Renormalized chiral critical dynamics and fluctuations

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Introduction and motivation

- Extensive studies of **QCD phase diagram**. Focus on **phase transition** between QGP (deconfinement, restored chiral symmetry) and hadron gas (confinement, broken chiral symmetry). Special interest in conjectured **QCD critical point**;
- **Stochastic dynamics of the chiral order parameter**, whose fluctuations are expected to show extraordinary behavior around criticality;
- In stochastic models, UV divergences manifest as **unphysical lattice spacing (dx) dependence** in numerical calculations.

Improve treatment of this problem with **lattice renormalization techniques**.

Relaxational model (no conserved quantities)

Stochastic relaxation equation of chiral order parameter

$$\frac{\partial^2\varphi}{\partial t^2} - \nabla^2\varphi + \eta\frac{\partial\varphi}{\partial t} + \frac{\partial V_{\rm eff}}{\partial\varphi} = \xi$$

Ginzburg-Landau effective potential, ϵ encodes phase transition

$$V_{\rm eff}(\varphi) = \frac{1}{2}\epsilon\varphi^2 + \frac{1}{4}\lambda\varphi^4$$

White thermal noise

$$\langle\xi(\vec{x},t)\rangle = 0 \quad \text{and} \quad \langle\xi(\vec{x},t)\xi(\vec{x}',t')\rangle = 2\eta T\,\delta(\vec{x}-\vec{x}')\delta(t-t')$$

Here, only mass renormalization is required; corrective counterterm for $V_{\rm eff}$

$$V_{\rm CT} = \left\{-\frac{3\lambda\Sigma}{4\pi}\frac{T}{{\rm d}x} + \frac{3}{8}\left(\frac{\lambda T}{\pi}\right)^2\left[\ln\left(\frac{6}{M{\rm d}x}\right) + \zeta\right]\right\}\frac{\varphi^2}{2}$$

The system

- Cubic lattice, sides $L = 20$fm, periodic boundary conditions;
- N cells in each direction, lattice spacing $dx = dy = dz = \frac{L}{N}$;
- $\lambda = 0.25$;
- $T = M = \eta = 1$;
- Quantities are made dimensionless.

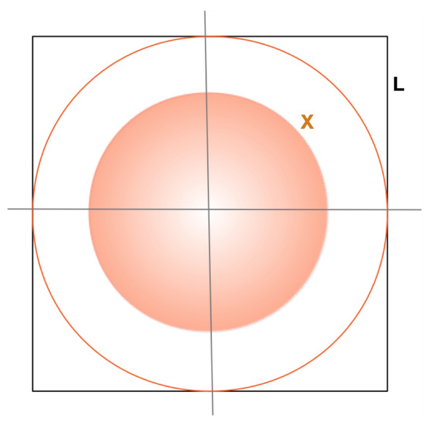
Observables

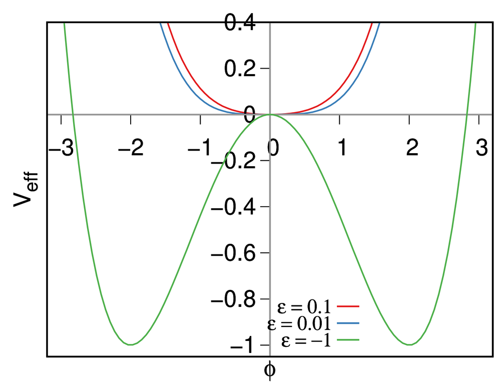
Volume average of φ for each noise configuration. Extract following observables from distribution over all configurations:

- Mean $\mathbf{\Phi}$;
- Variance $\boldsymbol{\sigma^2}$;
- Kurtosis as $\boldsymbol{\kappa\sigma^2}$.

Conditions

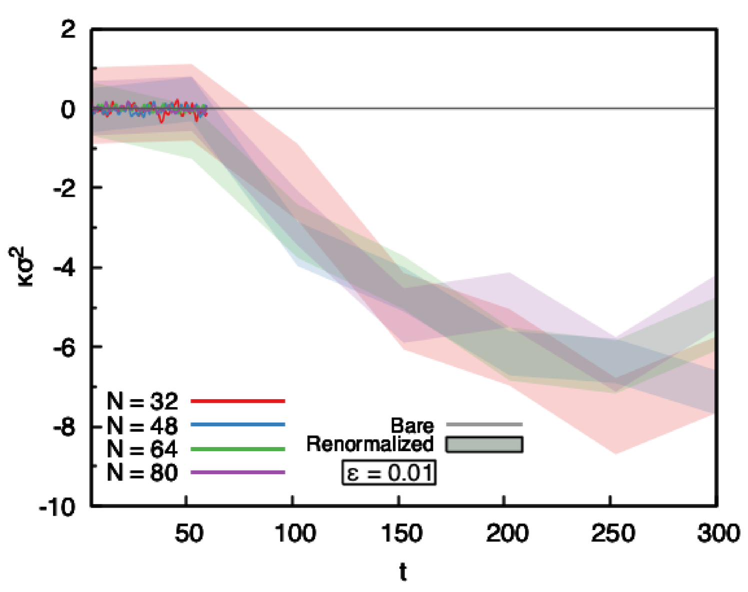
Bare (solid lines) and **renromalized** (dashed lines and shaded areas) systems.
Equilibrium at $t_{\rm fin}$, over volume varying with radius X.
Relaxation at fixed radius $X = 8$, over $t = 0 \rightarrow t_{\rm fin}$.





