Lucien HEURTIER

IPHC - Strasbourg

Primordial Black Hole Archaeology

Durham University

UKRI Science and Technology Facilities Council

References

- A. Cheek (Warsaw), L.H., Y.F. Perez-Gonzalez (Durham), J. Turner (Durham)

- *Phys.Rev.D 105 (2022) 1, 015022* [PRD Highlights]

- *Phys.Rev.D 105 (2022) 1, 015023* [PRD Highlights]

- Phys.Rev.D *106 (2022) 10, 103012*

- ArXiv: 2212.03878 Dec 2022, Submitted to PRD

- K.R. Dienes (Arizona), L.H., F. Huang (Israel), D. Kim (Texas), B. Thomas, and T.M.P. Tait (California)

- ArXiv: 2212.01369 Dec 2022, Submitted to PRD

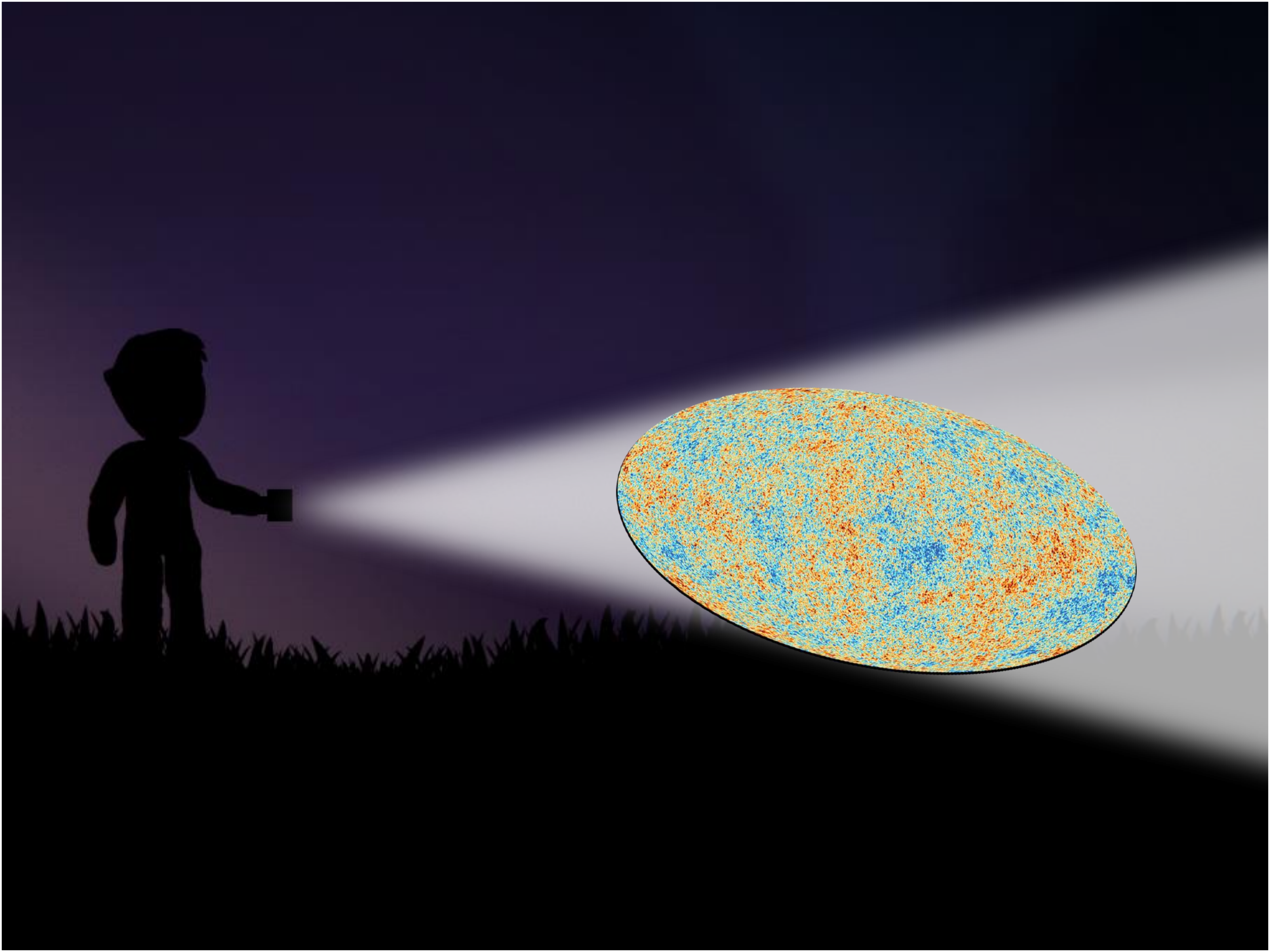
- A. Ghoshal (Warsaw), Y. Gouttenoire (Israel), L.H., P. Simakachorn (Spain)

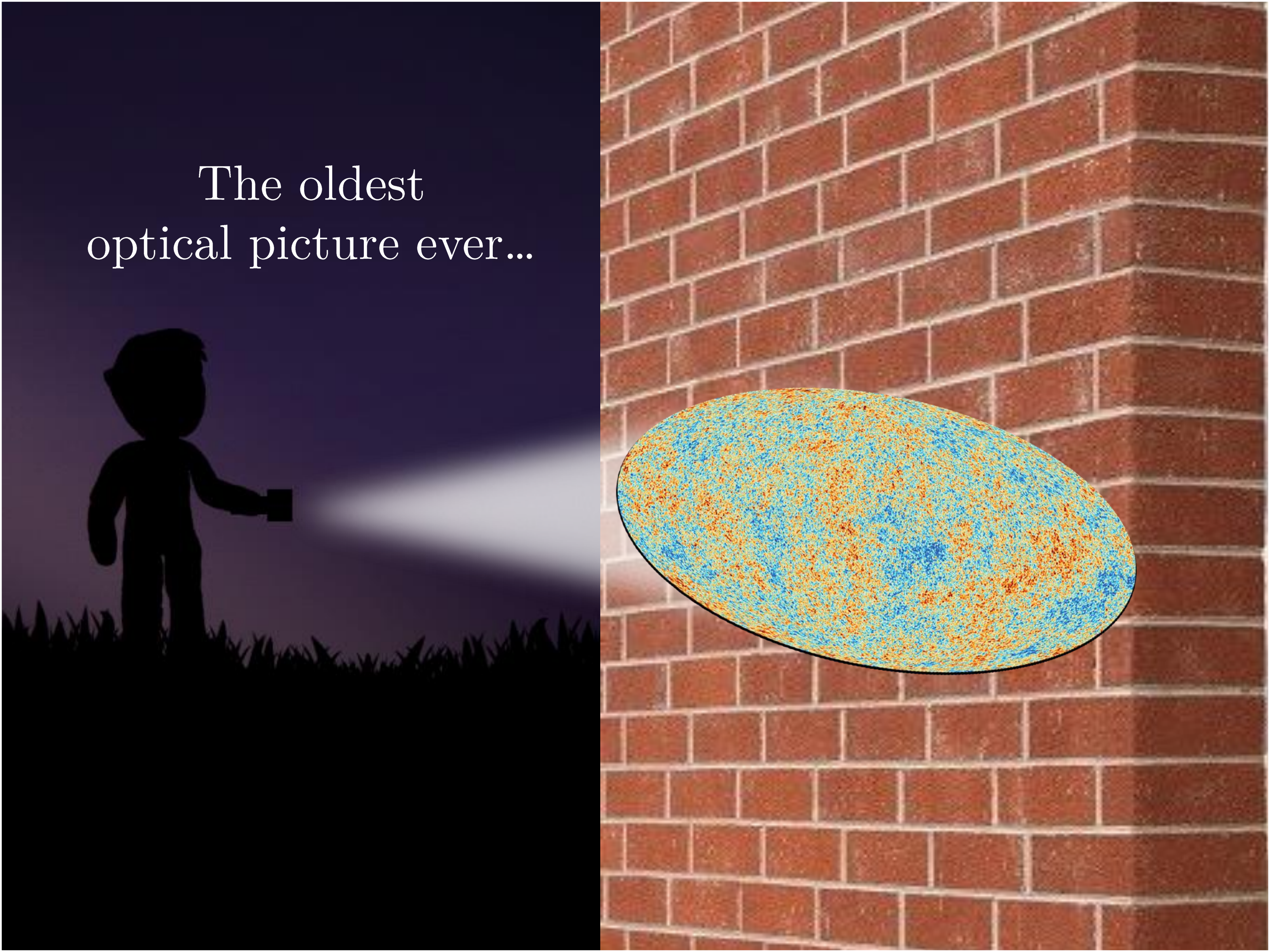
- ArXiv: 2302.XXXX Feb 2023, *to appear*

- S. Mishra (Nottingham), L.H., and V. Vennin (Paris)

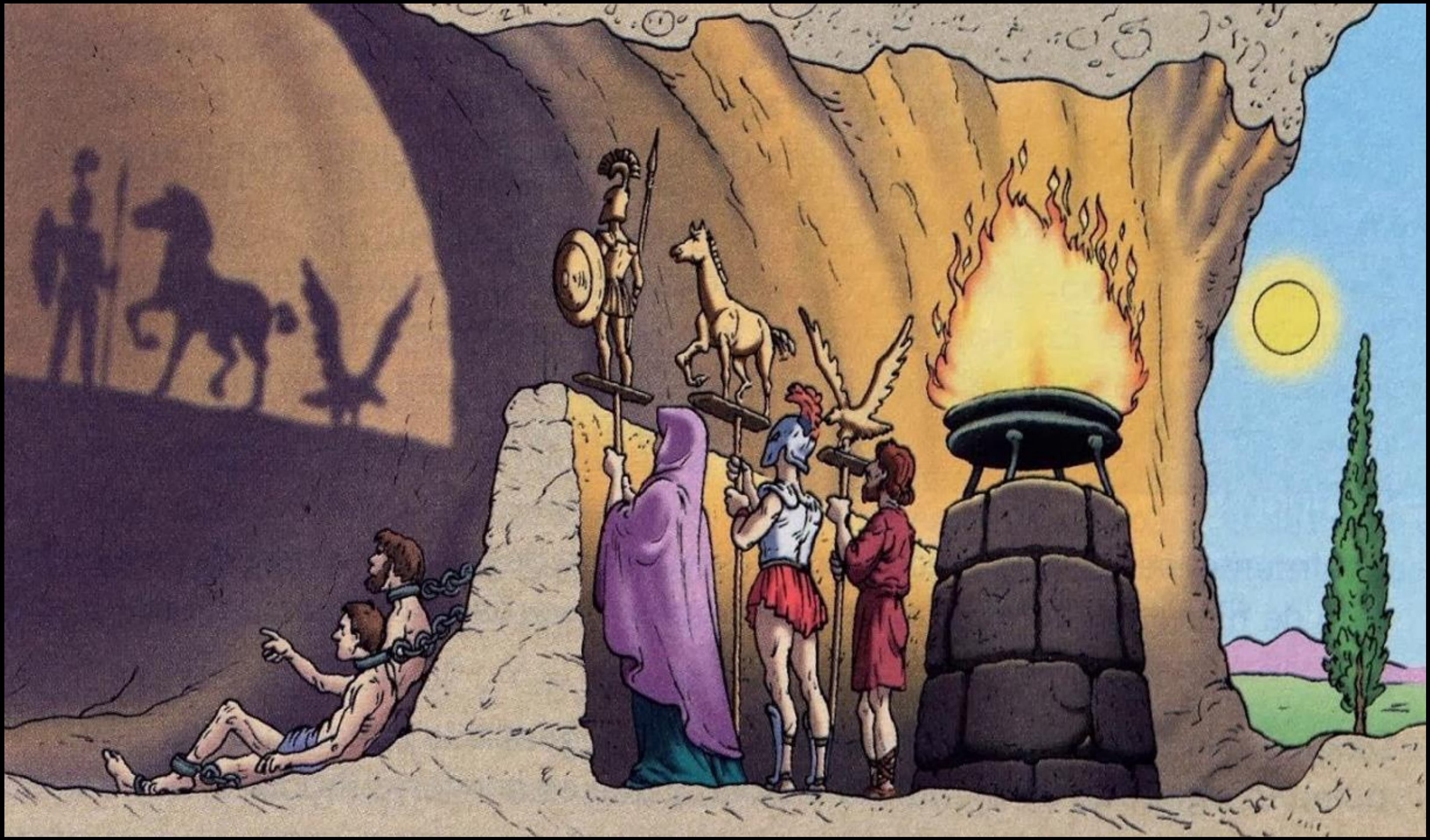
- ArXiv: 23XX.XXXX 2023, *on-going project*

WHY
Primordial
Black Holes?



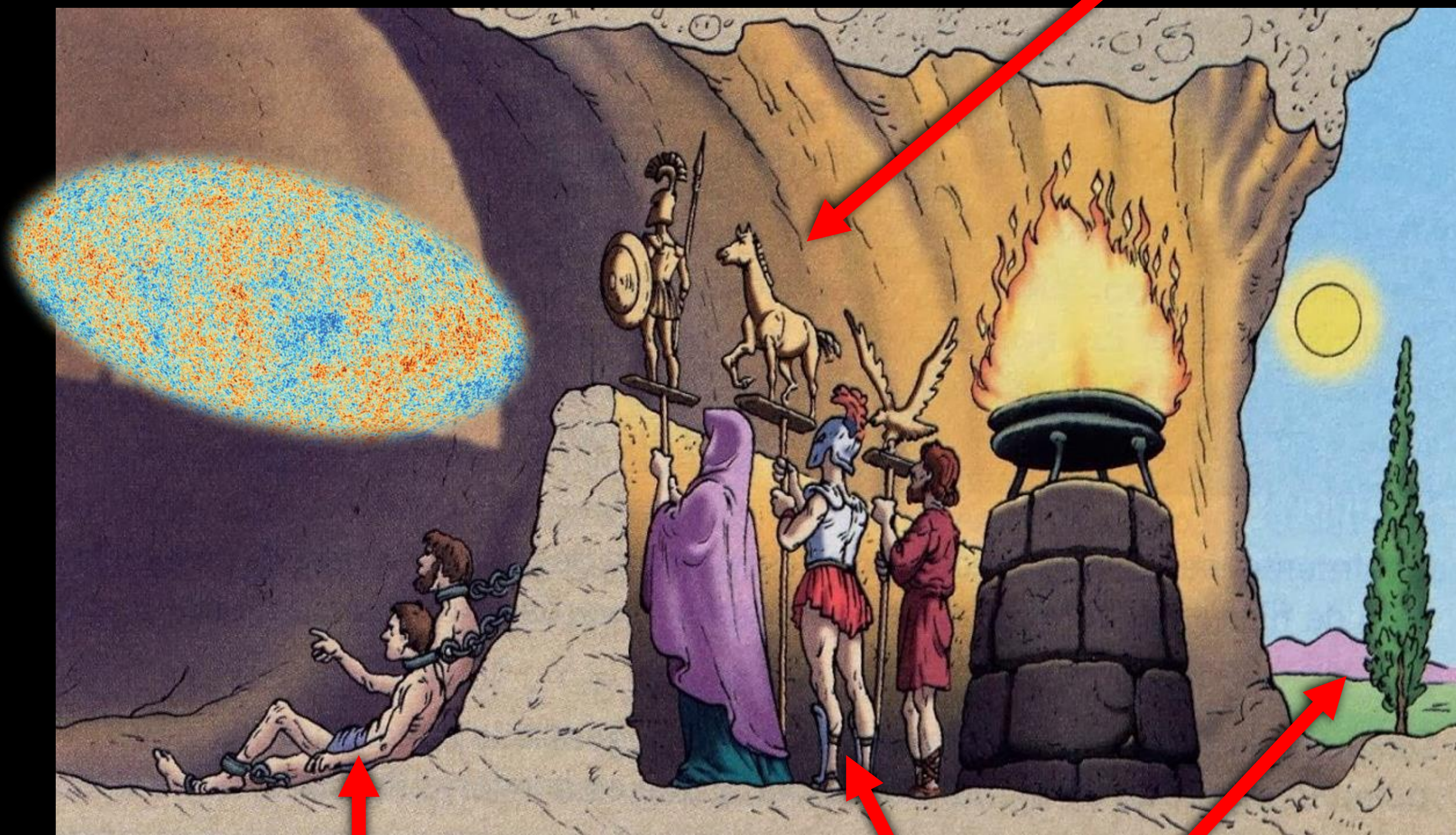

The oldest
optical picture ever...

Allegory of the Cave...



Allegory of the Cave...

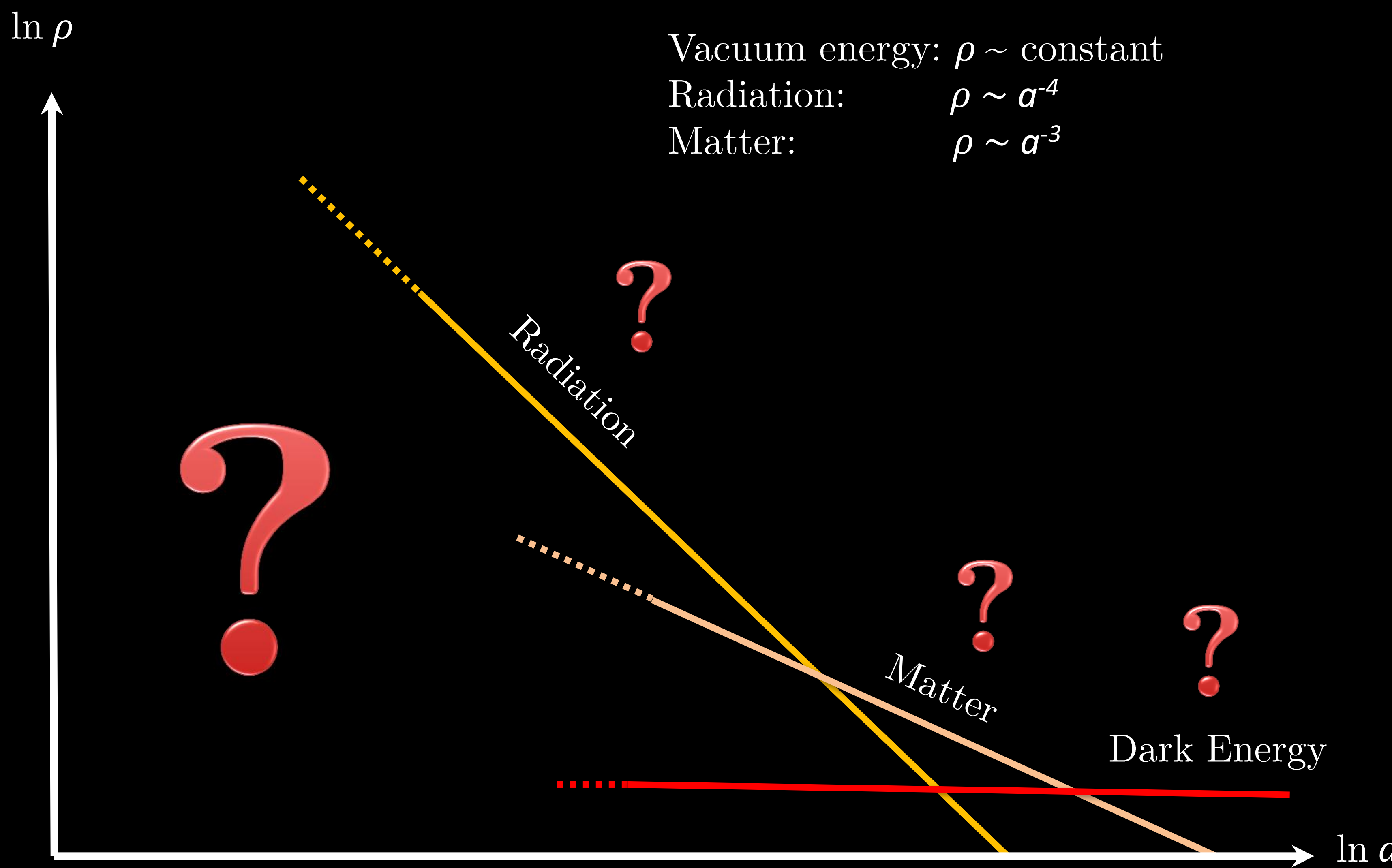
ΛCDM



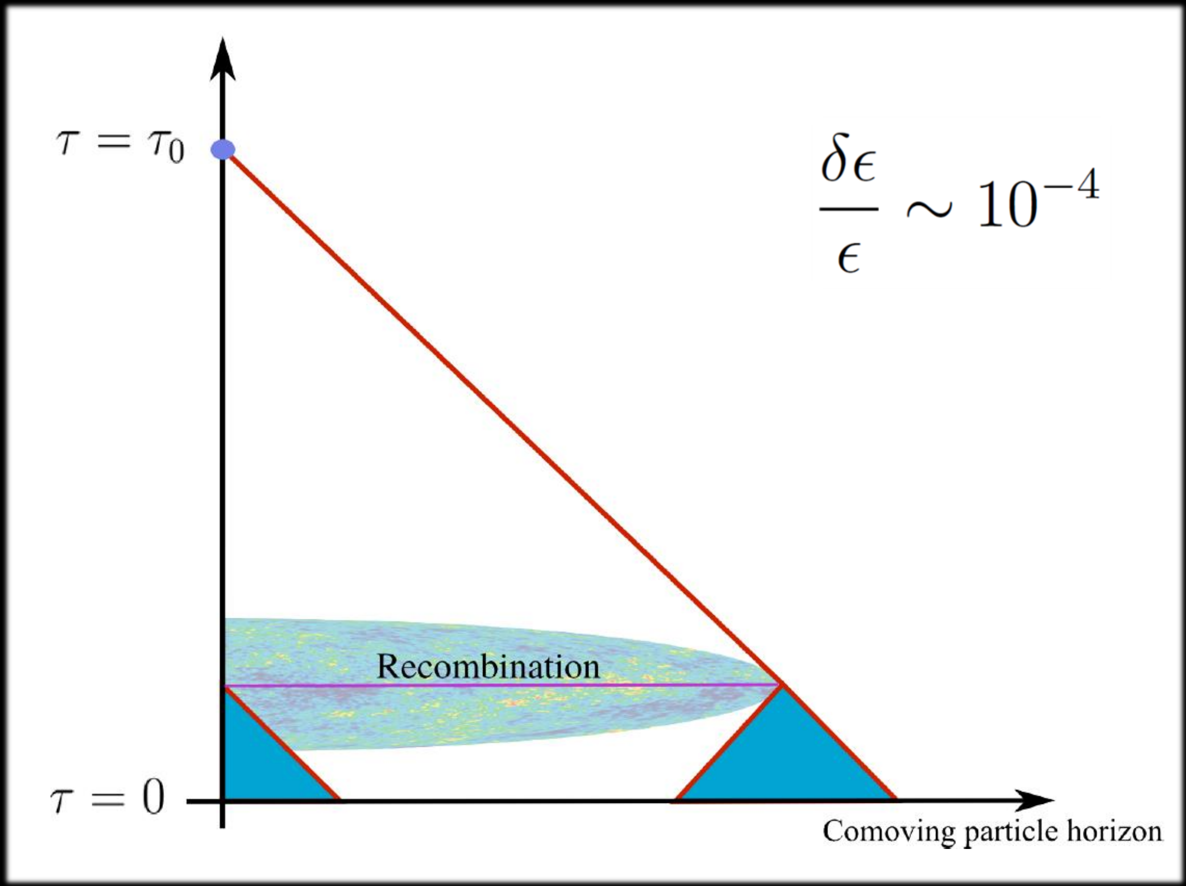
Us

Nature

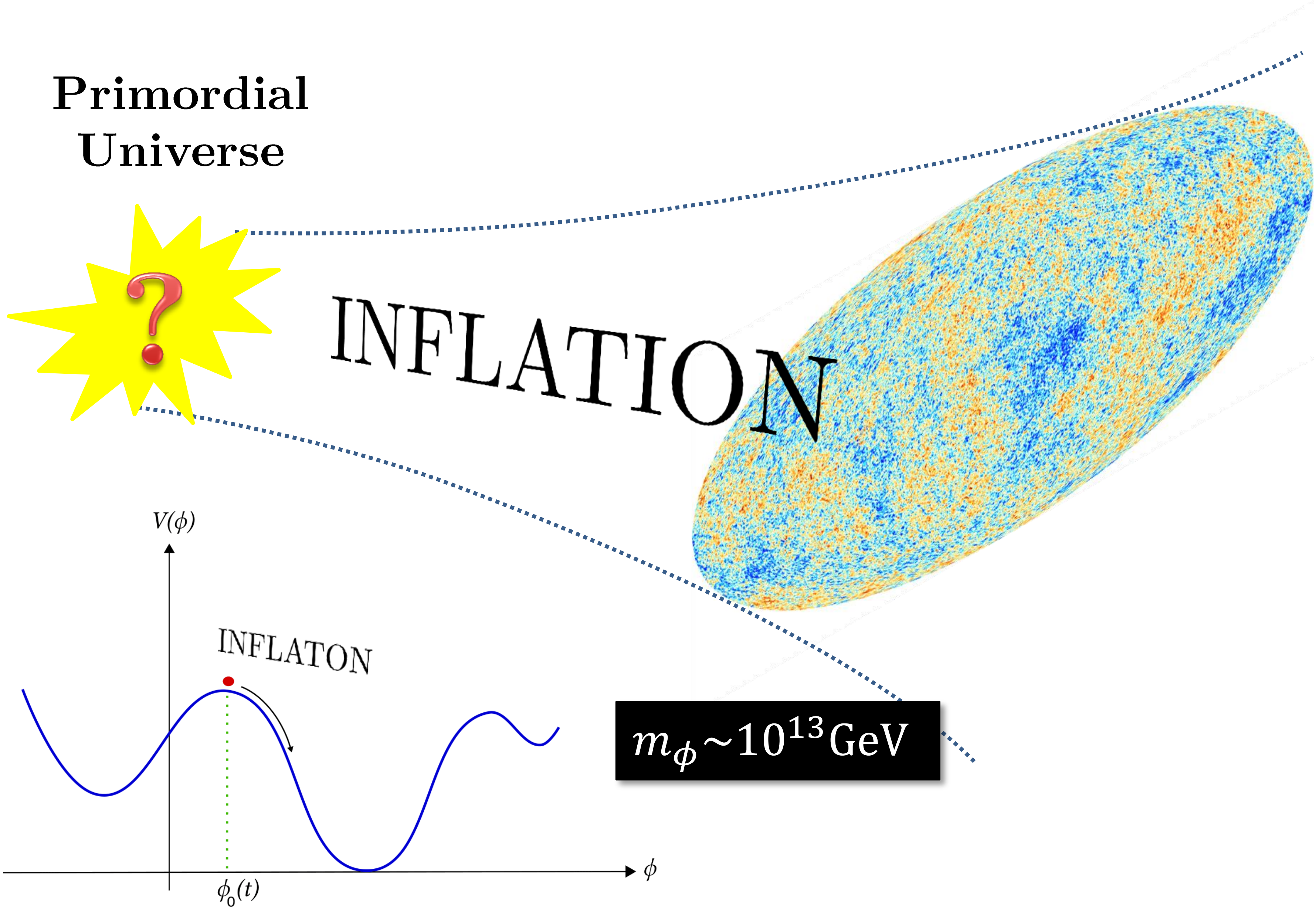
The ΛCDM Ideology



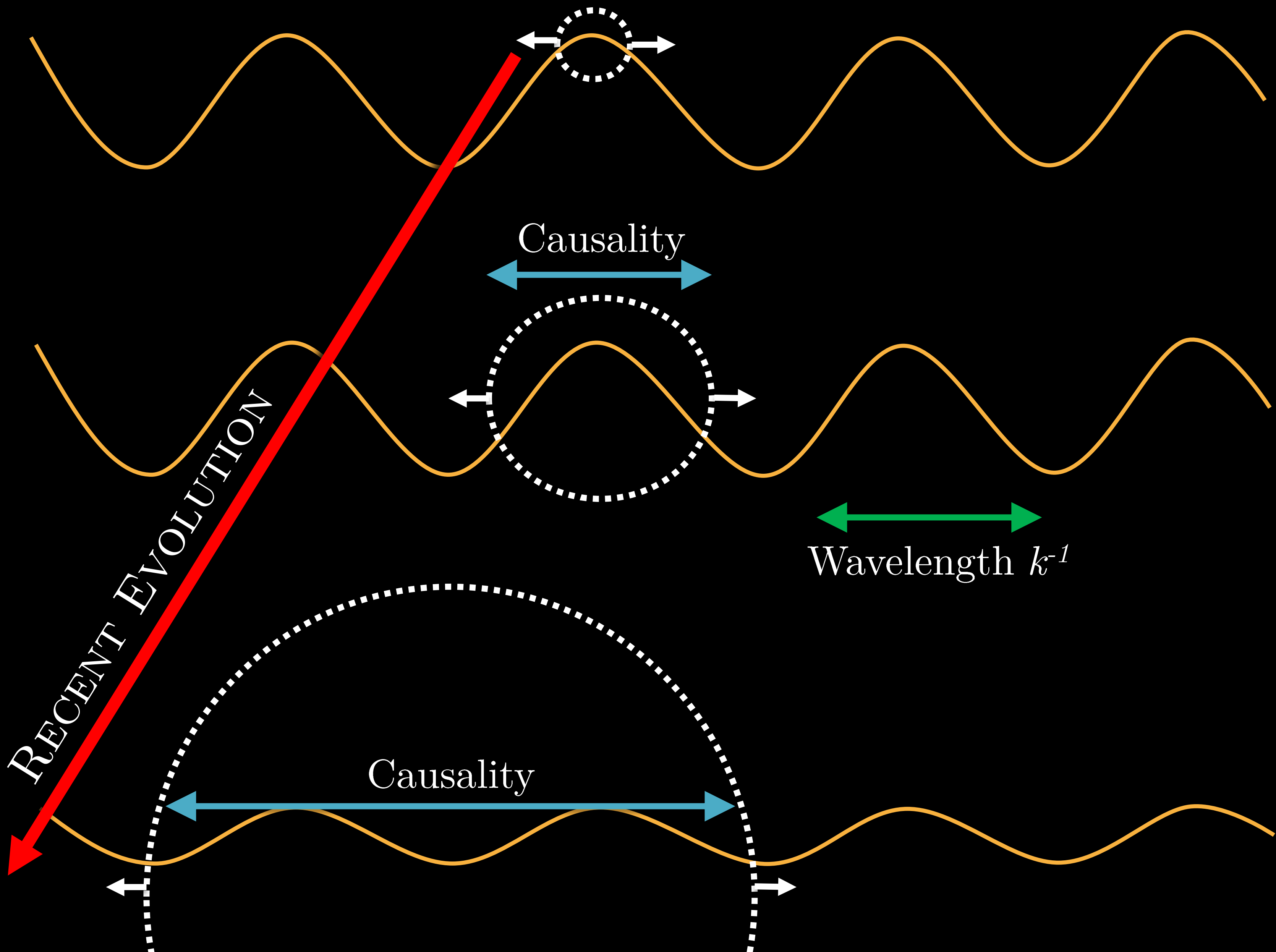
The *Horizon* Problem



Cosmic Inflation



Primordial Perturbation Evolution



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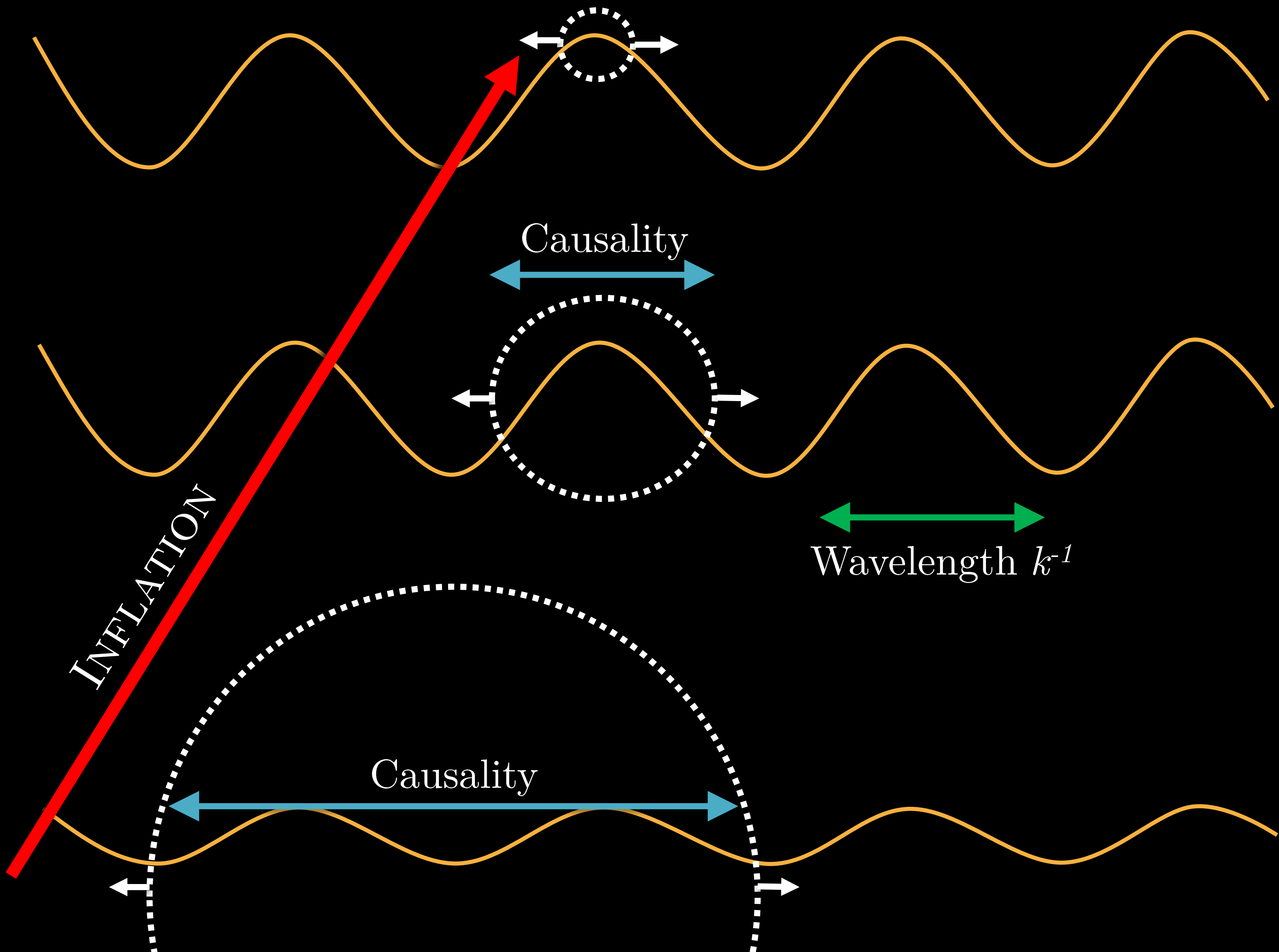

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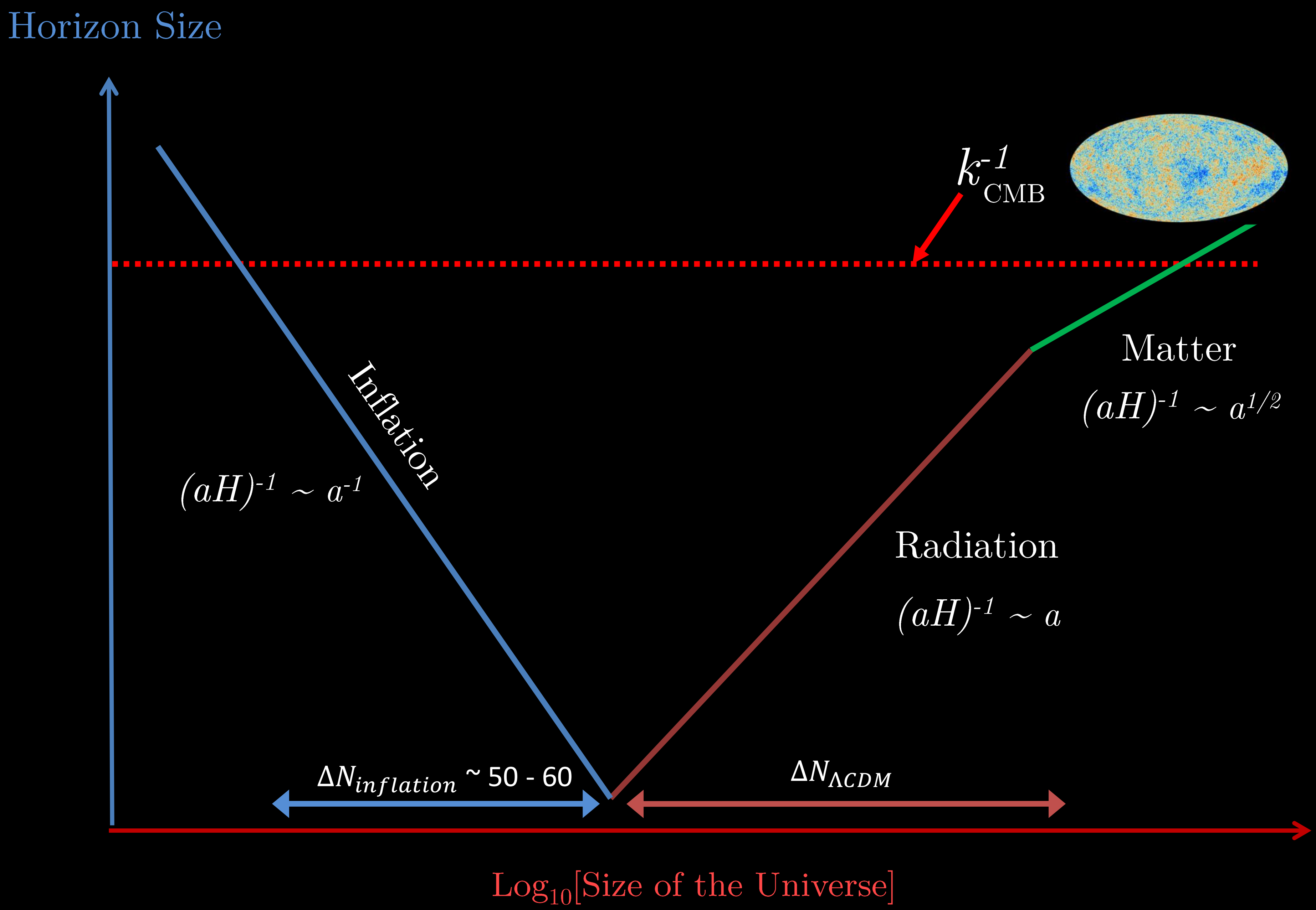

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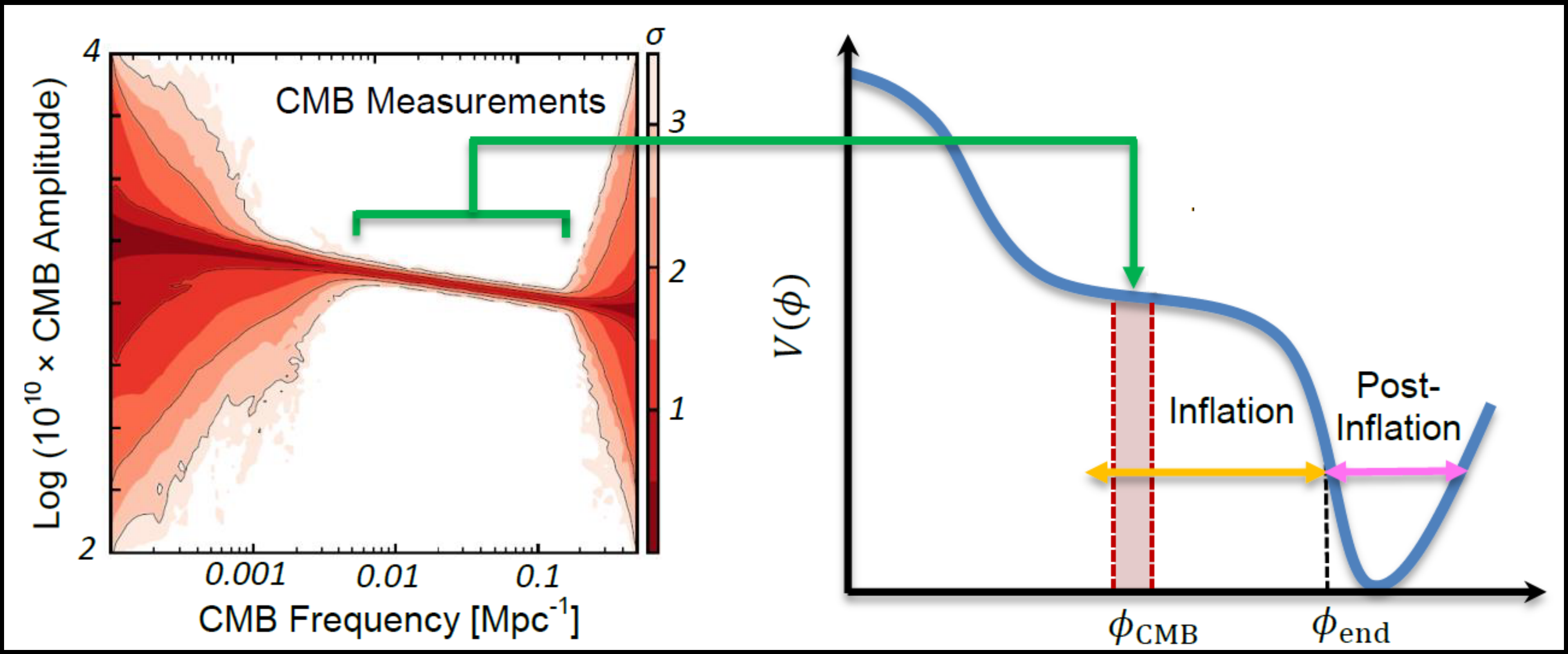


Perturbation Horizon Crossing

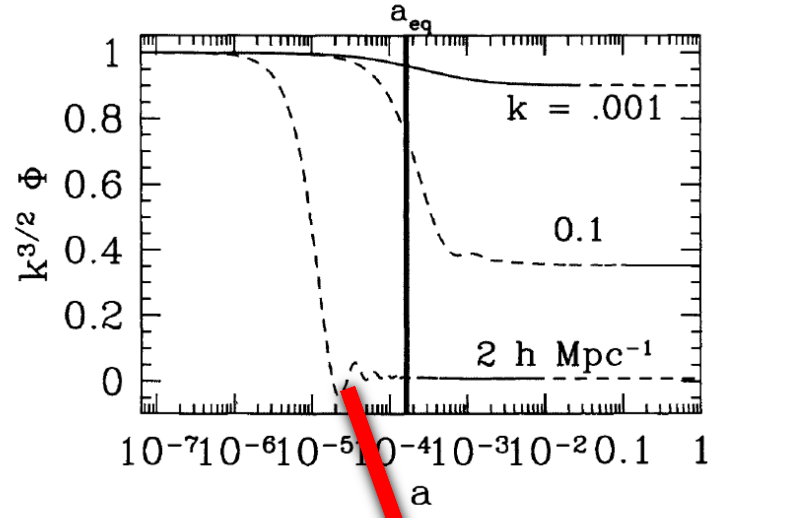


Figure 7.2. The linear evolution of the gravitional potential Φ. Dashed line denotes that the mode has entered the horizon. Evolution through the shaded region is described by the transfer function. The potential is unnormalized, but the relative normalization of the three modes is as it would be for scale-invariant perturbations. Here baryons have been neglected, $\Omega_m = 1$, and $h = 0.5$.

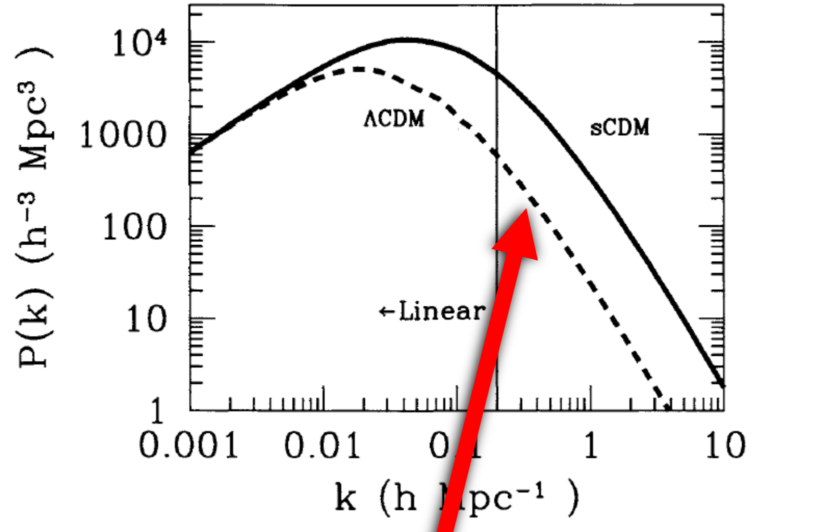
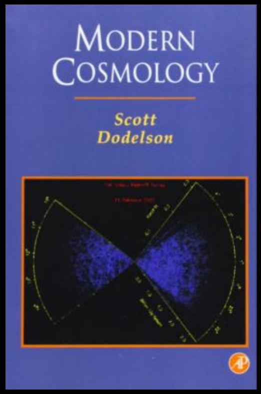


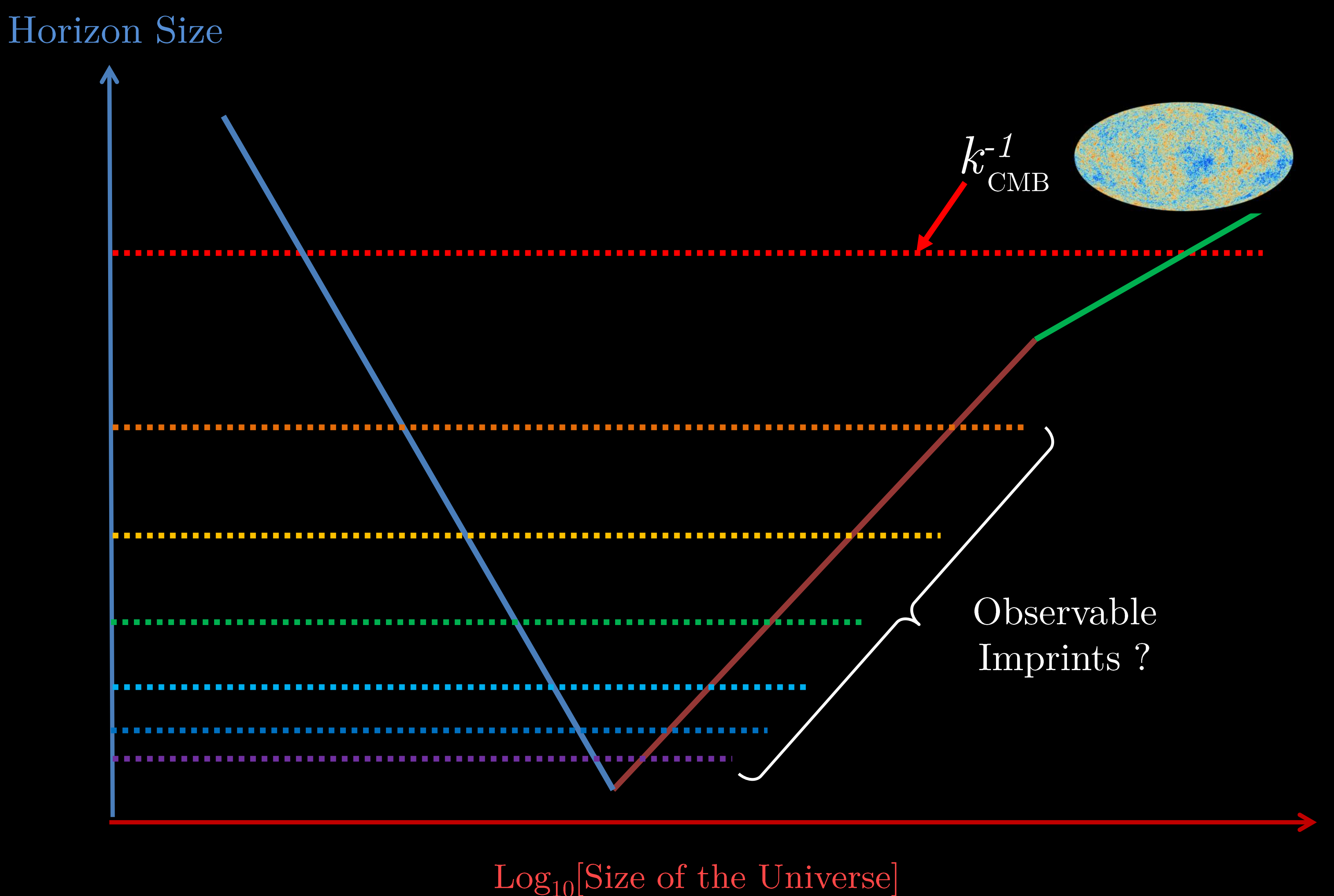
Figure 7.4. The power spectrum in two Cold Dark Matter models, with (ΛCDM) and without (sCDM) a cosmological constant. The spectra have been normalized to agree on large scales. The spectrum in the cosmological constant model turns over on larger scales because of a later a_{eq}. Scales to the left of the vertical line are still evolving linearly.



