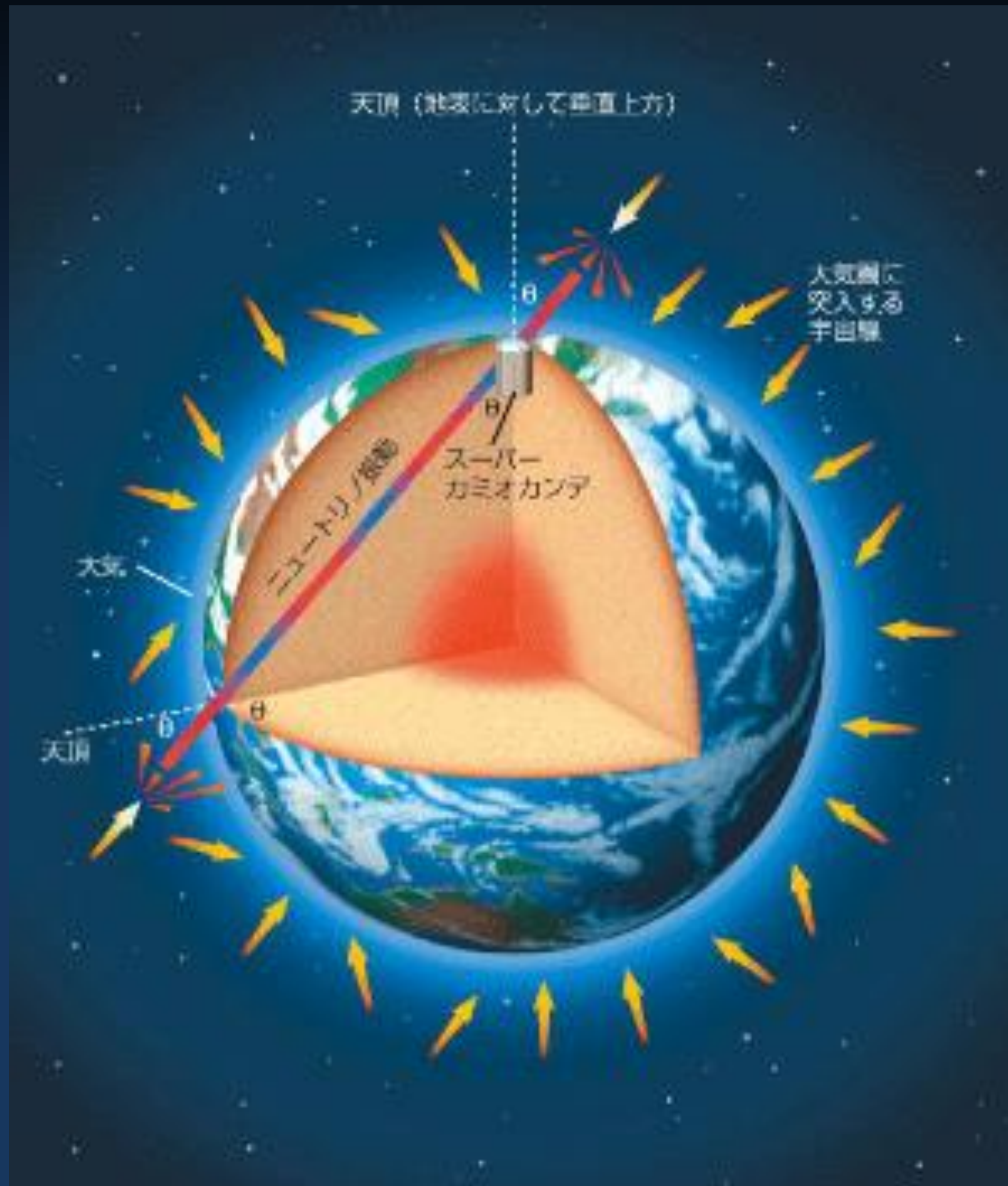
The Data: Accelerator Neutrinos



Atmospheric Neutrinos

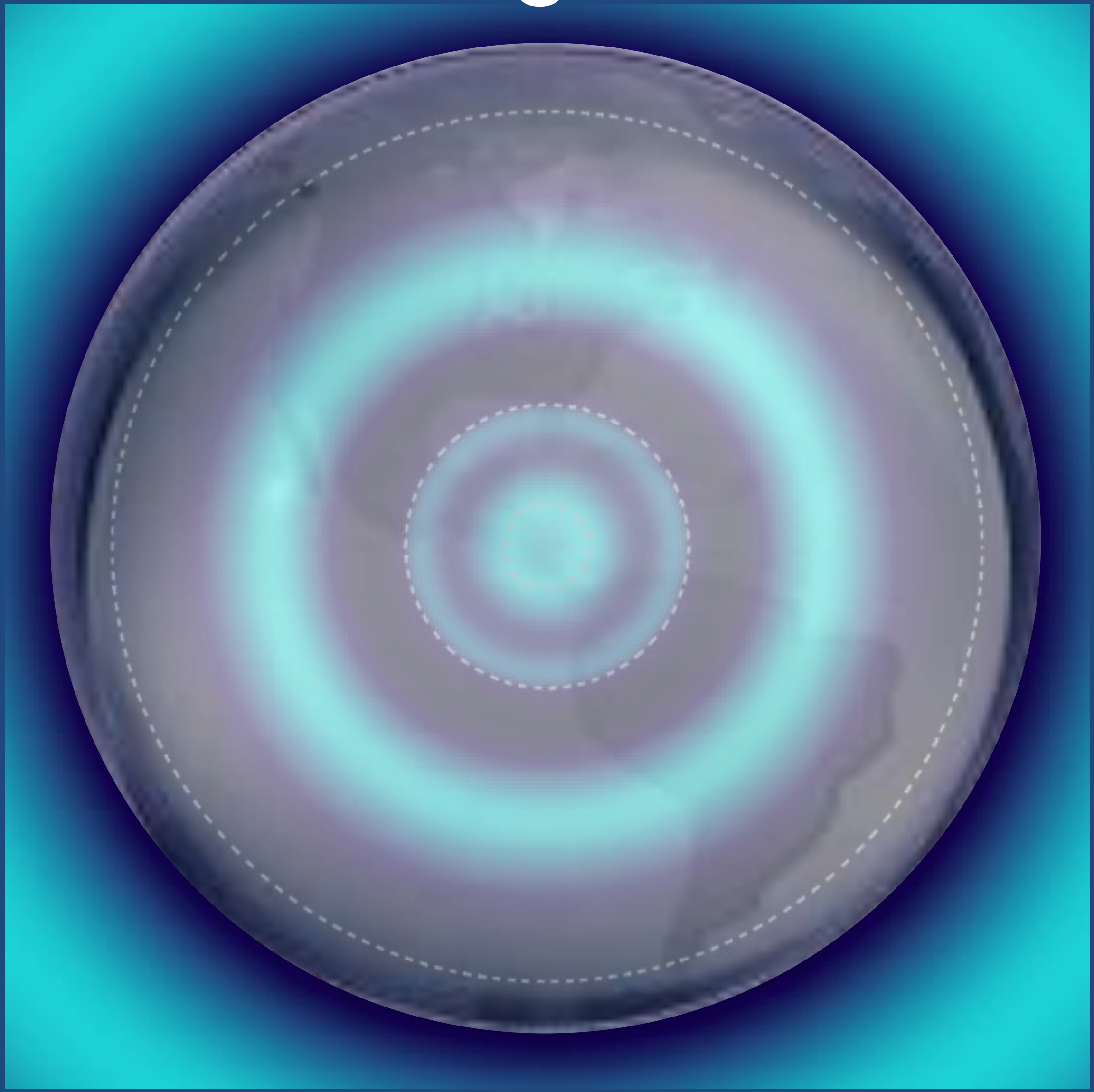


Atmospheric Neutrinos



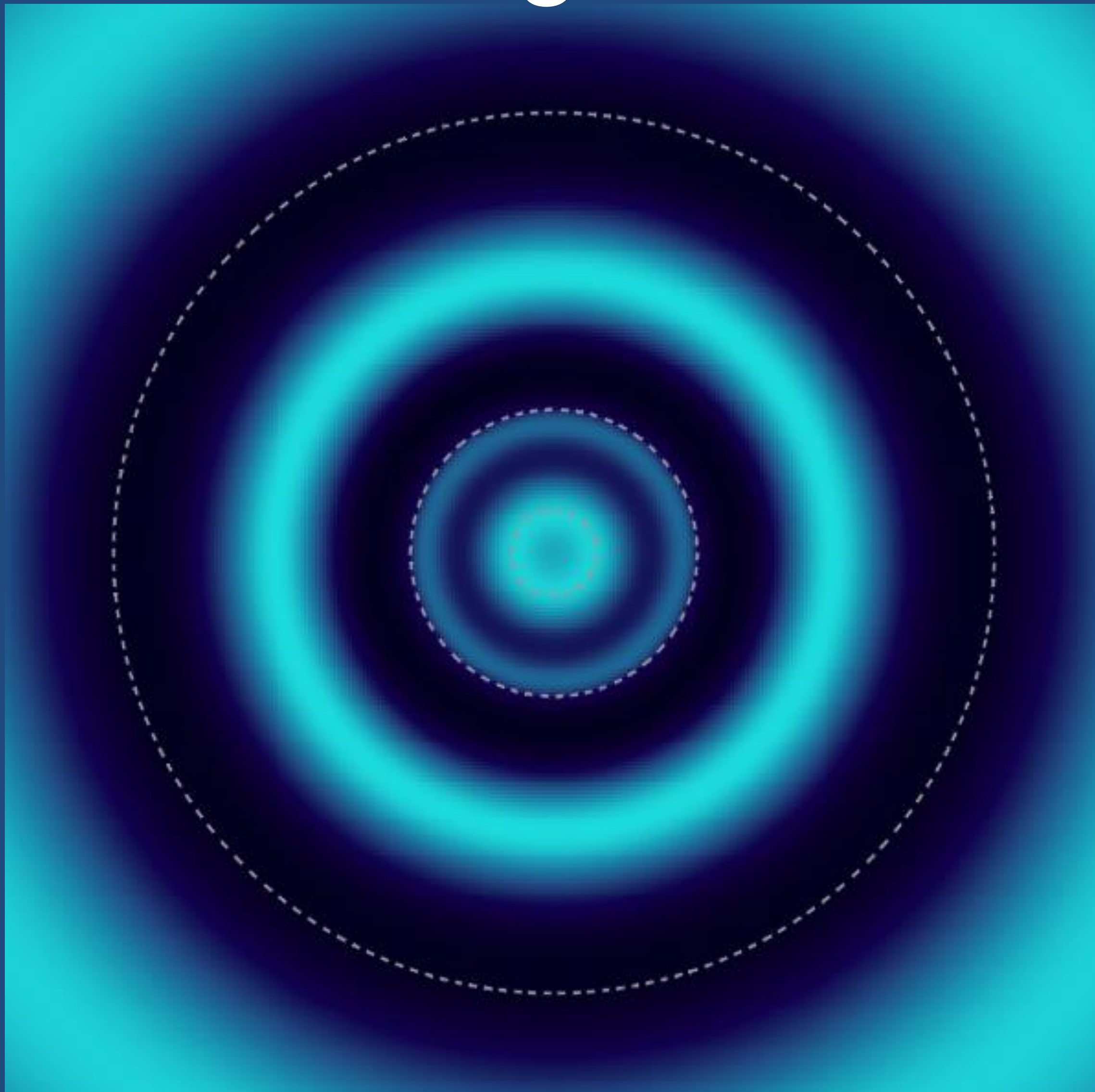
Looking Down for Neutrinos

Looking Down for Neutrinos

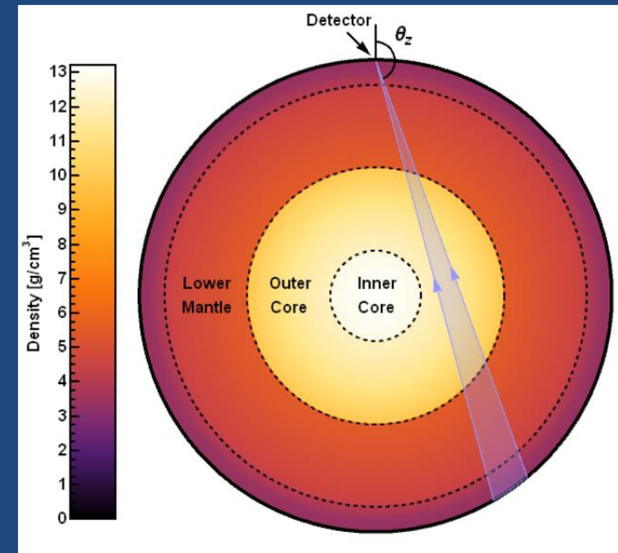


Muon neutrinos at 4 GeV

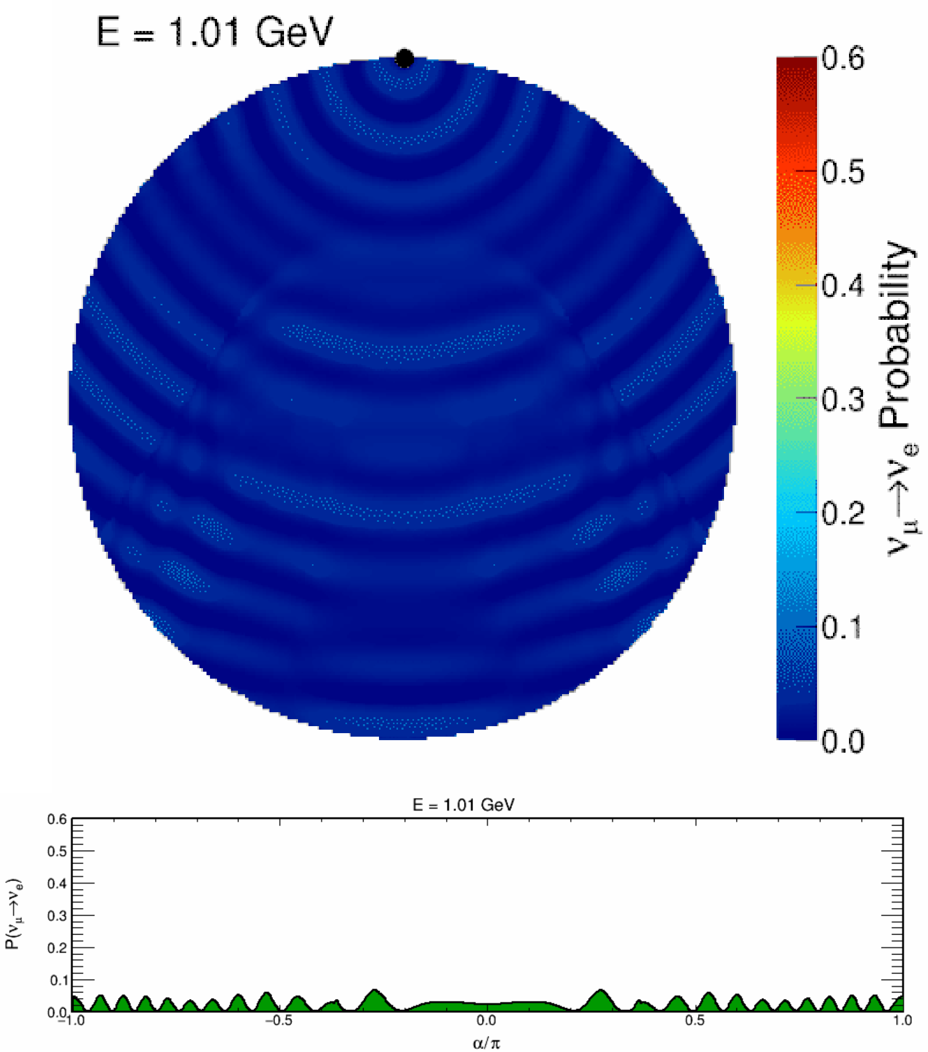
Looking Down for Neutrinos



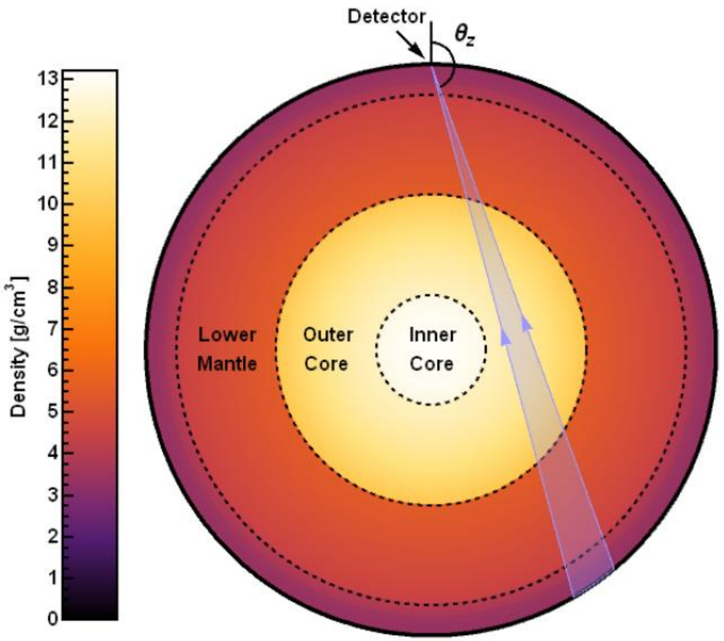
Muon neutrinos at 4 GeV



Atmospheric Neutrinos



Nue appearance at the surface

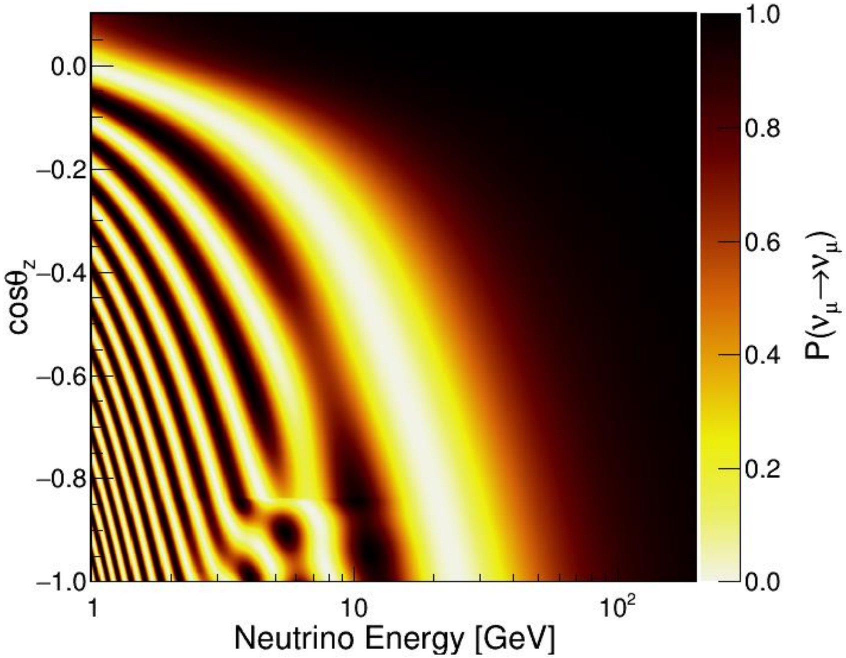


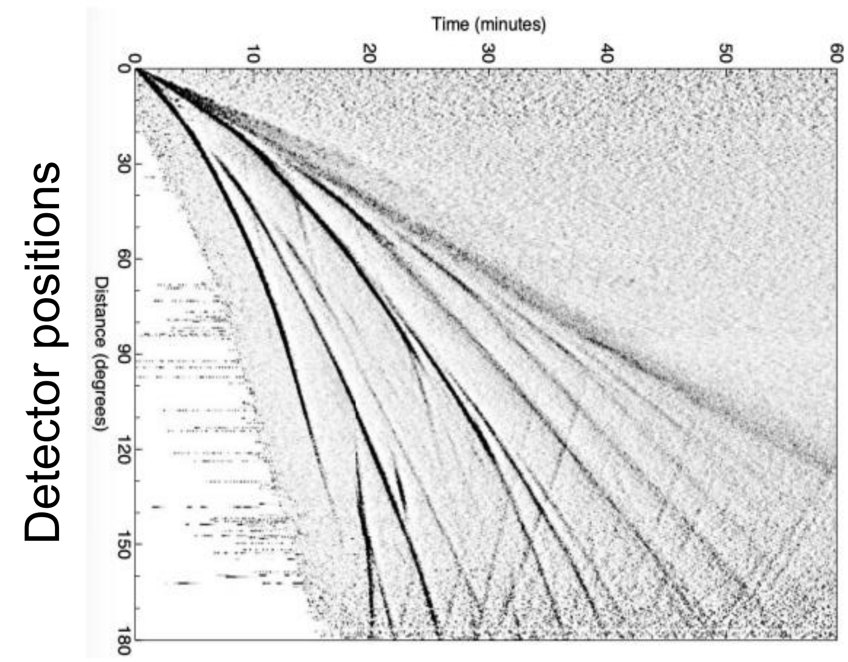
Oscillations are **resonant** at certain energies

E_{res} ~ 7 GeV in Mantle
E_{res} ~ 3 GeV in Core

Time is Energy

- Neutrino oscillograms are not so different from travel time curves
- Seismic waves have many detectors. Neutrinos many sources.
- The question is how do we go from these to an Earth model?





But Data is Hard

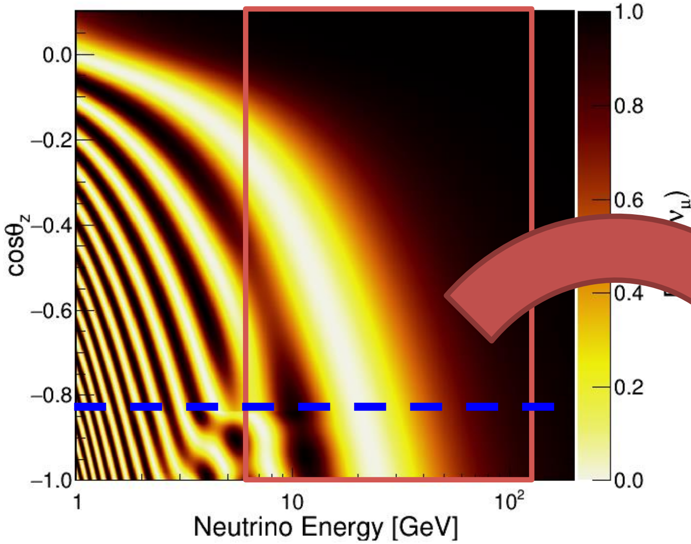
