Dynamics of the chiral critical point in QCD, diffusion coefficient in Model G

Eduardo Grossi
Florence University & INFN Florence

E.G., A.Soloviev, D. Teaney, F. Yan PRD (2020)
E.G., A. Soloviev, D. Teaney, F. Yan PRD (2021)
A. Florio, E.G., A. Soloviev, D. Teaney PRD (2022)
A. Florio, E.G., D. Teaney PRD (2023)
A.Florio, E.G. A.Mazeliauskas, A. Soloviev, D. Teaney in preparation

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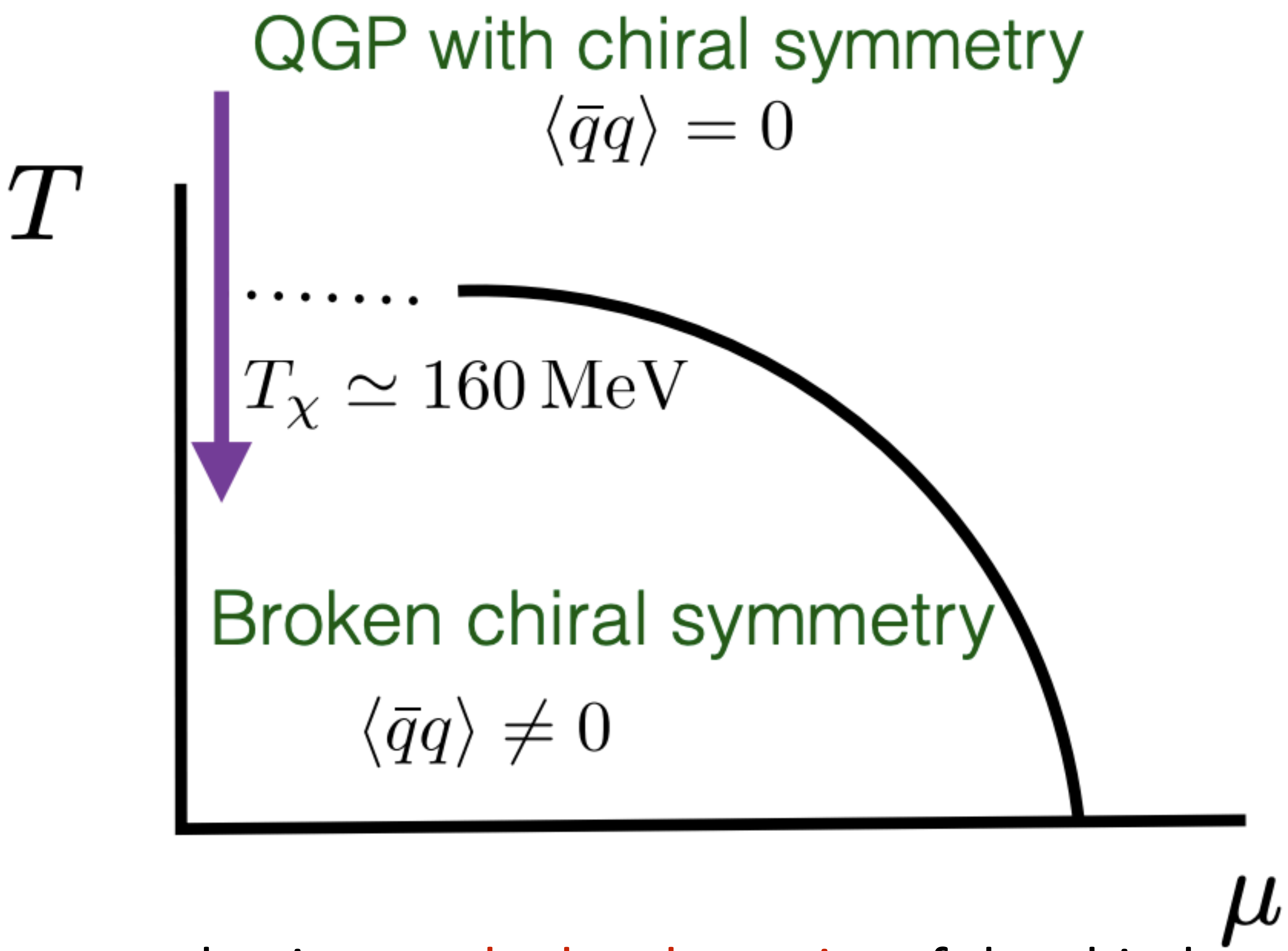
Motivation

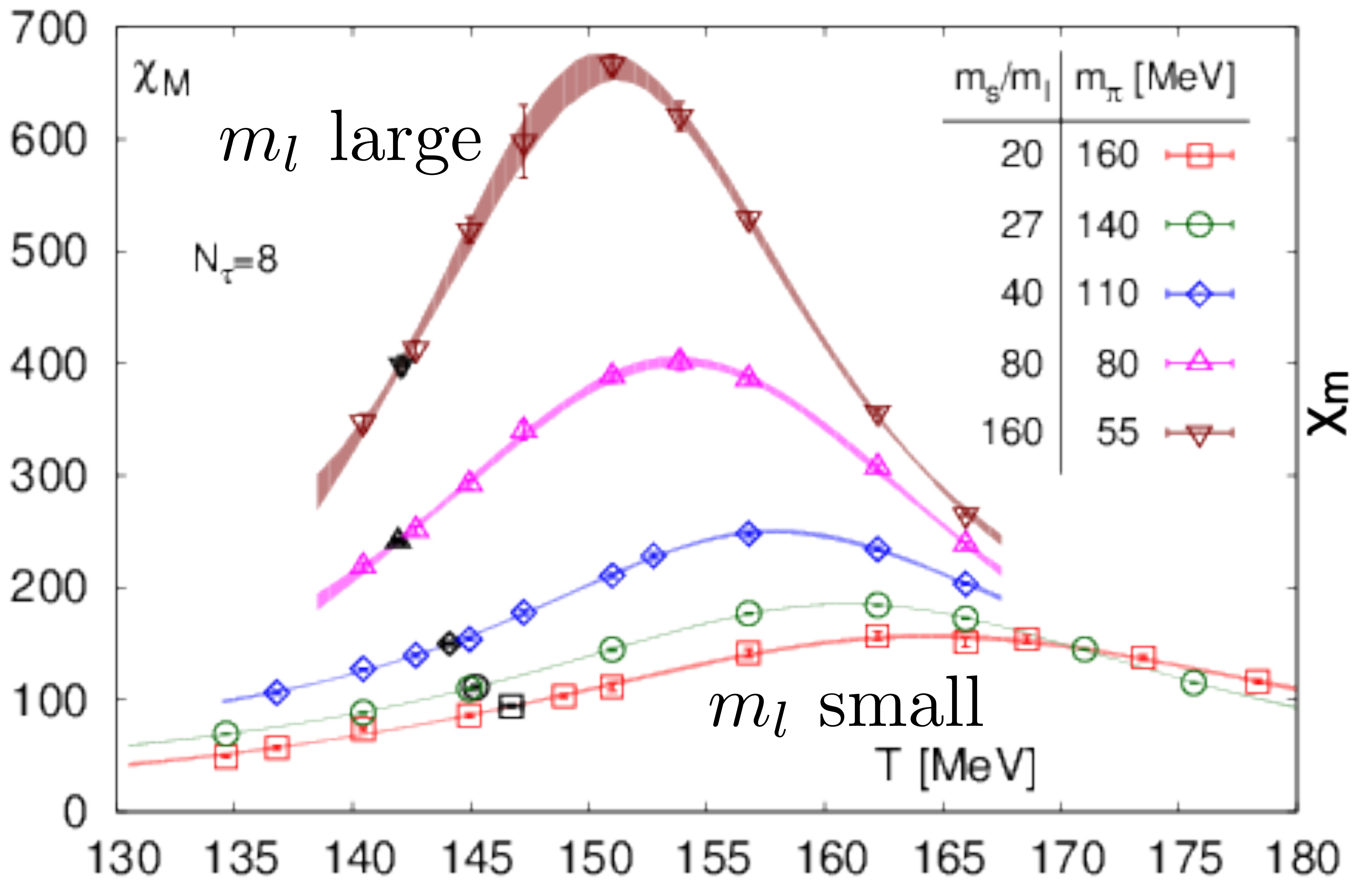


We are neglecting any hydro-dynamics of the chiral condensate !

