

# Phenomenology of Identified Particle Spectra in Heavy-Ion Collisions at LHC Energies

## Zubarev Meets Bayes

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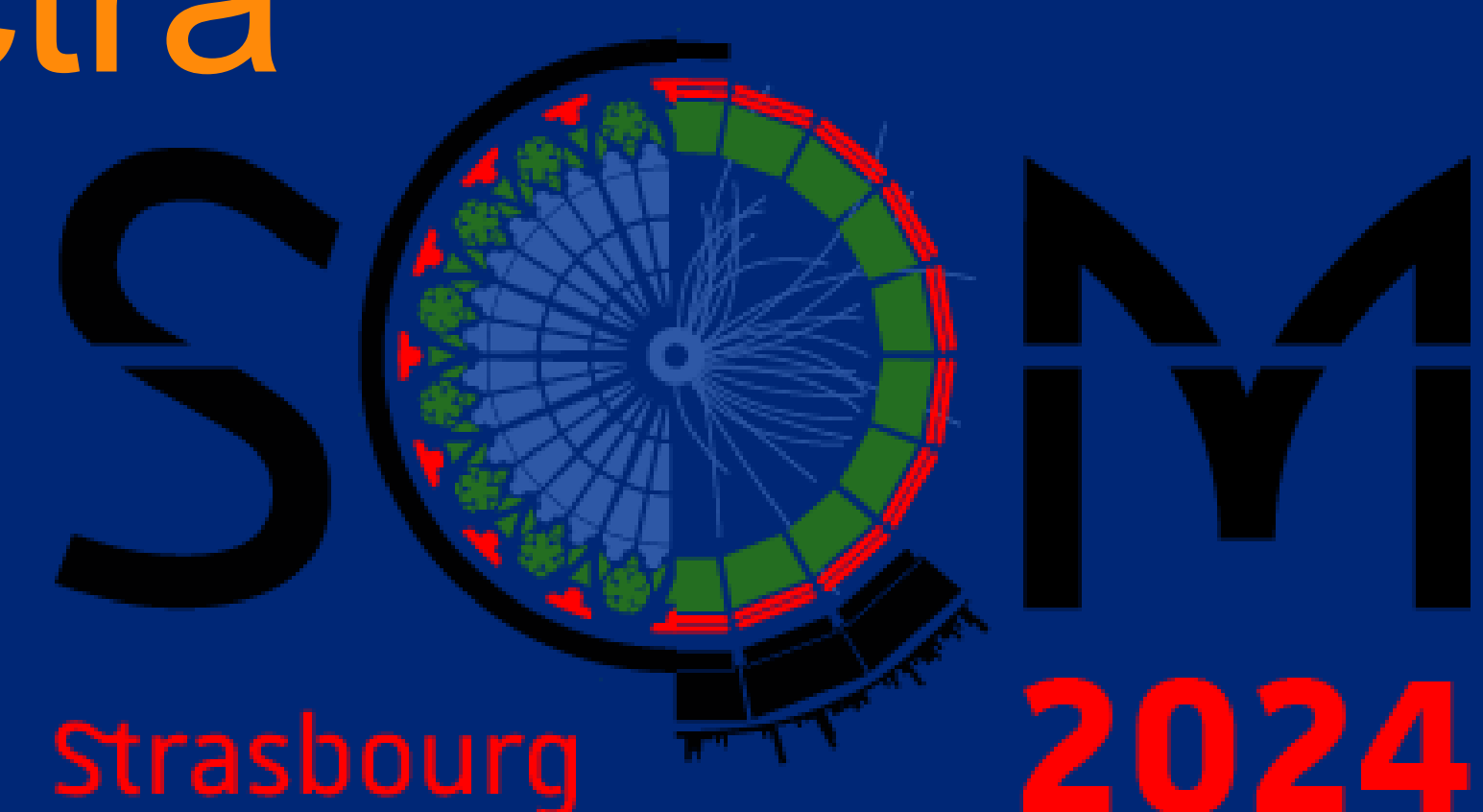
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### Introduction