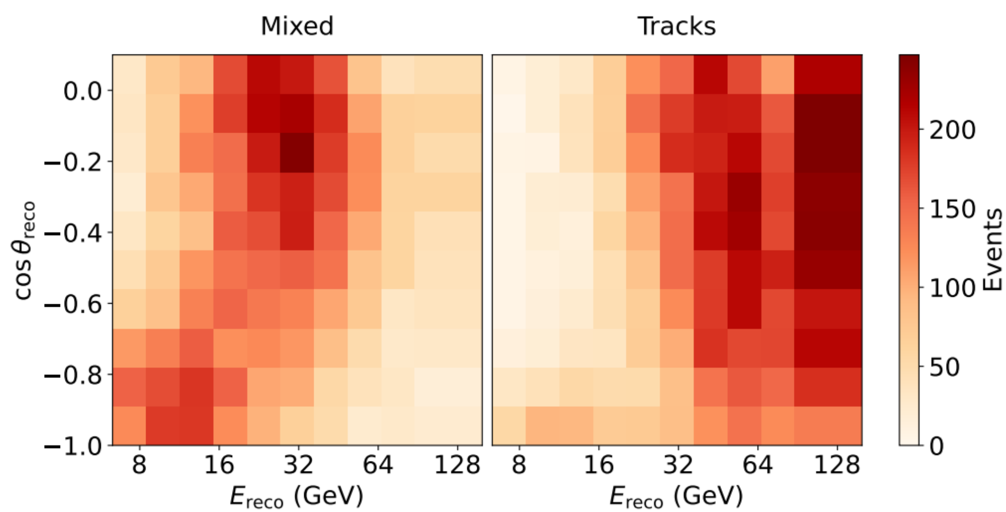


TABLE V. Best-fit number of events with 7.5 years of livetime for each neutrino flavor and interaction type, as well as atmospheric μ, along with the observed counts from the data. The rate is also given for comparison to other experiments.

Type	Events	Rates $[1/10^6 s]$
$\nu_\mu + \bar{\nu}_\mu$ CC	17656	75.03
$\nu_e + \bar{\nu}_e$ CC	1820	7.74
$\nu_\tau + \bar{\nu}_\tau$ CC	603	2.56
$\nu_{all} + \bar{\nu}_{all}$ NC	1222	5.19
Atmospheric μ	711	3.02
Total (best-fit)	22012	93.54
Observed	21914	93.08

Forward problem: Generate prediction
Already computationally expensive



An inverse problem
We don’t usually do this!

FIG. 17. Observed number of data events in the analysis binning for the full 8 years of livetime.

Double Inversion?

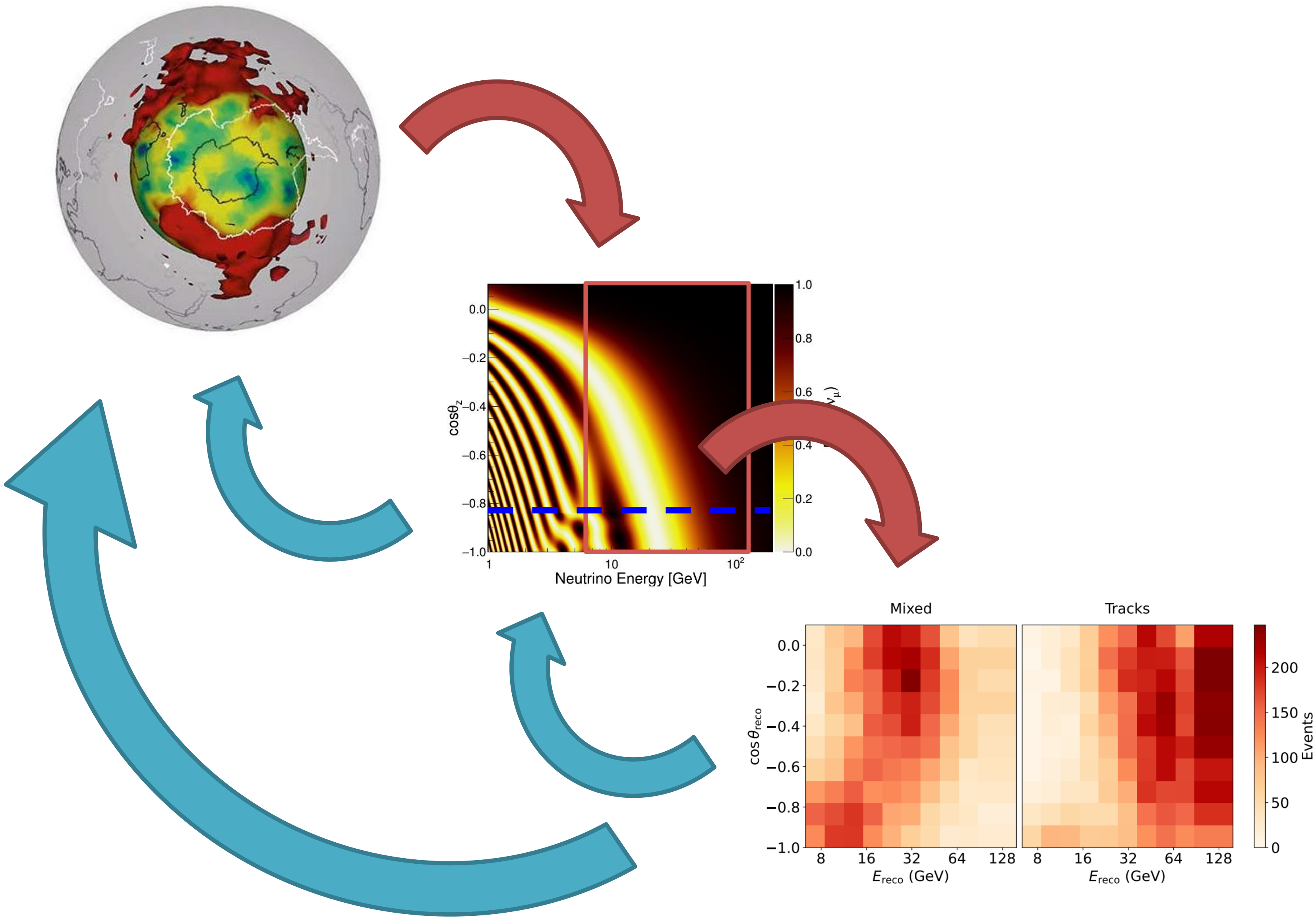
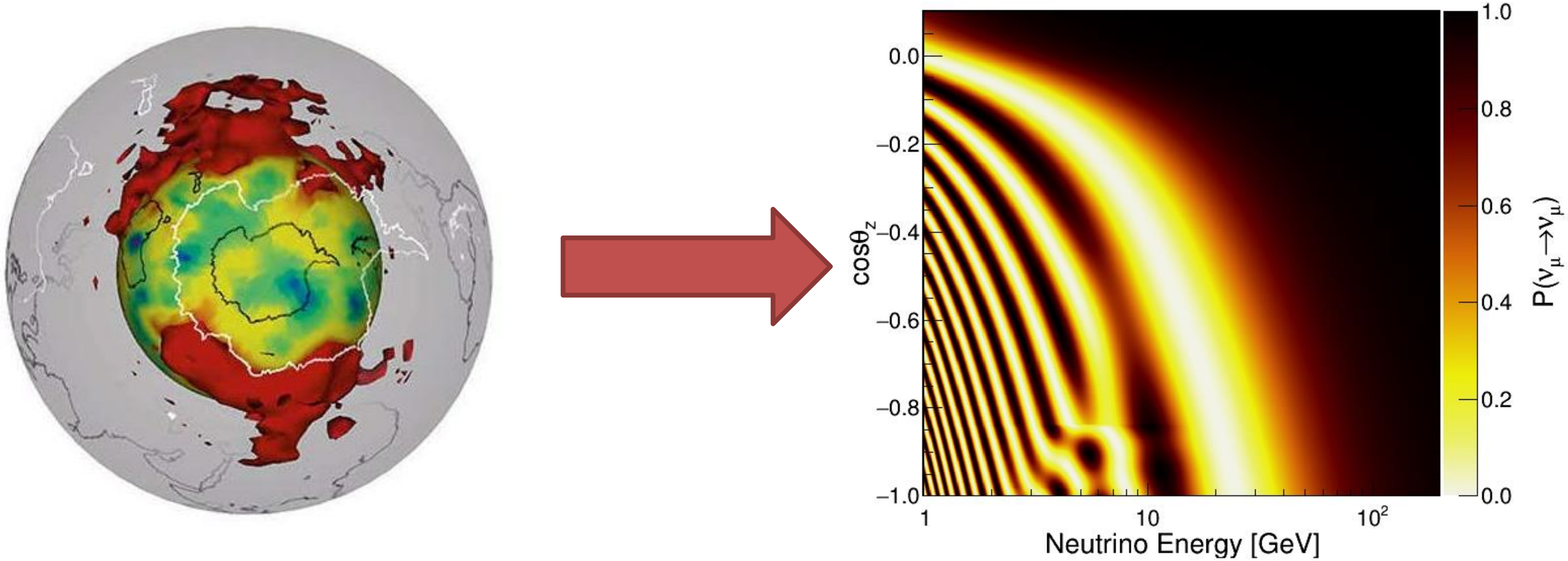


FIG. 17. Observed number of data events in the analysis binning for the full 8 years of livetime.