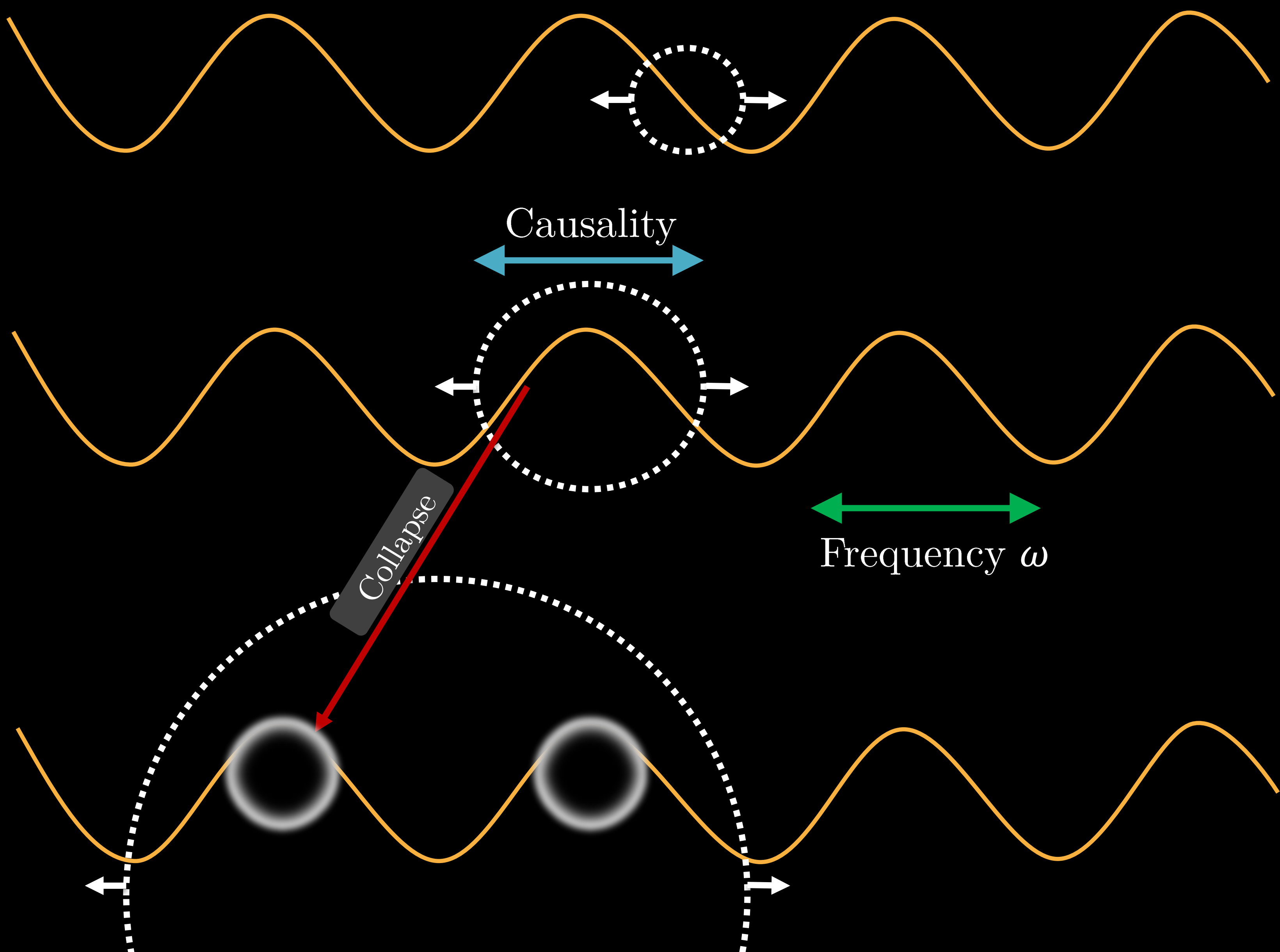
Modes entering the horizon BEFORE matter-radiation equality DECAY...

Causality erases small-scale structures

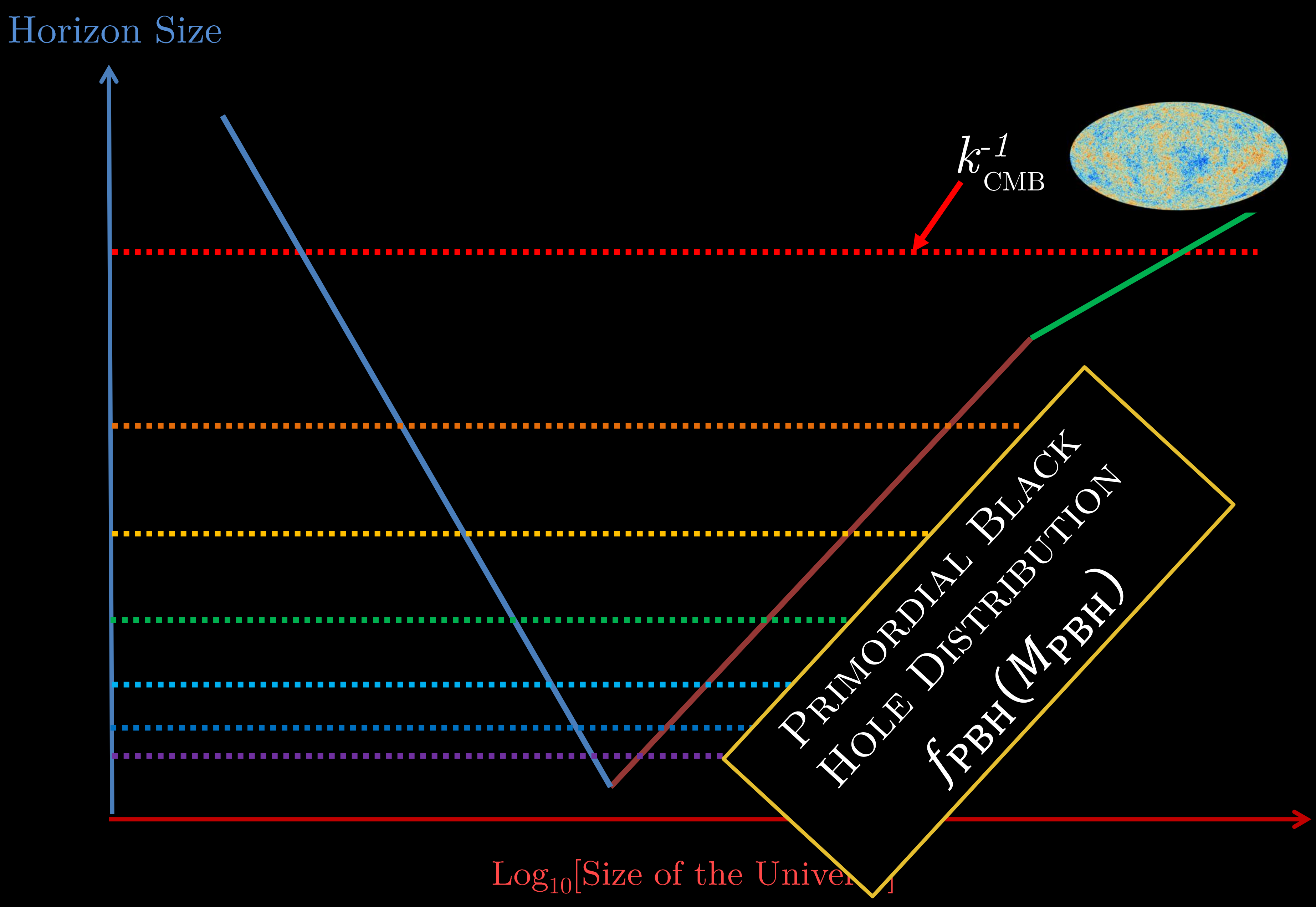
Primordial Perturbation Evolution



Why Mini Primordial Black Holes?



Primordial Perturbation Evolution



How Many Should We Expect ?

ΛCDM model

« Pure » Inflation → « Pure » Radiaton → « Pure » Matter → ...

> Very few PBHs expected
>
> Very little information about the primordial Universe

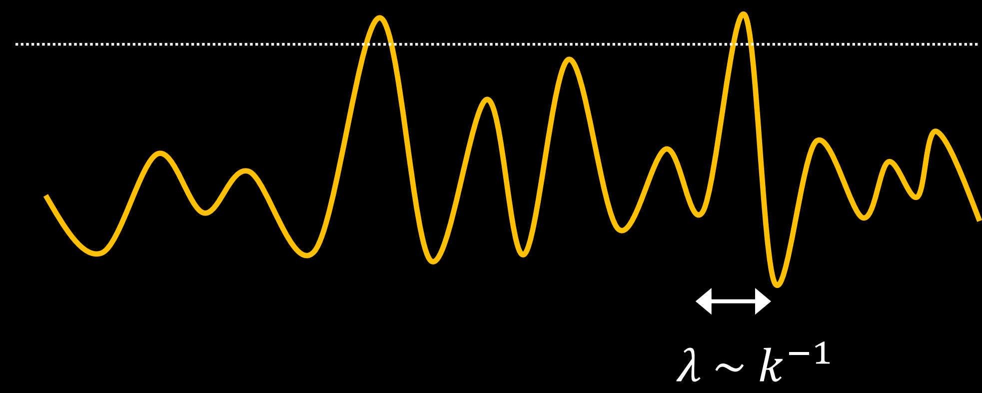
(To infinity and) beyond the SM

> In explicit UV theories, deviations from ΛCDM can affect the formation of PBHs and their Hawking evaporation...

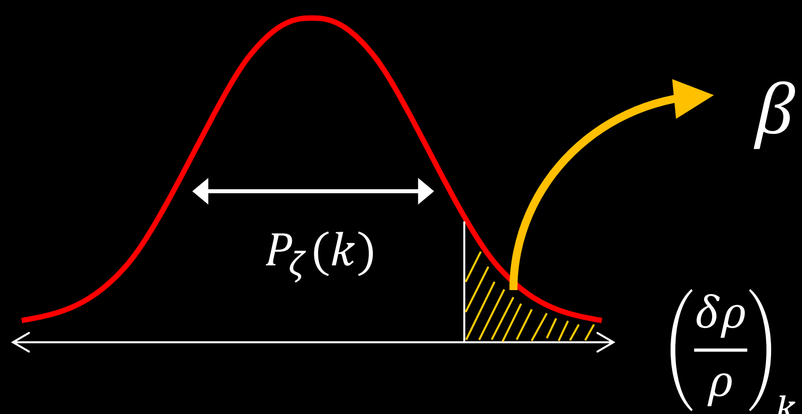
PBH Formation

Collapse of Overdensities

$$\left|\frac{\delta\rho}{\rho}\right| > \rho_c$$



Key Ingredients : 1) Scalar curvature *Power Spectrum* $P_\zeta(k)$



$$\beta(M_k) \sim \exp\left(-\frac{1}{P_\zeta(k)}\right)$$

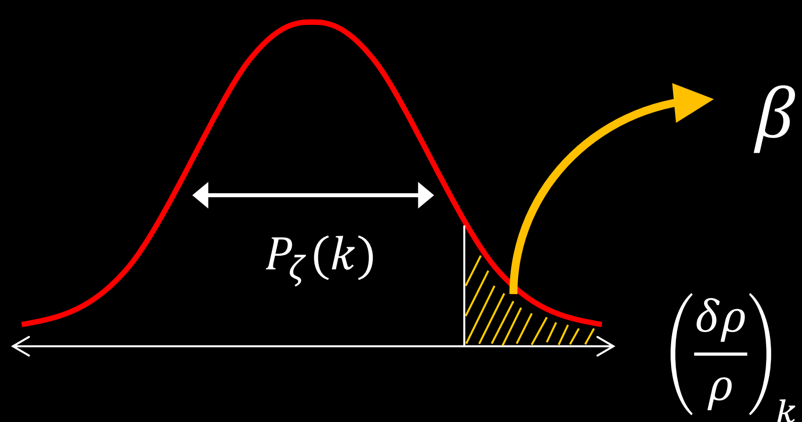
Small Power Spectrum → Small density of PBHs formed

PBH Formation

Key Ingredients : 1) Scalar curvature *Power Spectrum* $P_\zeta(k)$

2) Equation of State in the Universe $\boldsymbol{w}$

$$\delta_c(k) = \frac{3(1+w)}{5+3w}\sin^2\left(\frac{\pi\sqrt{w}}{1+3w}\right)$$



$$\beta(M_k) \sim \exp\left(-\frac{1}{P_\zeta(k)}\right)$$

Small Power Spectrum → Small density of PBHs formed

PBH Formation

Key Ingredients : 1) Scalar curvature *Power Spectrum* $P_\zeta(k)$

2) Equation of State in the Universe $\boldsymbol{w}$

→

BSM Physics

can affect

BOTH

PBH Formation

Key Ingredients : 1) Scalar curvature *Power Spectrum* $P_\zeta(k)$

L'*inflaton* parti, les *perturbations* dansent...

During inflation, small perturbations may be generated at scales $k \sim k_{\mathrm{CMB}}$

After inflation, larger perturbations may be sourced at scales $k \gg k_{\mathrm{CMB}}$

- Bumpy potentials (Ultra-Slow-Roll period)
- Colliding scalar-field bubbles
- Enhancement due to early matter domination
- ...

PBH Formation

Key Ingredients : 2) Equation of State in the Universe $\boldsymbol{w}$

The post-inflationary Universe, quèsaco ?

PBH Formation

Key Ingredients : 2) Equation of State in the Universe $\boldsymbol{w}$

Cosmological moduli may start to oscillate

→ Early Matter Domination ($\boldsymbol{w} = 0$)

- String Theory compactification
- Transverse directions in SUGRA
- Axion-like particle models
- ...

PBH Formation

Key Ingredients : 2) Equation of State in the Universe w

Compact Extra Dimensions may place the Universe in *Stasis*

→ Mixed Matter/Radiation state ($w \in [0\,,1/3]$)

A tower of states with regular spectrum

Energy densities with similar pattern

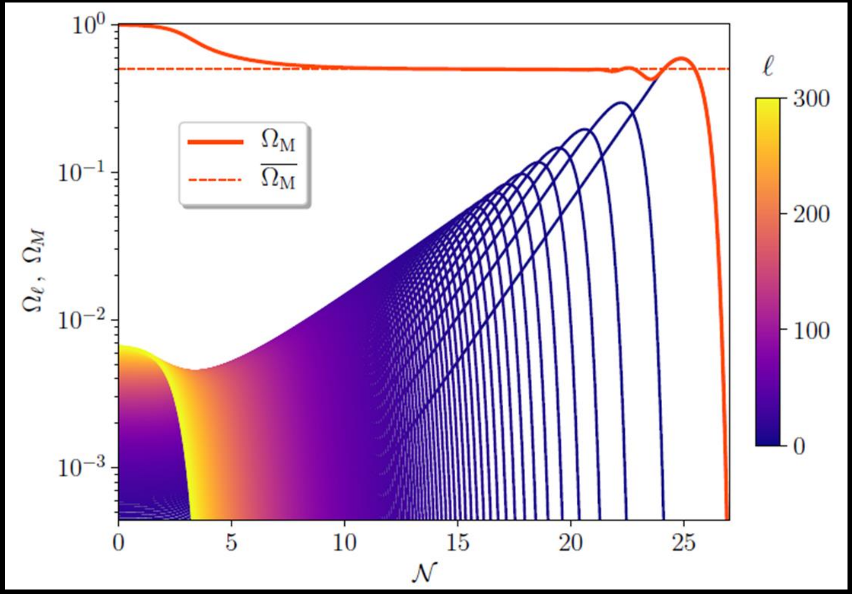
→ The ensemble is attracted to a mixed-state system...

$$m_\ell = m_0 + (\Delta m)\ell^\delta$$

$$\Gamma_\ell = \Gamma_0 \left(\frac{m_\ell}{m_0}\right)^\gamma \qquad \Omega_\ell^{(0)} = \Omega_0^{(0)} \left(\frac{m_\ell}{m_0}\right)^\alpha$$

$$\overline{\Omega}_M = \frac{2\gamma\delta - 4(1+\alpha\delta)}{2\gamma\delta - (1+\alpha\delta)} \,.$$

[Dienes, LH, Huang, Kim, Tait, Thomas ‘21]



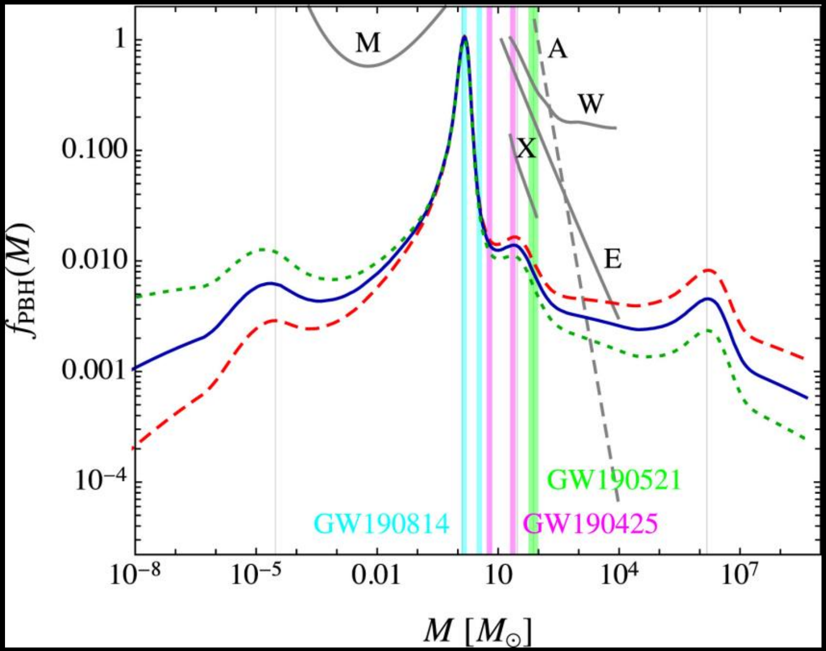
PBH Formation

Key Ingredients : 2) Equation of State in the Universe $\boldsymbol{w}$

Fluctuation of w : During phase transitions (QCD) may fluctuate ⟶ variations of $\beta(M)$

Dynamics of w leaves an imprint in the PBH spectrum

Probing its shape ⟶ Reading the spectrum pattern



[Byrnes, Hindmarsh, Young, Hawkins ‘18]
[Carr, Clesse, García-Bellido, Kühnel ’20]
[Juan, Serpico, Abellán ‘22]
[Musco, Jedamzik, Young ‘23]

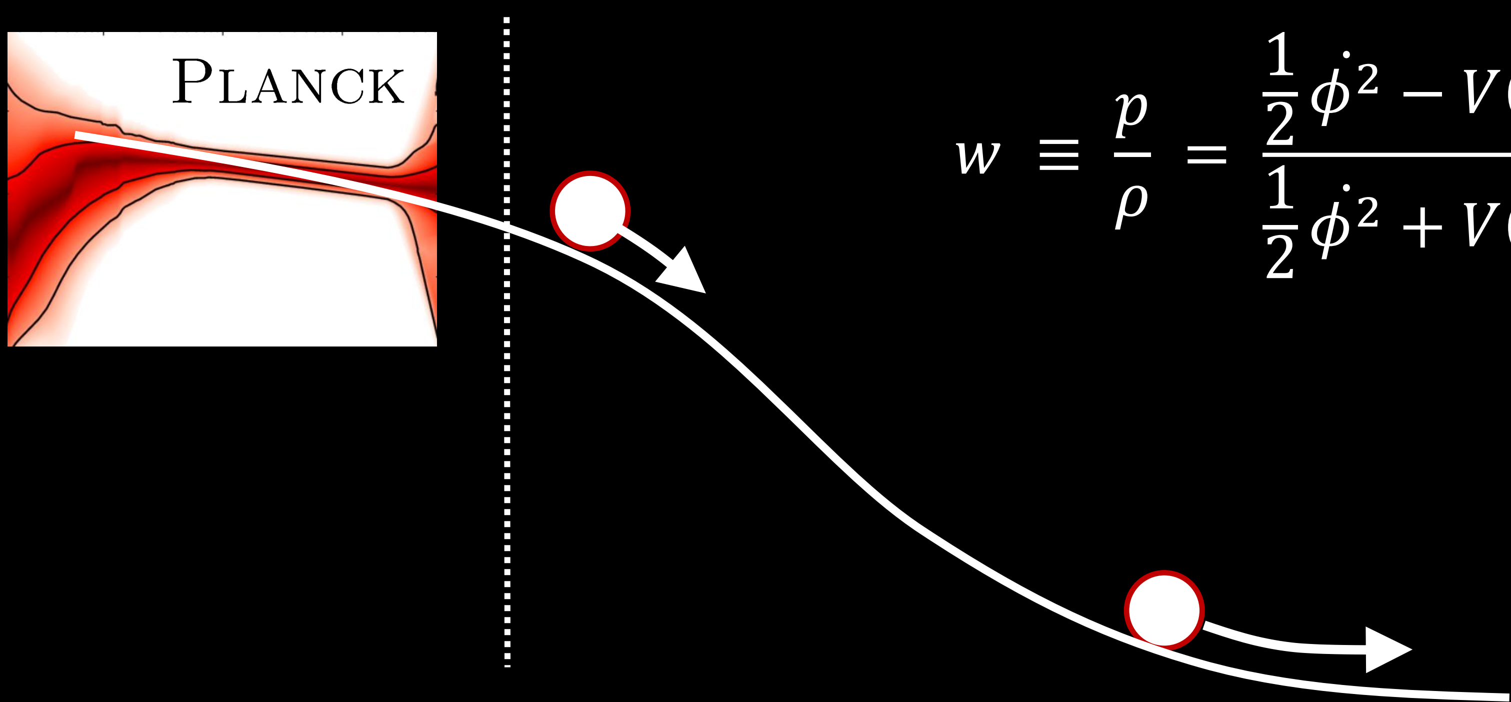
PBH Formation

Key Ingredients : 2) Equation of state in the Universe $\boldsymbol{w}$

The post-inflationary Universe, quèsaco ?

Runaway directions : Quintessential (non-oscillatory) inflation

→ Kination ($\boldsymbol{w} = 1$)



$$w \equiv \frac{p}{\rho} = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)}$$

PBH Formation

Key Ingredients : 2) Equation of state in the Universe $\boldsymbol{w}$

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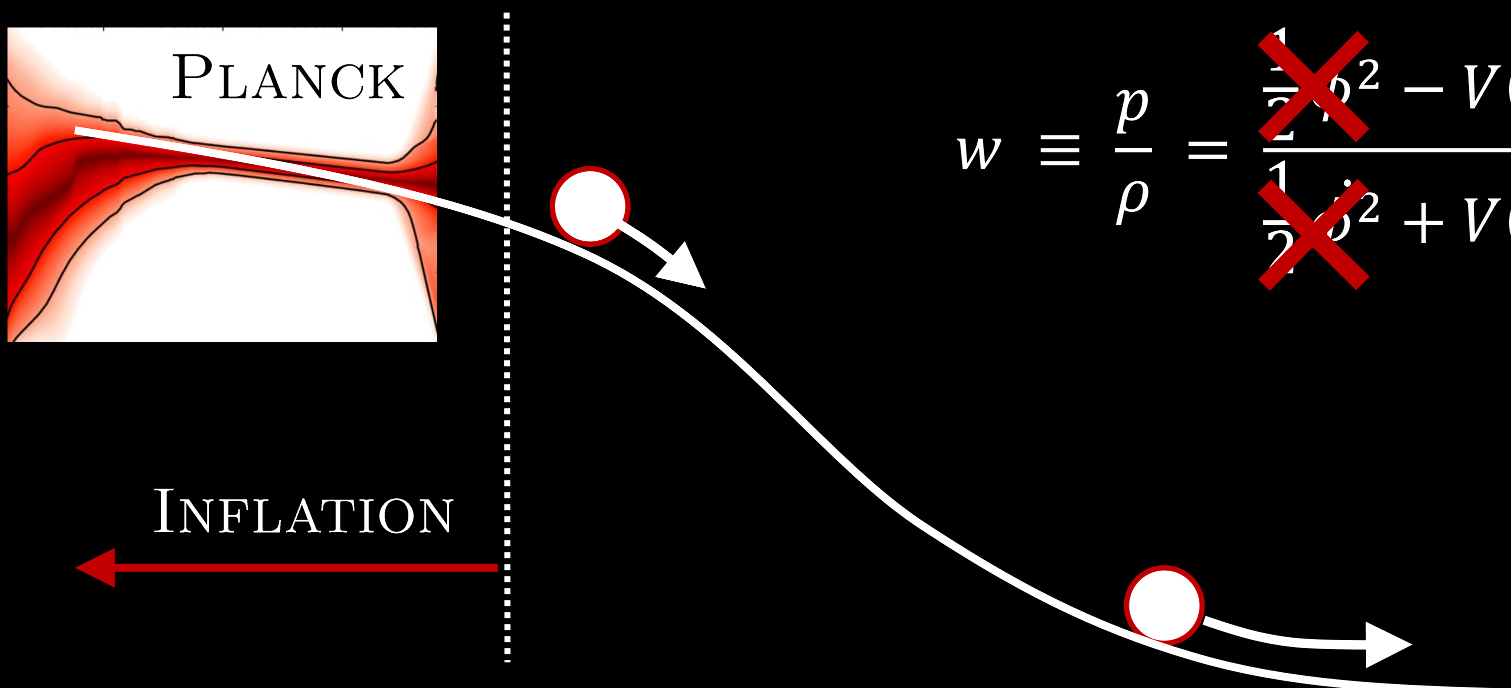


$$w \equiv \frac{p}{\rho} = \frac{\cancel{\frac{1}{2}\dot{\phi}^2} - V(\phi)}{\cancel{\frac{1}{2}\dot{\phi}^2} + V(\phi)} \approx -1$$

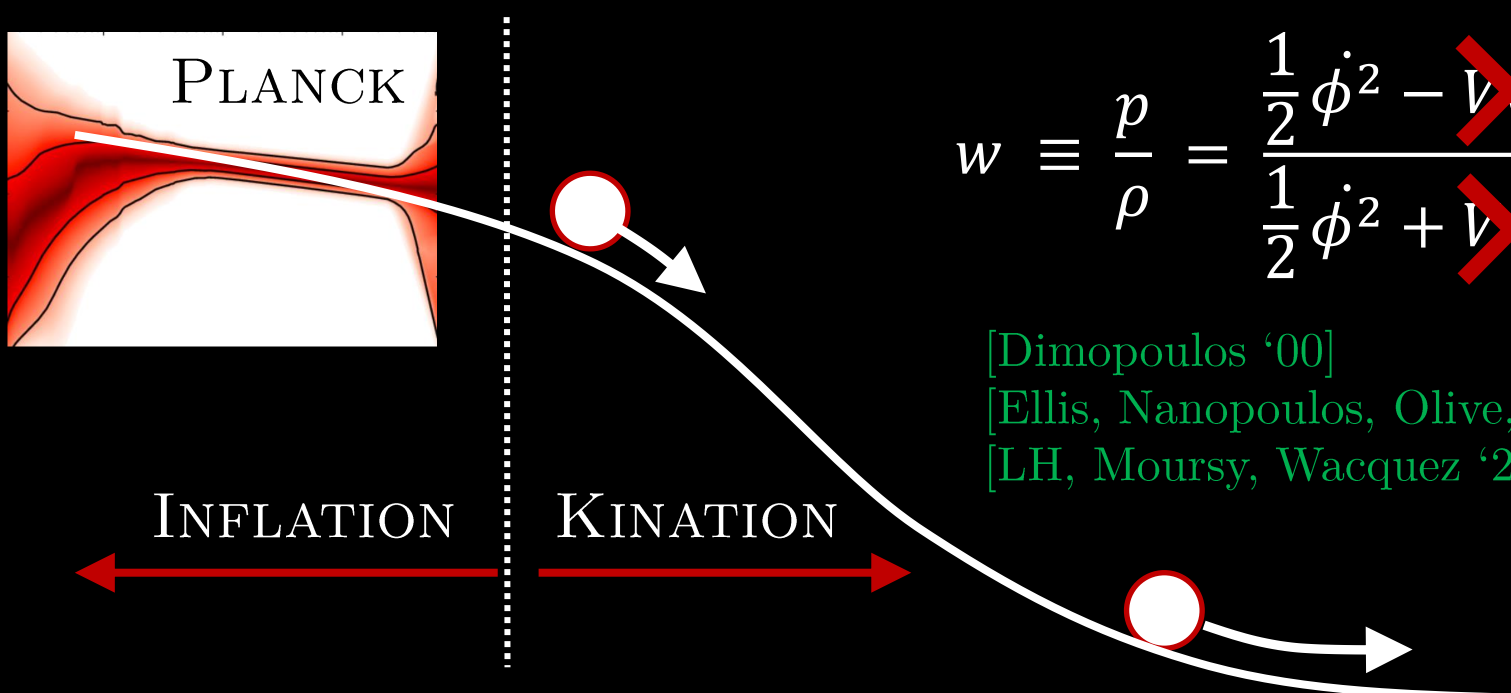
PBH Formation

Key Ingredients : 2) Equation of state in the Universe $\boldsymbol{w}$

The post-inflationary Universe, quèsaco ?

Runaway directions : Quintessential (non-oscillatory) inflation

→ Kination ($\boldsymbol{w} = 1$)



$$w \equiv \frac{p}{\rho} = \frac{\frac{1}{2}\dot{\phi}^2 - \cancel{V(\phi)}}{\frac{1}{2}\dot{\phi}^2 + \cancel{V(\phi)}} \approx 1$$

[Dimopoulos '00]
[Ellis, Nanopoulos, Olive, Verner '20]
[LH, Moursy, Wacquez '22] ...

PBH Formation

An example : Sgoldstino-less Inflation

Spontaneously broken SUSY → Goldstino superfield S

How to get read of massless scalar ?

→ Use a version of SUGRA where $S^2 = 0$ ...

Introduced to match linearly vs. non linearly realized SUSY
[Rocek '78],[Lindström, Rocek '79]

Realized in certain models of String theory exhibiting non linear SUSY [Kallosh, Wrase '14],[Bergshoeff, Dasgupta, Kallosh, Van Proeyen, Wrase '15],[Kallosh, Quevedo, Uranga '15]

Or use effective theories where the sgoldstino decouples
[Komargodski, Seiberg '09],[Kallosh, Karlsson, Murli '15]

PBH Formation

An example : Sgoldstino-less Inflation

$$K = |S|^2 - \frac{c}{\Lambda^2}|S|^4 \,,$$
$$W = f\, S\,.$$

$$S = s + \sqrt{2}\theta\psi_s + \theta^2 F_s$$

Limit $c \to \infty$: Integrate out the scalar s

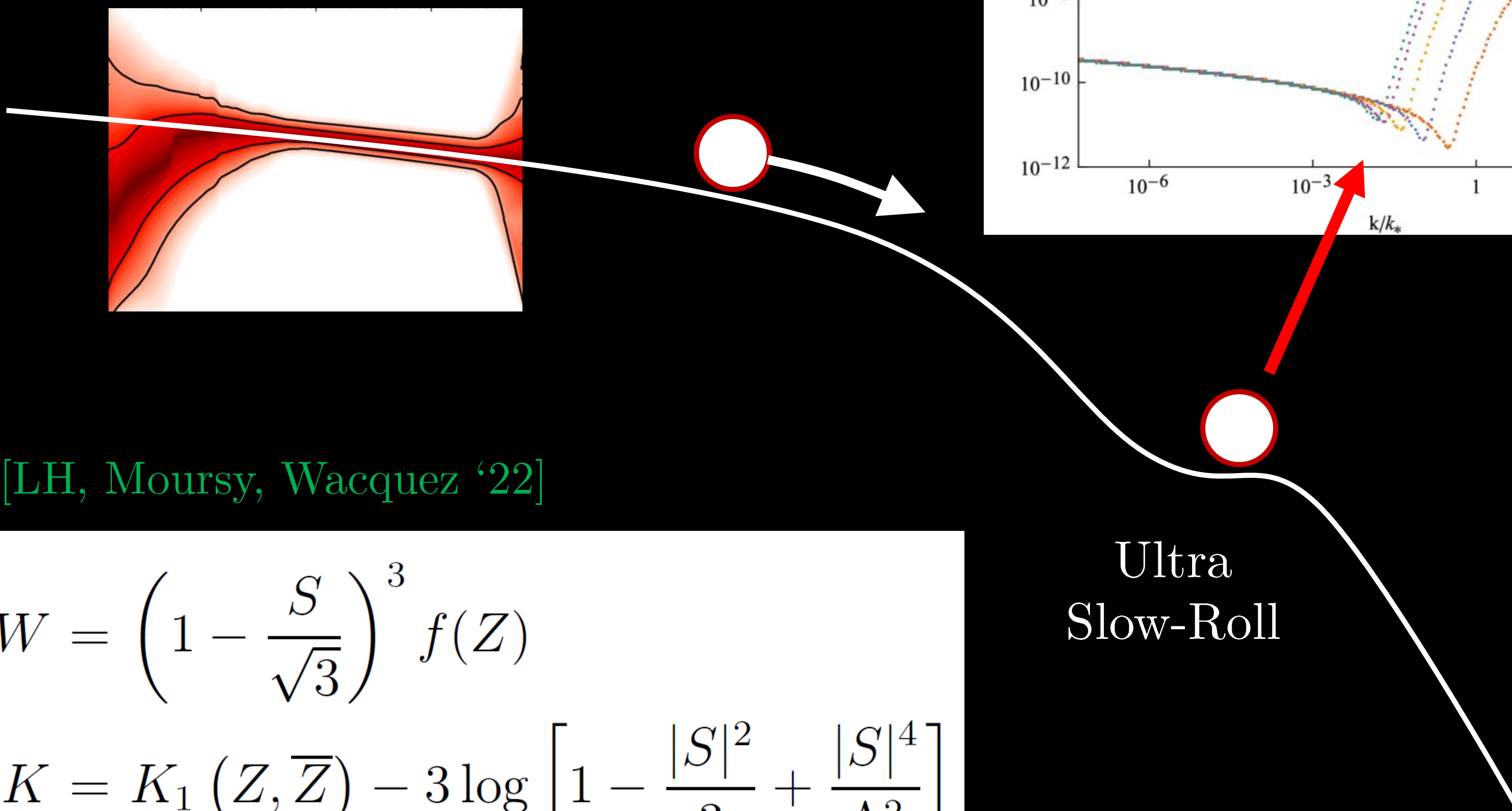
$$\frac{\delta \mathcal{L}}{\delta s^*} = 0 \quad \Rightarrow \quad s = \frac{\psi_s \psi_s}{2F_s} \quad \Rightarrow \quad s^2 = 0 \quad \textbf{and} \quad S^2 = 0$$

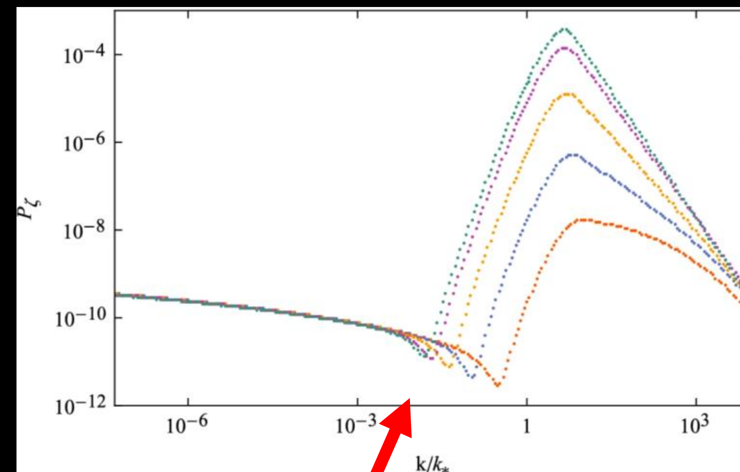
[Komargodski, Seiberg '09]

→ Many consequences for inflation
[Dudas, LH, Wieck, Winkler ‘15]
[Gonzalo, LH, Moursy ‘16]
[Argurio, Dries, LH, Mariotti ‘16]

PBH Formation

Planck



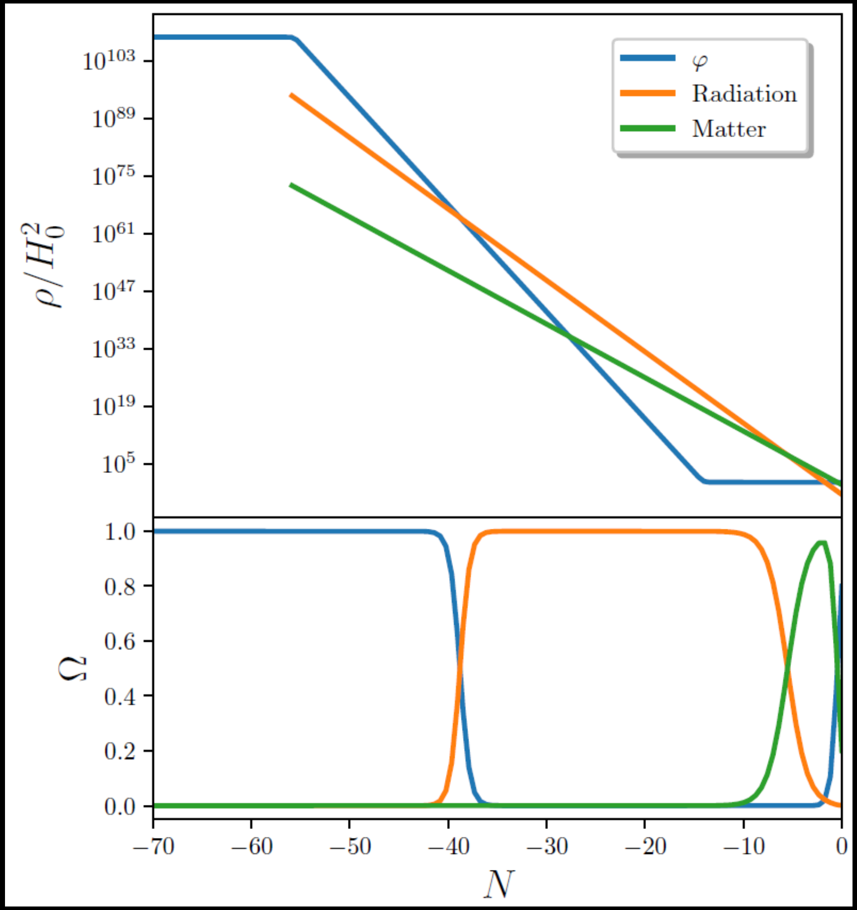


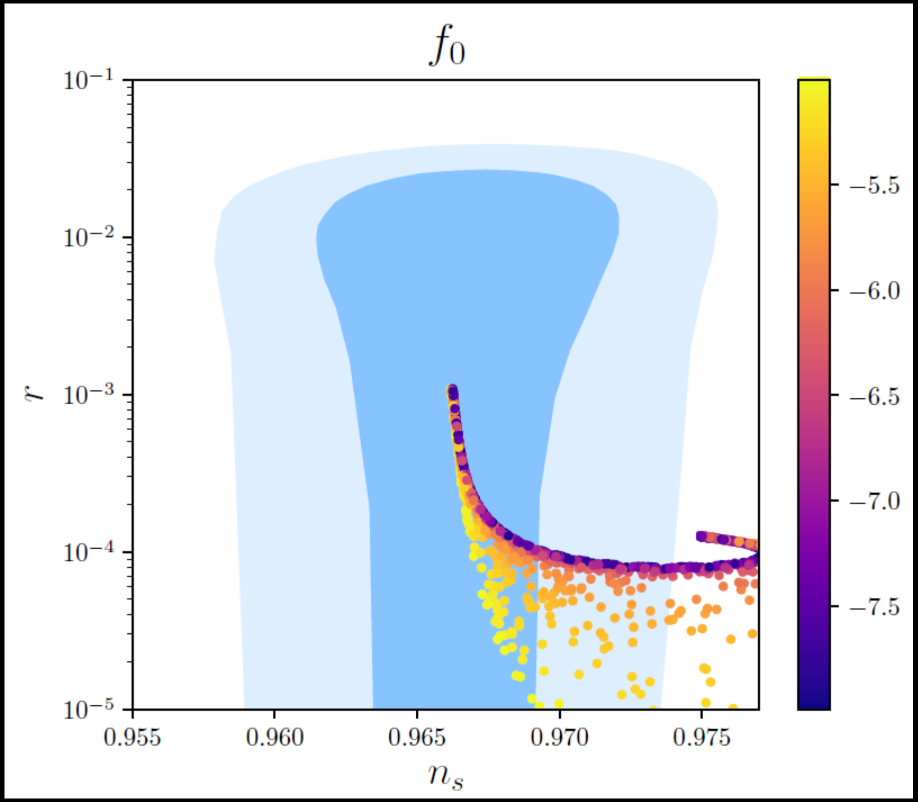
[LH, Moursy, Wacquez ‘22]

$$W = \left(1 - \frac{S}{\sqrt{3}}\right)^3 f(Z)$$

$$K = K_1\left(Z, \overline{Z}\right) - 3\log\left[1 - \frac{|S|^2}{3} + \frac{|S|^4}{\Lambda^2}\right]$$

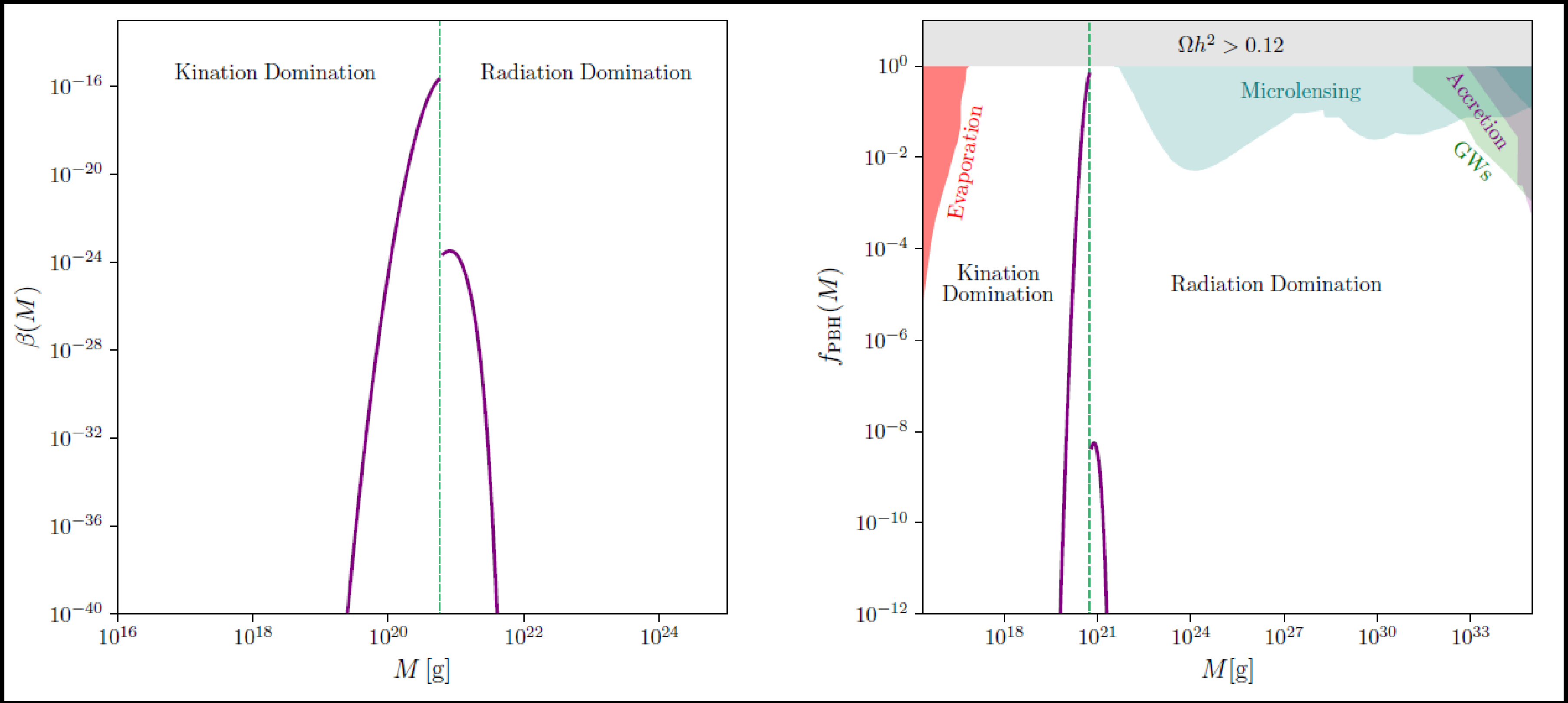
PBH Formation





[LH, Moursy, Wacquez '22]

