Bayesian constraints on the high density QCD EoS for nuclear matter from Heavy-ion collision data

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Abstract. A Bayesian framework is introduced to constrain the density dependence of the Equation of State (EoS) for nuclear matter. The EoS is inferred using existing measurements of elliptic flow and the mean transverse kinetic energy of protons in the beam energy range of $\sqrt{s_{\mathrm{NN}}} = 2$–$10$ GeV. Tight constraints are obtained for densities up to 4 times the nuclear saturation density. However, the results are highly sensitive to the choice of observables, highlighting the need for consistent, high-precision measurements in this energy range.

1 Introduction

Relativistic nuclei-nuclei collisions generate extremely hot and/or dense nuclear matter, which cools, expands and eventually freezes out into stable hadrons, traversing the phase diagram of quantum chromodynamics (QCD), providing a unique opportunity for laboratory exploration of the phase structure of strongly interacting nuclear matter. In theoretical calculations, the dynamics and evolution of such a hot and/or dense system are governed by the Equation of State (EoS), which relates the system's pressure to its energy density and net-baryon density. The EoS decides the location of phase boundaries, the nature of transitions between the phases and the critical regions in the QCD phase diagram, making its extraction one of the primary objectives of heavy-ion collision (HIC) experiments worldwide.

At vanishing chemical potential, first-principles lattice QCD calculations predict a smooth crossover for both chiral and deconfinement transitions [1–3]. However, at finite baryon densities, the fermionic sign problem prevents direct lattice QCD calculations, necessitating a more comprehensive approach which would involve comparing experimental data with QCD inspired, effective model calculations to infer the EoS [4–8]. Due to the lack of a definitive observable for the unambiguous extraction of the EoS, a global fit to multiple EoS-sensitive observables is necessary to systematically constrain or infer the EoS. Several machine learning methods have also been proposed to study the EoS [9–12].

For the beam energies used to study high baryon density matter ($\sqrt{s_{\mathrm{NN}}} < 15$ GeV), another challenge has been the absence of a consistent theoretical framework that describes

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both the early compression and subsequent expansion phases of the collision using a consistent EoS. This issue was recently addressed in [13, 14], where a realistic chiral mean-field EoS (CMF) was incorporated into both the non-equilibrium and equilibrium phases of the collision within the microscopic UrQMD [15, 16] model. By extending this method, arbitrary density-dependent EoSs can be implemented in UrQMD. Leveraging this approach, the present work [17] constrains the high-density EoS through a Bayesian inference procedure, utilizing existing experimental data. This not only provides the first quantitative analysis of the constraints that current data impose on the density dependence of the EoS but also establishes a robust analysis framework that can be employed for future experimental data.

2 Methodology

To consistently describe the entire evolution using a consistent EoS, we employ the microscopic UrQMD model, which has been augmented to incorporate a density-dependent potential. In the model, the density-dependent potential energy V enters the QMD equations of motion, and can be directly related to the pressure of the system [18], and thus to the EoS.

For our problem, we parameterise the potential energy V as a 7^{th} degree polynomial for densities above 2 times saturation density (n_0), below which we fix it to be the CMF EoS,

$$i.e., V(n_B) = \sum_{i=1}^{7} \theta_i \left(\frac{n_B}{n_0} - 2\right)^i + h \tag{1}$$

where h=-22.07 MeV is set to ensure that the potential energy is a continuous function at $2n_0$.

Given model parameters $\boldsymbol{\theta}$ and experimental data $\mathbf{D}$, Bayes' theorem estimate the posterior $P(\boldsymbol{\theta}|\mathbf{D})$ by scaling our prior knowledge $P(\boldsymbol{\theta})$ based on the likelihood $P(\mathbf{D}|\boldsymbol{\theta})$ which quantifies how effectively the parameters describe the data. The parameters of the model to be constrained are the polynomial coefficients $\boldsymbol{\theta} = \{\theta_1, \theta_2, ..., \theta_7\}$. In order to constrain them, we will be using the experimental data $\mathbf{D}$ comprising elliptic flow (v_2) of mid rapidity protons from mid-central Au-Au collisions at $\sqrt{s_{\mathrm{NN}}}$ = 2.24, 2.32, 2.4, 2.42, 2.51, 3.0, 3.32, 3.84, 4.23 and 4.72 GeV [19–25] and the mean transverse kinetic energy ($\langle m_T \rangle - m_0$) in central Au-Au collisions at $\sqrt{s_{\mathrm{NN}}}$ = 3.83, 4.29, 6.27, 7.7 and 8.86 GeV [26–28].

To calculate the likelihood, numerous UrQMD simulations must be performed with a wide range of EoSs to generate predictions for comparison with the data. However, this approach can be computationally slow and infeasible. To address this, we train Gaussian Process (GP) models as fast emulators for the UrQMD simulations. Once trained, the GP emulator accurately predicts the observables, v_2 and $\langle m_T \rangle - m_0$.

