The W-boson mass and the Standard Model

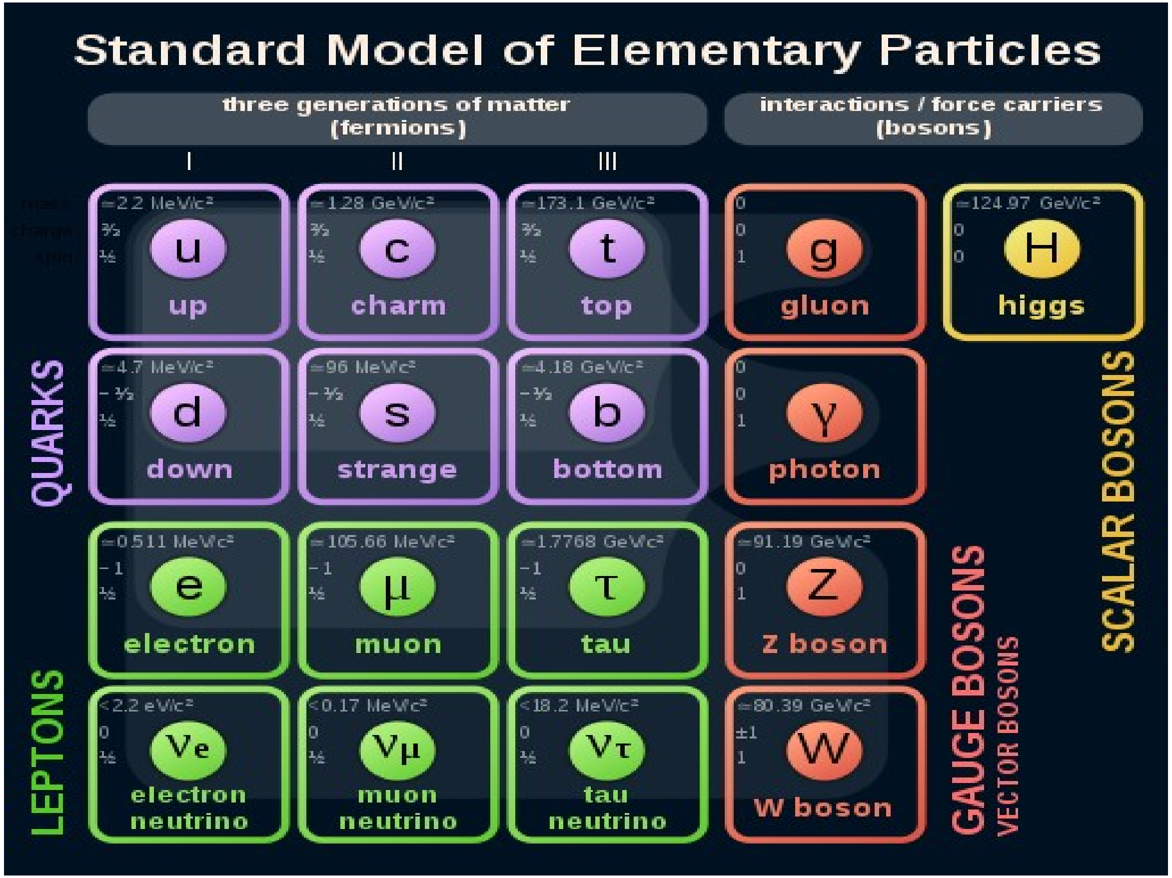
Maarten Boonekamp

CEA/IRFU and Guest Scientist at Helmholtz Institut, Mainz

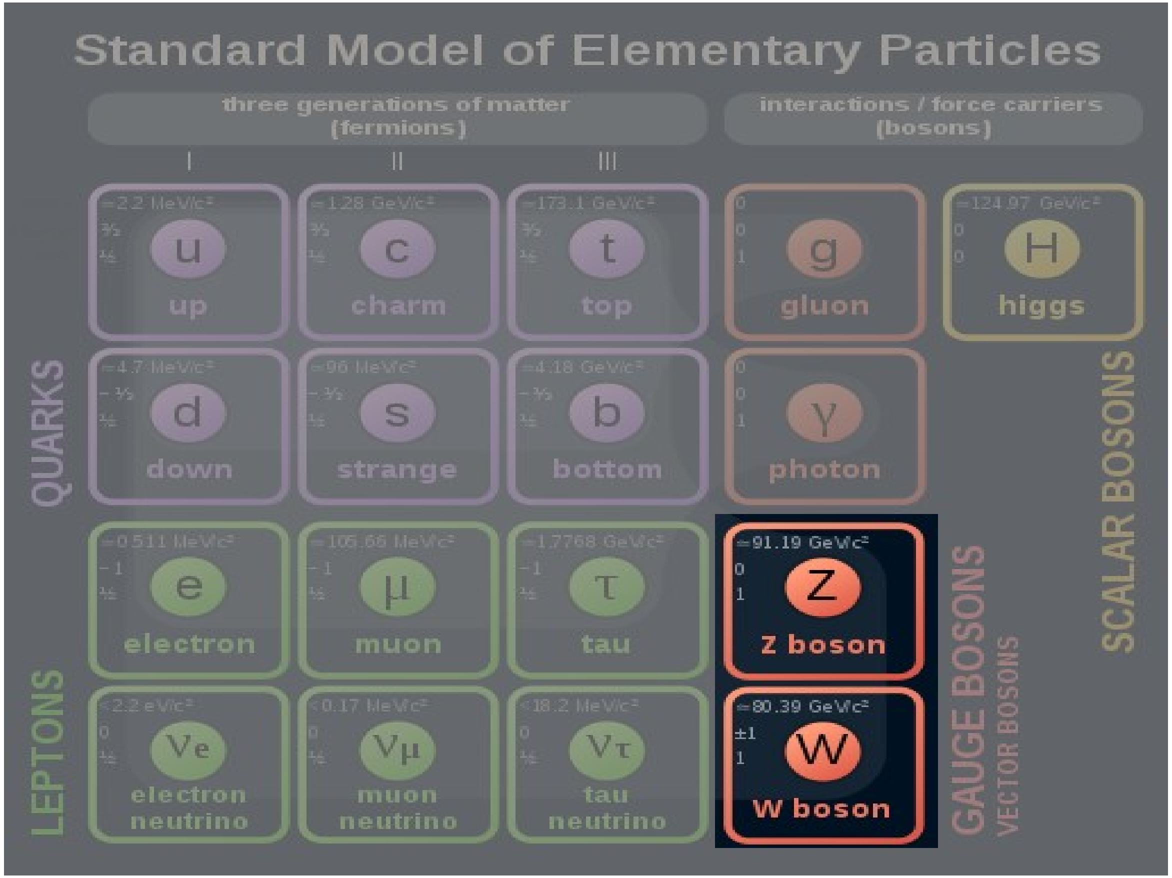
Outline

- Tests of the electroweak theory
- Very basics of electroweak phenomenology in the gauge sector (fundamental relations and quantum corrections)
- The W-boson mass
  - General ideas, issues, results/prospects
- Consistency of the Standard Model?
  - Before that : consistency between experiments?
  - Not discussed : which new physics, if not?

