Quarkyonic solution to the hyperon puzzle

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Abstract. We show that Quarkyonic Matter can mitigate the hyperon puzzle. The key observation is that the hyperon threshold is shifted to a higher density by a factor of constituent strange quark mass. We illustrate this effect by using the ideal dual Quarkyonic (IdylliQ) model with multiple flavors.

1 Introduction

The existence of hyperons in neutron star cores remains as an open question. Hyperons are expected to appear since it is at some point more likely to add a particle with different flavor due to Pauli blocking than adding another nucleon. The onset of hyperons leads to the problematic reduction of the maximum mass of a neutron star, which is at odds with the presence of heavy pulsars. This problem is known as *hyperon puzzle*.

Meanwhile, we believe that nuclear matter undergoes deconfinement and eventually turns into quark matter with increasing baryon density. It is natural to expect so since nucleons in nuclear matter overlap at a few times the saturation density, so quarks become relevant degrees of freedom. The liberation of quarks at high density is led by screening of the confinement potential by quarks in the medium, which is based on the weak-coupling analysis [1].

However, as was pointed out in Ref. [2] based on the large-N_c QCD, the deconfinement at high density may not be as simple as this conventional picture. Namely, in the large-N_c limit of QCD, the screening mass for gluons from quark loops is suppressed by the factor $1/N_c$. This should be contrasted to QCD at finite temperatures in which the thermal screening of the confinement potential is efficient enough to form the deconfined quark-gluon plasma.

Such an observation in large-N_c QCD at high density led to the notion of Quarkyonic matter [2]: The fundamental degrees of freedom in dense large-N_c QCD matter are confined baryons because the confinement interaction is never screened from the argument above. At the same time, matter can be described in terms of quarks in the weakly-coupled regime because the fundamental degrees of freedom in the weak-coupling calculations are quarks. This can be interpreted as the duality between baryonic and quark degrees of freedom.

Quarkyonic matter has successfully been applied to reproduce the semi-quantitative feature of the neutron star equation of state [3]. The key feature of Quarkyonic matter is the nontrivial occupation number in the phase space. This has been recently derived in Ref. [4] by using the ideal dual Quarkyonic (IdylliQ) model, which explicitly takes into account the dual aspect of Quarkyonic matter.

In this contribution, we show that the onset of hyperons in the IdylliQ matter is shifted to a higher density compared to the conventional picture, so the hyperon puzzle is alleviated.

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2 Ideal dual Quarkyonic (IdylliQ) model

In this section, we briefly sketch the IdylliQ model of the symmetric nuclear matter developed in [4, 5] and discuss its solution [4].

From the dual nature of Quarkyonic matter, we expect the thermodynamic quantities to be described simultaneously in terms of baryons and quarks. Here, we specifically consider the baryon density n_B and the energy density ε. We introduce the phase-space density for baryons $f_B(k)$ and quarks $f_q(q)$. The quark distribution f_q is defined for a fixed color, $f_q \equiv f_q^R = f_q^G = f_q^B$. In such a setup, the baryon distribution is given by

$$n_B = 4\int_k f_B(k) = 4\int_q f_q(q)\,, \tag{1}$$

where the factor 4 arises from the spin and isospin degeneracy, and $\int_k$ and $\int_q$ are the shorthand notation for the integration over the momentum $\int \frac{d^3\boldsymbol{k}}{(2\pi)^3}$ and $\int \frac{d^3\boldsymbol{q}}{(2\pi)^3}$, respectively. The energy density is

$$\varepsilon_B[f_B] = 4\int_k E_B(k) f_B(k)\,, \qquad \varepsilon_q[f_q] = 4\int_q E_q(q)[N_c f_q(q)]\,, \tag{2}$$

where we fixed the dispersion relation for a baryon as $E_B(k) = \sqrt{M_N^2 + k^2}$ and M_N is the nucleon mass. Duality means $\varepsilon = \varepsilon_B[f_B] = \varepsilon_q[f_q]$. Each description is related through the following relation, which effectively takes into account the quark confinement inside baryons:

$$f_q(q) = \int_k \varphi\left(\boldsymbol{q} - \frac{\boldsymbol{k}}{N_c}\right) f_B(k)\,. \tag{3}$$

The function φ describes a momentum distribution of a single quark in a single baryon state, so the physical requirement for φ is to have a peak at $\boldsymbol{q} - \boldsymbol{k}/N_c = 0$ with the finite width to account for the quark confinement effect. Here, for φ, we choose

$$\varphi(\boldsymbol{q}) = \frac{2\pi^2}{\Lambda^3}\frac{e^{-q/\Lambda}}{q/\Lambda}\,, \tag{4}$$

where the parameter Λ is an effective confinement scale. This specific choice (4) was made so that the model becomes analytically soluble while fulfilling the physical requirement.