Forward Pass 1



Quantum Evolution

Schrödinger: $i\frac{\partial}{\partial t}\mathcal{U} = H\,\mathcal{U}$

$$\mathcal{P}_{\alpha\to\beta} = |\langle\beta|\,\mathcal{U}(t)\,|\alpha\rangle|^2$$

Time-independent H:

$$\mathcal{U}(t) = e^{-iHt}$$

$$H = V H_D V^\dagger$$ ← **Main problem**

$$\mathcal{U}(t) = V e^{-iH_D t} V^\dagger$$ ← **Easy to compute**

$$V\,H_D\,V^\dagger$$

V: **Eigenvectors**; H_D: **Eigenvalues**

Neutrino Hamiltonian in Vacuum

PMNS Matrix = Vacuum Eigenvectors

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{CP}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{CP}} & 0 & c_{23} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Mass basis

$$H_0 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{\Delta m^2_{21}}{2E} & 0 \\ 0 & 0 & \frac{\Delta m^2_{31}}{2E} \end{bmatrix}$$

Flavour basis

$$H = U H_0 U^\dagger$$

Easy to solve

Already know diagonalisation

(Eigenvectors and Eigenvalues)

Neutrino Hamiltonian in Matter

$$H_{eff} = U \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{\Delta m^2_{21}}{2E} & 0 \\ 0 & 0 & \frac{\Delta m^2_{31}}{2E} \end{bmatrix} U^\dagger + V_e \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

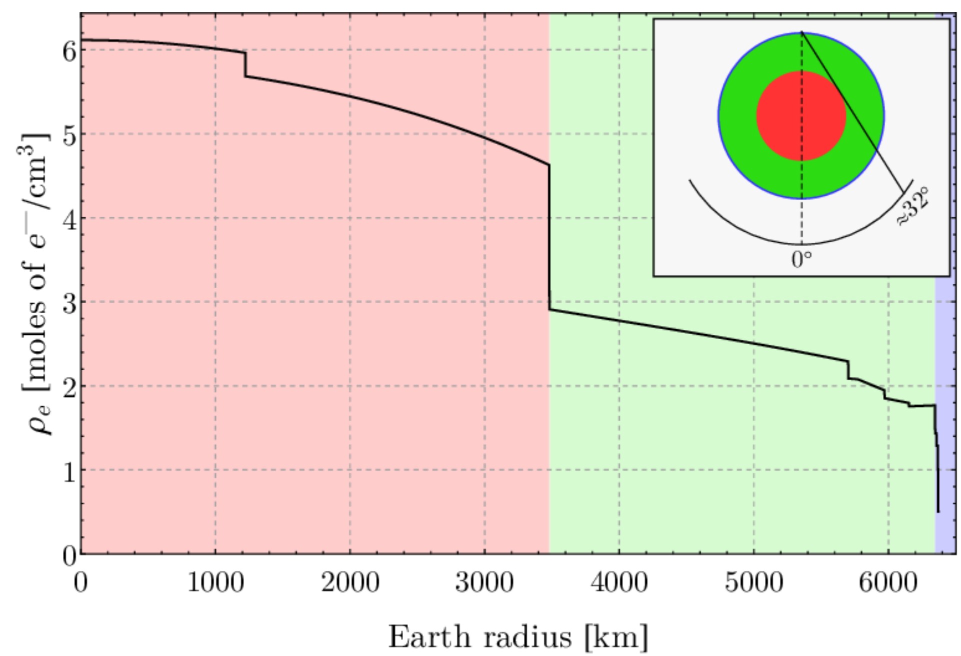
Time Dependent

$V_e \equiv \pm\sqrt{2} G_F n_e$ (+ for ν, – for $\overline{\nu}$)

Relevant when $V_e \gtrsim \frac{\Delta m^2}{2E} \sim 1/L$

In general, $1/V_e \sim 1700$ km $\times \left(\frac{3 \text{ g/cm}^3}{\rho}\right)$

Comparable to $\frac{\Delta m^2_{31}}{2E}$ for $E \sim 10$ GeV



Quantum Evolution

Schrödinger: $i\frac{\partial}{\partial t}\mathcal{U} = H\,\mathcal{U}$

$$\mathcal{P}_{\alpha\to\beta} = |\langle\beta|\mathcal{U}(t)\,|\alpha\rangle|^2$$

Time-independent H:

$$\mathcal{U}(t) = e^{-iHt}$$

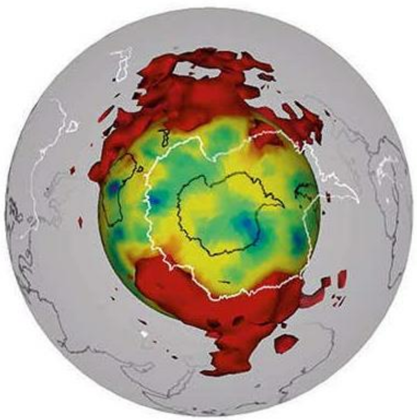
$$H = V H_D V^\dagger$$

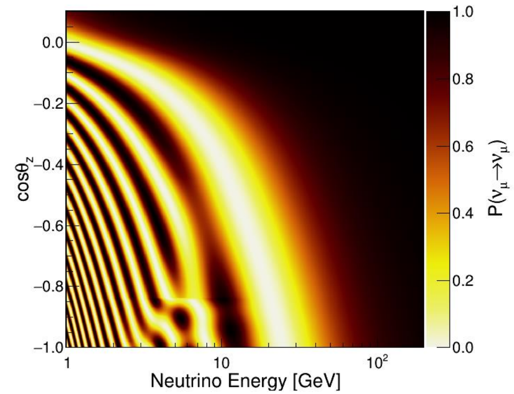
$$\mathcal{U}(t) = V e^{-iH_D t} V^\dagger$$

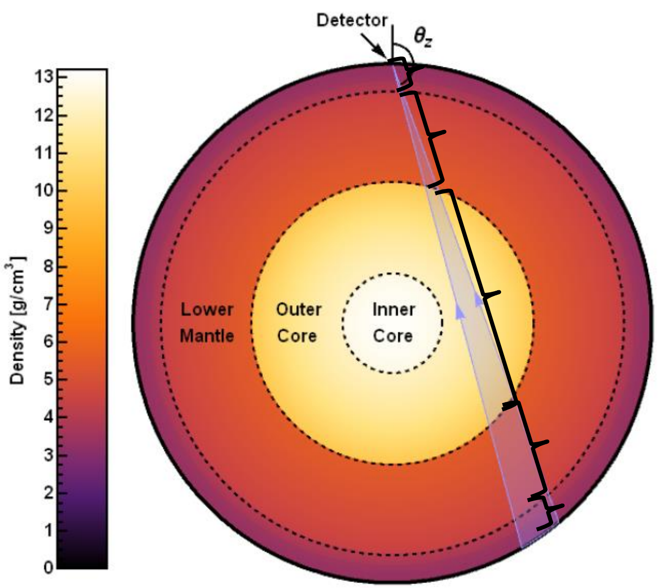
Time-dependent H:

$$\mathcal{U}(t) = \mathcal{T} e^{-i\int_0^t H(t')dt'} \approx \prod_k e^{-iH(t_k)\Delta t}$$

Forward Pass 1







- Trace neutrino path through the Earth
- Break path into N segments of similar electron density
- Compute neutrino evolution through each segment with constant density assumption

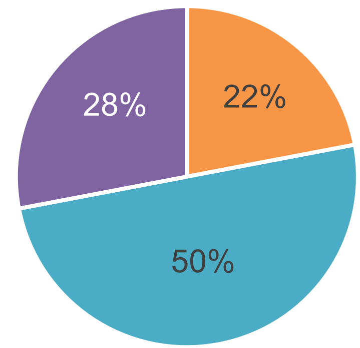
OscProb Package

- Diagonalises Hamiltonian to obtain **exact probabilities**
- Three step process:
  - **Build Hamiltonian** from parameters
  - **Solve Hamiltonian**
    - Fast algorithm from GLoBES for 3 neutrinos*
  - **Propagate neutrino** state
- Repeat for each step of constant matter in neutrino path
- **PremModel** class has built-in Earth layers model

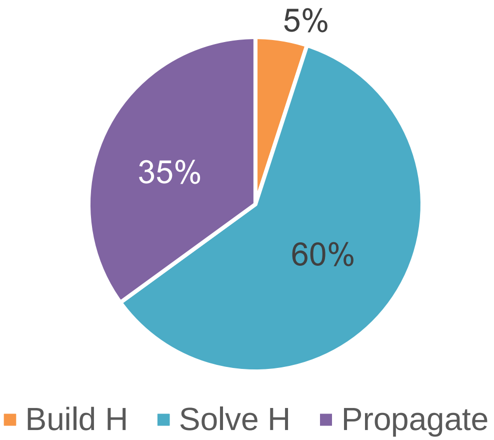
- **For a 3D model with 1M bins: ~ 1 second?**
- Probably depends on IO scalability

https://github.com/joaoabcoelho/OscProb

Single Step (2.2μs)



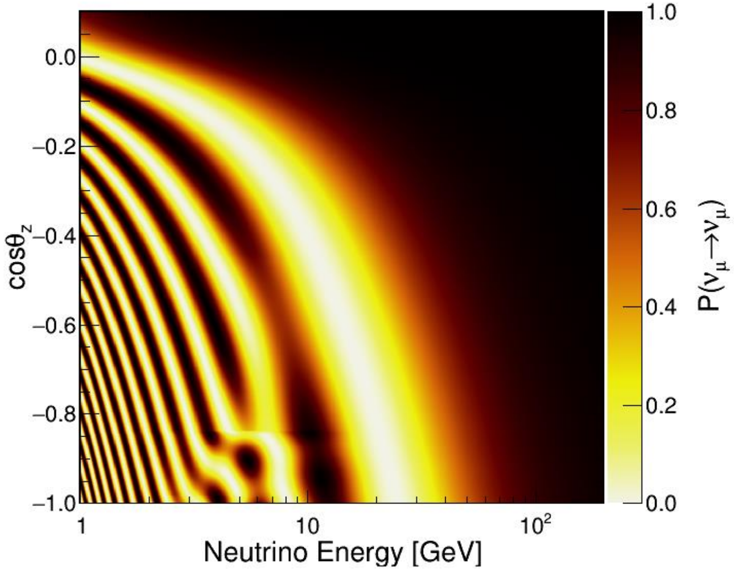
85 steps (110μs) †

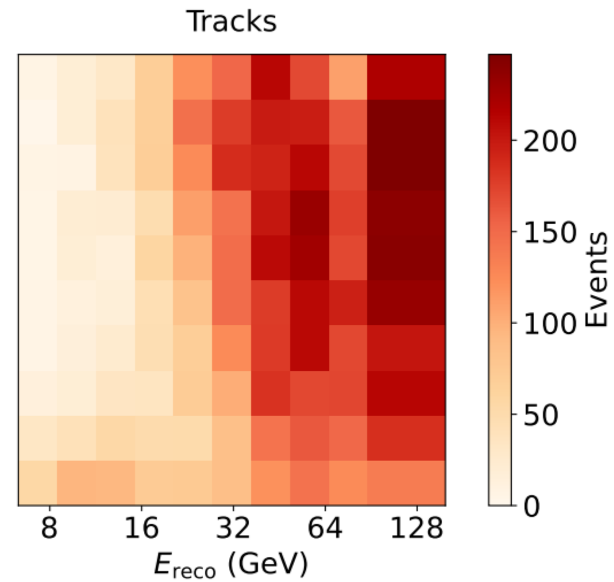


† Up-going (42+2 layers)

*J. Kopp, Int. J. Mod. Phys. C, **19**, 523 (2008)

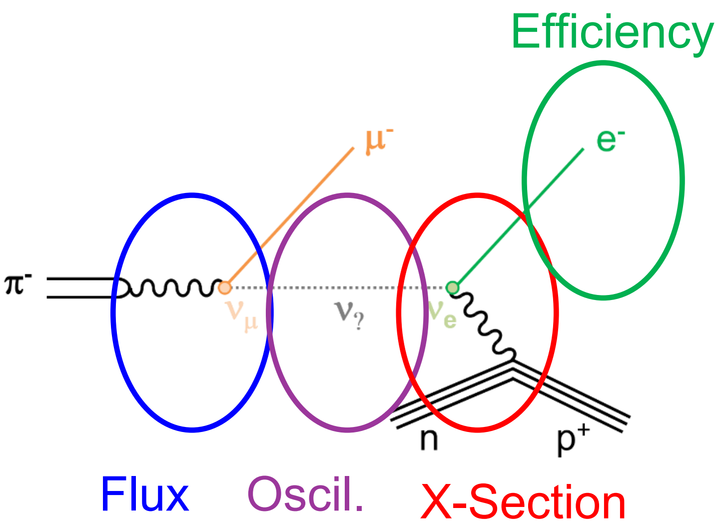
Forward Pass 2

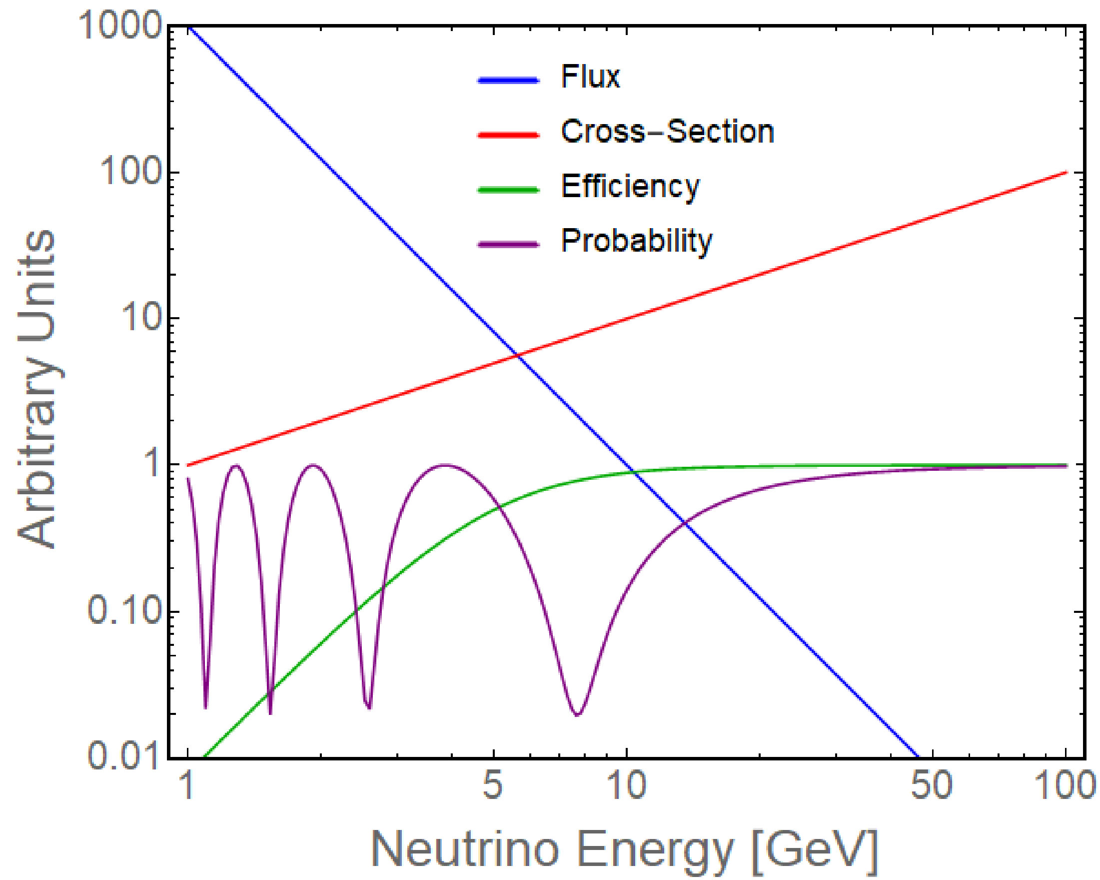




A Chain of Events

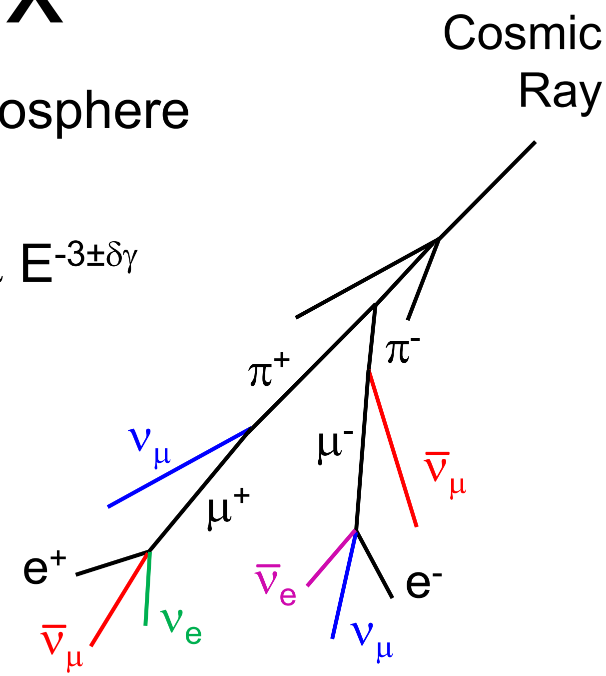
- Neutrino rates are a product of source, propagation and detection
- Rate = Flux × Oscillation Probability × Cross-Section × Efficiency
- Lets go through a toy model