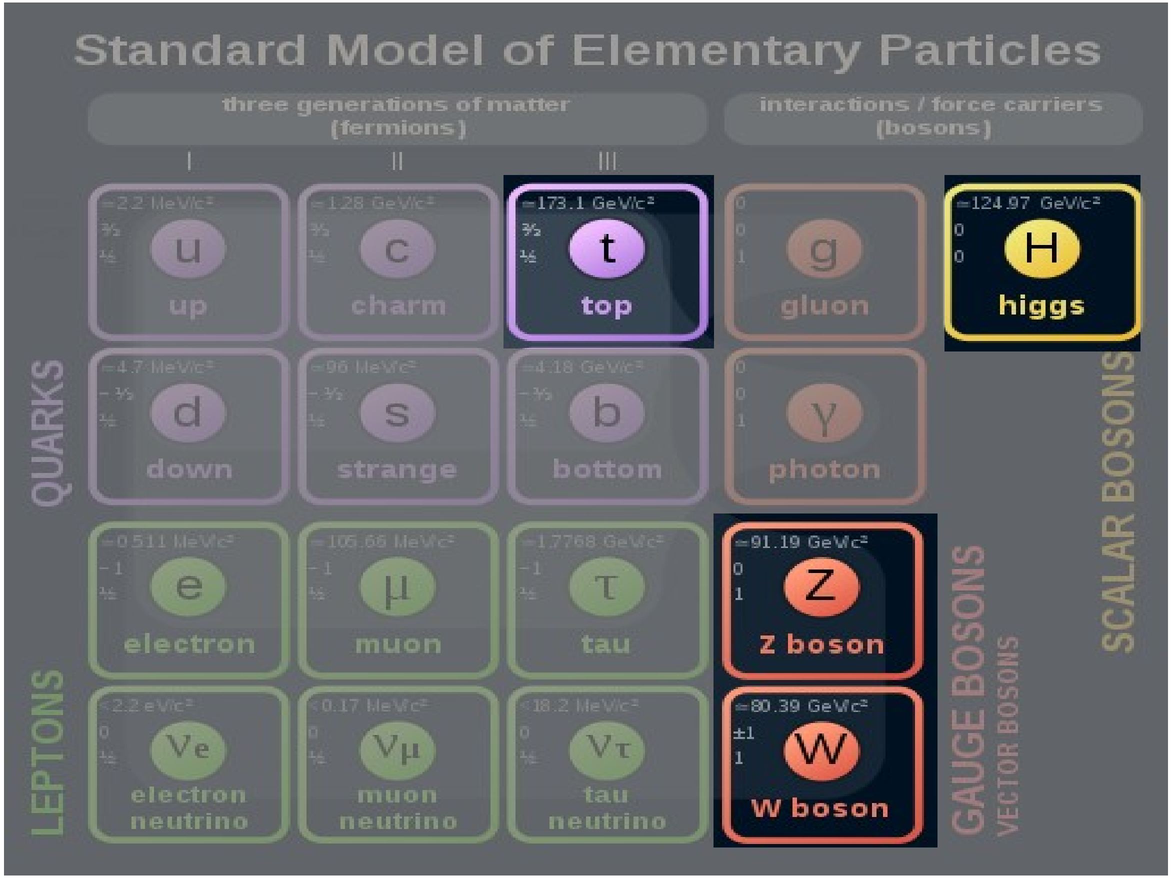
Elementary particles



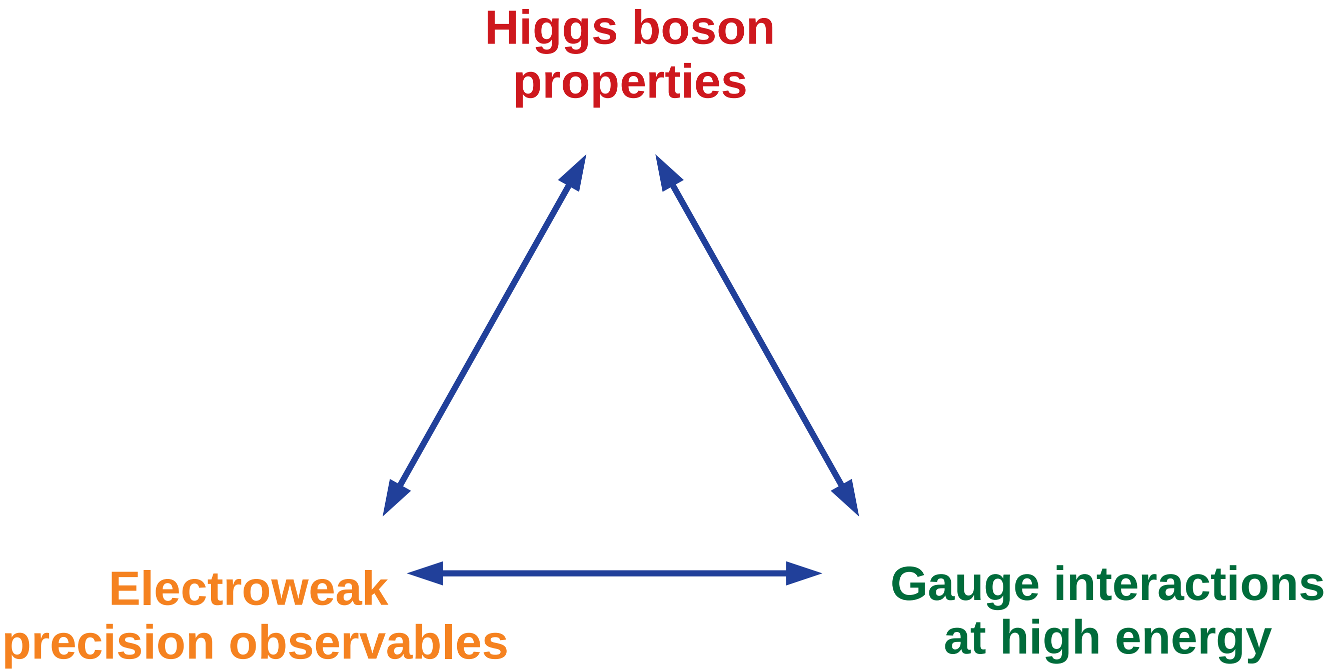
Elementary particles



Elementary particles

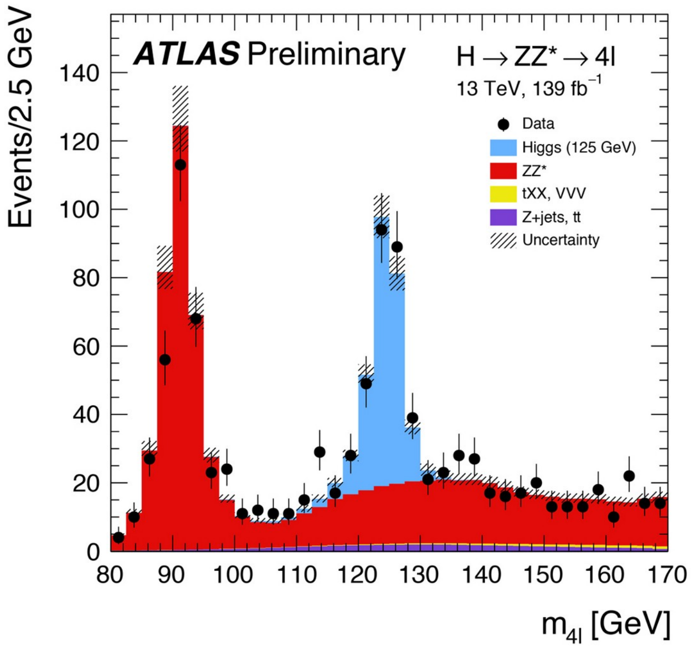


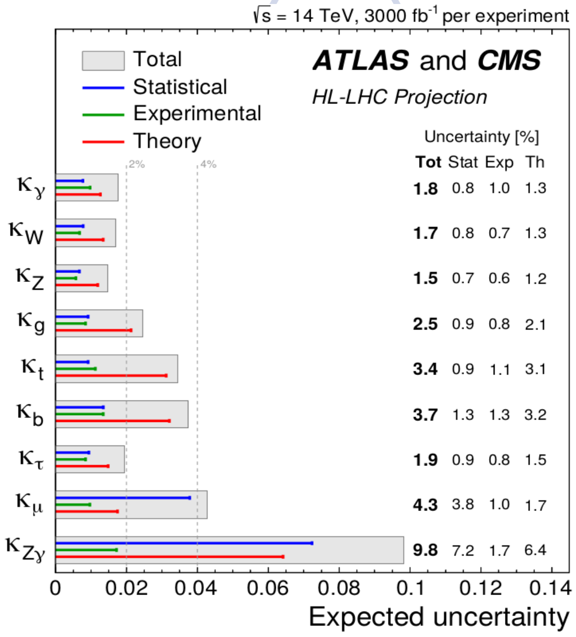
Tests of the electroweak theory

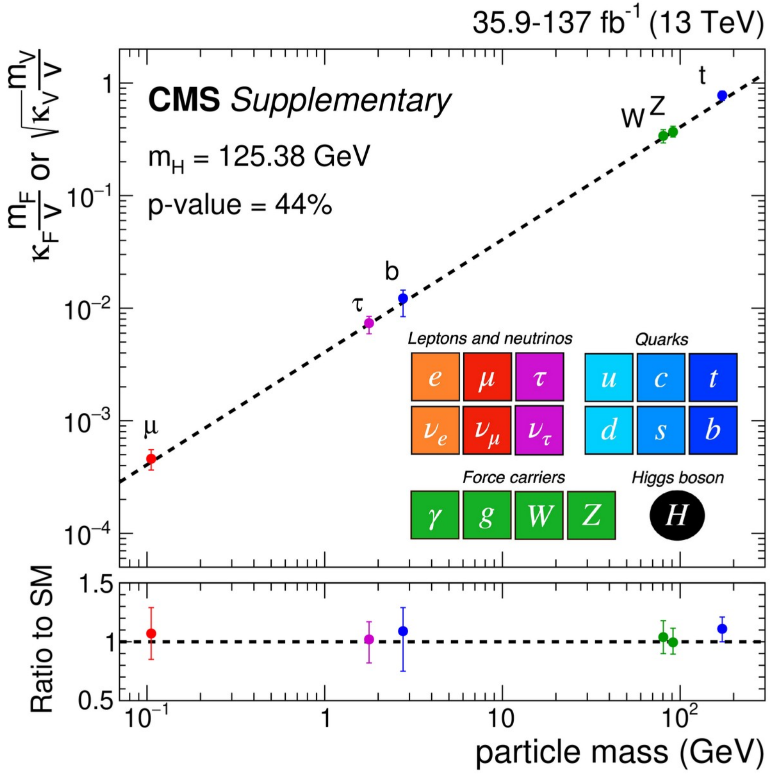


Tests of the electroweak theory

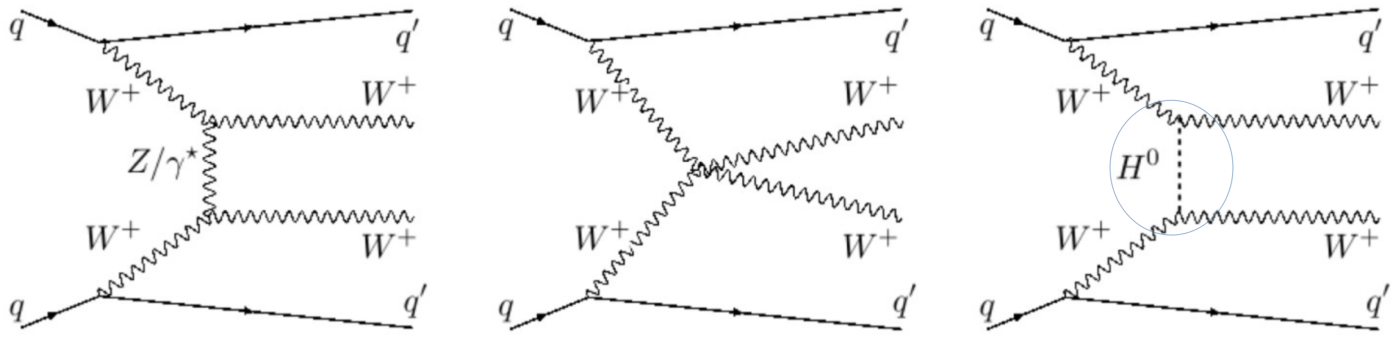
Higgs boson properties

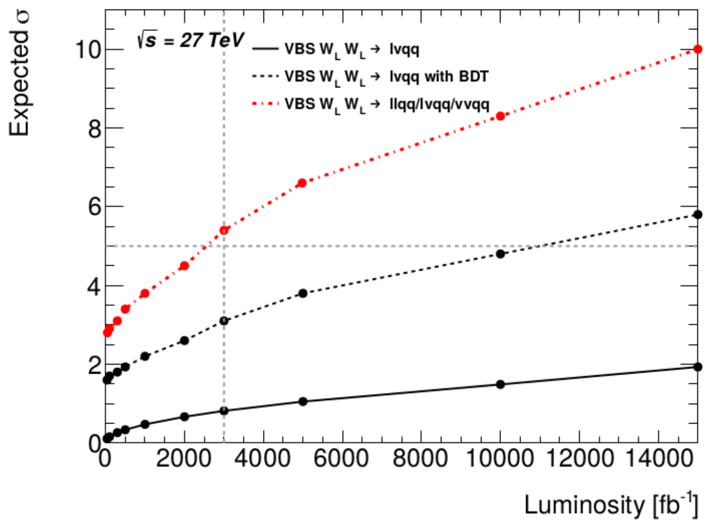






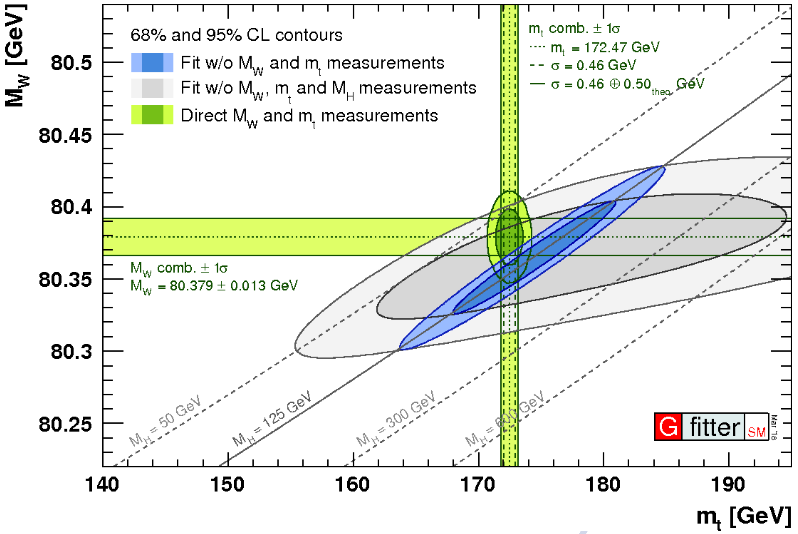
Tests of the electroweak theory



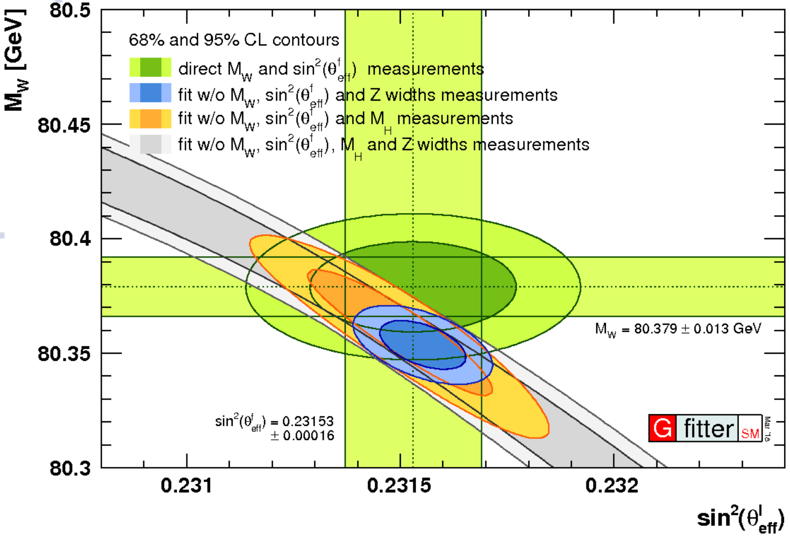


Gauge interactions at high energy

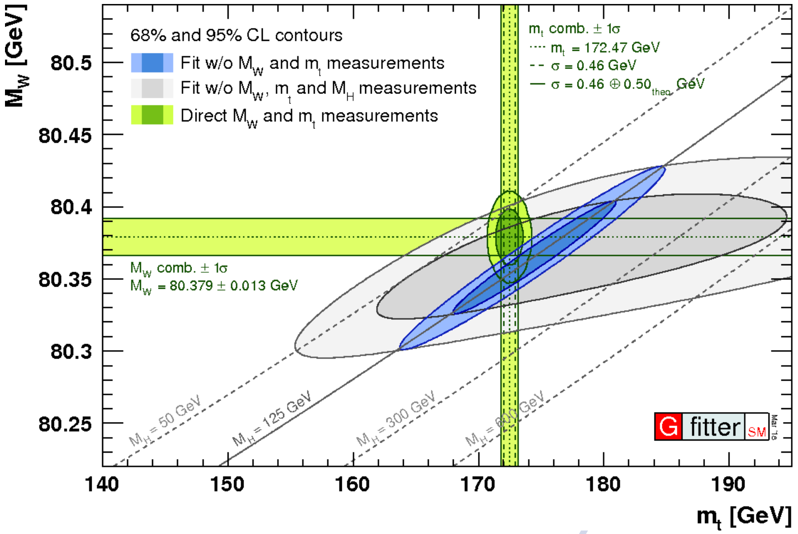
Tests of the electroweak theory

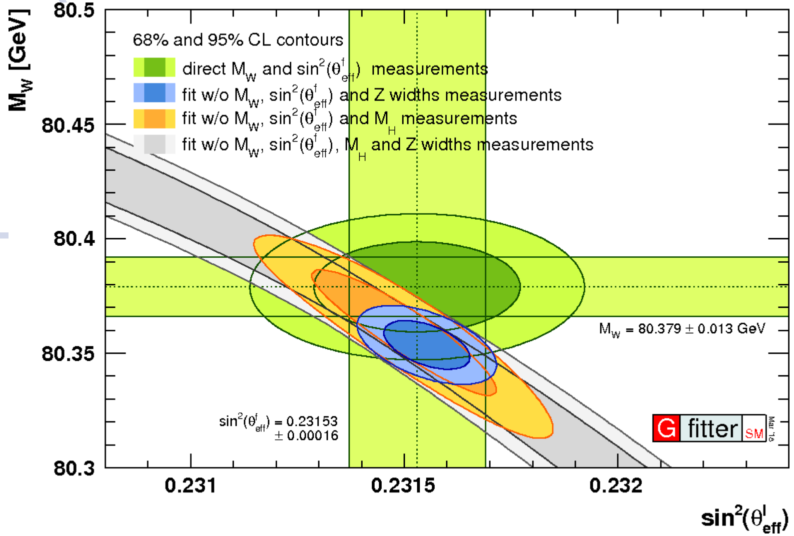


Electroweak precision observables



Tests of the electroweak theory





Electroweak precision observables

This talk

Electroweak predictions in leading order

- The electroweak gauge sector of the SM is constrained by three precisely known parameters :
  - The electromagnetic coupling constant : α = 1/137035999206(11)
  - The muon decay constant : G_μ = 1.1663787(6) GeV^{-2}
  - The Z boson mass : m_Z = 91.1876(21) GeV

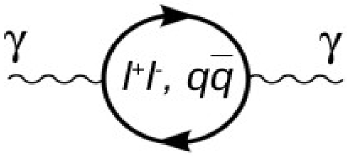
- The W boson mass is given by

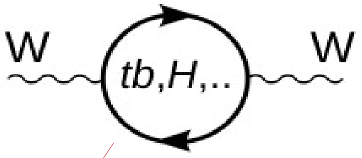
$$m_W^2 = \frac{m_Z^2}{2}\left(1+\sqrt{1-4\frac{\pi\alpha}{\sqrt{2}G_\mu m_Z^2}}\right)$$

Quantum corrections : m_W

- Higher-order corrections, predominantly the boson self-energies, modify the leading-order relations to

$$m_W^2 = \frac{m_Z^2}{2}\left(1+\sqrt{1-4\frac{\pi\alpha}{\sqrt{2}G_\mu m_Z^2}\frac{1}{1-\Delta r}}\right)$$

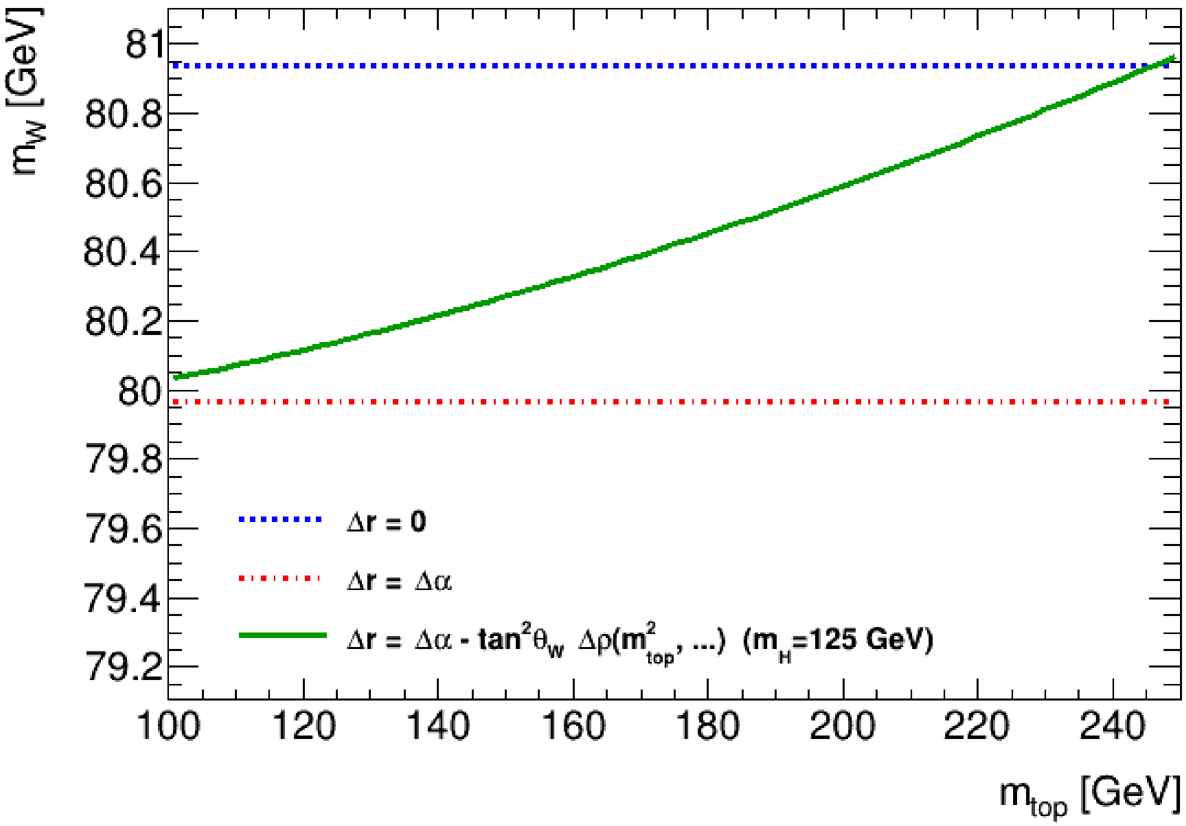




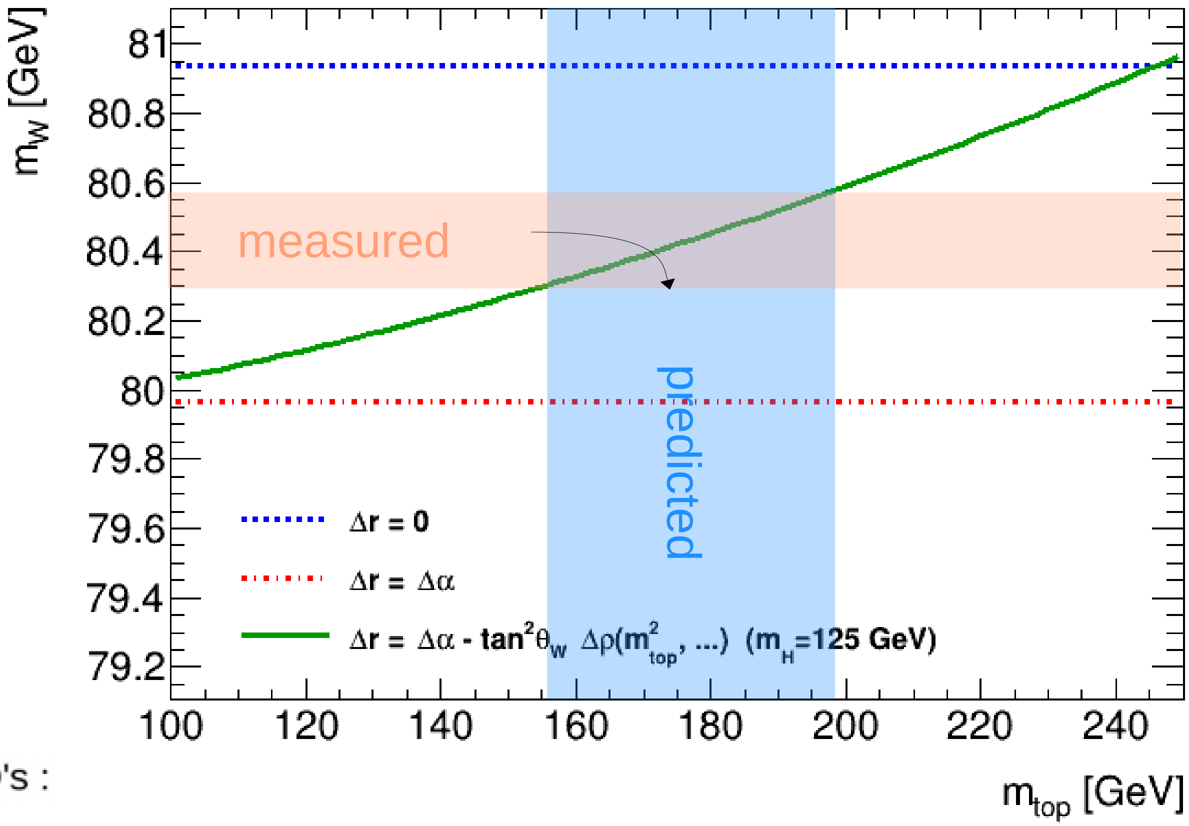
$$\Delta r = \Delta\alpha - \tan^2\theta_W \Delta\rho = \sim 0.059 - \frac{3G_\mu m_W^2}{8\sqrt{2}\pi^2}\left[\frac{m_{top}^2}{m_W^2}\cot^2\theta_W - \left(\ln\frac{m_H^2}{m_W^2} - \frac{5}{6}\right) + \ldots\right]$$

$\alpha(0) \sim 1/137.. \rightarrow \alpha(m_Z) \sim 1/128.9$

Quantum corrections : m_W



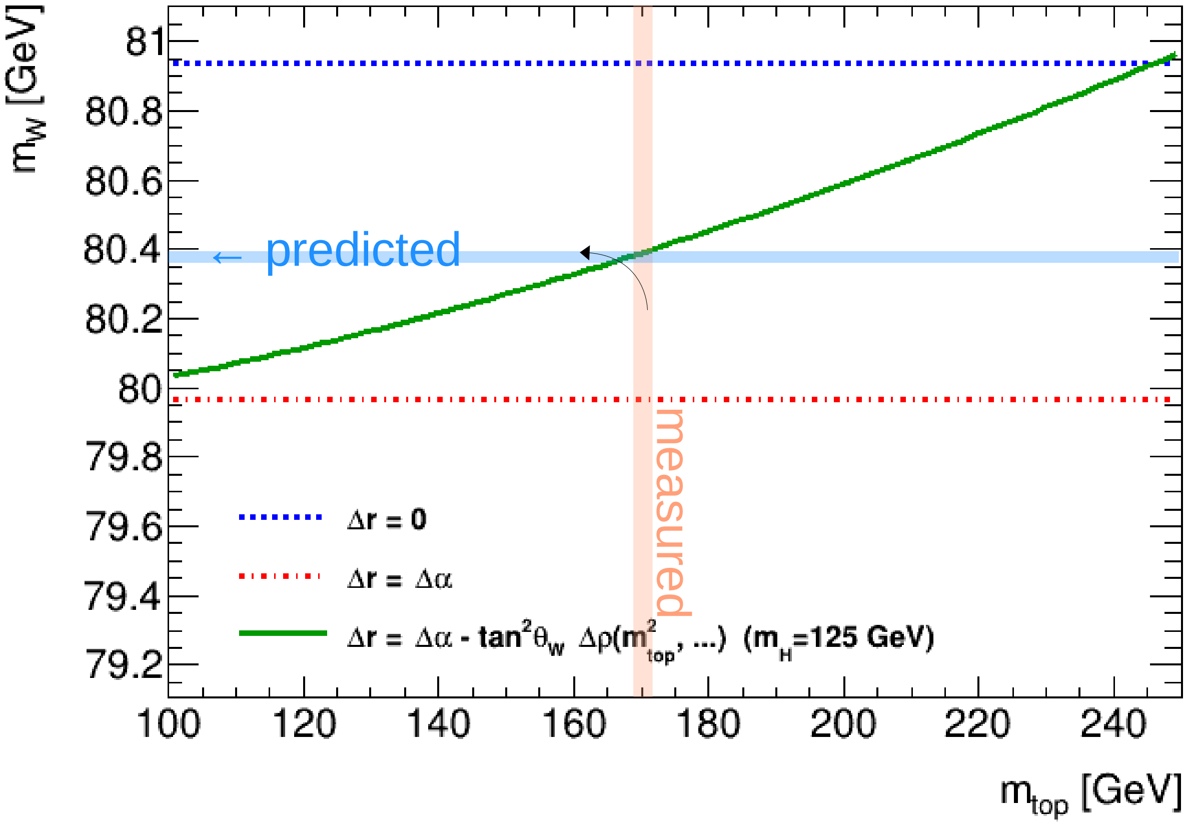
Quantum corrections : m_W



Situation in the early 90's :

- Inputs : δm_Z ~ 2 MeV (LEP); δm_W ~ 200 MeV; ~40 GeV < m_H < 1 TeV
- Output : **160 GeV < m_{top} < 190 GeV** (before the top quark discovery!)

