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Effective Field Theory: a neutrino focused introduction

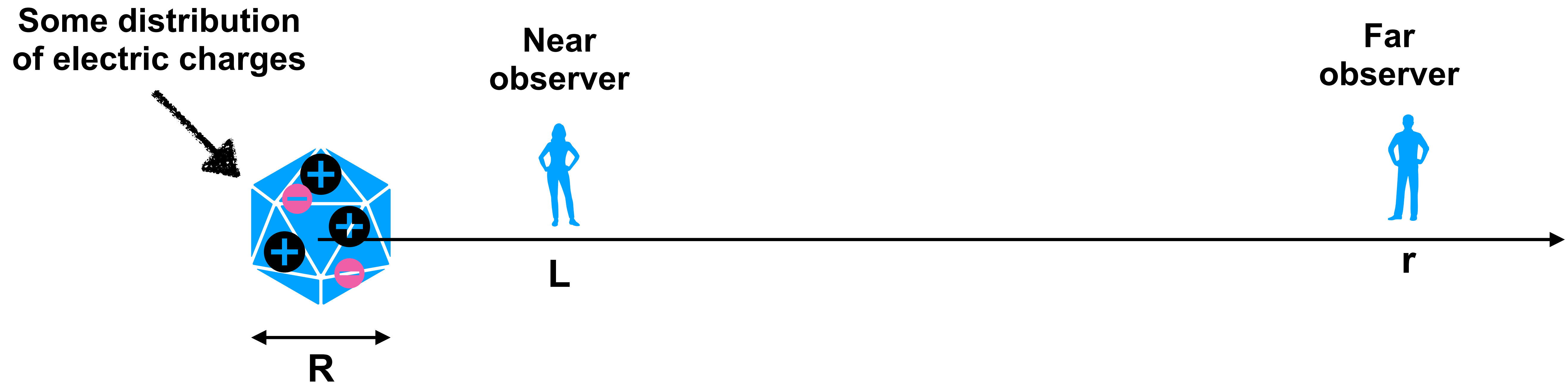
Talk given at the 3rd BSM-nu workshop in IJClab Orsay

24 May 2023

Part 1

Brief Philosophy of EFT

Role of scale in physical problems



Near observer, L~R, needs to know the position of every charge to describe electric field in her proximity

Far observer, $r \gg R$, can instead use multipole expansion:

$$V(\vec{r}) = \frac{Q}{r} + \frac{\vec{d} \cdot \vec{r}}{r^3} + \frac{Q_{ij} r_i r_j}{r^5} + \ldots$$

$$\sim 1/r \qquad \sim R/r^2 \qquad \sim R^2/r^3$$

Higher order terms in the multipole expansion are suppressed by powers of the small parameter (R/r).
One can truncate the expansion at some order depending on the value of (R/r) and experimental precision

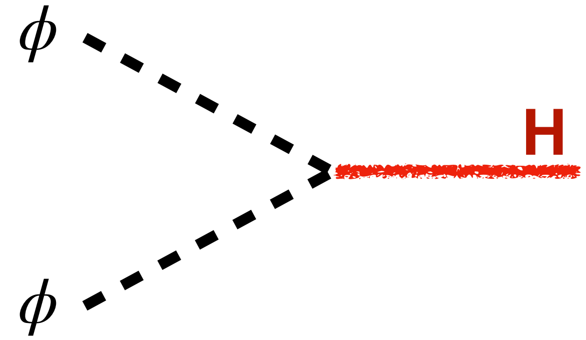
Far observer is able to describe electric field in his vicinity using just a few parameters:
the total electric charge Q, the dipole moment $\vec{d}$, eventually the quadrupole moment Q_{ij}, etc….

On the other hand, far observer can only guess the "fundamental" distributions of the charges,
as many distinct distributions lead to the same first few moments

Far observer may discover that he has been using EFT all his life

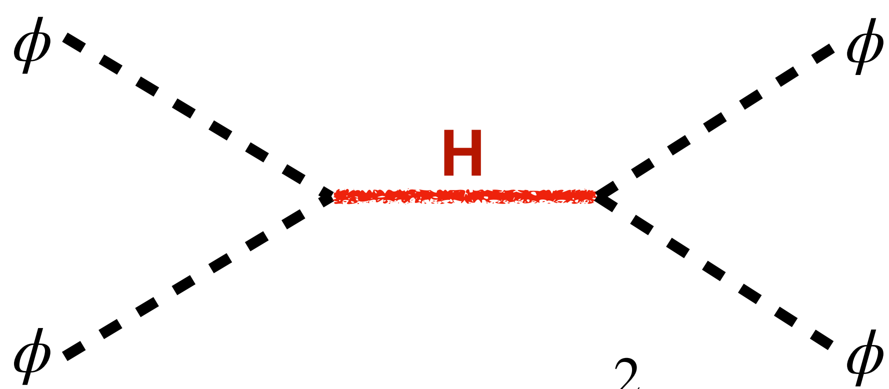
Role of scale in quantum field theory

Consider a theory of a light particle ϕ interacting with a heavy particle H



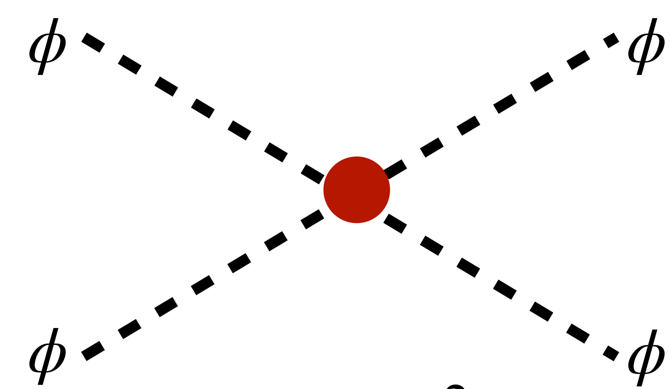
Heavy particle H propagator in momentum space:

$$P(p^2) = \frac{1}{p^2 - m_H^2 + i\epsilon} \approx \begin{cases} \dfrac{1}{p^2 + i\epsilon} & p^2 \gg m_H^2 \\ -\dfrac{1}{m_H^2} & p^2 \ll m_H^2 \end{cases}$$



$$\mathscr{M} \sim \frac{g^2}{p^2 + i\epsilon}$$

At large momentum scales, $p^2 >> m_H^2$, we see propagation of the heavy particle H. Long range force acting between light particles ϕ

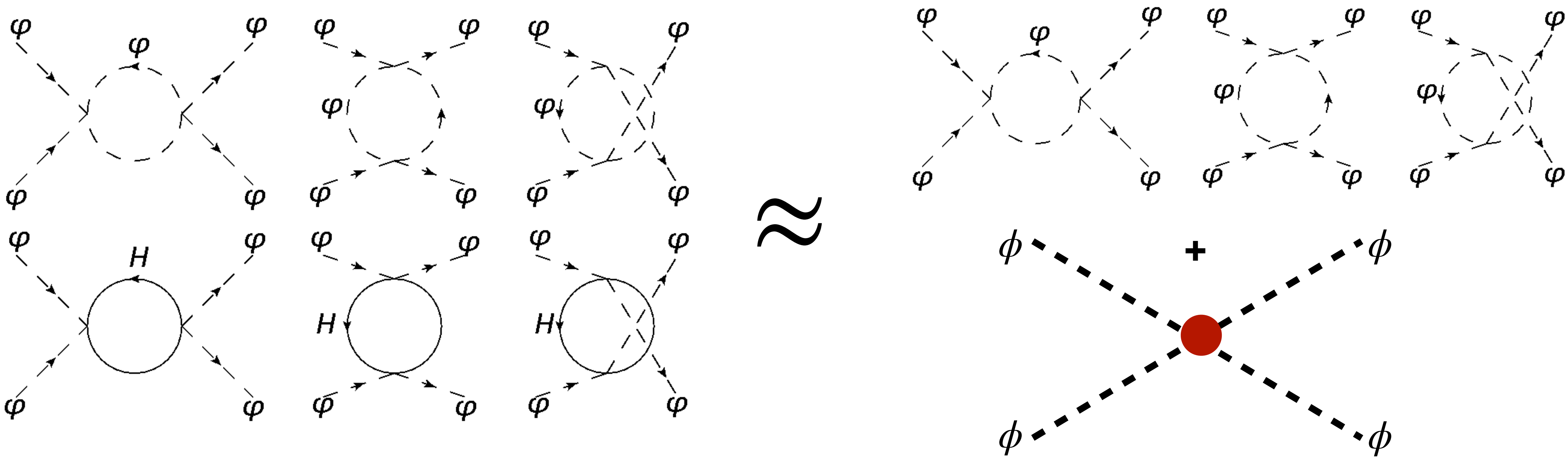


$$\mathscr{M} \sim \frac{g^2}{m_H^2}$$

At small momentum scales, $p^2 << m_H^2$, propagation of the heavy particle H effectively leads to a contact interaction between light particles ϕ

Role of scale in quantum field theory

Effective theory approach works beyond tree level



This works also for higher loops, and with both heavy and light particles in the loops

Effective field theory

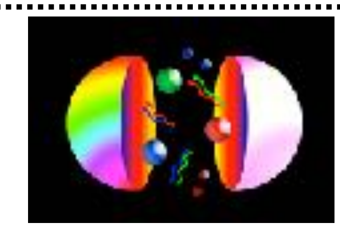
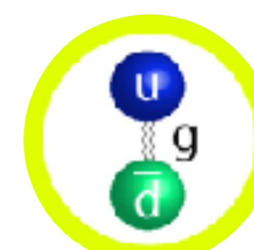
How to build an EFT

Bottom up

Top down

**Starting with a set of particles
we build the Lagrangian
describing all their possible interactions
obeying a prescribed set of symmetries
and organised in a consistent expansion**

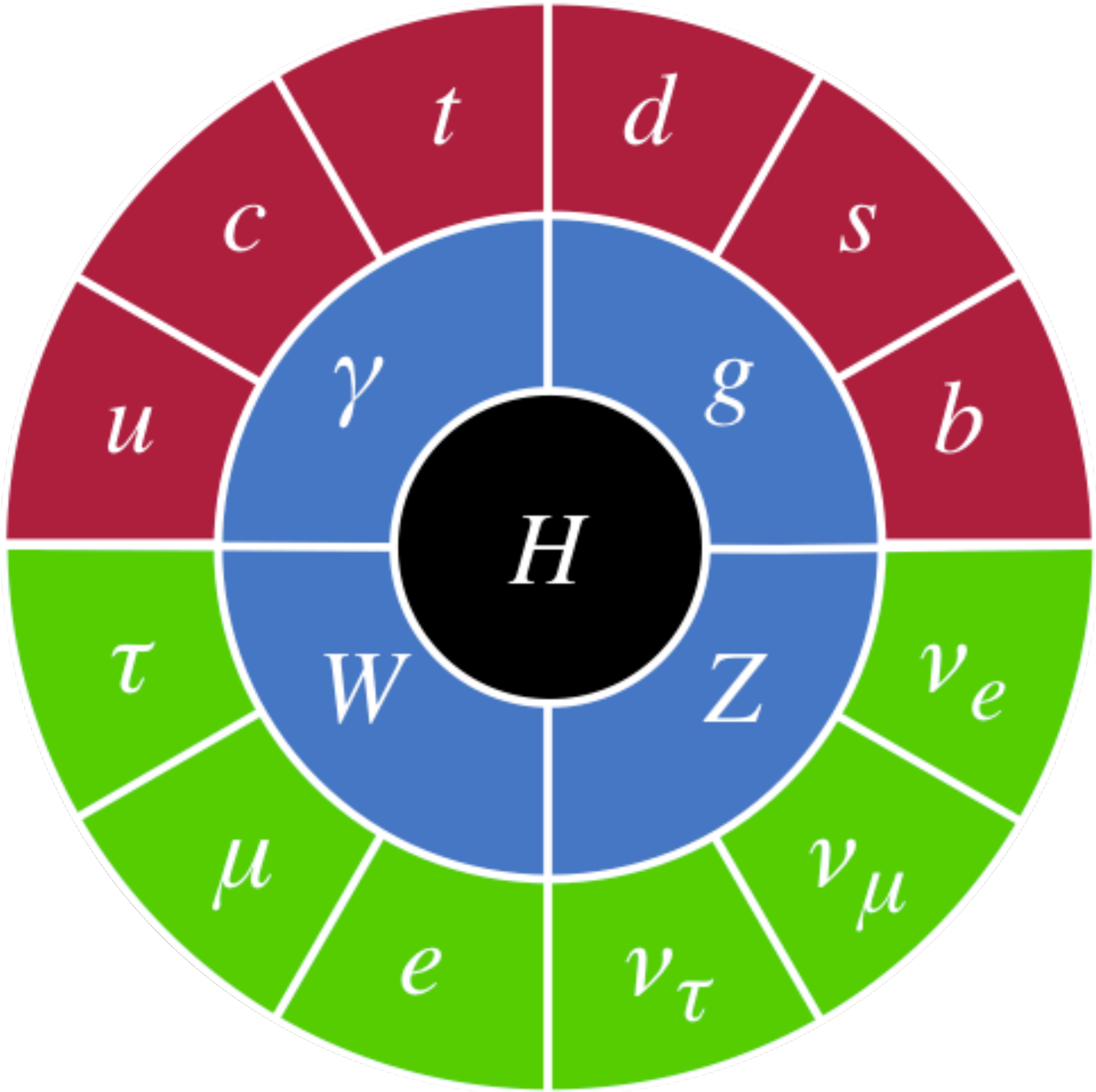
**Starting with a given theory
(effective or fundamental)
we integrate out degrees of freedom
heavier than some prescribed mass scale**

UV	10 TeV	**Dragons**	
SMEFT	100 GeV	$\gamma, g, W, Z, \nu_i, e, \mu, \tau$ + **u, d, s, c, b, t** + **h**	
WEFT5	5 GeV	$\gamma, g, \nu_i, e, \mu, \tau$ + **u, d, s, c, b**	
WEFT4	2 GeV	$\gamma, g, \nu_i, e, \mu, \tau$ + **u, d, s, c**	
ChRT	1 GeV	γ, ν_i, e, μ + **hadrons**	
ChPT	100 MeV	$\gamma, \nu_i, e, \mu, \pi, K$	
QED	1 MeV	γ, ν_i, e	
EH	0.01 eV	γ, ν_i γ	