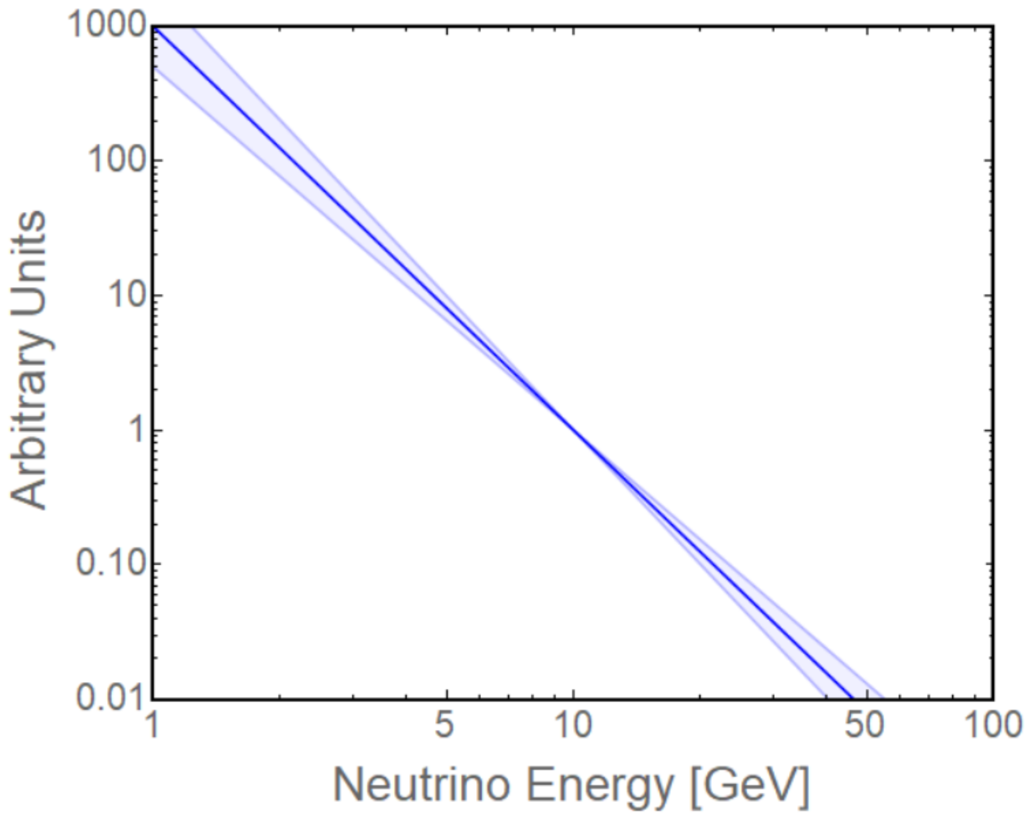


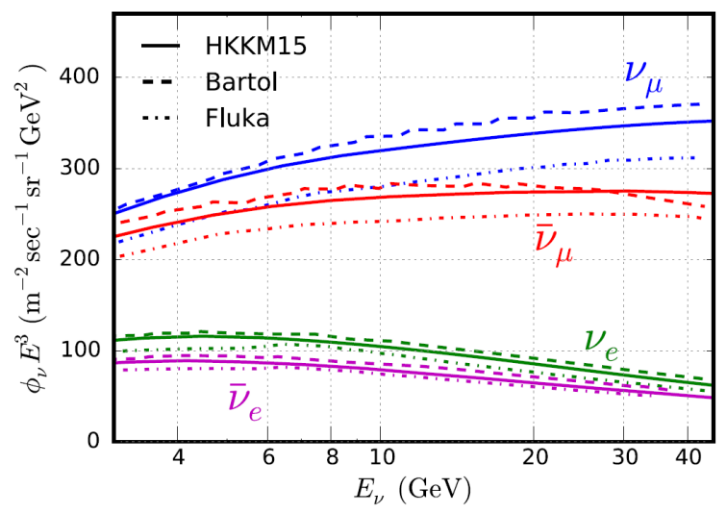


Neutrino Flux

- Neutrino flux from cosmic rays hitting the atmosphere
- Follows similar power law spectrum: $\phi \propto E^{-3}$
- For our toy model, lets add an uncertainty $\phi \propto E^{-3\pm\delta\gamma}$

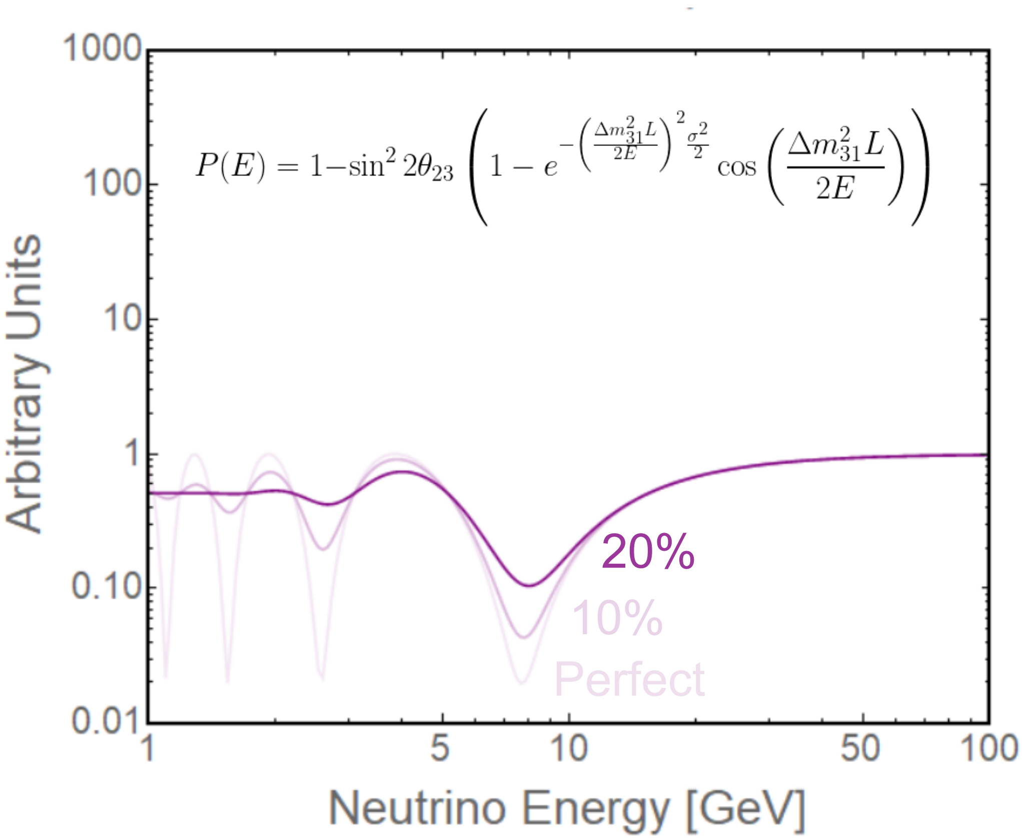




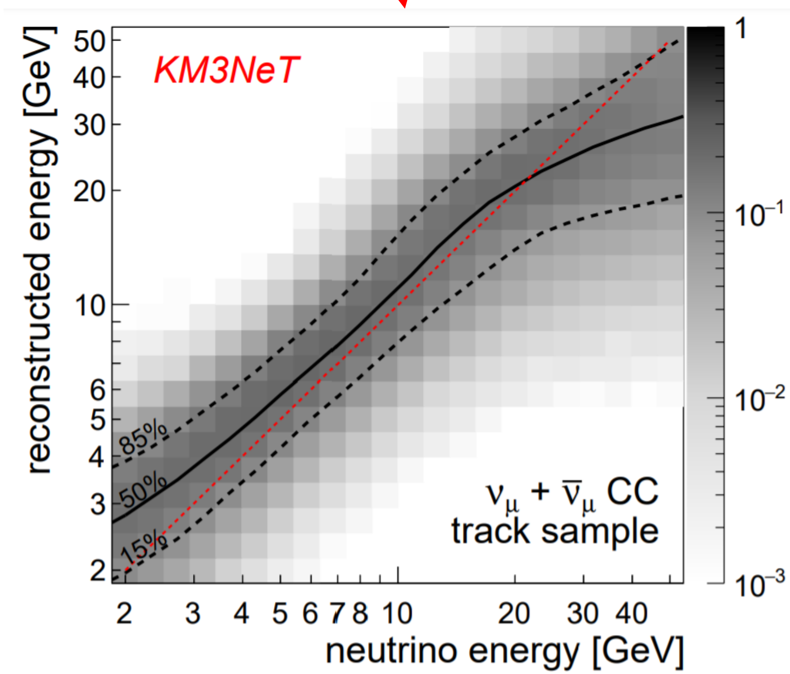


Oscillation & Resolution

- Oscillations will be our main phenomena of interest
- Energy resolution will degrade our ability to see fast oscillations
- We will implement an energy resolution of 20% at the probability level

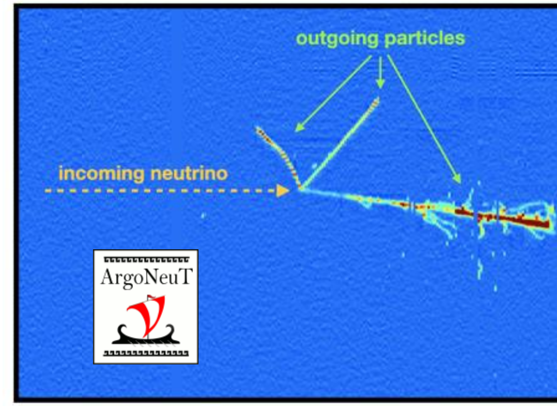


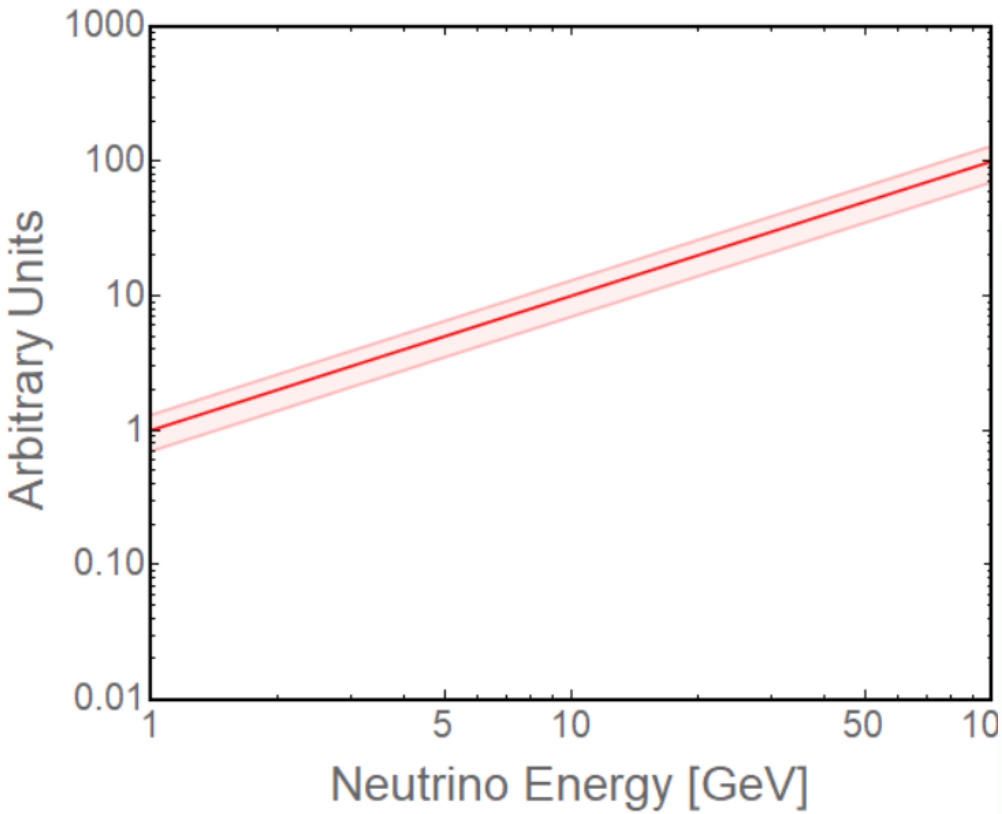
$$P(E_{reco}) = \int R(E_{reco}, E_{true})\, P(E_{true})\, dE_{true}$$



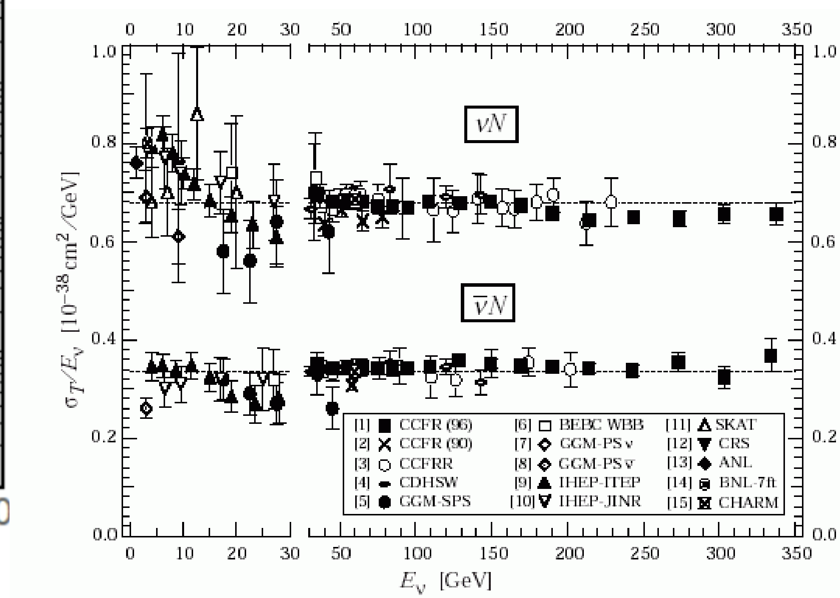
Neutrino X-Section

- Interaction rate at detector is given by the x-section
- At GeV energies, x-section is linear in energy: $\sigma \propto E$
- For our toy model, lets add an uncertainty on the normalisation