Motivation 2

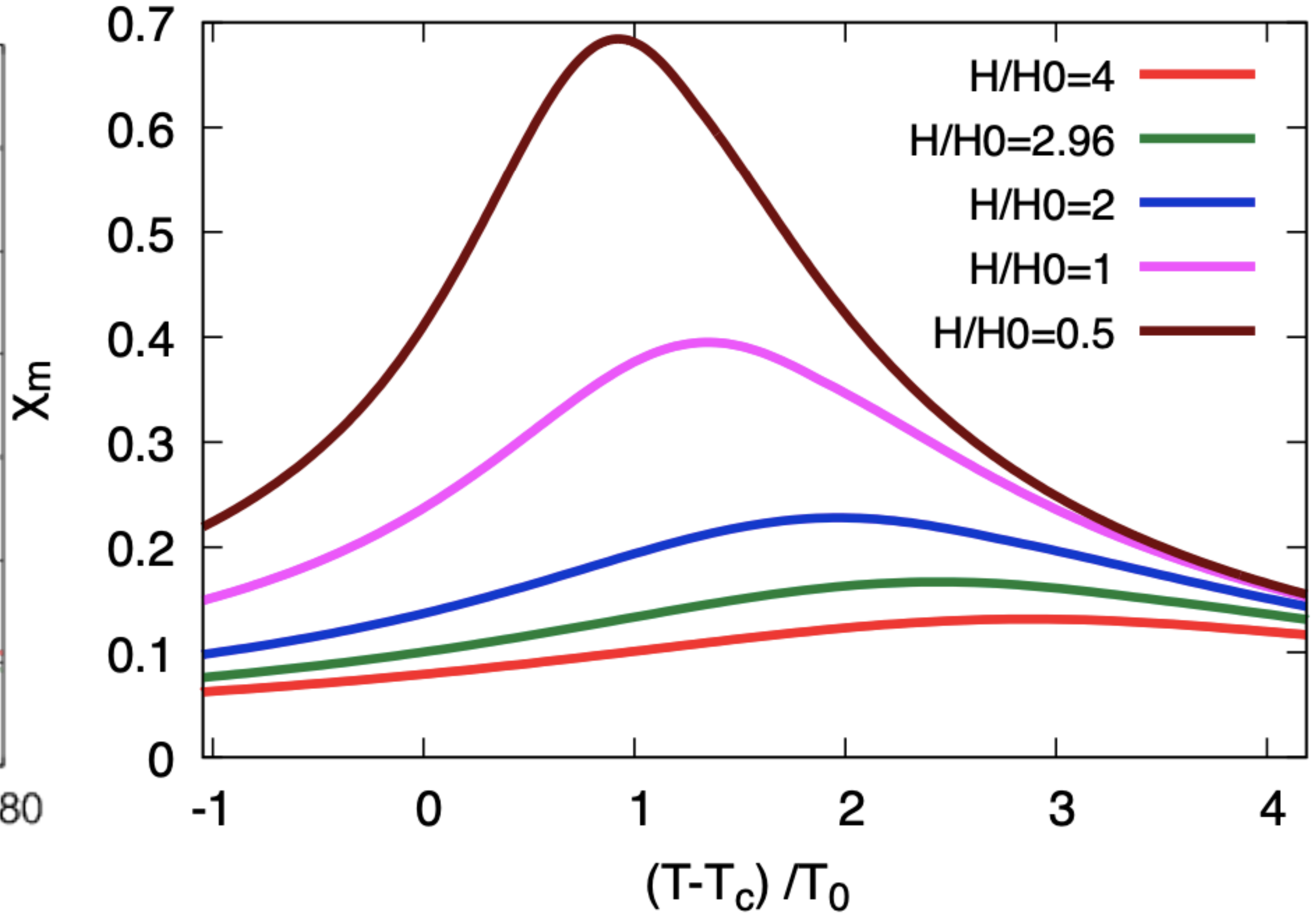
$$\chi_M = \frac{\partial \langle \bar{q} q \rangle}{\partial m_l}$$

[Ding et al PRL 2019]



O(4) scaling form

Engels et al., Nucl.Phys.B 655 (2003) 277-299



The chiral susceptibility seems to respect the scaling as predicted from O(4) universality class in d=3

[HotQCD, 2019, 2020]

[Cuteri, Philipsen, Sciara, 2021]

[Kotov, Lombardo, Trunin, 2021]

[Kaczmarek, Mazur, Sharma, 2021]

Setup: the O(4) phase transition

The (approximated) conserved quantities of 2 flavour QCD are

$T^{\mu\nu}$	J_V^μ	J_A^μ
Stress	Iso-vector (isospin)	Iso-axial
(T, u^μ)	μ_V	μ_A
	$\bar{q}\gamma^0 t_I q$	$\bar{q}\gamma^0\gamma_5 t_I q$

The approximate flavour symmetry $SU(2)_L \times SU(2)_R \sim O(4)$

The order parameter is the chiral condensate

$$\langle \bar{q}q \rangle \sim \phi_\alpha = (\sigma, \varphi_\alpha) = (\text{sigma}, \text{pions})$$

We need the hydrodynamic theory of the charge and the order parameter

Rajagopal Wilczek (93)
Son and Stephanov (02)

Equation of motion (Model G)

Rajagopal Wilczek (93)

Chiral condensate ϕ_a + Axial and Vector charge $n_{ab} = \chi_0 \mu_{ab}$

$$\partial_t \phi_a + g_0\, \mu_{ab}\phi_b = \Gamma_0 \nabla^2 \phi_a - \Gamma_0 (m_0^2 + \lambda\phi^2)\phi_a + \Gamma_0 H_a + \theta_a\,,$$

$$\partial_t n_{ab} + g_0\, \nabla \cdot (\nabla \phi_{[a}\phi_{b]}) + H_{[a}\phi_{b]} = D_0 \nabla^2 n_{ab} + \partial_i \Xi^i_{ab}\,.$$

Ideal part — Dissipative part — Gaussian Noise

- The ideal part is charge conservation and Josephson constraint
- Two dissipative coefficient Γ_0 and D_0 and noise
- The simulation of the stochastic process is done with an ideal step and metropolis update.