Part 2

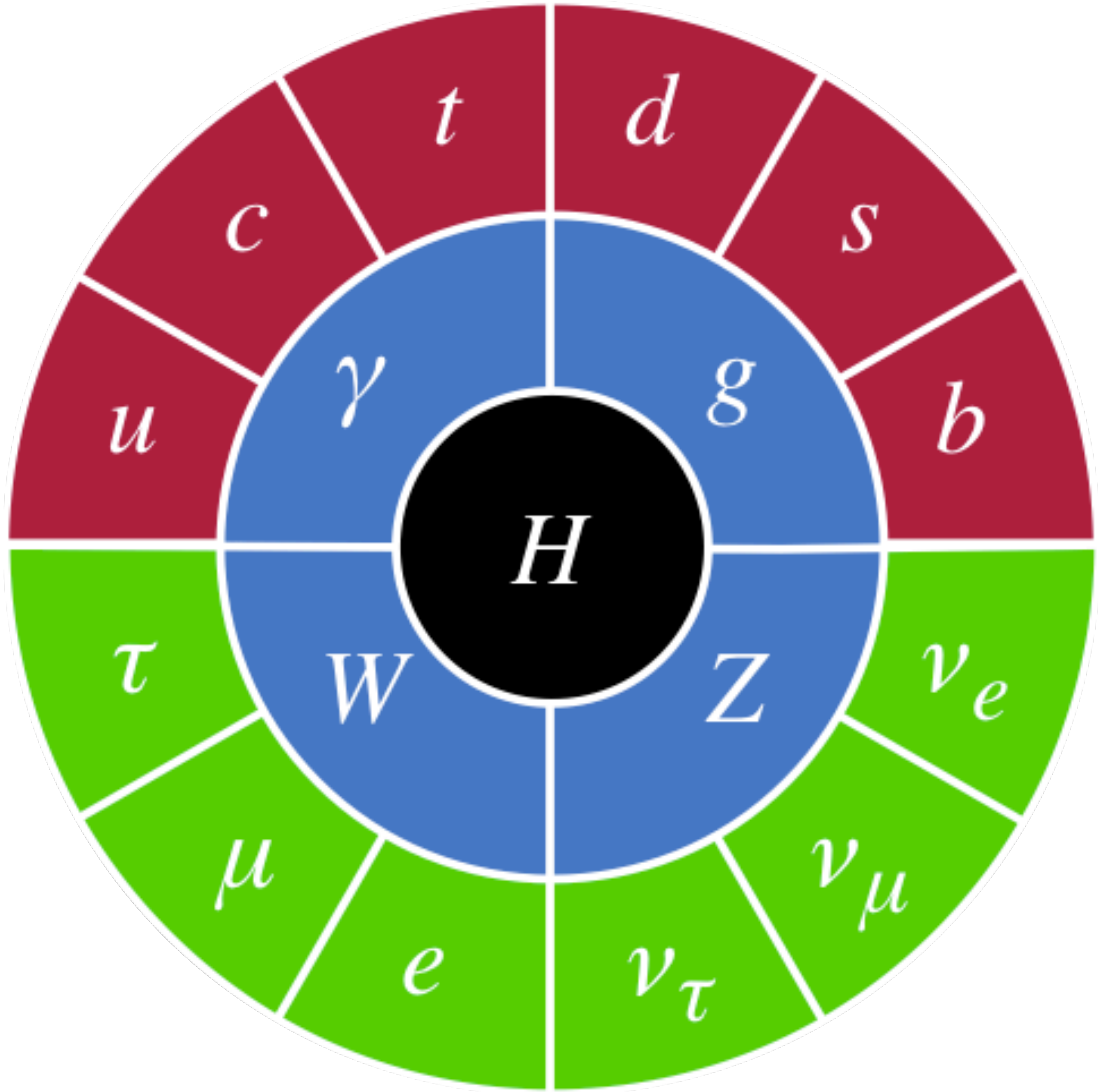
Introducing SMEFT

Elementary particles we know today



This set of particles are the propagating degrees of freedom (at least) right above the electroweak scale, that is at $E \sim$ 100 GeV - 1 TeV

Elementary particles we know today



In these lectures gravity is decoupled and ignored (good assumption in most of laboratory experiments). Otherwise the relevant EFT is called GRSMEFT.

SMEFT

SMEFT is an effective theory for these degrees of freedom:

Field	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	Name	Spin	Dimension
G_μ^a	$\mathbf{8}$	$\mathbf{1}$	0	Gluon	1	1
W_μ^k	$\mathbf{1}$	$\mathbf{3}$	0	Weak SU(2) bosons	1	1
B_μ	$\mathbf{1}$	$\mathbf{1}$	0	Hypercharge boson	1	1
Q	$\mathbf{3}$	$\mathbf{2}$	1/6	Quark doublets	1/2	3/2
U^c	$\bar{\mathbf{3}}$	$\mathbf{1}$	-2/3	Up-type anti-quarks	1/2	3/2
D^c	$\bar{\mathbf{3}}$	$\mathbf{1}$	1/3	Down-type anti-quarks	1/2	3/2
L	$\mathbf{1}$	$\mathbf{2}$	-1/2	Lepton doublets	1/2	3/2
E^c	$\mathbf{1}$	$\mathbf{1}$	1	Charged anti-leptons	1/2	3/2
H	$\mathbf{1}$	$\mathbf{2}$	1/2	Higgs field	0	1

incorporating certain physical assumptions:

1. **Locality, unitarity, Poincaré symmetry**
2. **Mass gap: absence of non-SM degrees of freedom at or below the electroweak scale**
3. **Gauge symmetry: local SU(3)xSU(2)xU(1) symmetry strictly respected by all interactions and spontaneously broken to SU(3)xU(1) by a VEV of the Higgs field**

Dimensional analysis

Using the unit system where $c = \hbar = 1$. Then all objects can be assigned mass dimension

$$[m] = [E] = \text{mass}^1 \Rightarrow [x] = [t] = \text{mass}^{-1} \Rightarrow [\partial_\mu] \equiv \left[\frac{\partial}{\partial x^\mu}\right] = \text{mass}^1$$

Canonical dimension of fields follow from canonically normalized action:

$$S = \int d^4x \mathscr{L} = \int d^4x \left\{ \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + i\bar{\psi}\bar{\sigma}^\mu\partial_\mu\psi - \frac{1}{2}[\partial_\mu A_\nu - \partial_\nu A_\mu]\partial^\mu A^\nu \right\}$$

Action is dimensional
(because path integral contains $e^{iS/\hbar}$)

$\Rightarrow$

$$[\phi] = \text{mass}^1$$

$$[\psi] = \text{mass}^{3/2}$$

$$[A] = \text{mass}^1$$

These rules allows one to determine dimensions of any interaction term, e.g.

$$\mathscr{L} \supset \lambda|H|^4 + C_H|H|^6 + C_\psi(\psi\psi)(\bar{\psi}\bar{\psi}) + \ldots \Rightarrow [\lambda] = \text{mass}^0 \quad [C_H] = \text{mass}^{-2} \quad [C_\psi] = \text{mass}^{-2}$$

Power counting

1. **Locality, unitarity, Poincaré symmetry**
2. **Mass gap: absence of non-SM degrees of freedom at or below the electroweak scale**
3. **Gauge symmetry: local SU(3)xSU(2)xU(1) symmetry strictly respected by all interactions**

In EFT, any interaction allowed by symmetries and general principles is present in the Lagrangian

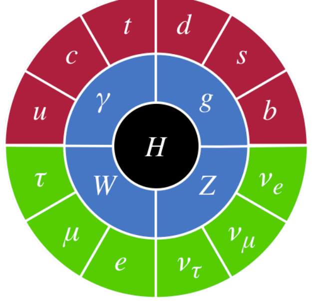
For practical reasons, we need an organizing principle to decide a-priori which interactions are more important, and which are less important

For example, SMEFT Lagrangian contains $|H|^4$ as well as $|H|^{12}$ Higgs self-interactions

Which is more important?

The answer is given by power counting

SMEFT power counting



1. Locality, unitarity, Poincaré symmetry
2. Mass gap: absence of non-SM degrees of freedom at or below the electroweak scale
3. Gauge symmetry: local SU(3)xSU(2)xU(1) symmetry strictly respected by all interactions

We can organize the SMEFT Lagrangian in a dimensional expansion:

$$\mathscr{L}_{\rm SMEFT} = \mathscr{L}_{D=2} + \mathscr{L}_{D=3} + \mathscr{L}_{D=4} + \mathscr{L}_{D=5} + \mathscr{L}_{D=6} + \mathscr{L}_{D=7} + \mathscr{L}_{D=8} + \dots$$

Each $\mathscr{L}_D$ is a linear combination of SU(3)xSU(2)xU(1) invariant interaction terms (operators) where D is the sum of canonical dimensions of all the fields entering the interaction

Since Lagrangian has mass dimension $[\mathscr{L}] = 4$, by dimensional analysis the couplings (Wilson coefficients) of interactions in $\mathscr{L}_D$ have mass dimension $[C_D] = 4 - D$

Standard SMEFT power counting: $C_D \sim \dfrac{c_D}{\Lambda^{D-4}}$ where $c_D \sim 1$,
and Λ is identified with the mass scale of the UV completion of the SMEFT,

In the spirit of EFT, each $\mathscr{L}_D$ should include a <u>complete</u> and <u>non-redundant</u> set of interactions

SMEFT power counting

$$\mathscr{L}_{\rm SMEFT} = \mathscr{L}_{D=2} + \mathscr{L}_{D=3} + \mathscr{L}_{D=4} + \mathscr{L}_{D=5} + \mathscr{L}_{D=6} + \mathscr{L}_{D=7} + \mathscr{L}_{D=8} + \dots$$

SM Lagrangian

Higher-dimensional $SU(3)_C \times SU(2)_L \times U(1)_Y$ invariant interactions added to the SM

At sufficiently high energies, such that we can ignore particle masses, amplitudes for physical processes take the form

$$\mathscr{M}_{\rm SMEFT} = \mathscr{M}_{\rm SM} + C_{D=5}E + C_{D=6}E^2 + C_{D=7}E^3 + C_{D=8}E^4 + \dots$$

$$\sim \mathscr{M}_{\rm SM} + \frac{c_5 E}{\Lambda} + \frac{c_6 E^2}{\Lambda^2} + \frac{c_7 E^3}{\Lambda^3} + \frac{c_8 E^4}{\Lambda^4} + \dots$$

Standard SMEFT power counting sets up the rules for expanding the amplitudes and observables in powers of the new physics scale Λ.

For $E \ll \Lambda$ expansion can be truncated at some D, depending on the desired precision

SMEFT at dimension 2

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{D=2} + \mathcal{L}_{D=3} + \mathcal{L}_{D=4} + \mathcal{L}_{D=5} + \mathcal{L}_{D=6} + \mathcal{L}_{D=7} + \mathcal{L}_{D=8} + \ldots$$

Only a single D=2 operator can be build from the SM fields:

$$\mathcal{L}_{D=2} = \mu_H^2 H^\dagger H$$

Philosophy of EFT: $\mu_H \sim \Lambda \gtrsim 1\ \text{TeV}$

Experiment: $\mu_H \sim 100\ \text{GeV}$

Unsolved mystery why $\mu_H^2 \ll \Lambda^2$,
which is called the hierarchy problem

From the point of view of EFT, the hierarchy problem is a breakdown of dimensional analysis

SMEFT at dimension 3

$$\mathscr{L}_{\mathrm{SMEFT}} = \mathscr{L}_{D=2} + \mathscr{L}_{D=3} + \mathscr{L}_{D=4} + \mathscr{L}_{D=5} + \mathscr{L}_{D=6} + \mathscr{L}_{D=7} + \mathscr{L}_{D=8} + \dots$$

$$\mathscr{L}_{D=3} = 0$$

Simply, no gauge invariant operators made of SM fields exist at canonical dimension D=3

The absence of D=3 operators is a feature of SMEFT, but not a law of nature.
E.g. in νSMEFT, where one also has singlet (right-handed) neutrino, one can write down

$$\mathscr{L}_{D=3}^{\nu\mathrm{SMEFT}} = \frac{1}{2}\nu^c M_\nu \nu^c + \mathrm{h.c.}$$

These are mass terms of the singlet neutrinos

SMEFT at dimension 4

$$\mathcal{L}_{\rm SMEFT} = \mathcal{L}_{D=2} + \mathcal{L}_{D=3} + \mathcal{L}_{D=4} + \mathcal{L}_{D=5} + \mathcal{L}_{D=6} + \mathcal{L}_{D=7} + \mathcal{L}_{D=8} + \dots$$

D=4 is special because it doesn't contain an explicit scale (marginal interactions)

$$\mathcal{L}_{D=4} = -\frac{1}{4}\sum_{V\in B,W^i,G^a} V_{\mu\nu}V^{\mu\nu} + \sum_{f\in Q,L} i\bar{f}\bar{\sigma}^\mu D_\mu f + \sum_{f\in U,D,E} if^c\sigma^\mu D_\mu \bar{f}^c$$
$$-\left(U^c Y_u \tilde{H}^\dagger Q + D^c Y_d H^\dagger Q + E^c Y_e H^\dagger L + \mathrm{h.c.}\right) + D_\mu H^\dagger D^\mu H - \lambda (H^\dagger H)^2$$
$$+\tilde{\theta} G^a_{\mu\nu}\tilde{G}^a_{\mu\nu},$$

$$\tilde{H}_a = \epsilon^{ab} H^*_b$$
$$V^a_{\mu\nu} = \partial_\mu V^a_\nu - \partial_\nu V^a_\mu - g f^{abc} V^b_\mu V^c_\nu$$
$$D_\mu f = \partial_\mu f + i g_s G^a_\mu T^a f + i g_L W^i_\mu \frac{\sigma^i}{2} f + i g_Y B_\mu Y f$$
$$\tilde{G}^a_{\mu\nu} \equiv \frac{1}{2}\epsilon_{\mu\nu\alpha\beta} G^{\alpha\beta\, a}$$

$$U^c = \begin{pmatrix} u^c \\ c^c \\ t^c \end{pmatrix} \qquad Q = \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} u \\ d \end{pmatrix} \\ \begin{pmatrix} c \\ s \end{pmatrix} \\ \begin{pmatrix} t \\ b \end{pmatrix} \end{pmatrix}$$

$$D^c = \begin{pmatrix} d^c \\ s^c \\ b^c \end{pmatrix}$$

$$E^c = \begin{pmatrix} e^c \\ \mu^c \\ \tau^c \end{pmatrix} \qquad L = \begin{pmatrix} l_1 \\ l_2 \\ l_3 \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} \nu_e \\ e \end{pmatrix} \\ \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \\ \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix} \end{pmatrix}$$

Experiment: all these interactions at D=4 above have been observed, except for $\tilde{\theta}$

Strictly speaking, λ has not been observed directly. Its value is known within SM hypothesis, but not within SMEFT, without additional assumptions. Observation of double Higgs production (receiving contribution from cubic Higgs coupling) will be a direct proof that λ is there in the Lagrangian.

Note that $\theta_B B_{\mu\nu}\tilde{B}_{\mu\nu}$ has no physical consequences, while $\theta_W W^k_{\mu\nu}\tilde{W}^k_{\mu\nu}$ can be eliminated by chiral rotation

Note on fermion conventions

I am using the 2-component spinor formalism

A Dirac fermion is described by a pair of spinor fields f and $\bar{f}^c$ **with the kinetic and mass terms**

$$\mathscr{L} = i\bar{f}\bar{\sigma}^\mu D_\mu f + i f^c \sigma^\mu D_\mu \bar{f}^c - m f^c f - m \bar{f}\bar{f}^c$$

$$\sigma^\mu = (1, \boldsymbol{\sigma})$$
$$\bar{\sigma}^\mu = (1, -\boldsymbol{\sigma})$$
$$\bar{f} \equiv f^*$$

To translate to 4-component Dirac notation use

$$F = \begin{pmatrix} f \\ \bar{f}^c \end{pmatrix}, \qquad \bar{F} = \begin{pmatrix} f^c & \bar{f} \end{pmatrix}, \qquad \gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}$$

$$\bar{F} \equiv F^\dagger \gamma^0$$

For example

$$\bar{f}\bar{\sigma}^\mu \partial_\mu f = \bar{F}_L \gamma^\mu \partial_\mu F_L$$

$$f^c \sigma^\mu \partial_\mu \bar{f}^c = \bar{F}_R \gamma^\mu \partial_\mu F_R$$

$$f^c f = \bar{F}_R F_L$$

$$\bar{f}\bar{f}^c = \bar{F}_L F_R$$

See the spinor bible [arXiv:0812.1594] for more details

SMEFT at dimension-5

$$\mathscr{L}_{\rm SMEFT} = \mathscr{L}_{D=2} + \mathscr{L}_{D=4} + \mathscr{L}_{D=5} + \mathscr{L}_{D=6} + \mathscr{L}_{D=7} + \mathscr{L}_{D=8} + \ldots$$

Weinberg (1979)
Phys. Rev. Lett. 43, 1566

$$H \to \begin{pmatrix} 0 \\ \mathrm{v}/\sqrt{2} \end{pmatrix}$$

$$\mathscr{L}_{D=5} = (LH)C(LH) + \mathrm{h.c.} \to \frac{1}{2} \sum_{J,K=e,\mu,\tau} \mathrm{v}^2 C_{JK}(\nu_J \nu_K) + \mathrm{h.c.}$$