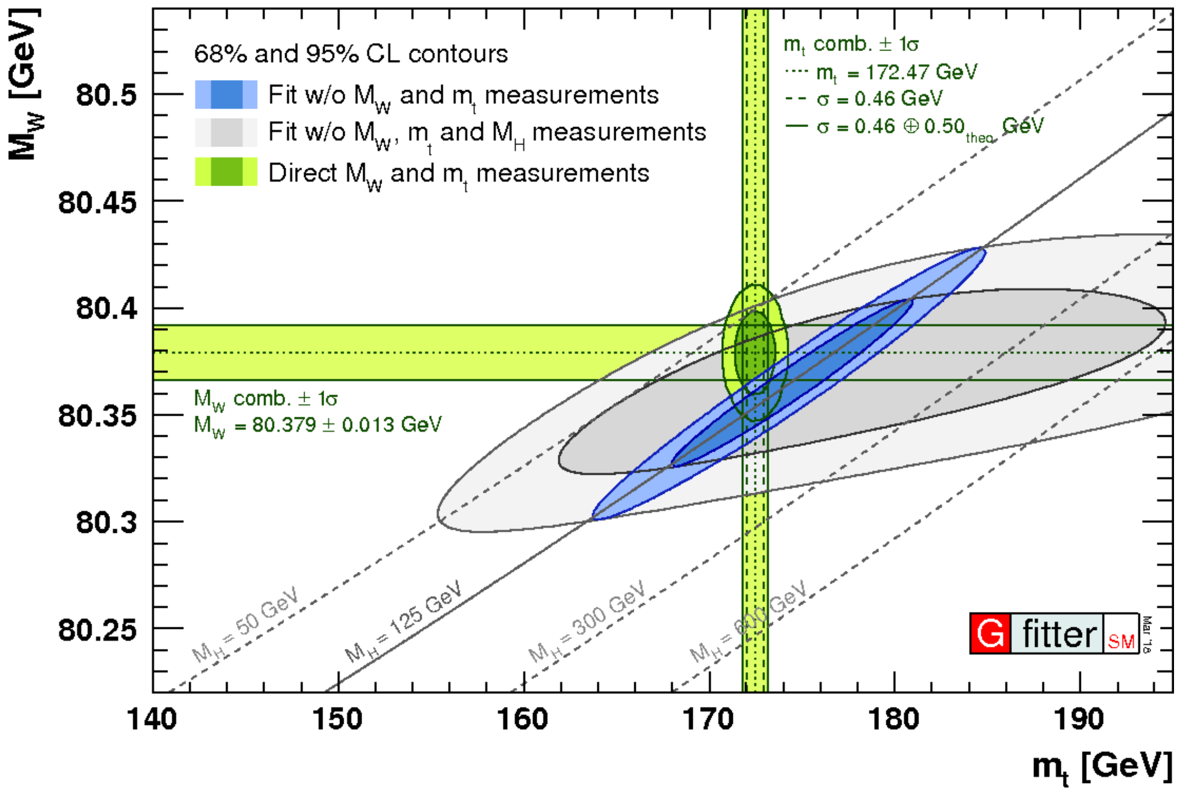
Quantum corrections : m_W



Nowadays:

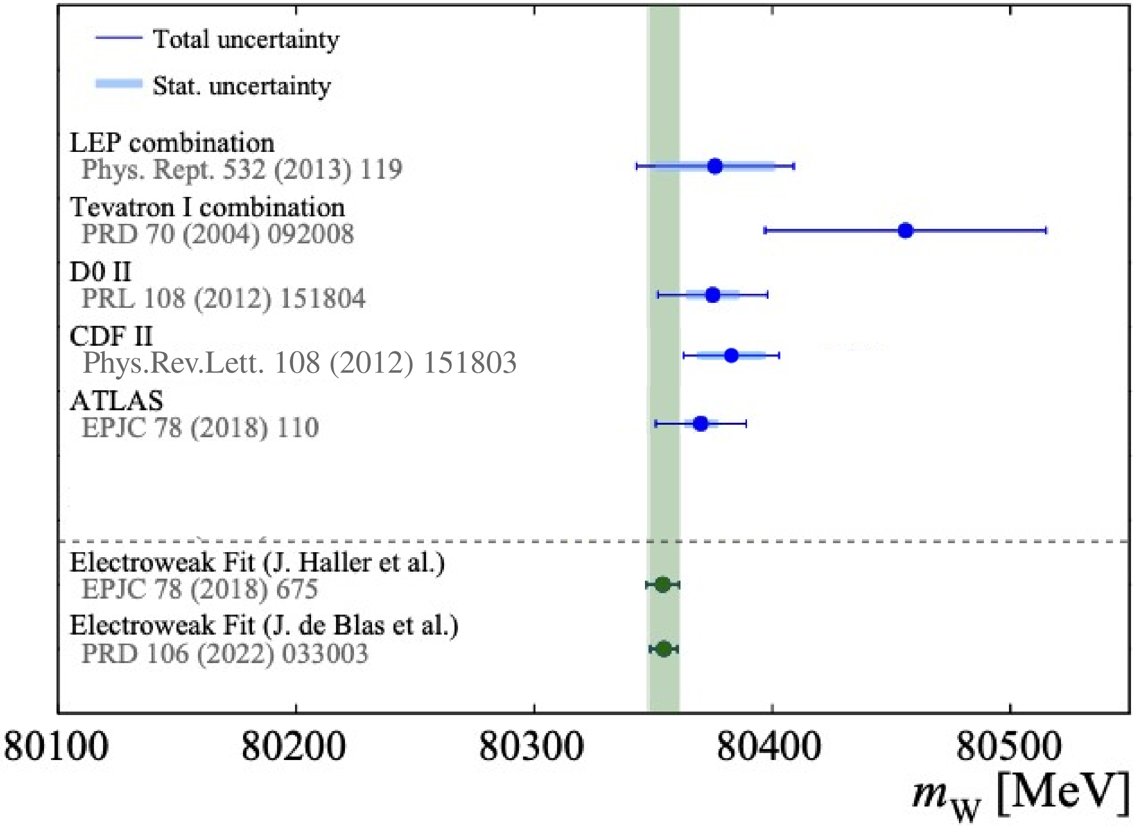
- Inputs : $\delta m_{top} \sim 0.7$ GeV $\delta m_H < 0.2$ GeV
- Output : **$m_W = 80.356 +/- 0.008$ GeV**

Prediction of m_W in the SM – a snapshot

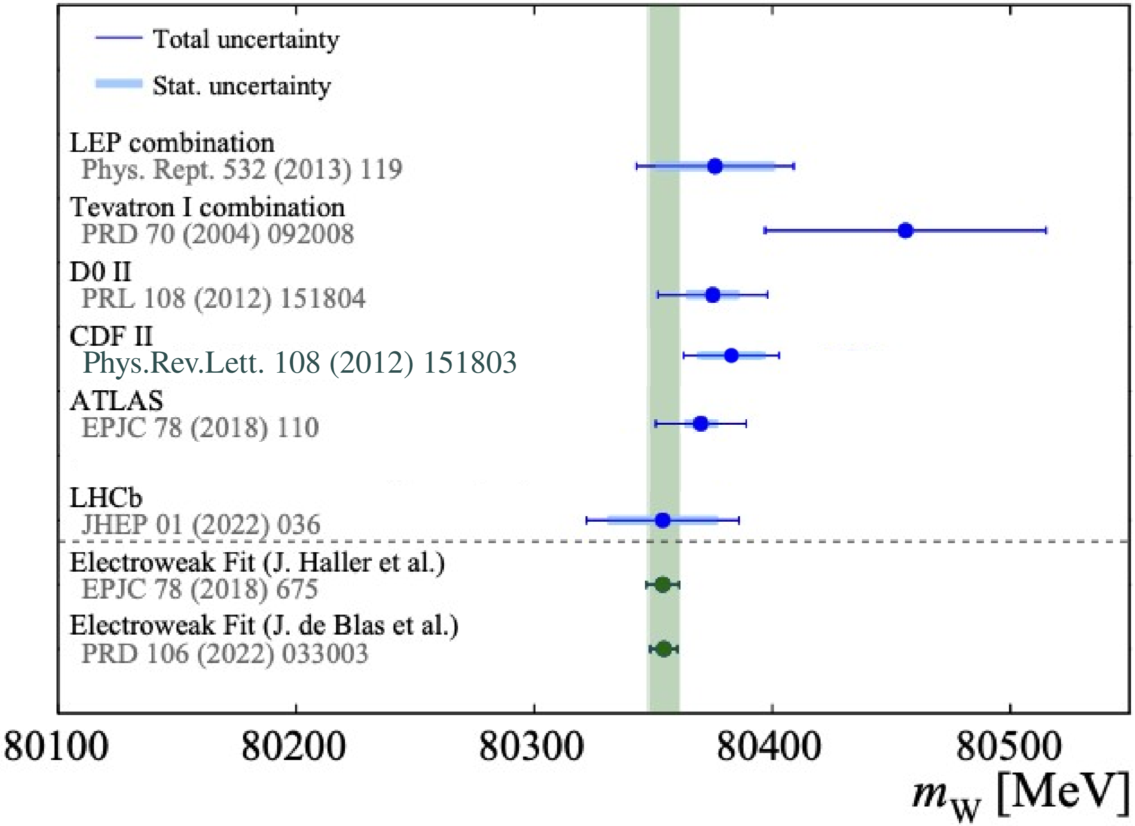


… the professional version of the same plot

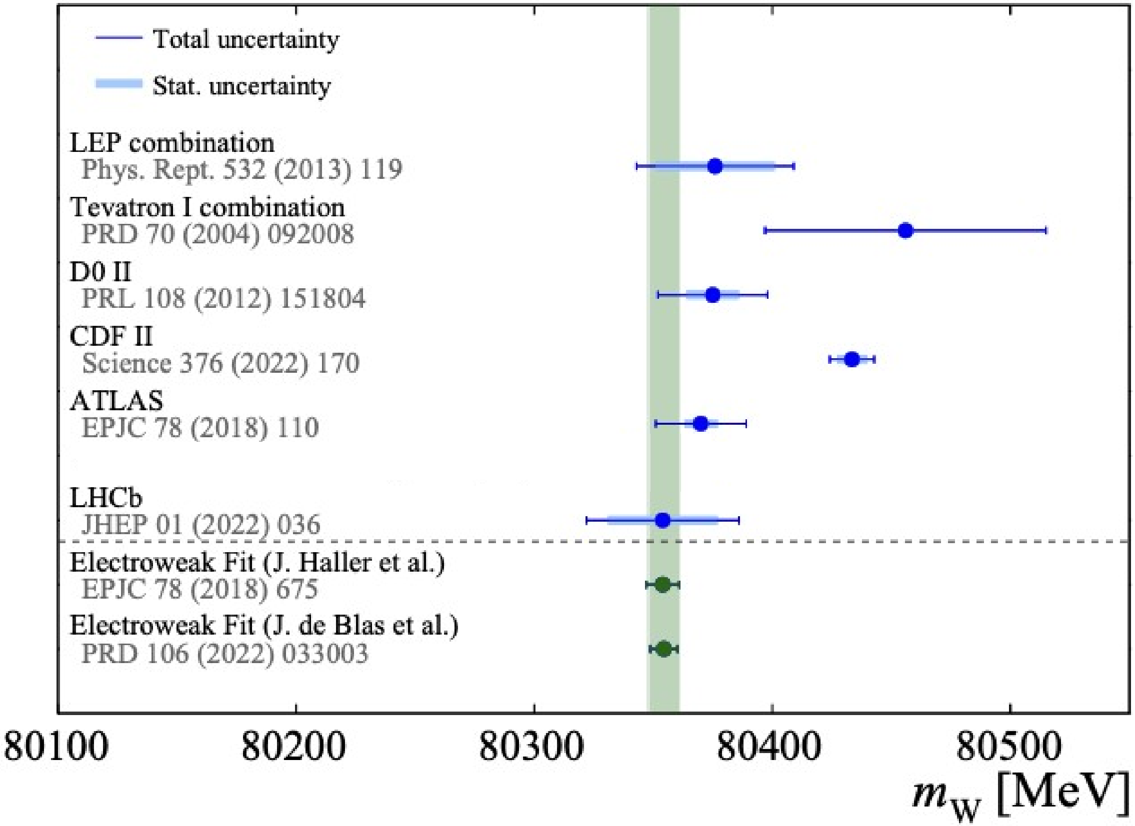
Measurements until 2020



2021


Total uncertainty
Stat. uncertainty
LEP combination
Phys. Rept. 532 (2013) 119
Tevatron I combination
PRD 70 (2004) 092008
D0 II
PRL 108 (2012) 151804
CDF II
Phys.Rev.Lett. 108 (2012) 151803
ATLAS
EPJC 78 (2018) 110
LHCb
JHEP 01 (2022) 036
Electroweak Fit (J. Haller et al.)
EPJC 78 (2018) 675
Electroweak Fit (J. de Blas et al.)
PRD 106 (2022) 033003
80100
80200
80300
80400
80500
m_W [MeV]

2022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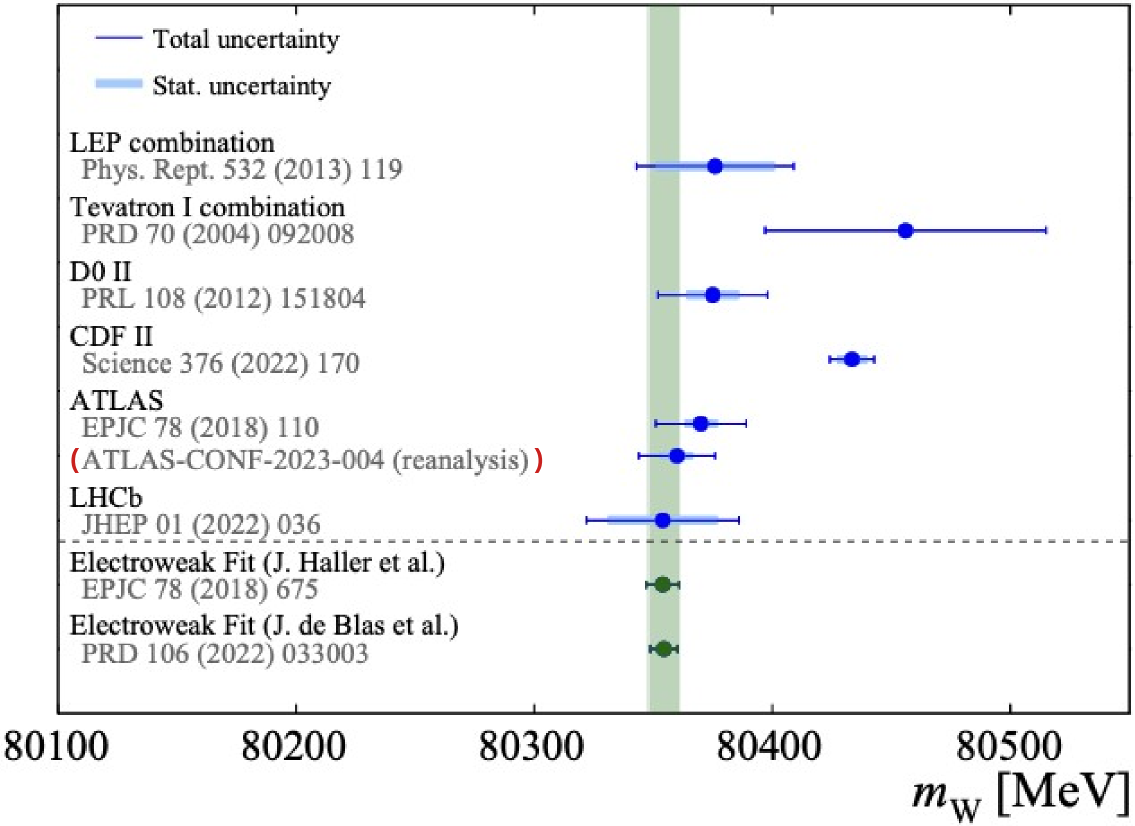
(2023)



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Science 376 (2022) 170
ATLAS
EPJC 78 (2018) 110
(ATLAS-CONF-2023-004 (reanalysis))
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80100
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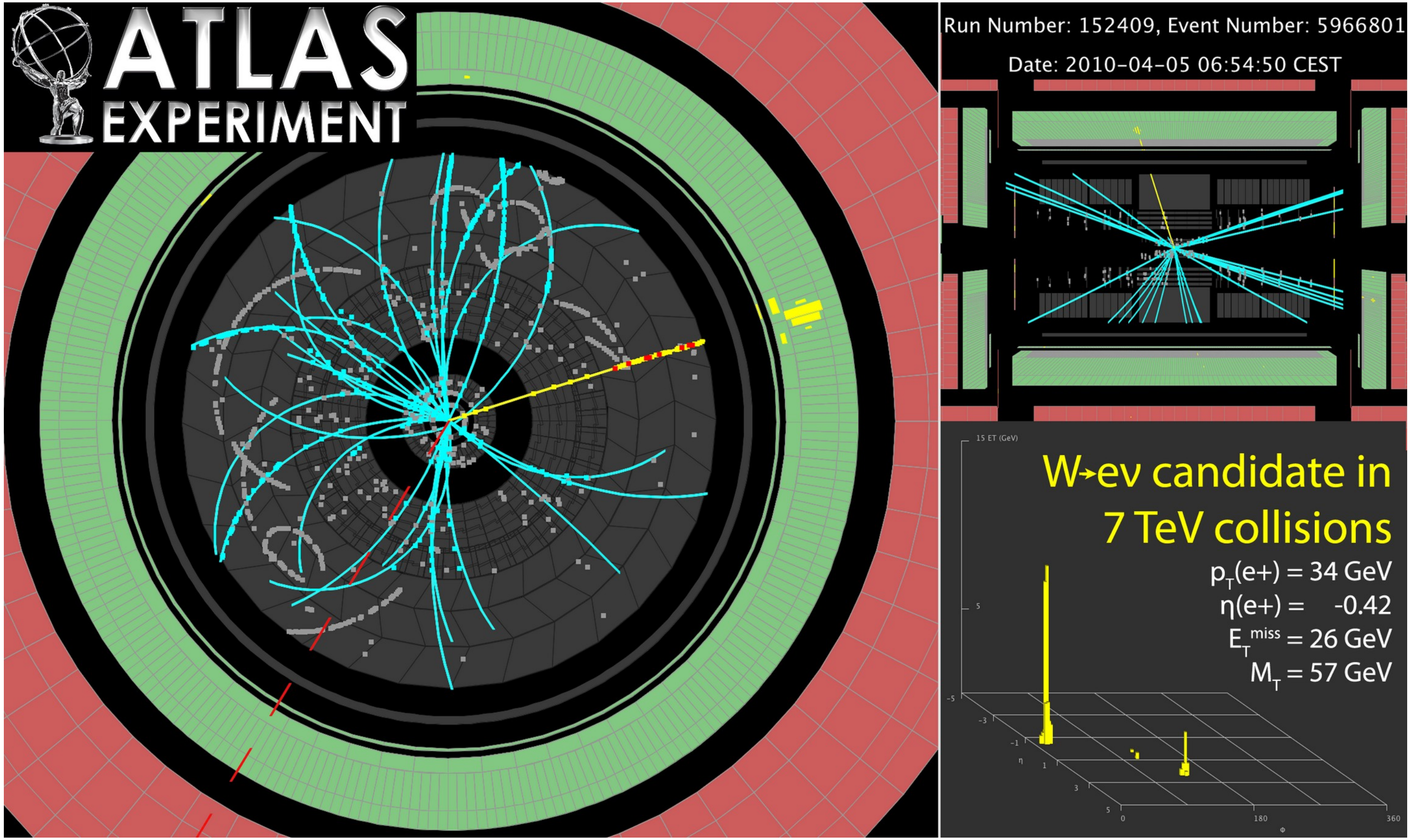
m_W [MeV]