PBH Formation



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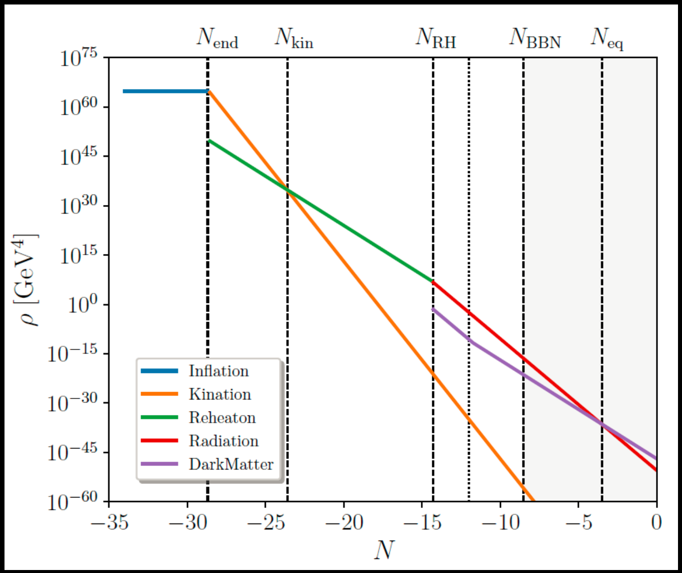
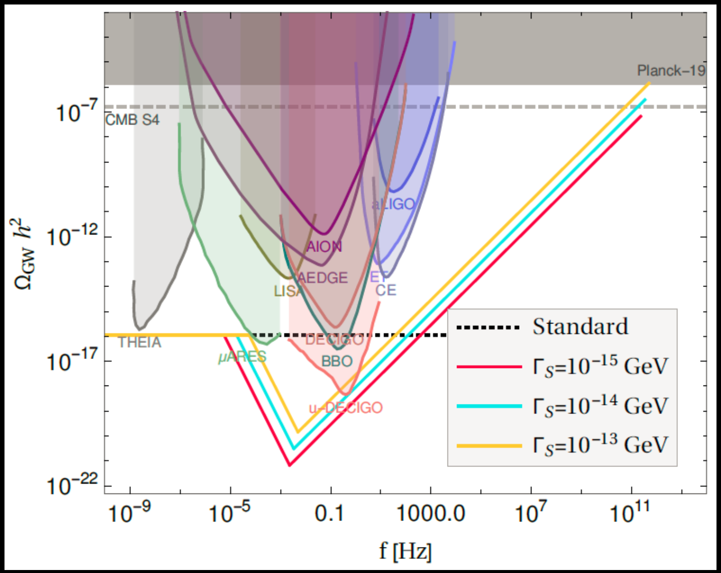
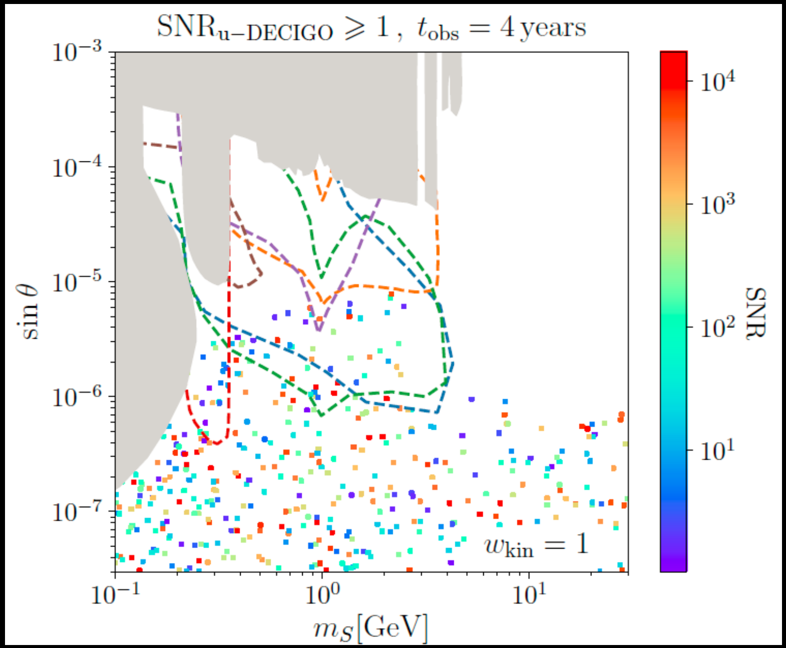
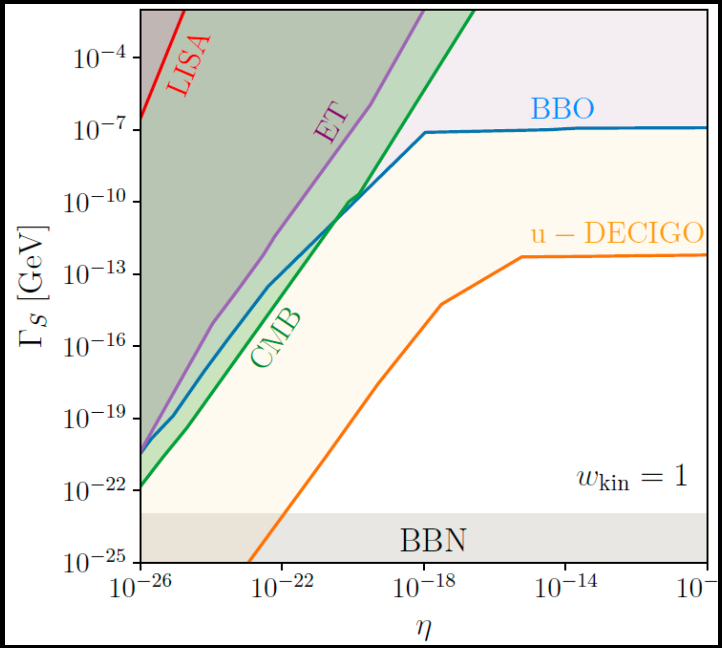
PBH Formation



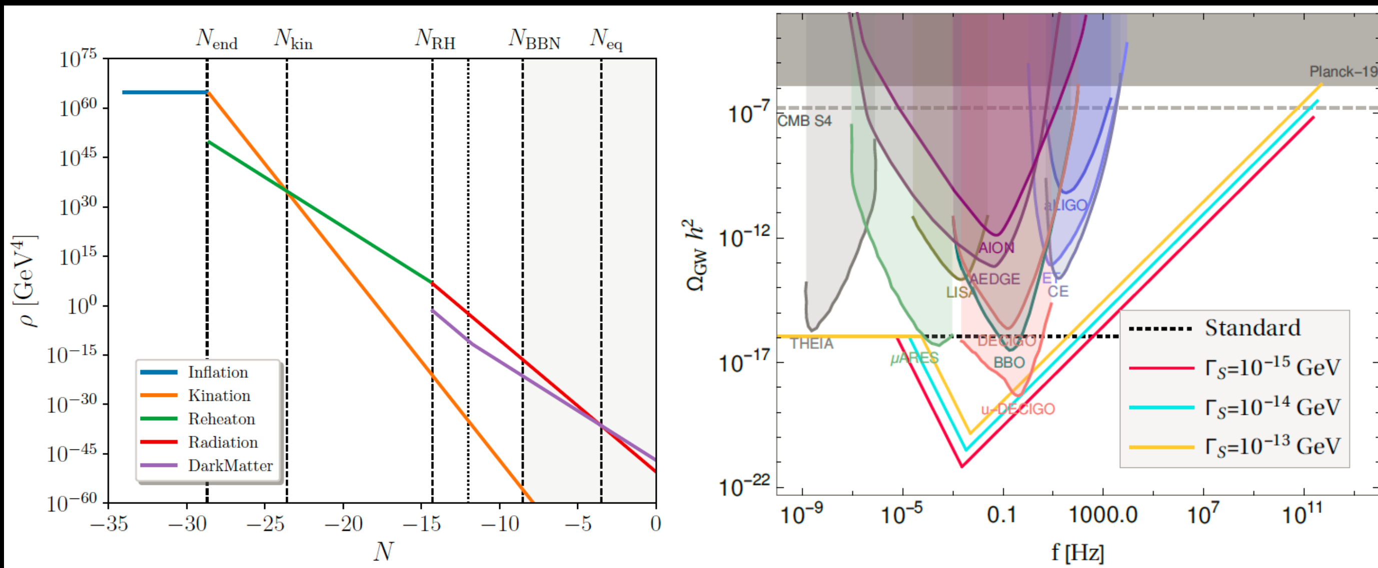


[Ghoshal, LH, Paul '22]

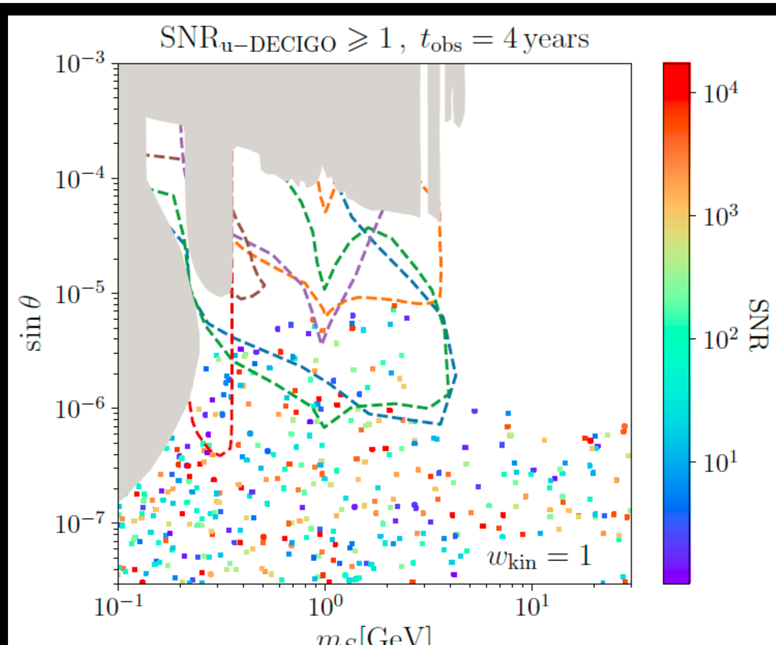


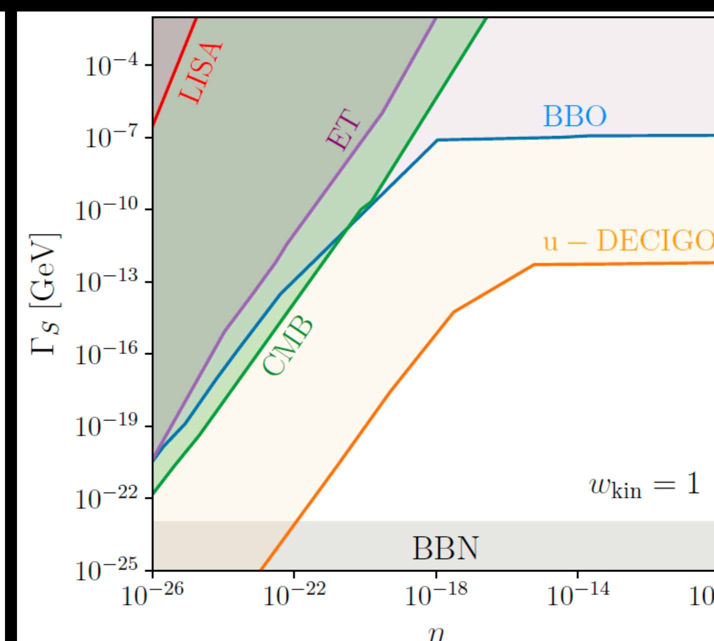


PBH Formation



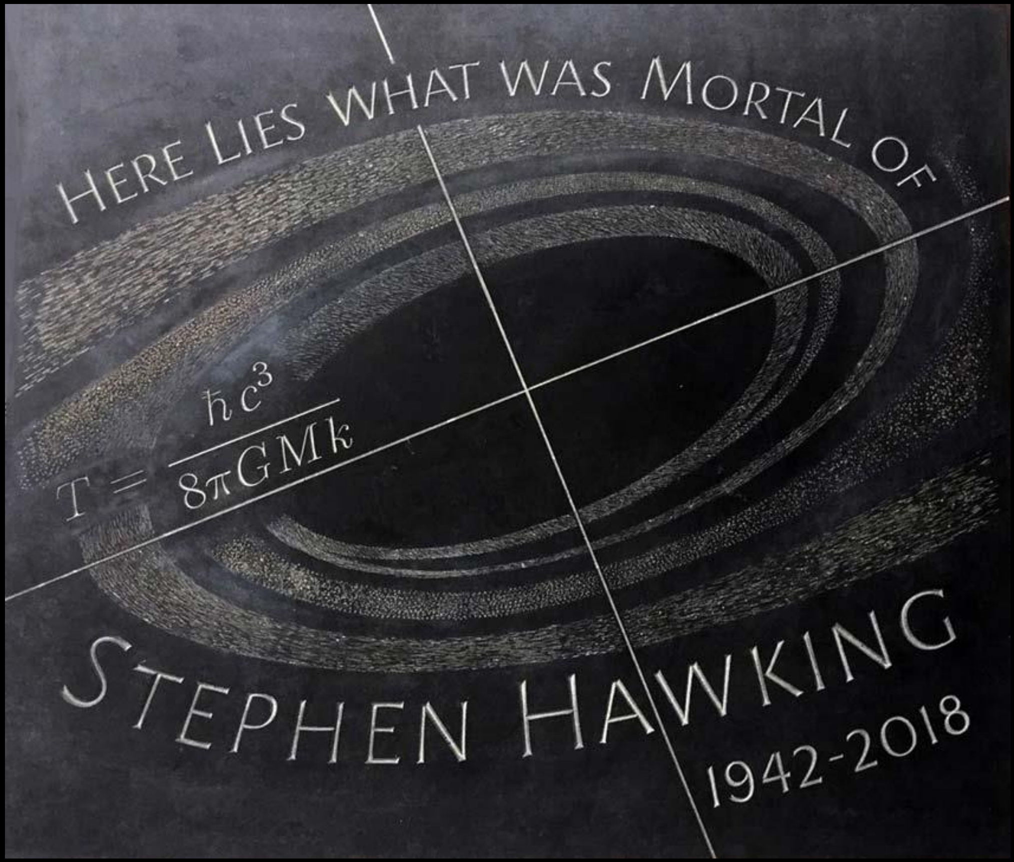
[Ghoshal, LH, Paul '22]





Black Holes Evaporate...

S. Hawking, 1974

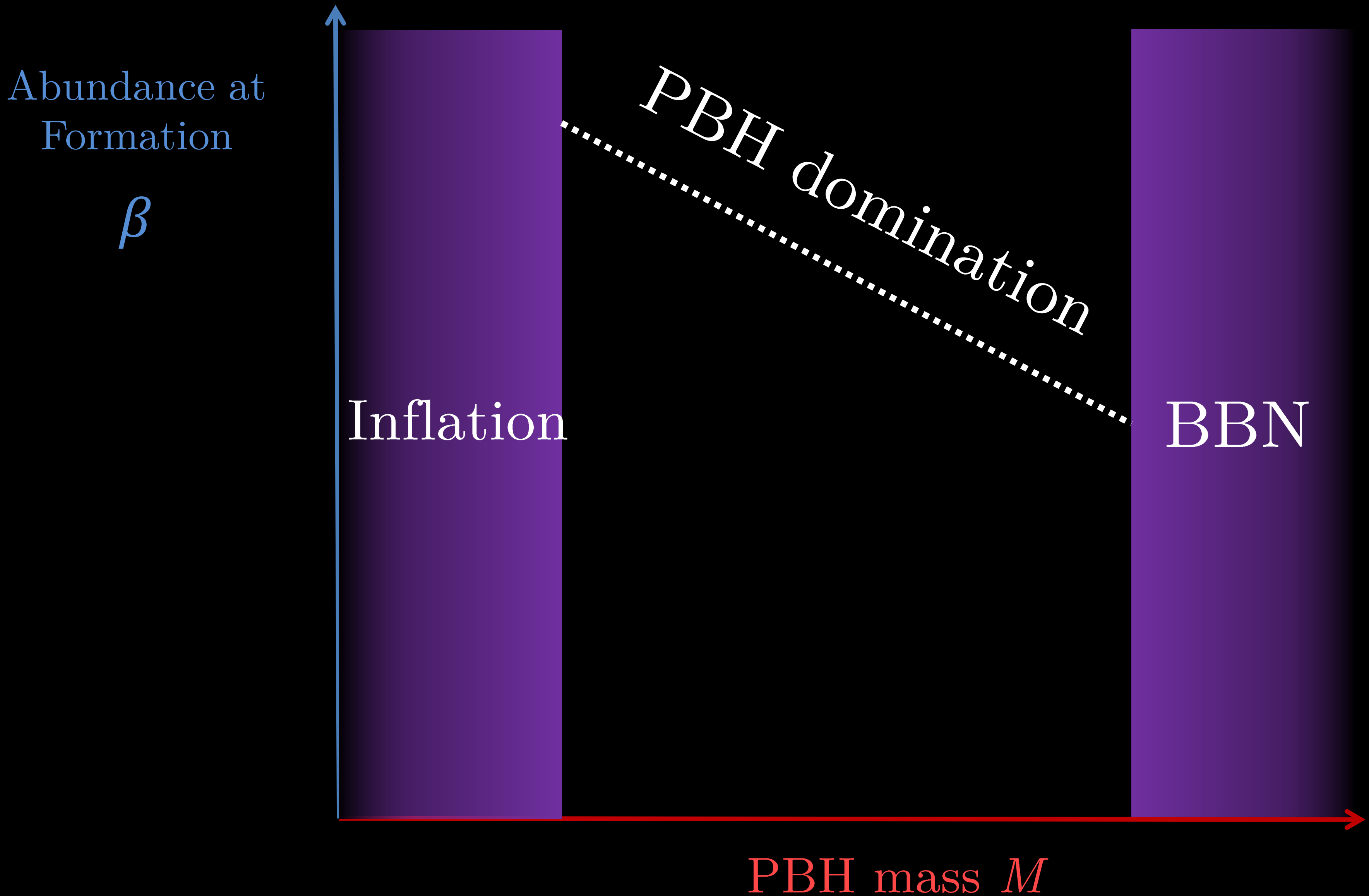


Primordial Black Hole Distribution

$f_{\mathrm{PBH}}(M_{\mathrm{PBH}})$

- Some may be stable and participate to the DM relic abundance ($M_{\mathrm{PBH}} \gtrsim 10^{15}\,\mathrm{g}$)
- Some may be unstable and evaporate after BBN ($10^{9}\,\mathrm{g} \lesssim M_{\mathrm{PBH}} \lesssim 10^{15}\,\mathrm{g}$)
- Some may be unstable and evaporate before BBN ($M_{\mathrm{PBH}} \lesssim 10^{9}\,\mathrm{g}$)

The Parameter Space



PBH EVAPORATION

$$\frac{\mathrm{d}M_{\mathrm{BH}}}{\mathrm{d}t} \equiv \sum_i \left.\frac{\mathrm{d}M_{\mathrm{BH}}}{\mathrm{d}t}\right|_i = -\sum_i \int_0^\infty E_i \frac{\mathrm{d}^2\mathcal{N}_i}{\mathrm{d}p\mathrm{d}t}\mathrm{d}p \;= -\varepsilon(M_{\mathrm{BH}})\frac{M_p^4}{M_{\mathrm{BH}}^2}$$

$$\frac{\mathrm{d}^2\mathcal{N}_i}{\mathrm{d}p\,\mathrm{d}t} = \frac{g_i}{2\pi^2}\frac{\sigma_{s_i}(M_{\mathrm{BH}},\mu_i,p)}{\exp\left[E_i(p)/T_{\mathrm{BH}}\right]-(-1)^{2s_i}}\frac{p^3}{E_i(p)}$$

$$\varepsilon(M_{\mathrm{BH}}) \equiv \sum_i g_i\varepsilon_i(z_i)$$

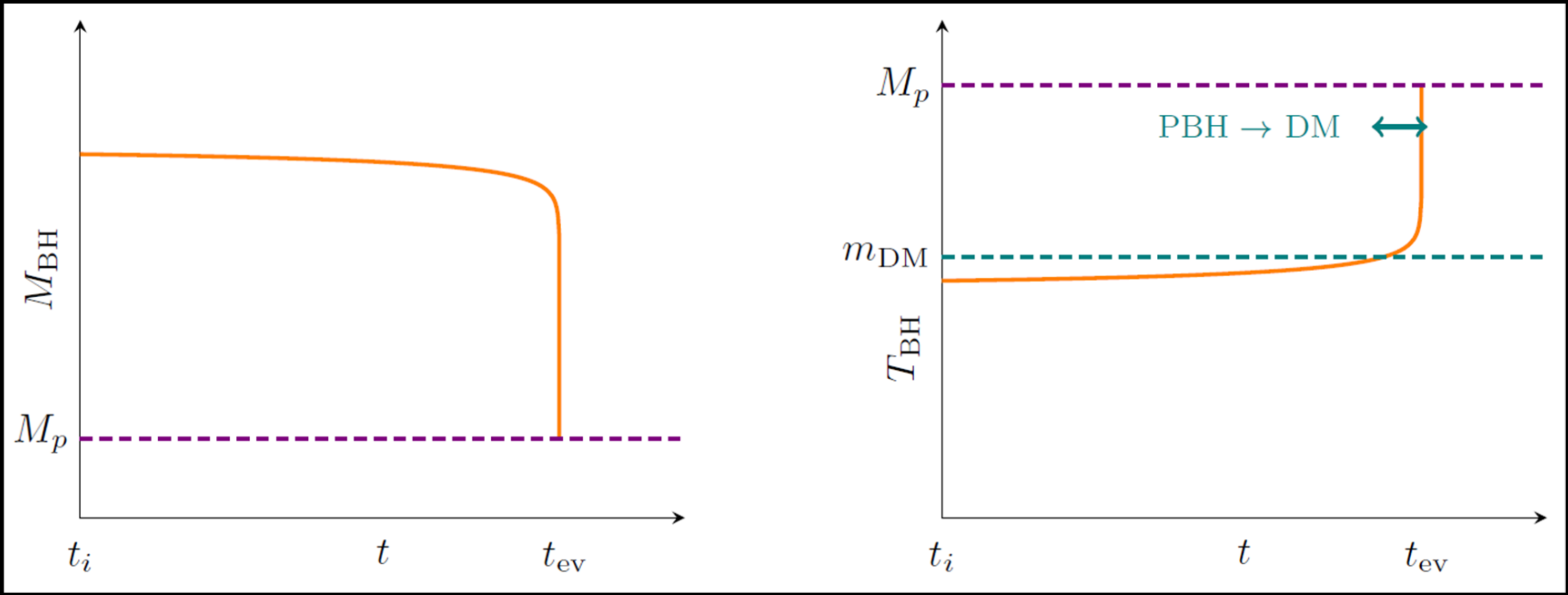
$$z_i \;=\; \mu_i/T_{\mathrm{BH}}$$

BSM Contributions?

$$T_{\mathrm{BH}} = \frac{1}{8\pi G M_{\mathrm{BH}}} \sim 1.06\ \mathrm{GeV}\left(\frac{10^{13}\ \mathrm{g}}{M_{\mathrm{BH}}}\right)$$

PBH EVAPORATION

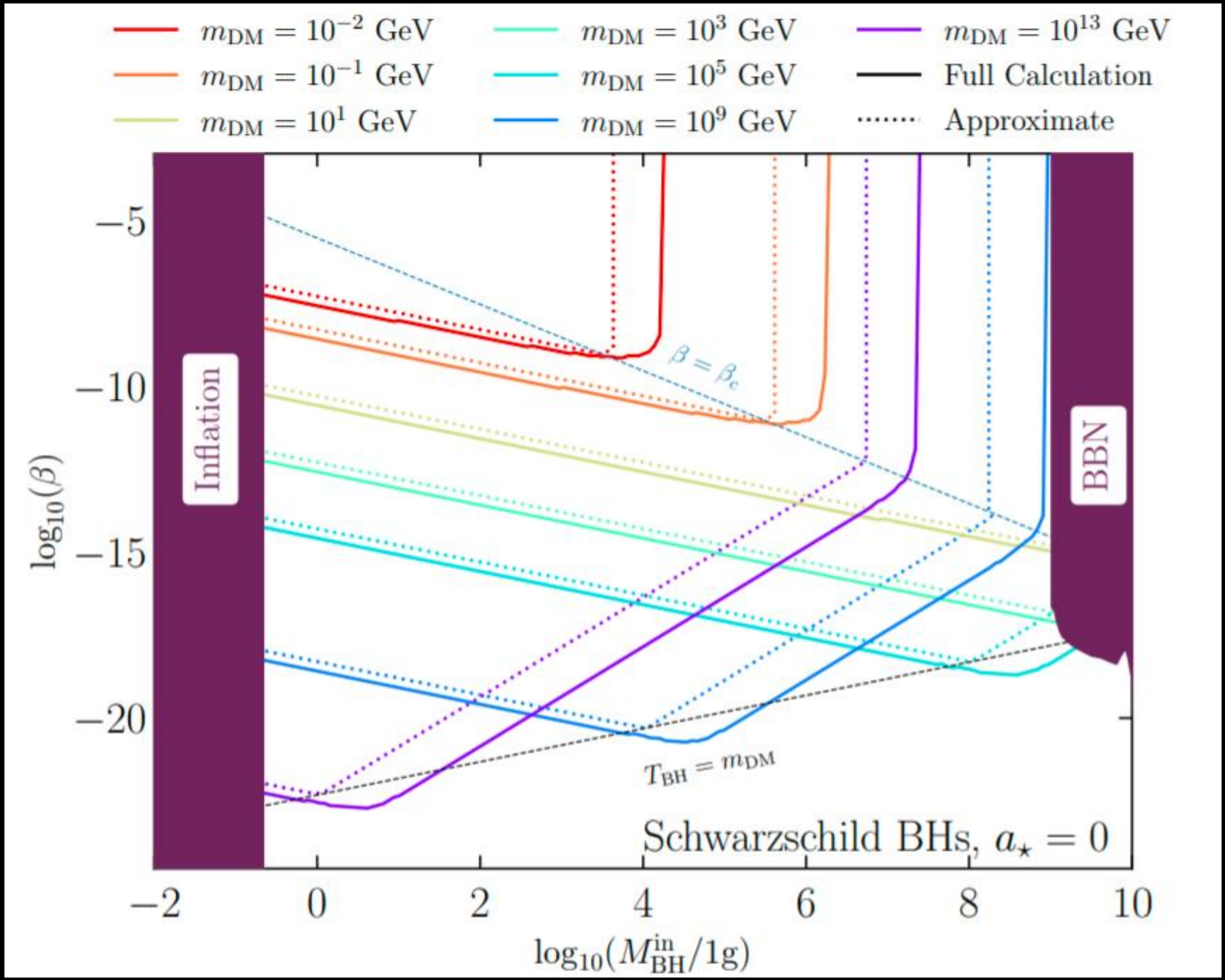
$$T_{\rm BH} = \frac{1}{8\pi G M_{\rm BH}} \sim 1.06\ \text{GeV} \left(\frac{10^{13}\ \text{g}}{M_{\rm BH}}\right)$$



→ More and more particles contribute to the evaporation

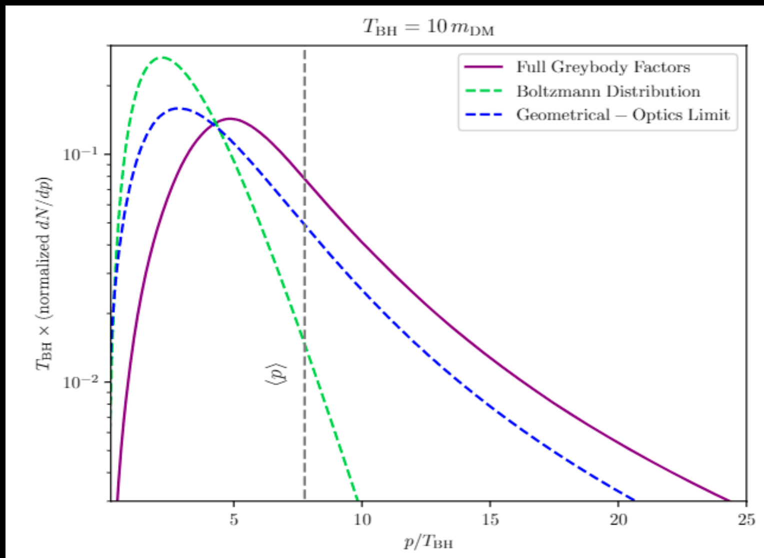
DM from Evaporation

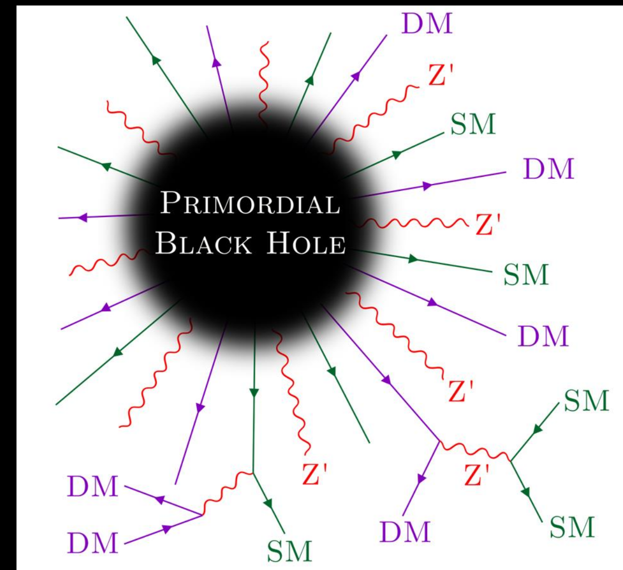
$$f_{\rm PBH}(M) = \delta(M - M_{\rm PBH})$$

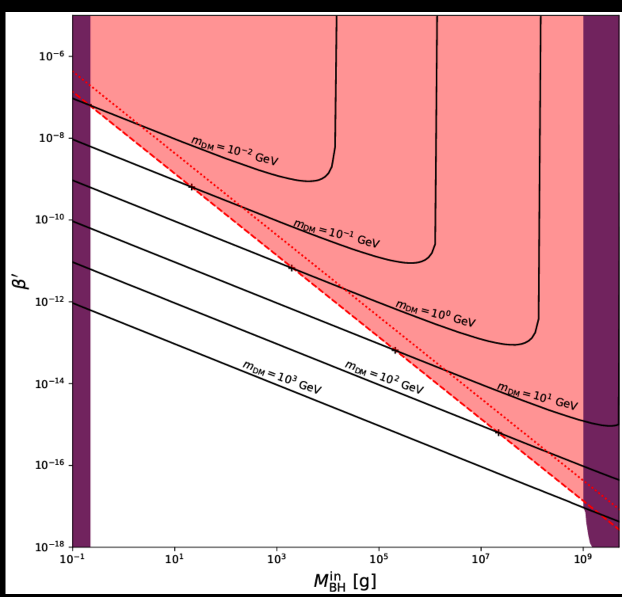


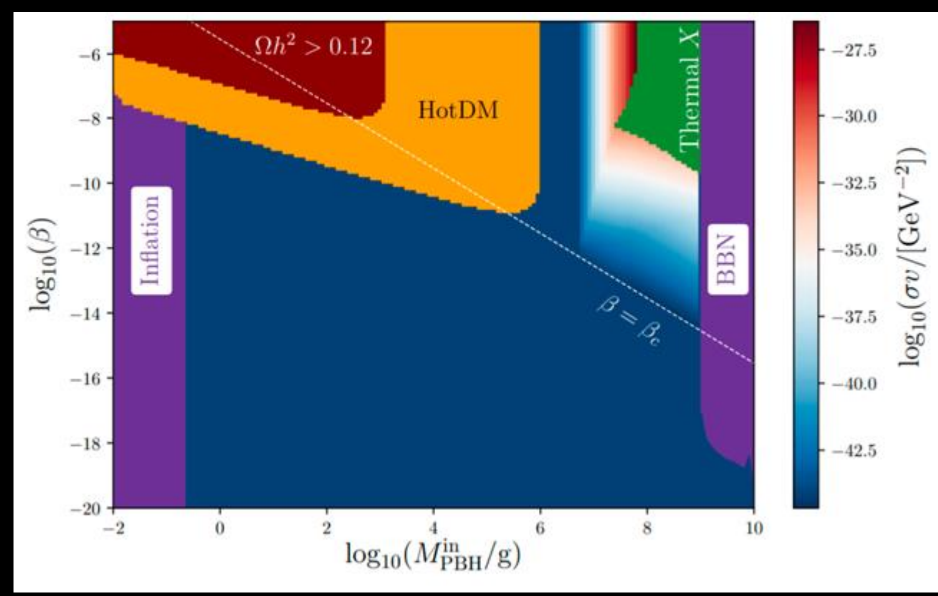
[Cheek, LH, Perez-Gonzalez and Turner '22] **Code Publicly available: FRISBHEE**

DM FROM EVAPORATION









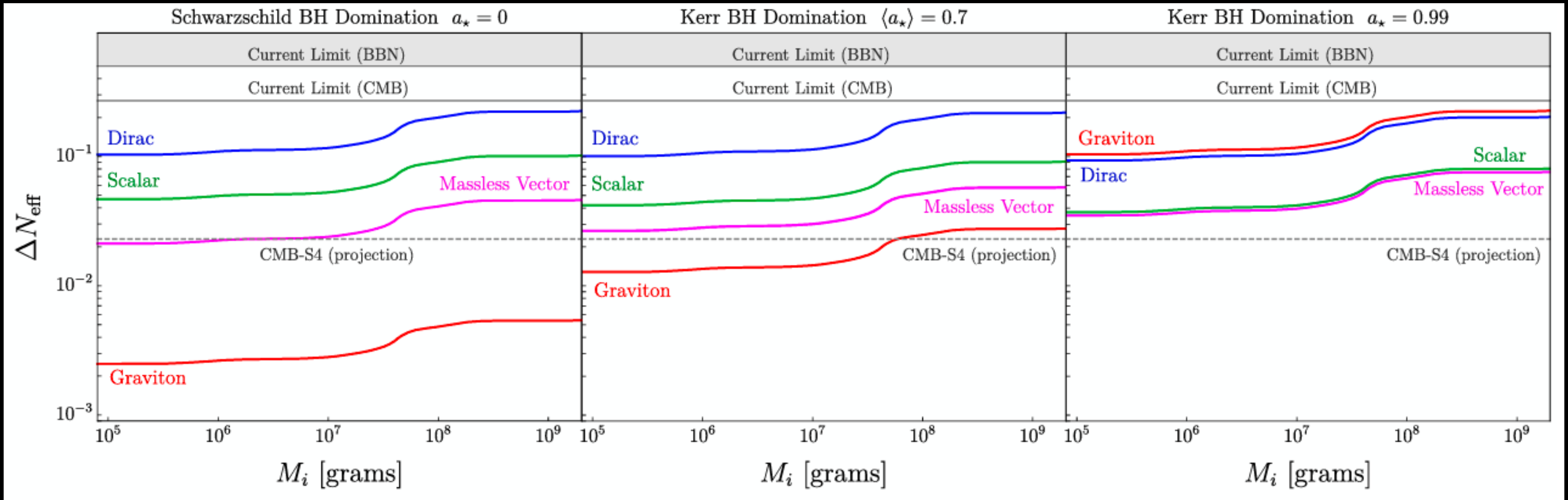
[Cheek, LH, Perez-Gonzalez and Turner '22]   Code Publicly available: **FRISBHEE**

Kerr PBHs and Dark Radiation

Dark particles with small masses can contribute to ΔNeff

Schwarzschild PBH → Negligible

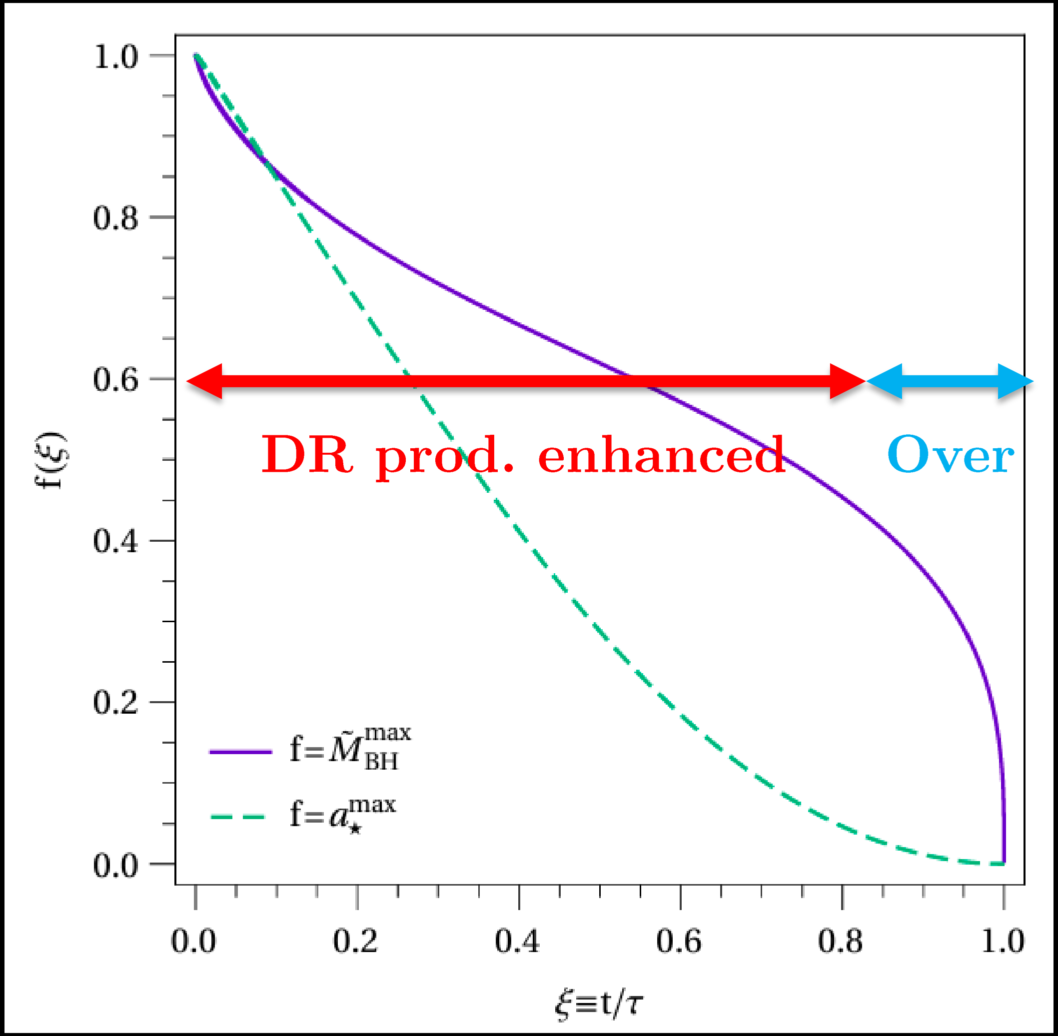
Kerr PBH → Argued to be critical



[Hooper et al '20]

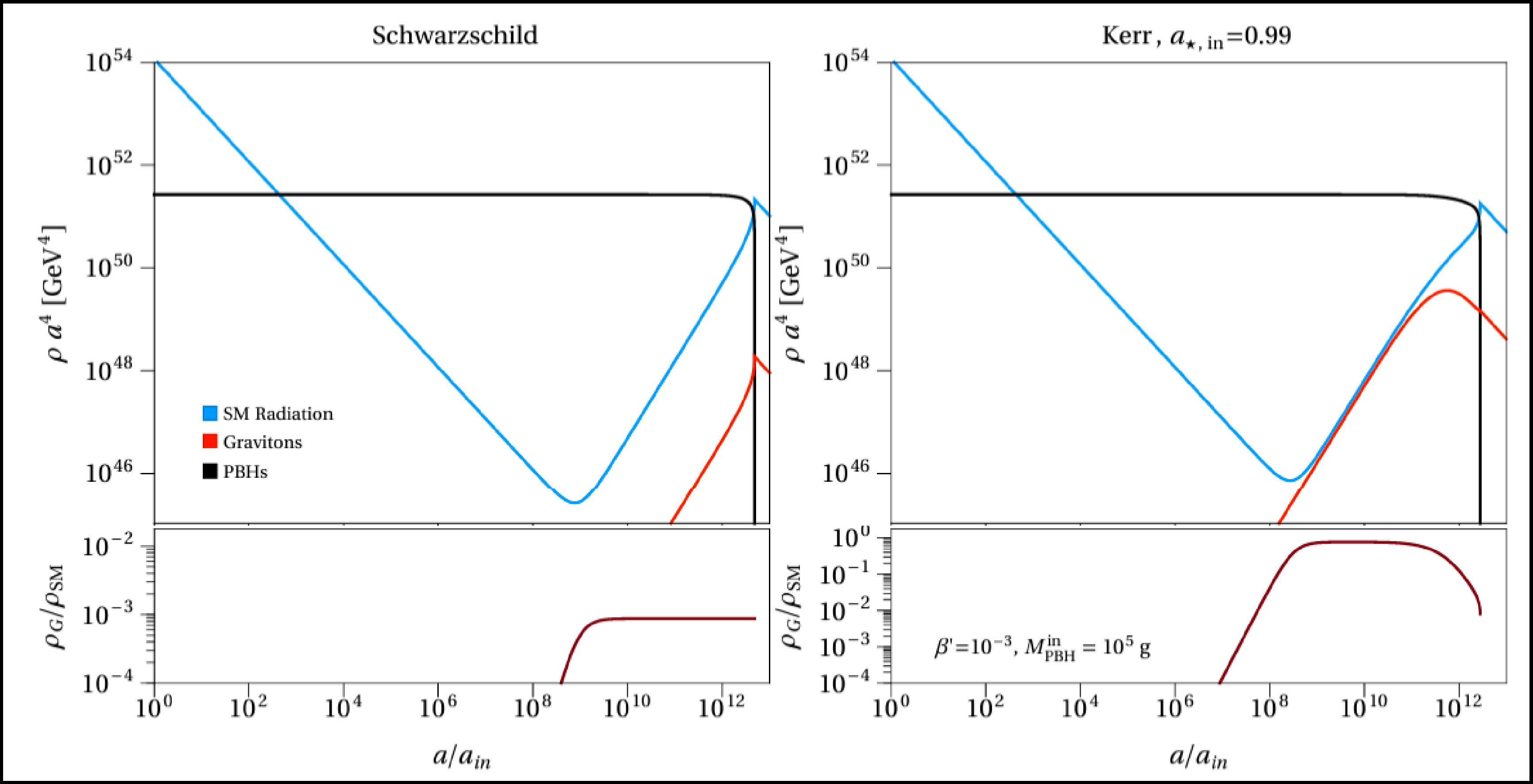
Kerr PBHs and Dark Radiation

Why ?



Kerr PBHs and Dark Radiation

Why ?



[Cheek, LH, Perez-Gonzalez and Turner ‘22] Code Publicly available: FRISBHEE

Kerr PBHs and Dark Radiation

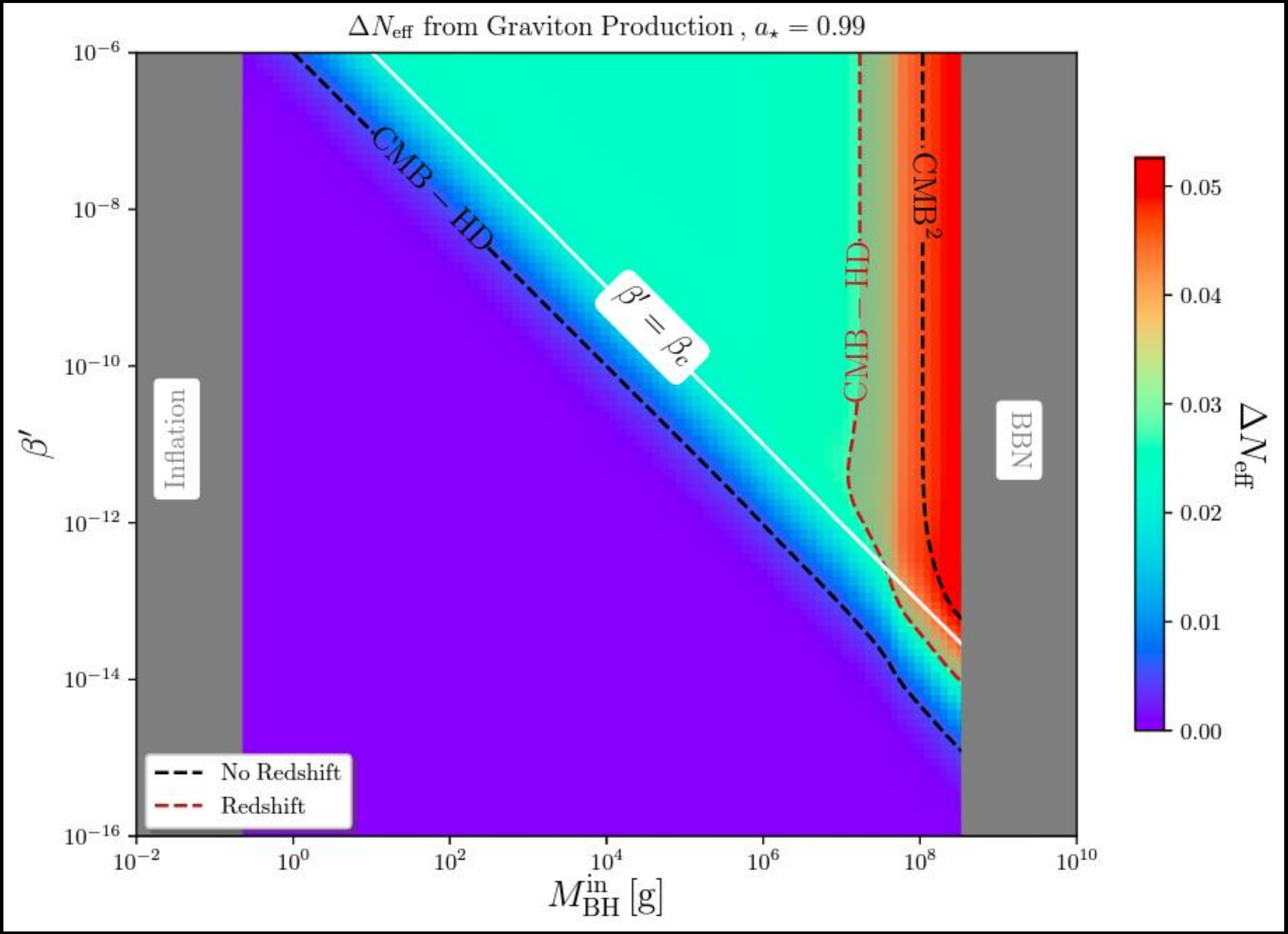
Why ?

$$\frac{\mathrm{d}\mathcal{N}_{\mathrm{DM}}}{\mathrm{d}p} = \int_0^{\tau} \mathrm{d}t' \, \frac{a(\tau)}{a(t')} \times \frac{\mathrm{d}^2\mathcal{N}_{\mathrm{DM}}}{\mathrm{d}p'\mathrm{d}t'} \left(p\frac{a(\tau)}{a(t')}, t' \right)$$

The correct one is better!