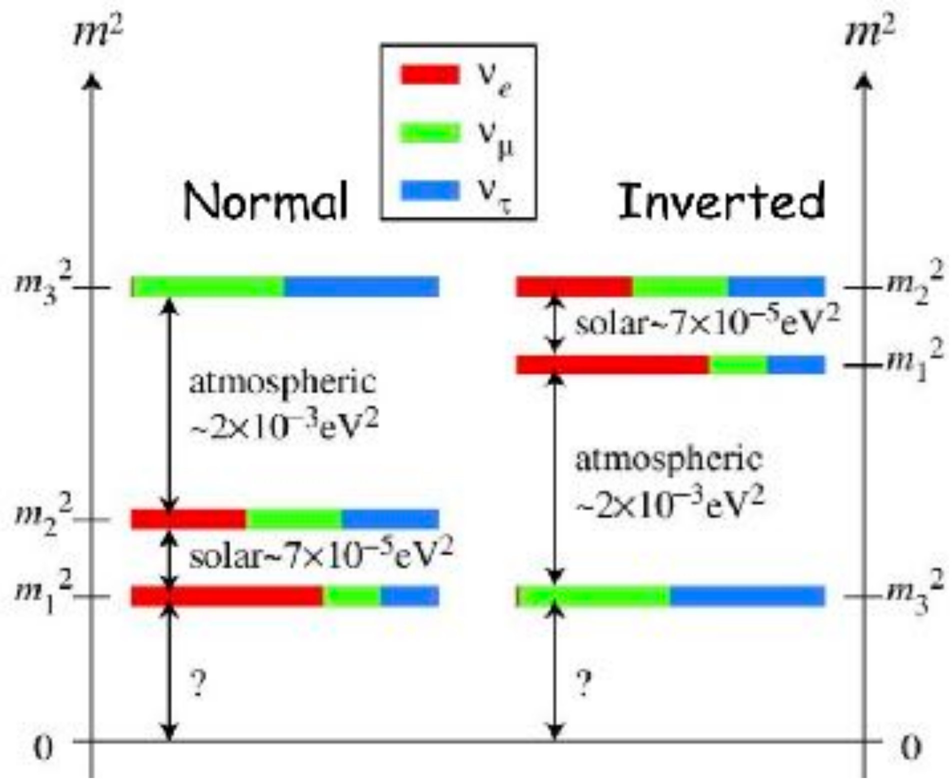
- At dimension 5, the only gauge-invariant operators one can construct are the so-called Weinberg operators, which break the lepton number
- After electroweak symmetry breaking they give rise to mass terms for the SM (left-handed) neutrinos with the mass matrix $M = -\mathrm{v}^2 C$. In the SMEFT scenario, neutrinos are purely Majorana.
- Neutrino oscillation experiments strongly suggest that these operators are present (unless new degrees of freedom exist at low energy scale , see later)

This is a huge success of the SMEFT paradigm: corrections to the SM Lagrangian predicted at the next order in the EFT expansion, are indeed observed in experiment!

SMEFT at dimension-5

$$\mathscr{L}_{\rm SMEFT} \supset -\frac{1}{2}(\nu M \nu) + {\rm h.c.} \qquad M = -\,{\rm v}^2 C$$

Neutrino masses or most likely in the 0.01 eV - 0.1 eV ballpark (though the lightest neutrino may even be massless)

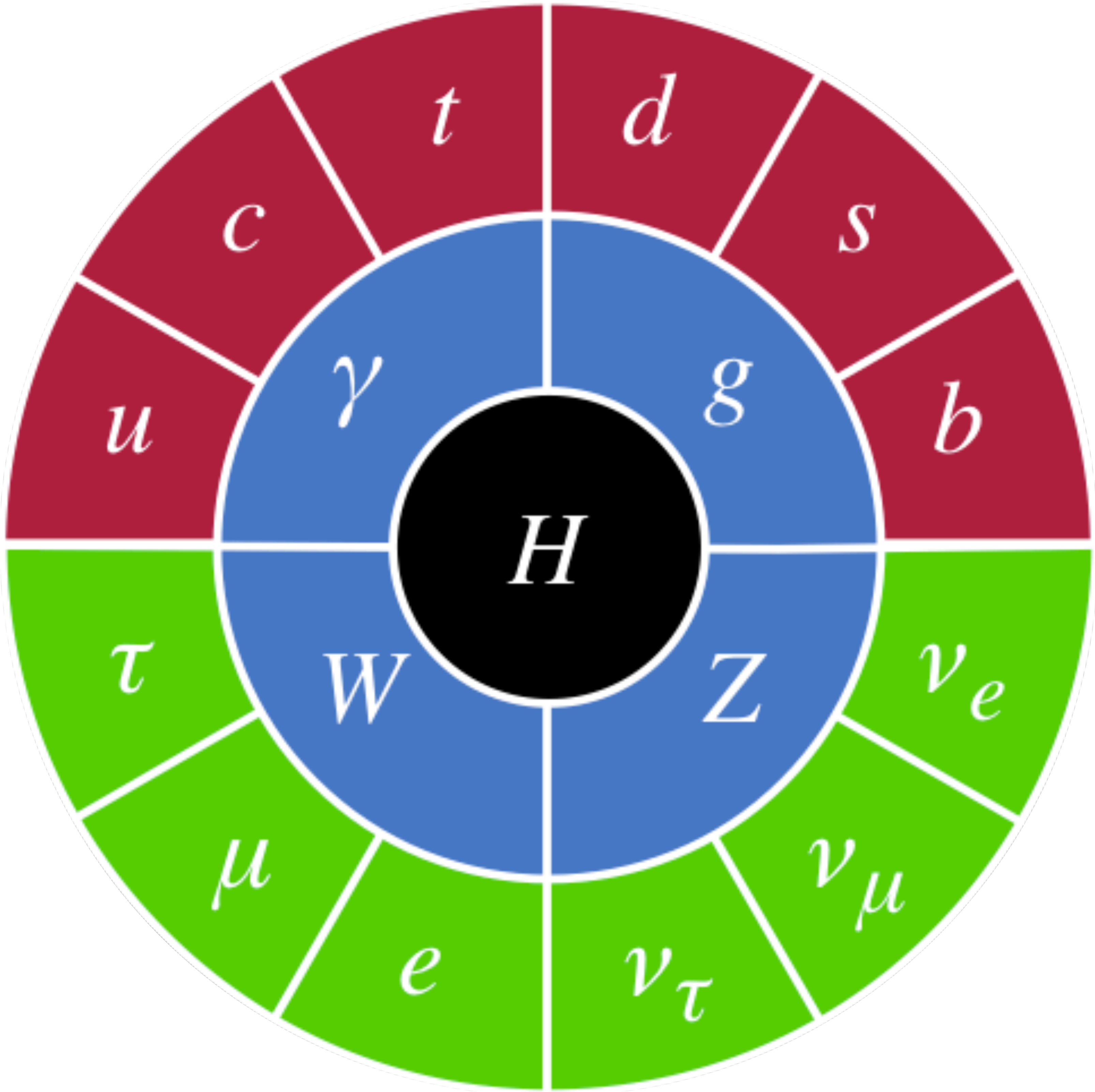


It follows that the dimension-5 Wilson coefficient is of order $C \sim \dfrac{1}{\Lambda}$ with $\Lambda \sim 10^{15}$ **GeV**

SMEFT paradigm points to an existence of a large scale in physics, independent of the Planck scale !

One one hand, that is perfect, because it suggests that the basic SMEFT assumption, $\Lambda \gg {\rm v}$, **is indeed satisfied**

Digression on nu-SMEFT



nu-SMEFT

nu-SMEFT is an effective theory for these degrees of freedom:

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D^c	$\bar{\mathbf{3}}$	$\mathbf{1}$	1/3	Down-type anti-quarks	1/2	3/2
L	$\mathbf{1}$	$\mathbf{2}$	-1/2	Lepton doublets	1/2	3/2
E^c	$\mathbf{1}$	$\mathbf{1}$	1	Charged anti-leptons	1/2	3/2
H	$\mathbf{1}$	$\mathbf{2}$	1/2	Higgs field	0	1
ν^c	**1**	**1**	**0**	**Singlet neutrinos**	**1/2**	**3/2**

incorporating certain physical assumptions:

1. **Locality, unitarity, Poincaré symmetry**
2. **Mass gap: absence of non-SM degrees of freedom at or below the electroweak scale**
3. **Gauge symmetry: local SU(3)xSU(2)xU(1) symmetry strictly respected by all interactions and spontaneously broken to SU(3)xU(1) by a VEV of the Higgs field**

SMEFT at dimension 3

$$\mathscr{L}_{\nu\mathrm{SMEFT}} = \mathscr{L}^{\nu\mathrm{SMEFT}}_{D=2} + \mathscr{L}^{\nu\mathrm{SMEFT}}_{D=3} + \mathscr{L}^{\nu\mathrm{SMEFT}}_{D=4} + \mathscr{L}^{\nu\mathrm{SMEFT}}_{D=5} + \mathscr{L}^{\nu\mathrm{SMEFT}}_{D=6} + \dots$$

In the presence of singlet (right-handed) neutrinos, one can write down their mass term at D=3:

$$\mathscr{L}^{\nu\mathrm{SMEFT}}_{D=3} = \frac{1}{2}\nu^c M_\nu \nu^c + \mathrm{h.c.}$$

Here M_ν is a 3x3 symmetric matrix containing a new mass scale.

Power counting suggests $M_\nu \sim \Lambda \gg \mathrm{v}$, but if that is the case, then we can integrate out the singlet neutrinos and return to SMEFT.

nu-sMEFT is worth considering only assuming $M_\nu \leq \mathrm{v}$, creating another violation of natural EFT power counting

nu-SMEFT at dimension 4

$$\mathscr{L}_{\nu\mathrm{SMEFT}} = \mathscr{L}^{\nu\mathrm{SMEFT}}_{D=2} + \mathscr{L}^{\nu\mathrm{SMEFT}}_{D=3} + \mathscr{L}^{\nu\mathrm{SMEFT}}_{D=4} + \mathscr{L}^{\nu\mathrm{SMEFT}}_{D=5} + \mathscr{L}^{\nu\mathrm{SMEFT}}_{D=6} + \ldots$$

D=4 is special because it doesn't contain an explicit scale (marginal interactions)

$$\mathscr{L}^{\nu\mathrm{SMEFT}}_{D=4} = -\frac{1}{4}\sum_{V\in B,W^i,G^a} V_{\mu\nu}V^{\mu\nu} + \sum_{f\in Q,L} i\bar{f}\bar{\sigma}^\mu D_\mu f + \sum_{f\in U,D,E} if^c\sigma^\mu D_\mu \bar{f}^c$$
$$-\left(U^cY_u\tilde{H}^\dagger Q + D^cY_dH^\dagger Q + E^cY_eH^\dagger L + \nu^cY_\nu\tilde{H}^\dagger L + \mathrm{h.c.}\right)$$
$$+D_\mu H^\dagger D^\mu H - \lambda(H^\dagger H)^2 + \tilde{\theta}G^a_{\mu\nu}\tilde{G}^a_{\mu\nu},$$

In nu-SMEFT at D=4 there are in addition Yukawa interactions with right-handed neutrinos
Together with the D=3 term, it gives neutrino masses

$$\mathscr{L}_{\nu\mathrm{SMEFT}} \supset \frac{1}{2}\nu^cM_\nu\nu^c - \frac{\mathrm{v}}{\sqrt{2}}\nu^cY_\nu\nu + \mathrm{h.c.}$$

As a result, neutrinos are generically mixed Majorana-Dirac

However, in the nu-SMEFT scenario the smallness of the neutrino masses does not have a natural explanation, and it only adds to mysteries of the SM (why are M_ν and Y_ν small) ?

nu-SMEFT at dimension 5

There are qualitatively new effects at D=5 in nu-SMEFT...

$$\mathscr{L}_{D=5}^{\nu\text{SMEFT}} \supset (\nu^c C_{NNH} \nu^c) H^\dagger H + (\nu^c C_{NNB} \sigma^{\mu\nu} \nu^c) B_{\mu\nu}$$

Another contribution to neutrino masses

Might also affect Higgs decays

Magnetic and electric Majorana dipole moment of neutrinos

Leads also to neutrino radiative decay

$$(\nu_J^c \sigma^{\mu\nu} \nu_K^c) B_{\mu\nu} = (\nu_K^c \sigma^{\nu\mu} \nu_J^c) B_{\mu\nu} = -(\nu_K^c \sigma^{\mu\nu} \nu_J^c) B_{\mu\nu}$$

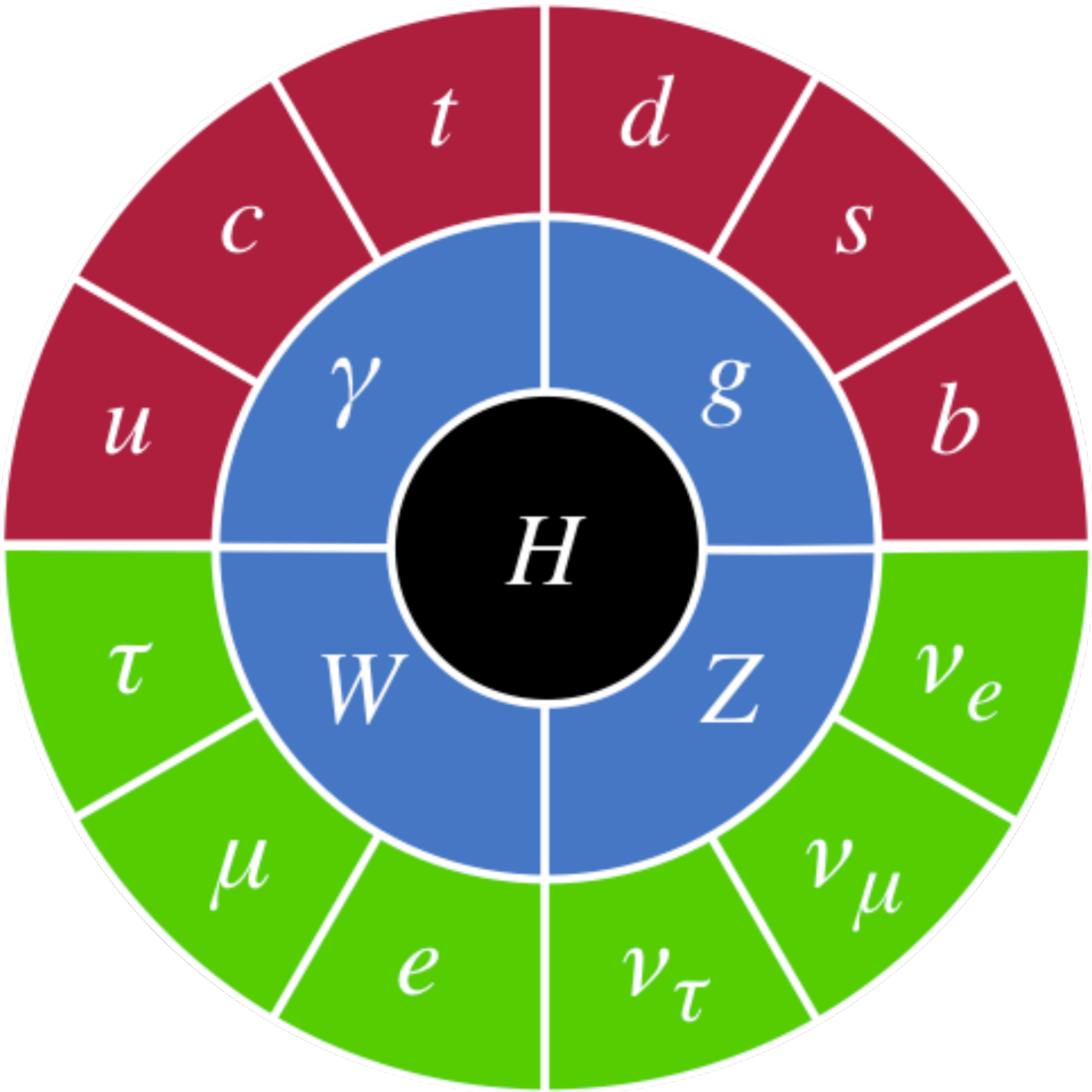
Therefore Majorana dipole moment involves necessarily 2 different neutrino flavours