The W boson mass in proton collisions



The W boson mass in proton collisions

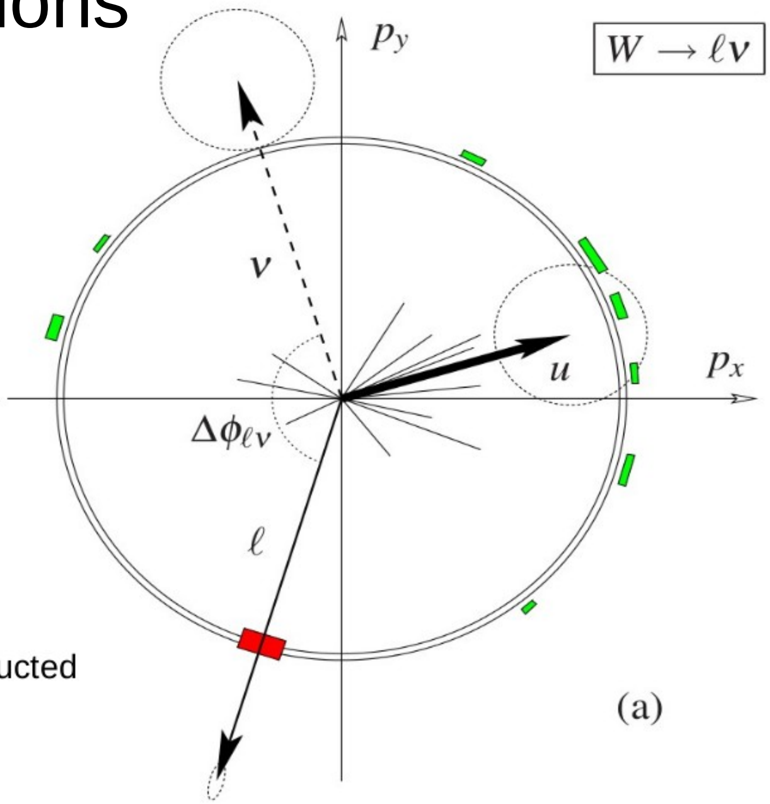
- Incomplete kinematics (missing neutrino!)
  → no invariant mass
  → rely on measured quantities, and exploit momentum conservation in the transverse plane

- Event representation :

  - Main signature :
    single electron or muon $\vec{p}_T^{\,l}$ p_T^l

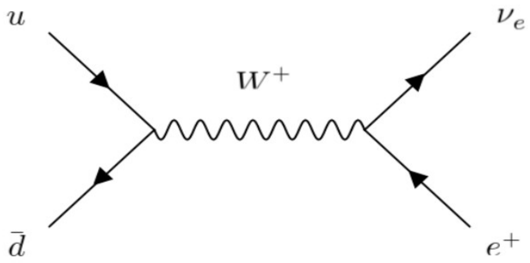
  - Recoil : sum of "everything else" reconstructed in the calorimeters; a measure of $p_T^{W,Z}$
    $$\vec{u}_T = \sum_i \vec{E}_{T,i}$$

  - Derived quantities : $\vec{p}_T^{\,miss} = -\left(\vec{p}_T^{\,\ell} + \vec{u}_T\right)$ $m_T = \sqrt{2 p_T^{\ell} p_T^{miss} (1 - \cos\Delta\phi)}$



The W boson mass in proton collisions

- The process at leading order, no width :



$$\hat{\sigma}_{u\bar{d}\to\ell^+\nu} = \frac{1}{3}\frac{|V_{ud}|^2}{3\pi}\left(\frac{G_F m_W^2}{\sqrt{2}}\right)^2 \delta(m^2 - m_W^2)$$

Unpolarized differential cross section (spin 1) :

$$\frac{d\hat{\sigma}_{u\bar{d}\to\ell^+\nu}}{d\cos\theta} \propto 1 + \cos^2\theta$$

$$p_T^\ell = \frac{m_W}{2}\sin\theta$$

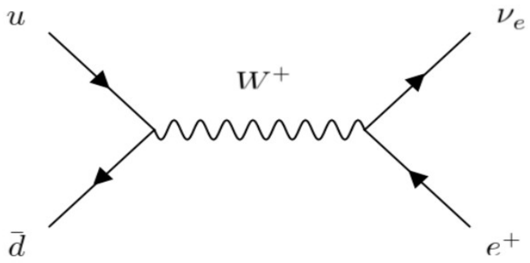
→

$$\frac{d\hat{\sigma}_{u\bar{d}\to\ell^+\nu}}{dp_T^\ell} \propto \frac{\left(1 - \frac{2p_T^\ell}{m_W^2}\right)}{\sqrt{1 - \frac{4p_T^\ell}{m_W^2}}}$$

→ the “Jacobian peak”

The W boson mass in proton collisions

- The process at leading order, no width :



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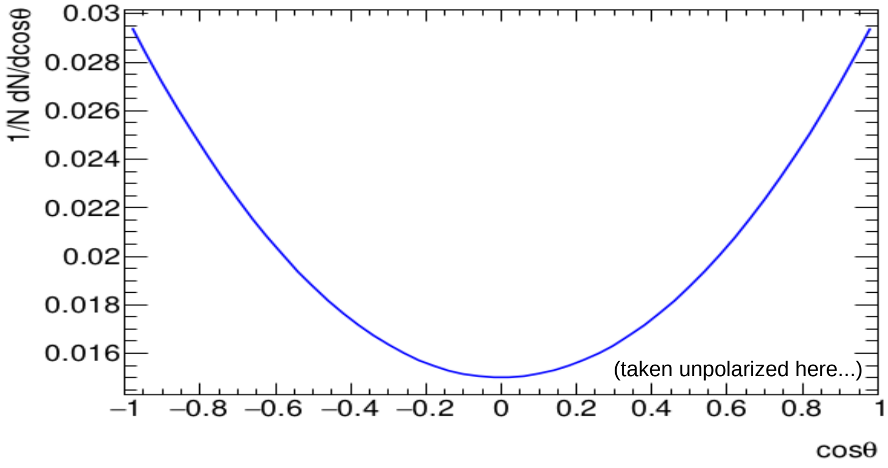
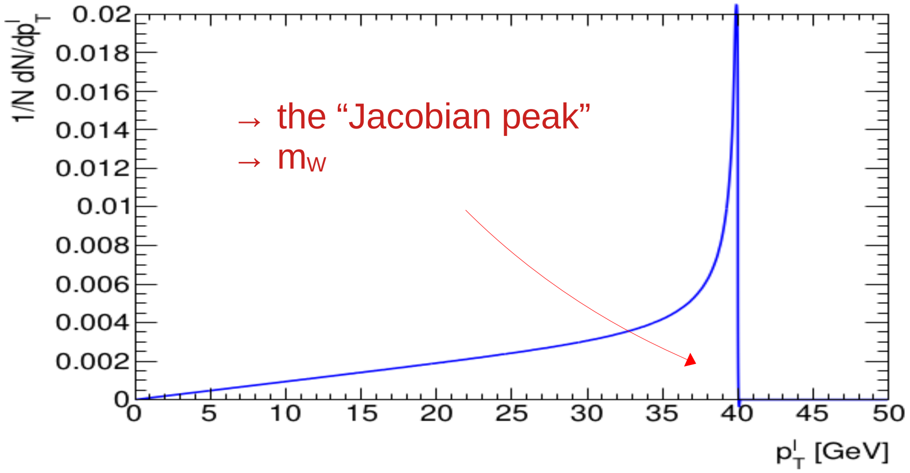
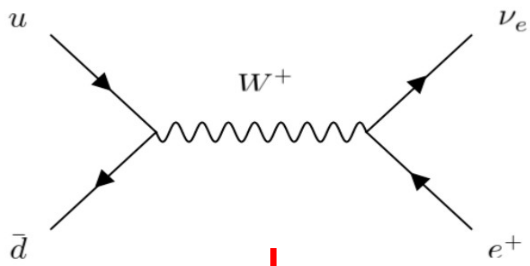


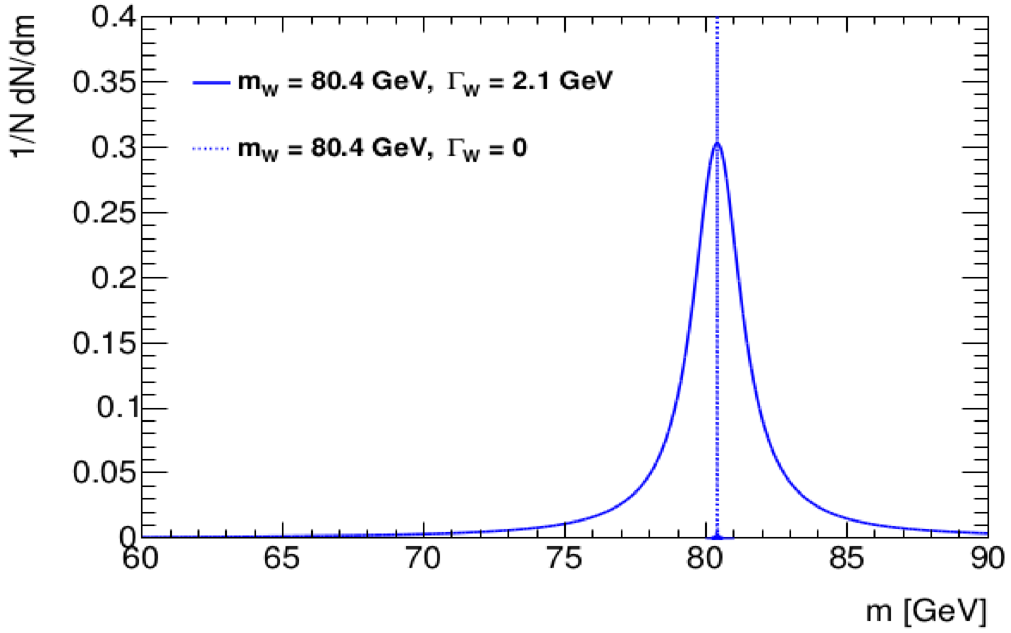


The W boson mass in proton collisions

- Natural width :



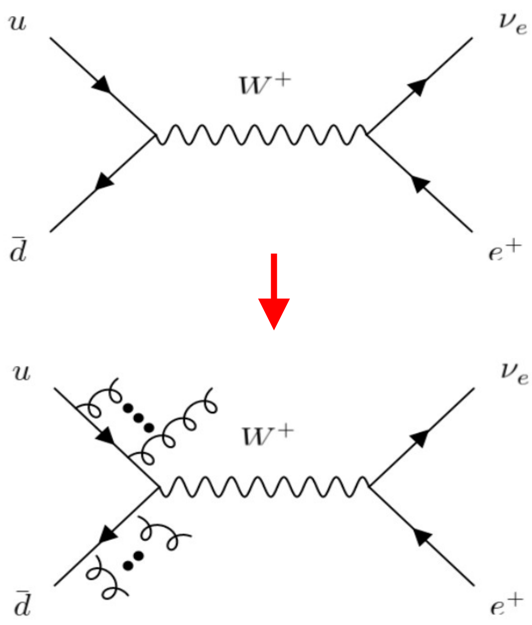
$$\delta(m^2 - m_W^2) \rightarrow \frac{m^2}{(m^2 - m_W^2)^2 + (m\Gamma_W/m_W)^2}$$

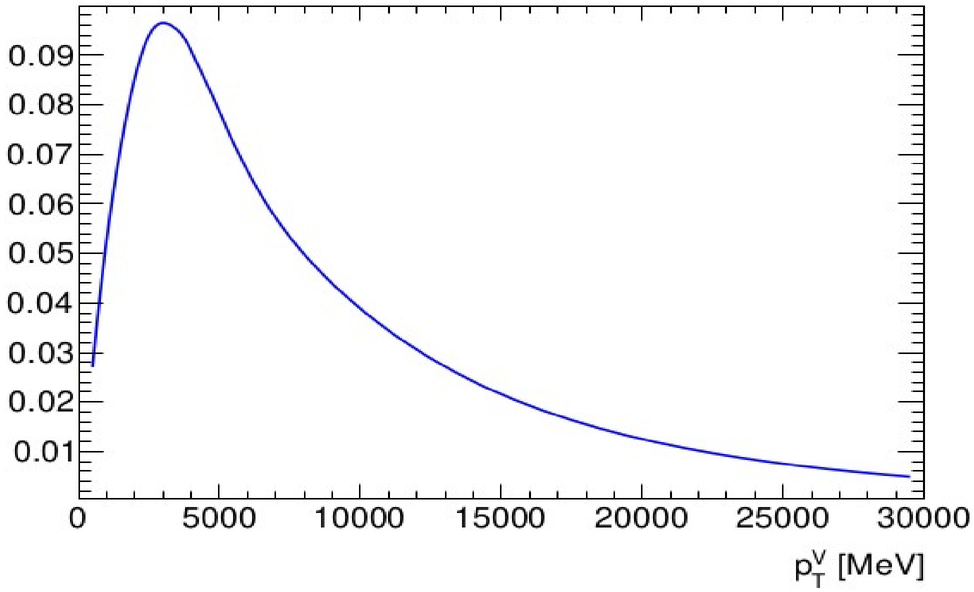


The W boson mass in proton collisions

- Radiation in the initial state (QCD)

  → non trivial transverse momentum distribution

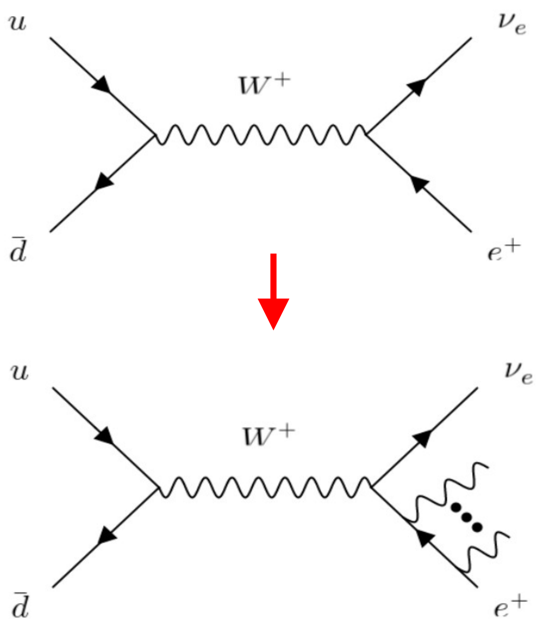


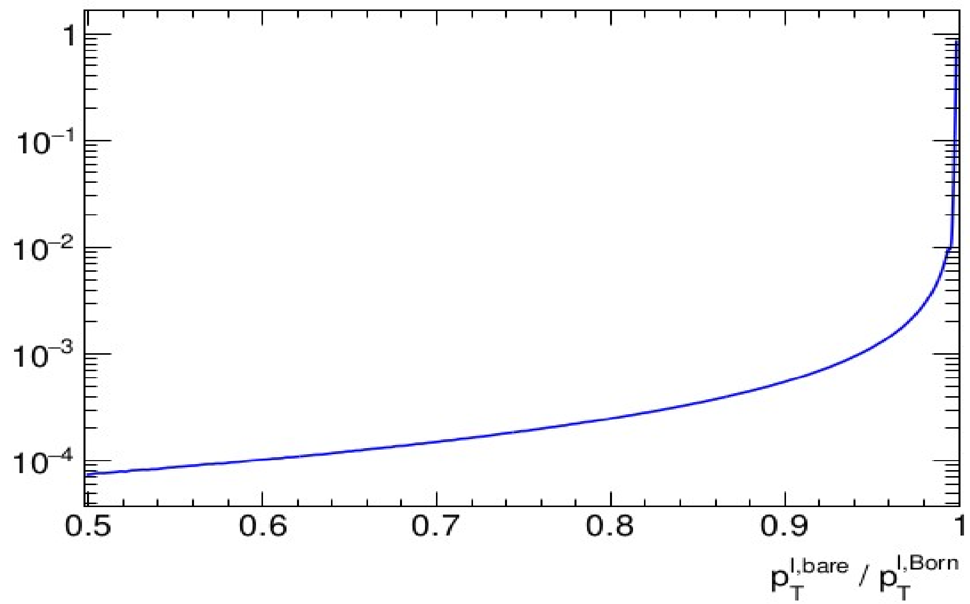


The W boson mass in proton collisions

Radiation in the final state (QED)