- At high temperature: charge diffusion
- At low temperature: pion propagation as the vev develops

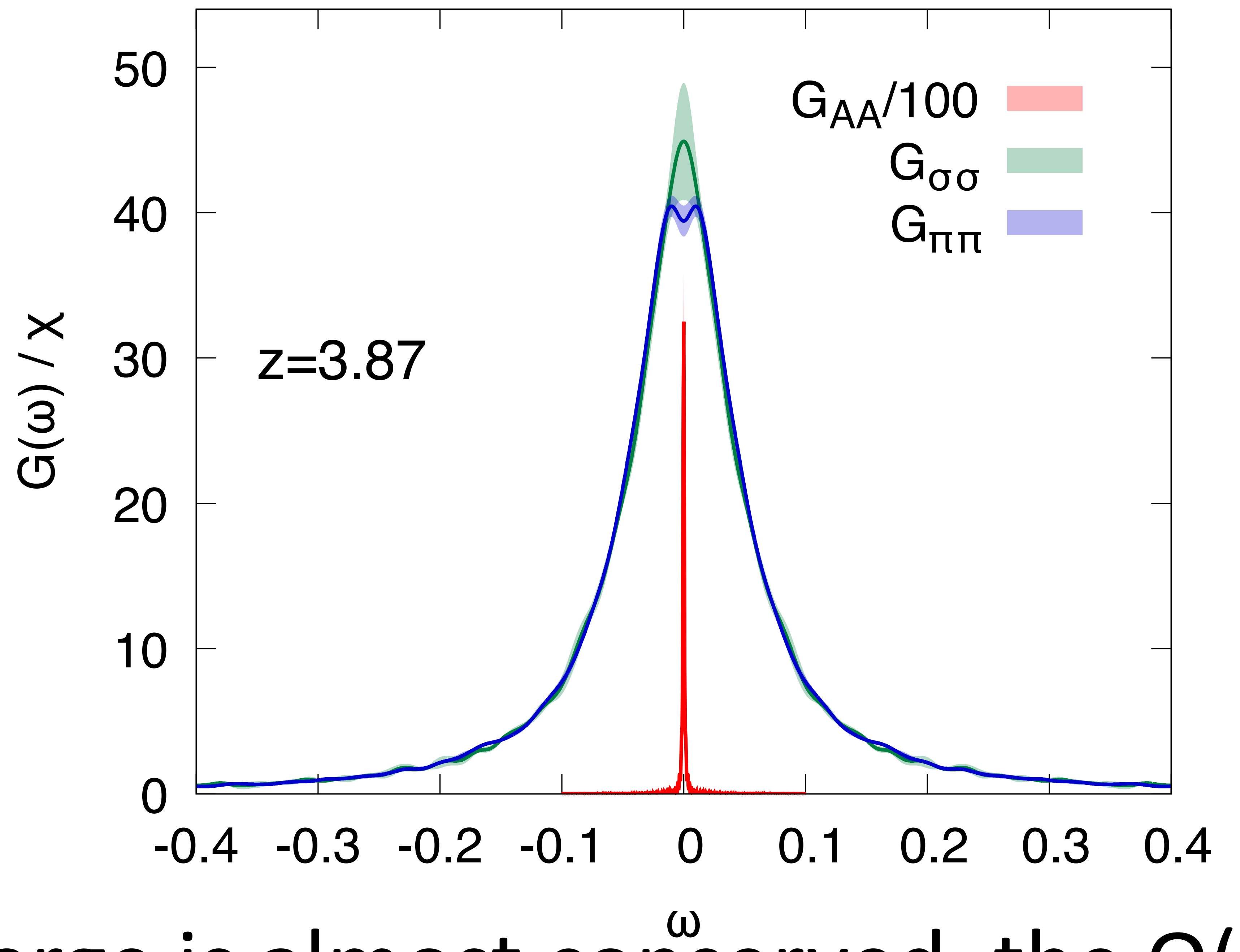
High temperature

$$G_{\sigma\sigma}(t,k) \equiv \frac{1}{V}\langle \sigma(t,\mathbf{k})\sigma(0,-\mathbf{k})\rangle_c\,,$$

$$G_{\pi\pi}(t,k) \equiv \frac{1}{3V}\sum_s \langle \pi_s(t,\mathbf{k})\pi_s(0,-\mathbf{k})\rangle_c\,,$$

$$G_{AA}(t,k) \equiv \frac{1}{3V}\sum_s \langle n_A^s(t,\mathbf{k})n_A^s(0,-\mathbf{k})\rangle_c\,,$$

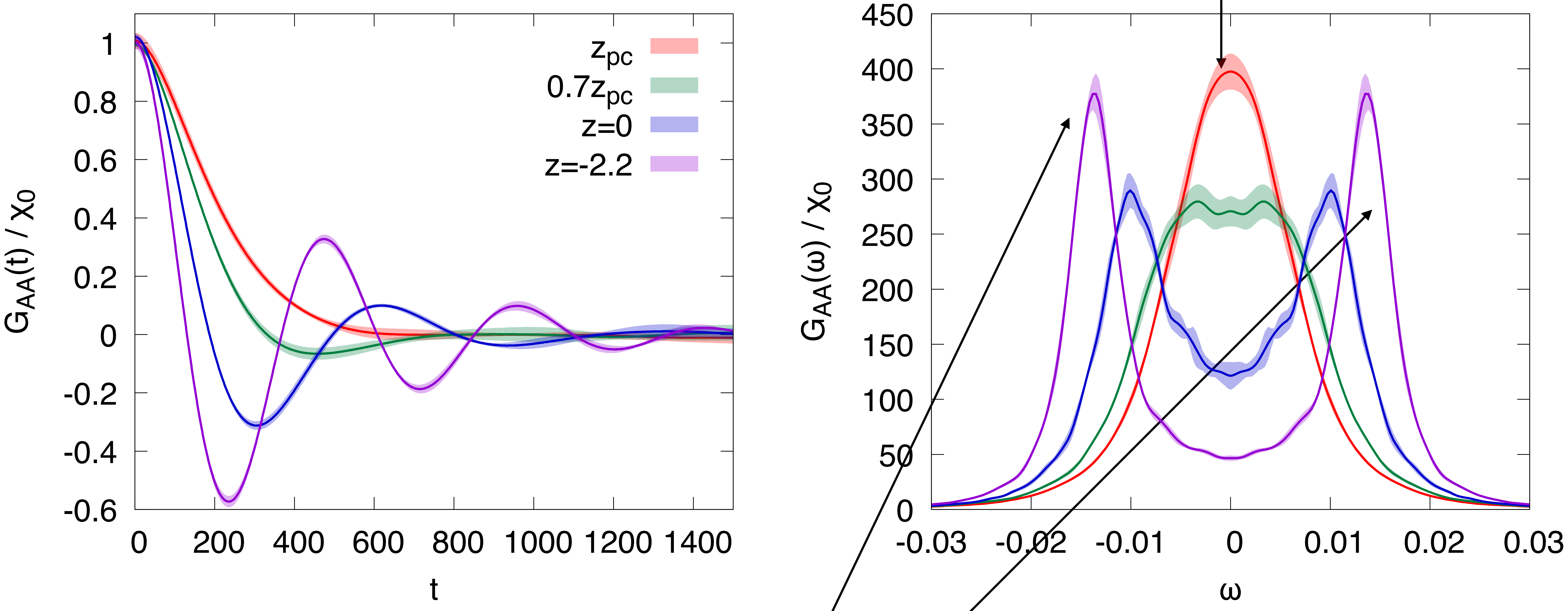
We focus on the statistical correlator at $k = 0$