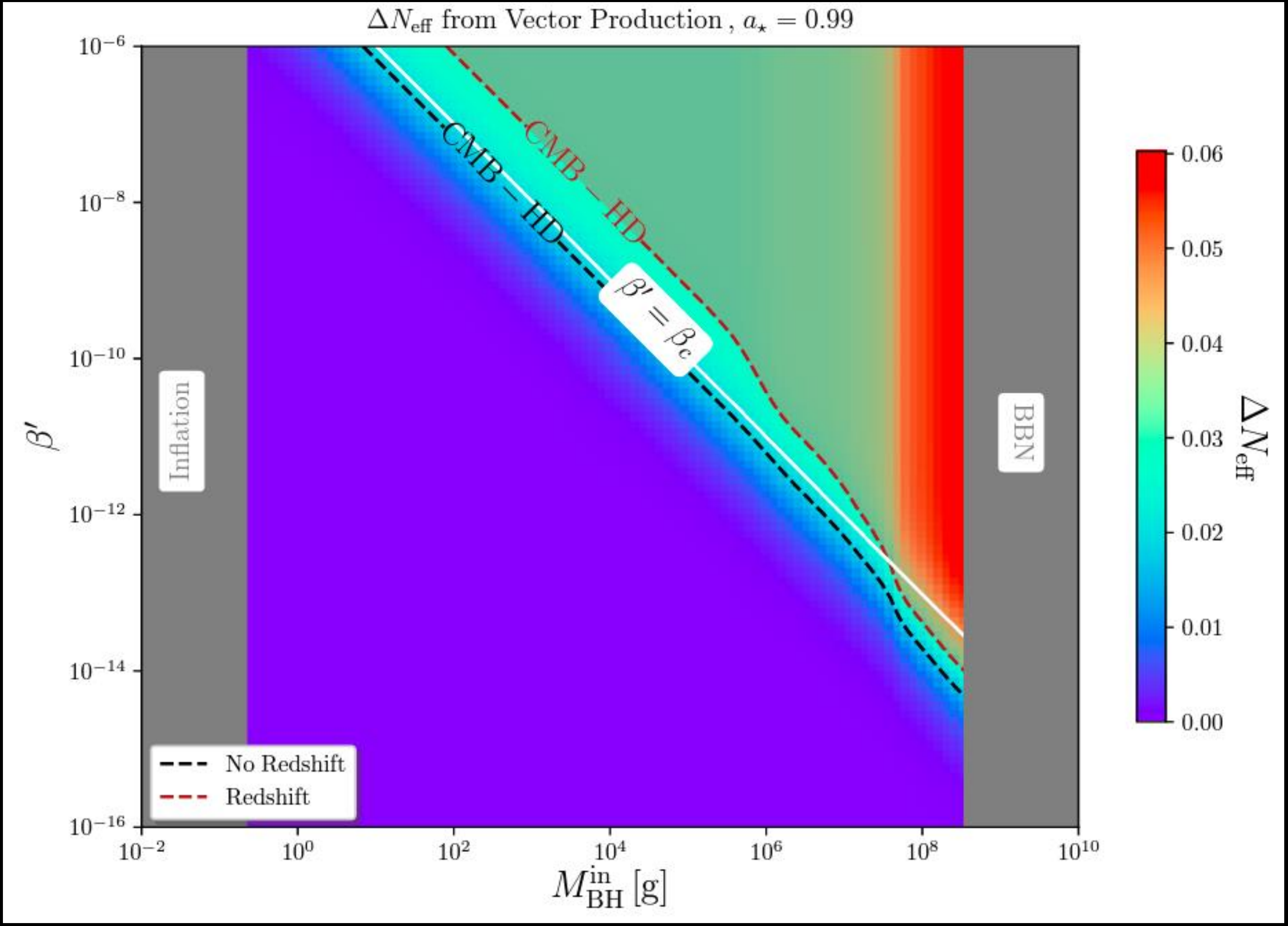
[Cheek, LH, Perez-Gonzalez and Turner '22] **Code Publicly available: FRISBHEE**

Kerr PBHs and Dark Radiation



[Cheek, LH, Perez-Gonzalez and Turner '22] **Code Publicly available: FRISBHEE**

Kerr PBHs and Dark Radiation



[Cheek, LH, Perez-Gonzalez and Turner ‘22] **Code Publicly available: FRISBHEE**

Evaporation of Extended Distributions

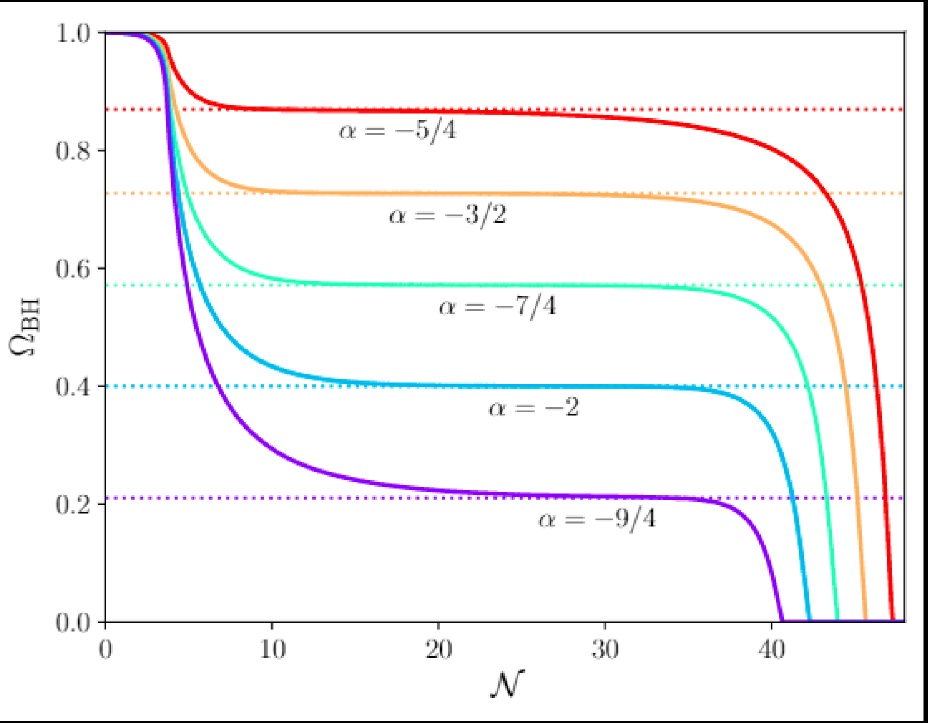
$$f_{\rm BH}(M) = \begin{cases} C M^{\alpha-1}, & \text{for } M_{\rm min} \le M \le M_{\rm max}; \\ 0, & \text{else.} \end{cases}$$

$$\alpha \equiv \frac{(-3w_{\rm form.} - 1)}{(w_{\rm form.} + 1)}$$

$$\frac{{\rm d}H}{{\rm d}t} = -\frac{1}{2}H^2(4 - \Omega_{\rm BH}),$$

$$\frac{{\rm d}\Omega_{\rm BH}}{{\rm d}t} = \Omega_{\rm BH}\left[\frac{\int_0^\infty f_{\rm BH}(M,t)\frac{{\rm d}M}{{\rm d}t}{\rm d}M}{\int_0^\infty f_{\rm BH}(M,t)M{\rm d}M}\right] + H\Omega_{\rm BH}(1 - \Omega_{\rm BH}).$$

$$= \frac{1+\alpha}{3(t-t_i)}$$

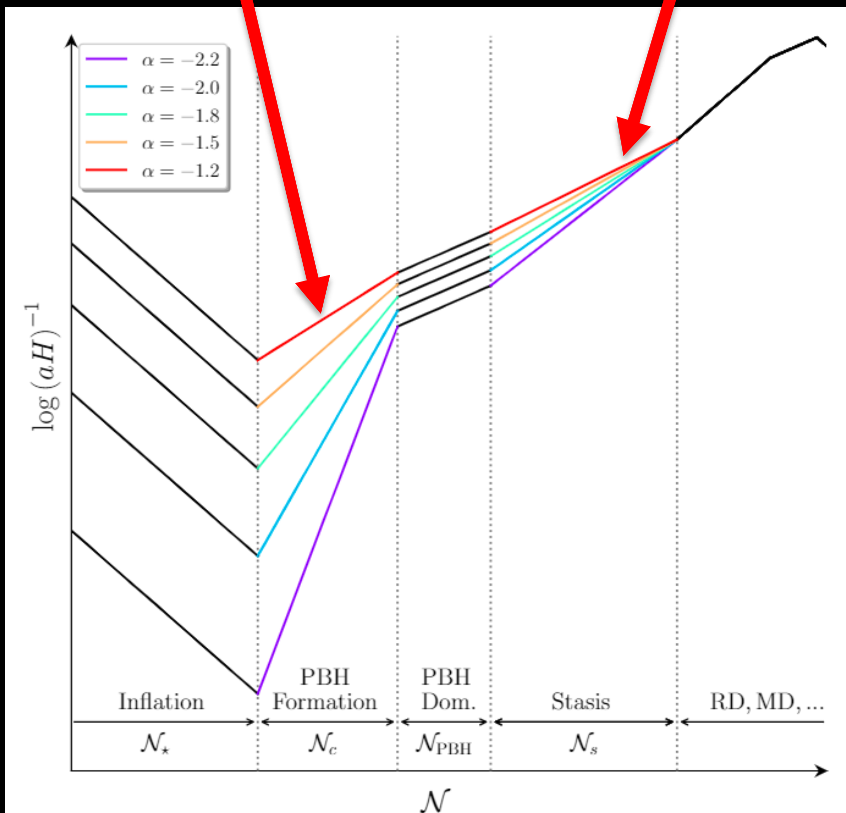


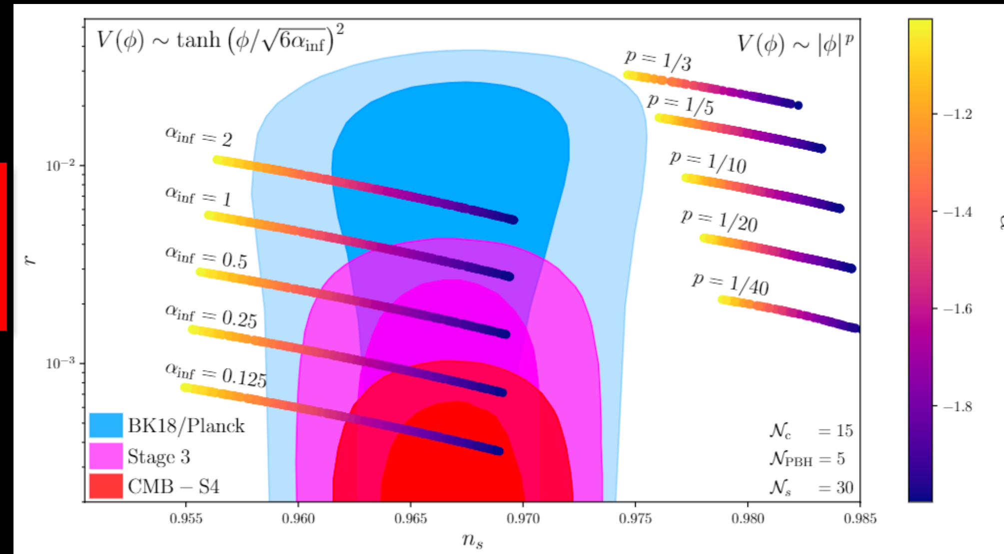
[Dienes, LH, Huang, Kim, Tait, Thomas '22]

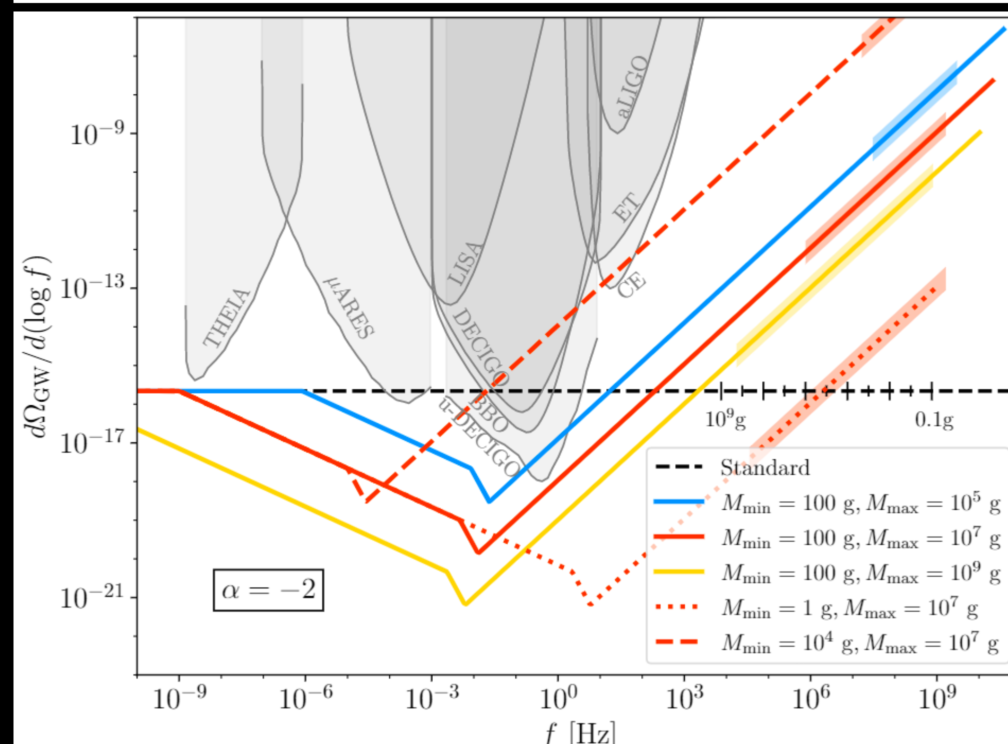
Gravitational Waves?

$$\alpha \equiv \frac{(-3w_{\rm form.} - 1)}{(w_{\rm form.} + 1)}$$

$$w_{\rm eff} = \frac{-\alpha - 1}{\alpha + 7}$$

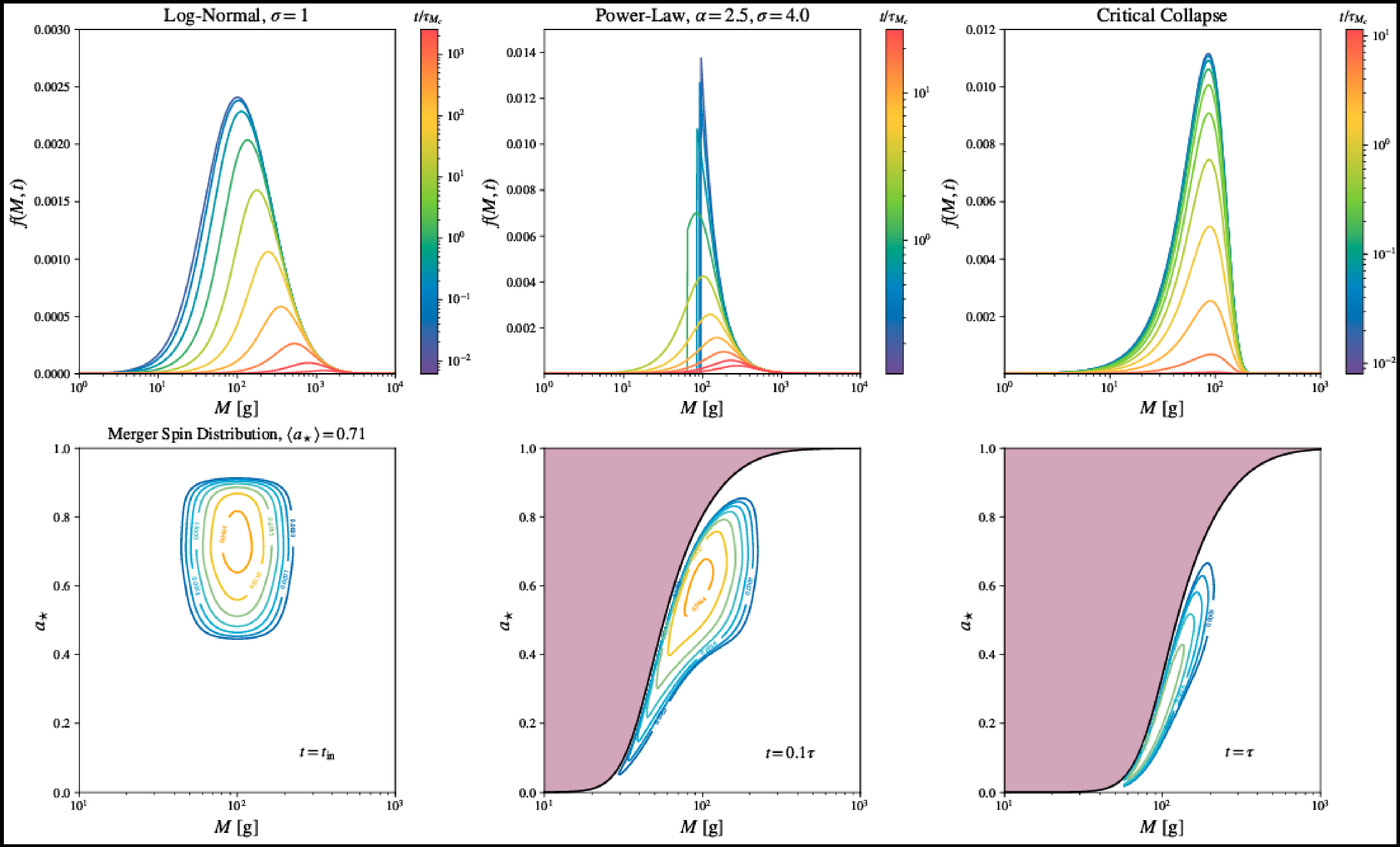






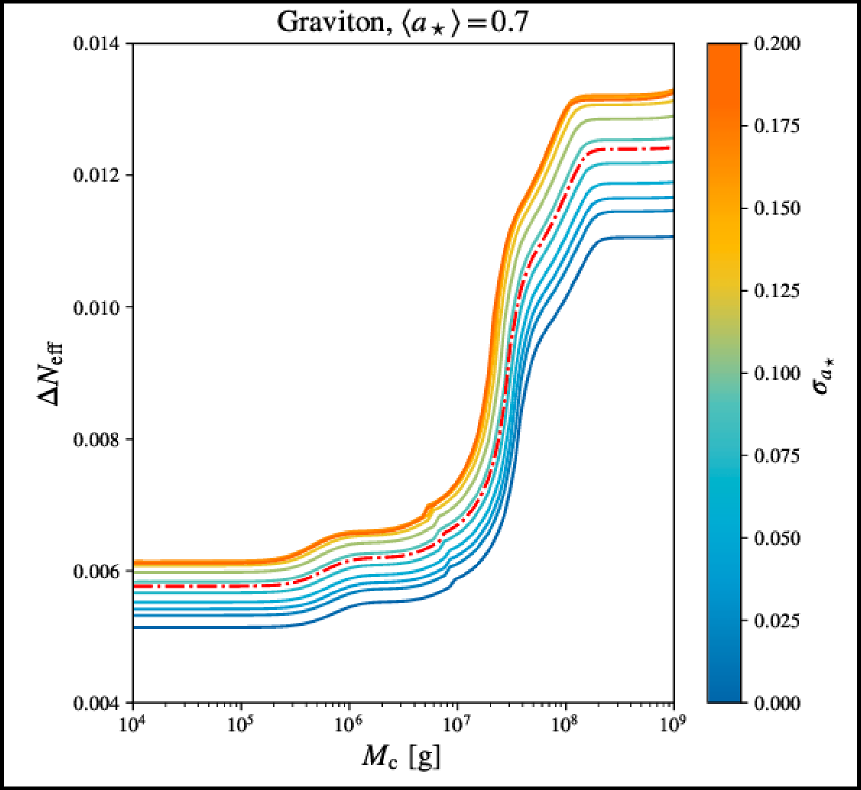
[Dienes, LH, Huang, Kim, Tait, Thomas ‘22]

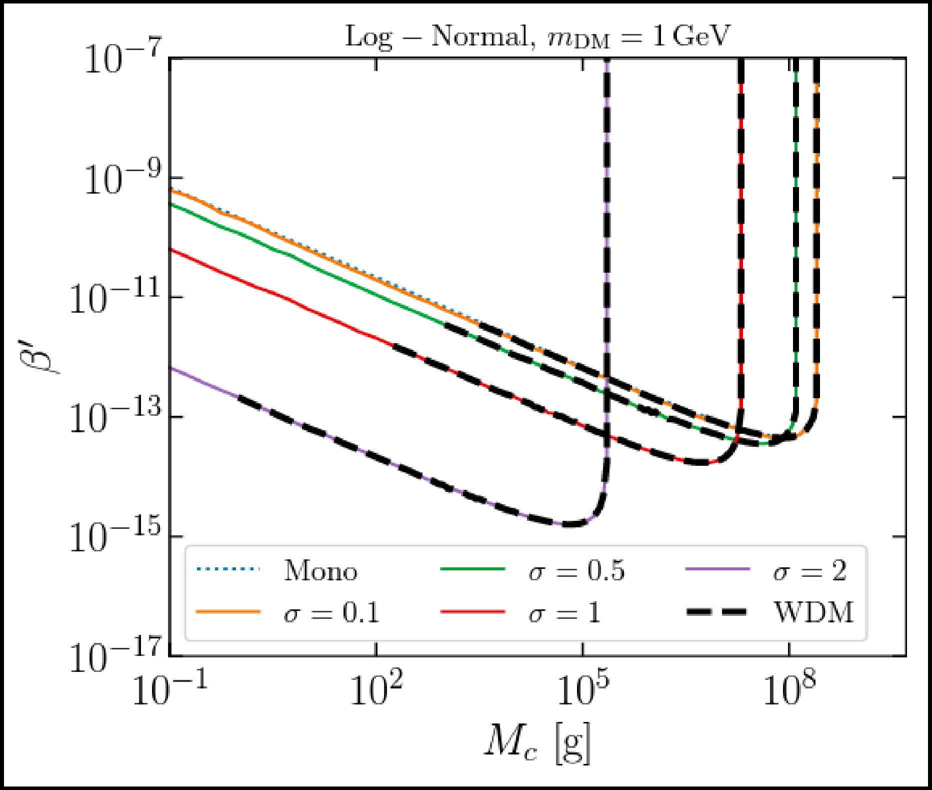
Evaporation of Extended Distributions



[Cheek, LH, Perez-Gonzalez, Turner ‘22]

III. Evaporation of Extended Distributions





[Cheek, LH, Perez-Gonzalez, Turner '22]

https://github.com/yfperezg/frisbhee

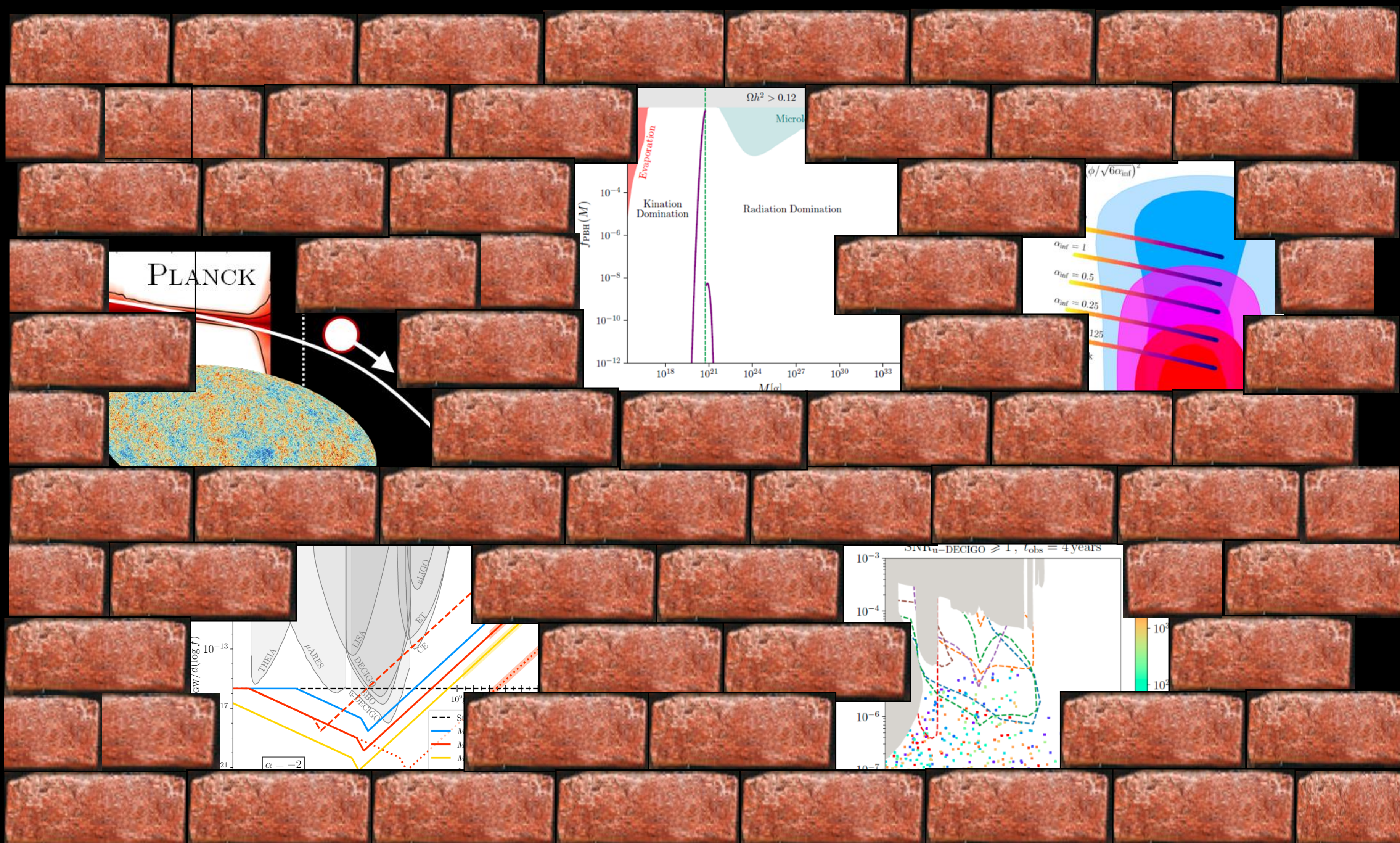
Conclusion

PBHs can leave several imprints in the early Universe

- Modify cosmology (EMD+ entropy inj.)
- Produce dark matter, leading to modified predictions for particle searches
- Particles produced from evaporation can be extremely boosted, which can lead to additionnal constraints from structure formation
- Kerr PBHs can lead to a large production of gravitons – existing results were refined
- Our code is accessible online: ?

https://github.com/yfperezg/frisbhee

Future Directions



Cosmic History is still to be unveiled!

Back up

PBHs likely to be produced AFTER Inflation

Collapse of Small-Scale Density Perturbations during Preheating in Single Field Inflation

Karsten Jedamzik* Martin Lemoine† and Jérôme Martin‡

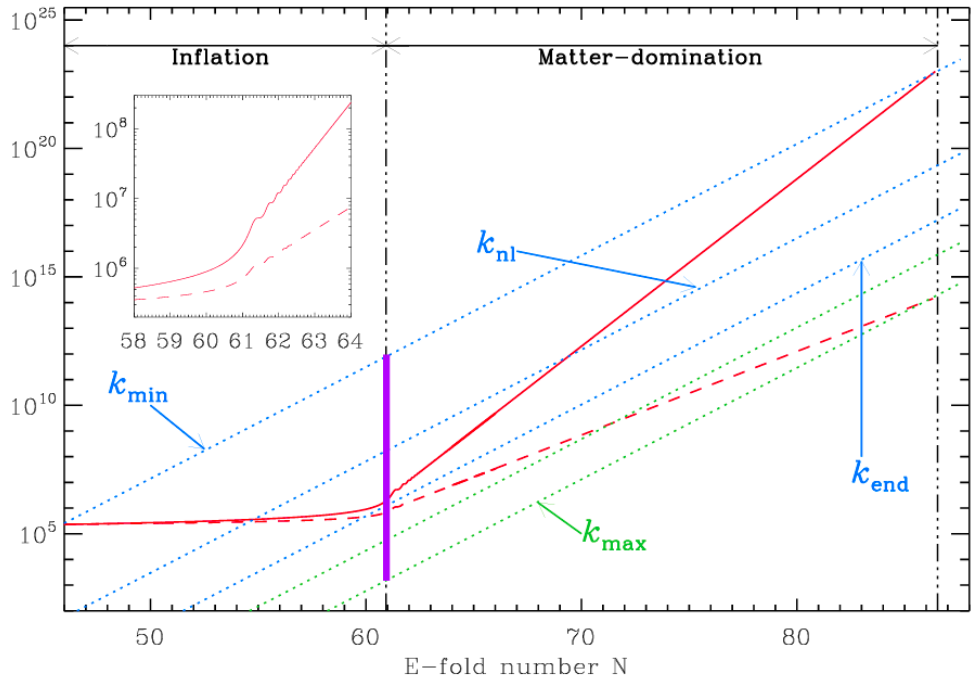
Primordial black holes from the preheating instability in single-field inflation

Jérôme Martin,[a] Theodoros Papanikolaou,[b] Vincent Vennin[b,a]

$$V(\phi) = \frac{m^2}{2}\phi^2 .$$

$$m = 2H_{\rm end}\frac{M_{\rm Pl}}{\phi_{\rm end}} .$$

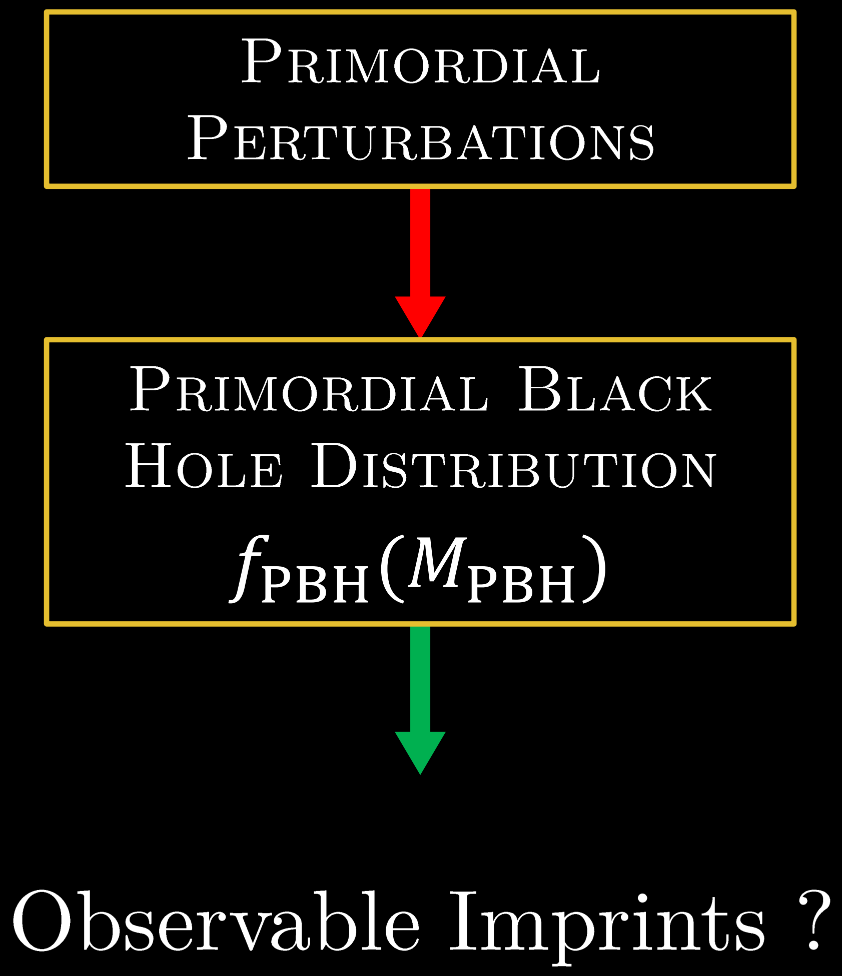
$$\phi(t) \simeq \phi_{\rm end}\left(\frac{a_{\rm end}}{a}\right)^{3/2}\sin(mt)$$



$$\frac{{\rm d}^2\tilde{v}_{\boldsymbol{k}}}{{\rm d}z^2}+\left[1+\frac{k^2}{m^2a^2}-\sqrt{6}\kappa\phi_{\rm end}\left(\frac{a_{\rm end}}{a}\right)^{3/2}\cos(2z)\right]\tilde{v}_{\boldsymbol{k}}=0\,,$$

where we have defined $z \equiv mt + \pi/4$. This equation is similar to a Mathieu equation

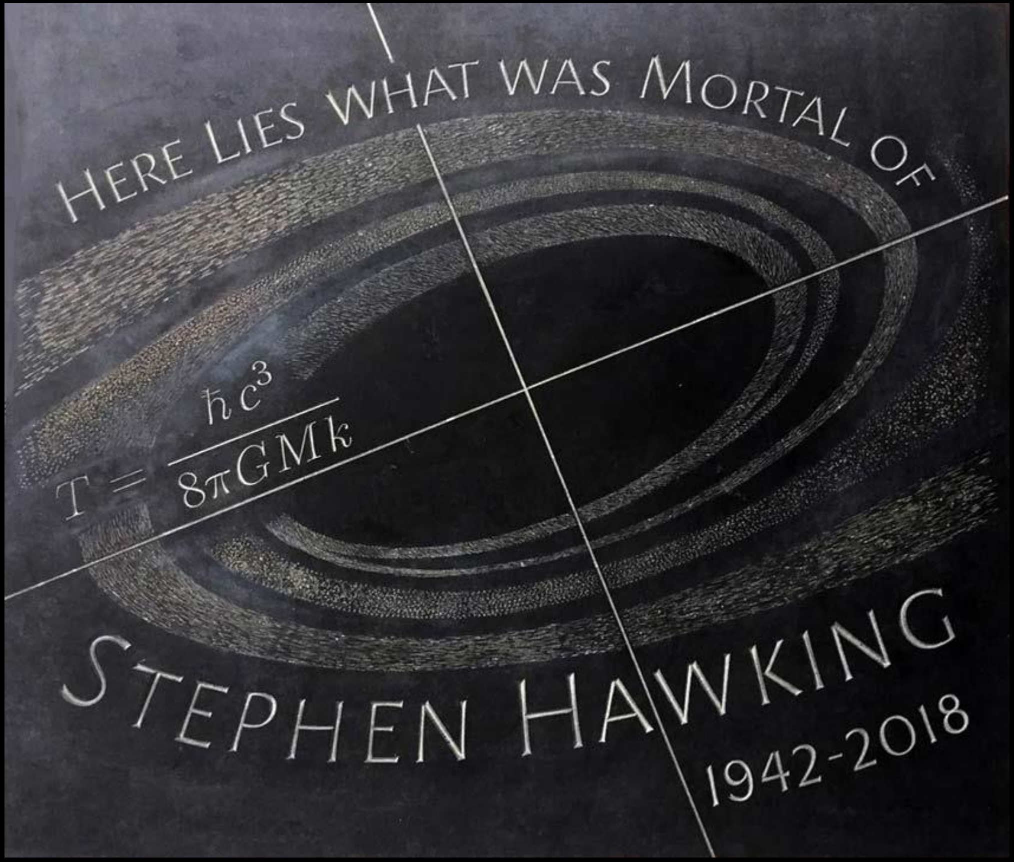
$$\frac{{\rm d}^2\tilde{v}_{\boldsymbol{k}}}{{\rm d}z^2}+\left[A_{\boldsymbol{k}}-2q\cos(2z)\right]\tilde{v}_{\boldsymbol{k}}=0 \qquad (13)$$



Primordial Perturbations
Primordial Black Hole Distribution
$f_{\rm PBH}(M_{\rm PBH})$
Observable Imprints ?

Black Holes Evaporate…

S. Hawking, 1974



Primordial Black Hole Distribution

$f_{\rm PBH}(M_{\rm PBH})$

- Some may be stable and participate to the DM relic abundance ($M_{\rm PBH} \gtrsim 10^{15}\,{\rm g}$)
- Some may be unstable and evaporate after BBN ($10^{9}\,{\rm g} \lesssim M_{\rm PBH} \lesssim 10^{15}\,{\rm g}$)
- Some may be unstable and evaporate before BBN ($M_{\rm PBH} \lesssim 10^{9}\,{\rm g}$)

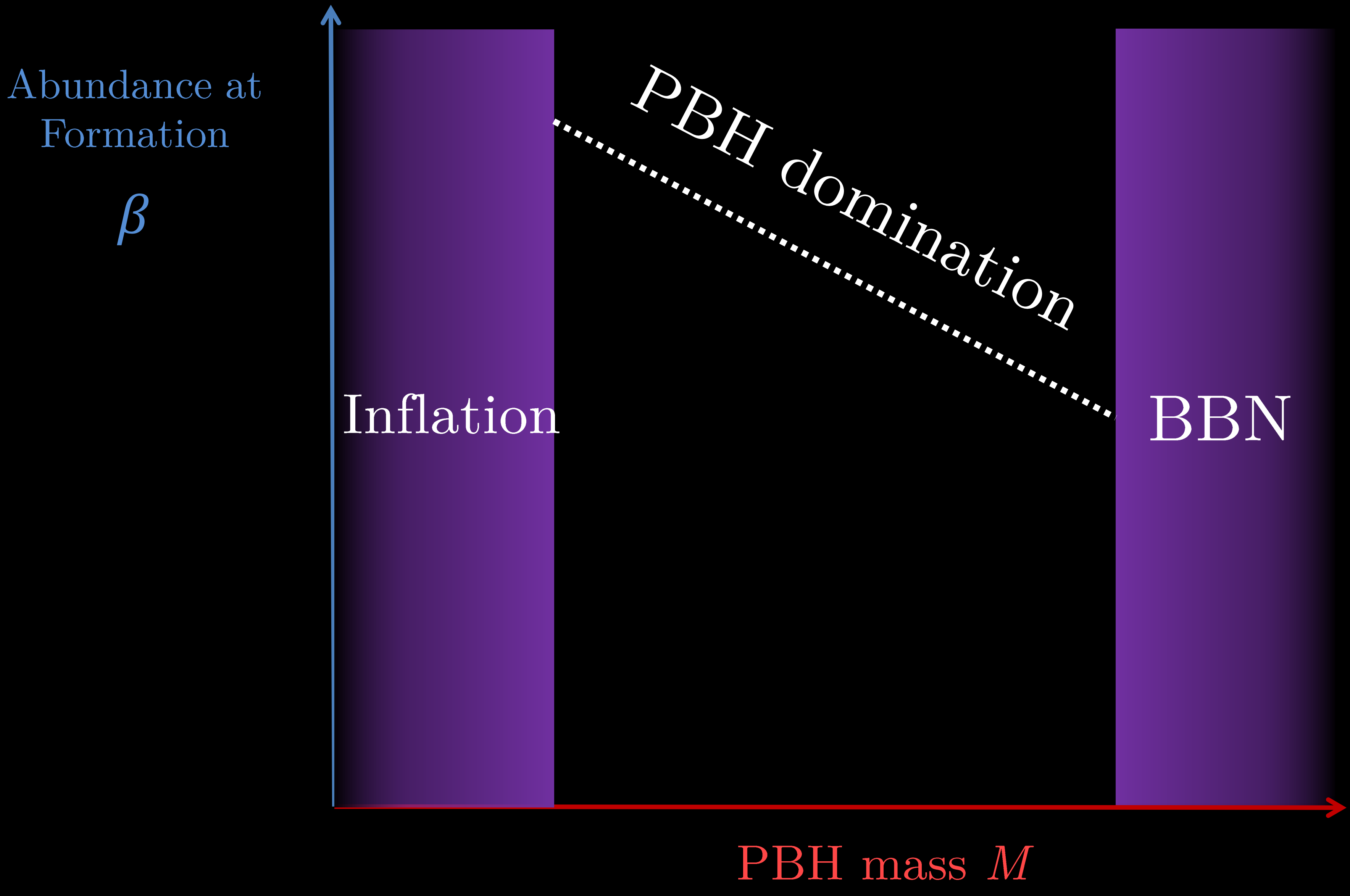
Primordial Black Hole Distribution

$f_{\mathrm{PBH}}(M_{\mathrm{PBH}})$

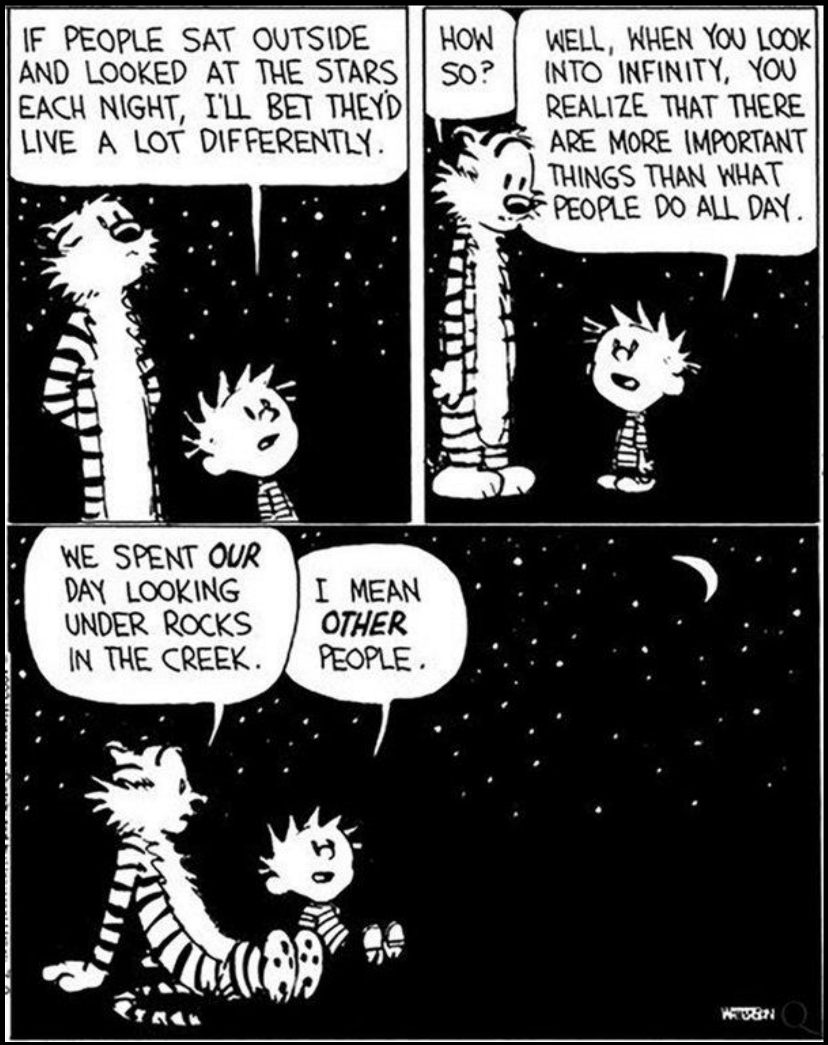
- Some may be stable and participate to the DM relic abundance ($M_{\mathrm{PBH}} \gtrsim 10^{15}\,\mathrm{g}$)
- Some may be unstable and evaporate after BBN ($10^{9}\,\mathrm{g} \lesssim M_{\mathrm{PBH}} \lesssim 10^{15}\,\mathrm{g}$)
- Some may be unstable and evaporate before BBN ($M_{\mathrm{PBH}} \lesssim 10^{9}\,\mathrm{g}$)

The topic of this talk ...

The Parameter Space



Is the SM still alone at the Planck scale?



Any BSM particle can be produced through PBH evaporation...

Outline

I. Effect of PBH evaporation on Dark-Matter Phenomenology

II. Kerr PBHs and Dark Radiation

III. Evaporation of Extended Distributions

IV. Gravitational Waves ?

I. Effect of PBH evaporation on Dark-Matter Phenomenology

If the DM relic density is made, at least partially, of particles, PBHs would contribute to its production.

