Signals of initial state quantum entanglement in relativistic particle collision

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Abstract. We present thermodynamic entropy calculations based on charged particle multiplicity data from proton-proton collisions measured by ALICE at the LHC in comparison to entanglement entropy calculations based on initial state parton distributions. The relative agreement of these distributions can be quantified by studying their higher order cumulants. The commonalities between the initial and final state entropy suggest that entanglement could be a possible source for the seemingly thermal and collective behavior in small systems.

1 Introduction

The deconfined phase of strongly interacting matter, generated in particle collisions at the RHIC and LHC, can be described by a combination of hydrodynamical and statistical models that trace the evolution of collisions from an initially thermalized system, even for elementary particle collisions [1], to the final state. While these models have successfully predicted particle yields with statistically significant accuracy, they rely on the assumption that the system reaches partial thermal equilibrium at very early times (less than 1 fm/c), which is highly improbable through particle collisions, such as e^+e^- or pp. Furthermore, this type of phenomenological approach fails to capture all the quantum dynamics of the initial system. Standard MC event generators, such as PYTHIA, describe an elementary proton-proton collision using single parton-parton interactions, thus ignoring the impact that the non-interacting partons, described by the proton wave function, might have on the reduced density matrix that captures the number of available final states. These deficiencies in the current models have led physicists to take new approaches relying on fundamental quantum principles to give a more accurate and complete description [2–4]. These approaches are supported by empirical evidence of quantum thermalization through entanglement found initially in condensed matter experiments at ultra-cold temperatures [5], but more recently also at higher temperatures and over finite volume [6].

2 Initial State Entropy

In a quantum system the degree of entanglement, or the information shared between a subsystem and the remainder of the system, can be quantified using the von Neumann entropy, defined for a subsystem A as:

$$S(A) = -\mathrm{Tr}(\rho_A \ln(\rho_A)), \quad (1)$$

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where ρ_A is the reduced density matrix of subsystem A. For a pure state, described by a single wavefunction, the von Neumann entropy of the complementary subsystem (the remainder of the system outside A) is equal in magnitude and opposite in sign, resulting in a total entropy of zero for the composite system. This entropy also known as entanglement entropy quantifies the level of disorder in the system and provides a quantum analog to thermodynamic entropy.

In proton-proton collisions, entropy in the initially pure system originates from a break in entanglement between the interacting parton and the remaining partonic spectator region. By tracing over the degrees of freedom in the reduced density matrix of the system, one can calculate the entanglement entropy between different regions. This calculation can be challenging due to the complex nature of the density matrix, however, in the limit of small momentum fractions ($x < 10^{-3}$), where the proton is described as a highly dense system of indistinguishable gluons, the relevant degrees of freedom can be effectively reduced to the parton number. In this scenario, the density matrix simplifies to:

$$\rho_A \approx \frac{1}{N} I_{N\times N} \tag{2}$$

where N represents the number of partons, and $I_{N\times N}$ represents the identity matrix in N×N dimensions. Consequently, by restricting the kinematics of our collision system to low x, we can simplify the calculation of the entanglement entropy, which reduces to:

$$S(A) \approx \ln(N) \tag{3}$$

Here, N is determined by integrating over the known parton probability distribution, with integration limits based on the spatial region measured in the final state. PDF's are provided in terms of the parton momentum fraction (or Bjorken-x) and momentum transfer Q^2. Our present analysis is limited to the central rapidity region covered by the Time Projection Chamber (TPC) in ALICE, which limits our x-range to values around 10^{-4}, corresponding to Q^2 values around unity. It is important to note here that in this low Q^2 regime higher order expansions of the QCD coupling constant give unreliable results for gluon distributions. Therefore, we only consider leading order (LO) parton distribution functions (PDFs)[7].