The axial charge is almost conserved, the O(4) field are simply dissipate with a broad width

Propagation of axial charge across the transition

Quark diffusion T>>Tc



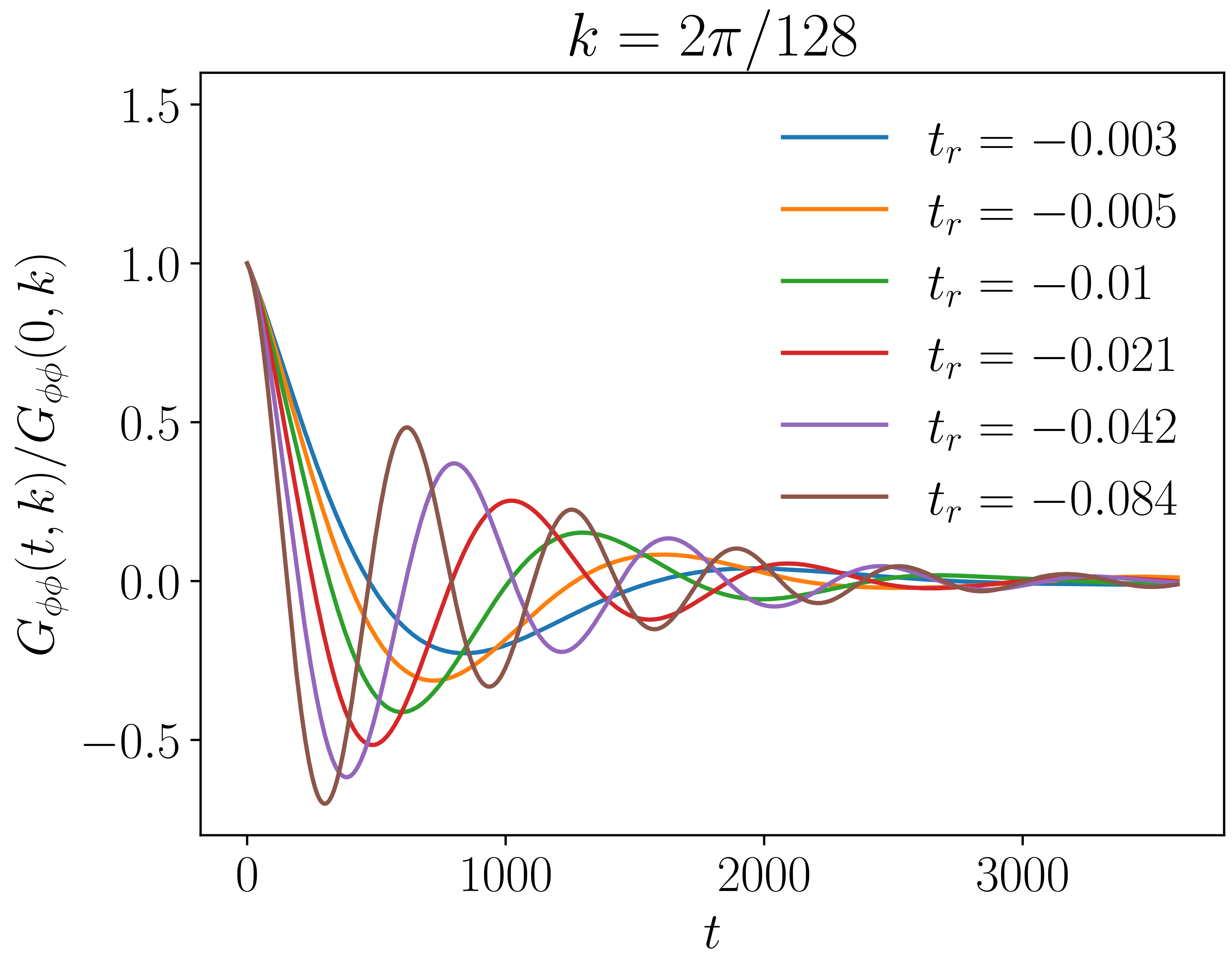
Quasiparticle Pions T<<Tc

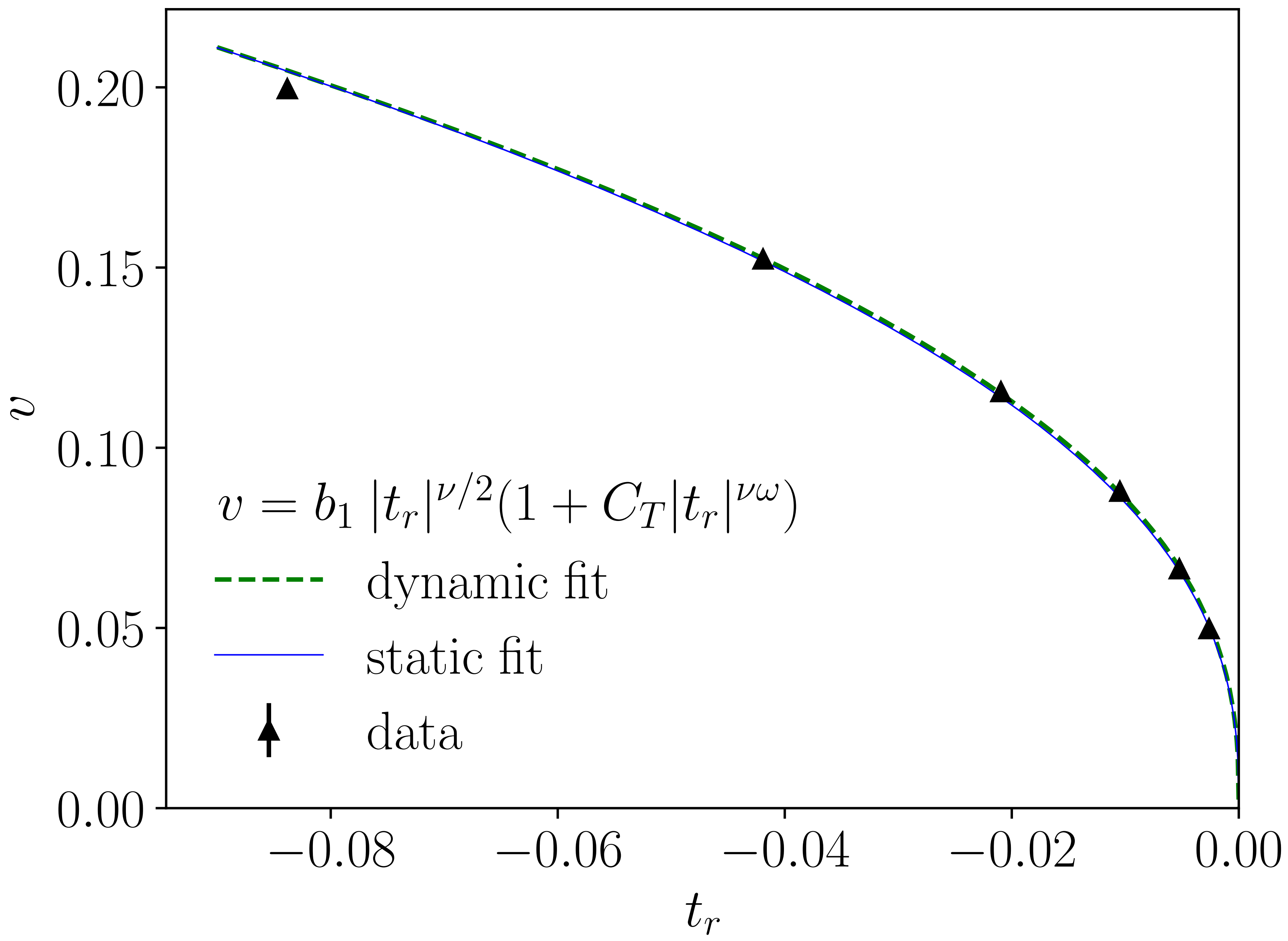
Around T_{pc} the axial charge starts changing form a diffusive form to a quasiparticle one