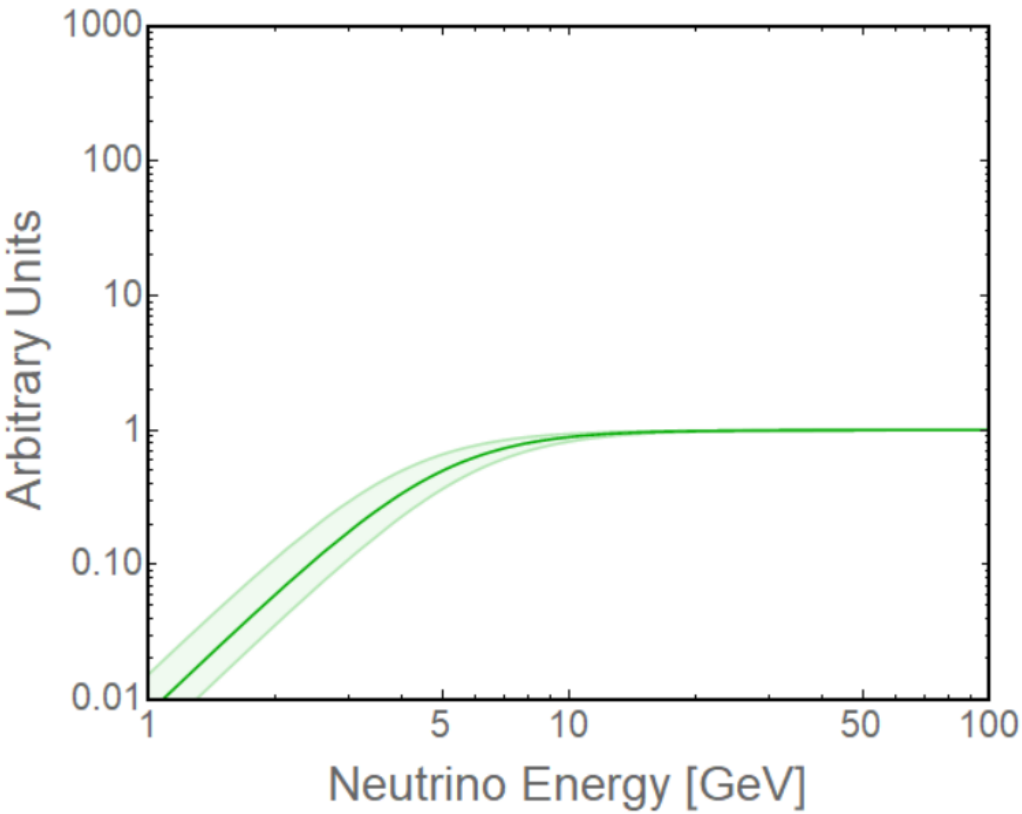


- $\nu/\bar{\nu}$ ratio is ~2 due to detector being made of matter, not antimatter

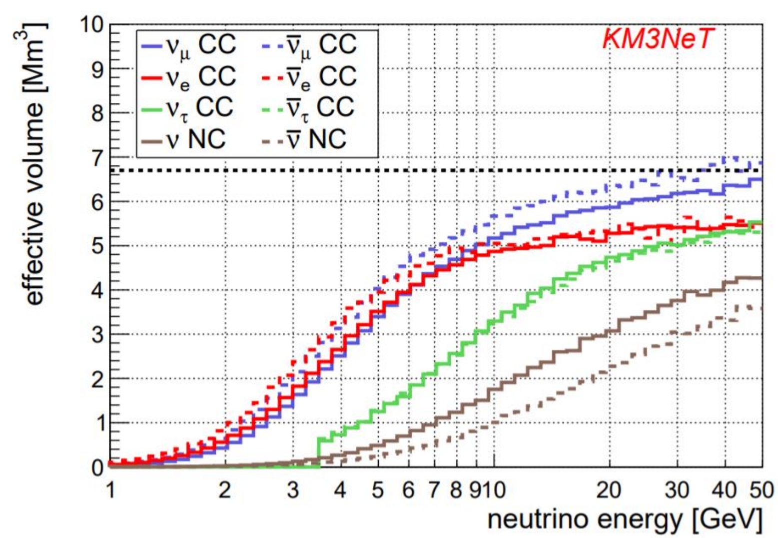


Detector Efficiency

- Detector needs to see enough light to trigger/reconstruct a neutrino
- More energy means more light. At low energies efficiency drops
- For our toy model, lets add an uncertainty on how much light we see for a given neutrino energy, i.e. what's the threshold

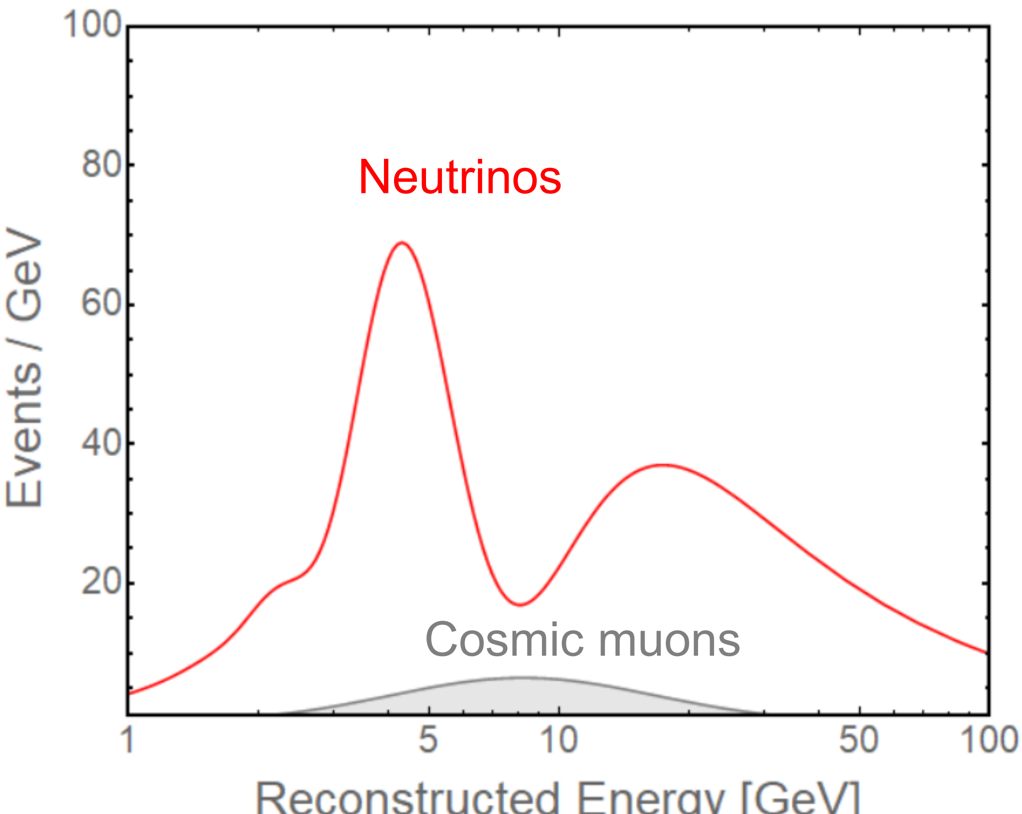


- In KM3NeT, we usually talk about the effective volume or mass
- Analogous to efficiency, but gives also number of target nuclei

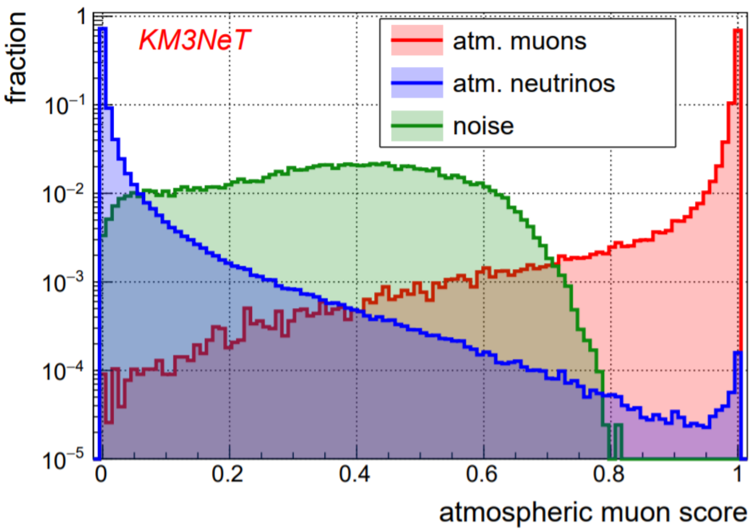


Background

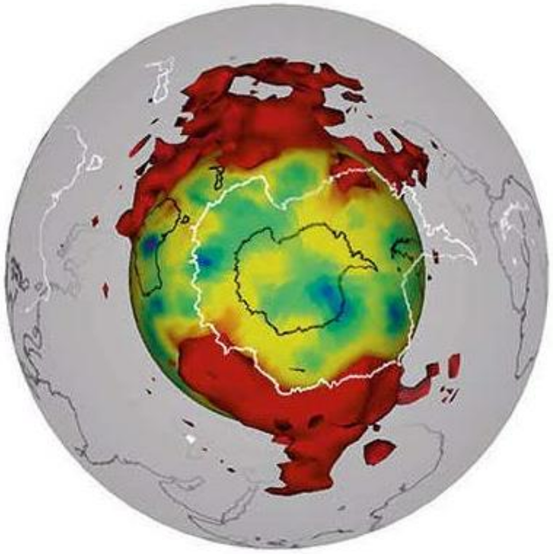
- We can now put it all together and we get an expected event rate
- Typically we will also have background sources, e.g. cosmic muons
- Lets model it as some small component added to our event rate
- In this toy, we will assume background events do not oscillate

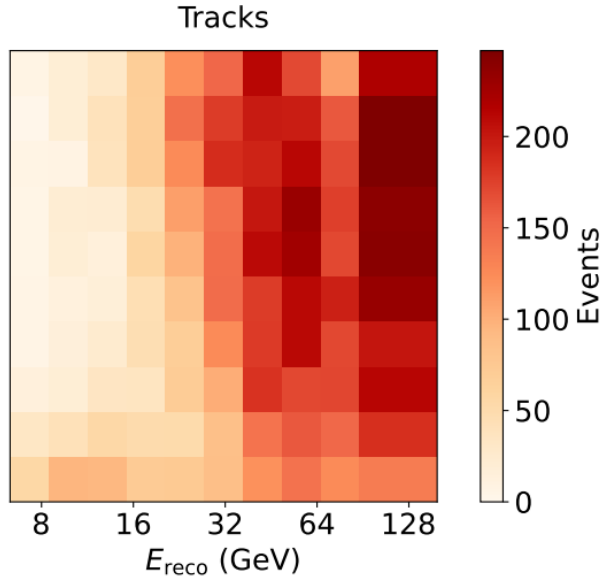


- We often use machine learning to distinguish between signal and background, and reduce the bkgd. contamination



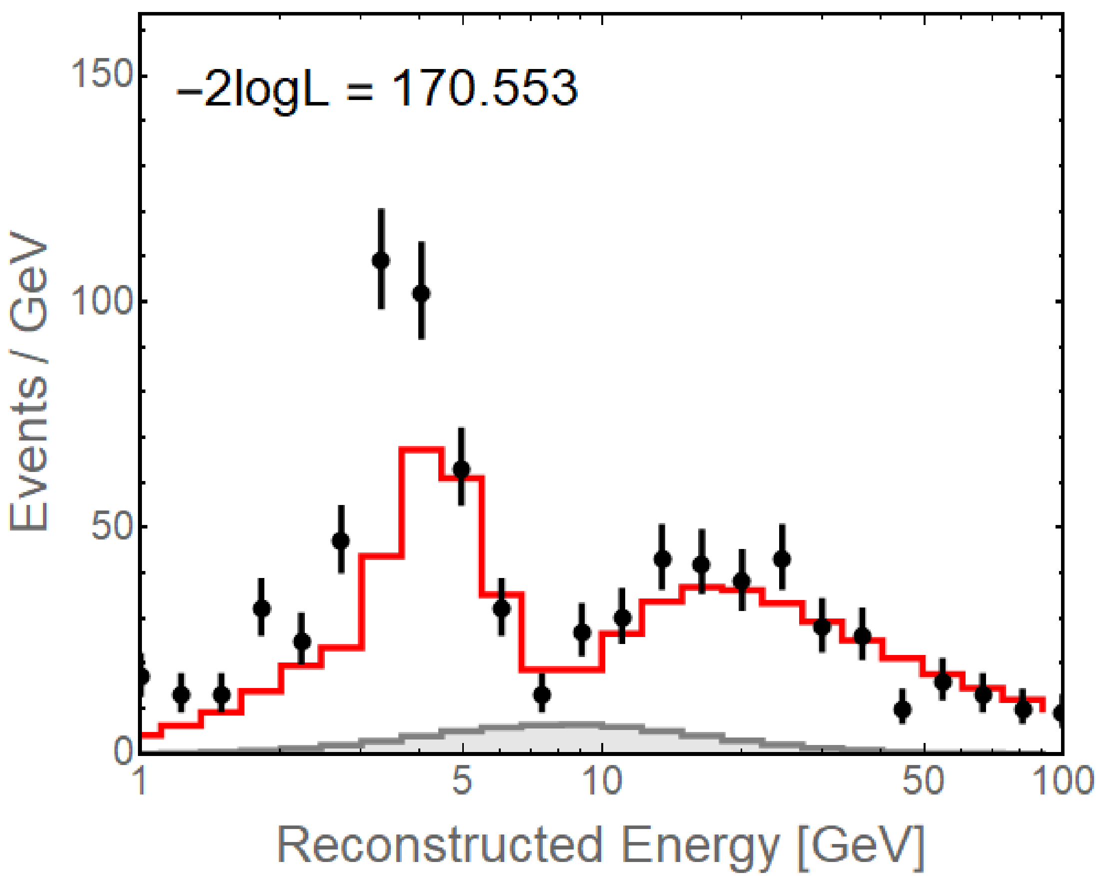
Inversion





Lets look at some data

- In general, our data doesn't agree with our predictions
- We quantify the disagreement by computing the likelihood of observing this data given the prediction we made
- Relatively simple problem when we're just counting events



Log Likelihood-Ratio

- The natural choice is to use the likelihood P(data | prediction)
- In general our metric is:

$$\lambda(data, pred) = -2 \log \left[\frac{P(data \mid pred)}{P(data \mid pred = data)_{*}} \right]$$

- If data is distributed as a Gaussian:

$$\lambda(d, p) = \chi^2 = \sum_i \frac{(p_i - d_i)^2}{\sigma_i^2}$$

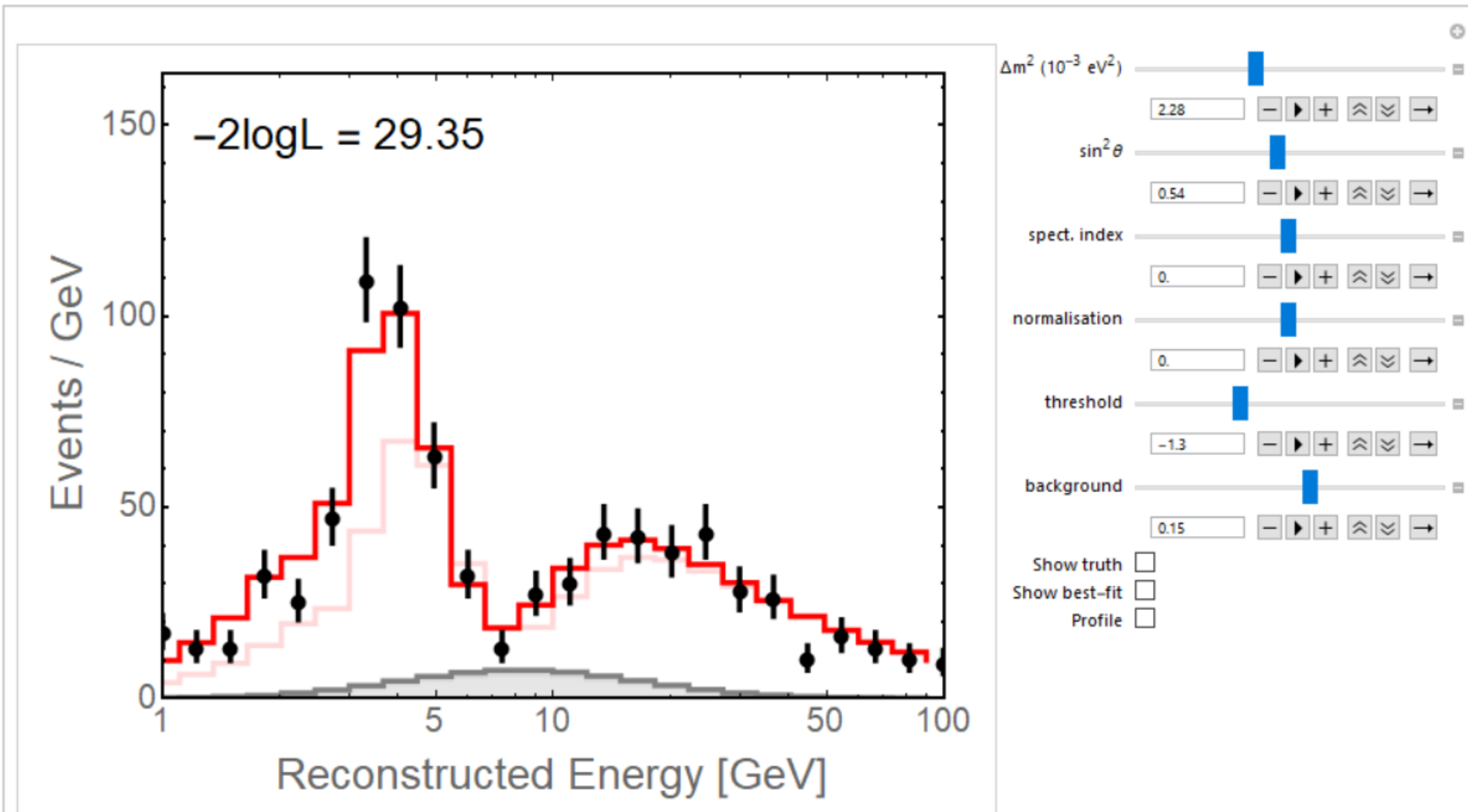
- For Poisson distributed data, this results in:

$$\lambda(d, p) = 2 \sum_i p_i - d_i + d_i \ log(d_i / p_i)$$

* Technically pred = best possible prediction, which may not be =data

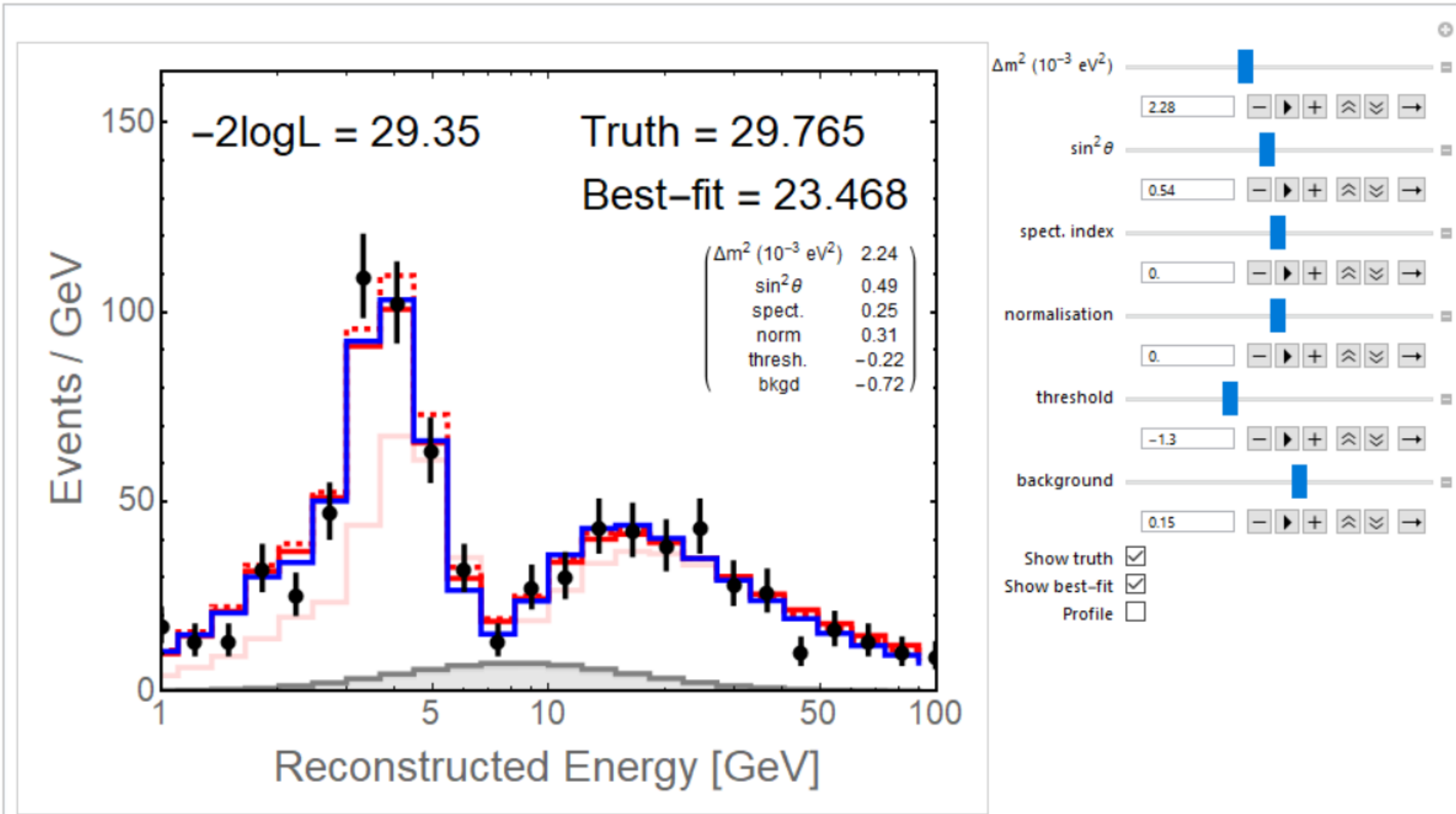
Now lets try to fix the prediction

- We can play around with multiple parameters to minimize -2logL



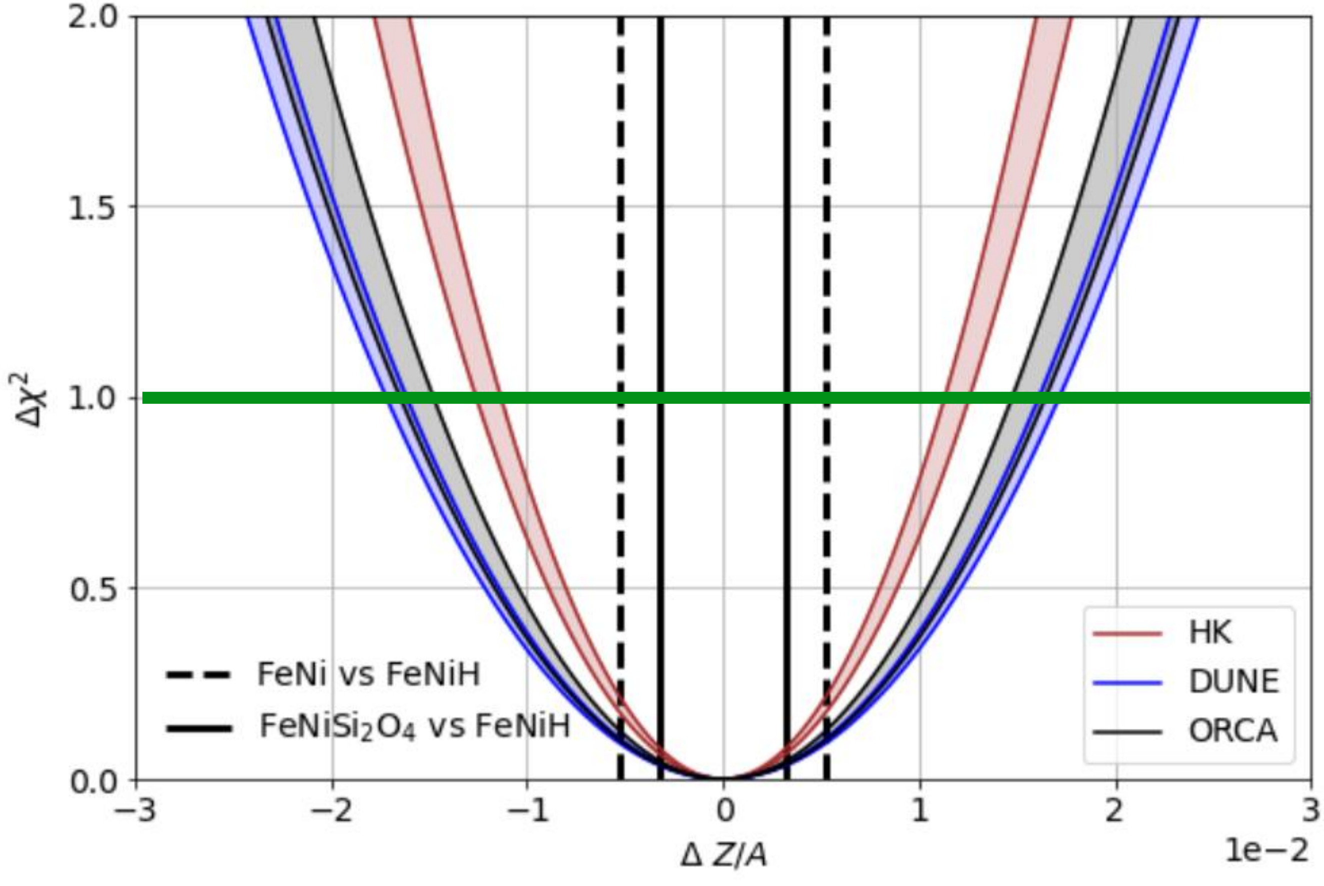
Now lets try to fix the prediction

- In practice, we use gradient descent and fit all parameters
- We can then build confidence regions around any parameter by considering what parameter values have $-2\Delta\log L < \alpha$



Now lets try to fix the prediction

- In practice, we use gradient descent and fit all parameters
- We can then build confidence regions around any parameter by considering what parameter values have $-2\Delta \log L < \alpha$

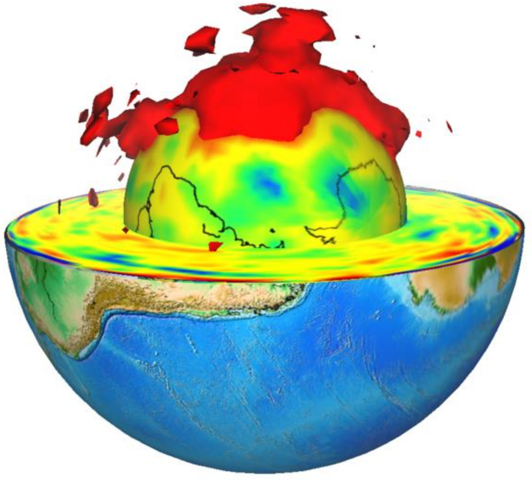


$\alpha = 1$
68% CL

The Real Challenge

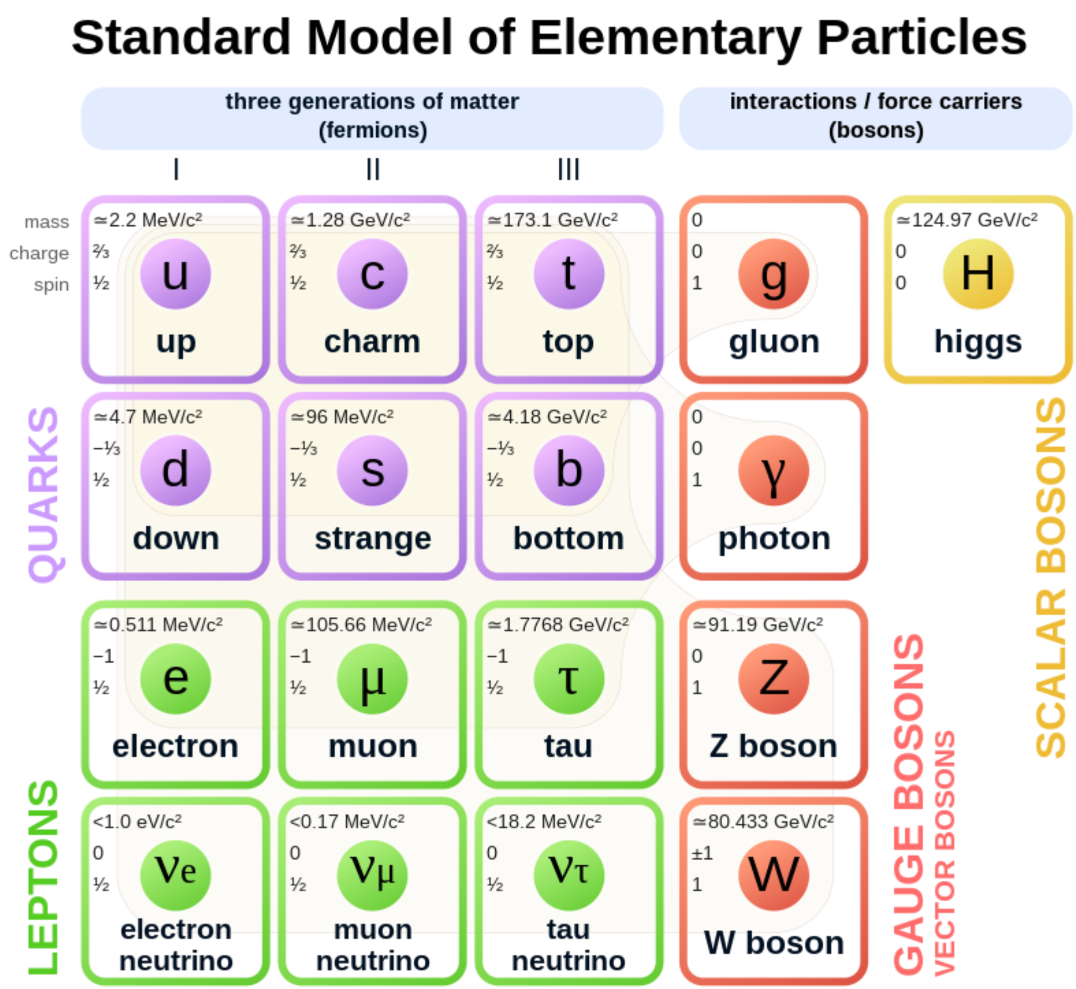
- Fitting 1M parameters is on a par with some Machine Learning problems
- Impossible to use 2nd order methods for gradient descent (Hessian too large)
- Computing the gradient requires at least 1M forward predictions **if we don't have a differentiable simulation model**
- At least 1 sec per prediction, so ~ 300 cpu-hours per gradient
- Without a Hessian, expect ~ 100 iterations to converge: **O(cpu-years)**
- And all of this is somewhat optimistic. Ignoring many other bottlenecks

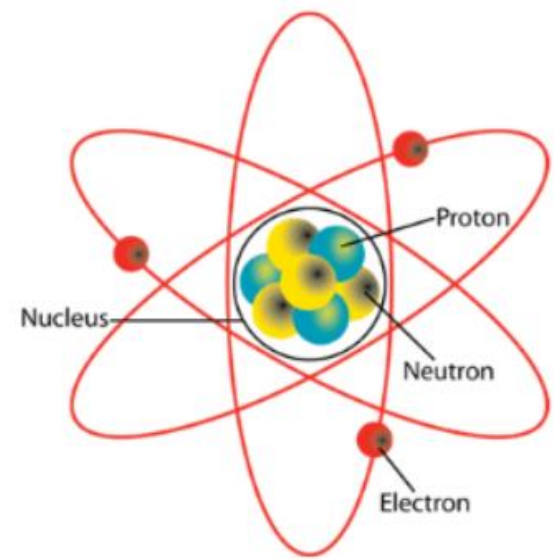
Of course, we can start with a less ambitious model of the Earth

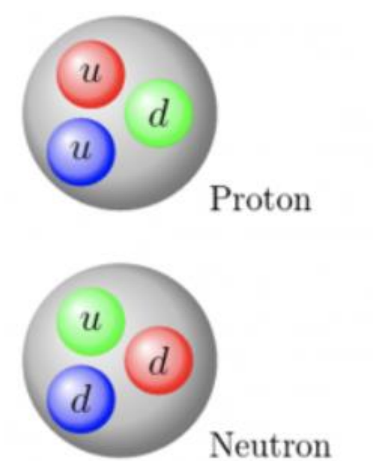


Thank you!