The more usual Dirac dipole moment arises only at D=6 in nu-SMEFT:

$$\mathscr{L}_{D=6}^{\nu\text{SMEFT}} \supset (\nu^c C_{\nu B} \tilde{H}^\dagger L) B_{\mu\nu} + (\nu^c C_{\nu B} \tilde{H}^\dagger \sigma^k L) W_{\mu\nu}^k + \text{h.c.}$$

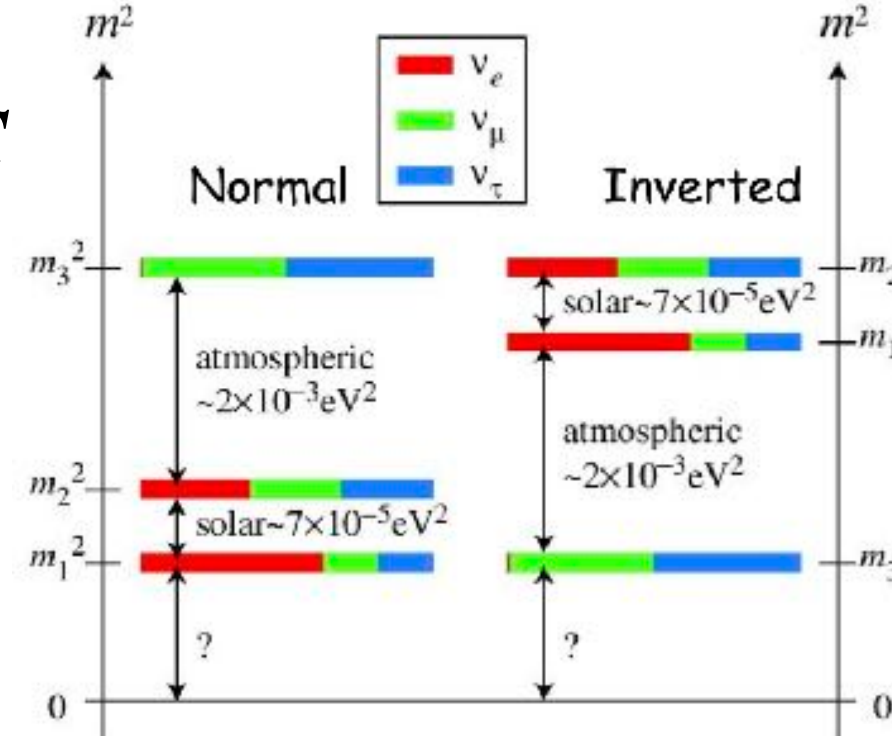
and in this case the dipole moments can be flavor diagonal

Back to SMEFT



Scales in SMEFT

$$\mathscr{L}_{\text{SMEFT}} \supset -\frac{1}{2}(\nu M \nu) + \text{h.c.} \quad M = -\text{v}^2 C$$



However, $\Lambda \sim 10^{15}$ GeV leads to a *psychological* problem

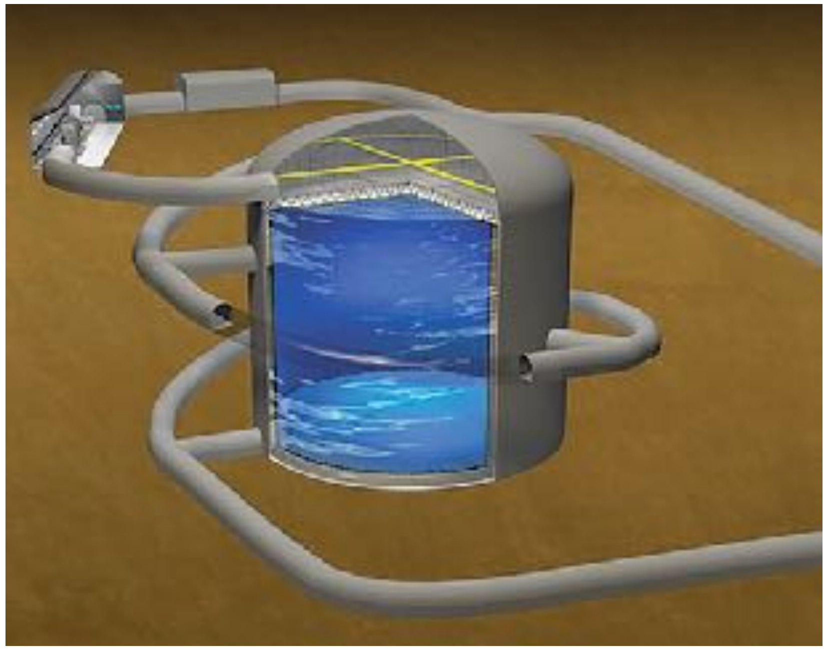
$$\mathscr{L}_{\text{SMEFT}} = \mathscr{L}_{D=2} + \mathscr{L}_{D=4} + \mathscr{L}_{D=5} + \mathscr{L}_{D=6} + \mathscr{L}_{D=7} + \mathscr{L}_{D=8} + \ldots$$

If $\mathscr{L}_{D=5} \sim \frac{1}{\Lambda}$ then naive SMEFT counting suggest $\mathscr{L}_{D=6} \sim \frac{1}{\Lambda^2}$, $\mathscr{L}_{D=7} \sim \frac{1}{\Lambda^3}$,

and so on

If this is really the correct estimate, then we will never see any other effects of higher-dimensional operators, except possibly of the baryon-number violating ones :/

Career opportunities



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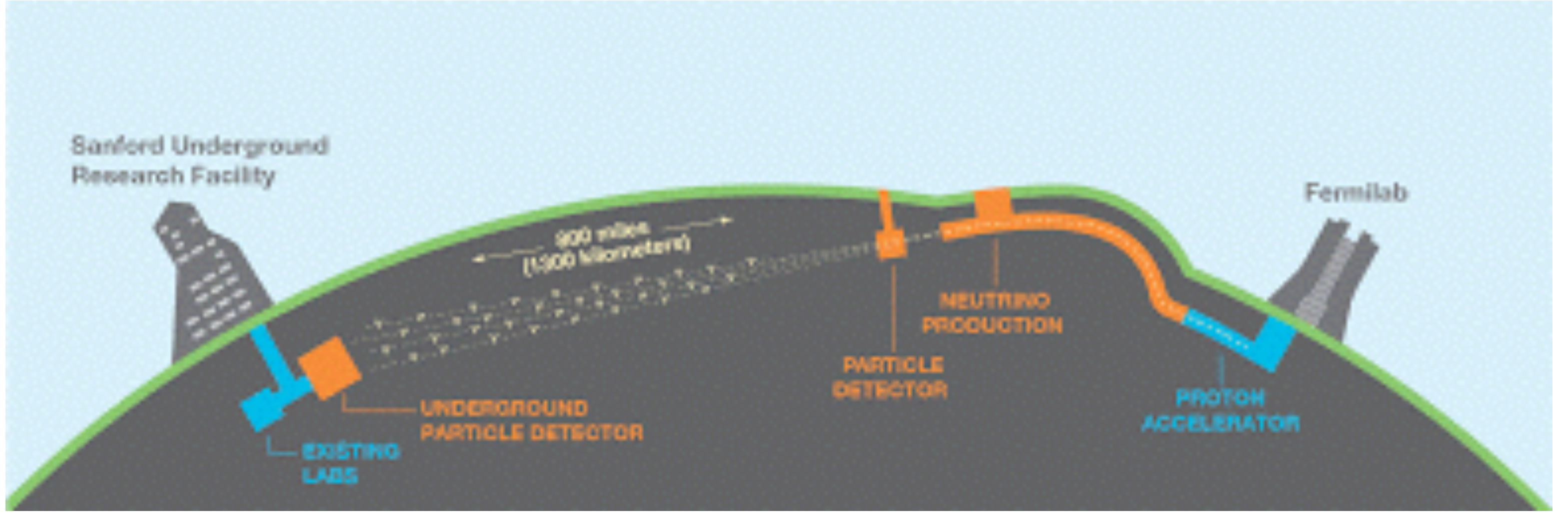
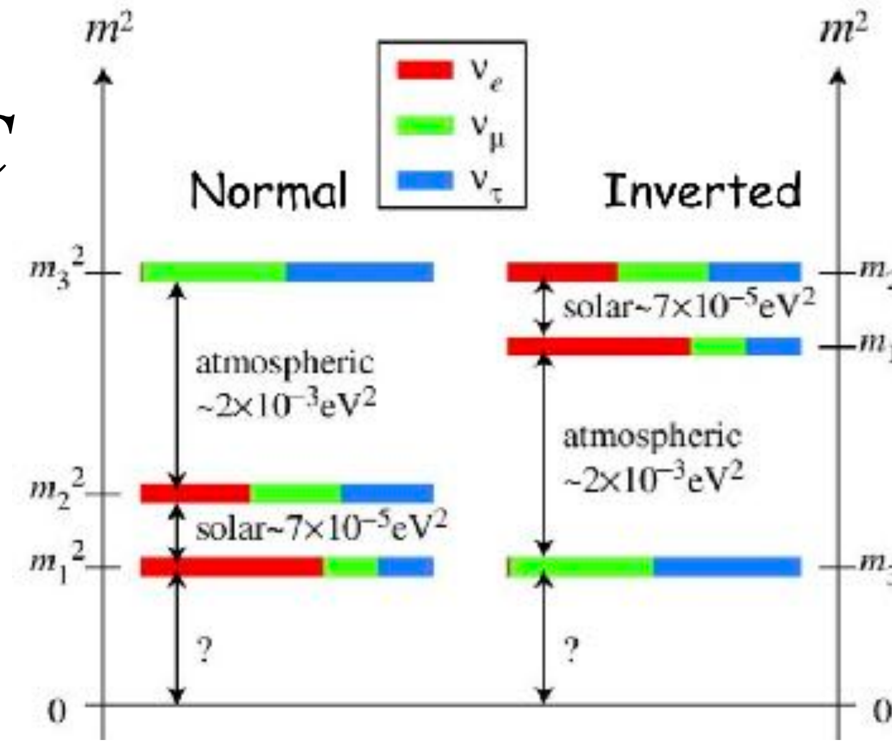


SMEFT at dimension-5

$$\mathscr{L}_{\rm SMEFT} \supset -\frac{1}{2}(\nu M \nu) + {\rm h.c.} \quad M = -{\rm v}^2 C$$



$$\mathscr{L}_{\rm SMEFT} = \mathscr{L}_{D=2} + \mathscr{L}_{D=4} + \mathscr{L}_{D=5} + \mathscr{L}_{D=6} + \mathscr{L}_{D=7} + \mathscr{L}_{D=8} + \dots$$

If $\mathscr{L}_{D=5} \sim \frac{1}{\Lambda}$ **then naive SMEFT counting suggest**

$$\mathscr{L}_{D=6} \sim \frac{1}{\Lambda^2}, \mathscr{L}_{D=7} \sim \frac{1}{\Lambda^3}, \dots$$

However, this conclusion is not set in stone

It is possible that the true new physics scale is not far from TeV, but its coupling to the lepton sector is very small

Alternatively, it is possible (and likely) that there is more than one mass scale of new physics

Dimension-5 interactions are special because they violate lepton number L.

More generally, all odd-dimension SMEFT operators violate B-L

If we assume that the mass scale of new particles with B-L-violating interactions is Λ_L**,**
and there is also B-L-conserving new physics at the scale $\Lambda \ll \Lambda_L$ **, then the estimate is**

$$\mathscr{L}_{D=5} \sim \frac{1}{\Lambda_L}, \mathscr{L}_{D=6} \sim \frac{1}{\Lambda^2}, \mathscr{L}_{D=7} \sim \frac{1}{\Lambda_L^3}, \mathscr{L}_{D=8} \sim \frac{1}{\Lambda^4}, \text{ and so on}$$

SMEFT at dimension-6

$$\mathscr{L}_{\text{SMEFT}} = \mathscr{L}_{D=2} + \mathscr{L}_{D=4} + \mathscr{L}_{D=5} + \mathscr{L}_{D=6} + \mathscr{L}_{D=7} + \mathscr{L}_{D=8} + \dots$$

Grządkowski et al
arXiv:1008.4884

At dimension-6 all hell breaks loose

$$\begin{aligned}
\mathscr{L}_{D=6} = {} & C_H (H^\dagger H)^3 + C_{H\Box}(H^\dagger H)\,\Box\,(H^\dagger H) + C_{HD}\,|H^\dagger D_\mu H|^2 \\
& + C_{HWB} H^\dagger \sigma^k H\, W^k_{\mu\nu} B_{\mu\nu} + C_{HG} H^\dagger H\, G^a_{\mu\nu} G^a_{\mu\nu} + C_{HW} H^\dagger H\, W^k_{\mu\nu} W^k_{\mu\nu} + C_{HB} H^\dagger H\, B_{\mu\nu} B_{\mu\nu} \\
& + + C_W \epsilon^{klm} W^k_{\mu\nu} W^l_{\nu\rho} W^m_{\rho\mu} + C_G f^{abc} G^a_{\mu\nu} G^b_{\nu\rho} G^c_{\rho\mu} \\
& + C_{H\widetilde{G}} H^\dagger H\, \widetilde{G}^a_{\mu\nu} G^a_{\mu\nu} + C_{H\widetilde{W}} H^\dagger H\, \widetilde{W}^k_{\mu\nu} W^k_{\mu\nu} + C_{H\widetilde{B}} H^\dagger H\, \widetilde{B}_{\mu\nu} B_{\mu\nu} + C_{H\widetilde{W}B} H^\dagger \sigma^k H\, \widetilde{W}^k_{\mu\nu} B_{\mu\nu} \\
& + C_{\widetilde{W}} \epsilon^{klm} \widetilde{W}^k_{\mu\nu} W^l_{\nu\rho} W^m_{\rho\mu} + C_{\widetilde{G}} f^{abc} \widetilde{G}^a_{\mu\nu} G^b_{\nu\rho} G^c_{\rho\mu} \\
& + H^\dagger H(\bar{L} H C_{eH} \bar{E}^c) + H^\dagger H(\bar{Q} \tilde{H} C_{uH} \bar{U}^c) + H^\dagger H(\bar{Q} H C_{dH} \bar{D}^c) \\
& + i H^\dagger \overleftrightarrow{D}_\mu H(\bar{L} C^{(1)}_{Hl} \bar{\sigma}^\mu L) + i H^\dagger \sigma^k \overleftrightarrow{D}_\mu H(\bar{L} C^{(3)}_{Hl} \bar{\sigma}^\mu \sigma^k L) + i H^\dagger \overleftrightarrow{D}_\mu H(E^c C_{He} \sigma^\mu \bar{E}^c) \\
& + i H^\dagger \overleftrightarrow{D}_\mu H(\bar{Q} C^{(1)}_{Hq} \bar{\sigma}^\mu Q) + i H^\dagger \sigma^k \overleftrightarrow{D}_\mu H(\bar{Q} C^{(3)}_{Hq} \bar{\sigma}^\mu \sigma^k Q) + i H^\dagger \overleftrightarrow{D}_\mu H(U^c C_{Hu} \sigma^\mu \bar{U}^c) \\
& + i H^\dagger \overleftrightarrow{D}_\mu H(D^c C_{Hd} \sigma^\mu \bar{D}^c) + \Big\{ i \tilde{H}^\dagger D_\mu H(U^c C_{Hud} \sigma^\mu \bar{D}^c) \\
& + (\bar{Q} \sigma^k \tilde{H} C_{uW} \bar{\sigma}^{\mu\nu} \bar{U}^c) W^k_{\mu\nu} + (\bar{Q} \tilde{H} C_{uB} \bar{\sigma}^{\mu\nu} \bar{U}^c) B_{\mu\nu} + (\bar{Q} \tilde{H} C_{uG} T^a \bar{\sigma}^{\mu\nu} \bar{U}^c) G^a_{\mu\nu} \\
& + (\bar{Q} \sigma^k H C_{dW} \bar{\sigma}^{\mu\nu} \bar{D}^c) W^k_{\mu\nu} + (\bar{Q} H C_{dB} \bar{\sigma}^{\mu\nu} \bar{D}^c) B_{\mu\nu} + (\bar{Q} H C_{dG} T^a \bar{\sigma}^{\mu\nu} \bar{D}^c) G^a_{\mu\nu} \\
& + (\bar{L} \sigma^k H C_{eW} \bar{\sigma}^{\mu\nu} \bar{E}^c) W^k_{\mu\nu} + (\bar{L} H C_{eB} \bar{\sigma}^{\mu\nu} \bar{E}^c) B_{\mu\nu} + \text{h.c.} \Big\} + \mathscr{L}^{4-\text{fermion}}_{D=6}
\end{aligned}$$