Broken phase m_q=0

A. Florio, E.G., D. Teaney PRD (2023)

In the broken phase one has pion waves



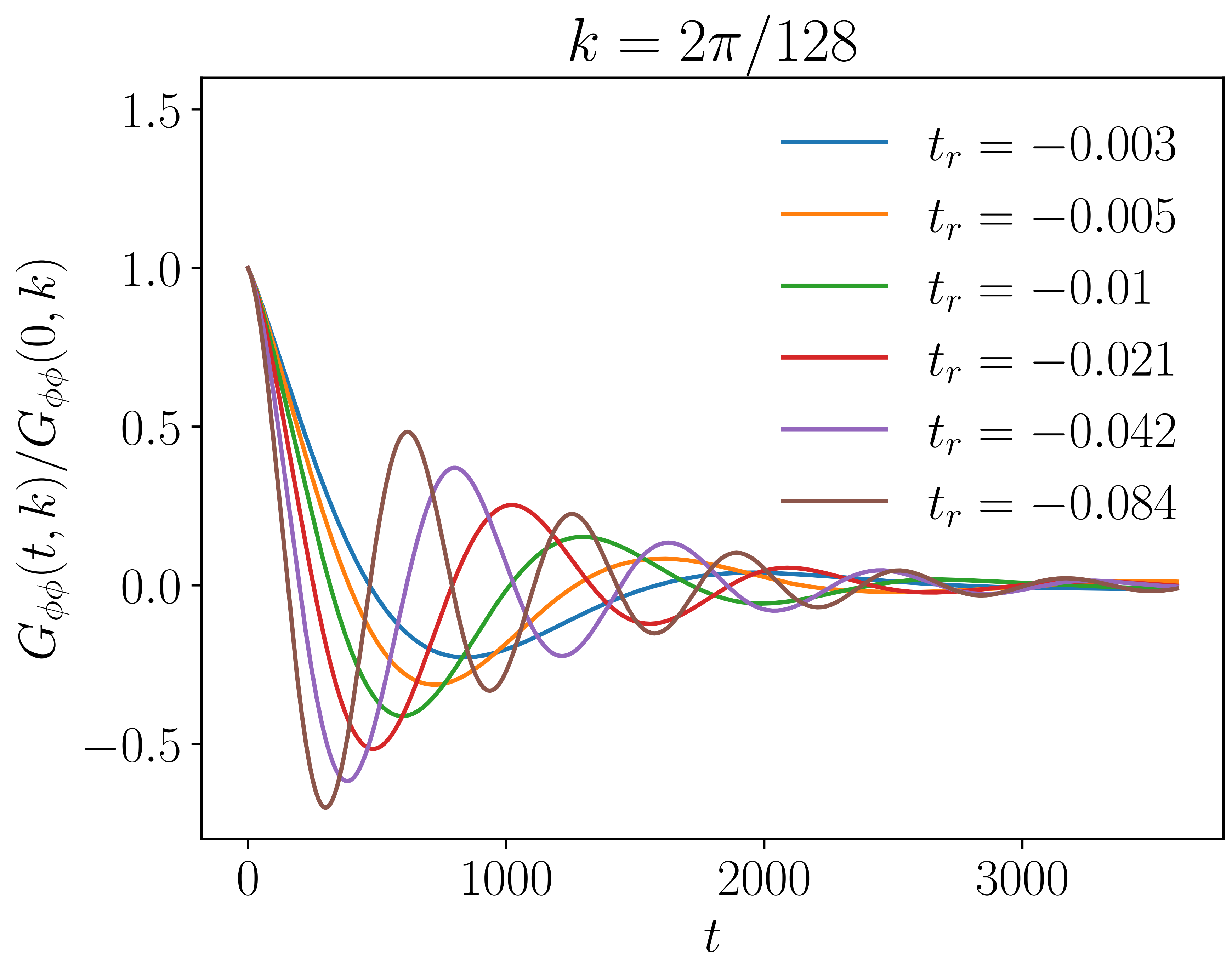


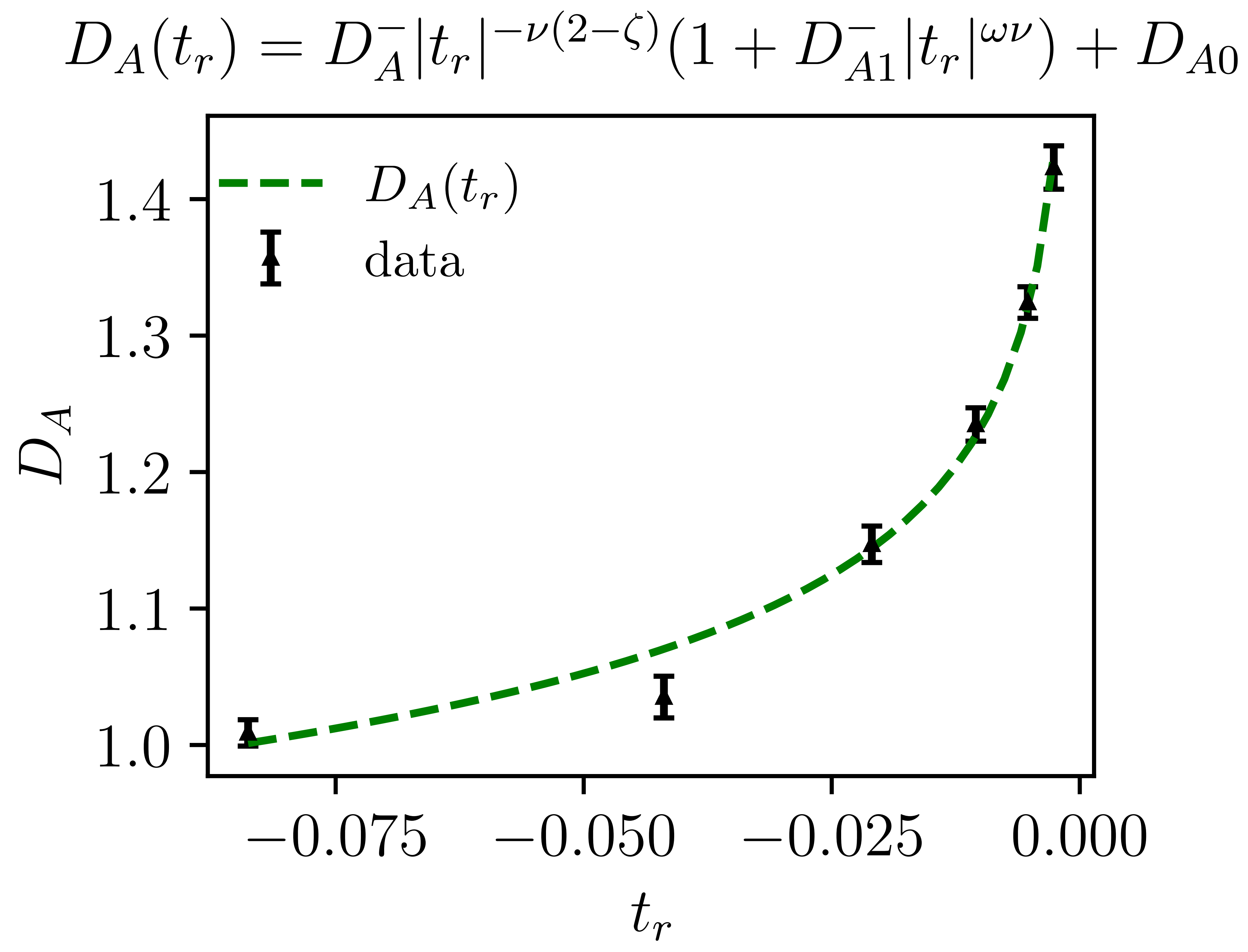
$$\omega(k) = vk + v_1 k^2.$$

The dispersion relation of the of the waves actually is determine by the GOR relation