Now, we find the optimal solutions of f_B and f_q by minimizing the ε at a given n_B and fulfilling the constraints $0 \le f_B \le 1$ and $0 \le f_q \le 1$. The minimum energy solution is $f_B(k) = \Theta(k_{\mathrm{F}} - k)$ as in the ordinary Fermi gas model when the corresponding quark distribution obtained through (3) is less than the unity. When $f_q(q) \ge 1$, the Fermi-Dirac distribution $f_B(k) = \Theta(k_{\mathrm{F}} - k)$ has to be modified. We find the minimum energy solution [4]

$$f_B(k) = \frac{1}{N_c^3}\Theta(k_{\mathrm{bu}} - k) + \Theta(k_{\mathrm{sh}} - k)\Theta(k - k_{\mathrm{bu}})\,, \tag{5}$$

where k_{bu} and k_{sh} refer to the edges of the bulk and the shell part, respectively. The corresponding quark distribution is

$$f_q(q) = \Theta(q_{\mathrm{bu}} - q) + f_q^{f_B=1}(q)\Theta(q_{\mathrm{sh}} - q)\Theta(q - q_{\mathrm{bu}}) + f_q^{f_B=0}(q)\Theta(q - q_{\mathrm{sh}})\,, \tag{6}$$

where $q_{\mathrm{bu}} = k_{\mathrm{bu}}/N_c$ and $q_{\mathrm{sh}} = k_{\mathrm{sh}}/N_c$. Also, $f_q^{f_B=1}(q) = N_c^3 + d_+\frac{e^{q/\Lambda}}{q/\Lambda} + d_-\frac{e^{-q/\Lambda}}{q/\Lambda}$ and $f_q^{f_B=0}(q) = c_-\frac{e^{-q/\Lambda}}{q/\Lambda}$. The coefficients c_- and $d_\pm$ are determined by the matching conditions at $q = q_{\mathrm{bu}}$ and q_{sh}. One can find the relation between q_{bu} and q_{sh} from the minimum energy condition:

$$\frac{\Lambda + q_{\mathrm{bu}}}{\Lambda + q_{\mathrm{bu}} - (\Lambda + q_{\mathrm{sh}})e^{-(q_{\mathrm{sh}} - q_{\mathrm{bu}})/\Lambda}} = N_c^3\,. \tag{7}$$

3 Hyperons in IdylliQ model

To address the hyperon puzzle, we consider including a hyperon on top of the pure neutron matter. For simplicity, we neglect the lepton contributions and limit ourselves to the charge-neutral hadron species. Now we extend the above model to the multi-flavor case [6]. In the above, we consider the symmetric nuclear matter, and the baryons are described by a single distribution f_B. Here, we consider the momentum distributions for neutrons f_n and for the strangeness -1 hyperons f_Y. To be more specific, we consider Λ^0 and Σ^0 hyperons and neglect their mass differences. The number density and energy density are modified as

$$n_n = 2\int_k f_n(k)\,, \qquad n_Y = 4\int_k f_Y(k)\,, \tag{8}$$

$$\varepsilon_B[f_n, f_Y] = 2\int_k \left[E_N(k) f_n(k) + 2E_Y(k) f_Y(k)\right]\,, \tag{9}$$

where the dispersion relations for neutrons and hyperons are $E_N(k) = \sqrt{k^2 + M_N^2}$ and $E_Y(k) = \sqrt{k^2 + M_Y^2}$, respectively. The baryon density is given by $n_B = n_n + n_Y$. In the flavor asymmetric case, the relation (3) is modified as

$$f_d(q) = \int_k \varphi\left(\boldsymbol{q} - \frac{\boldsymbol{k}}{N_c}\right)\left[\frac{2}{3} f_n(k) + \frac{2}{3} f_Y(k)\right]\,, \tag{10}$$

where the factor in front of f_n originates from the baryon number content $2/3$ in a neutron, and the factor in front of f_Y from the baryon number content $1/3$ in a hyperon times the mass degeneracy 2 for Λ^0 and Σ^0. One can write down the similar relation for u and s quarks.