$|H|^2 B_{\mu\nu}\widetilde{B}_{\mu\nu}$
$|H|^2 G^a_{\mu\nu}\widetilde{G}^a_{\mu\nu}$
$|H|^2 W^a_{\mu\nu}\widetilde{W}^a_{\mu\nu}$
$|H|^2 W^a_{\mu\nu}W^a_{\mu\nu}$
$|H|^2 B_{\mu\nu}B_{\mu\nu}$
$G^a_{\mu\nu}G^a_{\nu\rho}\widetilde{G}^a_{\rho\mu}$
$|H|^6$
$|H|^2 G^a_{\mu\nu}G^a_{\mu\nu}$

SMEFT at dimension-6

Bosonic operators

$$\mathscr{L}_{\text{SMEFT}} \supset \sum_X C_X O_X$$

$$O_H = (H^\dagger H)^3$$

$$O_{H\Box} = (H^\dagger H)\Box(H^\dagger H)$$

$$O_{HD} = |H^\dagger D_\mu H|^2$$

$$O_{HG} = H^\dagger H\, G^a_{\mu\nu} G^a_{\mu\nu} \qquad O_{H\widetilde{G}} = H^\dagger H\, G^a_{\mu\nu} \widetilde{G}^a_{\mu\nu}$$

$$O_{HW} = H^\dagger H\, W^k_{\mu\nu} W^k_{\mu\nu} \qquad O_{H\widetilde{W}} = H^\dagger H\, W^k_{\mu\nu} \widetilde{W}^k_{\mu\nu}$$

$$O_{HB} = H^\dagger H\, B_{\mu\nu} B_{\mu\nu} \qquad O_{H\widetilde{B}} = H^\dagger H\, B_{\mu\nu} \widetilde{B}_{\mu\nu}$$

$$O_{HWB} = H^\dagger \sigma^k H\, W^k_{\mu\nu} B_{\mu\nu} \qquad O_{HW\widetilde{B}} = H^\dagger \sigma^k H\, W^k_{\mu\nu} \widetilde{B}_{\mu\nu}$$

$$O_W = \epsilon^{klm} W^k_{\mu\nu} W^l_{\nu\rho} W^m_{\rho\mu} \qquad O_{\widetilde{W}} = \epsilon^{klm} W^k_{\mu\nu} W^l_{\nu\rho} \widetilde{W}^m_{\rho\mu}$$

$$O_G = f^{abc} G^a_{\mu\nu} G^b_{\nu\rho} G^c_{\rho\mu} \qquad O_{\widetilde{G}} = f^{abc} G^a_{\mu\nu} G^b_{\nu\rho} \widetilde{G}^c_{\rho\mu}$$

These are mostly relevant for Higgs physics and certain electroweak precision observables. The CP odd ones, affect important CP observables via loop effects, such as e.g. EDMs

SMEFT at dimension-6

$$\mathscr{L}_{\mathrm{SMEFT}} \supset \sum_{I,J=1}^{3} [O_{fH}]_{IJ}[C_{fH}]_{IJ} + \mathrm{h.c.}$$

Yukawa-like operators

$$O_{eH} = H^\dagger H(\bar{L}H\bar{E}^c)$$

$$O_{uH} = H^\dagger H(\bar{Q}\tilde{H}\bar{U}^c)$$

$$O_{dH} = H^\dagger H(\bar{Q}H\bar{D}^c)$$

These affect single Higgs boson couplings to SM fermions. Bounds depends on the flavor but typically don't exceed $|C| \lesssim \frac{1}{(1\ \mathrm{TeV})^2}$

SMEFT at dimension-6

Vertex-like operators

$$O_{Hl}^{(1)} = iH^\dagger \overleftrightarrow{D}_\mu H(\bar{L}\bar{\sigma}^\mu L)$$

$$O_{Hl}^{(3)} = iH^\dagger \sigma^k \overleftrightarrow{D}_\mu H(\bar{L}\bar{\sigma}^\mu \sigma^k L)$$

$$O_{He} = iH^\dagger \overleftrightarrow{D}_\mu H(E^c \sigma^\mu \bar{E}^c)$$

$$O_{Hq}^{(1)} = iH^\dagger \overleftrightarrow{D}_\mu H(\bar{Q}\bar{\sigma}^\mu Q)$$

$$O_{Hq}^{(3)} = iH^\dagger \sigma^k \overleftrightarrow{D}_\mu H(\bar{Q}\bar{\sigma}^\mu \sigma^k Q)$$

Affects W boson couplings to left-handed quarks and this way it may affect various neutrino experiments

$$O_{Hu} = iH^\dagger \overleftrightarrow{D}_\mu H(U^c \sigma^\mu \bar{U}^c)$$

$$O_{Hd} = iH^\dagger \overleftrightarrow{D}_\mu H(D^c \sigma^\mu \bar{D}^c)$$

$$O_{Hud} = i\tilde{H}^\dagger D_\mu H(U^c \sigma^\mu \bar{D}^c)$$

Induces W boson couplings to right-handed quarks and this way it may affect various neutrino experiments

These affect electroweak precision observables (W boson mass, Z branching fractions), which are measured at per-mille level at LEP

Bounds of order $|C| \lesssim \frac{1}{(10\ \mathrm{TeV})^2}$

SMEFT at dimension-6

$$\mathcal{L}^{\rm dipole}_{D=6} = (\bar{Q}\sigma^k \tilde{H} C_{uW} \bar{\sigma}^{\mu\nu} \bar{U}^c) W^k_{\mu\nu} + (\bar{Q}\tilde{H} C_{uB} \bar{\sigma}^{\mu\nu} \bar{U}^c) B_{\mu\nu} + (\bar{Q}\tilde{H} C_{uG} T^a \bar{\sigma}^{\mu\nu} \bar{U}^c) G^a_{\mu\nu}$$
$$+ (\bar{Q}\sigma^k H C_{dW} \bar{\sigma}^{\mu\nu} \bar{D}^c) W^k_{\mu\nu} + (\bar{Q} H C_{dB} \bar{\sigma}^{\mu\nu} \bar{D}^c) B_{\mu\nu} + (\bar{Q} H C_{dG} T^a \bar{\sigma}^{\mu\nu} \bar{D}^c) G^a_{\mu\nu}$$
$$+ (\bar{L}\sigma^k H C_{eW} \bar{\sigma}^{\mu\nu} \bar{E}^c) W^k_{\mu\nu} + (\bar{L} H C_{eB} \bar{\sigma}^{\mu\nu} \bar{E}^c) B_{\mu\nu} + \text{h.c.} \qquad ($$

These affect anomalous magnetic and electric moments of SM particles at tree level
Bounds depend on flavor and can be very strong, especially for the first generation

$$\sigma_{\mu\nu} = \frac{i}{2}\left[\sigma_\mu \bar{\sigma}_\nu - \sigma_\nu \bar{\sigma}_\mu\right] \qquad \bar{\sigma}_{\mu\nu} = \frac{i}{2}\left[\bar{\sigma}_\mu \sigma_\nu - \bar{\sigma}_\nu \sigma_\mu\right]$$

SMEFT at dimension-6

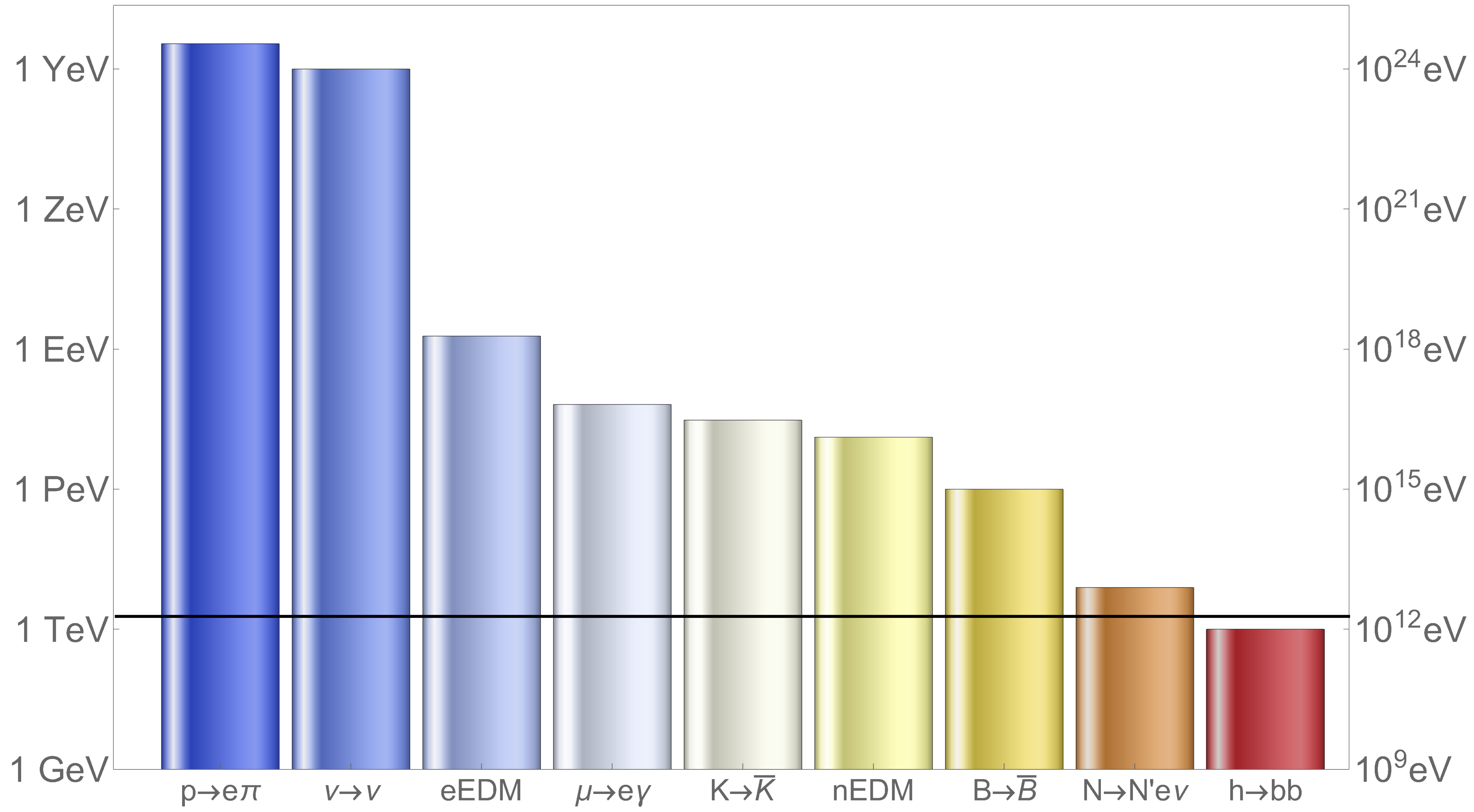
4-fermion operators

$$
\begin{aligned}
\mathcal{L}_{D=6}^{4-\text{fermion}} = & (\bar{L}\bar{\sigma}^\mu L)C_{ll}(\bar{L}\bar{\sigma}_\mu L) + (E^c\sigma_\mu\bar{E}^c)C_{ee}(E^c\sigma_\mu\bar{E}^c) + (\bar{L}\bar{\sigma}^\mu L)C_{le}(E^c\sigma_\mu\bar{E}^c) \\
& + (\bar{L}\bar{\sigma}^\mu L)C_{lq}^{(1)}(\bar{Q}\bar{\sigma}_\mu Q) + (\bar{L}\bar{\sigma}^\mu\sigma^k L)C_{lq}^{(3)}(\bar{Q}\bar{\sigma}_\mu\sigma^k Q) \\
& + (E^c\sigma_\mu\bar{E}^c)C_{eu}(U^c\sigma_\mu\bar{U}^c) + (E^c\sigma_\mu\bar{E}^c)C_{ed}(D^c\sigma_\mu\bar{D}^c) \\
& + (\bar{L}\bar{\sigma}^\mu L)C_{lu}(U^c\sigma_\mu\bar{U}^c) + (\bar{L}\bar{\sigma}^\mu L)C_{ld}(D^c\sigma_\mu\bar{D}^c) + (E^c\sigma_\mu\bar{E}^c)C_{eq}(Q\bar{\sigma}_\mu Q) \\
& + \left\{ (\bar{L}\bar{E}^c)C_{ledq}(D^c Q) + \epsilon^{kl}(\bar{L}^k\bar{E}^c)C_{lequ}^{(1)}(\bar{Q}^l\bar{U}^c) + \epsilon^{kl}(\bar{L}^k\bar{\sigma}^{\mu\nu}\bar{E}^c)C_{lequ}^{(3)}(\bar{Q}^l\bar{\sigma}^{\mu\nu}\bar{U}^c) + \text{h.c.} \right\} \\
& + (\bar{Q}\bar{\sigma}^\mu Q)C_{qq}^{(1)}(\bar{Q}\bar{\sigma}_\mu Q) + (\bar{Q}\bar{\sigma}^\mu\sigma^k Q)C_{qq}^{(3)}(\bar{Q}\bar{\sigma}_\mu\sigma^k Q) \\
& + (U^c\sigma_\mu\bar{U}^c)C_{uu}(U^c\sigma_\mu\bar{U}^c) + (D^c\sigma_\mu\bar{D}^c)C_{dd}(D^c\sigma_\mu\bar{D}^c) \\
& + (U^c\sigma_\mu\bar{U}^c)C_{ud}^{(1)}(D^c\sigma_\mu\bar{D}^c) + (U^c\sigma_\mu T^a\bar{U}^c)C_{ud}^{(8)}(D^c\sigma_\mu T^a\bar{D}^c) \\
& + (Q^c\sigma_\mu\bar{Q}^c)C_{qu}^{(1)}(U^c\sigma_\mu\bar{U}^c) + (Q^c\sigma_\mu T^a\bar{Q}^c)C_{qu}^{(8)}(U^c\sigma_\mu T^a\bar{U}^c)] \\
& + (Q^c\sigma_\mu\bar{Q}^c)C_{qd}^{(1)}(D^c\sigma_\mu\bar{D}^c) + (Q^c\sigma_\mu T^a\bar{Q}^c)C_{qd}^{(8)}(D^c\sigma_\mu T^a\bar{D}^c) \\
& + \left\{ \epsilon^{kl}(\bar{Q}^k\bar{U}^c)C_{quqd}^{(1)}(\bar{Q}^l\bar{D}^c) + \epsilon^{kl}(\bar{Q}^k T^a\bar{U}^c)C_{quqd}^{(1)}(\bar{Q}^l T^a\bar{D}^c) + \text{h.c.} \right\} \\
& + \left\{ (D^c U^c)C_{duq}(\bar{Q}\bar{L}) + (QQ)C_{qqu}(\bar{U}^c\bar{E}^c) + (QQ)C_{qqq}(QL) + (D^c U^c)C_{duu}(U^c E^c) + \text{h.c.} \right\}.
\end{aligned}
$$