→ decays leptons lose a fraction of their energy

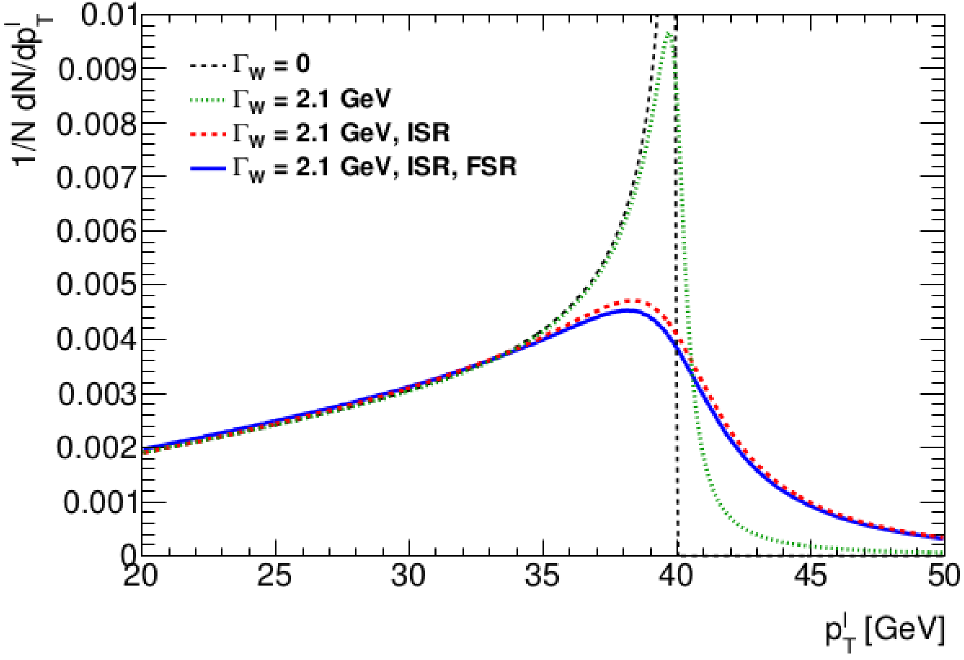


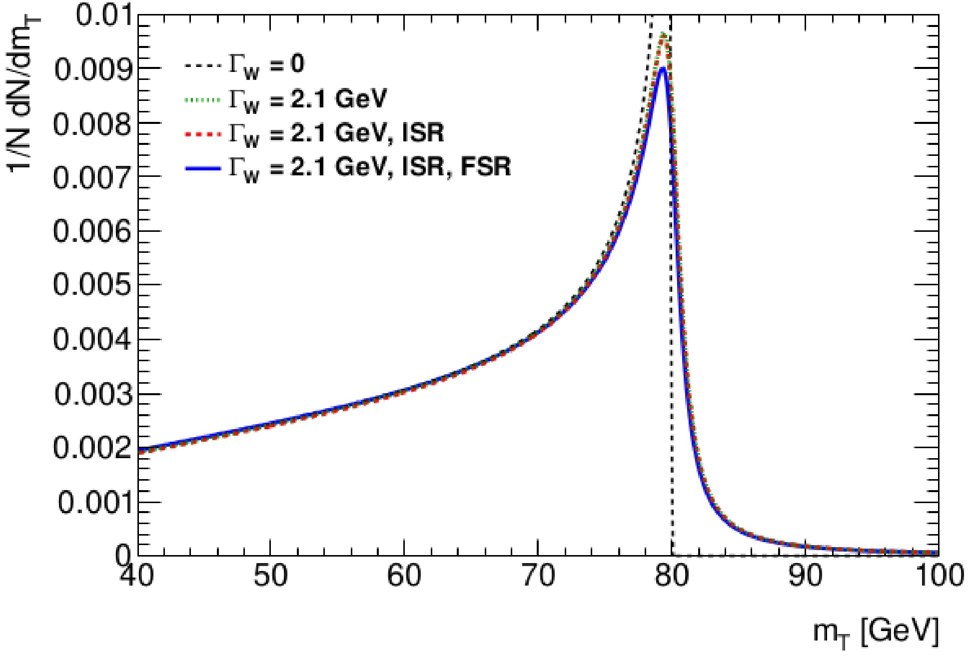


The W boson mass in proton collisions

- Summary of physics effects

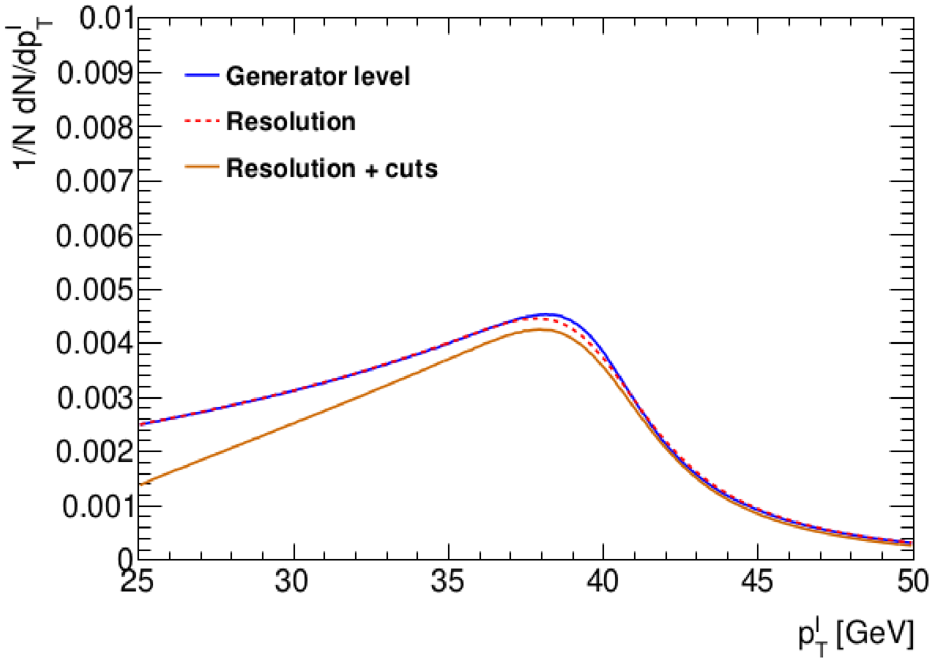
  → all carry uncertainties to be quantified!

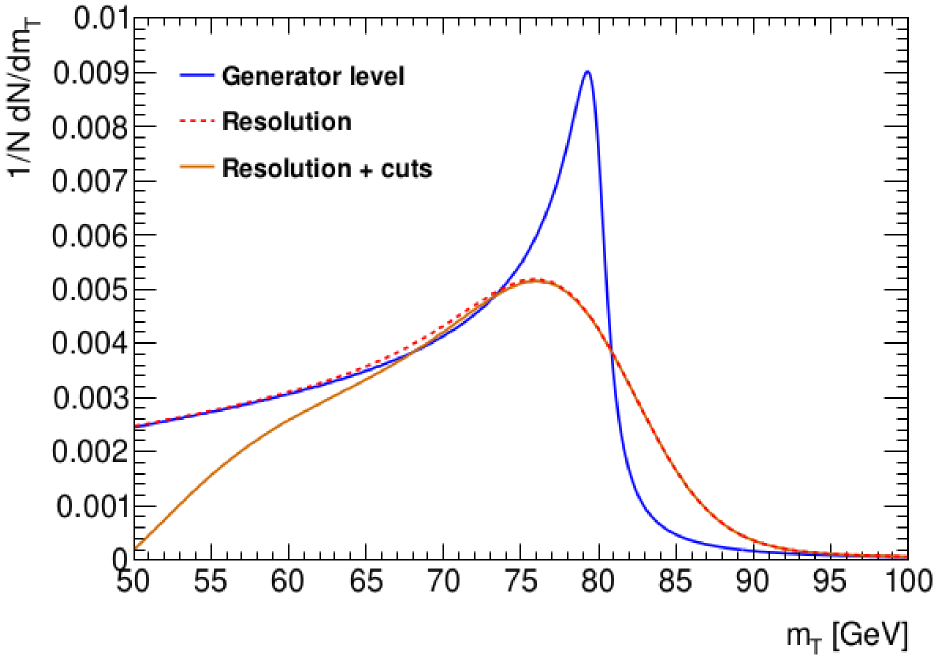




The W boson mass in proton collisions

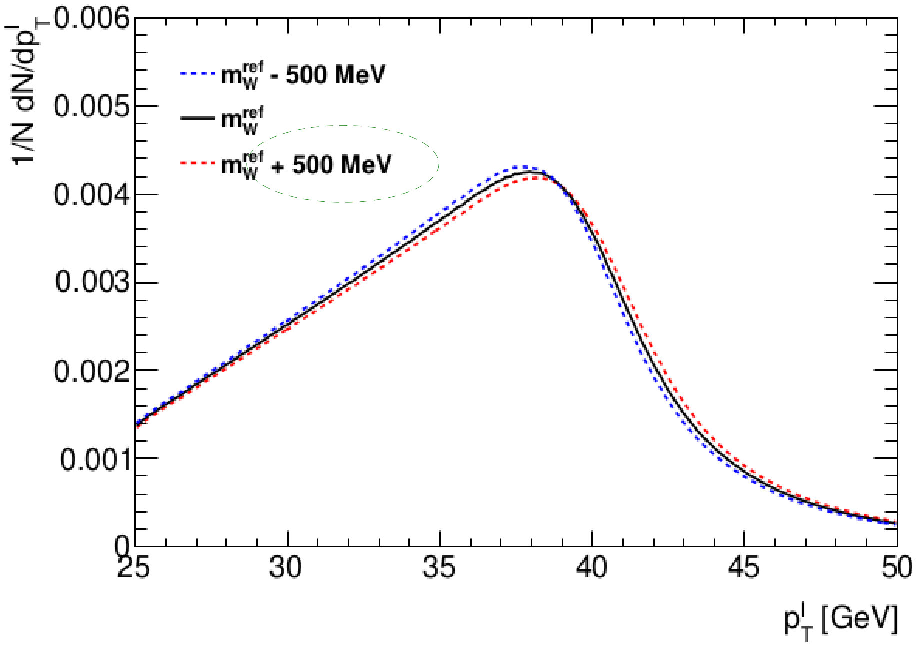
- Detector effects, also with uncertainties :
  - Lepton calibration and resolution;   Missing E_T resolution ~ 5 – 15 GeV
  - Efficiencies and acceptance ~15% (with non-trivial kinematic dependence!)

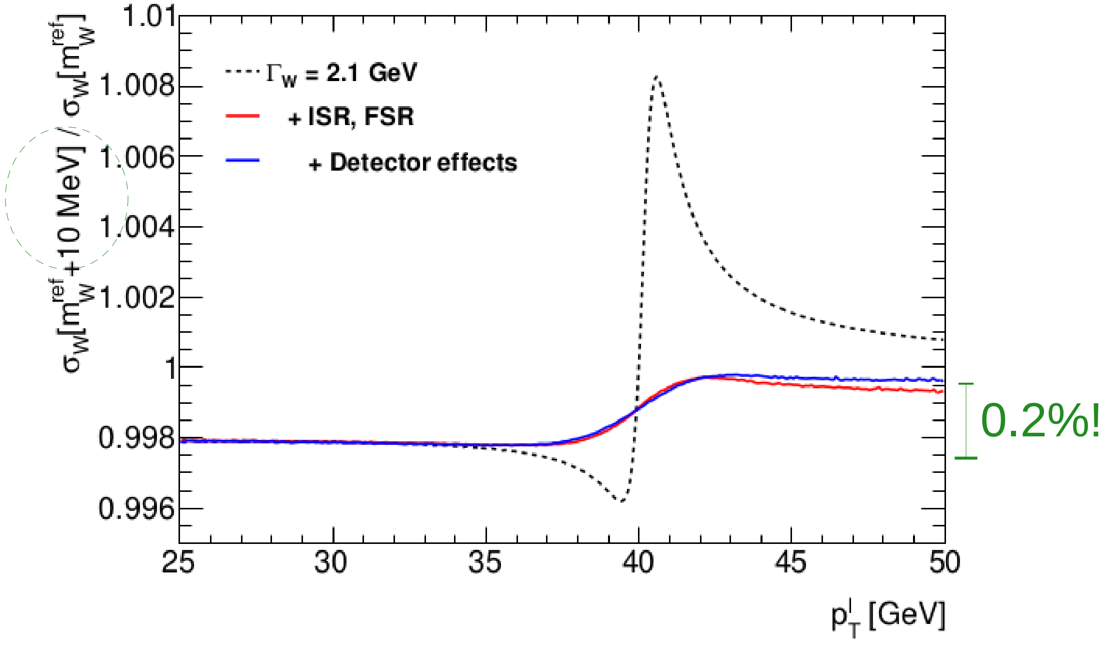




The W boson mass in proton collisions

- Mass measurement : produce models ("templates") of the final state distributions for different mass hypotheses; compare to data





Three slides on calibration

- The Z boson mass is perfectly well know on this scale of precision, so can be used to calibrate the absolute scale of the momentum measurements

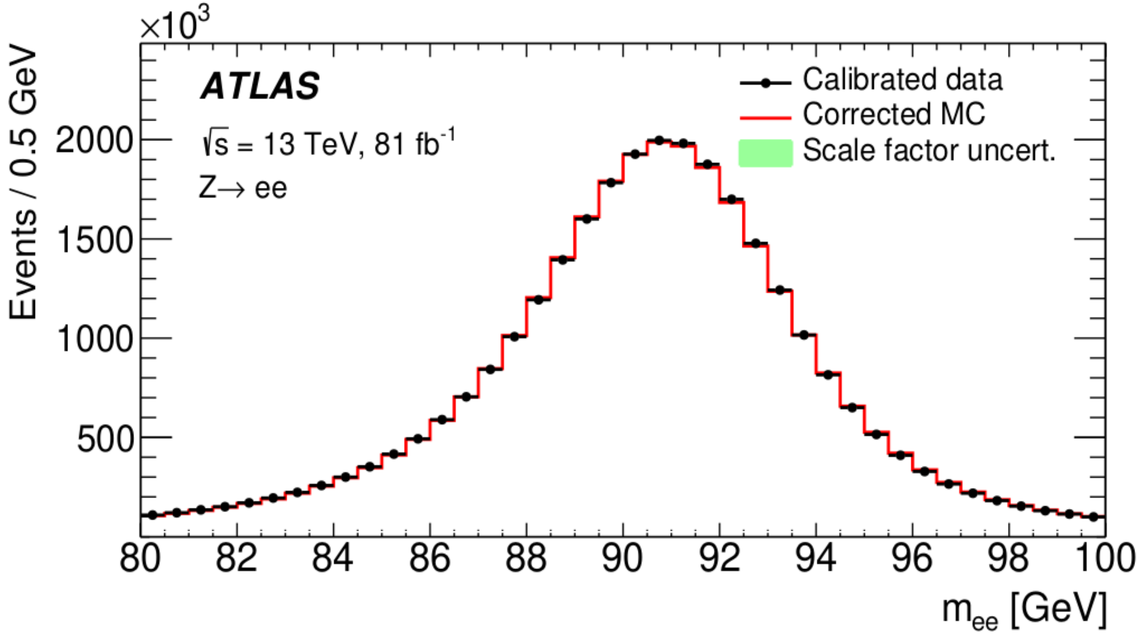
- Detector response derived from first principles to ~0.5% for calorimeters, ~0.05% for tracking detectors.

  ~0.01% is required here

- m_Z is known to ~0.002%,

  $m_{J/psi}$ to ~ 10^{-6}

  → used for final adjustments



Three slides on calibration

- Leptons calibration from “perfectly known” resonances

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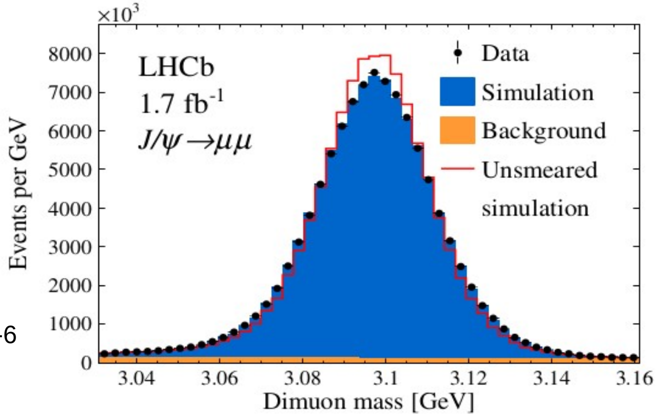
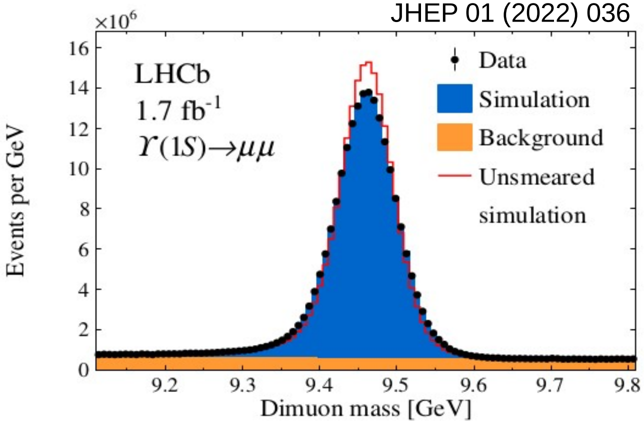
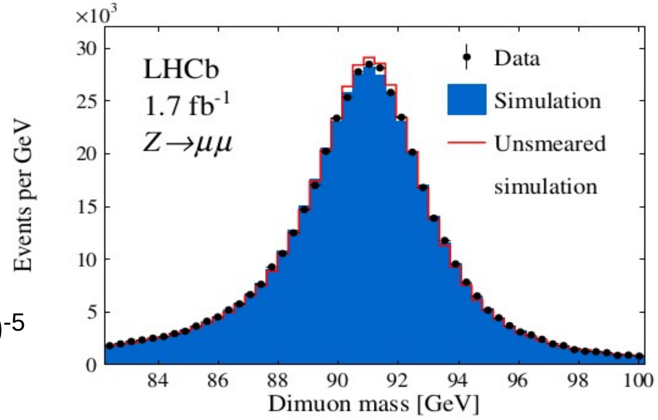