The Structure of the Universe

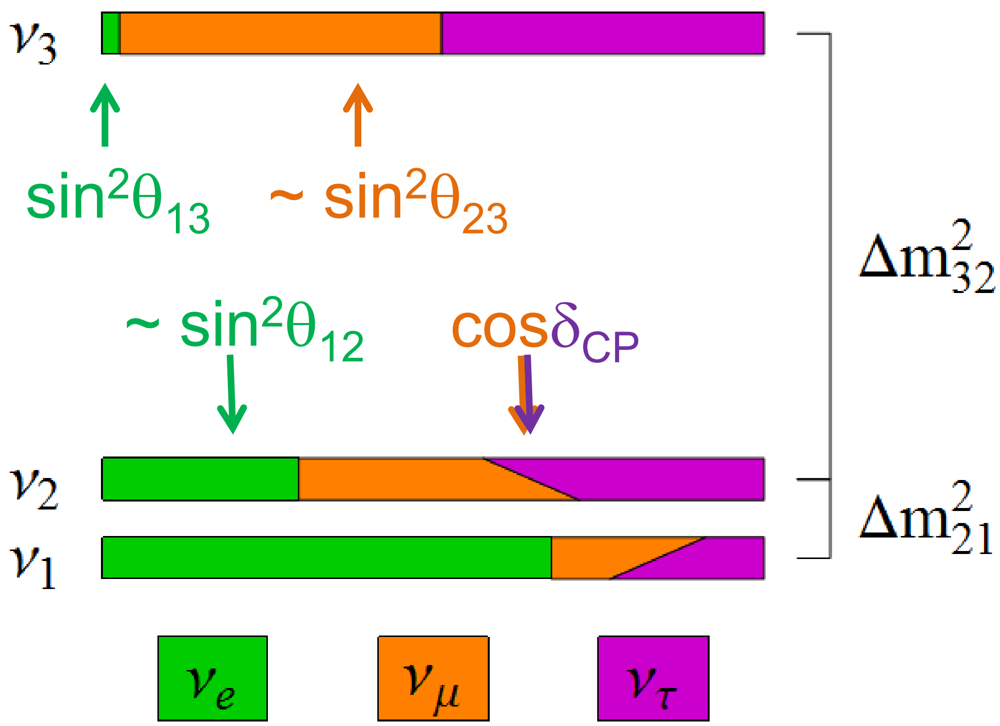






Neutrino Oscillations

- There are 3 neutrinos, so things are a bit more complicated
- Two independent differences in mass-squared (Δm^2_{21}, Δm^2_{32})
- 3 mixing angles (θ_{12}, θ_{13}, θ_{23}) and 1 CPV phase δ_{CP}

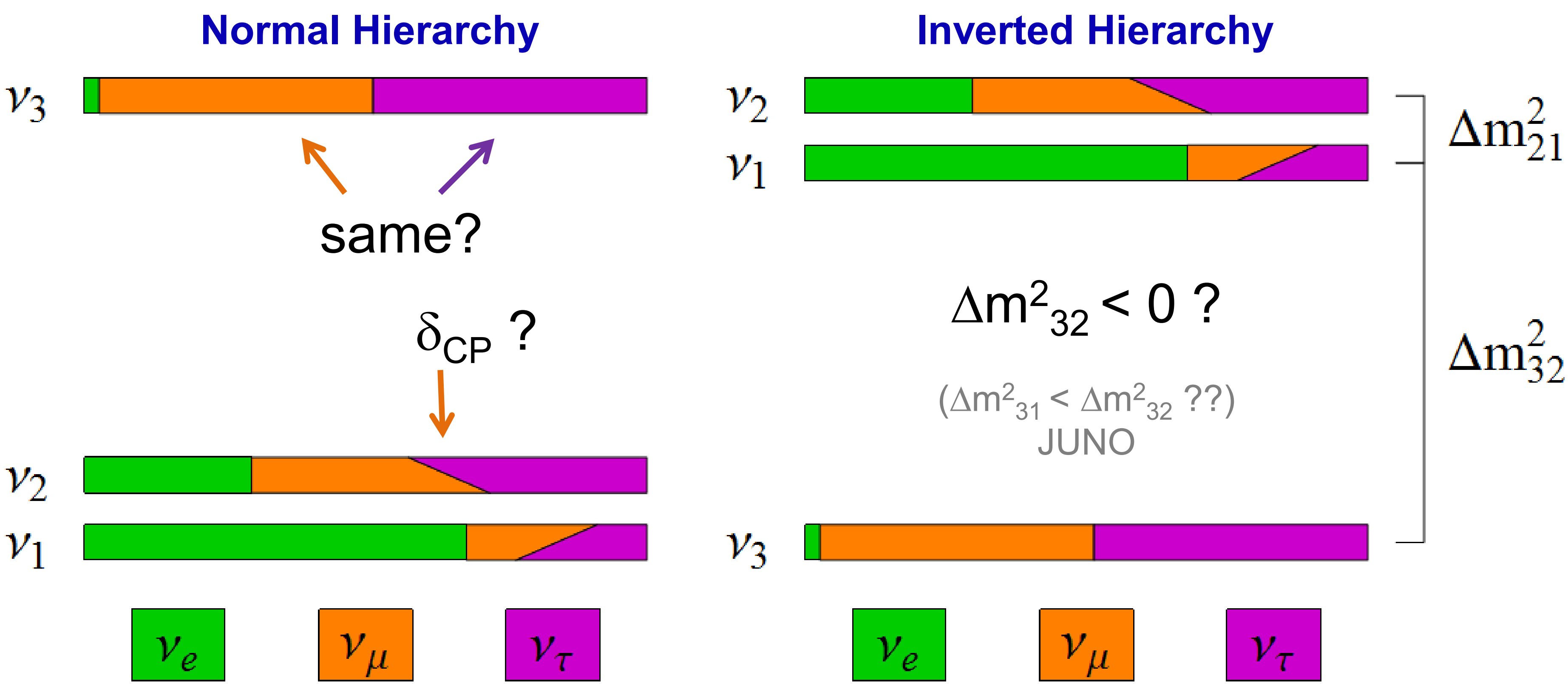


Missing Pieces

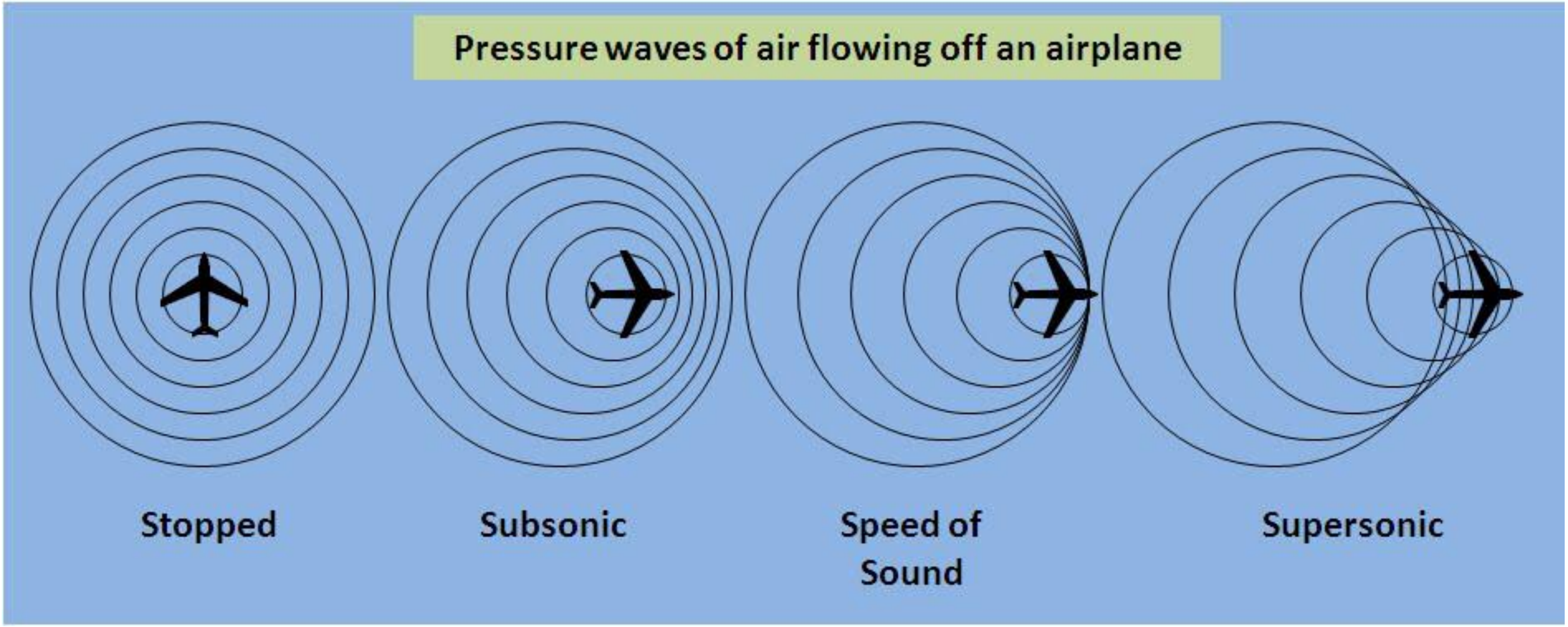
symmetries

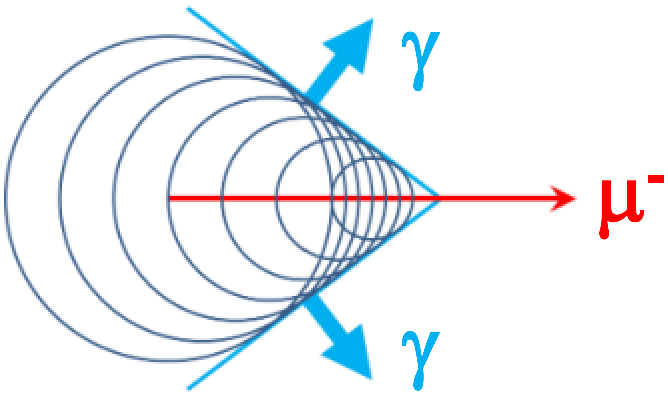
$$\sin^2 2\theta \times \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

- Is $\theta_{23} = \pi/4$? Underlying symmetry?
- Do neutrinos violate CP? (δ_{CP})
- **What is the mass ordering? (Mass Hierarchy)**



Cherenkov Radiation

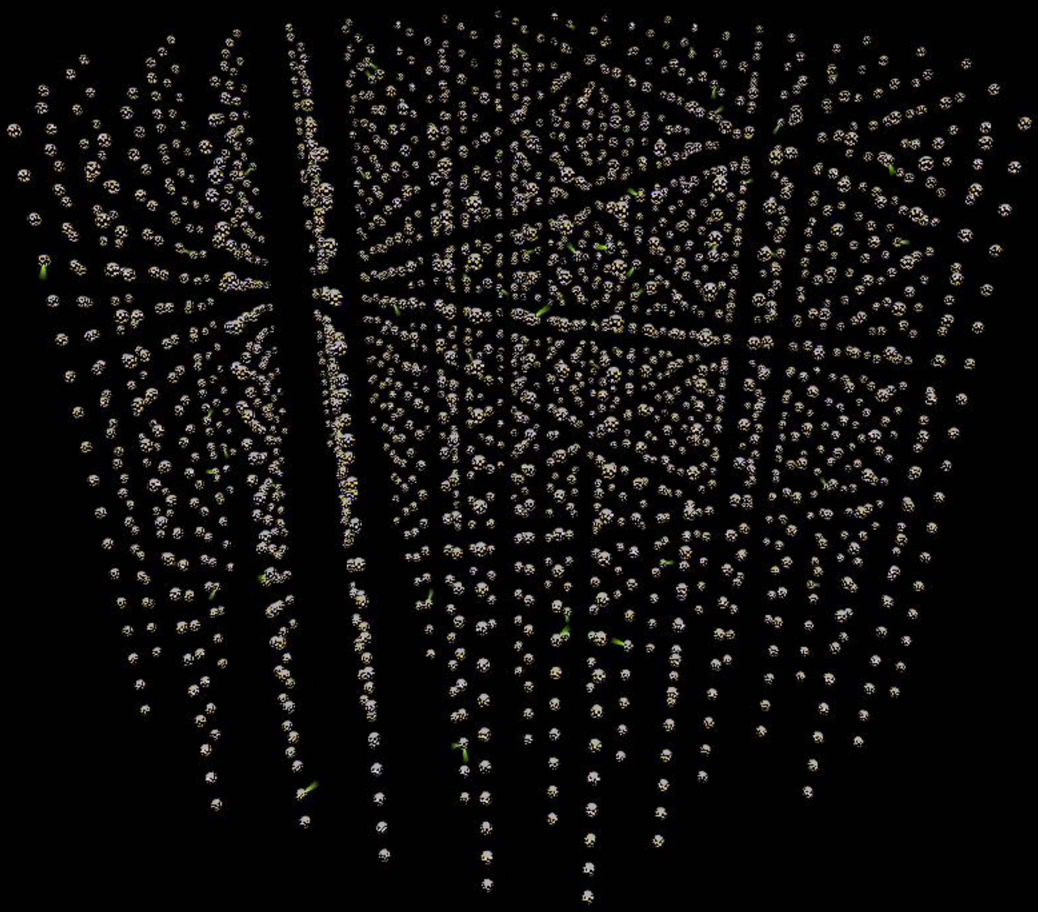


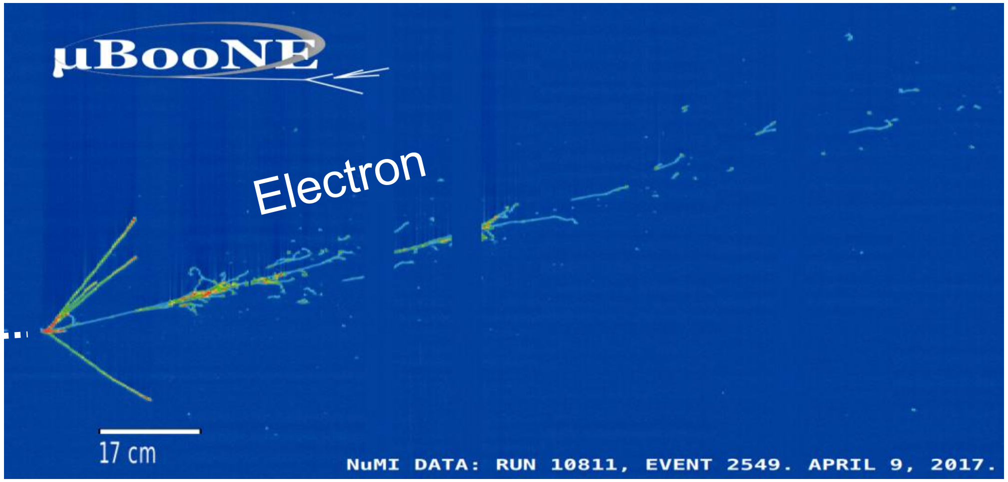


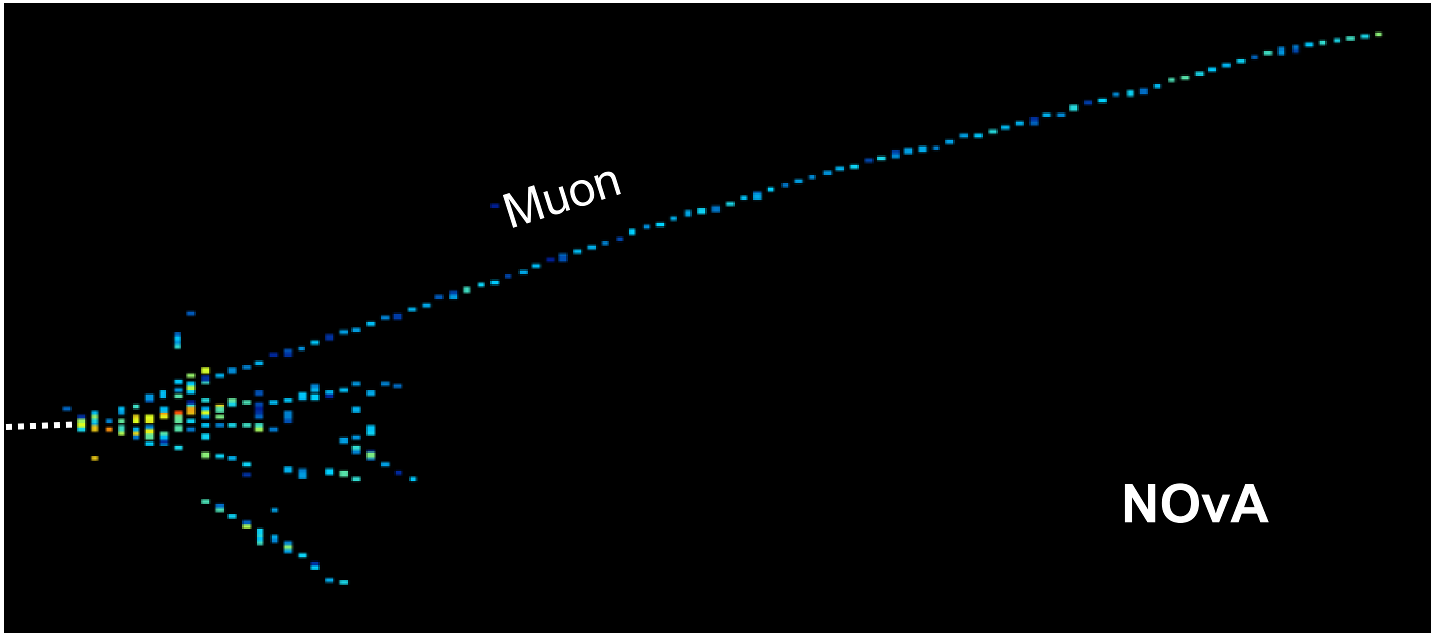
Muons and electrons can travel faster than light in water