Prior to the collision we have a Lorentz contracted proton moving near the speed of light. The partons exist in a complete coherent state (fully entangled) which contains zero von-Neumann entropy. At the onset of the collision there is a break of entanglement which is encoded in the reduced density matrix that defines all quantum states of constituent partons within the overlap region. After the initial quench of the wave function, the combined system of interacting partons interacts through flux tube or string generation along the beam axis, thereby generating an extensive volume [8].

We can begin to calculate an entropy in the low-x limit based on the number of interacting gluons calculated by summing over PDF's, although the need to extend this formalism to sea quark contributions at low Q^2 and moderate x was shown by Hentschinski et al. [9].

The authors also added a factor of 2/3 to account for unmeasured neutral states in the final distribution. The entropy equation then becomes:

$$S_{EE} = ln(2/3(N_{gluons} + N_{quarks})) = ln(N_{gluons} + N_{quarks}) + ln(2/3) \tag{4}$$

Finally, the entanglement entropy calculated this way is only taking into account the diagonal elements of the reduced density matrix and not the non-diagonal terms of the infinite density matrix. In the literature the inclusive calculation is known as the entropy of ignorance. Using an initial CGC model, theorists have been able to determine the ratio between this entropy of ignorance and the entanglement entropy [10]. In the calculation it is observed that at low Q^2 this ratio is quite large with an upper limit of about 1.4. For the mid-rapidity

ALICE measurements the ratio reaches around 1.24 for $\sqrt{s}$ = 0.9 TeV [10]. We apply this correction factor to the measured initial state entropy.

3 Final State Entropy

Since the fractional momentum and momentum transfer scale are not directly measurable in proton-proton collisions, an alternative way of comparing entropies has to be employed. In the final-state we can calculate a thermodynamic entropy using the probability distribution of produced particles over any range in pseudo-rapidity as long as we can define a mapping between η in the final state and the fractional momentum x in the initial state. Within the CGC model a common approximation is given by ln(1/x) = $y_{\text{initial proton}}$ - y_{hadron}.

This relation might break down at large x [11], but we are mostly interested in the low x, high gluon density region. ALICE has published mid-rapidity charged particle multiplicity data for four collision energies at $\sqrt{s}$ = 0.9, 2.76, 7, and 8 TeV [12]. As examples, the measured multiplicity distributions for 0.9 and 8 TeV are shown in Fig.1, and are well described by a Negative Binomial Distribution (NBD). Since both colliding protons contribute equally to the final state, the distribution is halved to obtain a meaningful comparison to single proton PDFs [4] and calculate the Shannon entropy of a single proton.

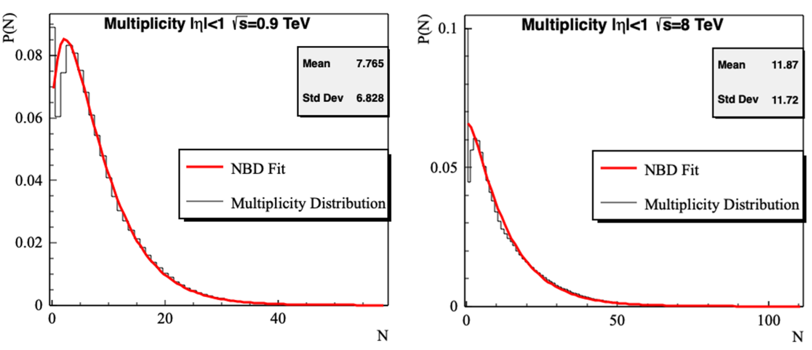

Figure 1. Measured hadron multiplicity distributions from ALICE data [12]

4 Results and Conclusions

Higher moments of the NBD distribution are used to compare a 1+1 toy model of non-linear QCD evolution of the BK equation as suggested by Kharzeev and Levin [2]. They demonstrated that one can construct a generating function that captures the non-linear interactions leading to dipole formation, in an entangled system, which sets an upper limit on the moments of the final state NBD distribution. We observe that the final state multiplicity distribution approaches the limit expected from a fully entangled state, see Fig.2.