As we will see below, since the system is d-quark abundant, only the d-quark distribution is saturated. Thus, the d-quark distribution is given by Eq. (6), and from the relation (10), the distribution at $k < k_{\mathrm{bu}}$ must fulfill the condition:

$$\frac{2}{3} f_n(k) + \frac{2}{3} f_Y(k) = \frac{1}{N_c^3}\,. \tag{11}$$

From this condition, we find that the optimal distributions for neutrons and hyperons:

$$f_n(k) = \frac{3}{2N_c^3}\Theta(k - k_Y)\Theta(k_{\mathrm{bu}} - k) + \Theta(k - k_{\mathrm{bu}})\Theta(k_{\mathrm{sh}} - k)\,, \tag{12}$$

$$f_Y(k) = \frac{3}{2N_c^3}\Theta(k_Y - k)\Theta(k)\,. \tag{13}$$

In Fig. 1, we plot f_n and f_Y, and the corresponding quark distributions, f_d, f_u, and f_s.

Now, from the relations above, we can compute the onset density of hyperons. This can be computed from the β-equilibrium condition, i.e., the chemical potential for neutrons μ_n and hyperons μ_Y should be the same. They can be computed from the thermodynamic relations $\mu_n = (\partial\varepsilon/\partial n_n)_{n_Y}$ and $\mu_Y = (\partial\varepsilon/\partial n_Y)_{n_n}$. We find that the onset of hyperons is at [6]

$$\mu_B = 2M_Y - M_N\,, \tag{14}$$

where μ_B is the baryon chemical potential and is equal to μ_n. The hyperon onset in the non-interacting baryon gas is at $\mu_B = M_Y$. Our estimate is shifted to much higher μ_B by $M_Y - M_N \simeq 0.18\,\mathrm{GeV}$.

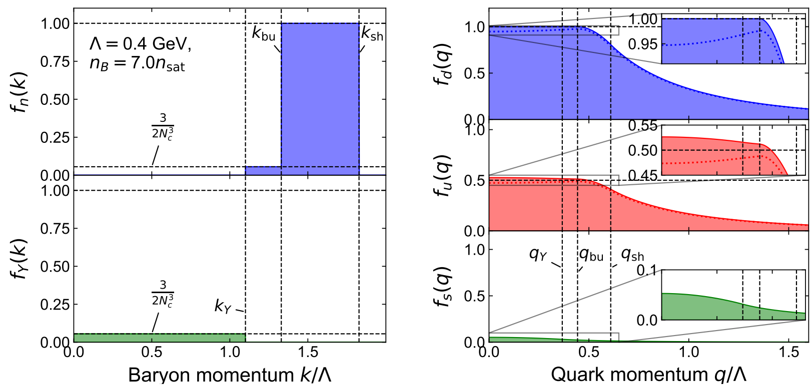

Figure 1. Phase-space density in the IdylliQ model. Left: The momentum distributions for neutrons (top) and hyperons (bottom). Right: The momentum distributions for d (top), u (middle), and s quarks.

4 Conclusions

We presented that the $S = -1$ hyperon threshold is shifted from $\mu_B = M_Y$ to higher density $2M_Y - M_N$ because of the Pauli exclusion effect of the quark substructure of hyperons. Since d-quark states are already filled by nucleons, hyperons that contains d quarks are less likely to appear. This makes the hyperon contribution to a neutron star less important and thus works toward the resolution of the hyperon puzzle; the shifted threshold is high enough so that it may not greatly affect the maximum mass of neutron stars. The $S = -2$ hyperon such as Ξ^0 can contribute at μ_B slightly above the $S = -1$ hyperon threshold; we could not mention it due to the limitation of the space. See Ref. [6] for further discussions.

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