$$v^2 \propto \langle \phi^2 \rangle$$

Axial Diffusion coefficient m_q=0





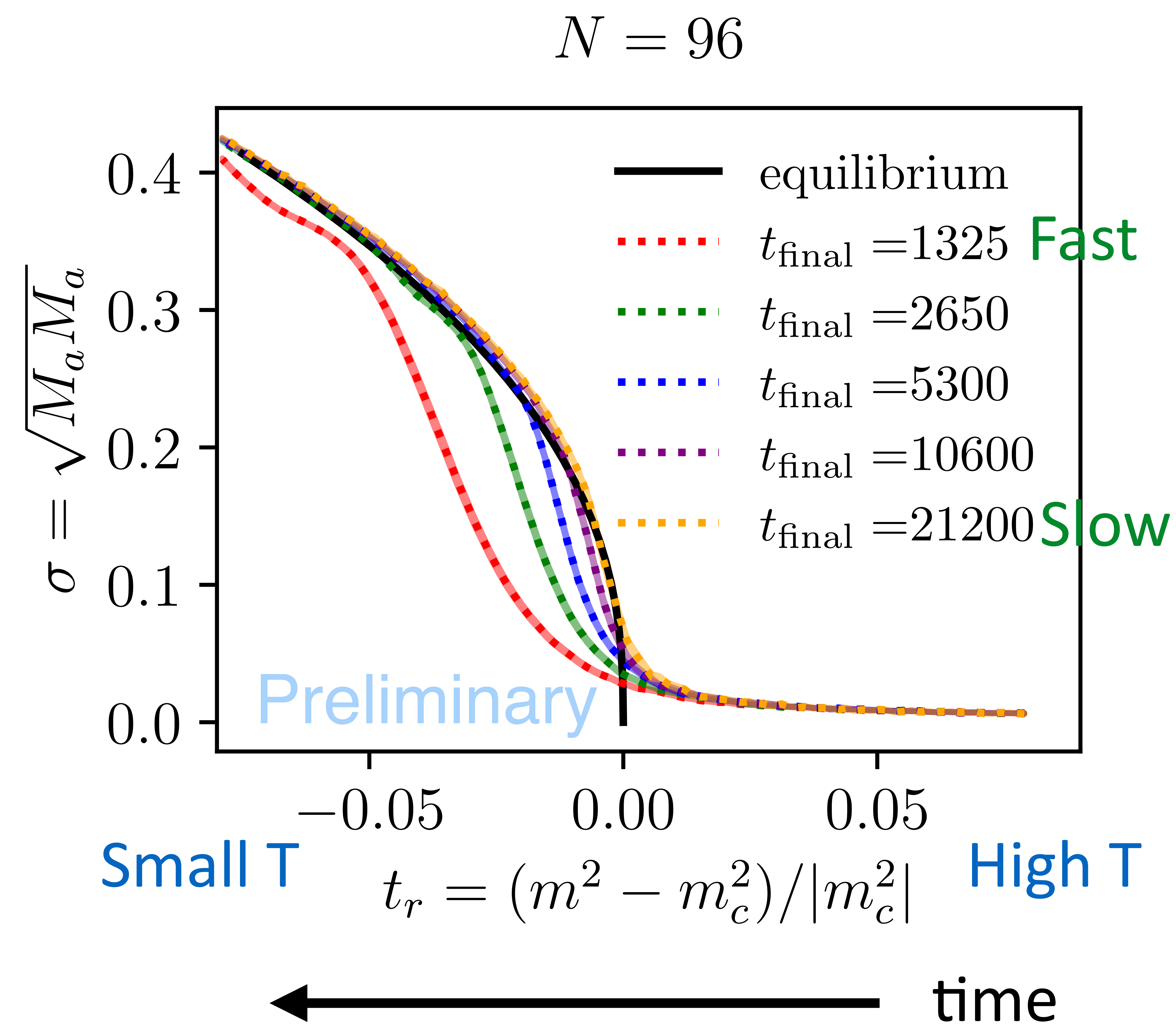
- The axial diffusion in the broken phase satisfies the scaling relation
- The constat part is big respect to the critical part
- There are severe dependencies form the finite volume effects, such as

Dumping rate of the pions $$\Gamma(k) = D_A k^2(1 + d_1/L)$$

Kibble-Zureck Protocol

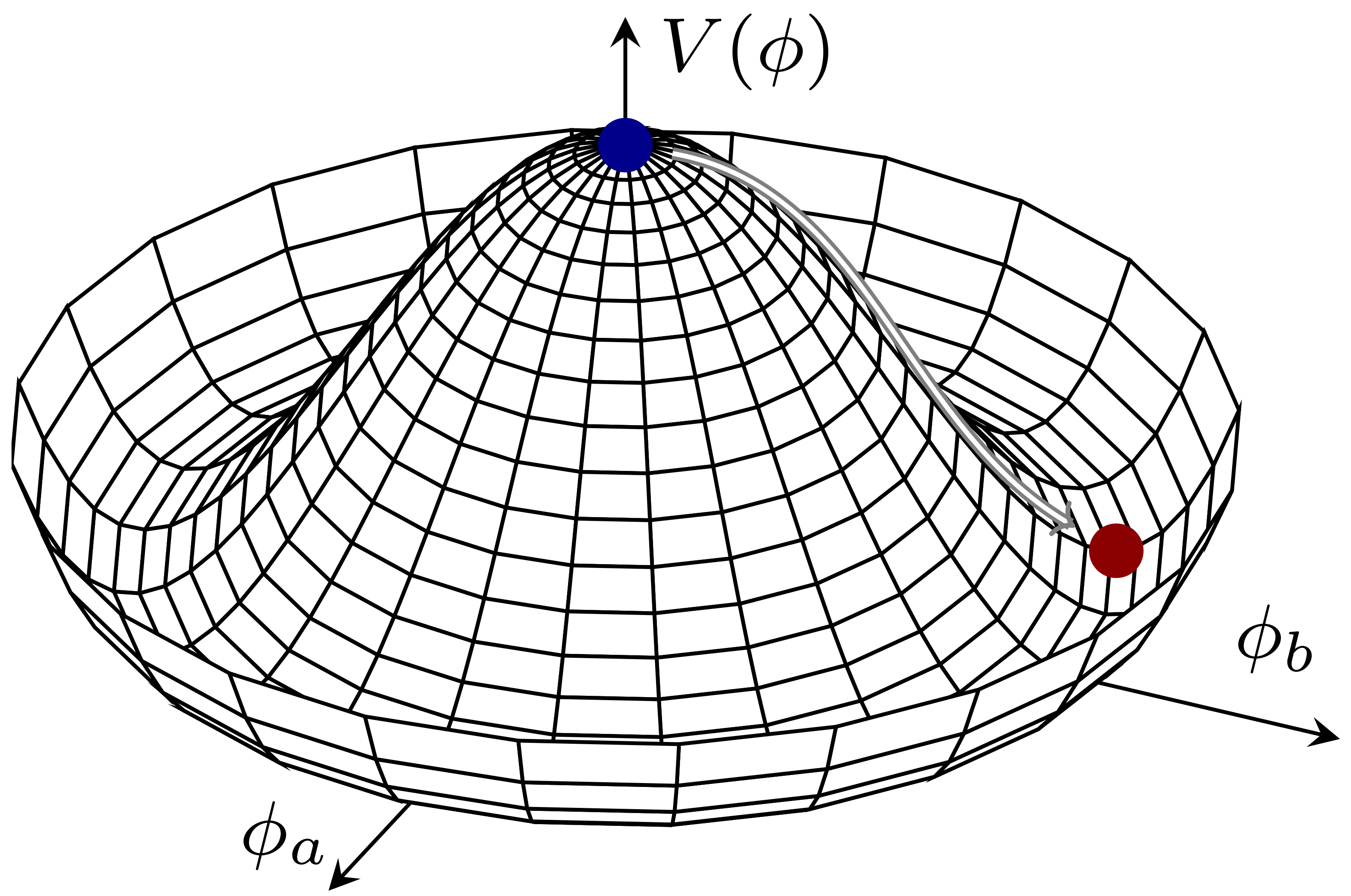
A.Florio, E.G., A.Mazeliauskas, A. Soloviev, D. Teaney in preparation