PBH evaporation

$$\frac{\mathrm{d}M_{\mathrm{BH}}}{\mathrm{d}t} \equiv \sum_i \left.\frac{\mathrm{d}M_{\mathrm{BH}}}{\mathrm{d}t}\right|_i = -\sum_i \int_0^\infty E_i \frac{\mathrm{d}^2\mathcal{N}_i}{\mathrm{d}p\mathrm{d}t}\mathrm{d}p \;= -\varepsilon(M_{\mathrm{BH}})\frac{M_p^4}{M_{\mathrm{BH}}^2}$$

$$\frac{\mathrm{d}^2\mathcal{N}_i}{\mathrm{d}p\,\mathrm{d}t} = \frac{g_i}{2\pi^2}\frac{\sigma_{s_i}(M_{\mathrm{BH}},\mu_i,p)}{\exp\left[E_i(p)/T_{\mathrm{BH}}\right]-(-1)^{2s_i}}\frac{p^3}{E_i(p)}$$

$$\varepsilon(M_{\mathrm{BH}}) \equiv \sum_i g_i\varepsilon_i(z_i)$$

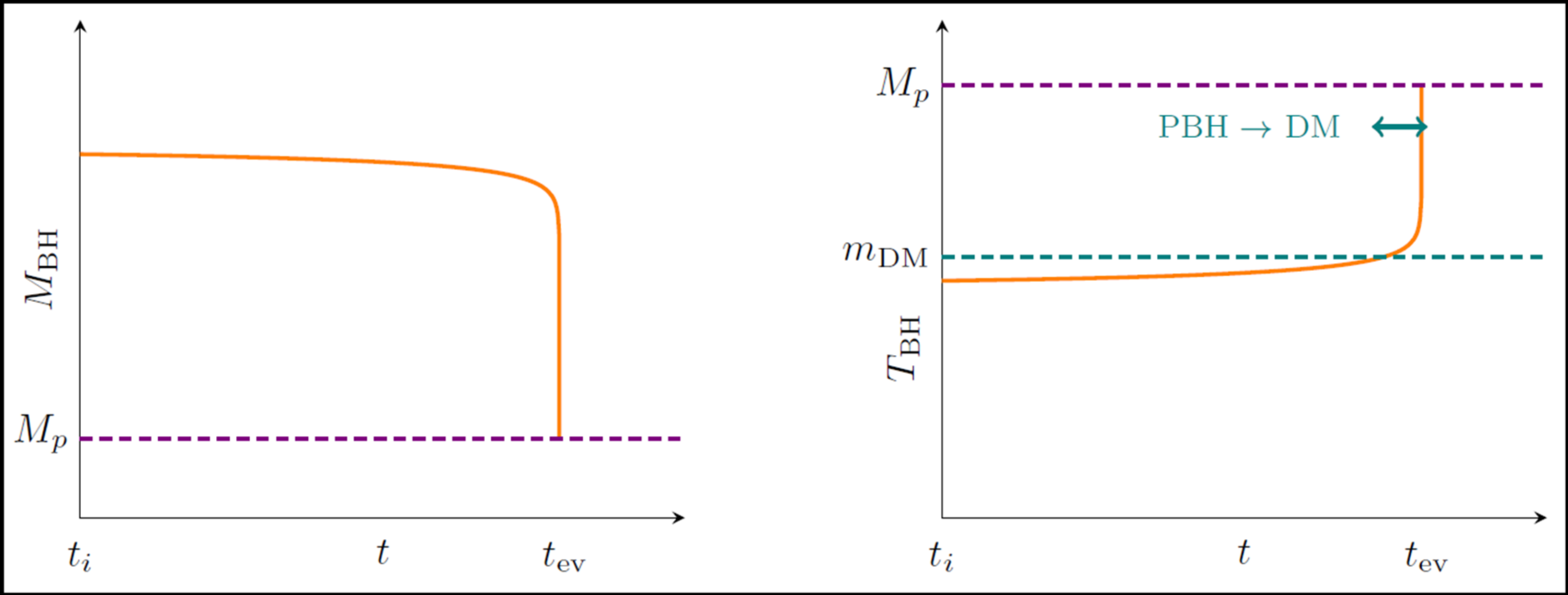
$$z_i \;=\; \mu_i/T_{\mathrm{BH}}$$

BSM Contributions?

$$T_{\mathrm{BH}} = \frac{1}{8\pi G M_{\mathrm{BH}}} \sim 1.06\ \mathrm{GeV}\left(\frac{10^{13}\ \mathrm{g}}{M_{\mathrm{BH}}}\right)$$

PBH evaporation

$$T_{\rm BH} = \frac{1}{8\pi G M_{\rm BH}} \sim 1.06\ \text{GeV} \left(\frac{10^{13}\ \text{g}}{M_{\rm BH}}\right)$$



→ More and more particles contribute to the evaporation

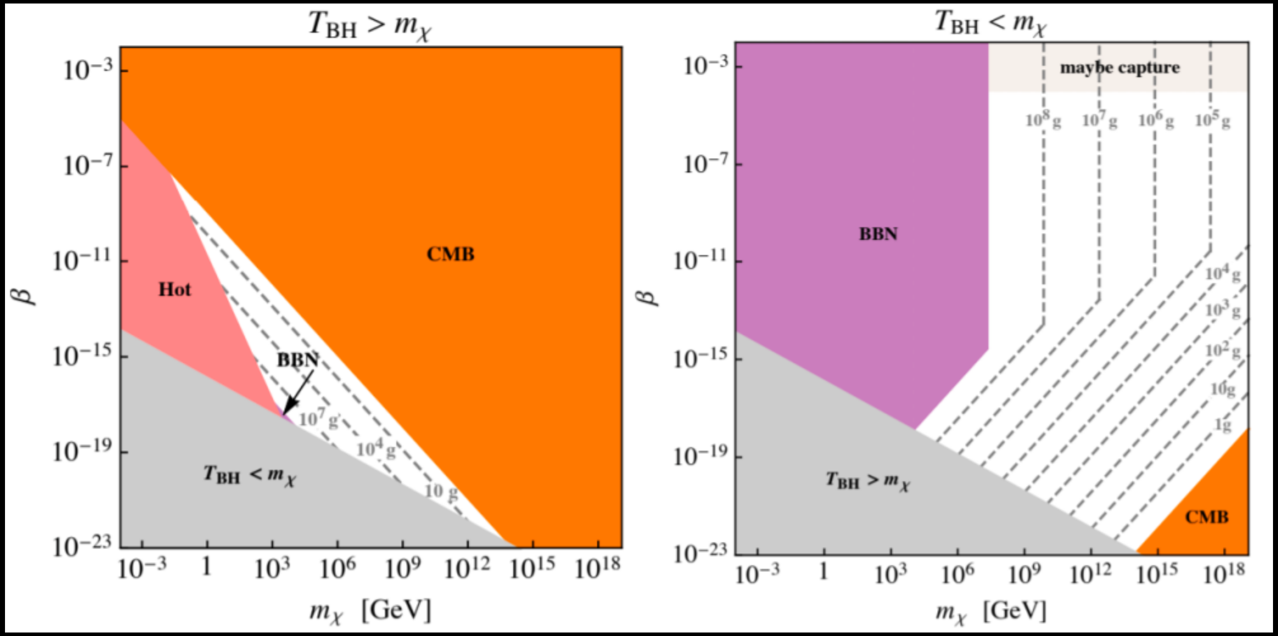
DM from Evaporation

$$\frac{\mathrm{d}^2\mathcal{N}_i}{\mathrm{d}p\,\mathrm{d}t} = \frac{g_i}{2\pi^2}\frac{\sigma_{s_i}(M_{\rm BH},\mu_i,p)}{\exp\left[E_i(p)/T_{\rm BH}\right]-(-1)^{2s_i}}\frac{p^3}{E_i(p)}$$

Very much used in the literature: the **geometrical-optics limit**

$$GM_{\rm BH}p \gg 1$$

$$\sigma_{s_i}(E,\mu)|_{\rm GO} = 27\pi G^2 M_{\rm BH}^2$$



[Gondolo, Sandick and Shams Es Haghi ‘20]

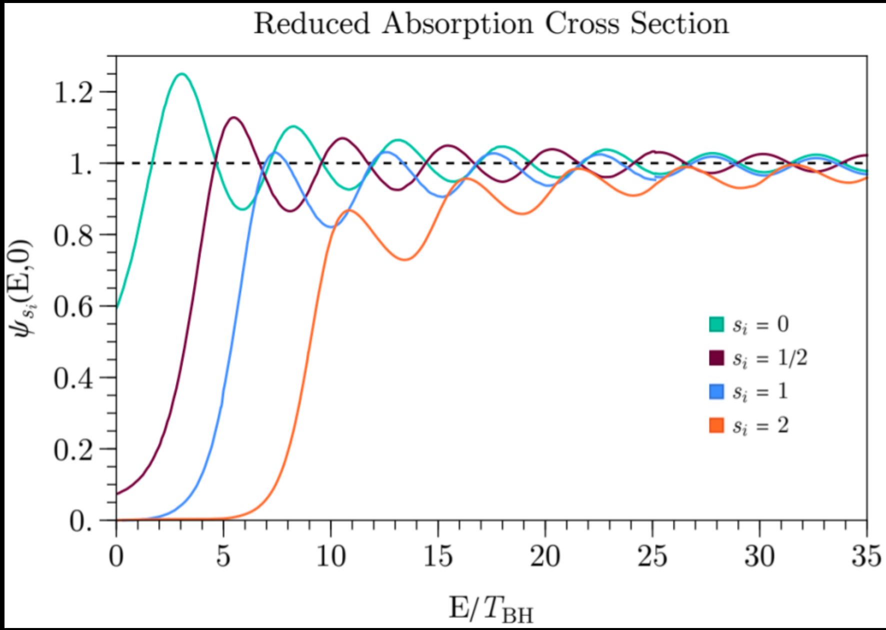
DM from Evaporation

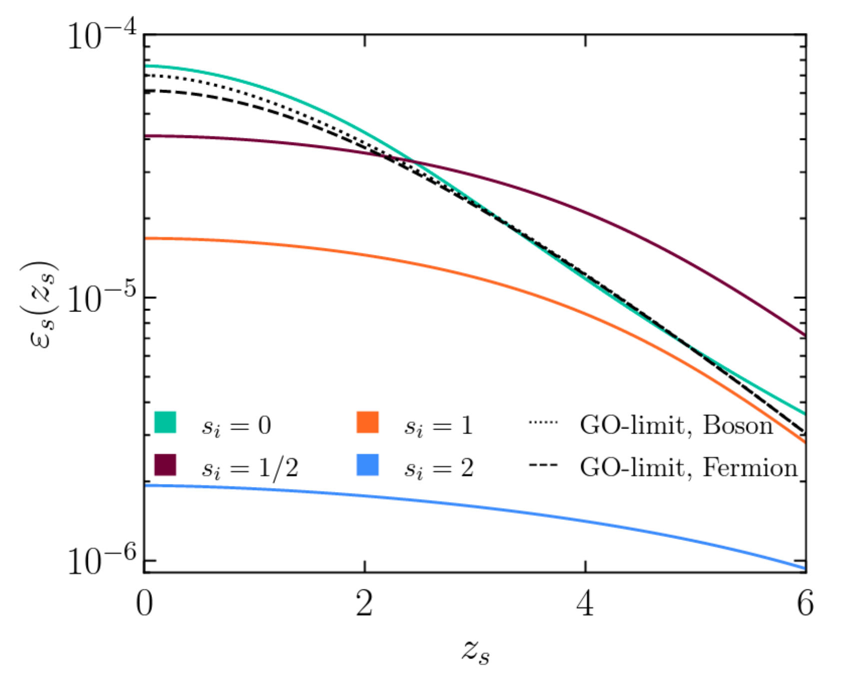
$$\frac{\mathrm{d}^2\mathcal{N}_i}{\mathrm{d}p\,\mathrm{d}t} = \frac{g_i}{2\pi^2}\frac{\sigma_{s_i}(M_{\rm BH},\mu_i,p)}{\exp\left[E_i(p)/T_{\rm BH}\right]-(-1)^{2s_i}}\frac{p^3}{E_i(p)}$$

Very bad approximation at (not too) low momentum...

$$\psi_{s_i}(E,\mu) \equiv \frac{\sigma_{s_i}(E,\mu)}{27\pi G^2 M_{\rm BH}^2}$$

$$\varepsilon_i(z_i) = \frac{27}{8192\pi^5}\int_{z_i}^{\infty}\frac{\psi_{s_i}(x)(x^2-z_i^2)}{\exp(x)-(-1)^{2s_i}}\,x\,\mathrm{d}x$$



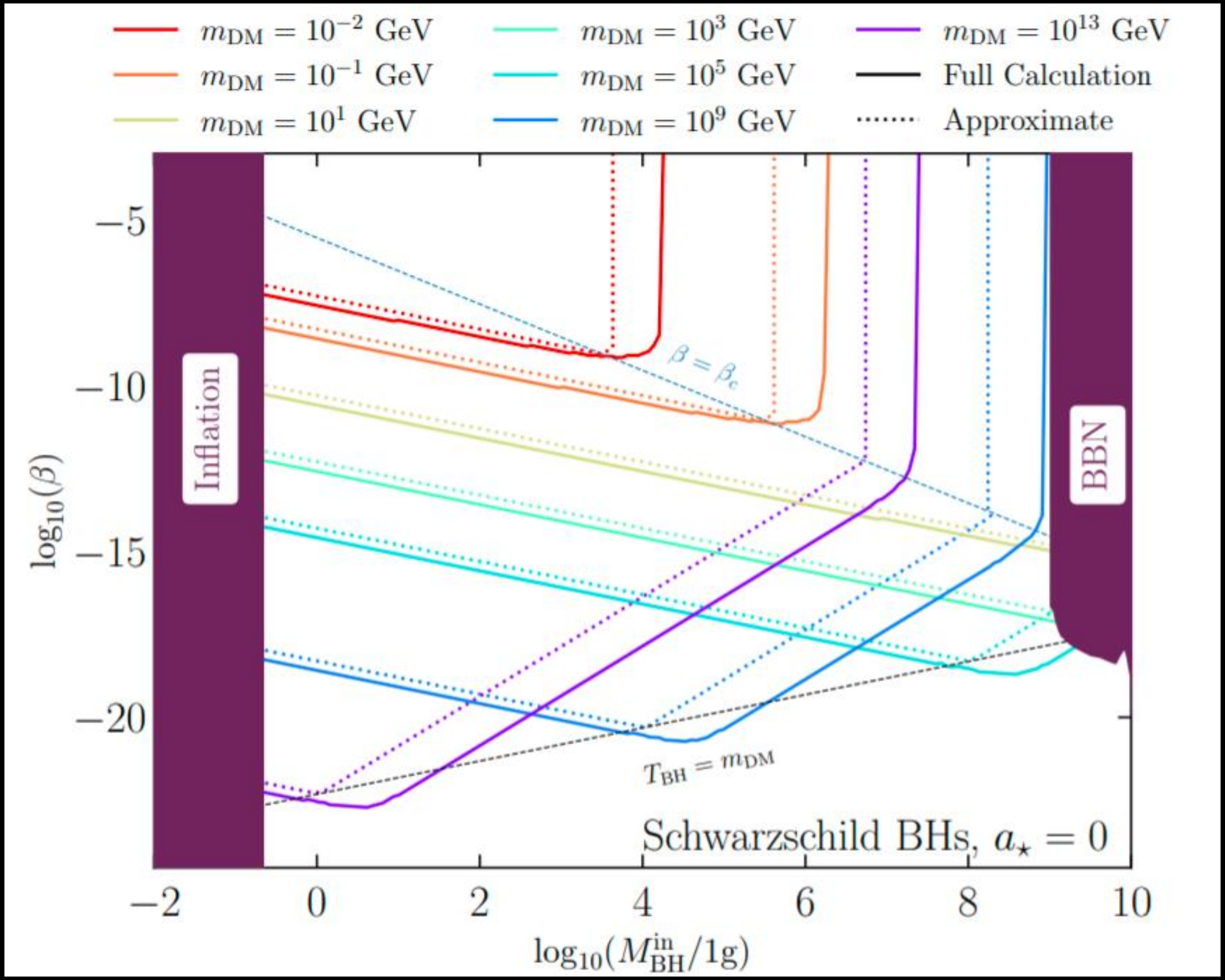


[Cheek, LH, Perez-Gonzalez and Turner '22]

Code Publicly available: FRISBHEE

DM from Evaporation

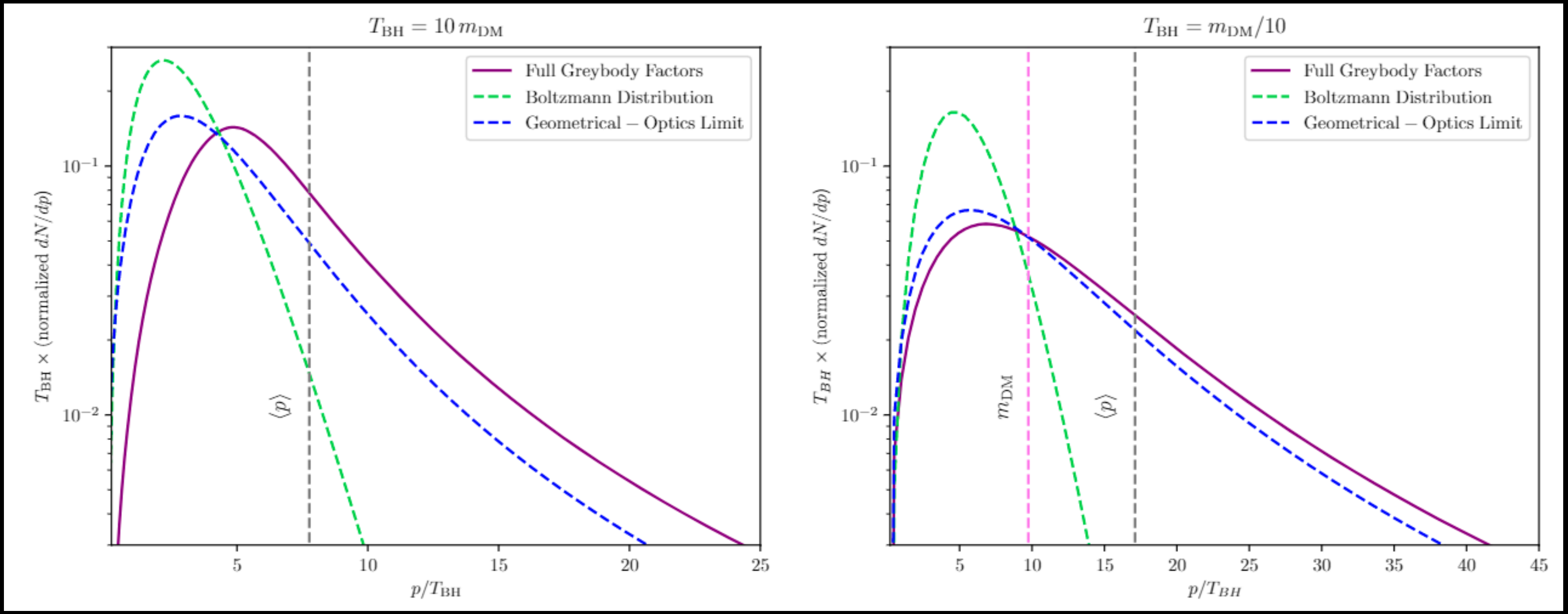
$$f_{\text{PBH}}(M) = \delta(M - M_{\text{PBH}})$$



[Cheek, LH, Perez-Gonzalez and Turner '22] **Code Publicly available: FRISBHEE**

DM from Evaporation

$$f_{\rm PBH}(M) = \delta(M - M_{\rm PBH})$$



[Cheek, LH, Perez-Gonzalez and Turner '22]   **Code Publicly available: FRISBHEE**

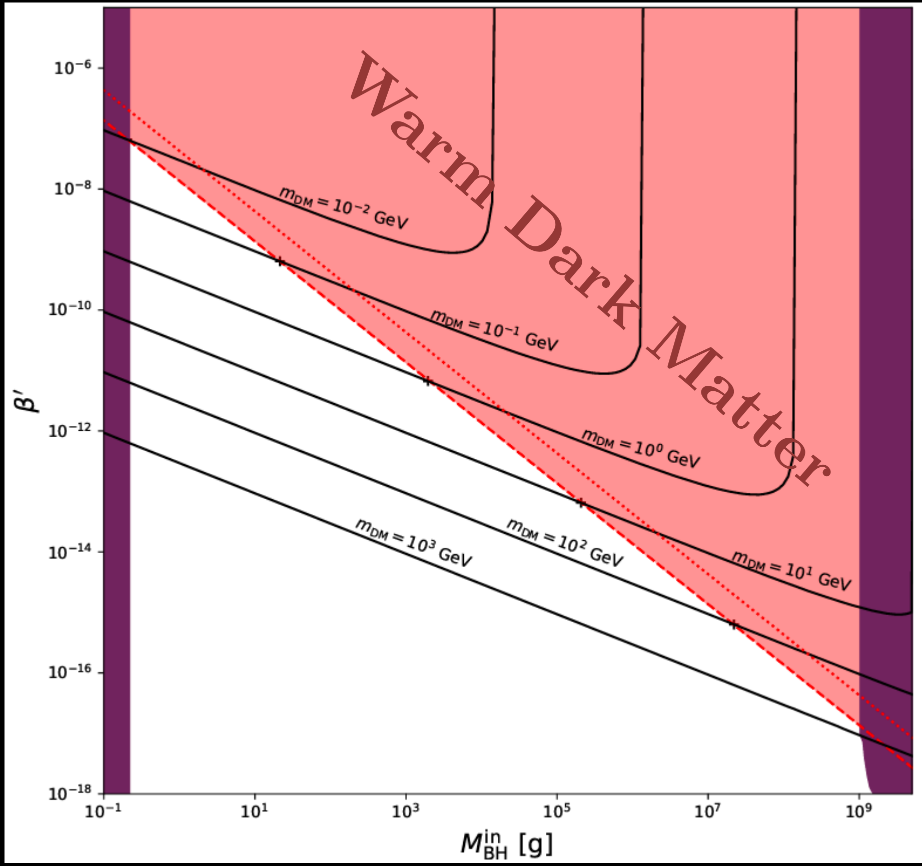
Kerr PBHs and Warm Dark Matter

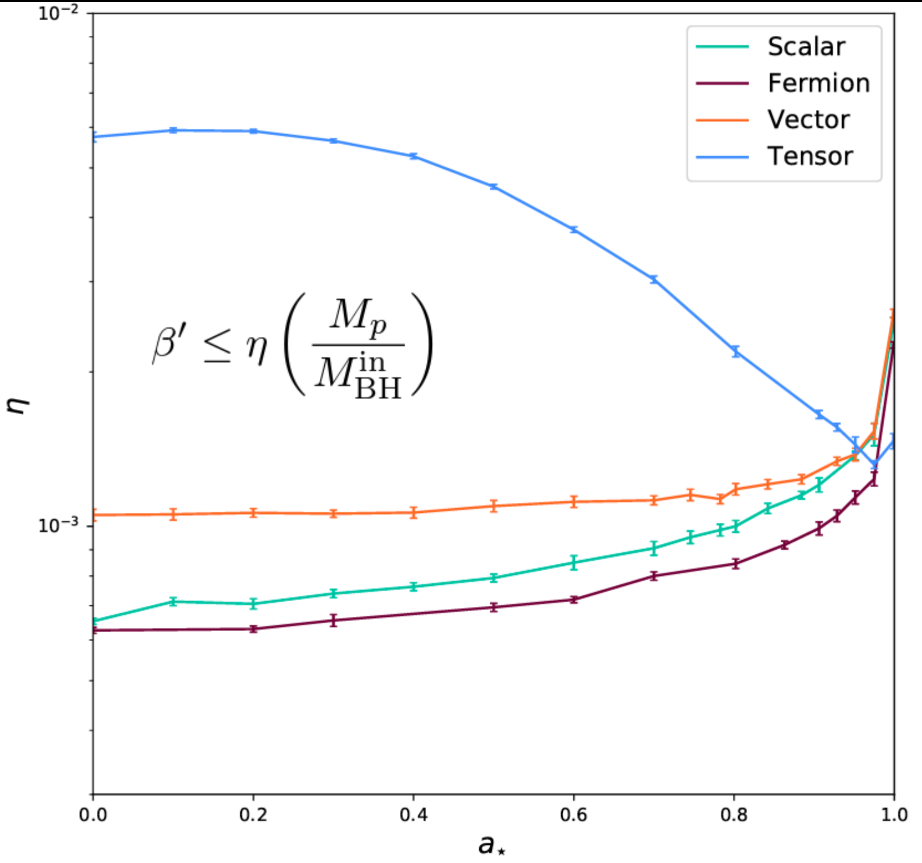
Using CLASS: expected matter power spectrum

$$P(k) = P_{\rm CDM}(k)T^2(k)$$

$$T(k) = (1 + (\alpha k)^{2\mu})^{-5/\mu}$$

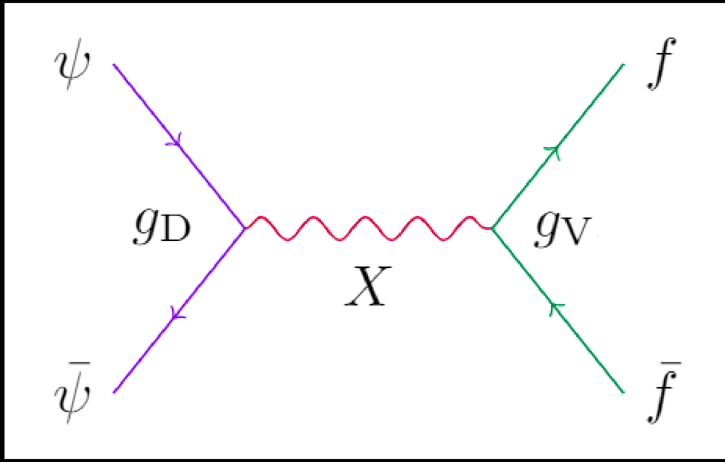
Saturated at $\alpha = 1.3 \times 10^{-2}\,{\rm Mpc}\,h^{-1}$

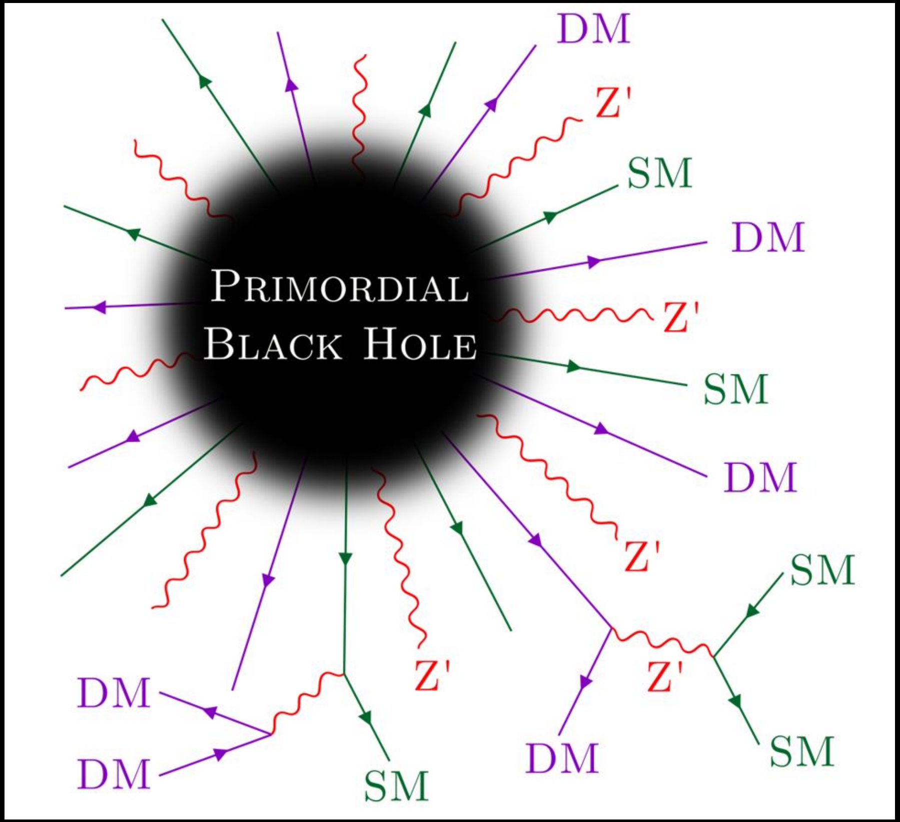




Thermal Production of DM

- DM may interact with SM particles and be produced in the early universe through thermal processes...
- Freeze-In or Freeze-Out





Thermal Production of DM

DM Annihilation, X decay

PBH evaporation

$$\dot{n}_{\rm DM} + 3Hn_{\rm DM} = g_{\rm DM}\int C[f_{\rm DM}]\frac{{\rm d}^3p}{(2\pi)^3} + \left.\frac{{\rm d}n_{\rm DM}}{{\rm d}t}\right|_{\rm BH},$$

$$\dot{n}_X + 3Hn_X = g_X\int C[f_X]\frac{{\rm d}^3p}{(2\pi)^3} + \left.\frac{{\rm d}n_X}{{\rm d}t}\right|_{\rm BH},$$

$$\dot{\rho}_{\rm SM} + 4H\rho_{\rm SM} = \left.\frac{{\rm d}M}{{\rm d}t}\right|_{\rm SM}.$$

PBHs evaporate **non-trivial distributions** of DM and X particles

Non-trivial evolution of the full distributions $f_X(p)$ and $f_{DM}(p)$

$$\left.\frac{{\rm d}n_i}{{\rm d}t}\right|_{\rm BH} = n_{\rm BH}g_i\int\left.\frac{\partial f_i}{\partial t}\right|_{\rm BH}\frac{p^2{\rm d}p}{2\pi^2}$$

Simplified approch...

Thermal Production of DM

- If PBHs evaporate **before FO**:
  → Assume Instantaneous thermalization

- If PBHs evaporate **after FO**:
  → Assume No thermalization

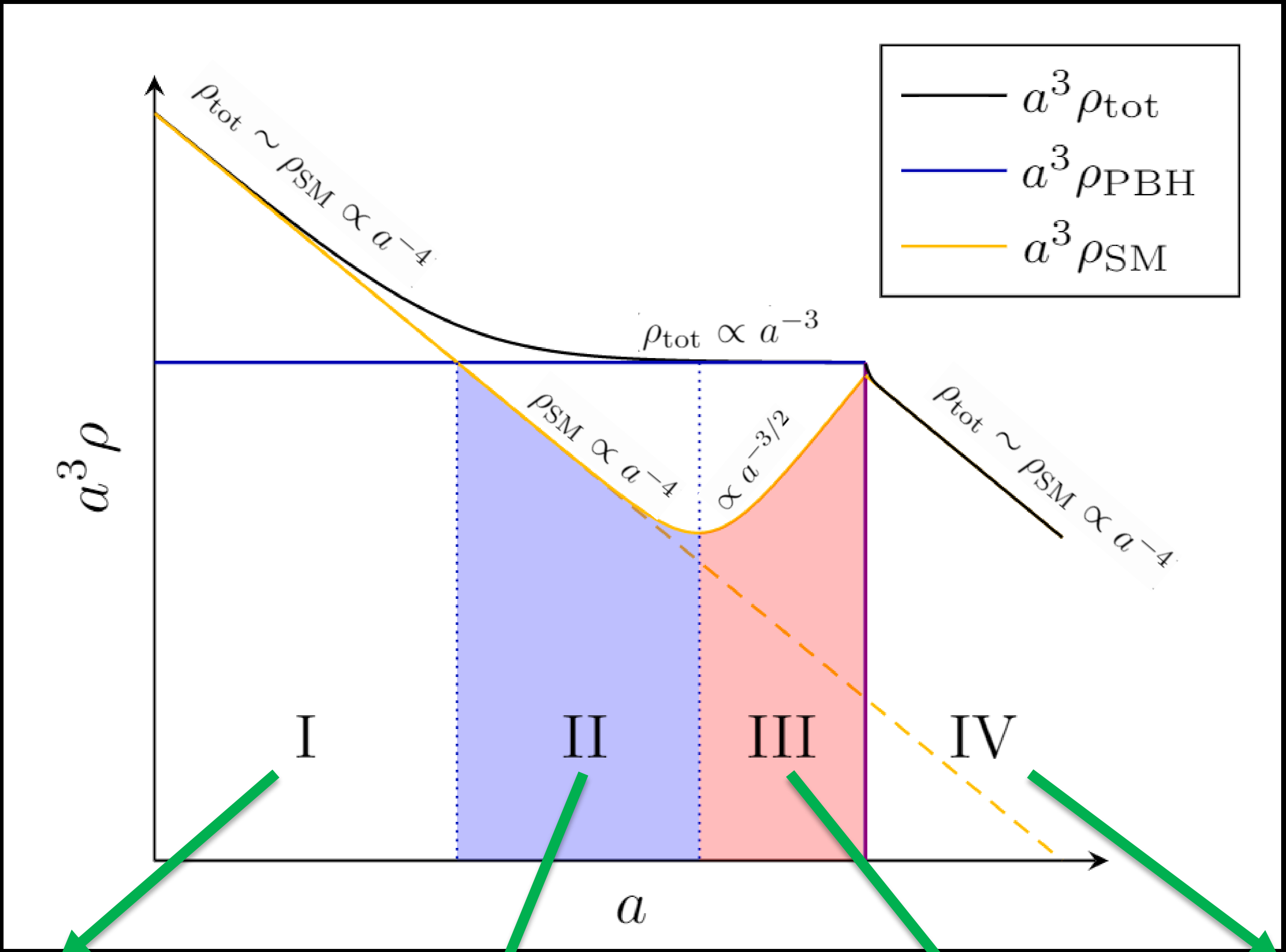
- **FI case**: assume No thermalization

→ Check those assumptions by evaluating at all time

$$\Gamma_{\rm th+ev} \equiv \frac{\langle \sigma \cdot v \rangle_{\rm th+ev} \times n^{\rm th}}{H}$$

$$\langle \sigma \cdot v \rangle_{\rm th+ev} \equiv \frac{\int \sigma \cdot v_{\rm moll} f_{\rm ev} f_{\rm th} d^3\vec{p}_1 d^3\vec{p}_2}{\left[\int d^3\vec{p}_1 f_{\rm ev}\right]\left[\int d^3\vec{p}_2 f_{\rm th}\right]}.$$

Modified Cosmology



FI/FO + entropy dilution

Matter-Dominated FI/FO

FI/FO during entropy injection

Regular FI/FO

Results

Freeze-Out [Cheek, LH, Perez-Gonzalez and Turner '22]

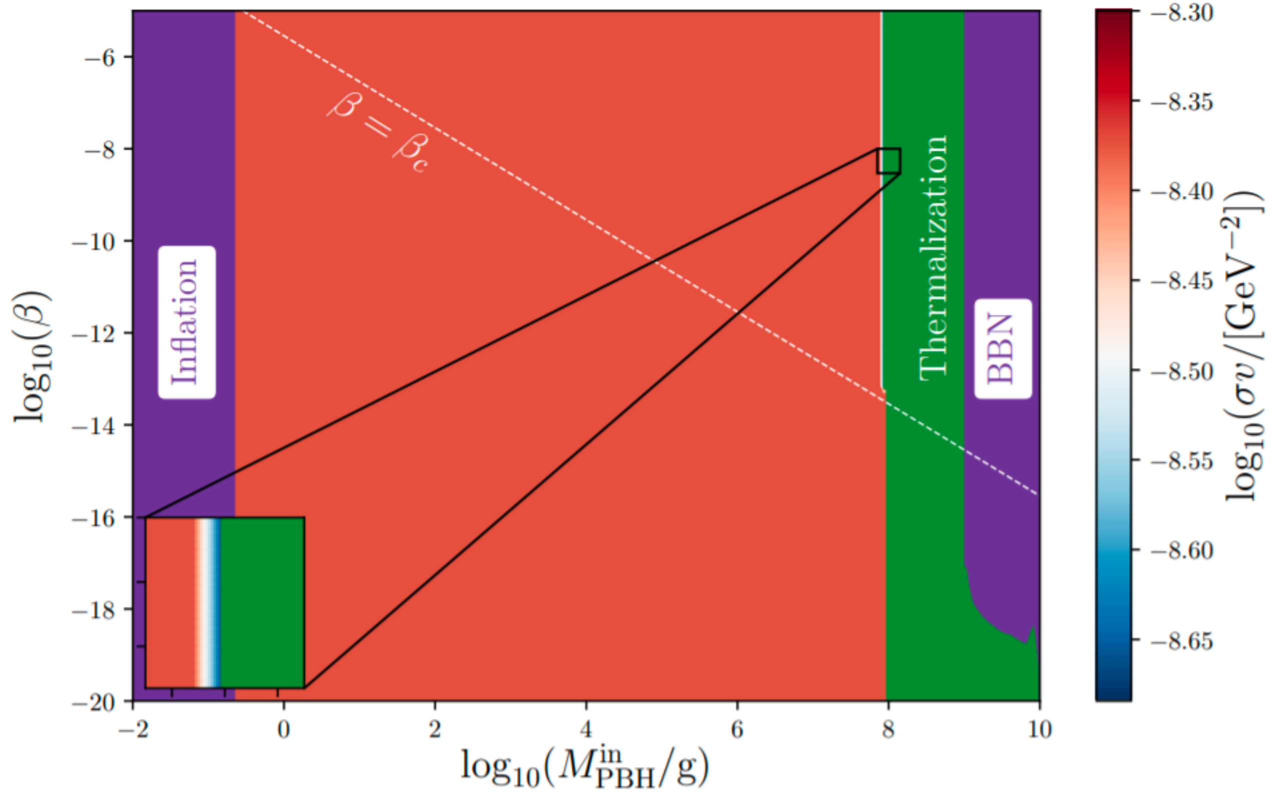


Fig. 7. Two-dimensional scan over the PBH fraction β and mass $M_{\rm BH}$ for a mediator mass $m_X = 10\,{\rm GeV}$ and a dark matter mass $m_{\rm DM} = 1\,{\rm GeV}$, and ${\rm Br}(X \to {\rm DM}) = 0.5$. The color map indicates the value of the non-relativistic cross-section of DM annihilation leading to the correct relic abundance in the Freeze-Out case. See the main text for a description of the different constraints.

Results

Freeze-In

[Cheek, LH, Perez-Gonzalez and Turner '22]

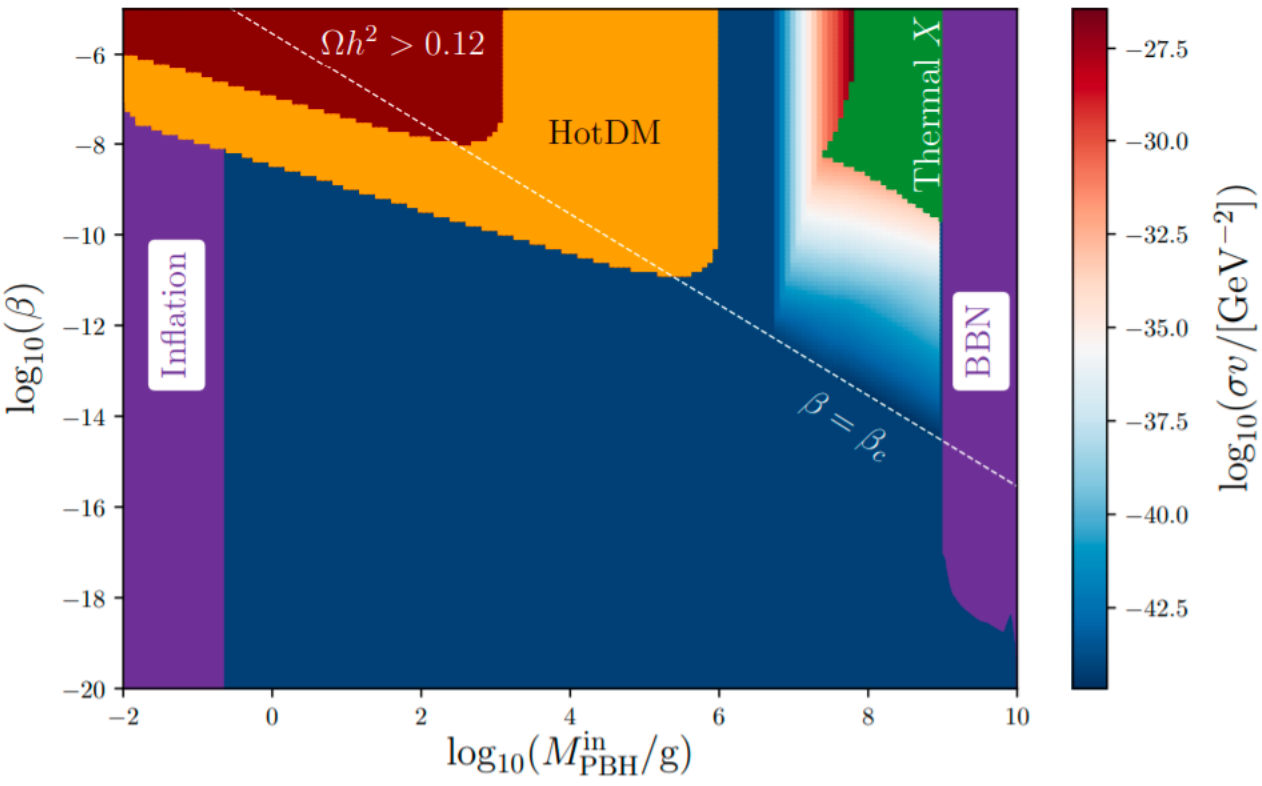


Fig. 11. Two-dimensional scan over the PBH fraction β and mass $M_{\rm BH}$ for a mediator mass $m_X = 1\,{\rm TeV}$, a dark matter mass $m_{\rm DM} = 1\,{\rm MeV}$, and ${\rm Br}(X \to {\rm SM}) = 10^{-7}$. The color map indicates the value of the non-relativistic cross-section of DM annihilation leading to the correct relic abundance in the Freeze-In case. See the main text for a description of the different constraints.

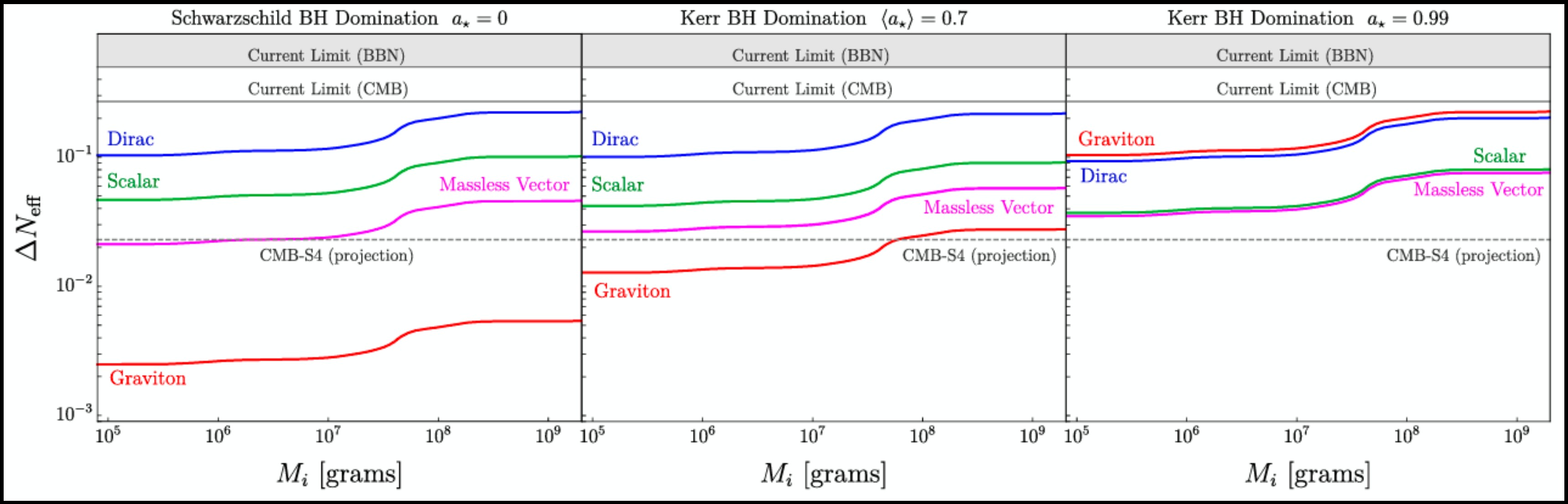
II. Kerr PBHs and Dark Radiation

Kerr PBHs and Dark Radiation

Dark particles with small masses can contribute to ΔNeff

Schwarzschild PBH → Negligible

Kerr PBH → Argued to be critical



[Hooper et al '20]

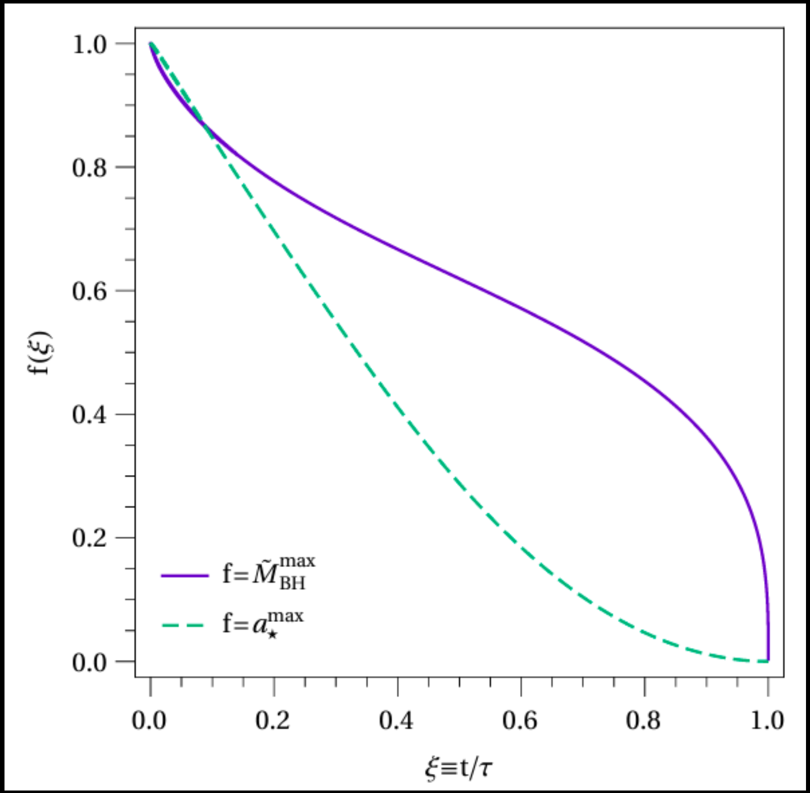
Kerr PBHs and Dark Radiation

$$\frac{\mathrm{d}^2\mathcal{N}_{ilm}}{\mathrm{d}p\mathrm{d}t} = \frac{\sigma_{s_i}^{lm}(M_{\mathrm{BH}}, p, a_\star)}{\exp\left[(E_i - m\Omega)/T_{\mathrm{BH}}\right] - (-1)^{2s_i}} \frac{p^3}{E_i}$$

$$\frac{dM_{\mathrm{BH}}}{dt} = -\epsilon(M_{\mathrm{BH}}, a_\star)\frac{M_p^4}{M_{\mathrm{BH}}^2}\,,$$

$$\frac{da_\star}{dt} = -a_\star[\gamma(M_{\mathrm{BH}}, a_\star) - 2\epsilon(M_{\mathrm{BH}}, a_\star)]\frac{M_p^4}{M_{\mathrm{BH}}^3}\,,$$

Kerr PBHs and Dark Radiation



Major effects:

Spin loss faster than mass loss

→ Shorter lifetime

→ Different ratio Dark Radiation / Radiation

How to calculate ΔNeff ?

Kerr PBHs and Dark Radiation

In the Standard Model

$$\rho_{\rm R}^{\rm SM} = \rho_\gamma \left[1 + \frac{7}{8}\left(\frac{T_\nu}{T_\gamma}\right) N_{\rm eff}^{\rm SM}\right] ,$$

$$T_\nu = (4/11)^{1/3} T_\gamma$$

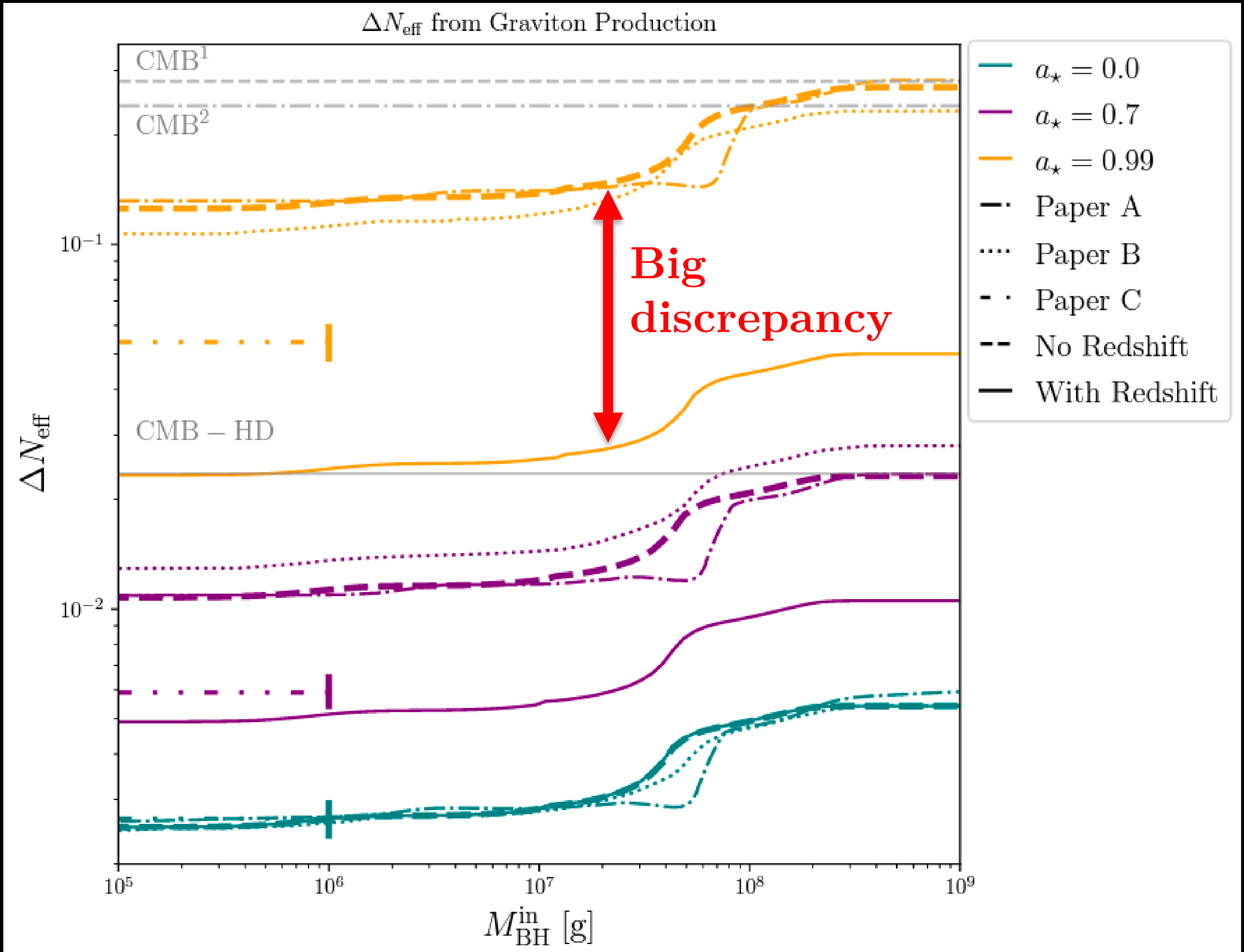
In the presence of Dark Radiation

$$\rho_{\rm R} \equiv \rho_\gamma \left[1 + \frac{7}{8}\left(\frac{T_\nu}{T_\gamma}\right)(N_{\rm eff}^{\rm SM} + \Delta N_{\rm eff})\right]$$

$$\Delta N_{\rm eff} = \left\{\frac{8}{7}\left(\frac{4}{11}\right)^{-\frac{4}{3}} + N_{\rm eff}^{\rm SM}\right\} \frac{\rho_{\rm DR}(T_{\rm ev})}{\rho_{\rm R}^{\rm SM}(T_{\rm ev})} \left(\frac{g_*(T_{\rm ev})}{g_*(T_{\rm eq})}\right) \left(\frac{g_{*S}(T_{\rm eq})}{g_{*S}(T_{\rm ev})}\right)^{\frac{4}{3}}$$

The quantity to evaluate

Kerr PBHs and Dark Radiation



Kerr PBHs and Dark Radiation

Why ?