Fig.3a shows final state entropy shown as red points compared with the initial-state entropy shown as bands, corrected for additional quark contributions, neutral particle contributions and the aforementioned entropy of ignorance factor. Entropies from two different corrected leading order parton distribution functions at the relevant x, Q^2 and α_s values are shown.

The final particle production can also be modeled by the simple fragmentation of strings, but as expected, even the most recent PYTHIA tunes underestimate the final state entropy

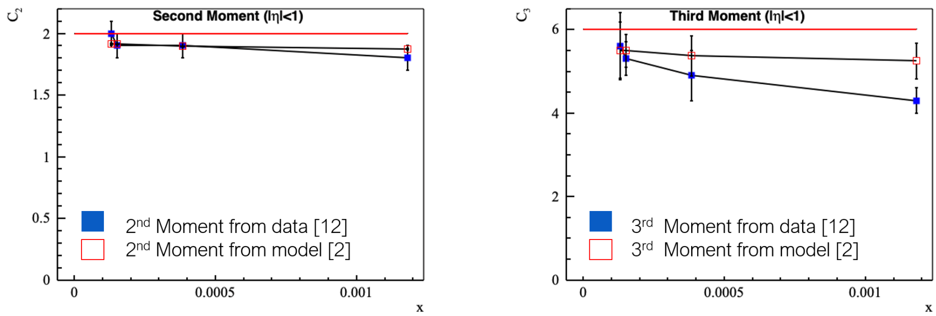

Figure 2. Comparison of theoretical moments from 1+1 toy model [2] final state moments of the multiplicity distribution [12]. Red lines represent the upper bound on entangled cumulants in the limit of $\bar{n}$ approaching infinity.

significantly when not taking into account some type of quantum effects. Lately these effects have been approximated via two distinct PYTHIA parameters and modes, namely the level of multi-parton interactions (MPI) and the color reconnection mode (CR). We show that these modes indeed help to bring the simple string fragmentation multiplicities to a level similar to the entangled initial state. Fig.3b shows the comparison to various PYTHIA modes.

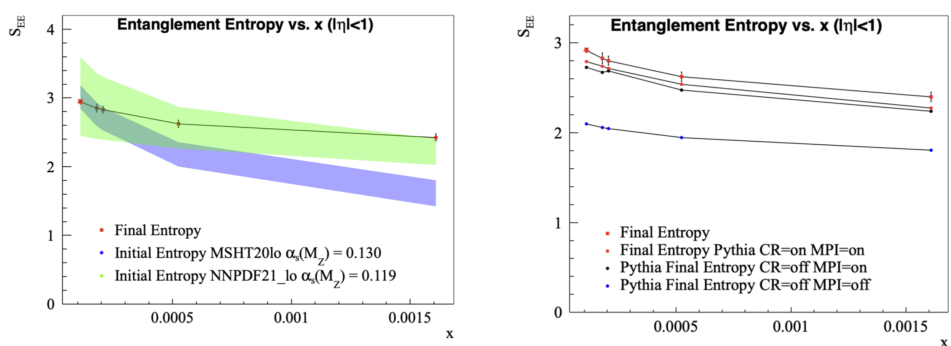

Figure 3. Left: Final state entropy including all corrections in red, and the initial state entropy shown by colored bands for two models (NNPDF [7] and MHST [13]), Right: Final state entropy calculated from data to PYTHIA using three different tunes.

We have shown that the entropies calculated from hadron multiplicity in the final state and parton multiplicity in the initial state approach one another at low-x. We observe better agreement between the initial state and final state for a lower strong coupling constant of 0.119. We have not yet seen any effect of gluon saturation in the covered x-range, but future measurements at forward rapidity at the LHC and over the full phase space at the EIC, will be highly relevant, since it is predicted that the entropy generation should follow the gluon saturation curve at very low x.

Overall our measurements give support to the theory that the quantum entangled state in the initial phase of a relativistic hadron collision could maintain its coherence throughout the early evolution and lead to a seemingly thermal final state, where the observed particle multiplicities are governed by the partition function of the initial parton states.

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