Starting form high temperature we performed a series of simulations where the temperature is lowerd at constat rate each time step



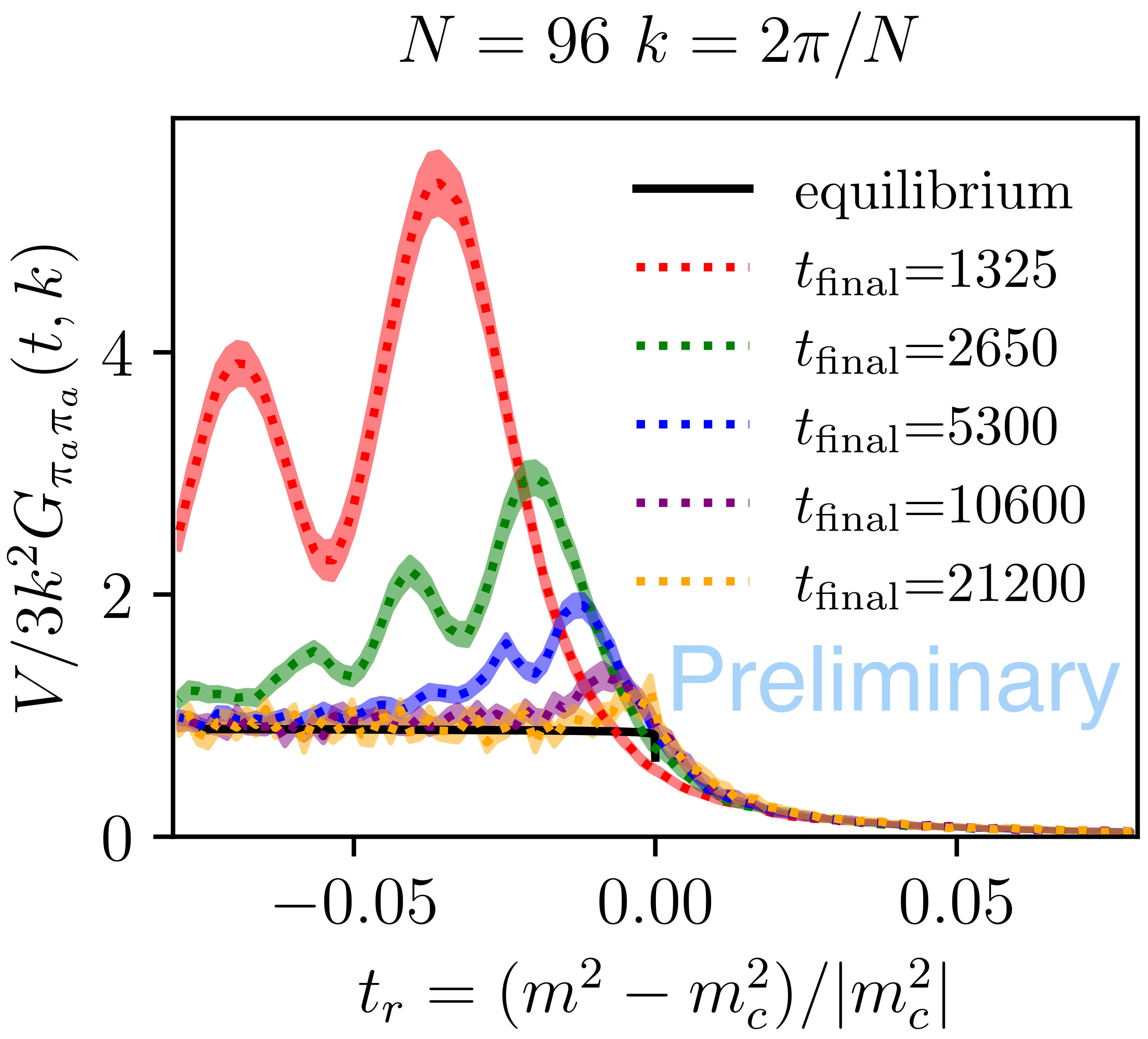
For slow rate the system has always time to requilibrate