The transverse momentum spectra of identified particles measured by the ALICE Collaboration [1] show an interesting feature, namely low- $p_T$  pion spectra show enhancement with respect to the predictions of various models. We apply the Zubarev approach of the non-equilibrium statistical operator [2] to develop a thermal particle generator that can account for the enhancement of soft pions by introducing an effective pion chemical potential. This is an alternative to the explanation of the low- $p_T$  enhancement by resonance decays. Bayesian inference methods are used to find the most probable sets of thermodynamic parameters at the freeze-out hypersurface.

### Zubarev Approach: Non-equilibrium Pion Distribution

The non-equilibrium state of the system is characterized by relevant observables  $\{B_n\}$  in addition to the standard set of conserved ones. We look for the distribution which maximizes the information entropy  $S_{\text{inf}} = -\text{Tr}\{\rho_{\text{rel}}(t) \ln \rho_{\text{rel}}(t)\}$ :

$$\rho_{\text{rel}}(t) = \frac{1}{Z_{\text{rel}}(t)} e^{-\sum_n F_n(t) B_n}, \quad Z_{\text{rel}}(t) = \text{Tr}\{e^{-\sum_n F_n(t) B_n}\},$$

where Lagrange multipliers  $F_n(t)$  are determined by the self-consistency conditions

$$\langle B_n \rangle^t = \langle B_n \rangle_{\text{rel}}^t = \text{Tr}\{\rho_{\text{rel}}(t) B_n\}.$$

In this work we assume the following:

- A state overpopulated by soft pions is formed at  $\tau < \tau_{\pi}^{\text{CFO}}$ .
- For  $\tau_{\pi}^{\text{CFO}} < \tau < \tau_{\pi}^{\text{FO}}$  the collisions conserve the particle number but evolve the distribution function to a thermal equilibrium distribution.

Under these assumptions, the pion number is quasi-conserved and can be chosen as a relevant observable. Then, the new self-consistency condition is:

$$\langle N_{\pi} \rangle_{\text{rel}}^t = \langle N_{\pi} \rangle^t.$$

This leads to the appearance of a non-equilibrium pion chemical potential [2]

$$f_{\pi} = \left( \exp \left[ \frac{E}{T} \right] - 1 \right)^{-1} \rightarrow f_{\pi} = \left( \exp \left[ \frac{E - \mu_{\pi}}{T} \right] - 1 \right)^{-1}$$

### Blast-Wave Model of Particle Freeze-out

Here, we consider chemical freeze-out on the cylindrical boost-invariant hypersurface of radius  $R$ , with constant temperature  $T$  at constant freeze-out proper time  $\tau = \sqrt{t^2 - z^2}$

$$\Sigma^{\mu} = (\tau \cosh \eta, r \cos \varphi, r \sin \varphi, \tau \sinh \eta), \text{ where } \eta = \frac{1}{2} \ln \frac{t+z}{t-z}.$$

The longitudinal expansion is assumed to be boost invariant, and the radial flow is parametrized as  $\beta_T = v(R/r)^n$  with the surface velocity  $v$  and profile exponent  $n$

$$u^{\mu} = (\cosh \rho \cosh \eta, \sinh \rho \cos \varphi, \sinh \rho \sin \varphi, \cosh \rho \sinh \eta), \text{ where } \rho = \text{atanh } \beta_T.$$

Then, for the particle with four-momentum

$$p^{\mu} = (m_T \cosh y, p_T \cos \psi, p_T \sin \psi, m_T \sinh y),$$

using the Cooper-Frye formula  $E \frac{d^3 N}{d^3 p} = \int_{\Sigma_{\text{FO}}} p^{\mu} d\Sigma_{\mu} f(x^{\mu}, p^{\mu} u_{\mu})$  one finds

$$\frac{d^6 N_i}{dp_T dy d\psi dr d\eta d\varphi} \propto \tau r p_T m_T \cosh(y - \eta) \times \left( \exp \left[ \frac{m_T \cosh \rho \cosh(y - \eta) - p_T \sinh \rho \cos(\varphi - \psi) - \mu_i}{T} \right] \pm 1 \right)^{-1}.$$

Here  $\tau, R, T, \{\mu_i\}, v$  and  $n$  are free model parameters.