Kerr PBHs and Dark Radiation

Why ?

$$\rho_{\rm R}^{\rm SM} = \rho_\gamma \left[1 + \frac{7}{8}\left(\frac{T_\nu}{T_\gamma}\right) N_{\rm eff}^{\rm SM}\right] \,,$$

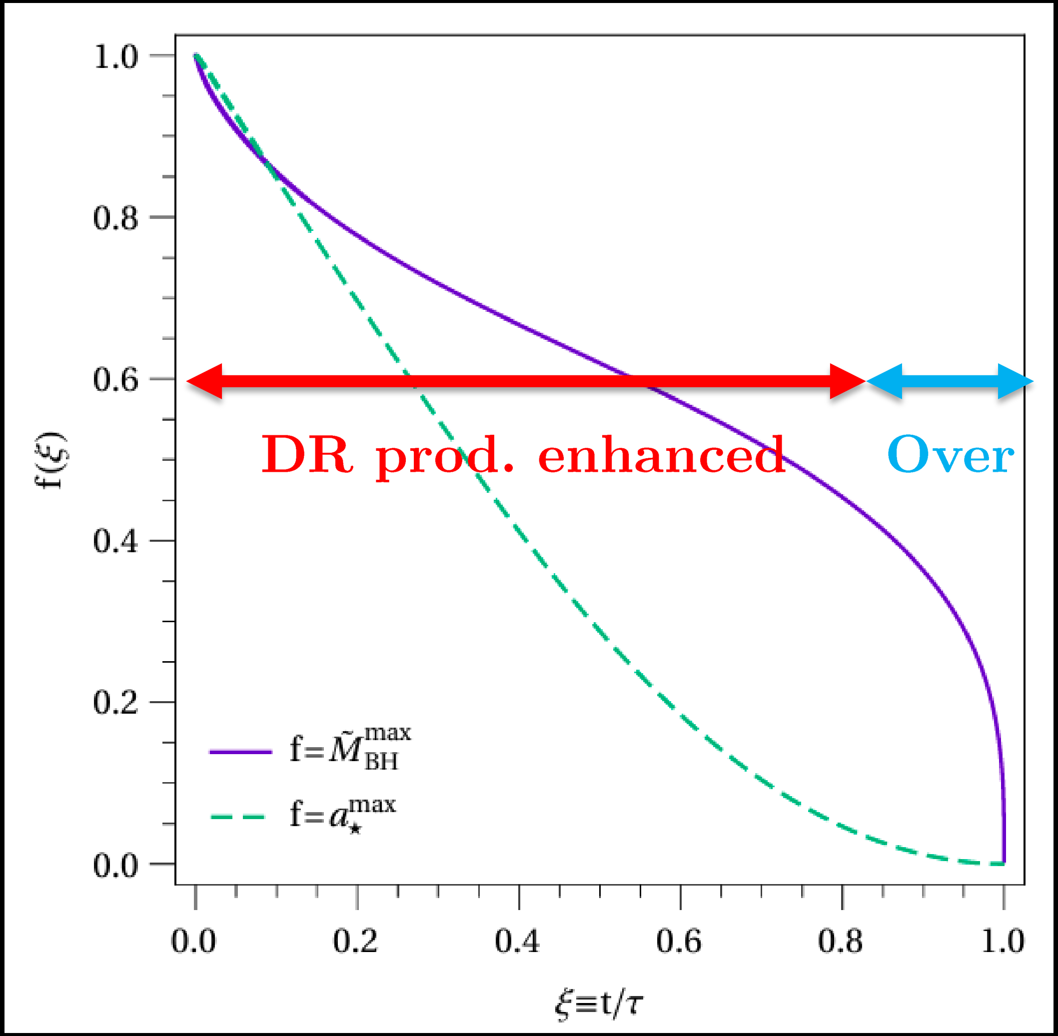
$$N_{\rm eff} \approx 3.045$$ (not just 3 ...)

The neutrino decoupling is NOT instantaneous

+ Temperature-dependent entropy transfer from electrons

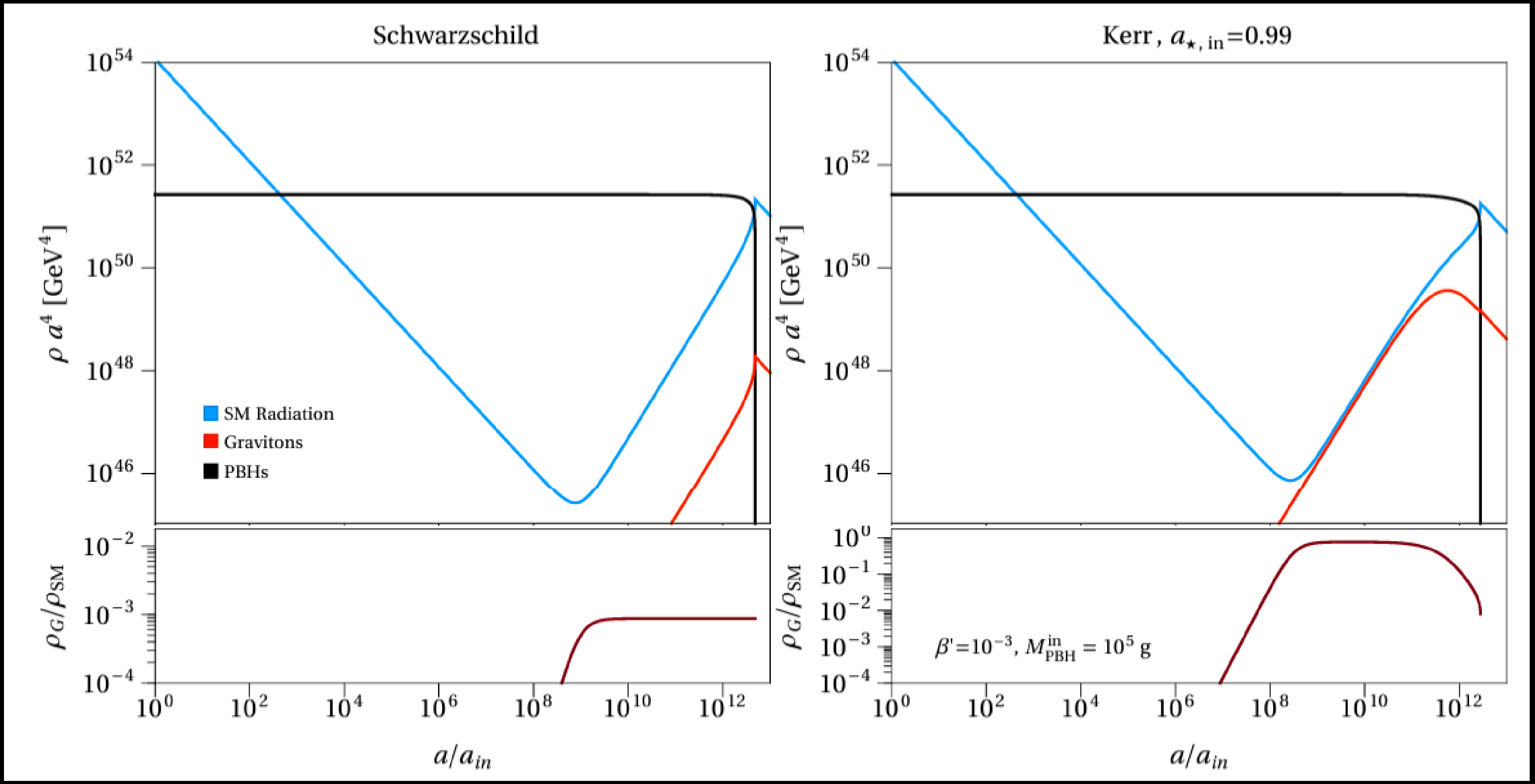
Kerr PBHs and Dark Radiation

Why ?



Kerr PBHs and Dark Radiation

Why ?



[Cheek, LH, Perez-Gonzalez and Turner ‘22] Code Publicly available: FRISBHEE

Kerr PBHs and Dark Radiation

Why ?

$$\frac{\mathrm{d}\mathcal{N}_{\mathrm{DM}}}{\mathrm{d}p} = \int_0^{\tau} \mathrm{d}t' \, \frac{a(\tau)}{a(t')} \times \frac{\mathrm{d}^2\mathcal{N}_{\mathrm{DM}}}{\mathrm{d}p'\mathrm{d}t'} \left(p \frac{a(\tau)}{a(t')}, t' \right)$$

some redshift is good

[Cheek, LH, Perez-Gonzalez and Turner '22] **Code Publicly available: FRISBHEE**

Kerr PBHs and Dark Radiation

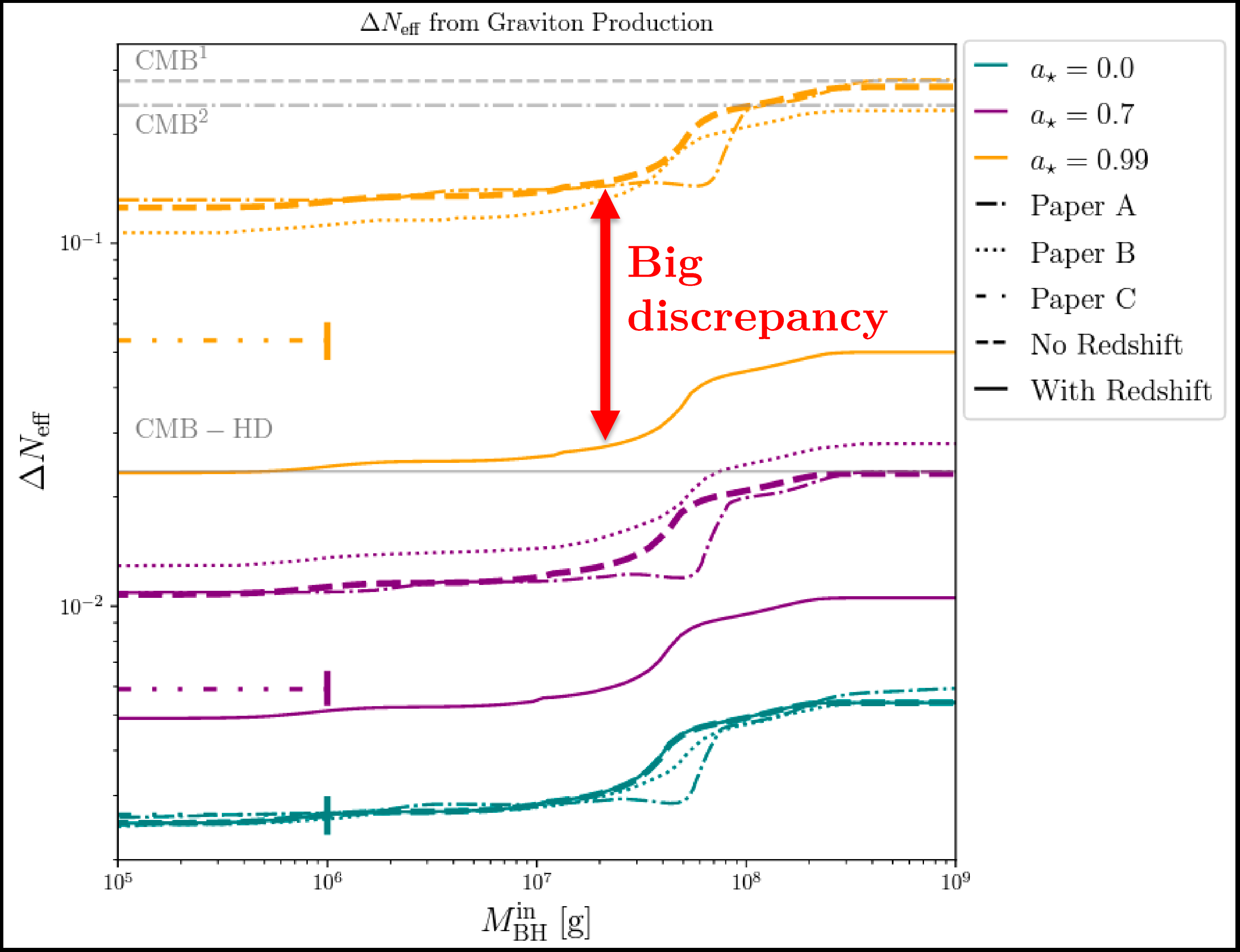
Why ?

$$\frac{\mathrm{d}\mathcal{N}_{\rm DM}}{\mathrm{d}p} = \int_0^{\tau} \mathrm{d}t' \, \frac{a(\tau)}{a(t')} \times \frac{\mathrm{d}^2\mathcal{N}_{\rm DM}}{\mathrm{d}p'\mathrm{d}t'} \left(p\frac{a(\tau)}{a(t')}, t' \right)$$

The correct one is better!

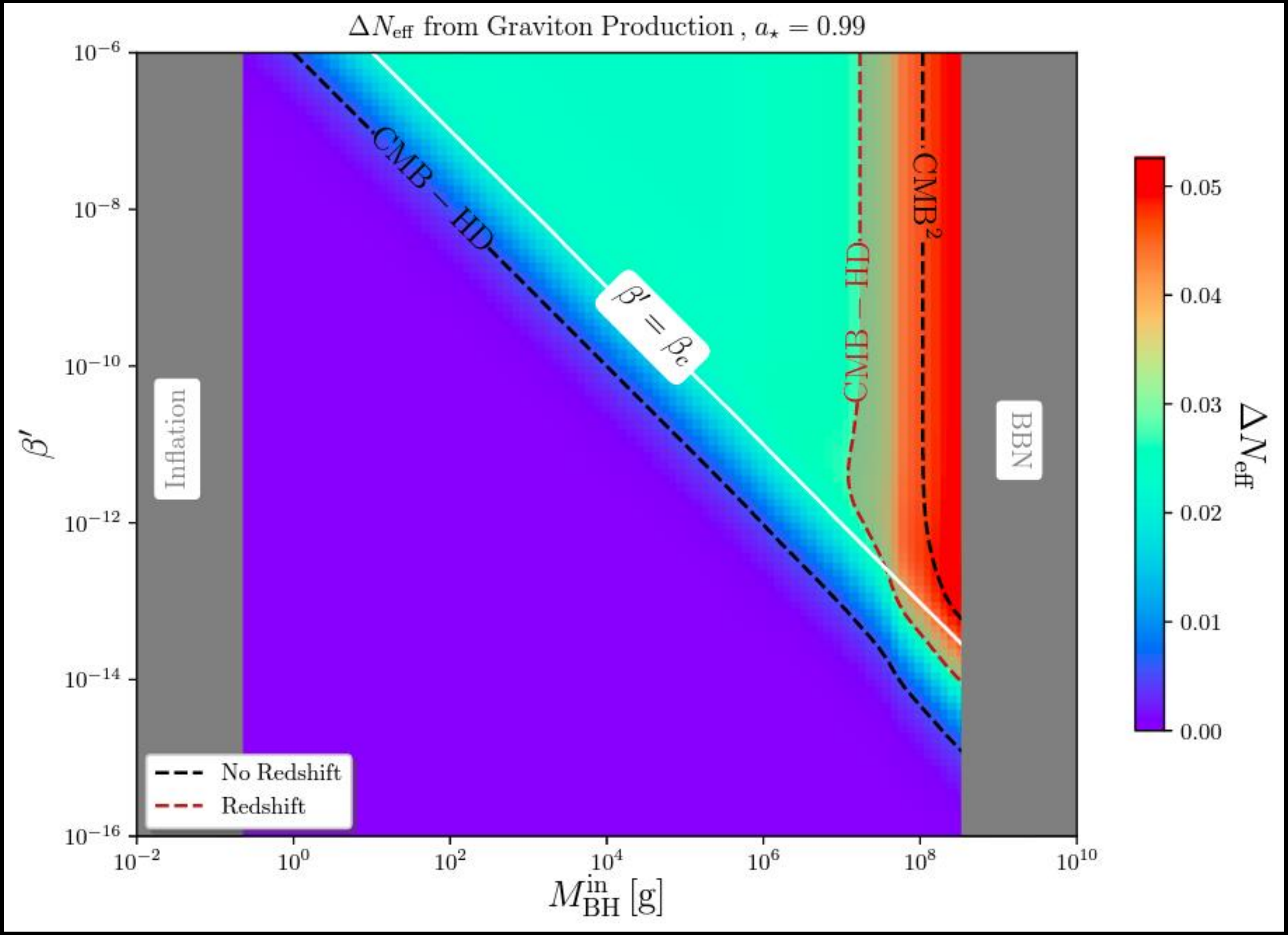
[Cheek, LH, Perez-Gonzalez and Turner '22] **Code Publicly available: FRISBHEE**

Kerr PBHs and Dark Radiation



[Cheek, LH, Perez-Gonzalez and Turner '22] **Code Publicly available: FRISBHEE**

Kerr PBHs and Dark Radiation



[Cheek, LH, Perez-Gonzalez and Turner '22] **Code Publicly available: FRISBHEE**

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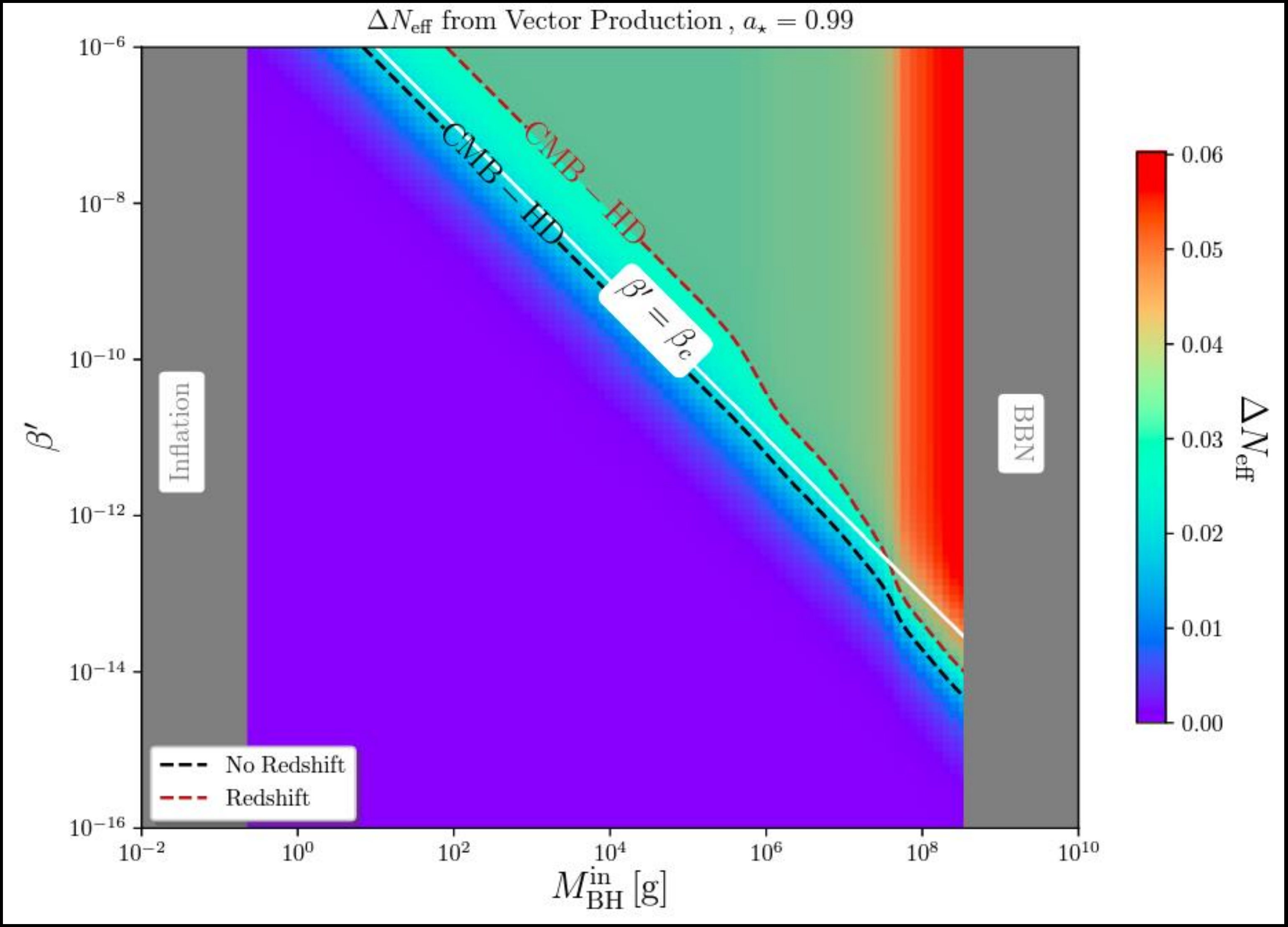

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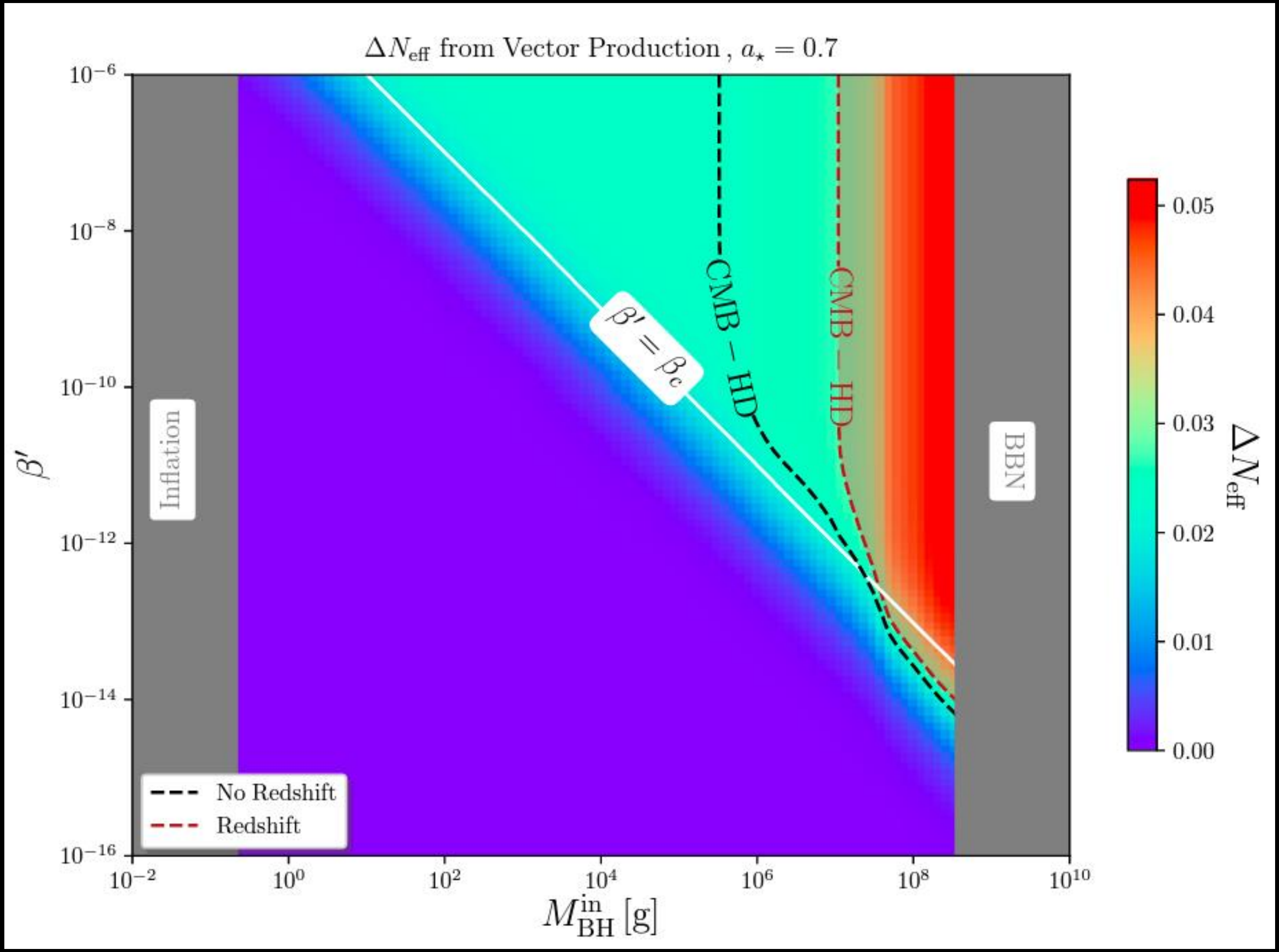
Kerr PBHs and Dark Radiation



[Cheek, LH, Perez-Gonzalez and Turner '22] **Code Publicly available: FRISBHEE**

III. Evaporation of Extended Distributions

III. Evaporation of Extended Distributions

In reality, PBHs don't all have the same mass...

$$f_{\mathrm{PBH}}(M, a) = \delta(M - M_{\mathrm{PBH}}) \times \delta(a - a_*)$$

$$\downarrow$$

$$f_{\mathrm{PBH}}(M, a) = F(M - M_{\mathrm{PBH}}) \times A(a - a_*)$$

III. Evaporation of Extended Distributions

[D. N. Page, Phys. Rev. D 14, 3260 (1976)]

$$\frac{dM_{\rm BH}}{dt} = -\epsilon(M_{\rm BH}, a_\star)\frac{M_p^4}{M_{\rm BH}^2} \,,$$

$$\frac{da_\star}{dt} = -a_\star[\gamma(M_{\rm BH}, a_\star) - 2\epsilon(M_{\rm BH}, a_\star)]\frac{M_p^4}{M_{\rm BH}^3} \,,$$

$$\frac{\mathrm{d}a}{\mathrm{d}t} = \frac{a}{M^3}\left[2f(a) - g(a)\right] \,,$$

$$\frac{\mathrm{d}M}{\mathrm{d}t} = -\frac{f(a)}{M^2} \,.$$

$$y \equiv -\ln(a)$$

$$z \equiv -\ln\left(\frac{M}{M_1}\right)$$

$$\tau \equiv M_1^{-3} t$$

$$\frac{\mathrm{d}z}{\mathrm{d}y} = \frac{f(a)}{g(a) - 2f(a)} \,,$$

$$\frac{\mathrm{d}\tau}{\mathrm{d}y} = \left(\frac{M}{M_1}\right)^3 \frac{1}{g(a) - 2f(a)} \,,$$

Generic solution (z, τ) for any M_1

$$M = M_i e^{z_i - z} \,,$$

$$(t - t_i) = M_i^3 e^{3z_i}(\tau - \tau_i) \,,$$

III. Evaporation of Extended Distributions

$$\begin{aligned} \mathrm{d}n_{\mathrm{BH}} &= f_{\mathrm{BH}}(M, a, t)\mathrm{d}M\mathrm{d}a \\ &= f_{\mathrm{BH}}(M_i, a_i, t_i)\mathrm{d}M_i\mathrm{d}a_i \,. \end{aligned}$$

$$\frac{\mathrm{d}\varrho_{\mathrm{BH}}}{\mathrm{d}t} = \int_0^{\infty} \frac{\mathrm{d}M}{\mathrm{d}t}\Theta(M) f_{\mathrm{BH}}(M_i, a_i, t_i)\mathrm{d}M_i\mathrm{d}a_i$$

Dynamics of the evaporation + Friedmann equations

→ Included in FRISBHEE

III. Evaporation of Extended Distributions

Examples:

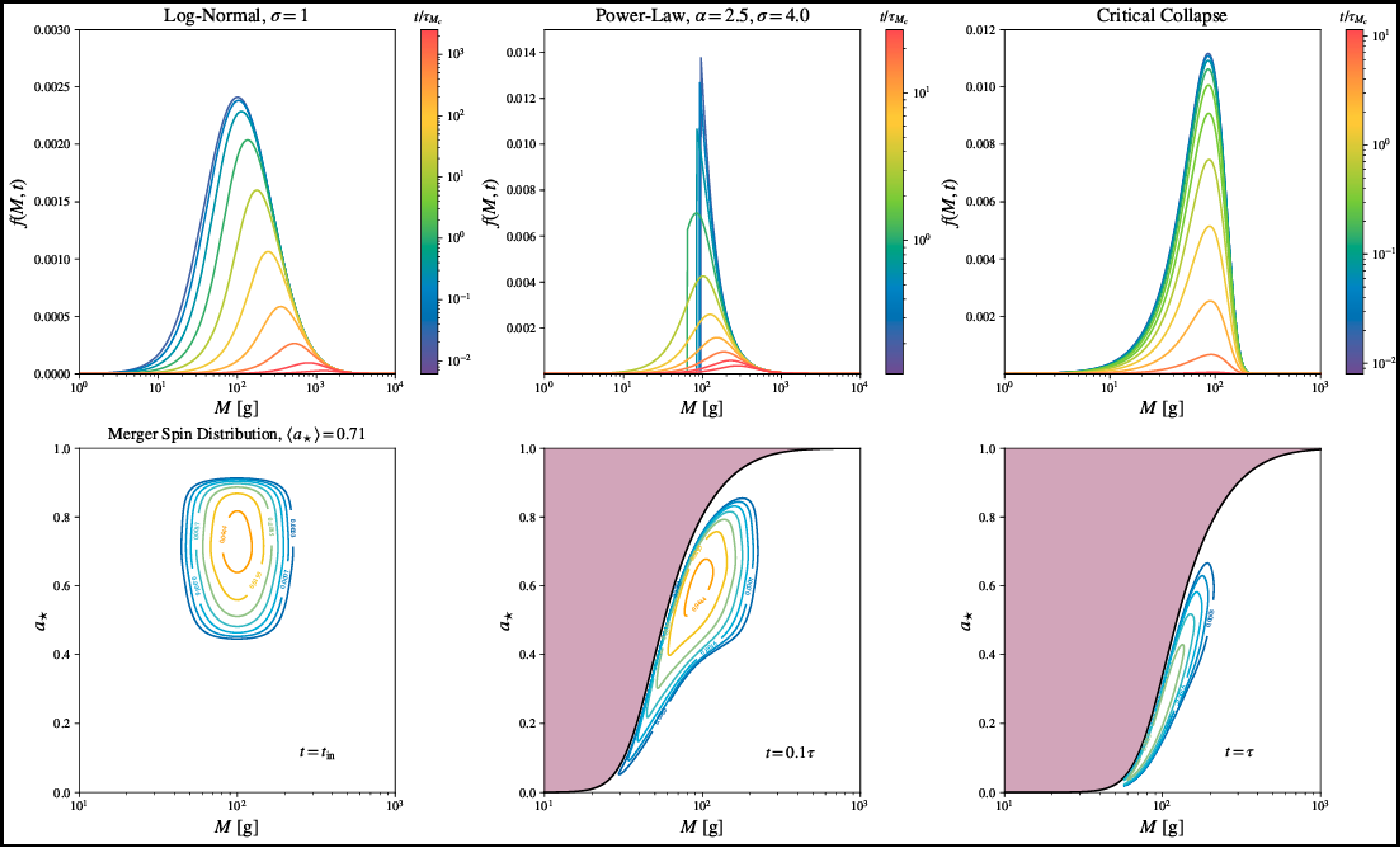
$$\frac{\mathrm{d}n}{\mathrm{d}M} \propto \frac{1}{M^2} \exp\left[-\frac{(\log M - \log M_\mathrm{c})^2}{2\,\sigma^2}\right]$$

Evaporation smeared around $\tau(M_c)$

$$\frac{\mathrm{d}n}{\mathrm{d}M} \propto M^{-\alpha} \quad \text{with} \quad \alpha = \frac{2\,(1+2\,w)}{1+w}$$

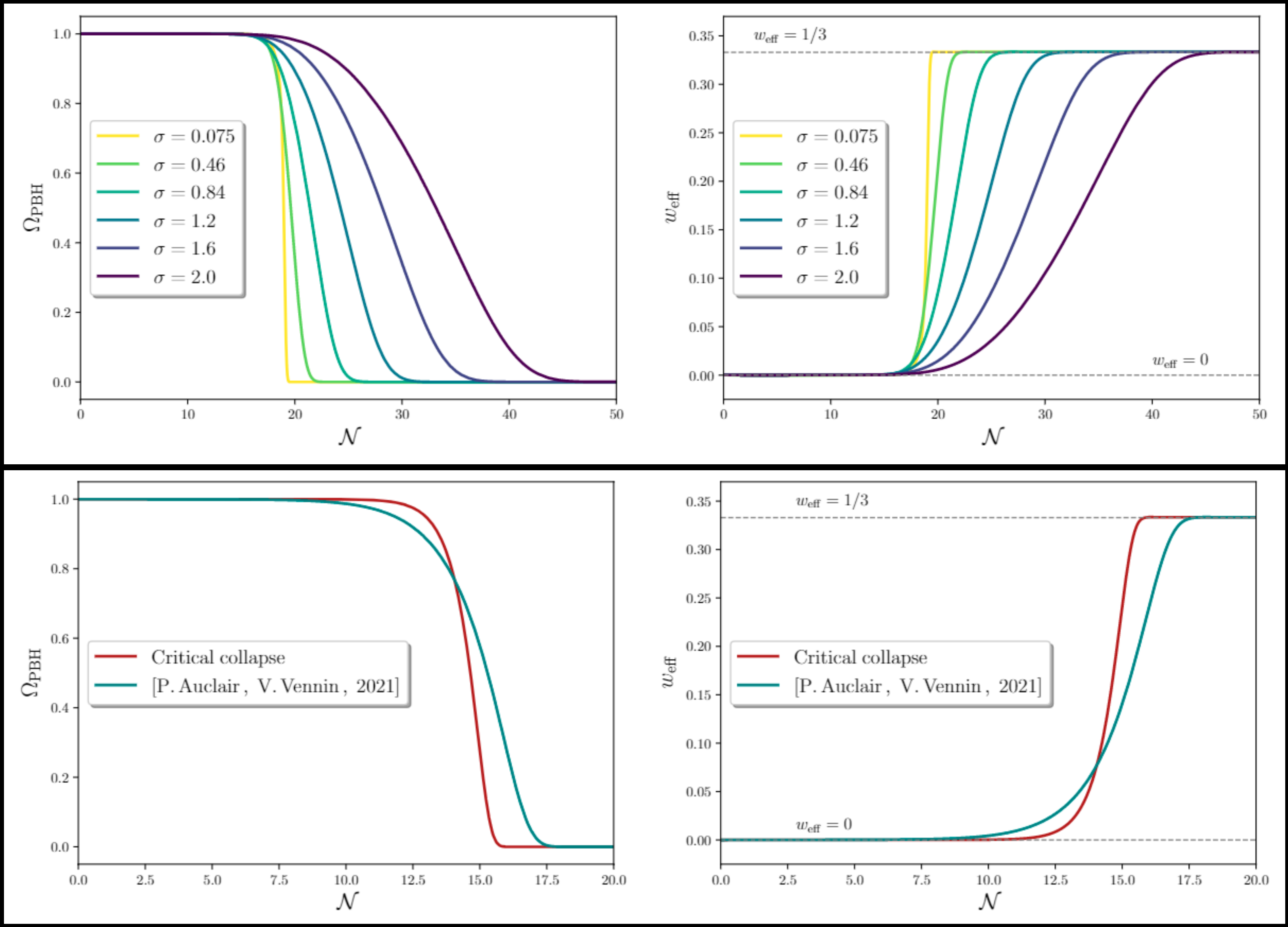
Regime of ‘Cosmological Stasis’

III. Evaporation of Extended Distributions



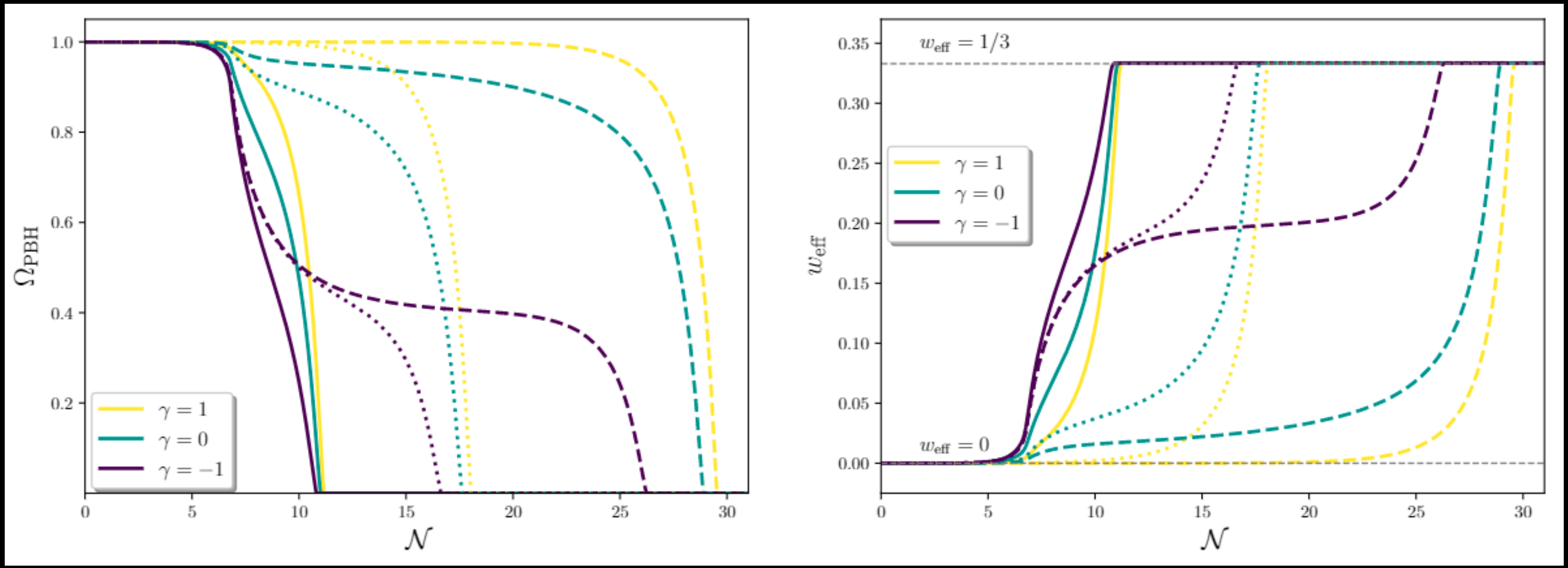
[Cheek, LH, Perez-Gonzalez, Turner '22]

III. Evaporation of Extended Distributions



III. Evaporation of Extended Distributions

$$\frac{dn}{dM} \propto M^{-\alpha} \quad \text{with} \quad \alpha = \frac{2\,(1 + 2\,w)}{1 + w}$$



'Stasis' regime reached for $0 < w \leq 1$

[Copeland, Liddle, Barrow '91]
[Dienes, LH, Huang, Kim, Tait, Thomas '22]
[Cheek, LH, Perez-Gonzalez, Turner '22]

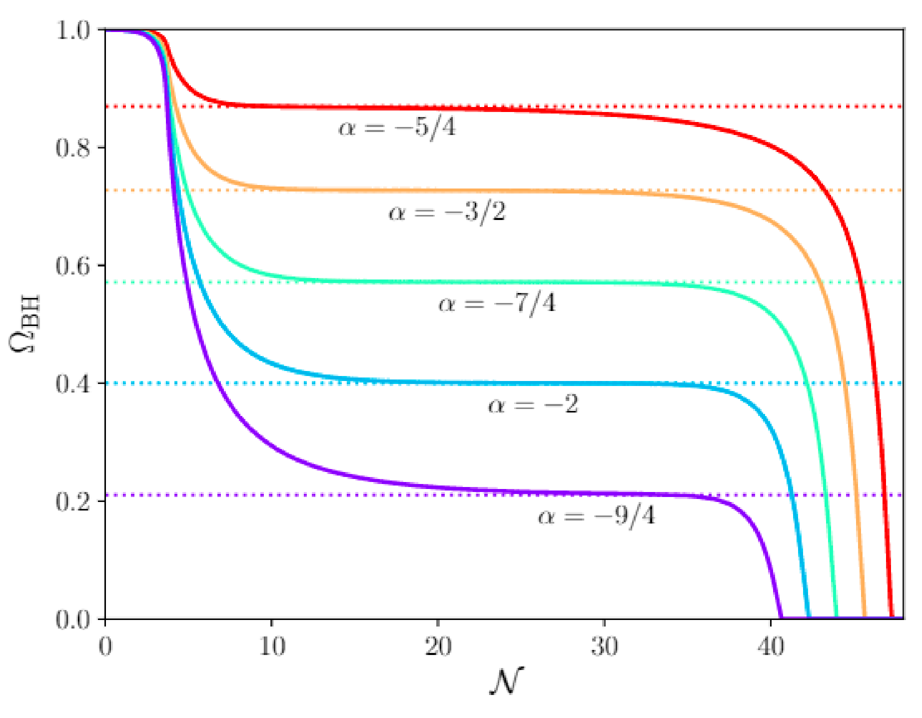
III. Evaporation of Extended Distributions

$$f_{\rm BH}(M) = \begin{cases} CM^{\alpha-1}, & \text{for } M_{\rm min} \le M \le M_{\rm max}; \\ 0, & \text{else.} \end{cases}$$

$$\alpha \equiv \frac{(-3w_{\rm form.} - 1)}{(w_{\rm form.} + 1)}$$

$$\frac{dH}{dt} = -\frac{1}{2}H^2(4 - \Omega_{\rm BH}),$$

$$\frac{d\Omega_{\rm BH}}{dt} = \Omega_{\rm BH}\left[\frac{\int_0^\infty f_{\rm BH}(M,t)\frac{dM}{dt}dM}{\int_0^\infty f_{\rm BH}(M,t)M\,dM}\right] + H\Omega_{\rm BH}(1 - \Omega_{\rm BH}).$$



$$= \frac{1+\alpha}{3(t - t_i)}$$

[Dienes, LH, Huang, Kim, Tait, Thomas '22]

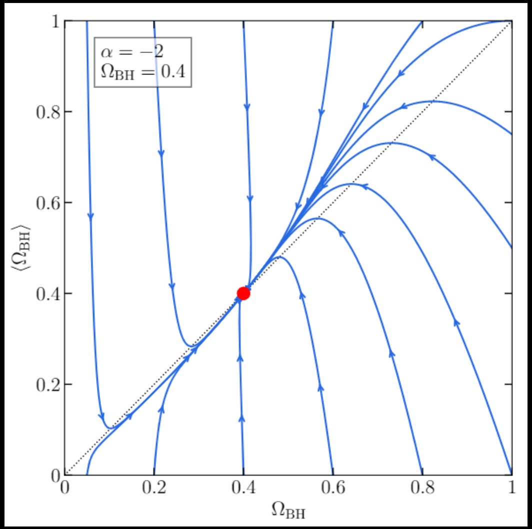
III. Evaporation of Extended Distributions

$$\begin{cases} \dfrac{d\Omega_{\rm BH}}{dt} = \dfrac{1}{t-t_i}\, f(\Omega_{\rm BH}, \langle\Omega_{\rm BH}\rangle) \\ \dfrac{d\langle\Omega_{\rm BH}\rangle}{dt} = \dfrac{1}{t-t_i}\, g(\Omega_{\rm BH}, \langle\Omega_{\rm BH}\rangle)\ , \end{cases}$$

$$f(\Omega_{\rm BH}, \langle\Omega_{\rm BH}\rangle) \equiv \Omega_{\rm BH}\left[\frac{1+\alpha}{3} + \frac{2(1-\Omega_{\rm BH})}{4-\langle\Omega_{\rm BH}\rangle}\right]$$

$$g(\Omega_{\rm BH}, \langle\Omega_{\rm BH}\rangle) \equiv \Omega_{\rm BH} - \langle\Omega_{\rm BH}\rangle\ ,$$

$$\langle\Omega_{\rm BH}\rangle \equiv \frac{1}{t-t_i}\int_{t_i}^{t} dt'\, \Omega_{\rm BH}(t')\ .$$



$$w_{\rm eff} = \frac{-\alpha - 1}{\alpha + 7}$$

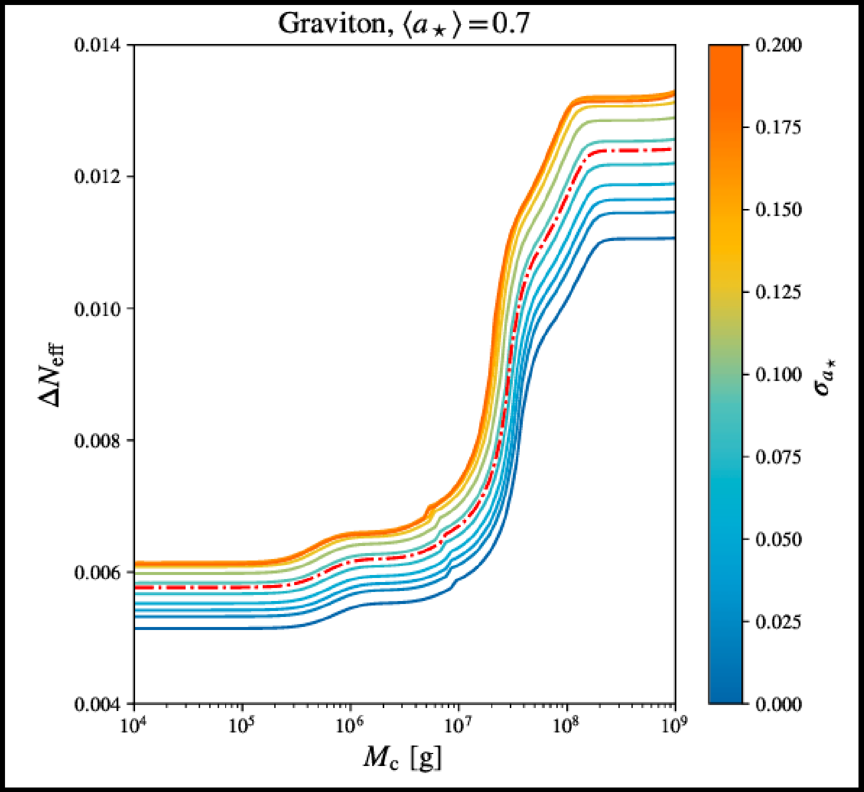
$$\Omega_{\rm BH} = \langle\Omega_{\rm BH}\rangle = \frac{4\alpha+10}{\alpha+7} \equiv \overline{\Omega}_{\rm BH}\ .$$

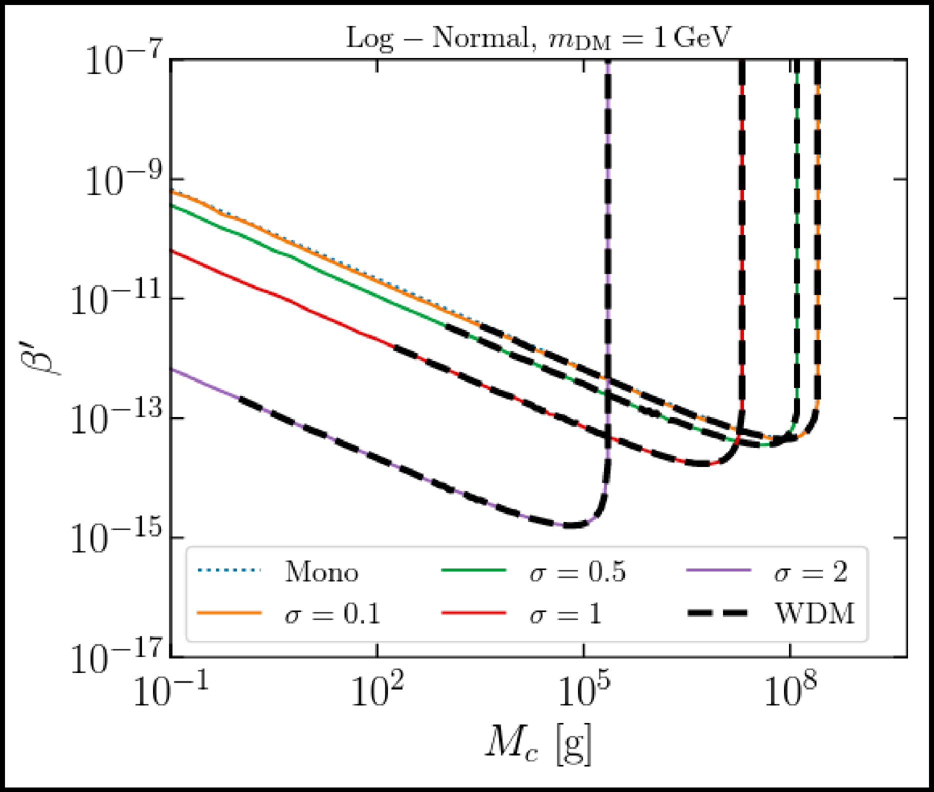
$$\mathcal{J} \equiv \frac{1}{t-t_i}\begin{pmatrix} \partial_{\Omega_{\rm BH}} f & \partial_{\langle\Omega_{\rm BH}\rangle} f \\ \partial_{\Omega_{\rm BH}} g & \partial_{\langle\Omega_{\rm BH}\rangle} g \end{pmatrix}$$

$$\lambda_\pm = \frac{1}{18}\left(-4\alpha \pm \sqrt{-19 - 4\alpha(2\alpha+19) - 59}\right)$$

[Dienes, LH, Huang, Kim, Tait, Thomas '22] [Copeland, Liddle, Barrow '91]

III. Evaporation of Extended Distributions





[Cheek, LH, Perez-Gonzalez, Turner '22]

Gravitational Waves?