These affect a wide range of physics, including neutrino physics.
Bounds can be very strong, especially for baryon-number violating operators and for certain flavor- or lepton-flavor-violating operators

SMEFT up to dimension-6

SMEFT Lagrangian up to dimension-6 provides a convenient framework for a bulk of precision physics happening today.
In particular, it allows one to quantify the strength of different observables



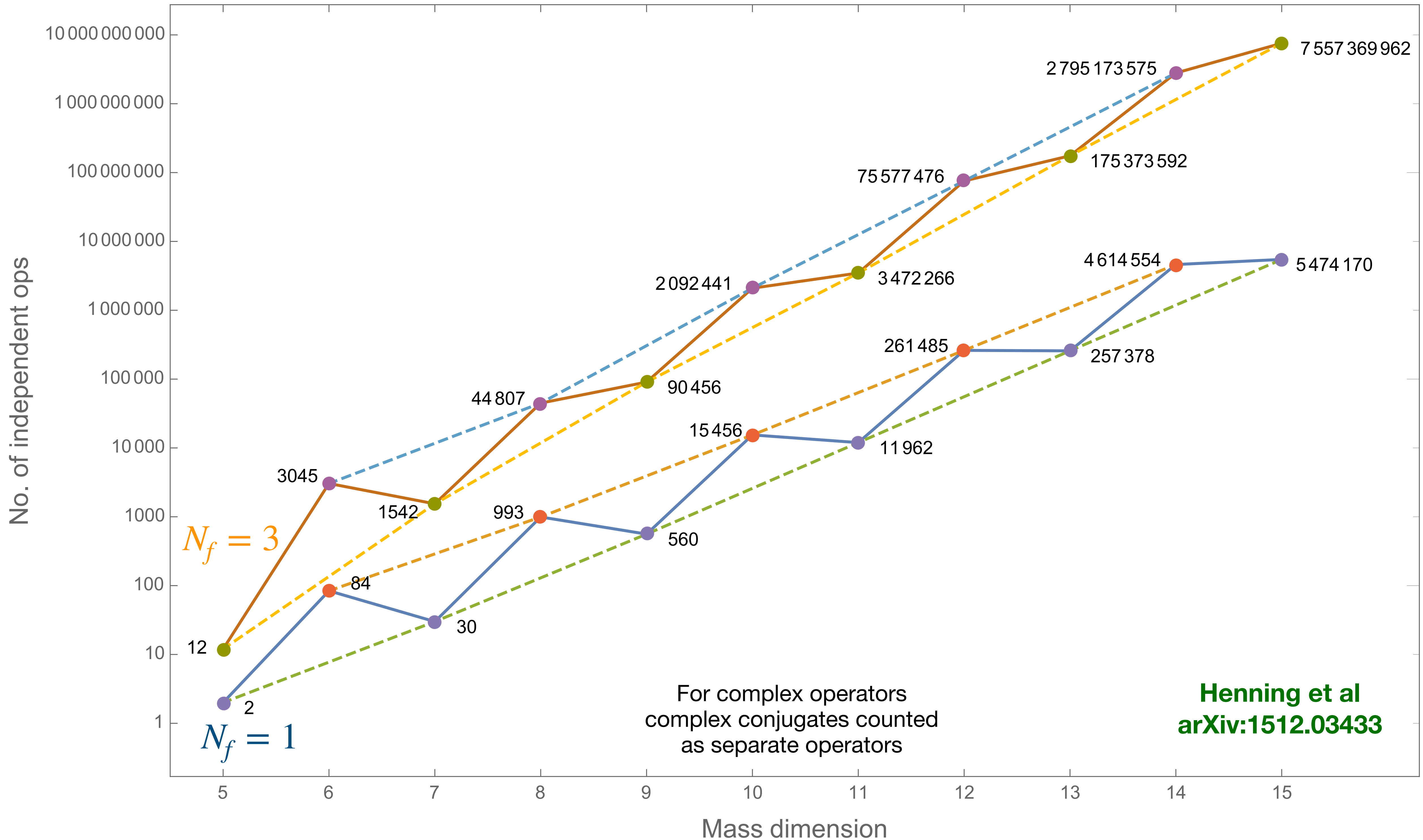
SMEFT up to dimension-6

SMEFT Lagrangian up to dimension-6 provides a convenient framework for a bulk of precision physics happening today.
Moreover, it leads to correlations between different observables, e.g. due to $SU(2)_W$ symmetry relating charged and neutral currents, and due to the interplay of tree- and loop-level contributions to observables

Importance of global fits collecting results from different types of experiments !

SMEFT at higher dimensions

$$\mathscr{L}_{\text{SMEFT}} = \mathscr{L}_{D=2} + \mathscr{L}_{D=4} + \mathscr{L}_{D=5} + \mathscr{L}_{D=6} + \mathscr{L}_{D=7} + \mathscr{L}_{D=8} + \ldots$$



Exponential growth of the number of operators with the canonical dimension D

SMEFT at higher dimensions

SMEFT at dimension-5: Weinberg (1979) Phys. Rev. Lett. 43, 1566

SMEFT at dimension-6: Grzadkowski et al arXiv: 1008.4884

SMEFT at dimension-7: Lehman arXiv: 1410.4193

SMEFT at dimension-8: Li et al arXiv: 2005.00008

SMEFT at dimension-9: Li et al arXiv: 2012.09188

SMEFT at dimension-10,11,12: Harlander, Kempkens, Schaaf arXiv: 2305.06832

Code to generate a basis at arbitrary dimension in SMEFT: Li et al arXiv:2201.04639

Beyond dimension-6

$$\mathscr{L}_{\rm SMEFT} = \mathscr{L}_{D=2} + \mathscr{L}_{D=4} + \mathscr{L}_{D=5} + \mathscr{L}_{D=6} + \mathscr{L}_{D=7} + \mathscr{L}_{D=8} + \dots$$

You need to be aware of the existence of higher-dimensional operators, whenever you need to argue validity of the EFT description

Moreover, a qualitatively new phenomenon may arise at higher dimensions

Electric and magnetic Majorana dipole moments of left-handed neutrinos arise at dimension-7

$$\mathscr{L}_{D=7} \supset (LH)\sigma^{\mu\nu}(LH)B_{\mu\nu} + \dots$$

At tree level, light-by-light scattering receives contribution from dimension-8, which in some situations may with lower order loop contributions

$$\mathscr{L}_{D=8} \supset (B_{\mu\nu}B_{\mu\nu})^2 + \dots$$

Neutron-antineutron oscillations arise at dimension-9

$$\mathscr{L}_{D=9} \supset \epsilon_{abc}\epsilon_{def}(\bar{d}_a\bar{d}_d)(q_b q_e)(q_c q_f) + \dots$$

In all such cases however, you need to argue validity of your EFT and why you don't expect any larger effects of new physics from operators of lower dimensions

Beyond dimension-6

$$\mathscr{L}_{\rm SMEFT} = \mathscr{L}_{D=2} + \mathscr{L}_{D=4} + \mathscr{L}_{D=5} + \mathscr{L}_{D=6} + \mathscr{L}_{D=7} + \mathscr{L}_{D=8} + \ldots$$

You need to be aware of the existence of higher-dimensional operators, whenever you need to argue validity of the EFT description

Moreover, a qualitatively new phenomenon may arise at higher dimensions

If experiment pinpoints a coefficient of some operators of dimension-6, then subleading dimension-8 operators will provide precious information

$$C_6 \sim \frac{g_*^2}{M^2}$$

Only determines coupling over mass scala of new physics

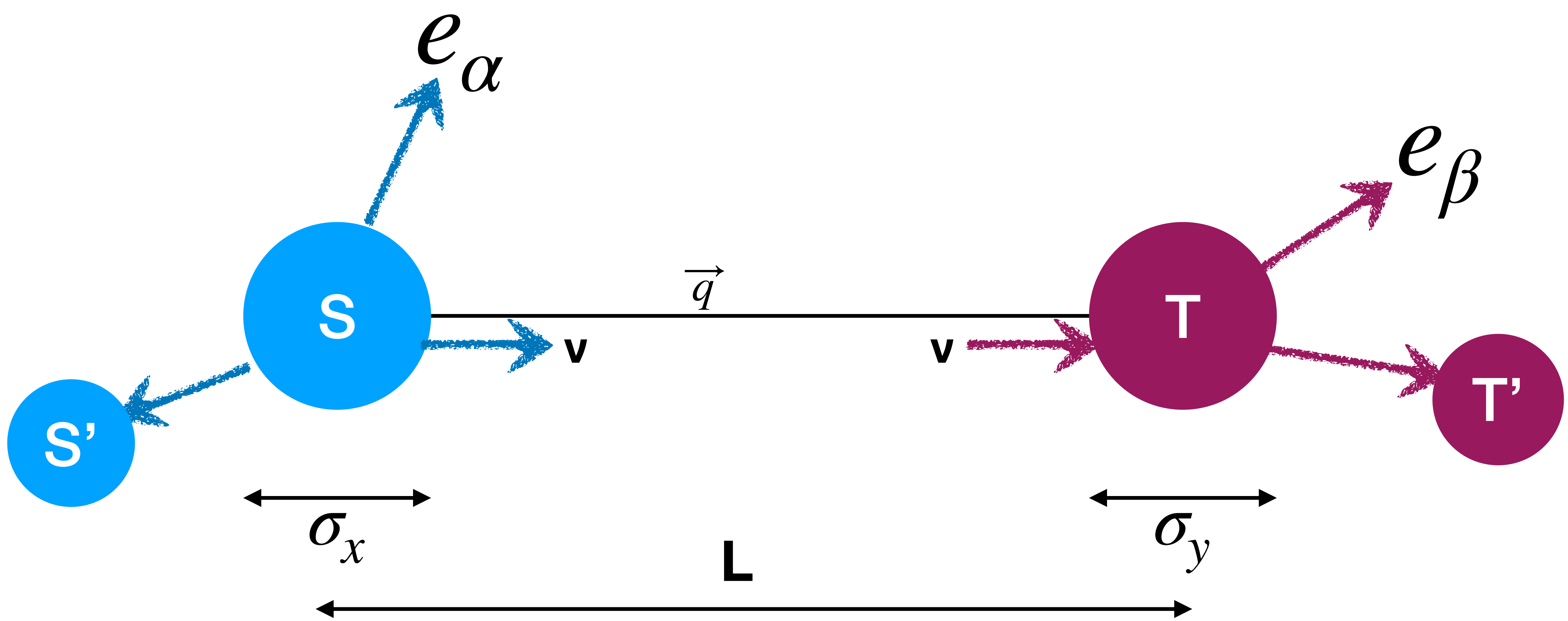
$$C_8 \sim \frac{g_*^2}{M^4}$$

May allow disentangle coupling and mass

Part 3

Some applications in neutrino physics

Neutrino oscillations in QFT



Process dominated by intermediate neutrinos close to mass shell, where amplitudes factorize into production and detection parts

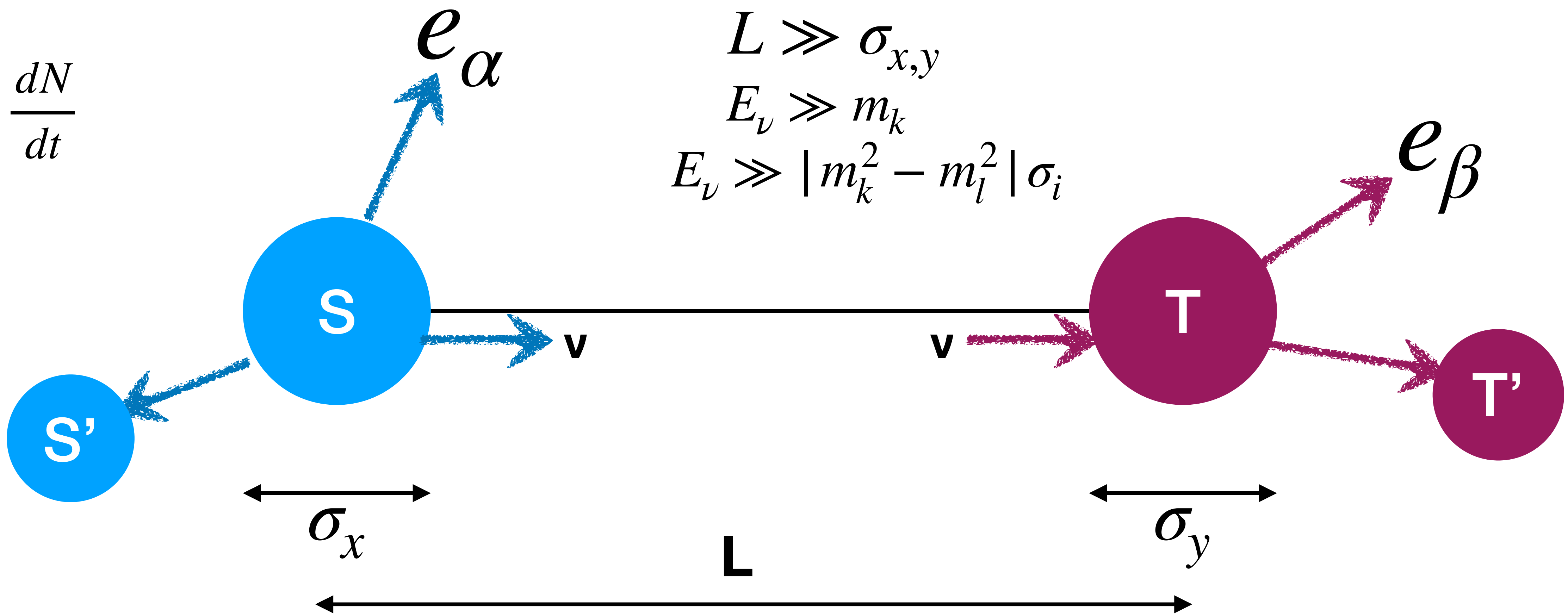
$$\mathcal{M}(ST \to S' e_\alpha T' e_\beta) = \sum_{k=1}^{3} \frac{\mathcal{M}(S \to S' e_\alpha \nu_k)\mathcal{M}(\nu_k T \to T e_\beta)}{q^2 - m_k^2 + i\epsilon} \equiv \sum_{k=1}^{3} \frac{\mathcal{M}^P_{\alpha k}\mathcal{M}^D_{\beta k}}{q^2 - m_k^2 + i\epsilon}$$

Oscillations due to interference between different neutrino mass eigenstates, possible thanks to momentum spread of source and target particles

Neutrino oscillations in QFT

$$R = \frac{dN}{dt}$$

$$L \gg \sigma_{x,y}$$
$$E_\nu \gg m_k$$
$$E_\nu \gg |m_k^2 - m_l^2| \sigma_i$$



$$\frac{dR_{\alpha\beta}}{dE_\nu} = \frac{N_S N_T}{32\pi L^2 m_S m_T} \sum_{k,l=1}^{3} \exp\left(-i\frac{L(m_k^2 - m_l^2)}{2E_\nu}\right) \int d\Pi'_P \mathcal{M}^P_{\alpha k} \bar{\mathcal{M}}^P_{\alpha l} \int d\Pi_D \mathcal{M}^D_{\beta k} \bar{\mathcal{M}}^D_{\beta l}$$

Observable rate

Geometric factor

Masses of source and target atoms

Oscillation phase

Production phase space (without integration over neutrino energy)

Detection phase space

AA, Gonzalez-Alonso, Tabrizi [arXiv:1910.02971]

Neutrino oscillations in QFT

$$\frac{dR_{\alpha\beta}}{dE_\nu} = \frac{N_S N_T}{32\pi L^2 m_S m_T} \sum_{k,l=1}^{3} \exp\left(-i\frac{L(m_k^2 - m_l^2)}{2E_\nu}\right) \int d\Pi'_P \mathcal{M}^P_{\alpha k} \bar{\mathcal{M}}^P_{\alpha l} \int d\Pi_D \mathcal{M}^D_{\beta k} \bar{\mathcal{M}}^D_{\beta l}$$