3 Results and Conclusions

The experimental observations and the trained GP models are now employed to construct the posterior distributions for the EoS. The left panel of Figure 1 illustrates the posterior distribution obtained when all 15 data points were included in the inference. As shown, tight constraints can be established for baryon densities up to 4 n_0. The "MEAN" EoS constructed by averaging the sampled EoS at different densities, suggests a stiff EoS at these densities. To evaluate the sensitivity of the inference to the choice of observables, we performed an additional analysis excluding the $\langle m_T \rangle - m_0$ values for $\sqrt{s_{\mathrm{NN}}}$ = 3.83 and 4.29 GeV in the inference. The resulting posterior distribution, displayed in the right panel of Figure 1, shows significant differences for densities above 3 n_0. Notably, the exclusion of these two data points results in a softening of the EoS in the density range of 3 - 5 n_0, which is consistent with signals of a first-order phase transition.

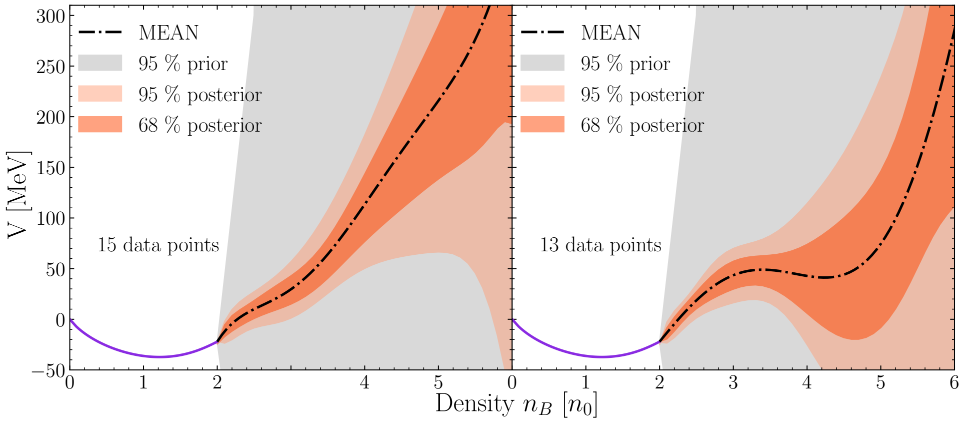

Figure 1. (Left) Posterior distribution for the EoS inferred using all 15 experimental observations. (Right) Posterior distribution for the EoS obtained excluding the $\langle m_T \rangle - m_0$ values for $\sqrt{s_{\rm NN}} = 3.83$ and 4.29 GeV (13 data points). The violet curve up to $2n_0$ represents the CMF EoS.

To investigate the significant deviation observed with the exclusion of just two data points, the MEAN EoSs extracted from both cases were implemented in UrQMD, and the v_2 and $\langle m_T \rangle - m_0$ values were calculated and compared with the experimental data. It was found that both EoSs reproduce the v_2 values reasonably well at low beam energies, with small deviations at higher energies ($\sqrt{s_{\rm NN}} > 3.5$ GeV). Interestingly, the $\langle m_T \rangle - m_0$ values for $\sqrt{s_{\rm NN}} > 6$ GeV showed better agreement with the experimental data when the two data points were excluded. This also resulted in an improved fit for v_2 for $\sqrt{s_{\rm NN}} > 3.5$ GeV. Conversely, when the two $\langle m_T \rangle - m_0$ data points were included in the inference, the fit to these specific points improved but led to a worse fit for v_2 at beam energies greater than 3.5 GeV. This indicates a potential tension between the v_2 and $\langle m_T \rangle - m_0$ data at around 4 GeV. Such a tension could either be a result of inconsistencies in the experimental measurements or is a limitation in the model's ability to accurately describe the physics at higher energies. For more details on the analysis and results, see [17].

In summary, using the bayesian framework, we find that tight constraints can be obtained for the density dependence of the EoS for densities up to 4 n_0 using the available heavy-ion collision data in in the beam energy range $\sqrt{s_{\rm NN}} = 2 - 10$ GeV. However, the results demonstrate significant sensitivity to the choice of observables, indicating that consistent data is critical for tightly constraining the EoS of nuclear matter at high densities. High-precision, consistent measurements are necessary for the unambiguous extraction of the EoS. Future data from the STAR-FXT program at RHIC, the CBM experiment at FAIR, and upcoming experiments at HIAF and NICA will be pivotal in achieving this goal. In future, the framework can be extended by incorporating additional observables (e.g., higher-order flow coefficients, differential flow observables) and introducing more parameters into the inference, such as a momentum dependence to the EoS, for a more comprehensive analysis.

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