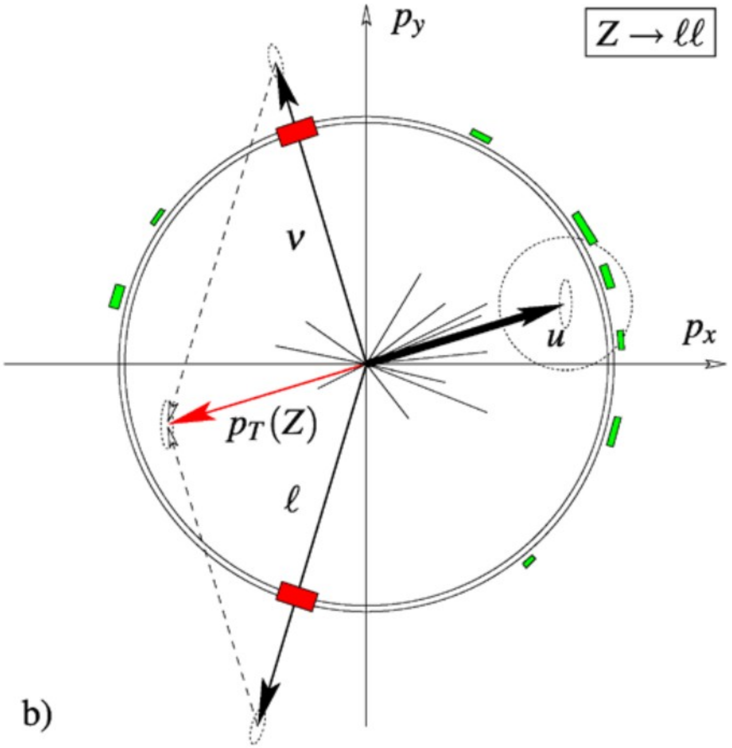
$\delta m_{J/psi} / m_{J/psi} \sim 10^{-6}$

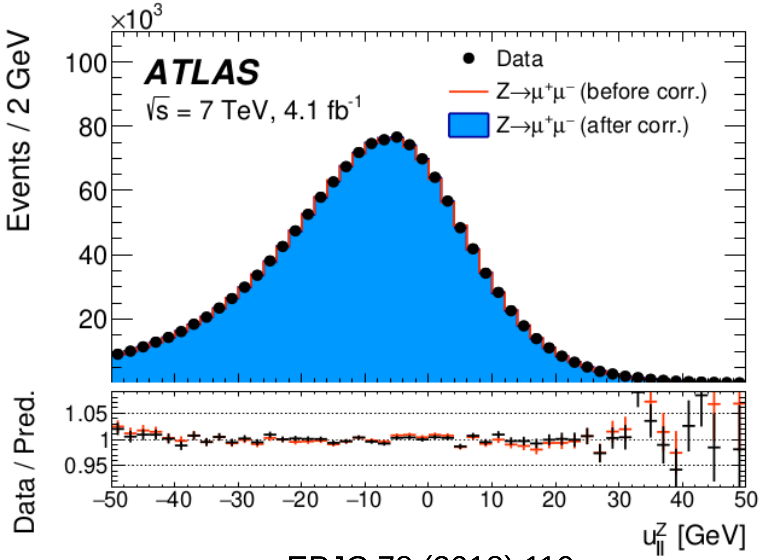


$\delta m_Z / m_Z \sim 2.10^{-5}$

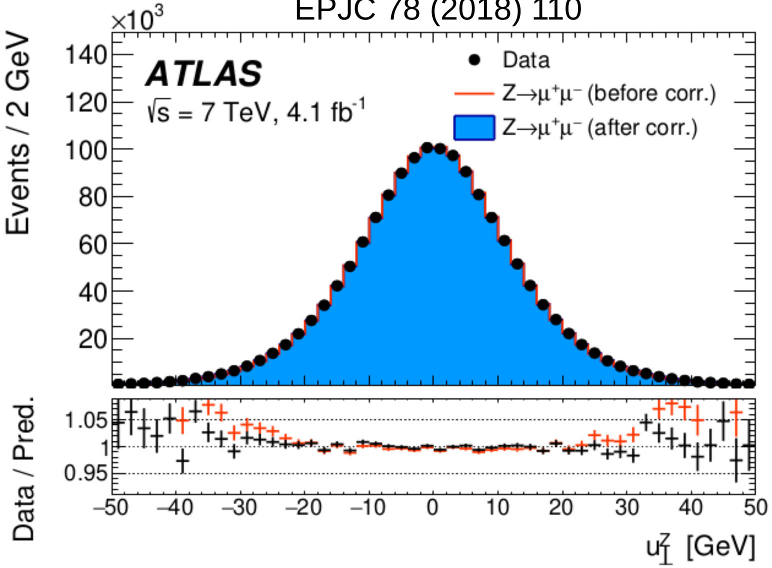
Three slides on calibration

- Recoil response & resolution calibrated using over-constrained kinematics in Z events





EPJC 78 (2018) 110



Vector-boson production at the LHC

- The magic formula, true to all orders in QCD:

$$\frac{d^5\sigma}{dp_1\,dp_2} = \frac{d^3\sigma}{dm\,dy\,dp_T}\left[\left(1+\cos^2\theta\right)+\sum_i A_i(p_T,y)\,f_i(\theta,\phi)\right]$$

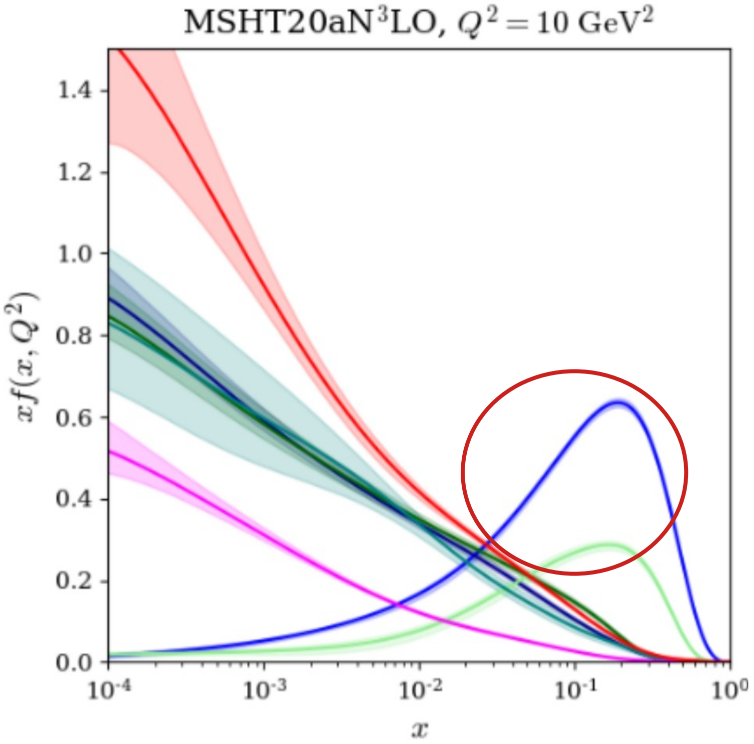
production | decay

Boson kinematics

Lepton angular distributions

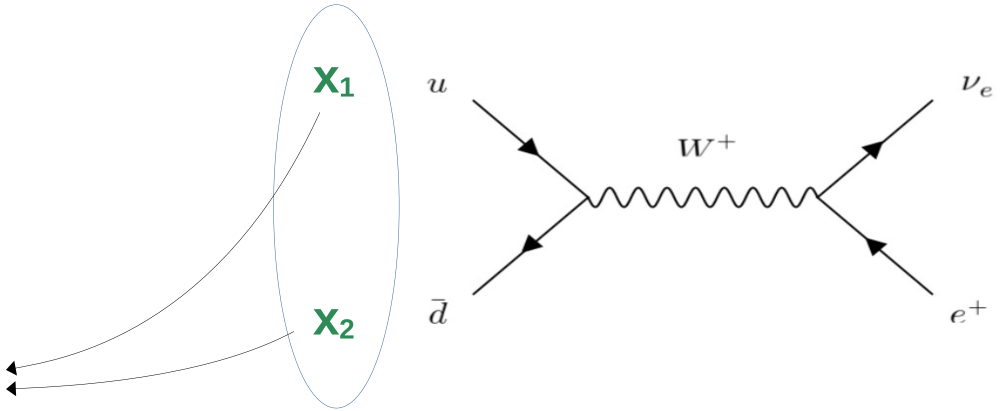
  - Not implemented in this way in generators (which evaluate matrix elements and PDFs) but useful to factor the different QCD modelling aspects, and describe each component using the most appropriate tool

Rapidity distribution and PDFs



m_W ~ 80 GeV $\quad$ $x_{1,2} = m/\sqrt{s}\, e^{\pm y}$

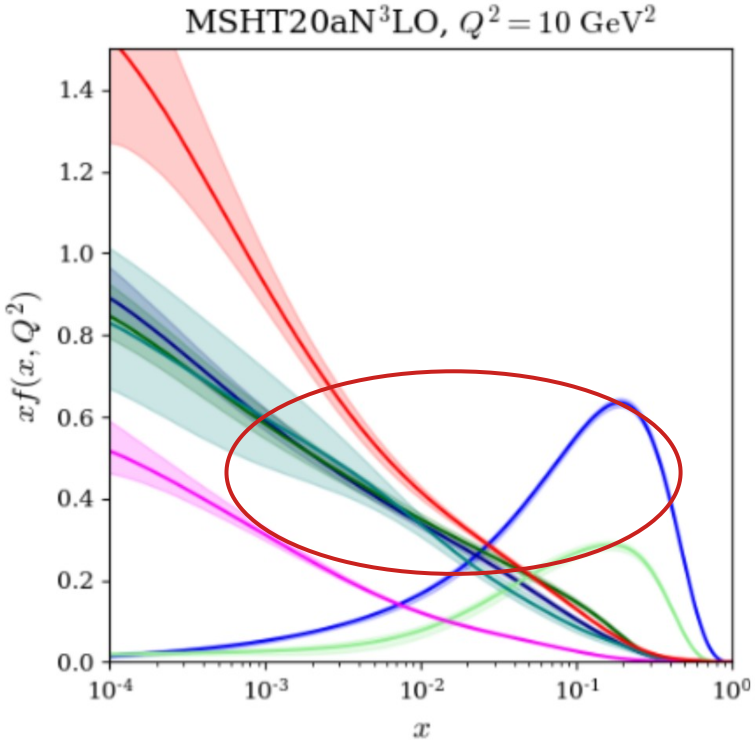
Tevatron	$\sqrt{s}$~ 2TeV	$p\bar{p}$	0<y<2	$x_{1,2} \sim 10^{-2} - 10^{-1}$
ATLAS	$\sqrt{s}$~ 7TeV	pp	0<y<3	$x_{1,2} \sim 10^{-3} - 10^{-1}$
LHCb	$\sqrt{s}$~13TeV	pp	y~4	$x_{1,2} \sim 10^{-4} - 10^{-1}$

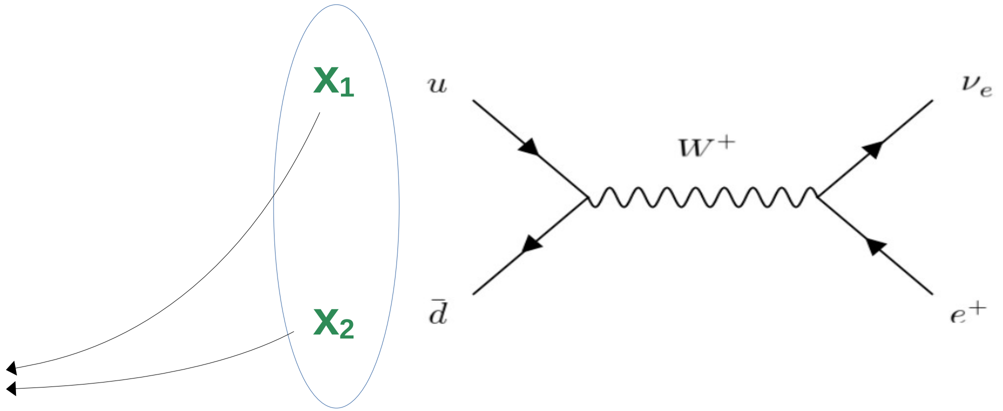


Rapidity distribution and PDFs

$m_W \sim 80$ GeV $\quad x_{1,2} = m/\sqrt{s}\, e^{\pm y}$

Tevatron	$\sqrt{s} \sim 2$TeV	$p\bar{p}$	$0<y<2$	$x_{1,2} \sim 10^{-2} – 10^{-1}$
ATLAS	$\sqrt{s} \sim 7$TeV	pp	$0<y<3$	$x_{1,2} \sim 10^{-3} – 10^{-1}$
LHCb	$\sqrt{s} \sim 13$TeV	pp	$y \sim 4$	$x_{1,2} \sim 10^{-4} – 10^{-1}$

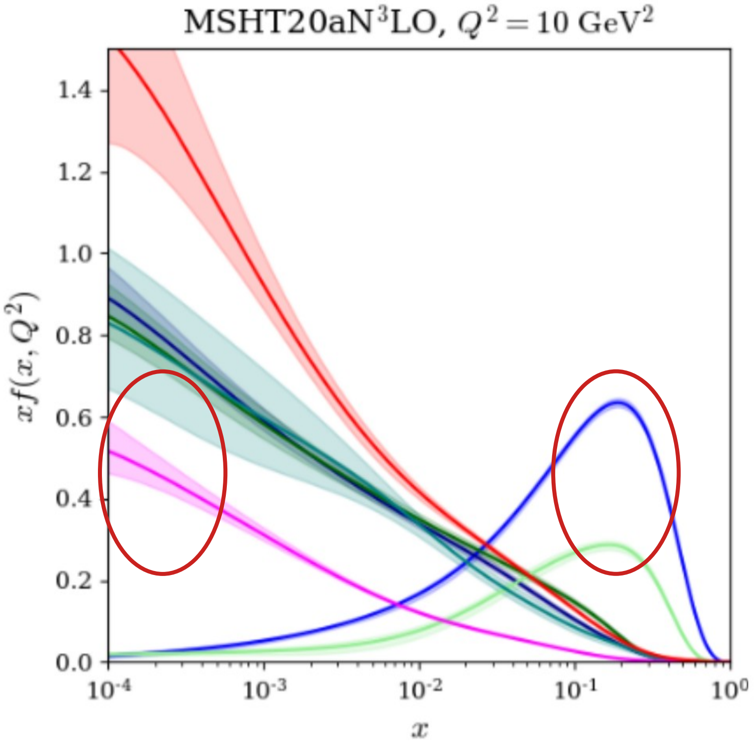


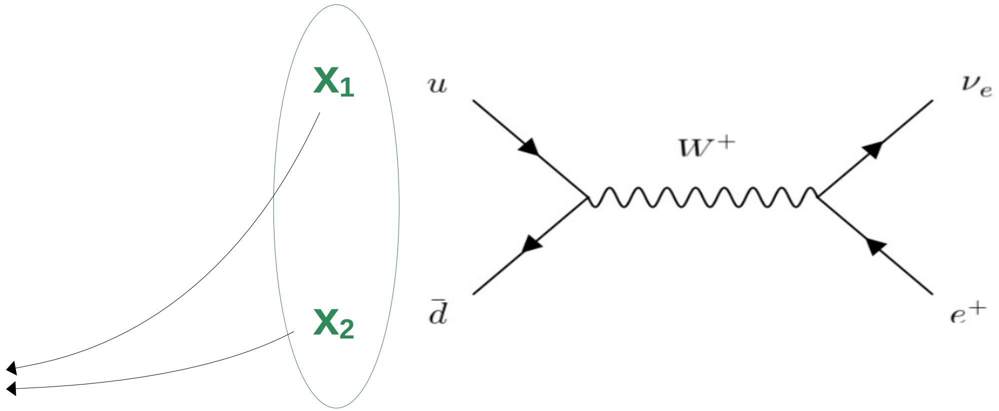


Rapidity distribution and PDFs

m_W ~ 80 GeV $\quad$ $x_{1,2} = m/\sqrt{s}\, e^{\pm y}$

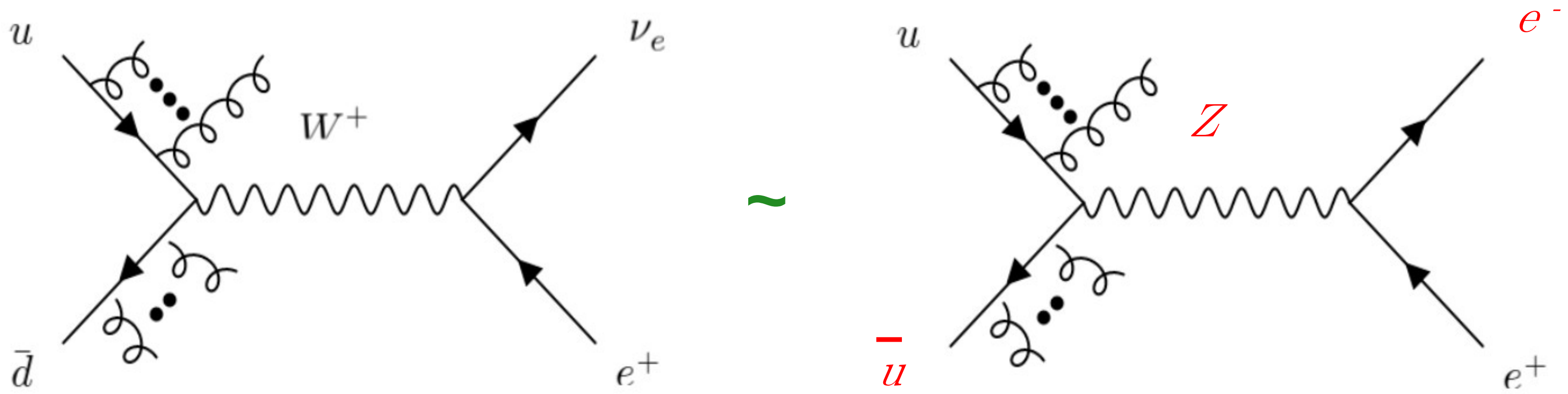
Tevatron	√s~ 2TeV	$p\bar{p}$	0<y<2	$x_{1,2}$ ~ 10^{-2} – 10^{-1}
ATLAS	√s~ 7TeV	pp	0<y<3	$x_{1,2}$ ~ 10^{-3} – 10^{-1}
LHCb	√s~13TeV	pp	y~4	$x_{1,2}$ ~ 10^{-4} – 10^{-1}