The rate above is already an observable in neutrino experiments, but to more easily compare to the commonly used language we can also define oscillation probability

$$P_{\alpha\beta} = \frac{R_{\alpha\beta}}{\Phi_\alpha \sigma_\beta}$$

Neutrino flux at the source (Φ_α)

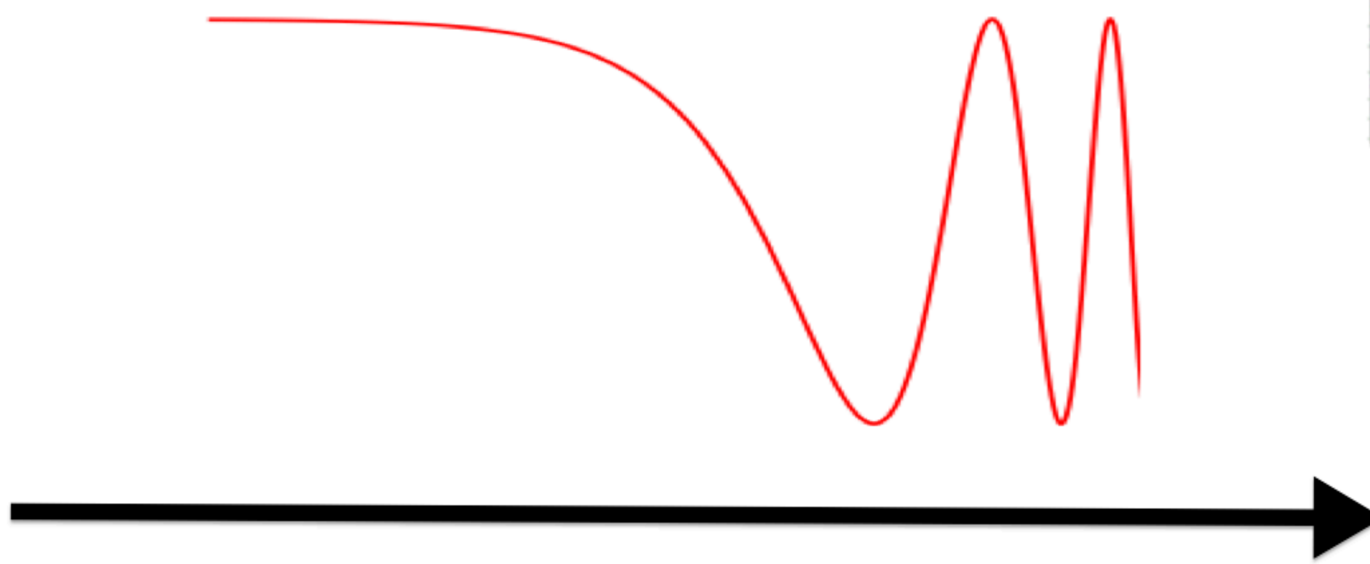
Neutrino cross section at the target (σ_β)

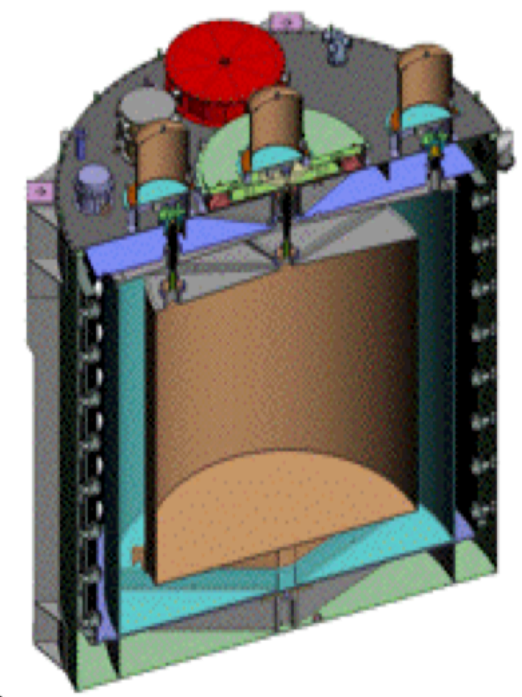
$$\frac{dP_{\alpha\beta}}{dE_\nu} = \frac{\sum_{k,l=1}^{3} \exp\left(-i\frac{L(m_k^2 - m_l^2)}{2E_\nu}\right) \int d\Pi'_P \mathcal{M}^P_{\alpha k} \bar{\mathcal{M}}^P_{\alpha l} \int d\Pi_D \mathcal{M}^D_{\beta k} \bar{\mathcal{M}}^D_{\beta l}}{\sum_{k,l=1}^{3} \int d\Pi'_P |\mathcal{M}^P_{\alpha k}|^2 \int d\Pi_D |\mathcal{M}^D_{\beta l}|^2}$$

At this point problem reduced to calculating Feynman diagrams and integrating over phase space

One phenomenological application:
electron antineutrino survival probability in reactor experiments

$$\bar{\nu}_e \to \bar{\nu}_e$$





[arXiv:1901.04553]
with Martin Gonzalez-Alonso and Zahra Tabrizi

Reactor neutrino oscillations in EFT

$$P_{\bar{\nu}_e \to \bar{\nu}_e} = \frac{\sum_{k,l=1}^{3} \exp\left(-i\frac{L(m_k^2 - m_l^2)}{2E_\nu}\right) \int d\Pi'_P \mathscr{M}_k^P \bar{\mathscr{M}}_l^P \int d\Pi_D \mathscr{M}_k^D \bar{\mathscr{M}}_l^D}{\int d\Pi'_P \sum_{k=1}^{3} |\mathscr{M}_k^P|^2 \int d\Pi_D \sum_{l=1}^{3} |\mathscr{M}_l^D|^2}$$

Short-baseline oscillations of electron antineutrinos produced in reactors

$\Rightarrow \Delta m_{21}^2 \approx 0$

Relevant for Daya Bay, RENO, Double Chooz

$$P_{\bar{\nu}_e \to \bar{\nu}_e} = 1 - \sin^2\left(\frac{\Delta m_{31}^2 L}{4E_\nu}\right) \sin^2\left(2\tilde{\theta}_{13} - \alpha_D \frac{m_e}{E_\nu - \Delta} - \alpha_P \frac{m_e}{f_T(E_\nu)}\right)$$
$$+\sin\left(\frac{\Delta m_{31}^2 L}{2E_\nu}\right) \sin(2\tilde{\theta}_{13})\left(\gamma_R + \beta_D \frac{m_e}{E_\nu - \Delta} - \beta_P \frac{m_e}{f_T(E_\nu)}\right) + \mathcal{O}(\epsilon_X^2) + \mathcal{O}(\Delta m_{21}^2)$$

Reactor neutrino oscillations in WEFT

Usual CP-conserving oscillation pattern (remains in SM limit)

$$P_{\bar{\nu}_e \to \bar{\nu}_e} = 1 - \sin^2\left(\frac{\Delta m_{31}^2 L}{4E_\nu}\right)\sin^2\left(2\tilde{\theta}_{13} - \alpha_D \frac{m_e}{E_\nu - \Delta} - \alpha_P \frac{m_e}{f_T(E_\nu)}\right)$$
$$+\sin\left(\frac{\Delta m_{31}^2 L}{2E_\nu}\right)\sin(2\tilde{\theta}_{13})\left(\gamma_R + \beta_D \frac{m_e}{E_\nu - \Delta} - \beta_P \frac{m_e}{f_T(E_\nu)}\right) + \mathcal{O}(\epsilon_X^2) + \mathcal{O}(\Delta m_{21}^2)$$

CP-violating oscillations (vanishes in SM limit)

Reactor neutrino oscillations in WEFT

$$P_{\bar\nu_e\to\bar\nu_e} = 1 - \sin^2\left(\frac{\Delta m^2_{31}L}{4E_\nu}\right)\sin^2\left(2\tilde\theta_{13} - \alpha_D\frac{m_e}{E_\nu-\Delta} - \alpha_P\frac{m_e}{f_T(E_\nu)}\right)$$
$$+\sin\left(\frac{\Delta m^2_{31}L}{2E_\nu}\right)\sin(2\tilde\theta_{13})\left(\gamma_R + \beta_D\frac{m_e}{E_\nu-\Delta} - \beta_P\frac{m_e}{f_T(E_\nu)}\right) + \mathcal{O}(\epsilon_X^2) + \mathcal{O}(\Delta m^2_{21})$$

Standard Θ_{13} mixing angle replaced by effective angle:

$$\tilde\theta_{13} = \theta_{13} + \mathrm{Re}[L] - \frac{3g_A^2}{3g_A^2+1}\mathrm{Re}[R]$$

$g_A \approx 1.25$

Original PMNS mixing angle

$$[X] \equiv e^{i\delta_{\rm CP}}\left(s_{23}[\epsilon_X]_{e\mu} + c_{23}[\epsilon_X]_{e\tau}\right)$$

$$\mathcal{L}_{WEFT} \supset -\frac{2V_{ud}}{v^2}\left[\left[1+\epsilon_L\right]_{e\beta}(\bar e\bar\sigma^\mu\nu_\beta)(\bar u\bar\sigma^\mu d) + \left[\epsilon_R\right]_{e\beta}(\bar e\bar\sigma_\mu\nu_\beta)(u^c\sigma^\mu\bar d^c) + \ldots\right] + \mathrm{h.c.}$$

Reactor neutrino oscillations in WEFT

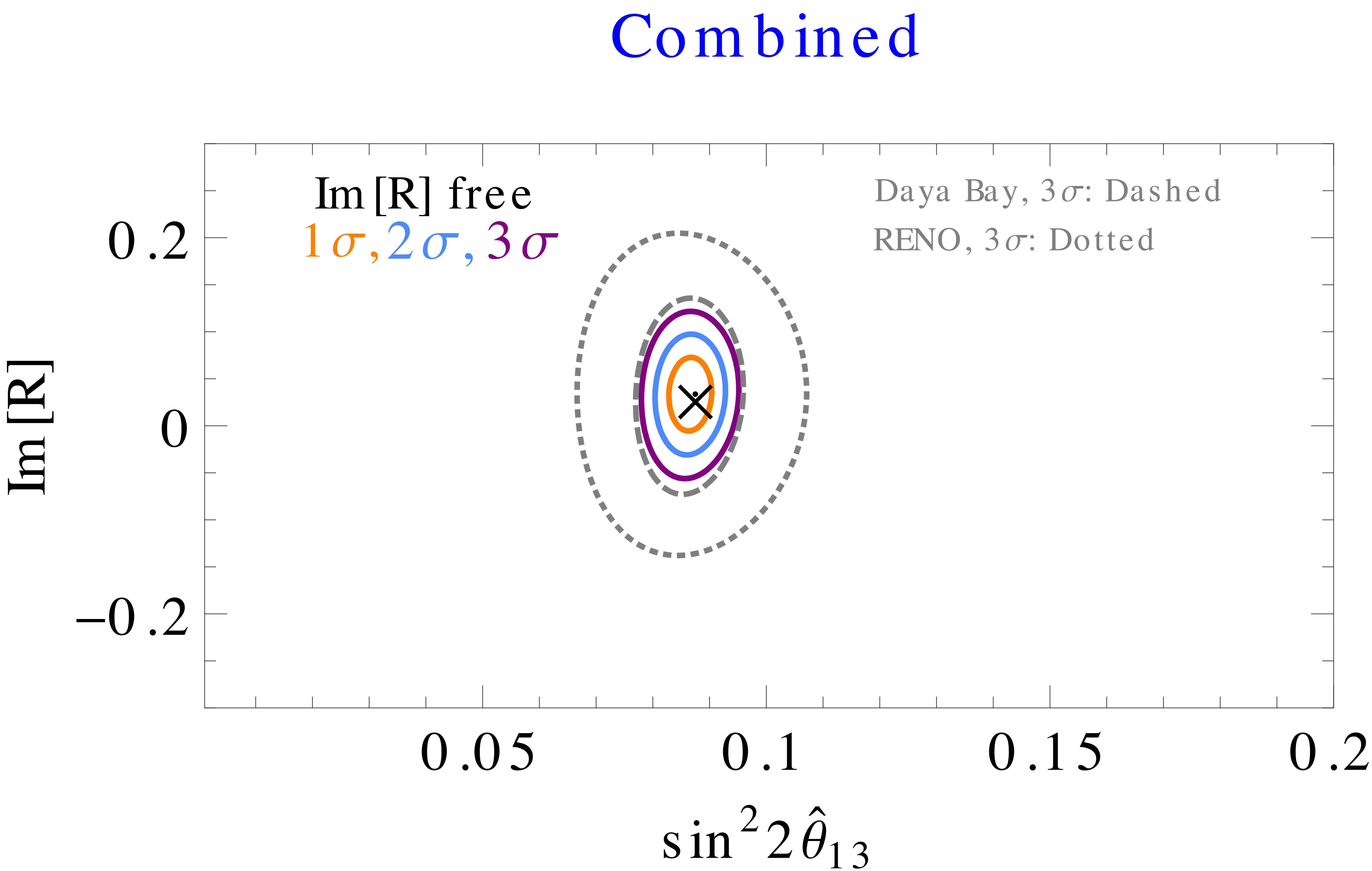
$$P_{\bar{\nu}_e\to\bar{\nu}_e} = 1 - \sin^2\left(\frac{\Delta m^2_{31}L}{4E_\nu}\right)\sin^2\left(2\tilde{\theta}_{13} - \alpha_D\frac{m_e}{E_\nu-\Delta} - \alpha_P\frac{m_e}{f_T(E_\nu)}\right)$$
$$+\sin\left(\frac{\Delta m^2_{31}L}{2E_\nu}\right)\sin(2\tilde{\theta}_{13})\left(\gamma_R + \beta_D\frac{m_e}{E_\nu-\Delta} - \beta_P\frac{m_e}{f_T(E_\nu)}\right) + \mathcal{O}(\epsilon_X^2) + \mathcal{O}(\Delta m^2_{21})$$

$$\gamma_R = -\frac{2}{3g_A^2+1}\mathrm{Im}\left[e^{i\delta_{\mathrm{CP}}}\left(s_{23}[\epsilon_R]_{e\mu} + c_{23}[\epsilon_R]_{e\tau}\right)\right]$$

Reactor neutrino oscillations are sensitive at linear level to flavor-off-diagonal WEFT Wilson coefficients ϵ_R

$$\mathcal{L}_{\mathrm{WEFT}} \supset -\frac{2V_{ud}}{v^2}\left[\left[1+\epsilon_L\right]_{e\beta}(\bar{e}\bar{\sigma}^\mu\nu_\beta)(\bar{u}\bar{\sigma}^\mu d) + \left[\epsilon_R\right]_{e\beta}(\bar{e}\bar{\sigma}_\mu\nu_\beta)(u^c\sigma^\mu\bar{d}^c) + \ldots\right] + \mathrm{h.c.}$$

Reactor neutrino oscillations in WEFT

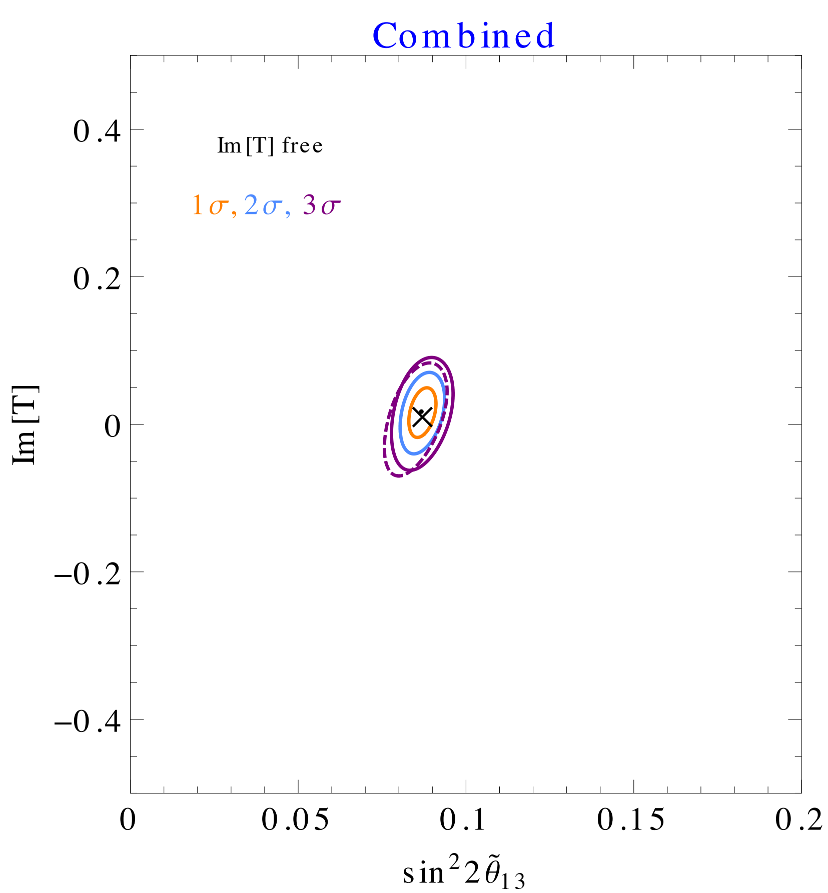


Note we cannot assume θ_{13} from SM fit.
SM and BSM have to be fit simultaneously!