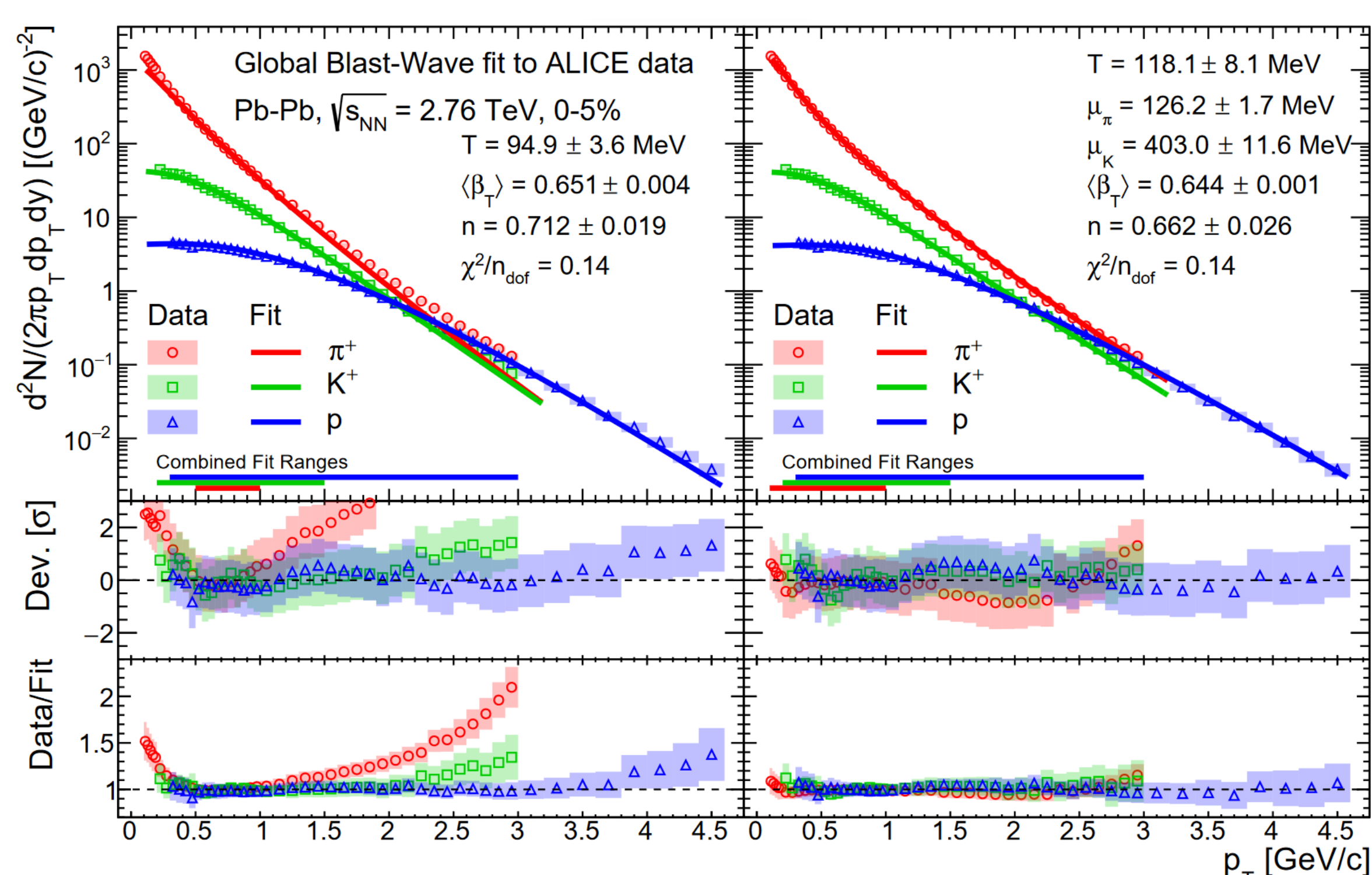


Figure 1. Left panel: The traditional blast-wave fit of ALICE data with Boltzmann statistics without chemical potentials. Right panel: Fit involving the Bose-Einstein statistics and chemical potentials of pions and kaons.

- ✓ Using two additional free parameters  $\mu_{\pi}$  and  $\mu_K$  one can achieve much better agreement between model and experimental data
- ✗ Feed-down and final state interactions are not taken into account explicitly
- ❖ Improved model: Thermal particle generator + Hadronic transport afterburner

### Bayesian Inference

For a model, which for an input parameter vector  $\vec{x} = (x_1, \dots, x_n)$  gives an output  $\vec{y} = \vec{y}(\vec{x}) = (y_1, \dots, y_m)$ , we want to find the “optimal” value of  $\vec{x}$  to describe the experimental data  $\vec{y}^{\text{obs}}$

$$P(\vec{x} | \vec{y}^{\text{obs}}) = \frac{\mathcal{L}(\vec{x}; \vec{y}^{\text{obs}}) P(\vec{x})}{P(\vec{y}^{\text{obs}})} \propto \underbrace{\mathcal{L}(\vec{x}; \vec{y}^{\text{obs}})}_{\text{Likelihood}} \times \underbrace{P(\vec{x})}_{\text{Prior}}$$

The goal is to find the Maximum a Posteriori (MAP) parameter, i.e. the point in parameter space which maximizes the posterior distribution.

**Model setup:**

- Blast-Wave thermal particle generator model with SMASH [3] afterburner
- Observables:  $p, \bar{p}, \pi^+, \pi^-, K^+, K^-$  spectra in 0-5% Pb-Pb@2.76 TeV collisions
- 200 training and 50 validation data sets
- Principal Component Analysis + Gaussian Processes emulator
- Markov Chain MC to recover posterior distribution
- Uniform prior