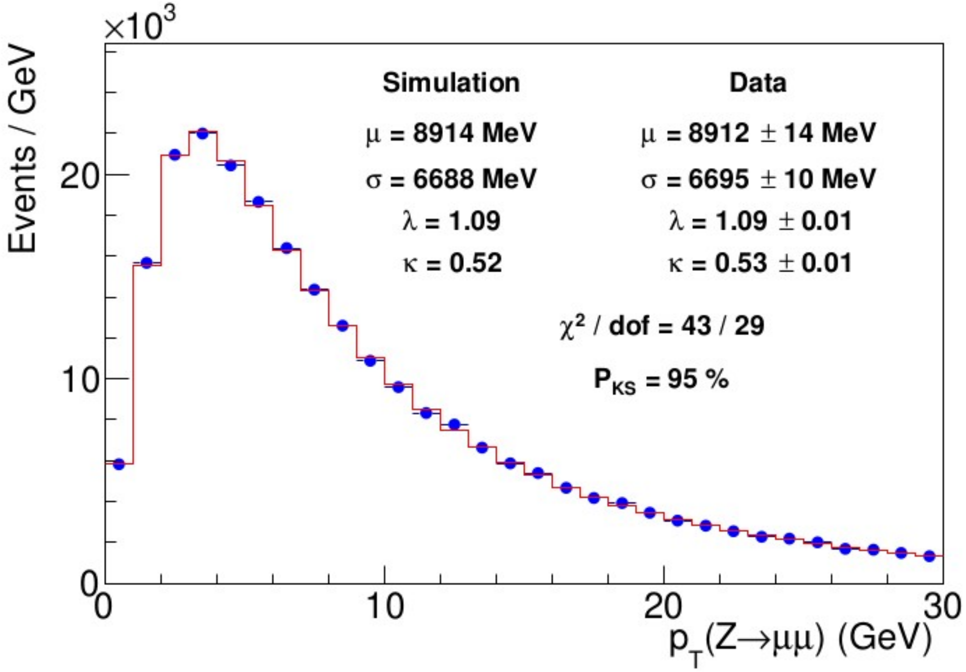
Transverse momentum distribution

- Initial state radiation involves large corrections, and is in part non-perturbative. W events are only partly measured (neutrino!)
- Approach : adjust model parameters using Z events, which are close to W's and can be measured precisely; extrapolate to W production



Transverse momentum distribution

- **Tevatron** : Z-based model tuning (Resbos); no extrapolation uncertainties, but validation with W events



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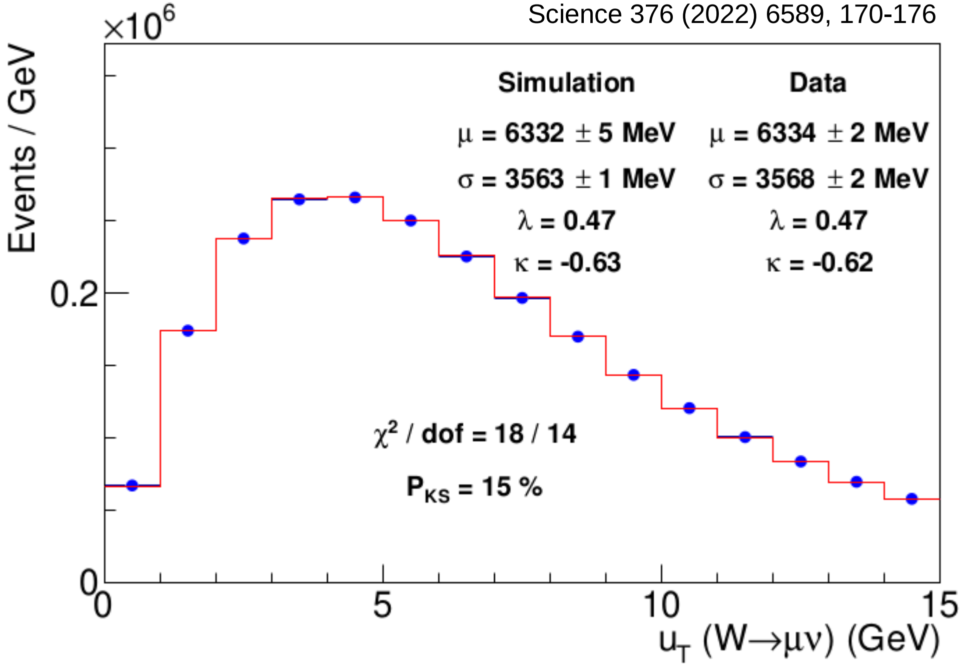
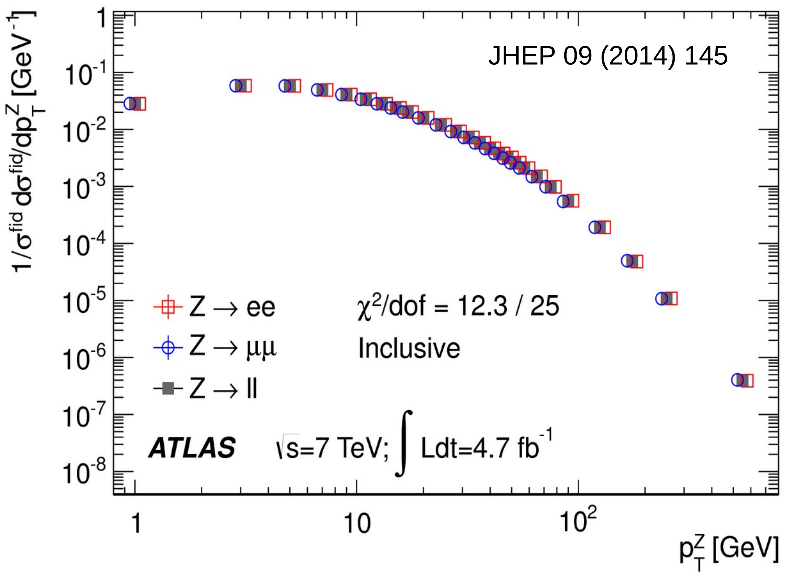
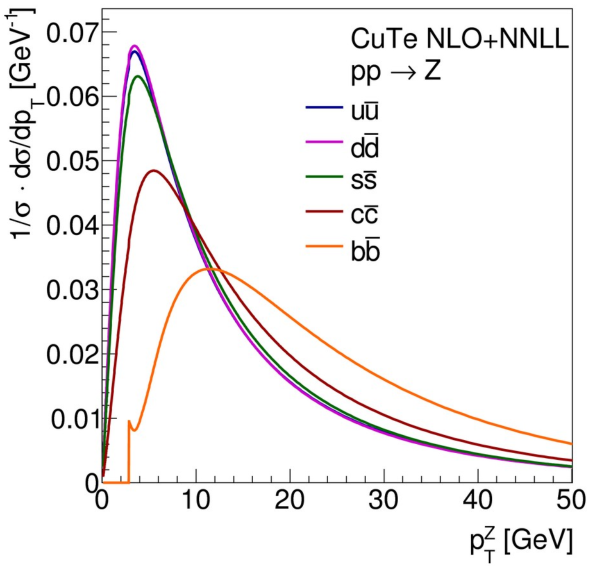


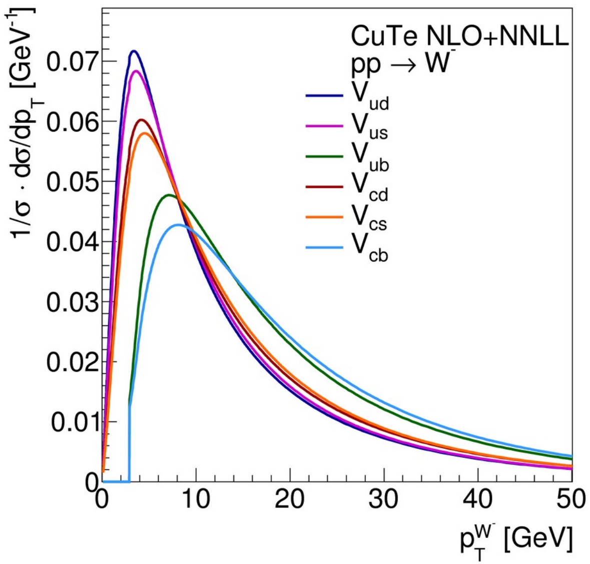
Transverse momentum distribution

- **ATLAS** : Z-based model tuning (Pythia) + Z→W extrapolation
- Corresponding uncertainties :
  - HQ mass treatment in showers and resummation
  - HQ PDFs

Measurement precision ~0.5%

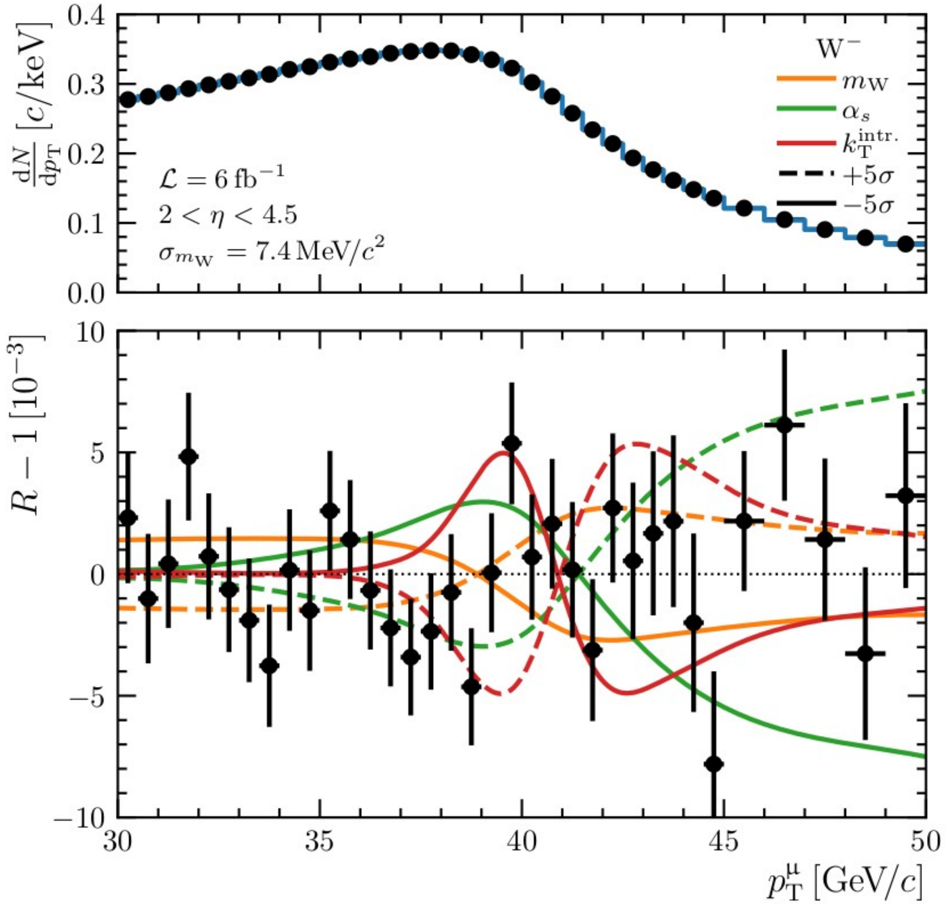






Transverse momentum distribution

- **LHCb** :
  - Z data
  - simultaneous fits to mW and pTW in W events
  - repeated for different theoretical models



After all is said and done...

- CDF, D0

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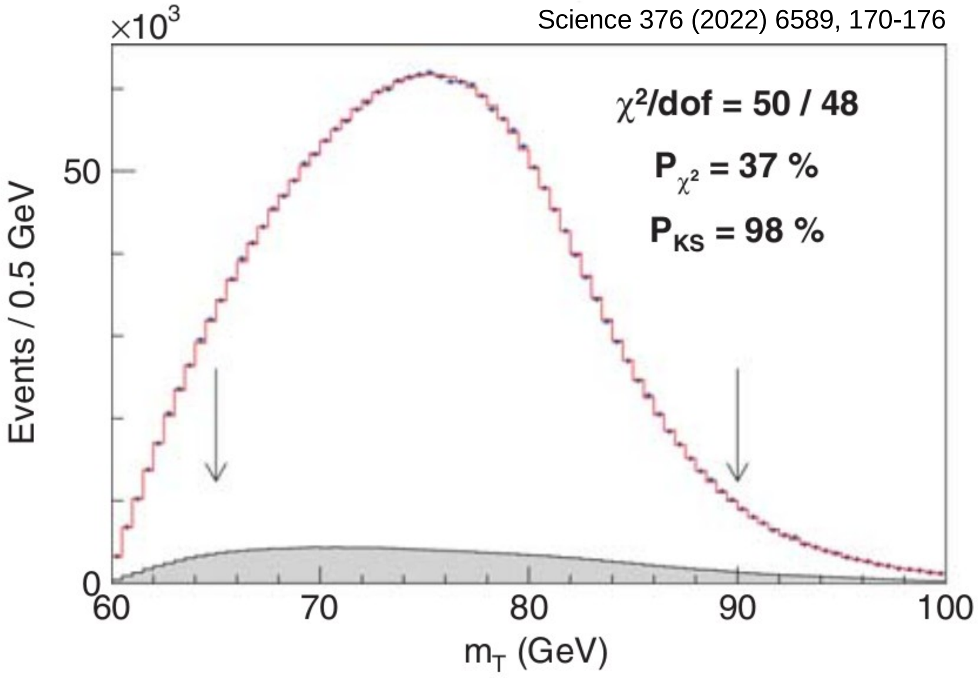

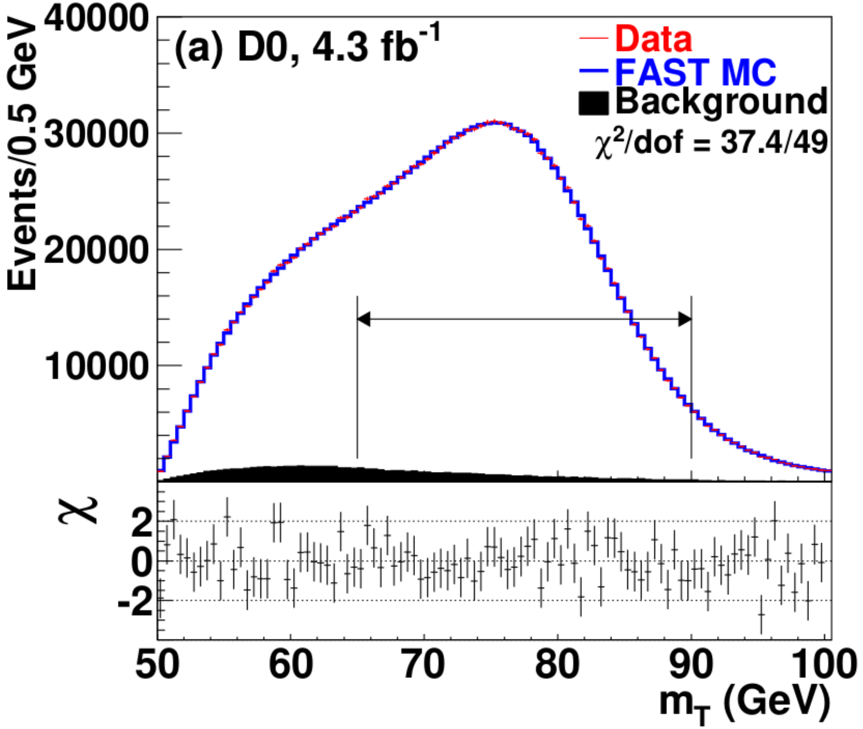

After all is said and done...