Reactor neutrino oscillations in WEFT

$$P_{\bar{\nu}_e\to\bar{\nu}_e} = 1 - \sin^2\left(\frac{\Delta m_{31}^2 L}{4E_\nu}\right)\sin^2\left(2\tilde{\theta}_{13} - \alpha_D\frac{m_e}{E_\nu - \Delta} - \alpha_P\frac{m_e}{f_T(E_\nu)}\right)$$
$$+\sin\left(\frac{\Delta m_{31}^2 L}{2E_\nu}\right)\sin(2\tilde{\theta}_{13})\left(\gamma_R + \beta_D\frac{m_e}{E_\nu - \Delta} - \beta_P\frac{m_e}{f_T(E_\nu)}\right) + \mathcal{O}(\epsilon_X^2) + \mathcal{O}(\Delta m_{21}^2)$$

The parameters $\alpha_{D,P}, \beta_{D,P}$ correspond to scalar and tensor 4-fermion interactions involving 1st generation quarks and leptons



E.g. tensor interaction

$$-\frac{2V_{ud}}{v^2}\frac{1}{4}\left[\epsilon_T\right]_{\alpha\beta}(\bar{e}^c_\alpha\sigma_{\mu\nu}\nu_\beta)(u^c\sigma^{\mu\nu}d)$$

SMEFT at dimension-6

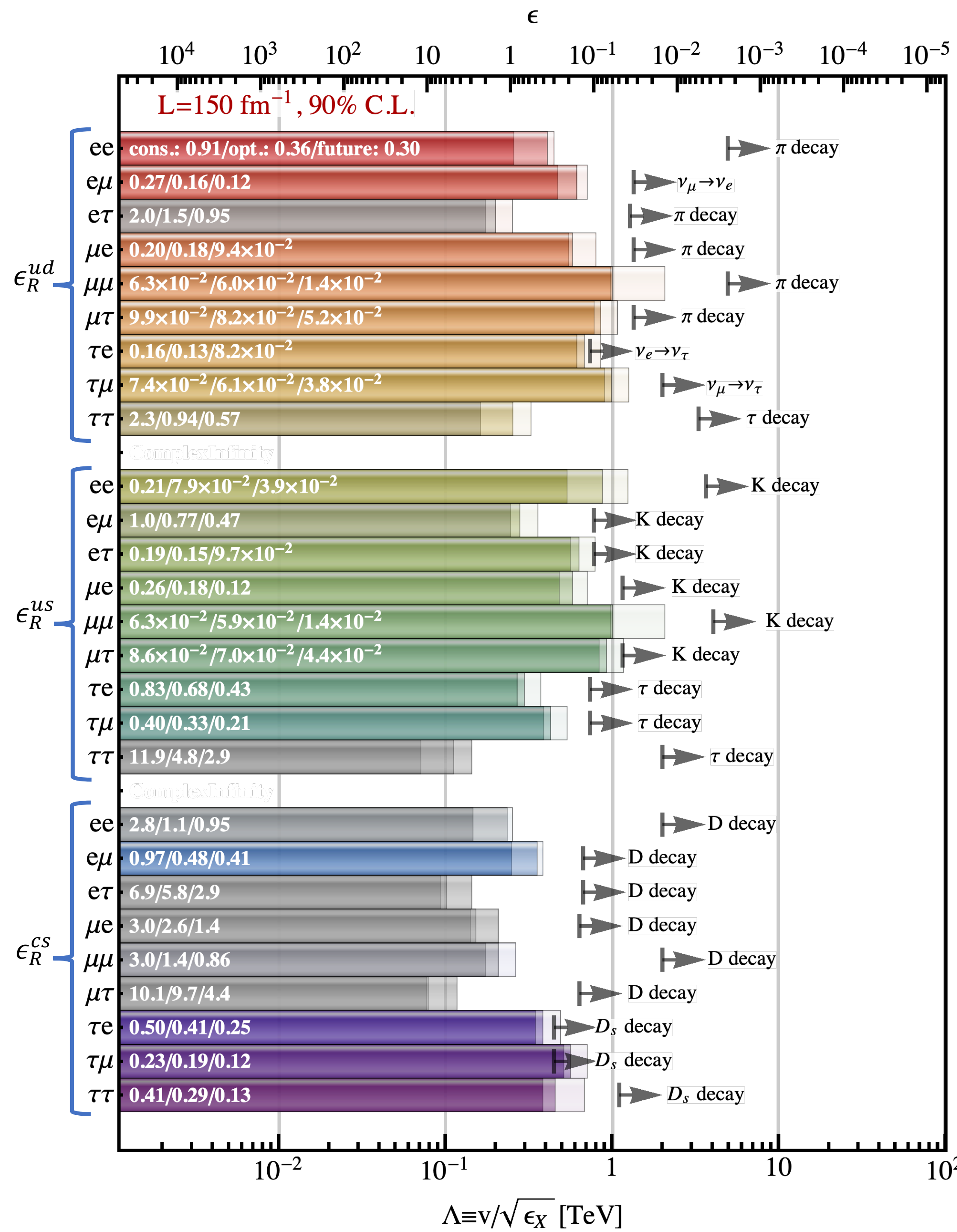
$$\mathscr{L}_{D=6}^{4-\text{fermion}} = (\bar{L}\bar{\sigma}^\mu L)C_{ll}(\bar{L}\bar{\sigma}_\mu L) + (E^c\sigma_\mu\bar{E}^c)C_{ee}(E^c\sigma_\mu\bar{E}^c) + (\bar{L}\bar{\sigma}^\mu L)C_{le}(E^c\sigma_\mu\bar{E}^c)$$
$$+(\bar{L}\bar{\sigma}^\mu L)C_{lq}^{(1)}(\bar{Q}\bar{\sigma}_\mu Q) + (\bar{L}\bar{\sigma}^\mu\sigma^k L)C_{lq}^{(3)}(\bar{Q}\bar{\sigma}_\mu\sigma^k Q)$$
$$+(E^c\sigma_\mu\bar{E}^c)C_{eu}(U^c\sigma_\mu\bar{U}^c) + (E^c\sigma_\mu\bar{E}^c)C_{ed}(D^c\sigma_\mu\bar{D}^c)$$
$$+(\bar{L}\bar{\sigma}^\mu L)C_{lu}(U^c\sigma_\mu\bar{U}^c) + (\bar{L}\bar{\sigma}^\mu L)C_{ld}(D^c\sigma_\mu\bar{D}^c) + (E^c\sigma_\mu\bar{E}^c)C_{eq}(Q\bar{\sigma}_\mu Q)$$
$$+\left\{(\bar{L}\bar{E}^c)C_{ledq}(D^cQ) + \epsilon^{kl}(\bar{L}^k\bar{E}^c)C_{lequ}^{(1)}(\bar{Q}^l\bar{U}^c) + \epsilon^{kl}(\bar{L}^k\bar{\sigma}^{\mu\nu}\bar{E}^c)C_{lequ}^{(3)}(\bar{Q}^l\bar{\sigma}^{\mu\nu}\bar{U}^c) + \mathrm{h.c.}\right\}$$
$$+(\bar{Q}\bar{\sigma}^\mu Q)C_{qq}^{(1)}(\bar{Q}\bar{\sigma}_\mu Q) + (\bar{Q}\bar{\sigma}^\mu\sigma^k Q)C_{qq}^{(3)}(\bar{Q}\bar{\sigma}_\mu\sigma^k Q)$$
$$+(U^c\sigma_\mu\bar{U}^c)C_{uu}(U^c\sigma_\mu\bar{U}^c) + (D^c\sigma_\mu\bar{D}^c)C_{dd}(D^c\sigma_\mu\bar{D}^c)$$
$$+(U^c\sigma_\mu\bar{U}^c)C_{ud}^{(1)}(D^c\sigma_\mu\bar{D}^c) + (U^c\sigma_\mu T^a\bar{U}^c)C_{ud}^{(8)}(D^c\sigma_\mu T^a\bar{D}^c)$$
$$+(Q^c\sigma_\mu\bar{Q}^c)C_{qu}^{(1)}(U^c\sigma_\mu\bar{U}^c) + (Q^c\sigma_\mu T^a\bar{Q}^c)C_{qu}^{(8)}(U^c\sigma_\mu T^a\bar{U}^c)]$$
$$+(Q^c\sigma_\mu\bar{Q}^c)C_{qd}^{(1)}(D^c\sigma_\mu\bar{D}^c) + (Q^c\sigma_\mu T^a\bar{Q}^c)C_{qd}^{(8)}(D^c\sigma_\mu T^a\bar{D}^c)$$
$$+\left\{\epsilon^{kl}(\bar{Q}^k\bar{U}^c)C_{quqd}^{(1)}(\bar{Q}^l\bar{D}^c) + \epsilon^{kl}(\bar{Q}^kT^a\bar{U}^c)C_{quqd}^{(1)}(\bar{Q}^lT^a\bar{D}^c) + \mathrm{h.c.}\right\}$$
$$+\left\{(D^cU^c)C_{duq}(\bar{Q}\bar{L}) + (QQ)C_{qqu}(\bar{U}^c\bar{E}^c) + (QQ)C_{qqq}(QL) + (D^cU^c)C_{duu}(U^cE^c) + \mathrm{h.c.}\right\}.$$

$$O_{Hud} = i\tilde{H}^\dagger D_\mu H(U^c\sigma^\mu\bar{D}^c)$$

Several SMEFT operators are probed by $\nu_e \to \nu_e$ measurements

All in all, short baseline reactor neutrino oscillations sensitive to 5 distinct linear combinations of dimension-6 SMEFT operators

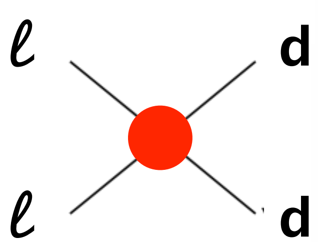
Projected FASERnu constraints



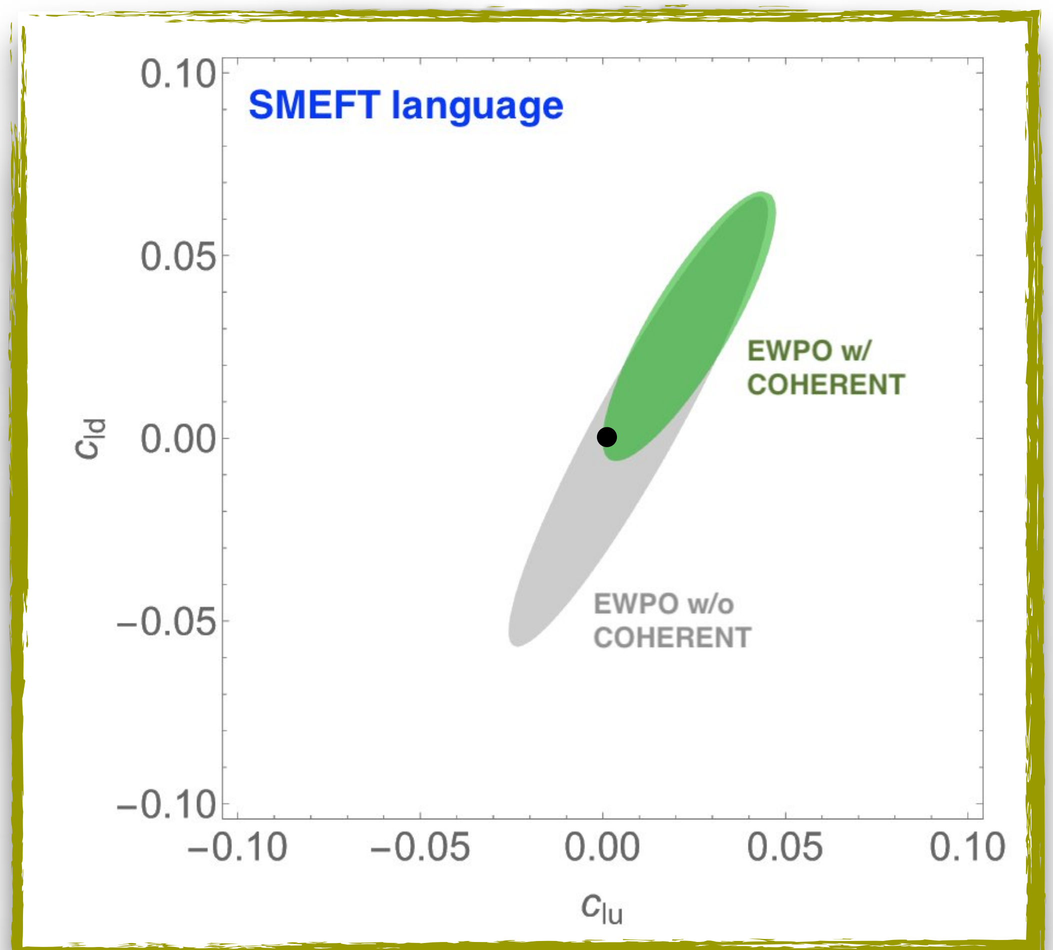
Multiple operators will be probed, although existing constraints are stronger in a 1-at-a-time analysis

[arXiv:2105.12136]
with Joachim Kopp, Martin Gonzalez-Alonso, Yotam Soreq and Zahra Tabrizi

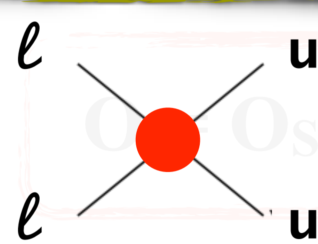
Current COHERENT constraints



$$\bar{\ell}\gamma_\mu \ell \;\; \bar{d}\gamma^\mu d$$



$$\bar{\ell}\gamma_\mu \ell \;\; \bar{u}\gamma^\mu u$$



Several neutral-current operators are probed. Constraints complementary to exist electroweak precision constraints and improve visibly the current global fits

[arXiv:2301.07036]
with Victor Breso-Pla, and Martin Gonzalez-Alonso

Summary

- EFT is a universal language to describe a multitude of low-energy and high-energy experiments, including neutrino oscillations and neutrino scattering on various targets

- SMEFT is perhaps the most popular EFT framework, as it does not assume existence of any particles beyond those of the Standard Model

- SMEFT allows one to combine constraints from different experiments and compare their sensitivity, in particular between neutrino experiments, LHC high-energy scattering, and LEP precision observables. All these inputs are vital to constrain the multi-dimensional parameter space of SMEFT

- A complete SMEFT analysis of neutrino oscillation experiments has not been done yet