For fast rate the system falls out of equilibirum

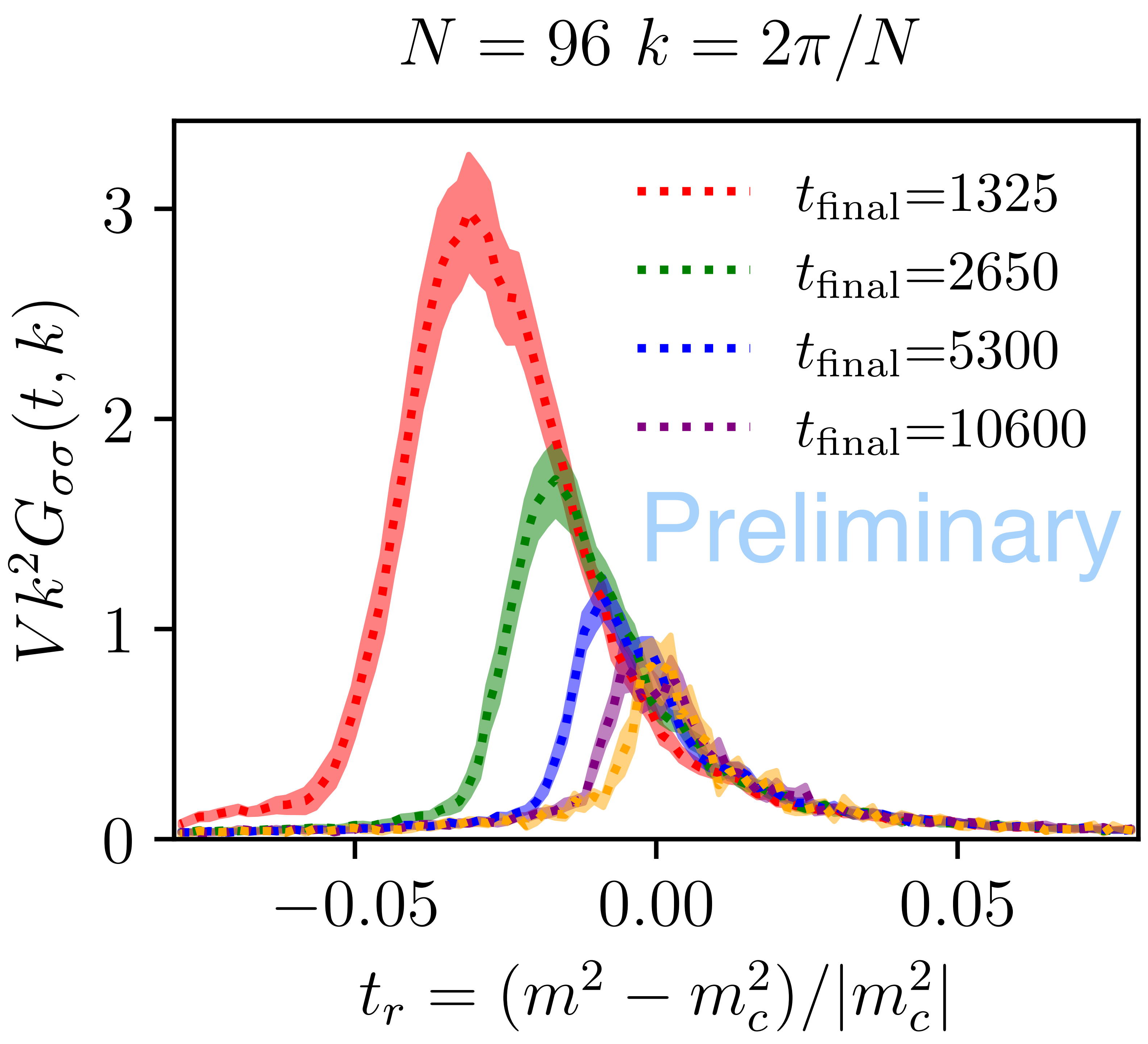


2-point function for small momenta

The equal time correlator



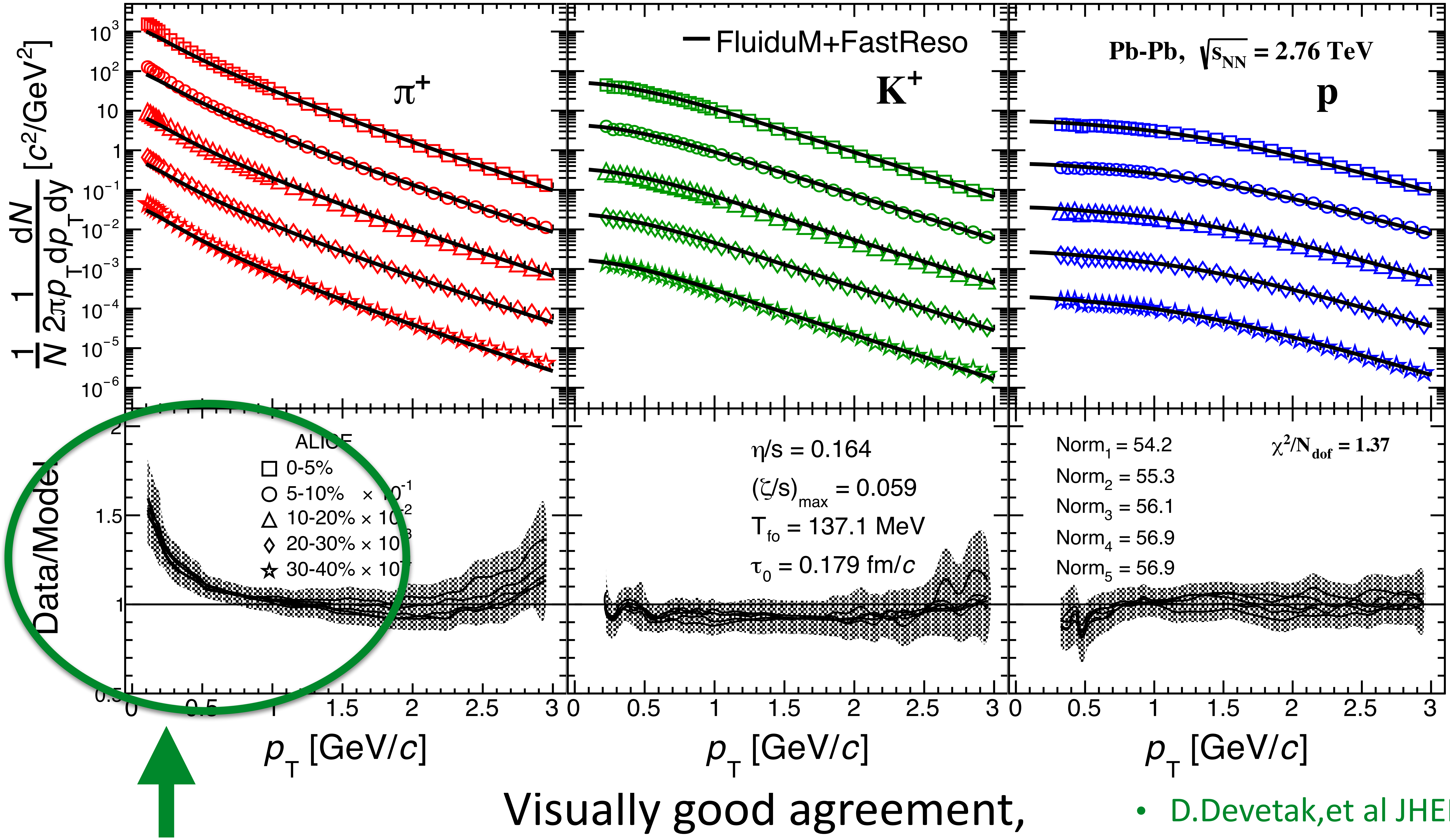
$$G_{\pi\pi}(t,k) \propto f_\pi(t,k)$$



- Both corrlator have an exponetial rise due to the rolling down of the potential
- The longitudianl model relaxes quickly to equilibrium
- The pion correlation starts oscillating and relaxes much slower
- The non-equilibirum contributions are big respect to equilibirium

Why this is important

Fit the pt spectra of pions kaons and protons in the first five centralities



Visually good agreement,

but statistically significant deviations

The main discrepancy is for pions at low pt

- D.Devetak,et al JHEP (2020)

Outlook

- We deeply and intensively study the real time dynamics of model G
- The model can accomadate the prodution of goldstone bosons in real time
- We study the Kibble-Zurek protocol. See also Mattis Harhoff Poster!
- Sudden quench into the broken phase induces pion pion correlation

More on phenomenology has to developed and studied

Open PhD position at Heidelberg University with Aleksas Mazeliauskas https://inspirehep.net/jobs/2786994