PBHs are known to source GWs in many different ways

Any GW spectrum gets affected by post-inflationary cosmology

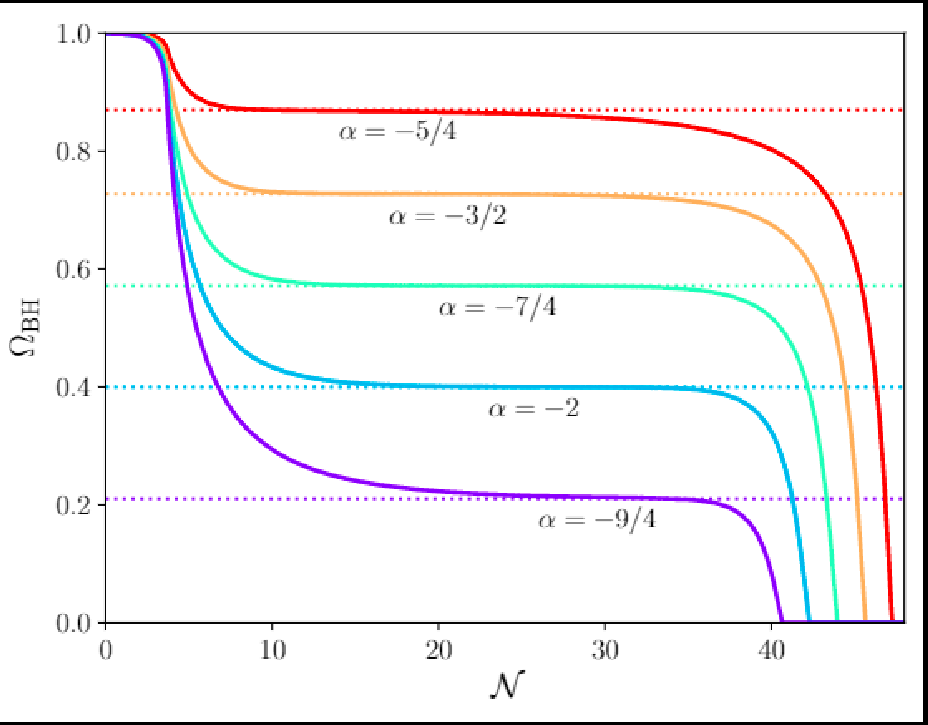
Measuring GWs can tell us a lot...

Gravitational Waves?

$$f_{\rm BH}(M) = \begin{cases} C M^{\alpha-1}, & \text{for } M_{\rm min} \leq M \leq M_{\rm max}; \\ 0, & \text{else.} \end{cases}$$

$$\alpha \equiv \frac{(-3w_{\rm form.} - 1)}{(w_{\rm form.} + 1)}$$

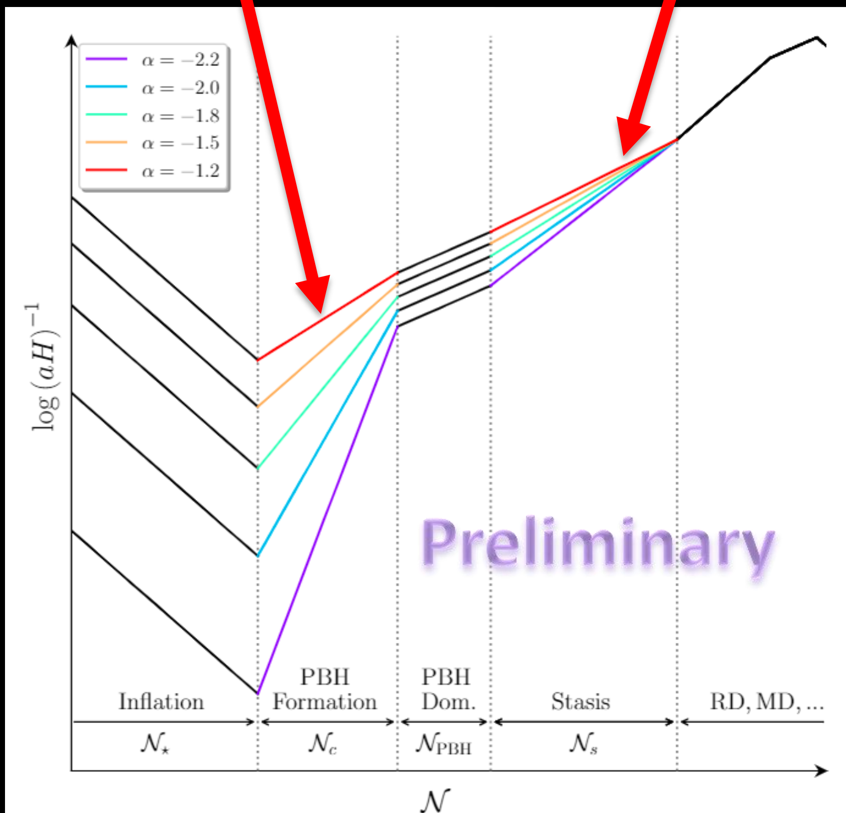
[Dienes, LH, Huang, Kim, Tait, Thomas '22]

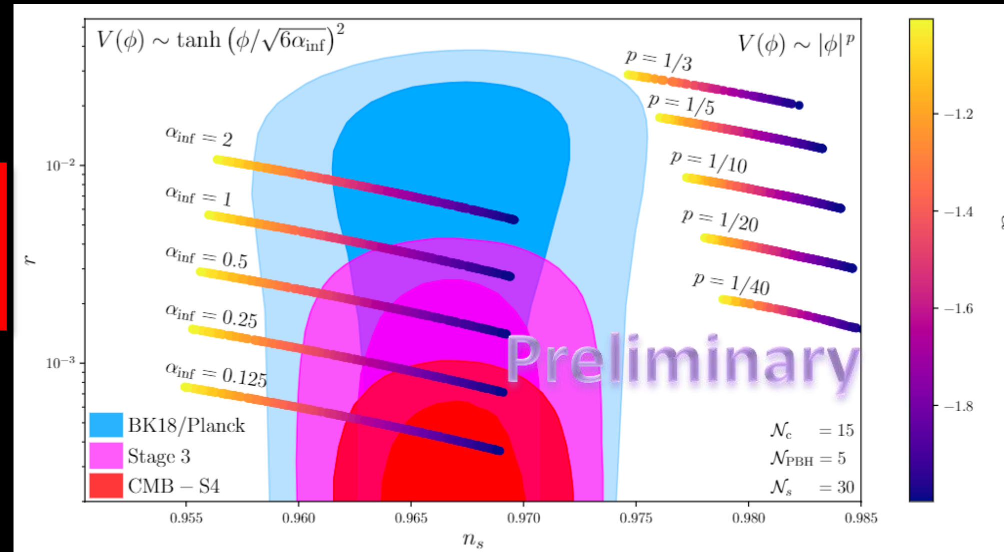


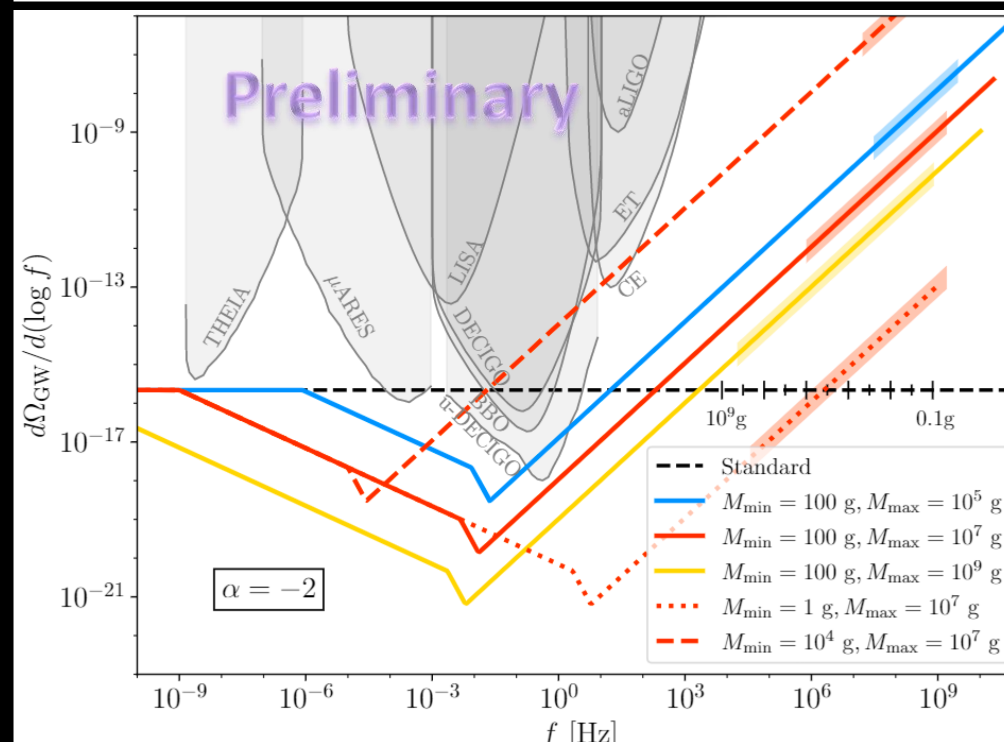
Gravitational Waves?

$$\alpha \equiv \frac{(-3w_{\text{form.}} - 1)}{(w_{\text{form.}} + 1)}$$

$$w_{\text{eff}} = \frac{-\alpha - 1}{\alpha + 7}$$



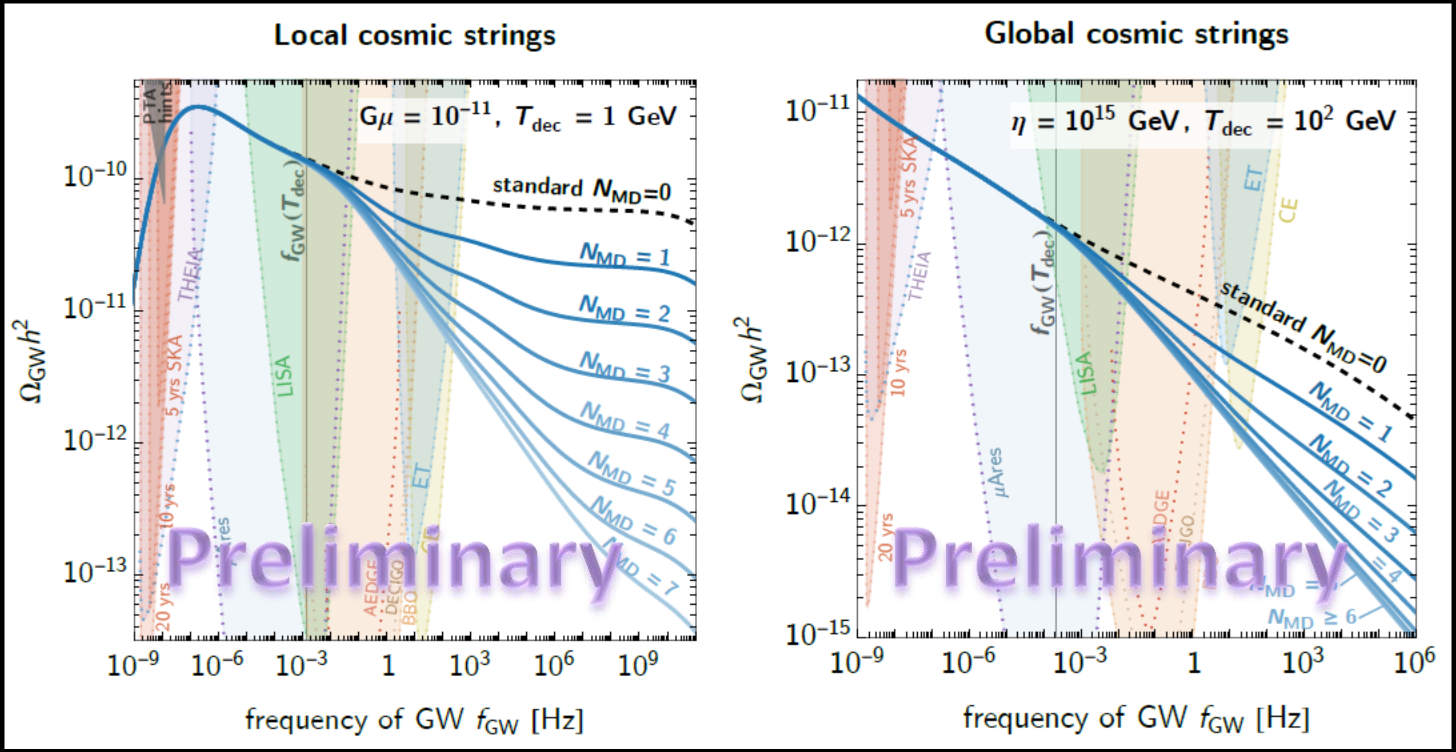




[Dienes, LH, Huang, Kim, Tait, Thomas ‘22]

Gravitational Waves?

GWs from cosmic strings?



[Ghoshal, Gouttenoire, LH, Simakachorn '23]

Gravitational Waves?

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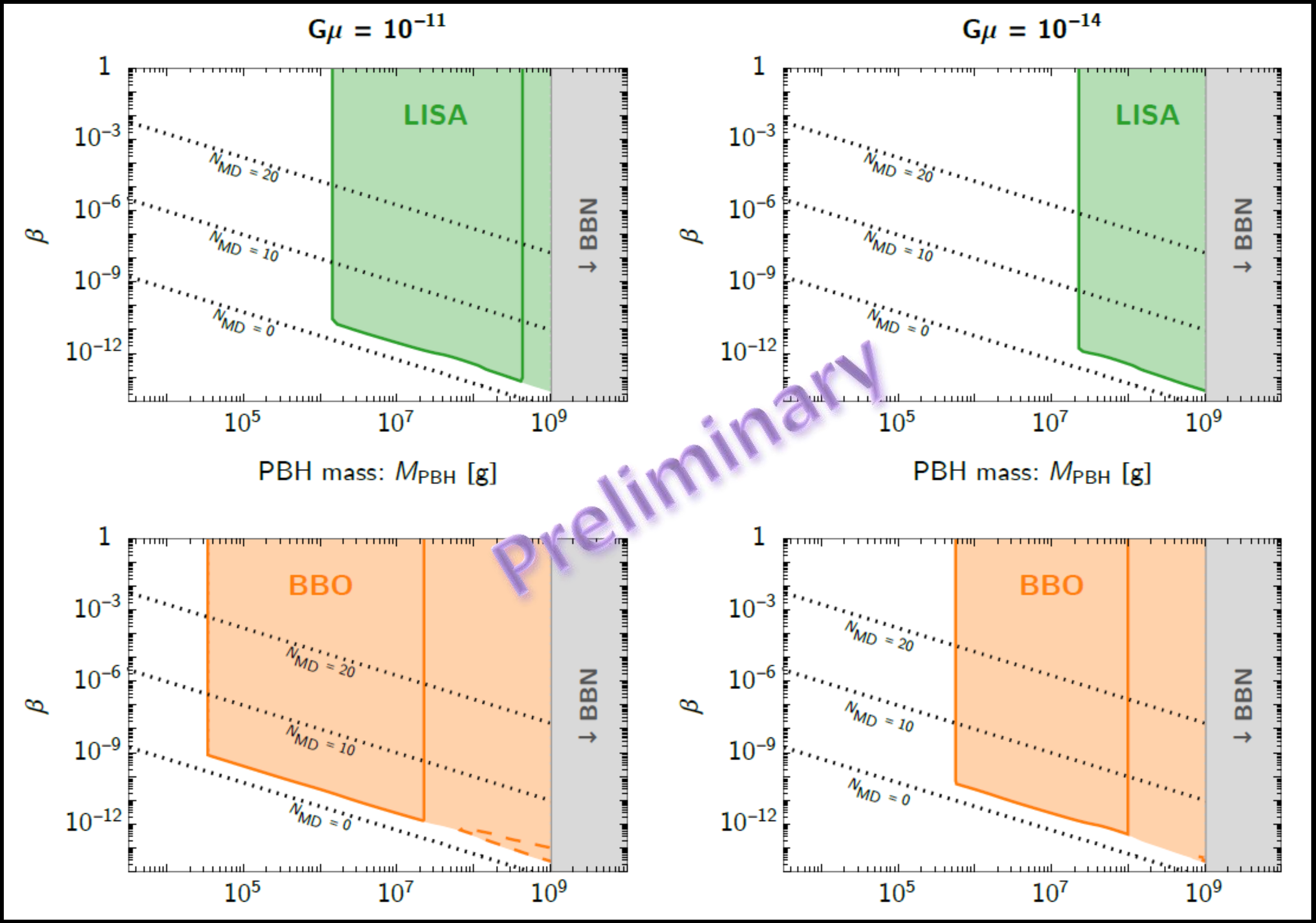
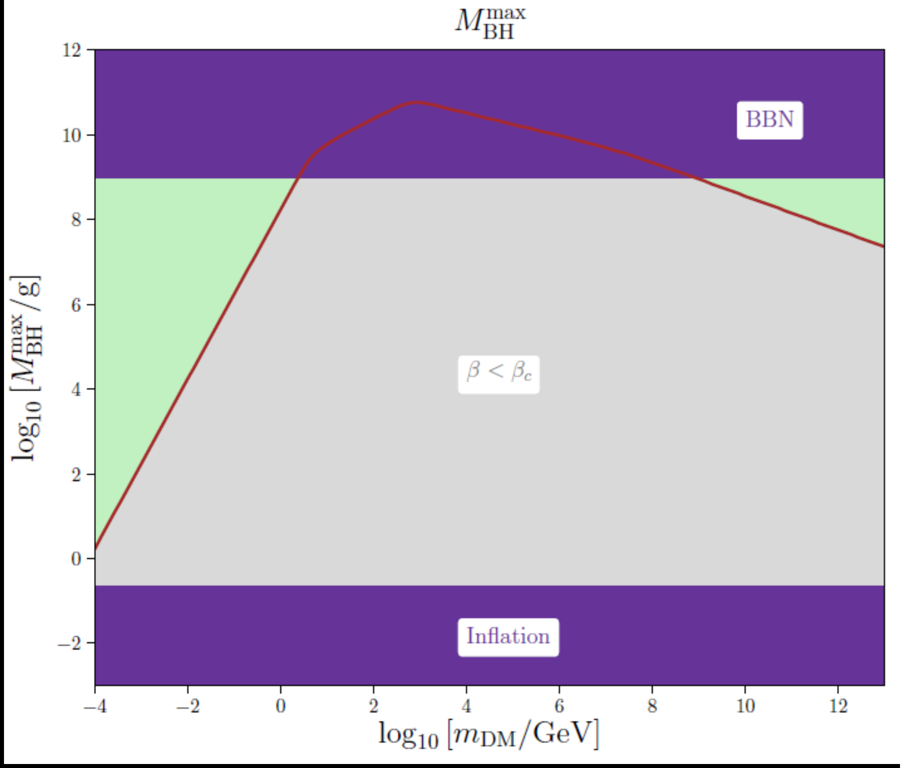
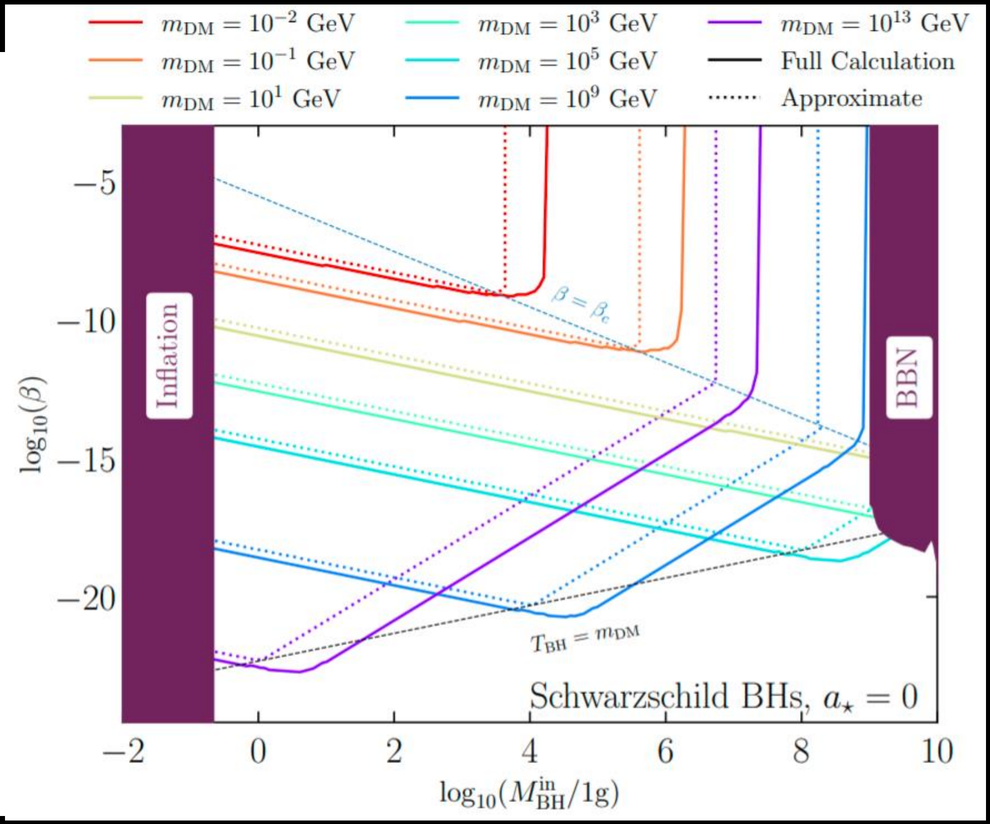


[Ghoshal, Gouttenoire, LH, Simakachorn '23]

Modified Cosmology





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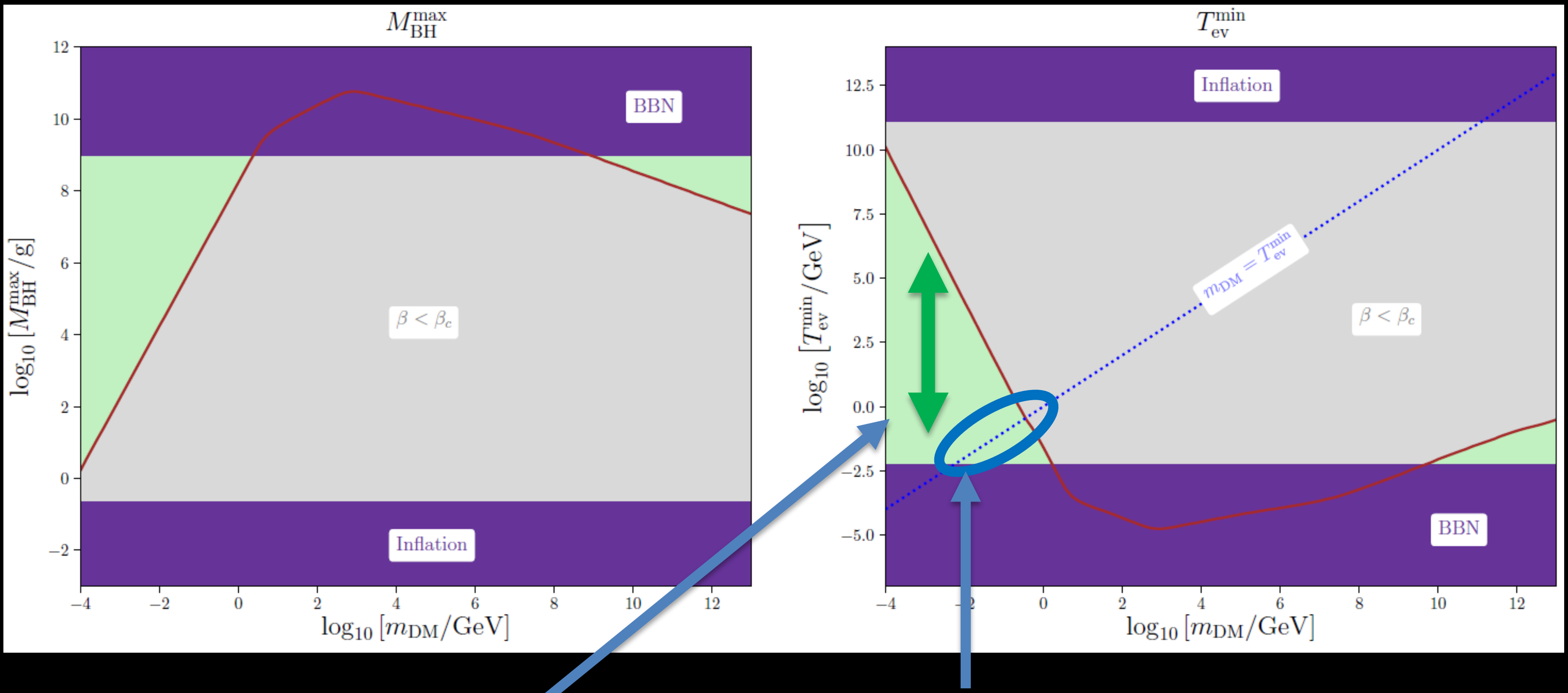
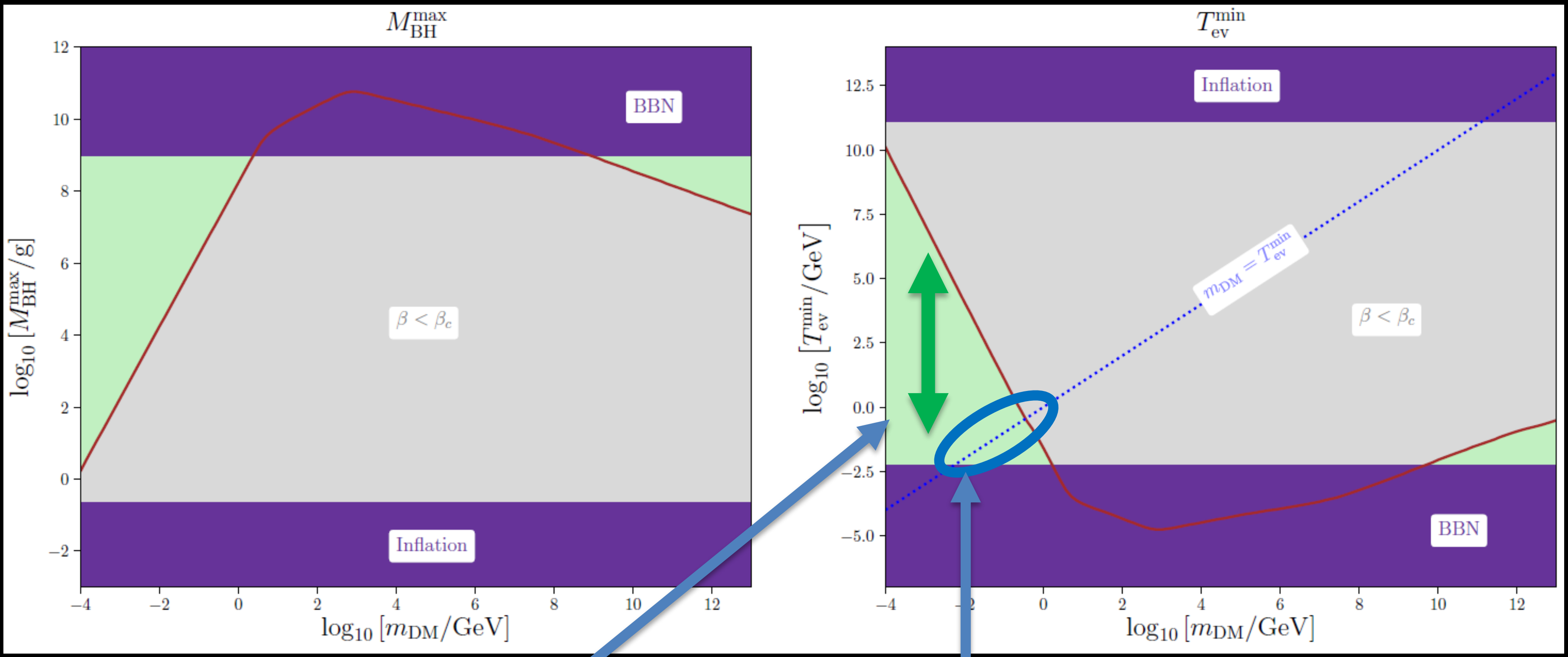


Region of interest for Freeze-In

Region of interest for Freeze-Out

Modified Cosmology

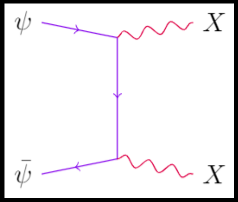


Region of interest for Freeze-In

~~Region of interest for Freeze-Out~~

Thermalization Of PBHs products…

TBH large +



Boltzmann Equations

Freeze–In case:

$$\dot{n}_{\rm DM}^{\rm th} + 3Hn_{\rm DM}^{\rm th} = \langle \sigma v \rangle_{\rm th} \left(n_{\rm DM,eq}^{2} - n_{\rm DM}^{\rm th\,2} \right)$$

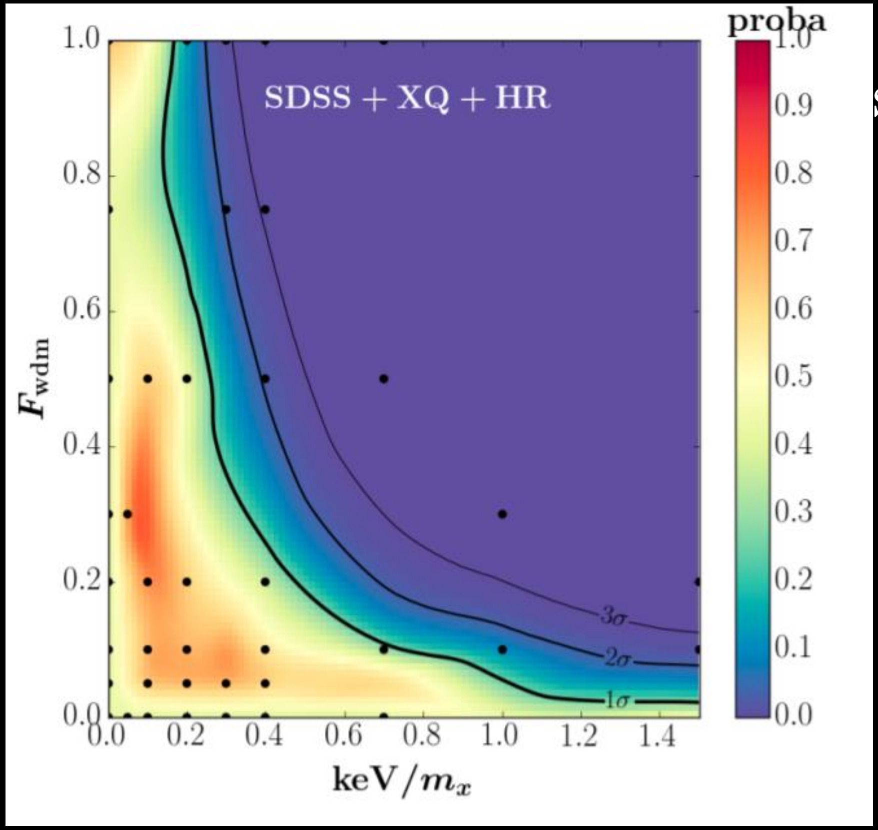
$$\dot{n}_{\rm DM}^{\rm ev} + 3Hn_{\rm DM}^{\rm ev} = \left. \frac{{\rm d}n_{\rm DM}^{\rm ev}}{{\rm d}t} \right|_{\rm BH} + 2\,\Gamma_{X\to {\rm DM}} \left\langle \frac{m_X}{E_X} \right\rangle_{\rm ev} n_X$$

$$\dot{n}_X + 3Hn_X = \left. \frac{{\rm d}n_X}{{\rm d}t} \right|_{\rm BH} - \Gamma_X \left\langle \frac{m_X}{E_X} \right\rangle_{\rm ev} n_X$$

$$\dot{\rho}_{\rm SM} + 4H\rho_{\rm SM} = \left. \frac{{\rm d}M}{{\rm d}t} \right|_{\rm SM} + 2m_X \Gamma_{X\to {\rm SM}} n_X$$

Non-cold Dark Matter

s

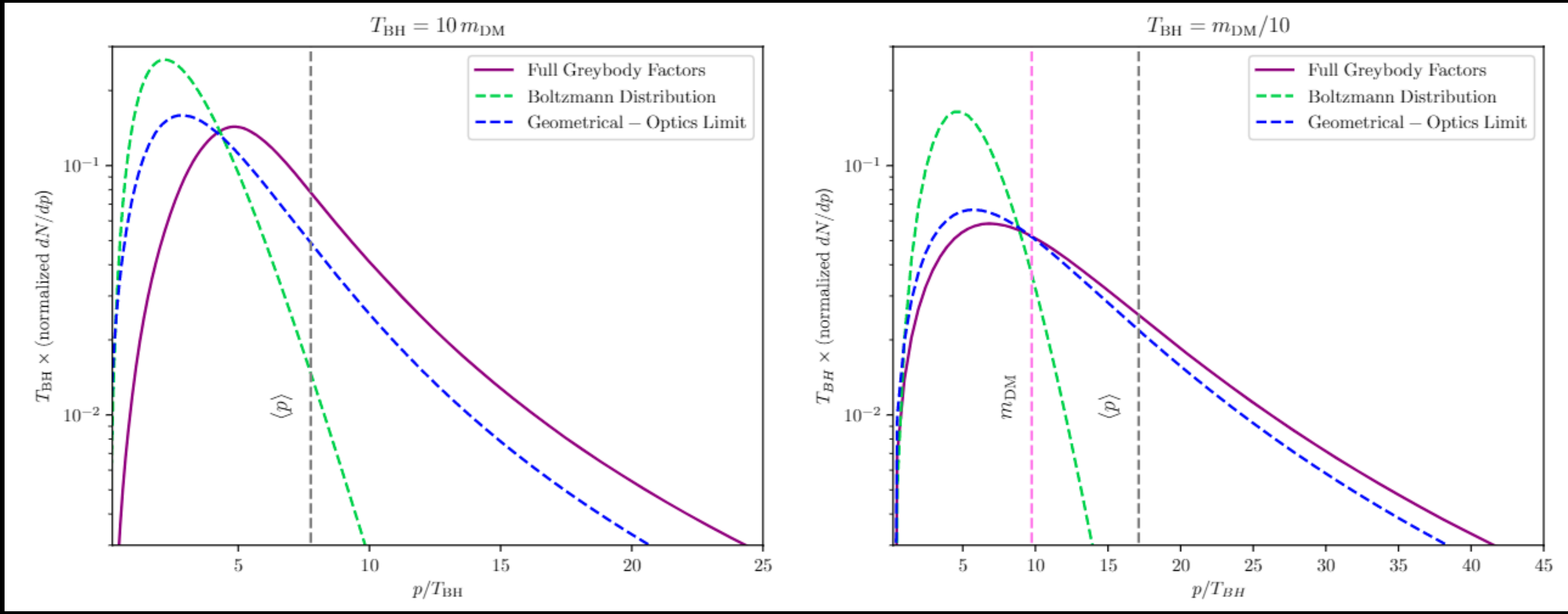


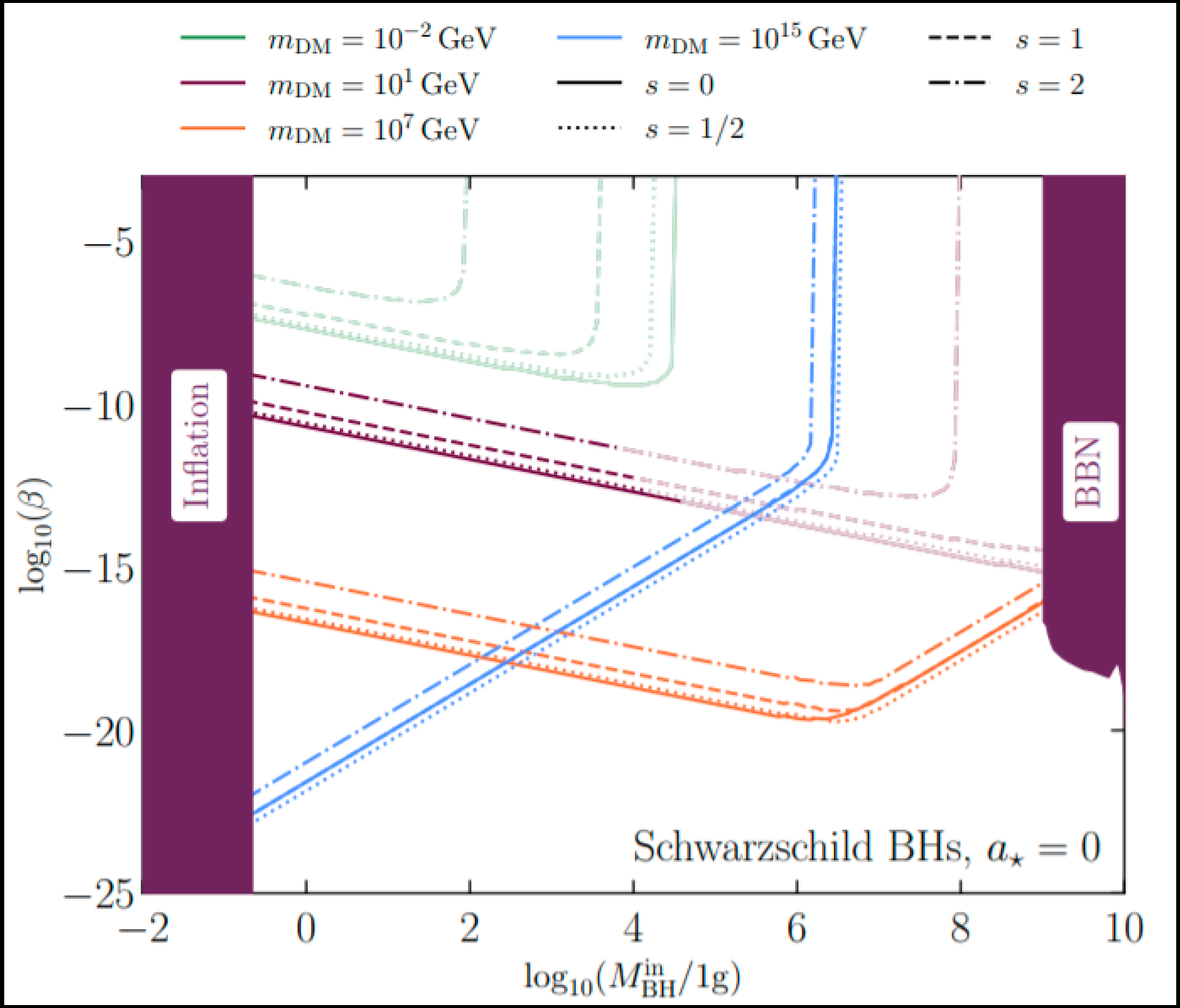
[Baur *et al.* 2017]

$$\langle v \rangle|_{t=t_0} = a_{\rm ev} \times \frac{\langle p \rangle|_{t=t_{\rm ev}}}{m_{\rm DM}}$$

DM from Evaporation

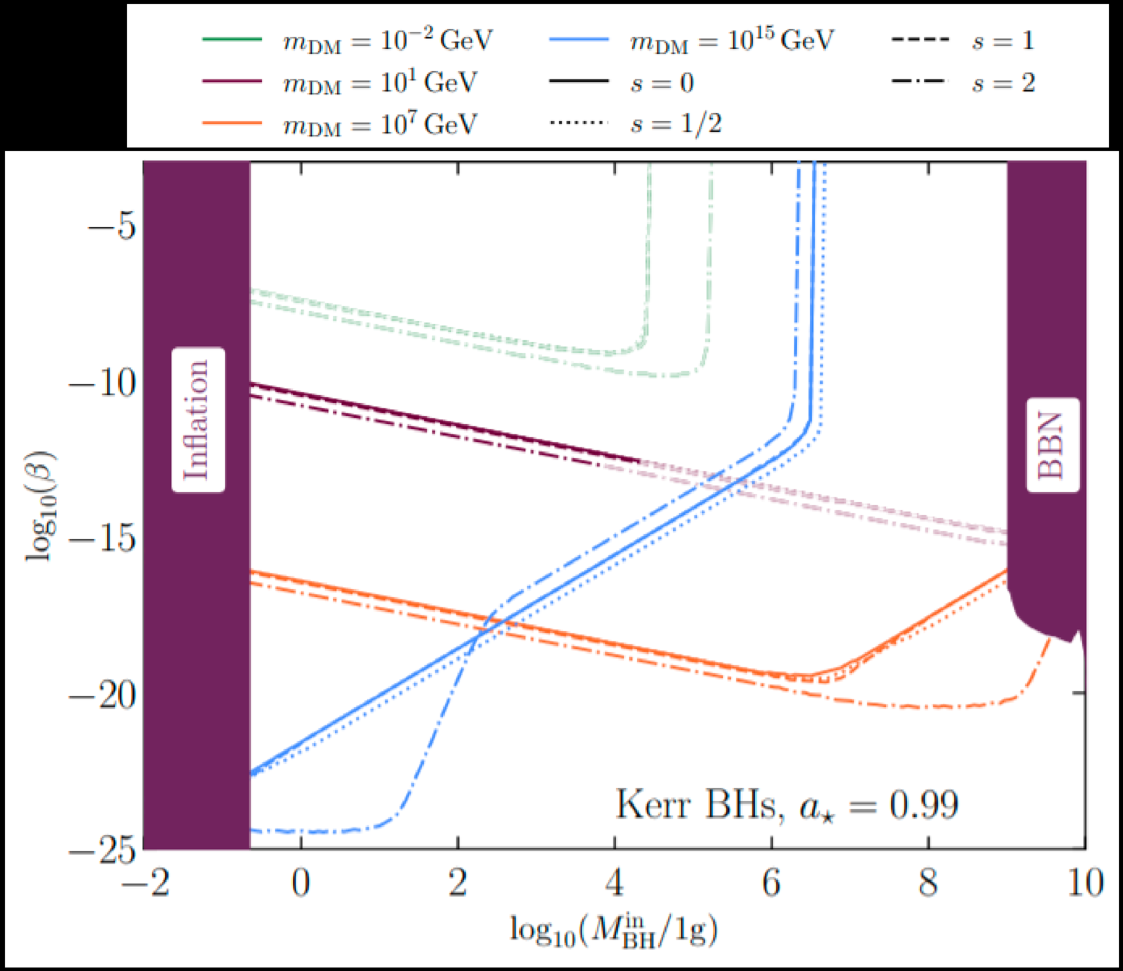
- Peculiar spectrum of evaporated DM particles
- Non-negligible difference between geometrical-optics limit and full distributions





$m_{\rm DM} = 10^{-2}$ GeV
$m_{\rm DM} = 10^{1}$ GeV
$m_{\rm DM} = 10^{7}$ GeV
$m_{\rm DM} = 10^{15}$ GeV
$s = 0$
$s = 1/2$
$s = 1$
$s = 2$
Inflation
BBN
Schwarzschild BHs, $a_\star = 0$
$\log_{10}(\beta)$
$\log_{10}(M_{\rm BH}^{\rm in}/1{\rm g})$
−5
−10
−15
−20
−25
−2
0
2
4
6
8
10