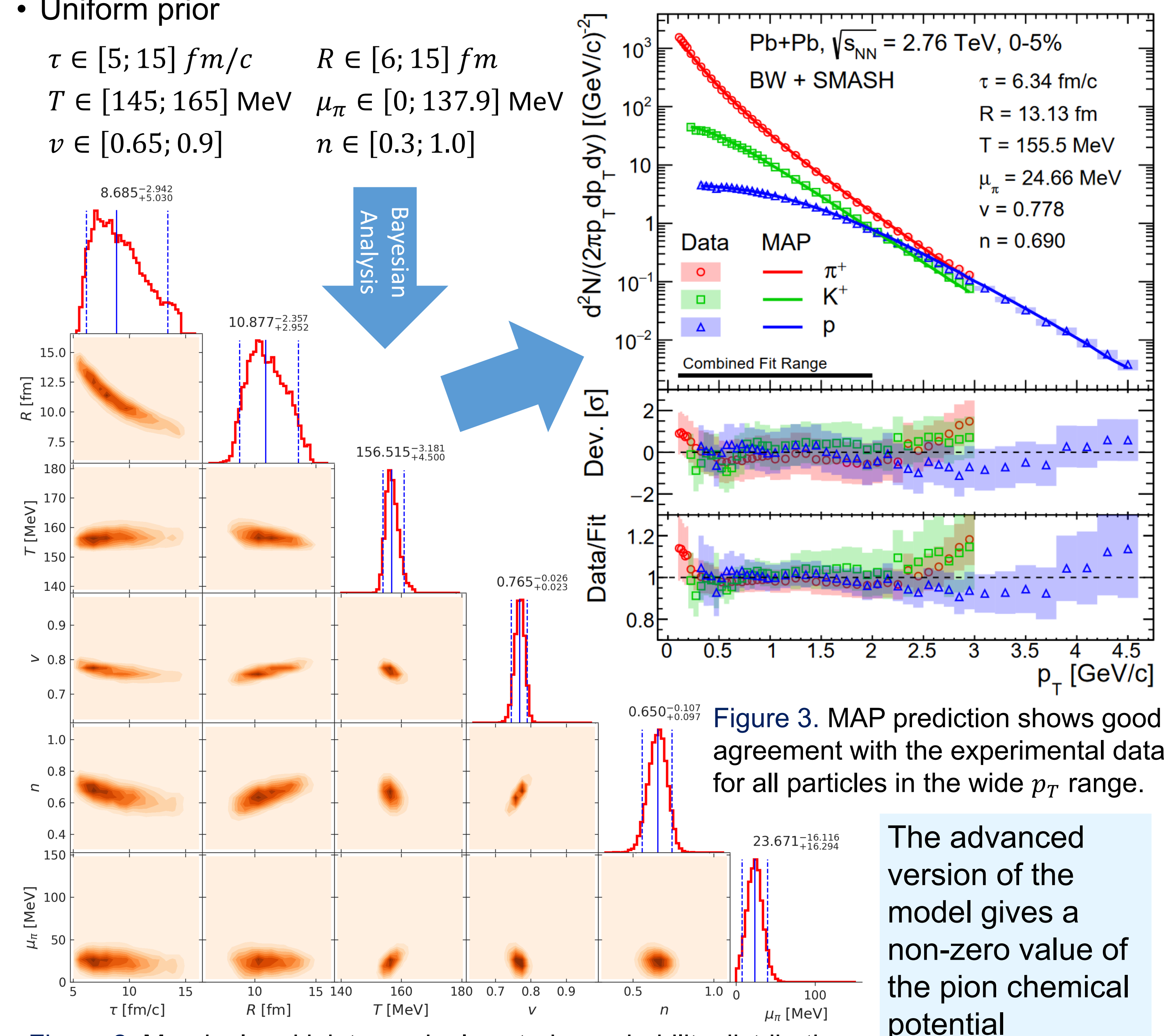


Figure 2. Marginal and joint marginal posterior probability distributions of model parameters. Blue solid lines indicate medians of the marginal distributions, while blue dashed lines show 90% credible intervals.

Figure 3. MAP prediction shows good agreement with the experimental data for all particles in the wide  $p_T$  range.

The advanced version of the model gives a non-zero value of the pion chemical potential  $\mu_{\pi} \approx 24$  MeV

### Summary

- The non-equilibrium process of pion production within the Zubarev approach of the non-equilibrium statistical operator leads to the appearance of a non-equilibrium pion chemical potential
- A naive model gives a value of non-equilibrium chemical potential close to the pion mass and can describe data well, but it does not take into account a feed-down from resonance decays and final state interactions
- A more sophisticated model gives smaller, but non-zero value of a non-equilibrium pion chemical potential  $\mu_{\pi} \approx 24$  MeV

### References

- [1] B. Abelev et al. (ALICE Collaboration), “Centrality dependence of  $\pi, K$  and  $p$  production in Pb-Pb collisions at  $\sqrt{s_{NN}} = 2.76$  TeV”, Phys. Rev. C 88, 044910 (2013)
- [2] D. Blaschke et al., “Nonequilibrium Pion Distribution within the Zubarev Approach”, Particles 2020, 3, 380–393
- [3] J. Weil et al., “Particle production and equilibrium properties within a new hadron transport approach for heavy-ion collisions”, Phys. Rev. C 94, 054905 (2016)