Emit light shockwave

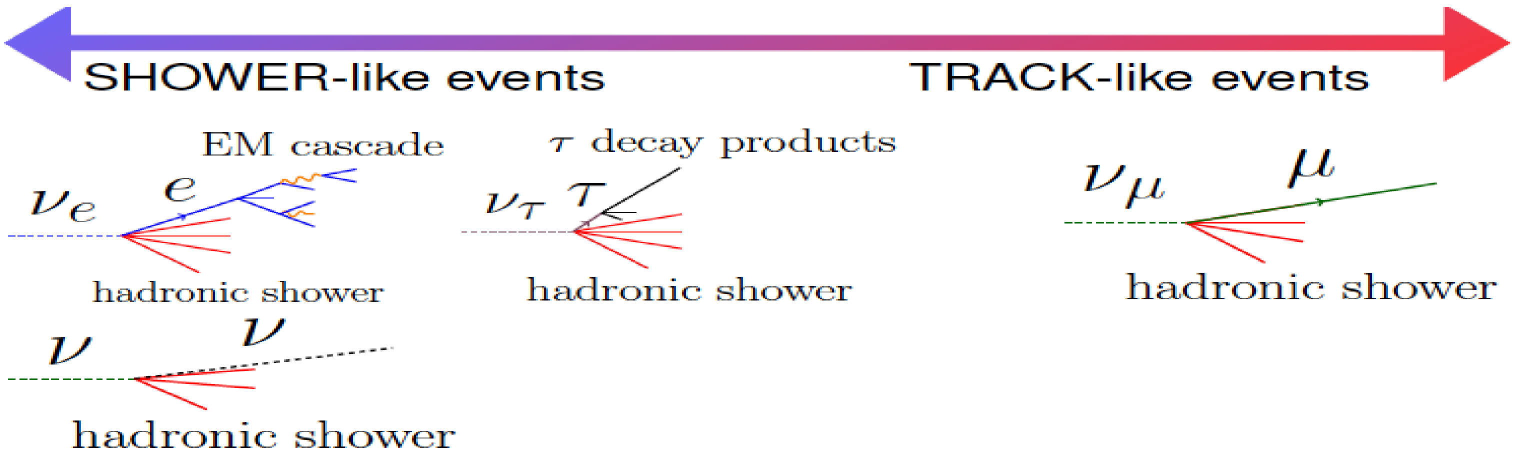
Neutrino Example

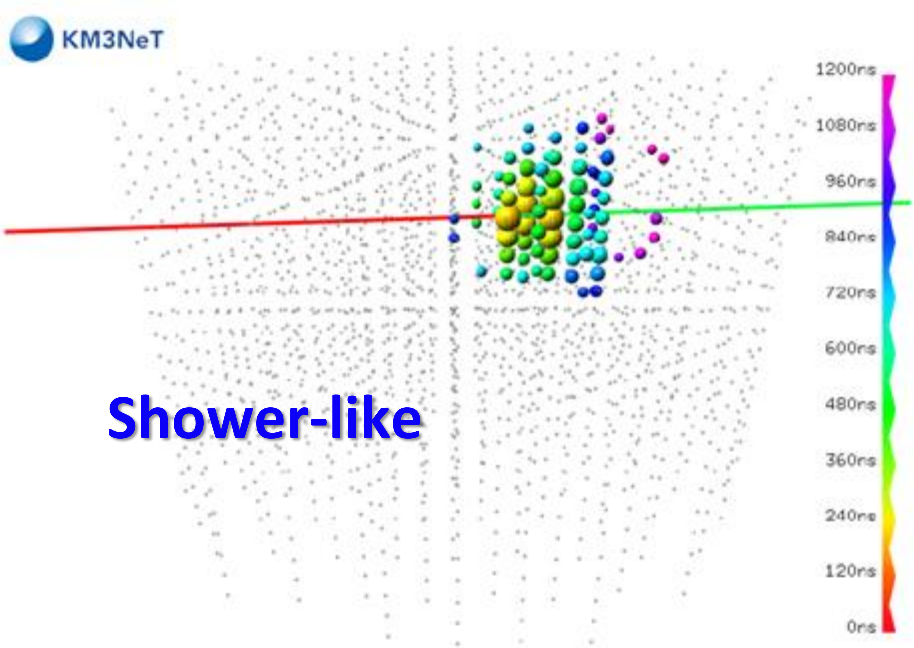


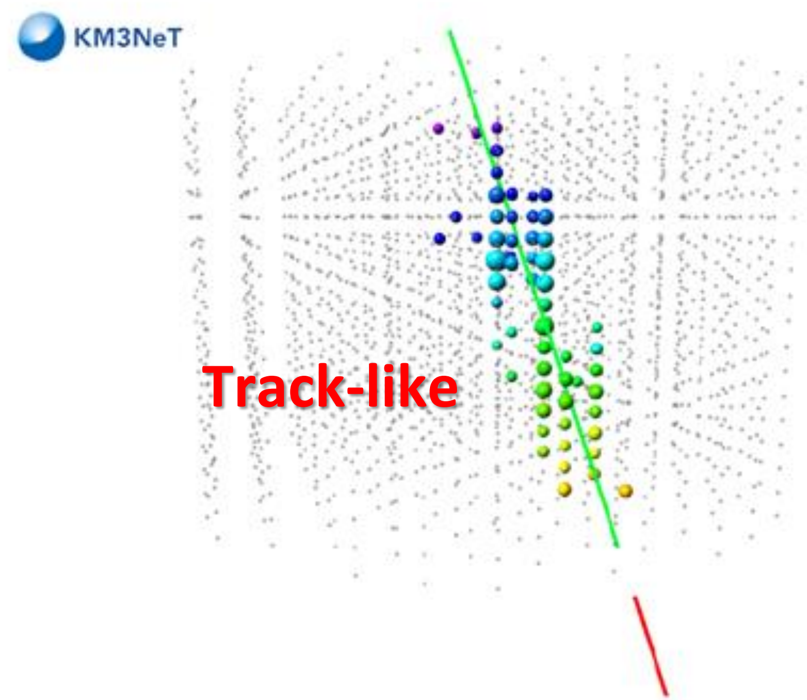

µBooNE
Electron
17 cm
NuMI DATA: RUN 10811, EVENT 2549. APRIL 9, 2017.


Muon
NOvA

Measuring Neutrinos

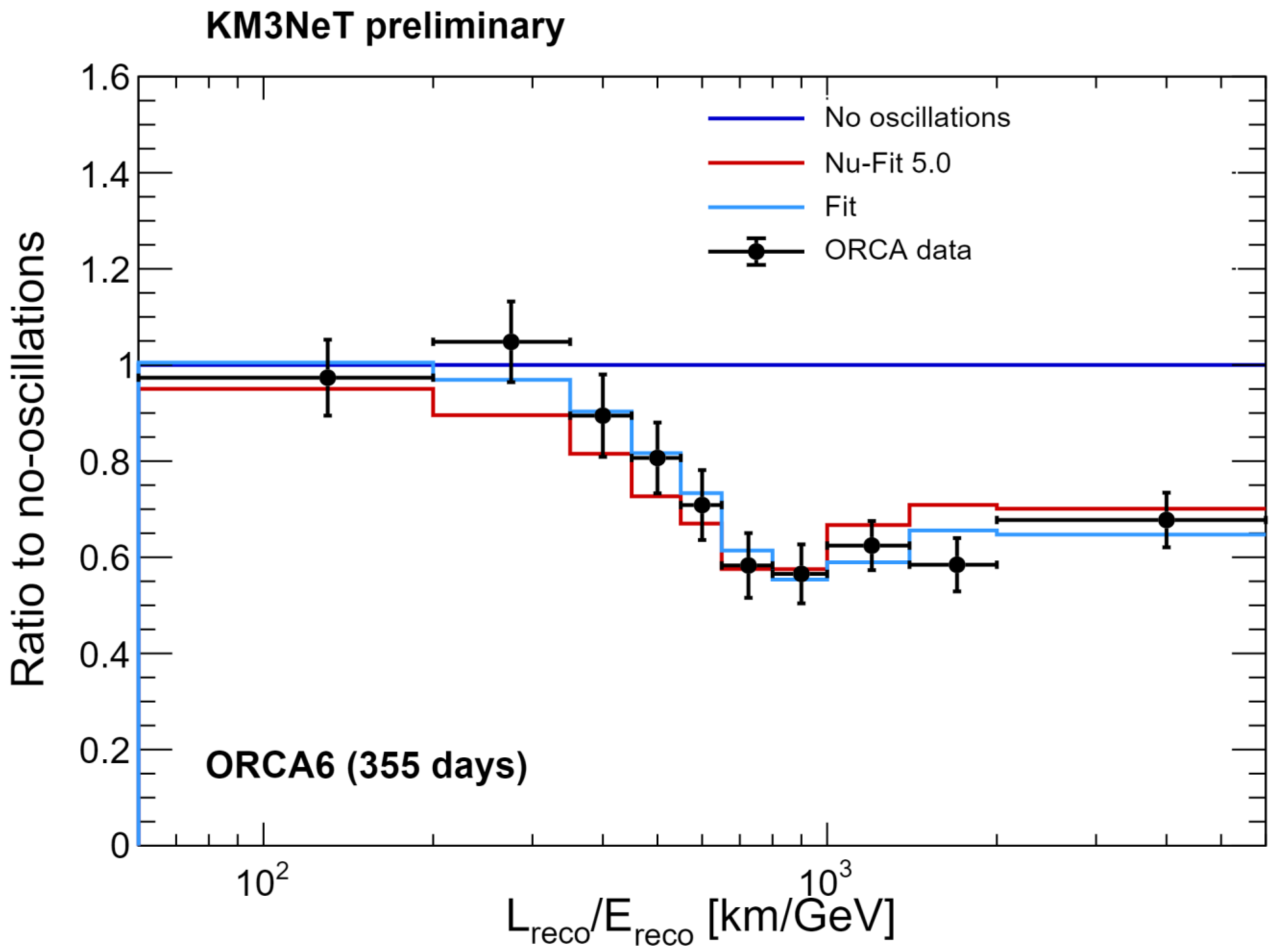






Example of L/E Plot

- **This exposure (300 kton-years) contains ~ 1200 neutrinos**
- **In total we expect ~ 0.5 M neutrinos in ORCA in 10 years (70 Mton-years)**



IceCube Response Function

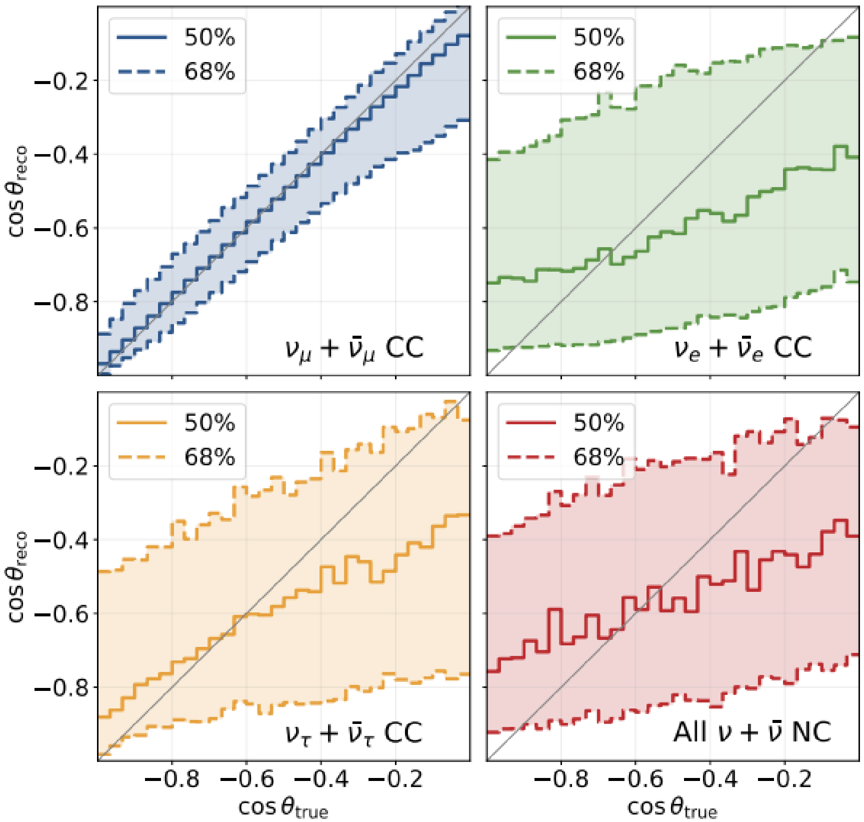


FIG. 10. Final level cosine zenith resolutions for different classes of neutrino events.

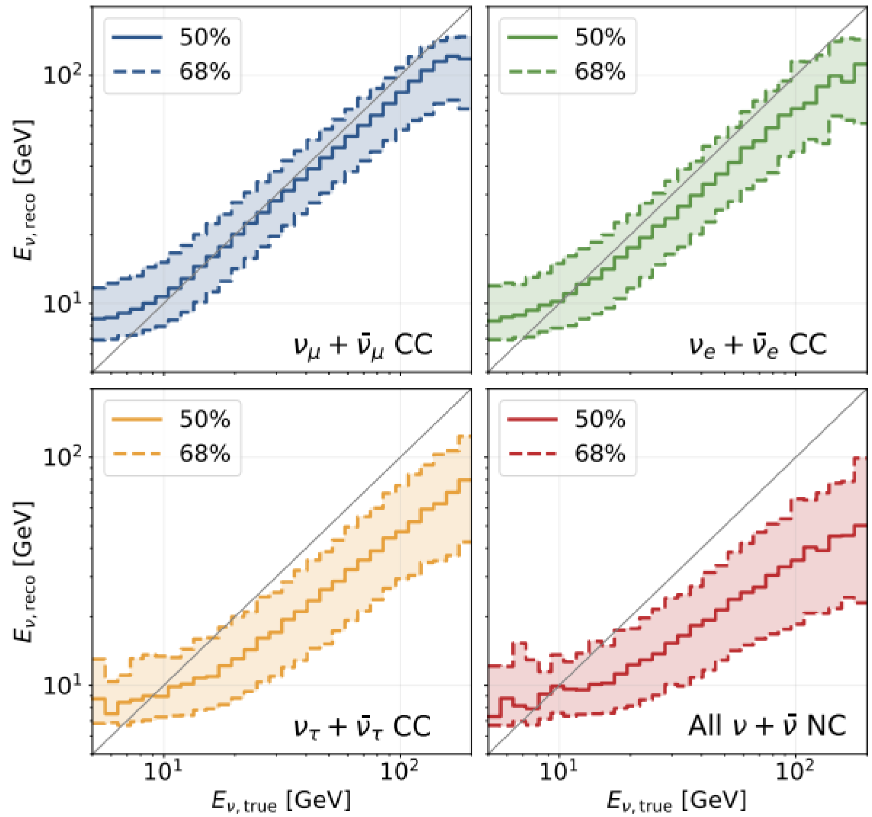


FIG. 11. Final level energy resolutions for different classes of neutrino events. All events are reconstructed using a track-plus-cascade hypothesis.