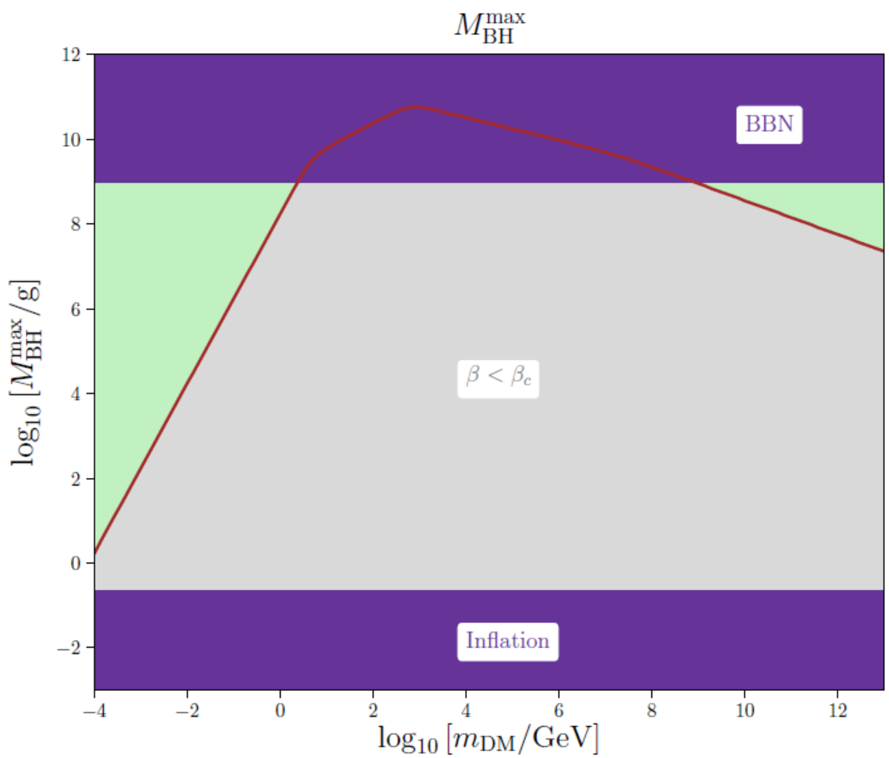
$$T_{\rm BH} = \frac{1}{4\pi G M_{\rm BH}} \frac{\sqrt{1-a_\star^2}}{1+\sqrt{1-a_\star^2}} \,,$$

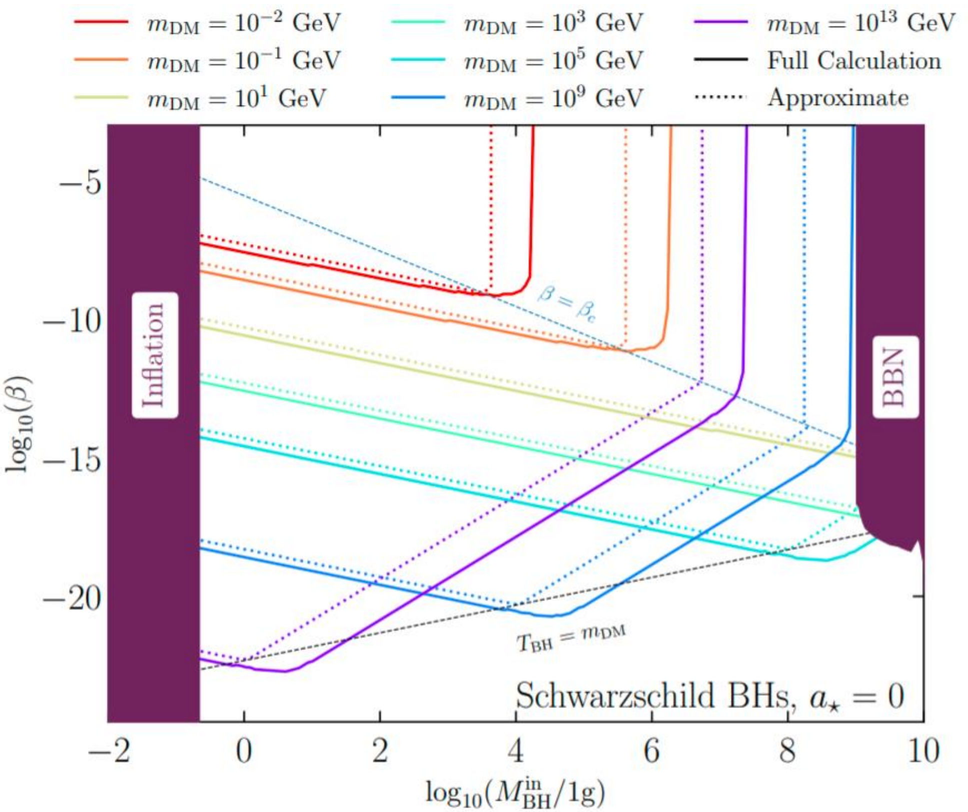


$$\frac{{\rm d}^2\mathcal{N}_{ilm}}{{\rm d}p{\rm d}t} = \frac{\sigma_{s_i}^{lm}(M_{\rm BH}, p, a_\star)}{\exp\left[(E_i - m\Omega)/T_{\rm BH}\right] - (-1)^{2s_i}} \frac{p^3}{E_i}$$

where $\Omega = (a_\star/2GM_{\rm BH})(1/(1+\sqrt{1-a_\star^2}))$

Modified Cosmology





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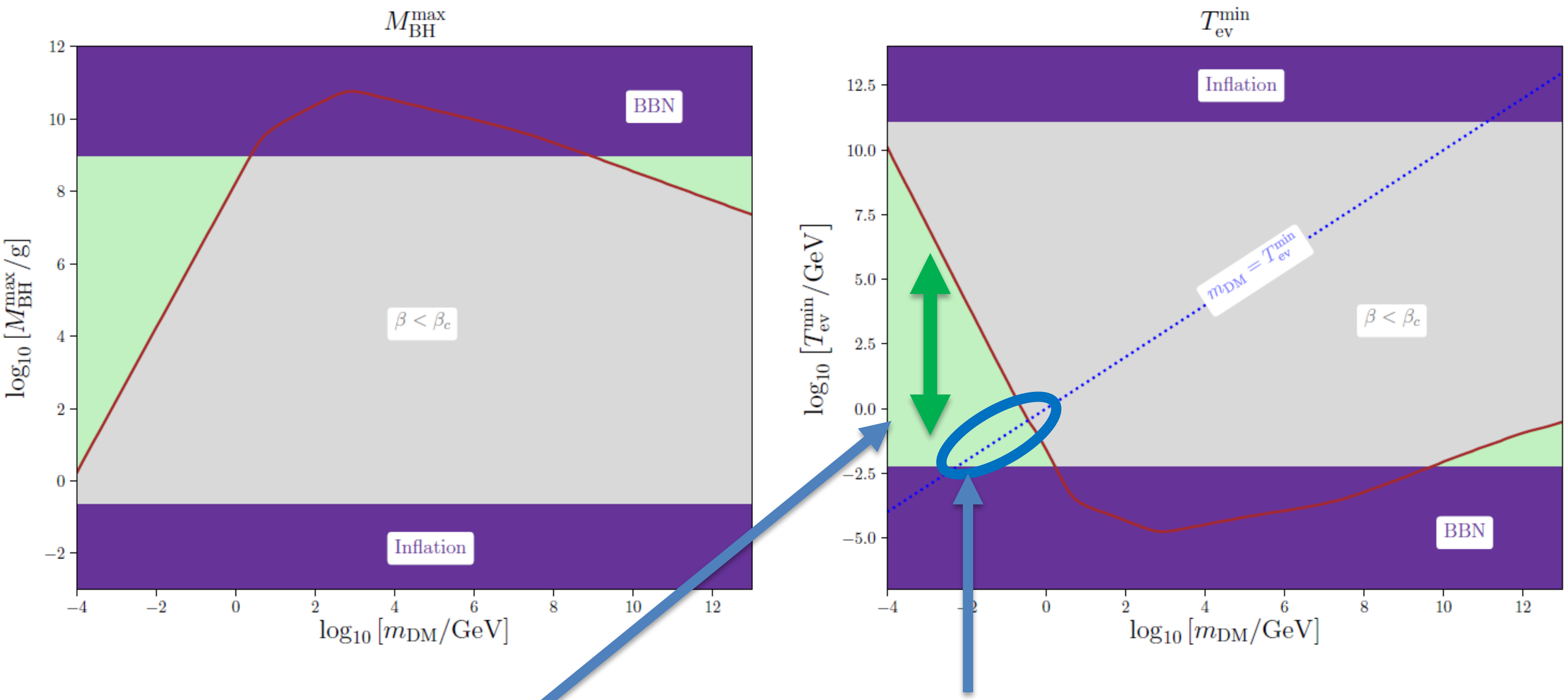
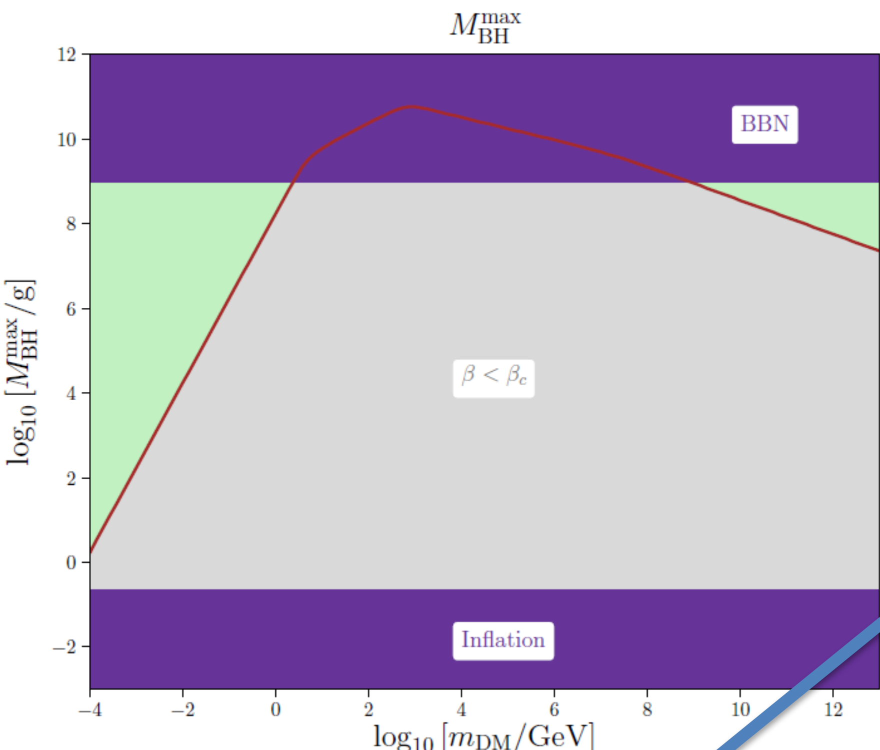
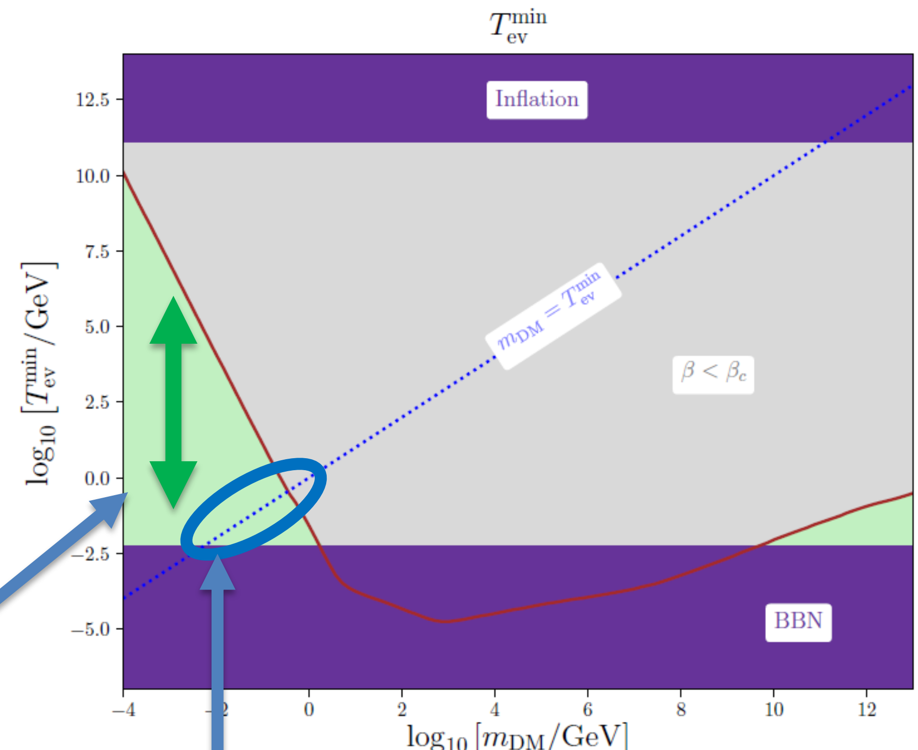


Region of interest
for Freeze-In

Region of interest
for Freeze-Out

Modified Cosmology





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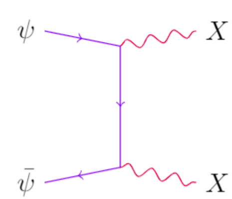
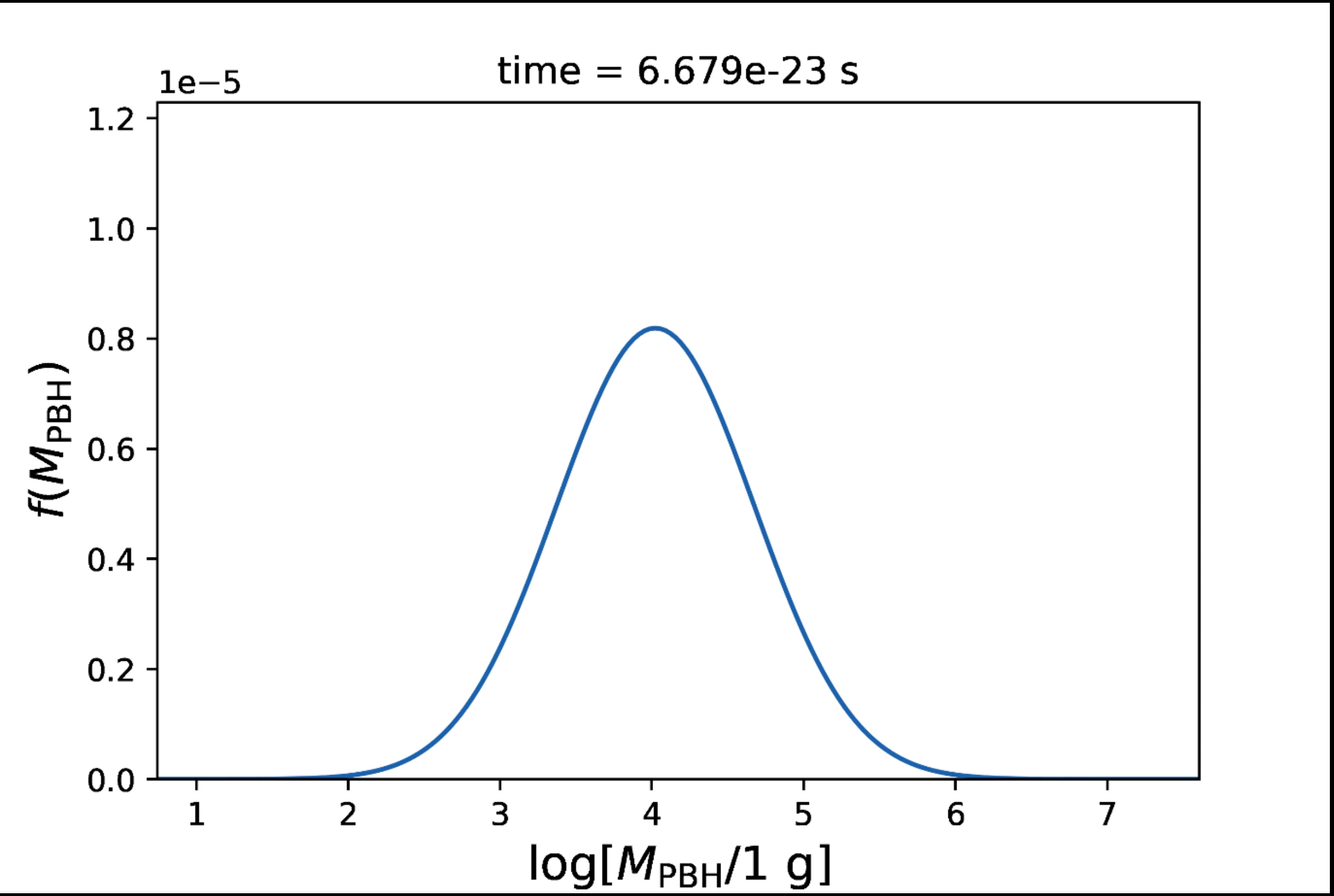
Of PBHs products…**

TBH large +



IV. Evaporation of Extended Distributions

$$\frac{dn}{dM} \propto \frac{1}{M^2} \exp\left[-\frac{(\log M - \log M_c)^2}{2\sigma^2}\right]$$



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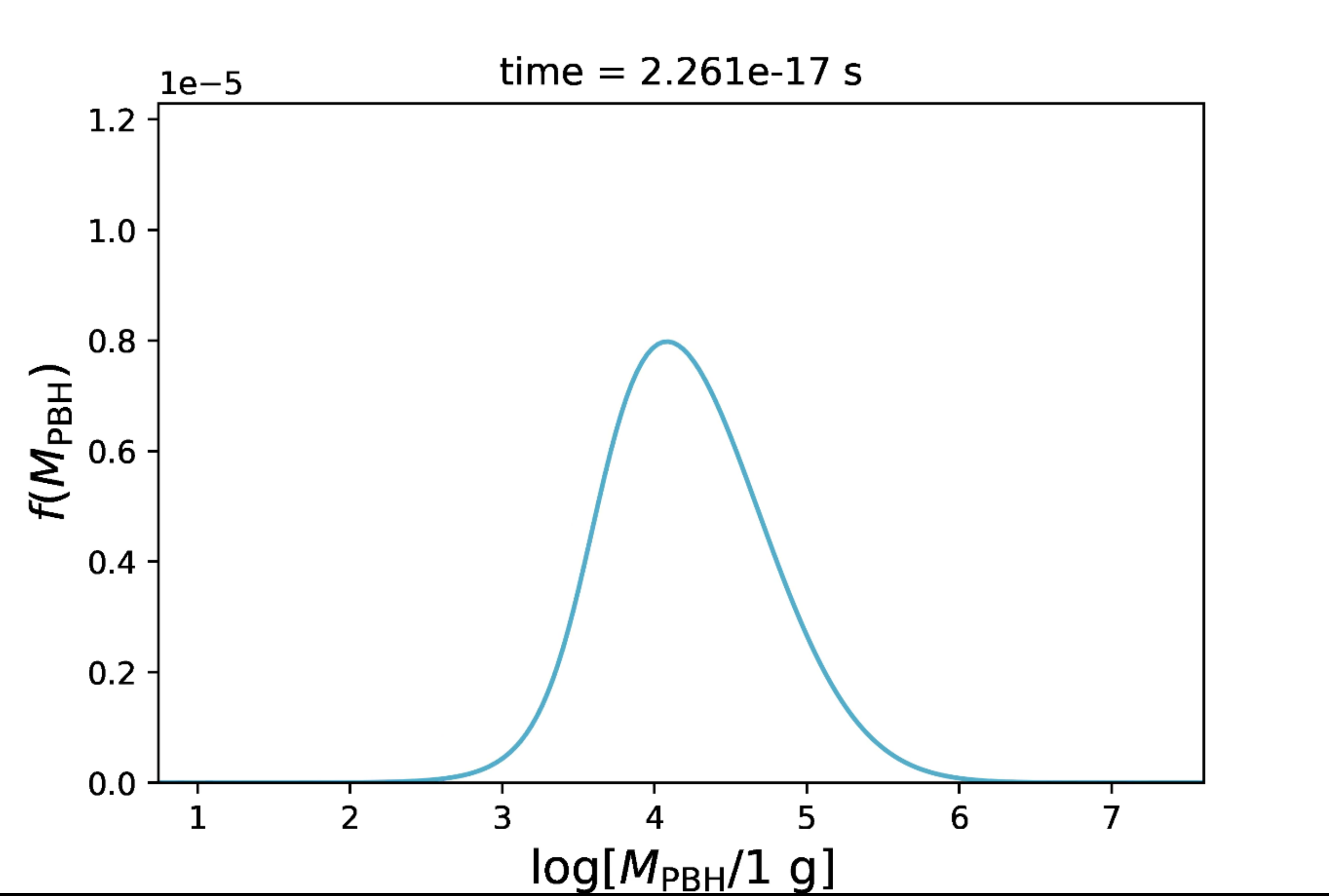

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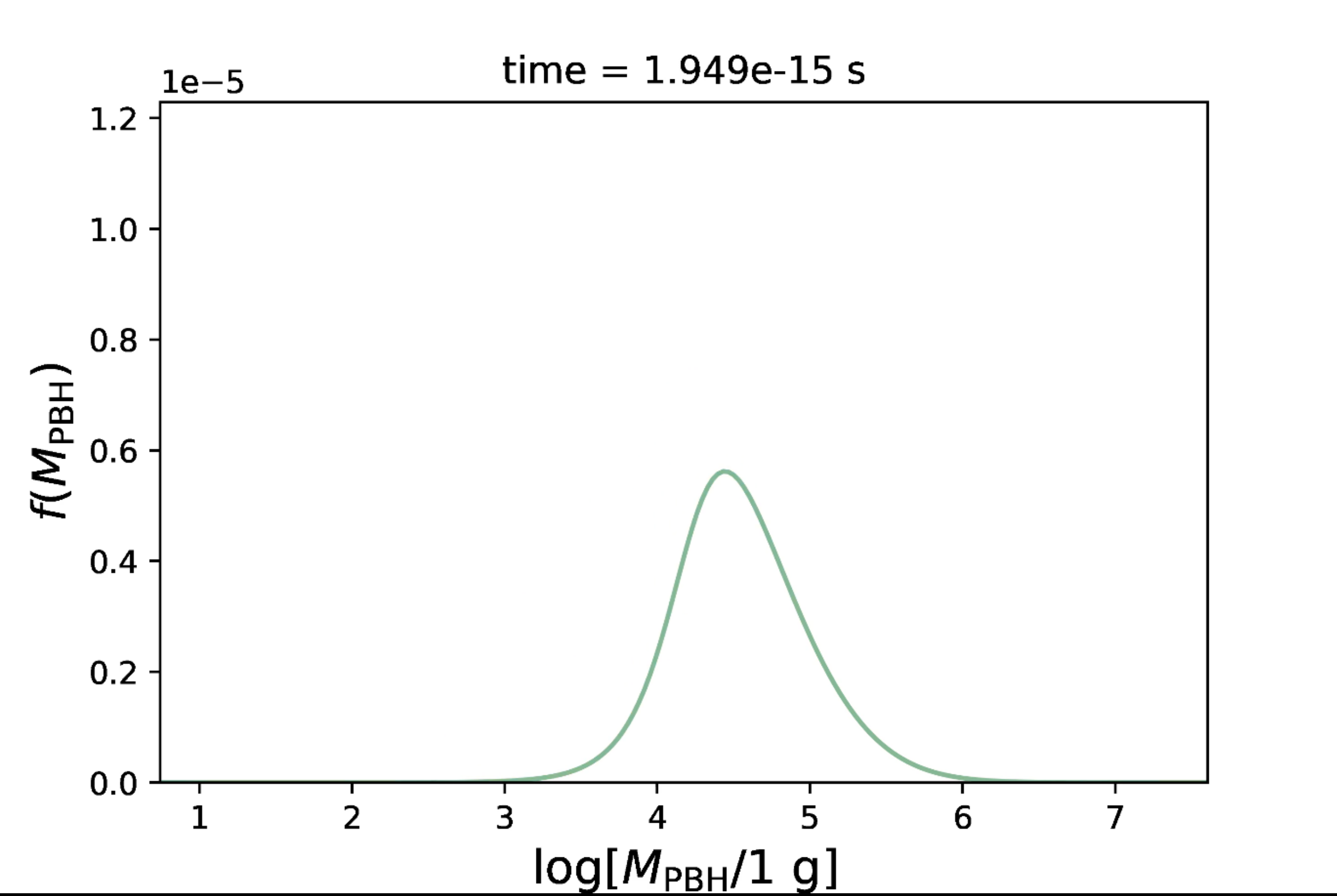

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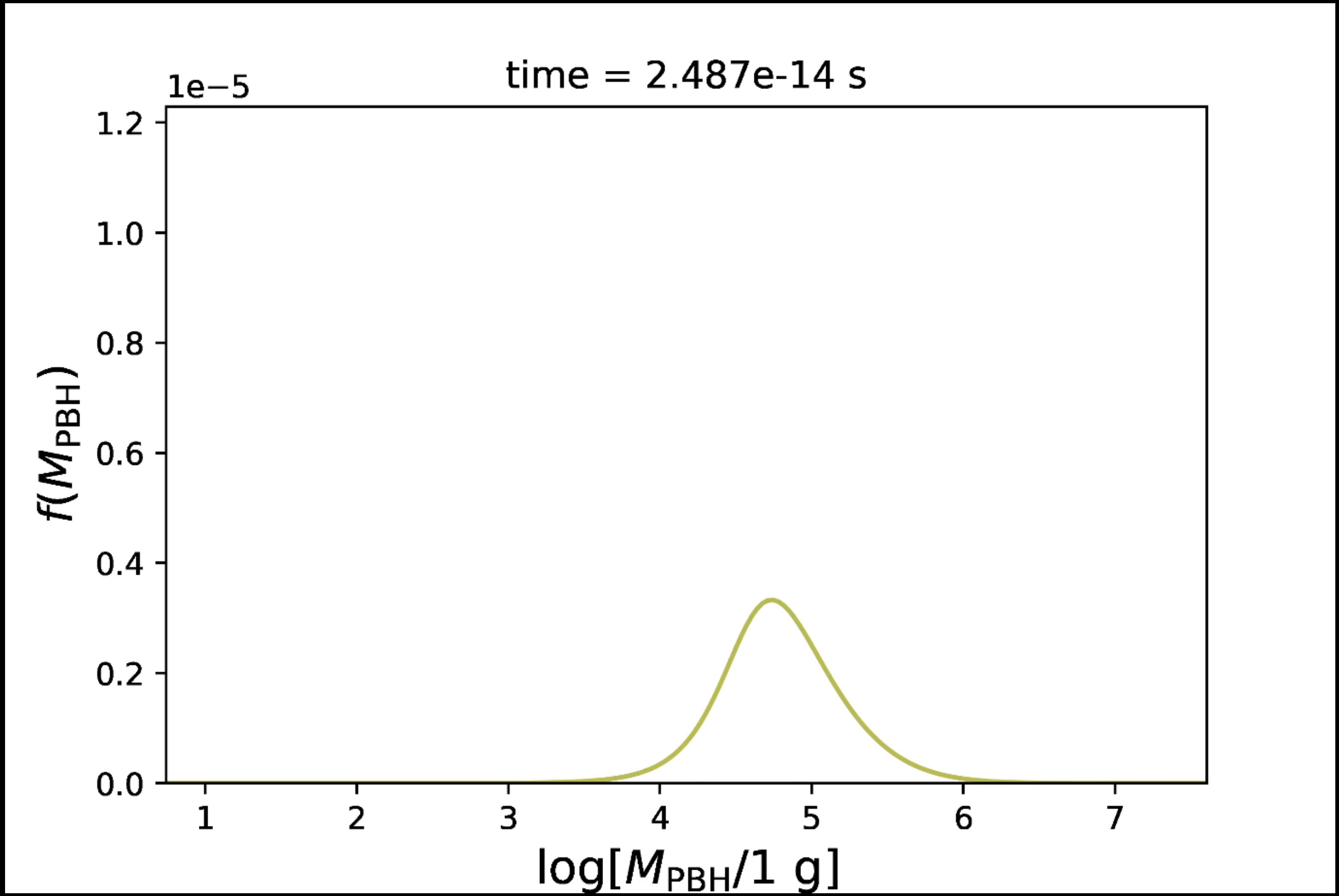

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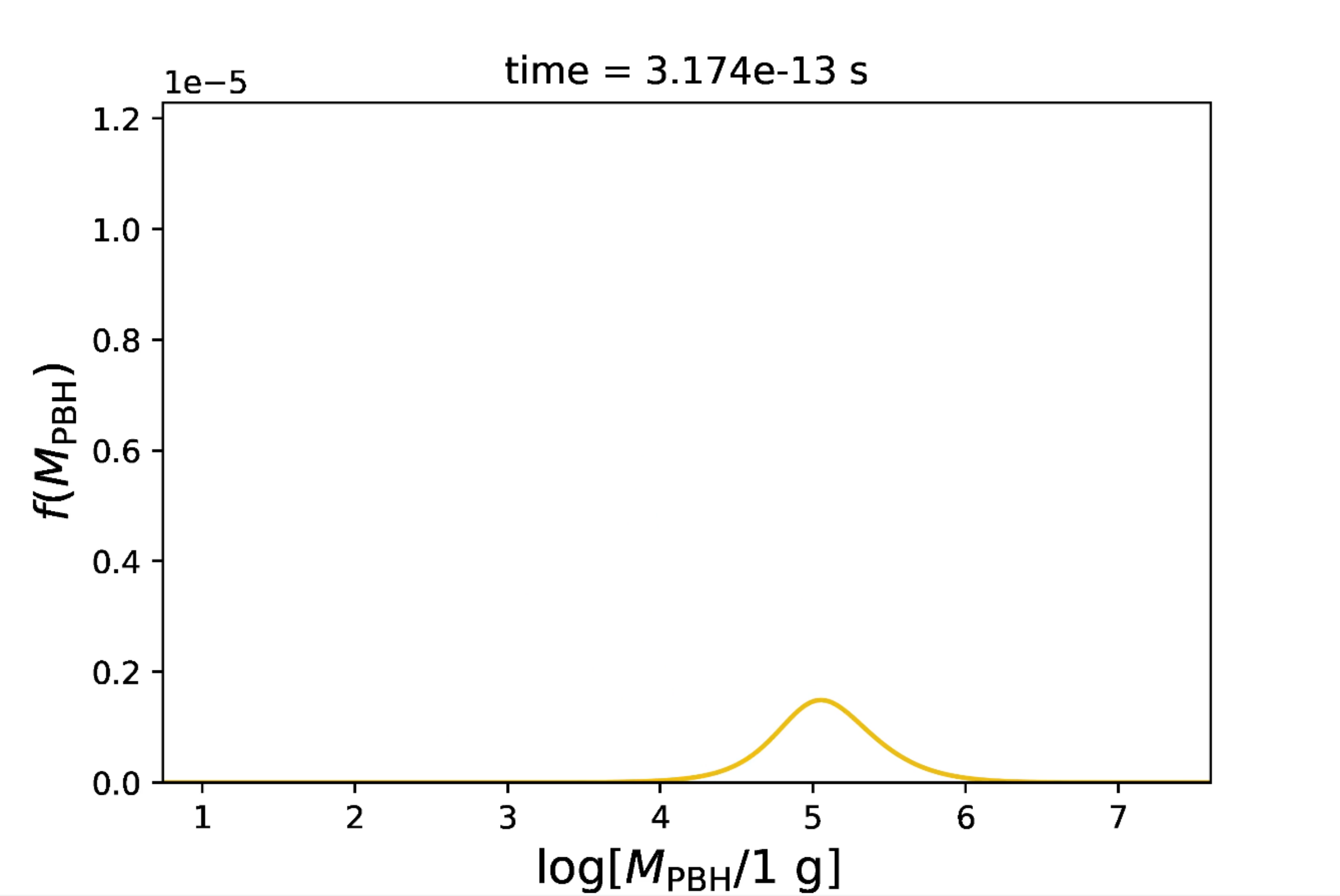

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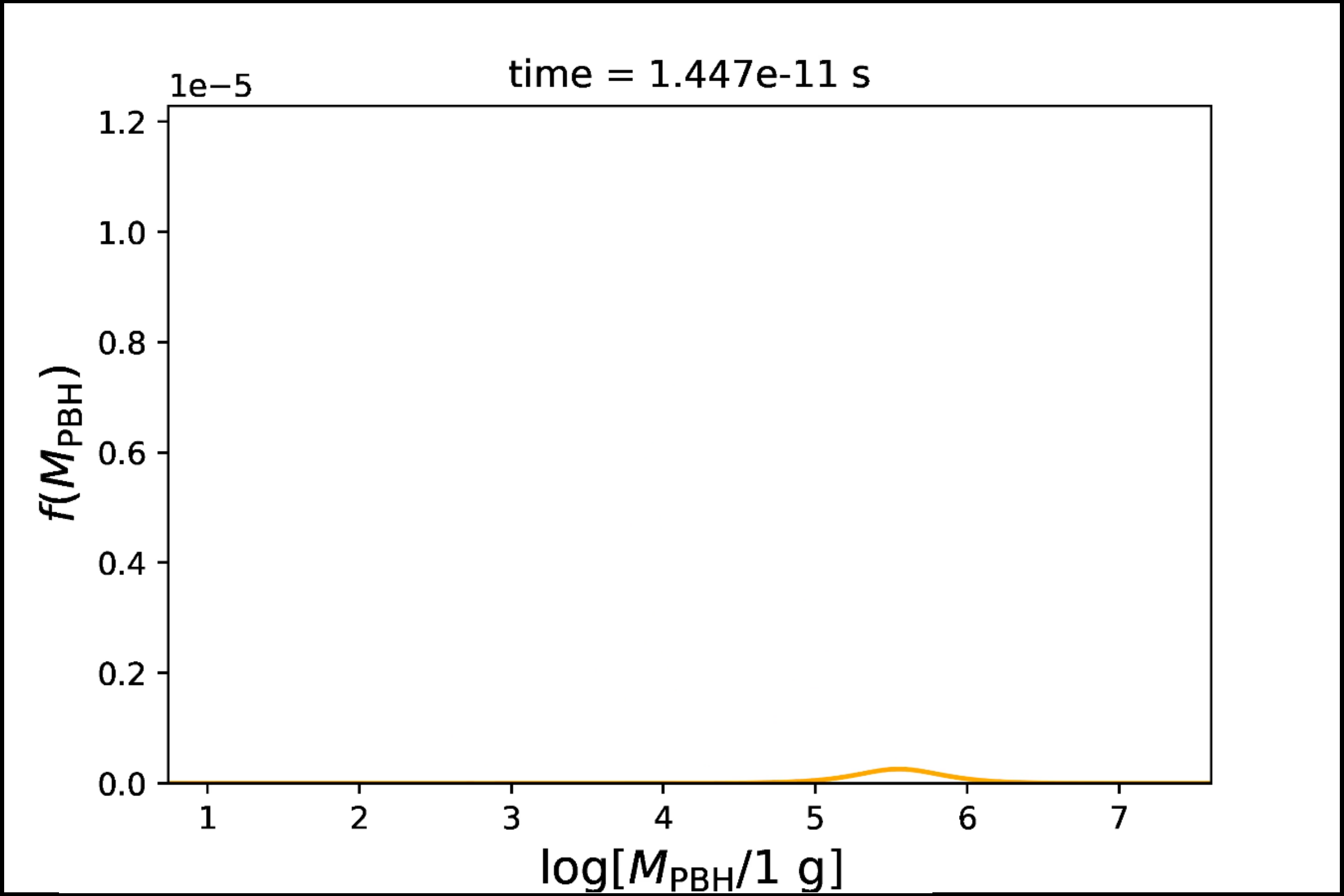

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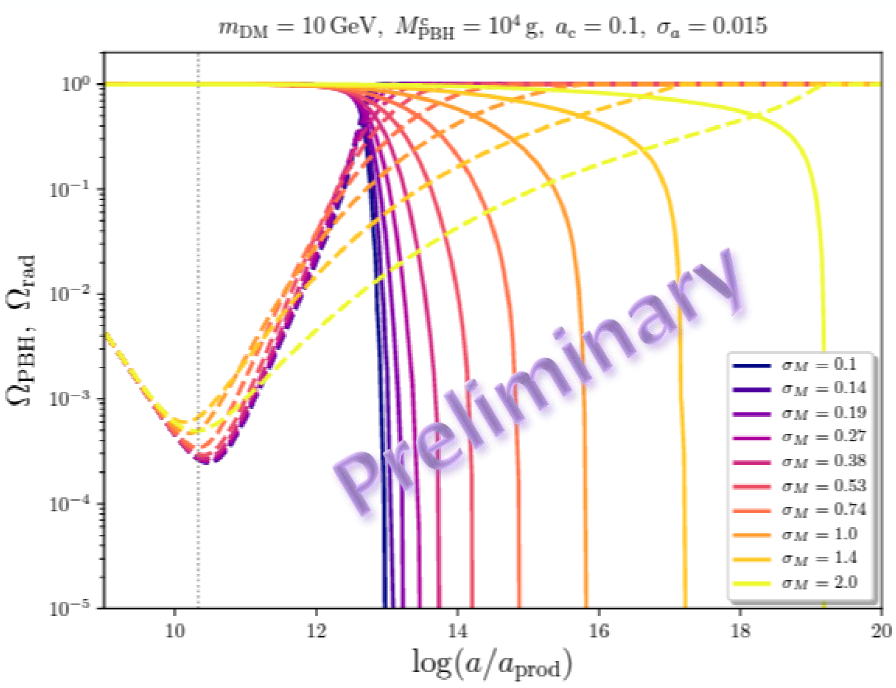

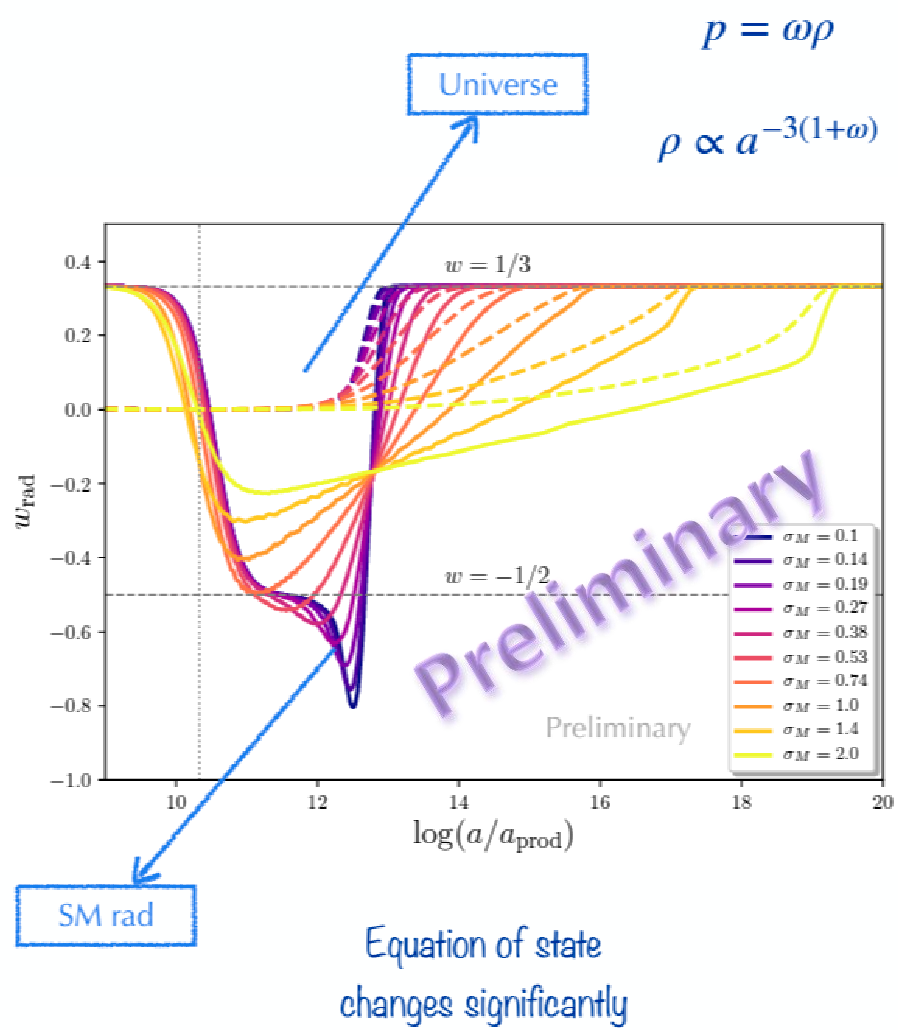

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$$\alpha \equiv \frac{(-3w_{\rm form.} - 1)}{(w_{\rm form.} + 1)}$$

$$\frac{\mathrm{d}H}{\mathrm{d}t} = -\frac{1}{2} H^2 (4 - \Omega_{\rm BH}),$$

$$\frac{\mathrm{d}\Omega_{\rm BH}}{\mathrm{d}t} = \Omega_{\rm BH} \left[\frac{\int_0^\infty f_{\rm BH}(M,t) \frac{\mathrm{d}M}{\mathrm{d}t} \mathrm{d}M}{\int_0^\infty f_{\rm BH}(M,t) M \mathrm{d}M} \right] + H \Omega_{\rm BH} (1 - \Omega_{\rm BH}).$$

$$= \frac{1+\alpha}{3(t - t_i)}$$

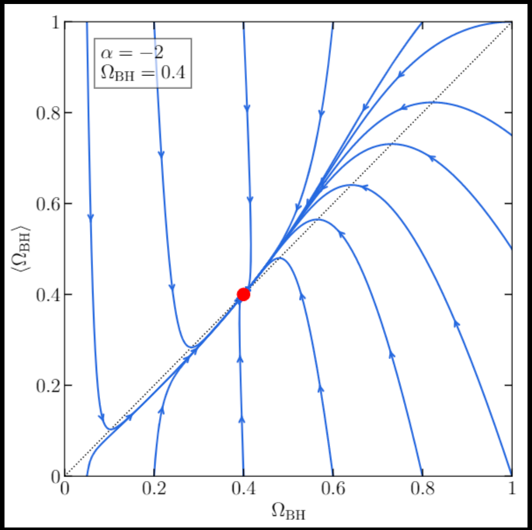
IV. Evaporation of Extended Distributions

$$\begin{cases} \dfrac{\mathrm{d}\Omega_{\mathrm{BH}}}{\mathrm{d}t} &= \dfrac{1}{t-t_i}\, f(\Omega_{\mathrm{BH}}, \langle\Omega_{\mathrm{BH}}\rangle) \\ \dfrac{\mathrm{d}\langle\Omega_{\mathrm{BH}}\rangle}{\mathrm{d}t} &= \dfrac{1}{t-t_i}\, g(\Omega_{\mathrm{BH}}, \langle\Omega_{\mathrm{BH}}\rangle)\ , \end{cases}$$

$$f(\Omega_{\mathrm{BH}}, \langle\Omega_{\mathrm{BH}}\rangle) \equiv \Omega_{\mathrm{BH}} \left[\frac{1+\alpha}{3} + \frac{2(1-\Omega_{\mathrm{BH}})}{4-\langle\Omega_{\mathrm{BH}}\rangle}\right]$$

$$g(\Omega_{\mathrm{BH}}, \langle\Omega_{\mathrm{BH}}\rangle) \equiv \Omega_{\mathrm{BH}} - \langle\Omega_{\mathrm{BH}}\rangle\ ,$$

$$\langle\Omega_{\mathrm{BH}}\rangle \equiv \frac{1}{t-t_i}\int_{t_i}^{t} dt'\, \Omega_{\mathrm{BH}}(t')\ .$$



$$w_{\mathrm{eff}} = \frac{-\alpha-1}{\alpha+7}$$

$$\Omega_{\mathrm{BH}} = \langle\Omega_{\mathrm{BH}}\rangle = \frac{4\alpha+10}{\alpha+7} \equiv \overline{\Omega}_{\mathrm{BH}}\ .$$

$$\mathcal{J} \equiv \frac{1}{t-t_i}\begin{pmatrix} \partial_{\Omega_{\mathrm{BH}}} f & \partial_{\langle\Omega_{\mathrm{BH}}\rangle} f \\ \partial_{\Omega_{\mathrm{BH}}} g & \partial_{\langle\Omega_{\mathrm{BH}}\rangle} g \end{pmatrix}$$

$$\lambda_{\pm} = \frac{1}{18}\left(-4\alpha \pm \sqrt{-19-4\alpha(2\alpha+19)-59}\right)$$

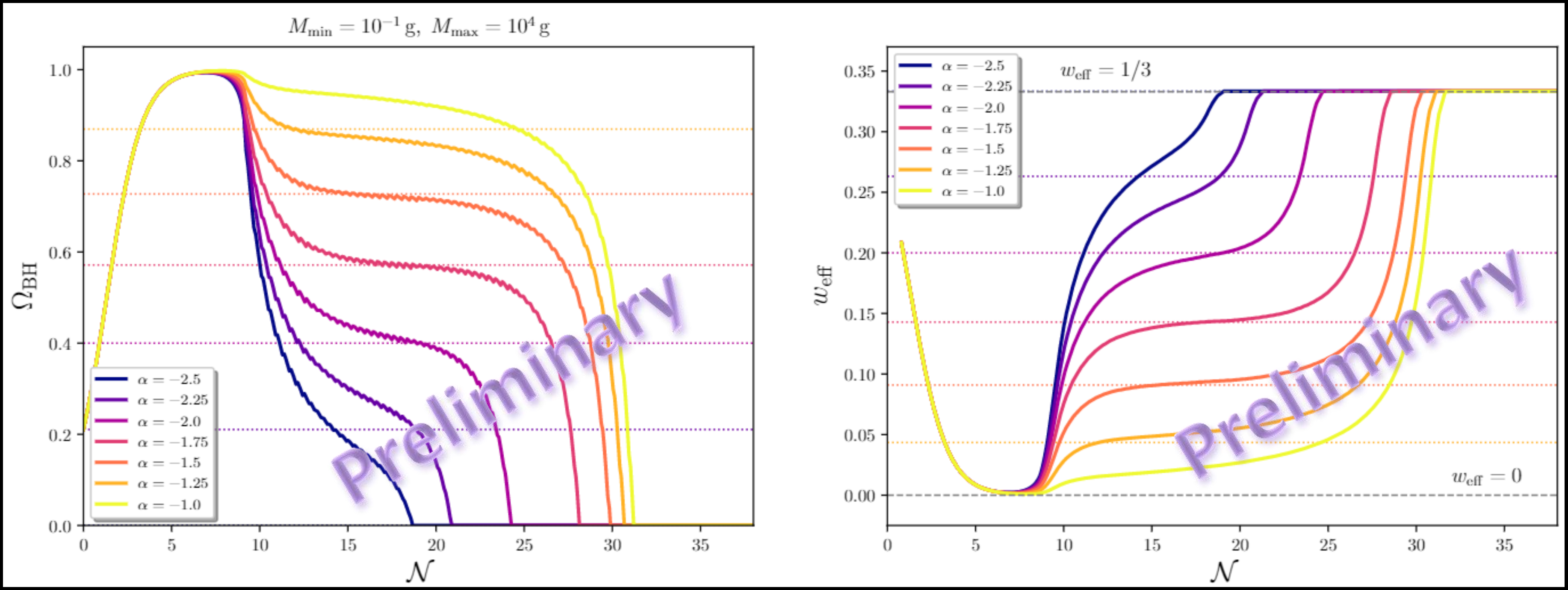
IV. Evaporation of Extended Distributions

$$w_{\rm eff} = \frac{-\alpha - 1}{\alpha + 7}$$

$$\alpha \equiv \frac{(-3w_{\rm form.} - 1)}{(w_{\rm form.} + 1)}$$

$$\Omega_{\rm BH} = \langle \Omega_{\rm BH} \rangle = \frac{4\alpha + 10}{\alpha + 7} \equiv \overline{\Omega}_{\rm BH} \, .$$

$$0 < w_{\rm form.} \leq 1 \quad \Rightarrow \quad 0 < \overline{\Omega}_{\rm BH} < 1$$



'Stasis' regime reached for $0 < w < 1$