- ATLAS

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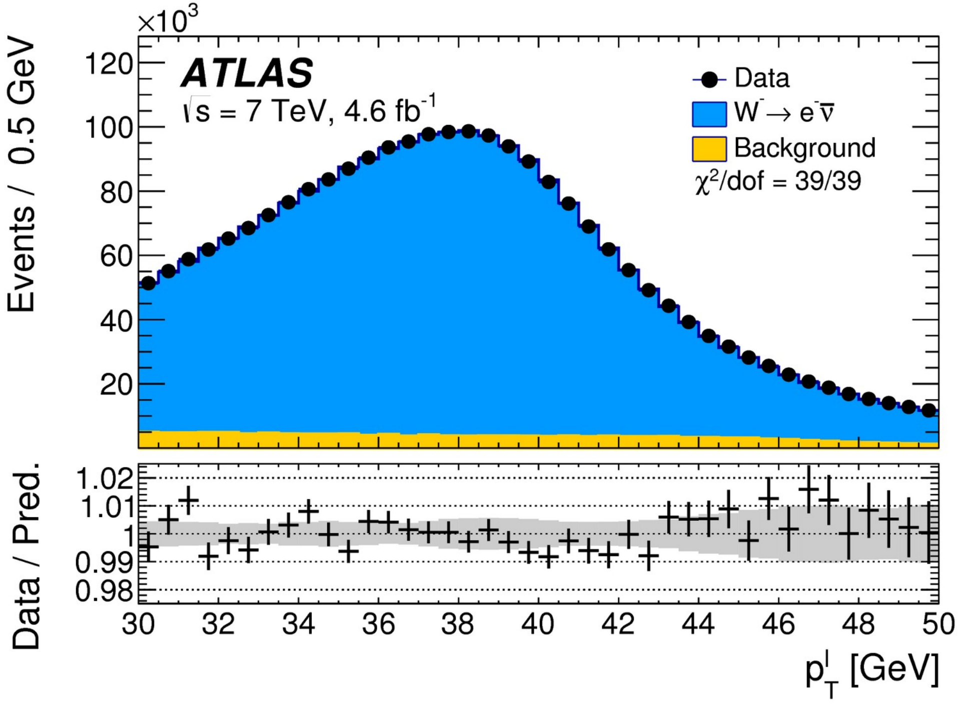
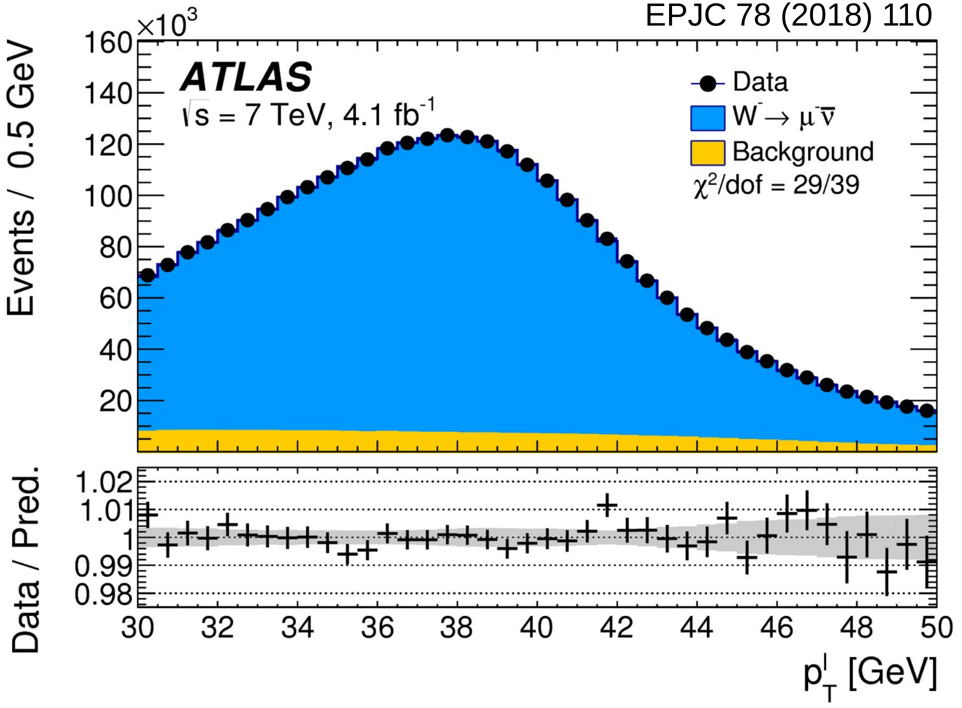
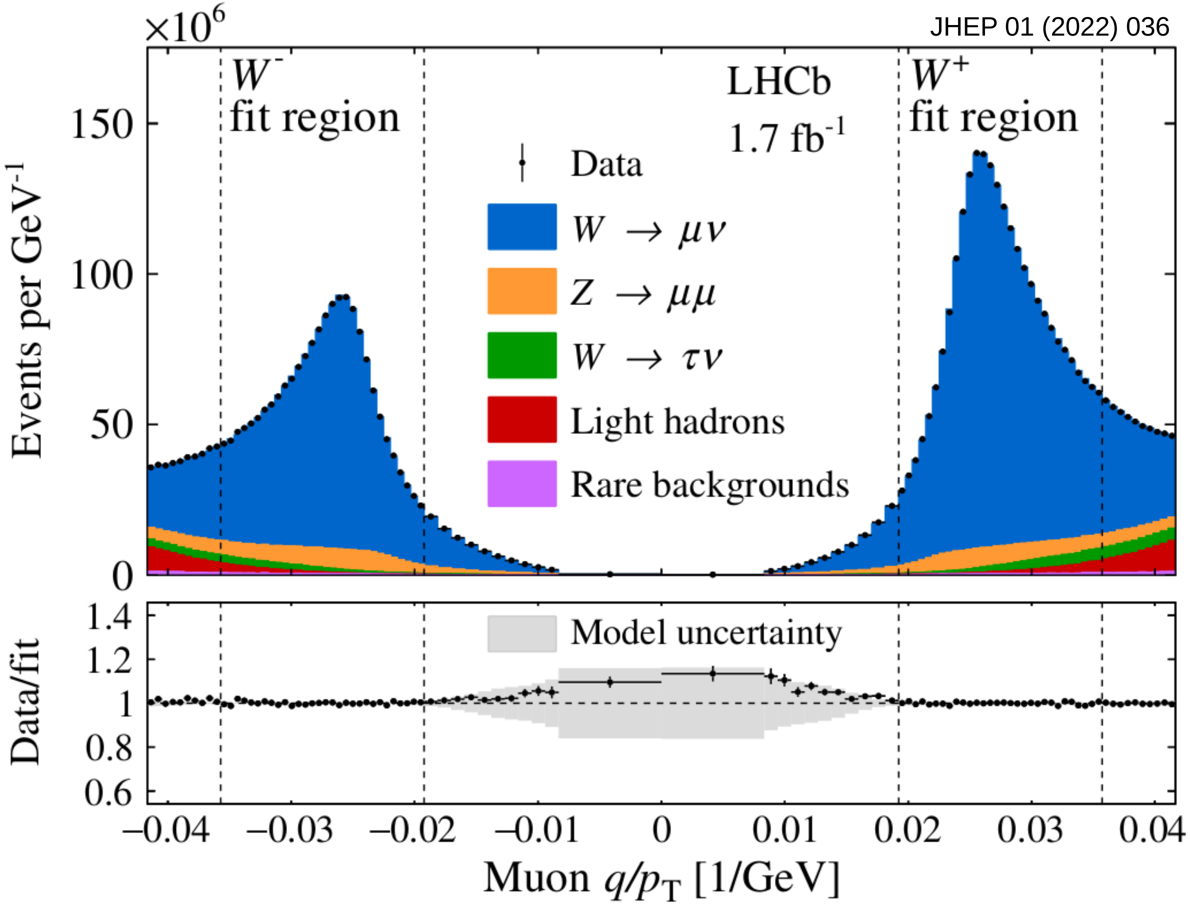




After all is said and done...

- LHCb



Experimental situation

- Last measurements:
  - D0 2013
    - m_W = 80375 ± 11 (stat.) ± 11 (exp.) ± 10 (theory) ± 10 (PDF)
  - ATLAS 2017

    m_W = 80370 ± 7 (stat.) ± 11 (exp.) ± 10 (theory) ± 9 (PDF)
  - LHCb 2021

    m_W = 80354 ± 23 (stat.) ± 10 (exp.) ± 17 (theory) ± 9 (PDF)
  - CDF 2022

    m_W = 80433 ± 6.4 (stat.) ± 4.5 (exp.) ± 3.5 (theory) ± 3.9 (PDF)

Experimental situation

- Last measurements:
  - D0 2013
    - $m_W = 80375 \pm 11$ (stat.) $\pm$ 11 (exp.) $\pm$ 10 (theory) $\pm$ 10 (PDF)
  - ATLAS 2017

    $m_W = 80370 \pm$ 7 (stat.) $\pm$ 11 (exp.) $\pm$ 10 (theory) $\pm$ 9 (PDF)
  - LHCb 2021

    $m_W = 80354 \pm$ 23 (stat.) $\pm$ 10 (exp.) $\pm$ 17 (theory) $\pm$ 9 (PDF)
  - CDF 2022

    $m_W = 80433 \pm 6.4$ (stat.) $\pm$ 4.5 (exp.) $\pm$ 3.5 (theory) $\pm$ 3.9 (PDF)

Experimental situation

- Last measurements:
  - D0 2013
    - $m_W = 80375 \pm 11\ (\text{stat.}) \pm 11\ (\text{exp.}) \pm 10\ (\text{theory}) \pm 10\ (\text{PDF})$
  - ATLAS 2017

    $m_W = 80370 \pm 7\ (\text{stat.}) \pm 11\ (\text{exp.}) \pm 10\ (\text{theory}) \pm 9\ (\text{PDF})$
  - LHCb 2021

    $m_W = 80354 \pm 23\ (\text{stat.}) \pm 10\ (\text{exp.}) \pm 17\ (\text{theory}) \pm 9\ (\text{PDF})$
  - CDF 2022

    $m_W =$ **80433** $\pm 6.4\ (\text{stat.}) \pm 4.5\ (\text{exp.}) \pm 3.5\ (\text{theory}) \pm 3.9\ (\text{PDF})$

Experimental compatibility? Combination? (preview)

- Measurements performed at different times, using different baseline PDFs and QCD tools : “translate” existing result to common baseline
- Two-step procedure :
  - correct to common PDF & QCD accuracy
  - combination including correlations

m_W^{CDF}	m_W^{D0}	m_W^{ATLAS}	m_W^{LHCb}
CTEQ6M(*)	CTEQ6.6	CT10nnlo	CT18/NNPDF/MSHT20
↓	↓	↓	↓
δm_W^{CDF}	δm_W^{D0}	δm_W^{ATLAS}	δm_W^{LHCb}

Common baseline

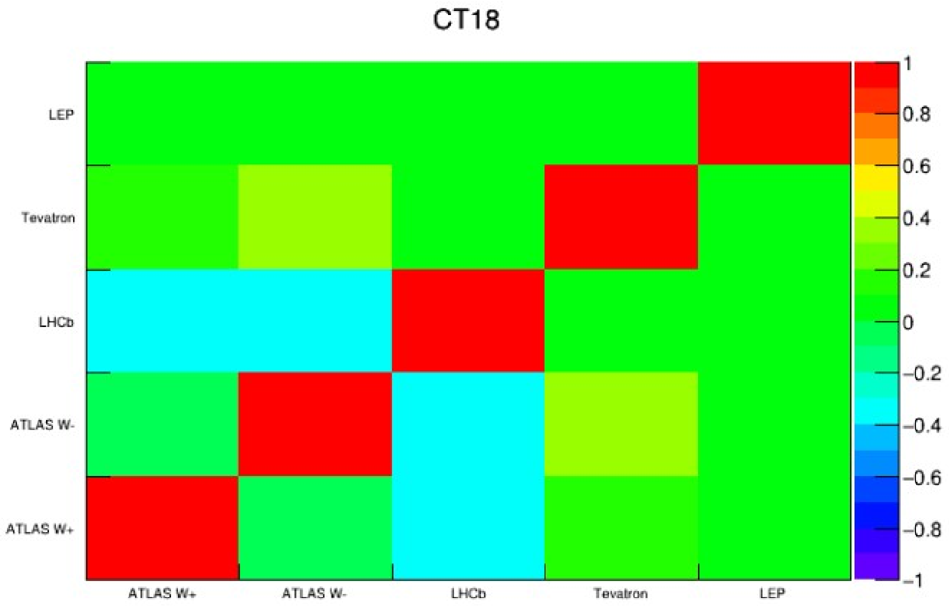
↓

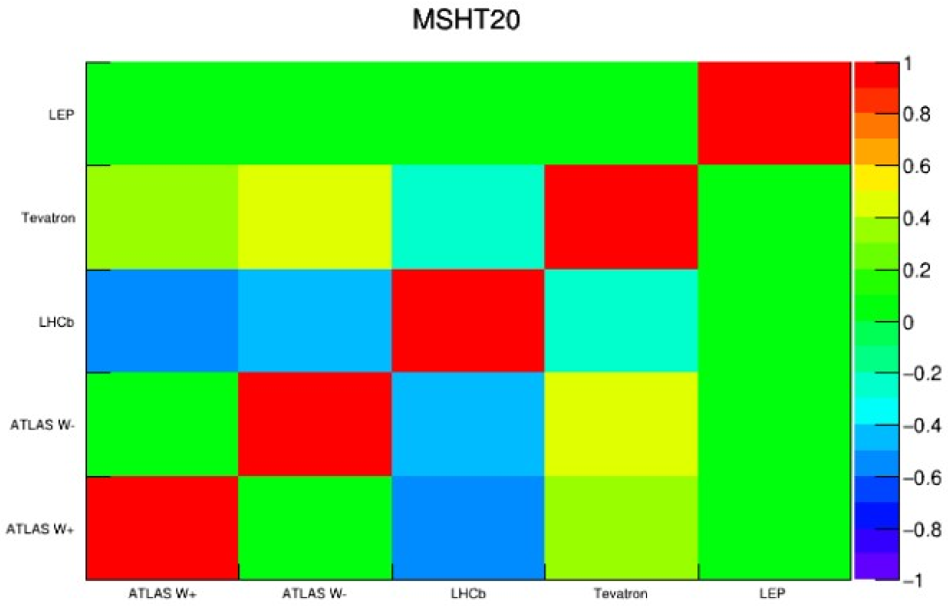
$m_W^{combined}$

… and repeat, for different PDFs

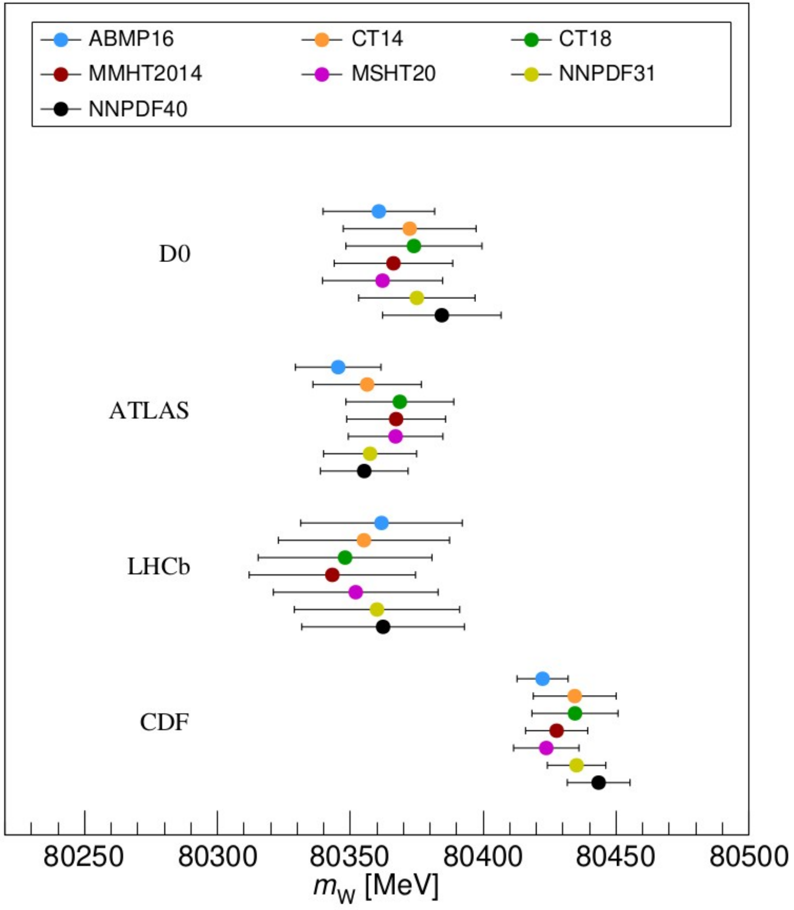
PDF correlations

- Non-trivial PDF correlations, with significant PDF model dependence!





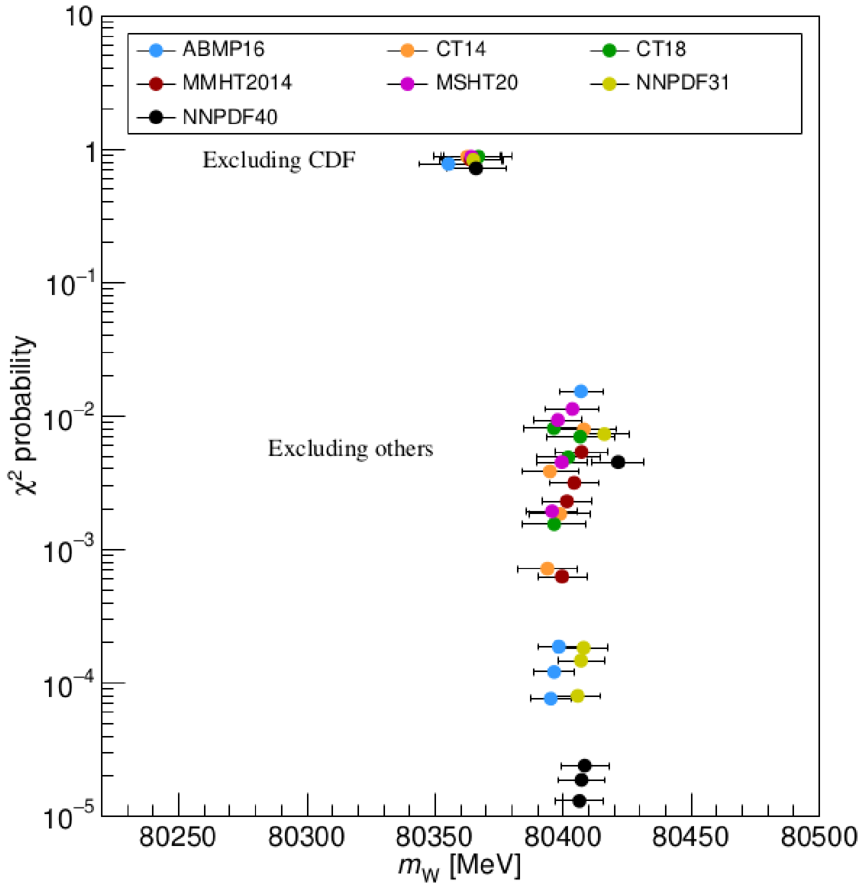
PDF dependence of individual measurements



TeVatron Summary	Value	Exp.	PDF	χ^2/N	Probability
ABMP16	80413.3 ± 9.1	8.2	4.0	8.7 / 2	0.0130
CT14	80426.4 ± 15.3	8.7	12.6	8.6 / 2	0.0135
CT18	80427.5 ± 16.0	8.5	13.6	8.3 / 2	0.0160
MMHT2014	80419.3 ± 11.3	8.2	7.8	8.6 / 2	0.0136
MSHT20	80415.3 ± 11.9	8.2	8.6	8.7 / 2	0.0130
NNPDF31	80426.9 ± 10.6	8.2	6.8	8.3 / 2	0.0159
NNPDF40	80435.2 ± 11.4	8.2	7.8	8.0 / 2	0.0181

LHC Summary	Value	Exp.	PDF	χ^2/N	Probability
ABMP16	80349.0 ± 14.2	13.9	2.9	0.2 / 1	0.6377
CT14	80355.9 ± 16.3	15.0	6.5	0.0 / 1	0.9760
CT18	80362.5 ± 16.6	15.3	6.3	0.3 / 1	0.6055
MMHT2014	80360.6 ± 15.4	14.5	5.1	0.4 / 1	0.5246
MSHT20	80363.1 ± 15.0	14.3	4.5	0.2 / 1	0.6843
NNPDF31	80358.0 ± 15.0	14.1	5.0	0.0 / 1	0.9430
NNPDF40	80356.8 ± 14.4	13.9	3.8	0.0 / 1	0.8392

Compatibility & combination



CT18	Value	Significance
ATLAS - Rest	-38.1 ± -23.2	1.6
CDF - Rest	67.6 ± 17.3	3.9
D0 - Rest	-23.9 ± -23.8	1.0
LEP - Rest	-20.5 ± -35.2	0.6
LHCb - Rest	-54.0 ± -35.3	1.5

All-CDF Summary	Value	Exp.	PDF	χ^2/N	Probability
ABMP16	80355.3 ± 11.2	10.9	2.6	1.8 / 4	0.7732
CT14	80362.6 ± 12.9	11.5	5.8	1.2 / 4	0.8841
CT18	80366.9 ± 13.3	11.8	6.2	1.2 / 4	0.8818
MMHT2014	80363.9 ± 12.1	11.2	4.7	1.5 / 4	0.8310
MSHT20	80364.2 ± 12.0	11.1	4.4	1.2 / 4	0.8824
NNPDF31	80364.9 ± 11.9	11.0	4.5	1.5 / 4	0.8280
NNPDF40	80366.0 ± 11.6	10.9	3.9	2.1 / 4	0.7103

Conclusions

- The W boson mass is arguably the most difficult measurement in HEP
  - Partial event reconstruction, incomplete kinematics
  - Calibrations
  - Physics modelling
  - Precision goal

  → so mistakes can be made..

- At present, it is difficult to quote a conclusive “world average”. The most precise measurement is also the most discrepant, and will likely stay forever
- Perspectives :
  - Ultimate goals of ATLAS, CMS, LHCb ~10 MeV each, with different experimental conditions and methods
  - News expected for the summer conferences, fingers crossed!