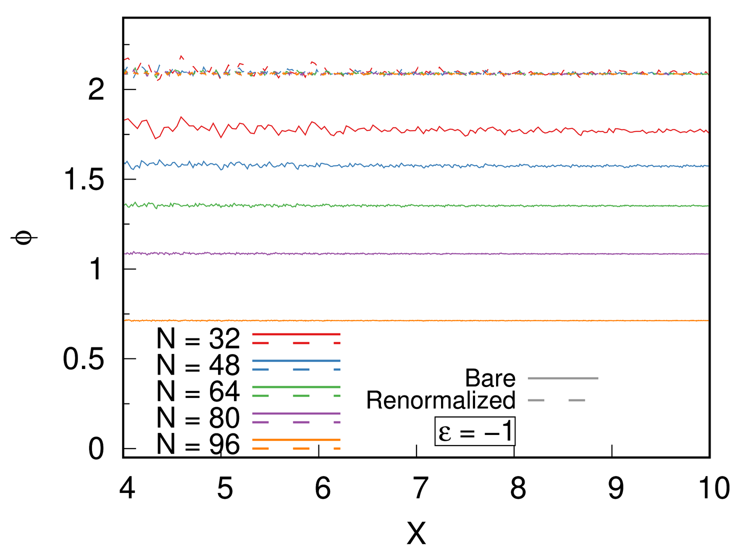
- $\boldsymbol{\epsilon = -1}$ broken symmetry. Φ and σ^2 at equilibrium and during relaxation;
- $\boldsymbol{\epsilon = 0.1}$, close to a critical point. Φ and σ^2 at equilibrium and during relaxation;
- $\boldsymbol{\epsilon = 0.01}$, closer to criticality. $\kappa\sigma^2$ during relaxation.

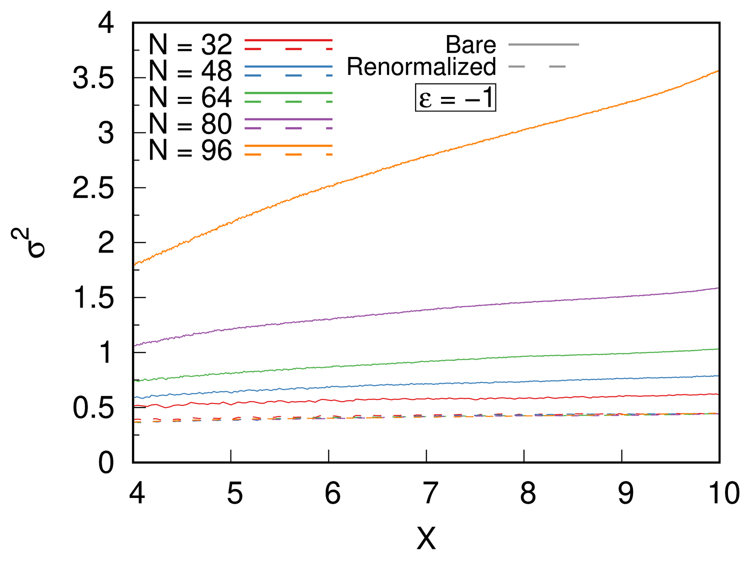
Dynamical relaxation of $\kappa\sigma^2$



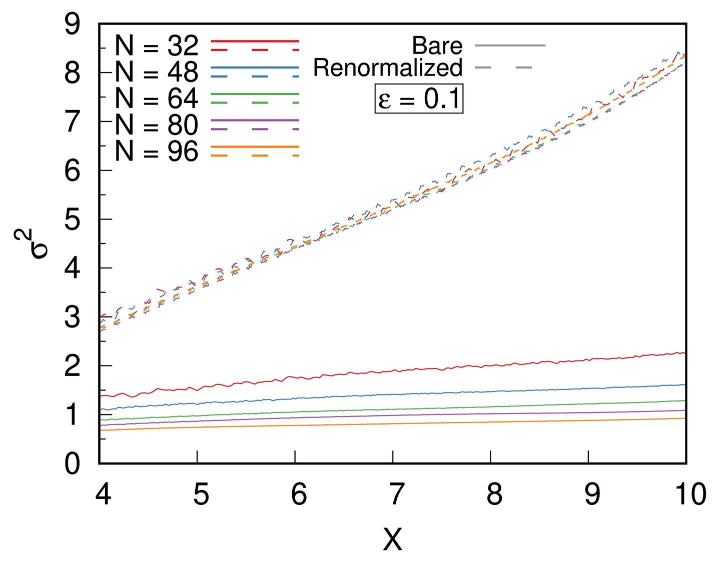
Closer to critical point, $t_{\rm fin} = 60$ sufficient in bare case, $t_{\rm fin} = 300$ needed for renormalized system.
Renormalization allows the restoration of expected non-Gaussianity; $\kappa\sigma^2$ takes finite non-zero values.

Φ and σ^2 at equilibrium



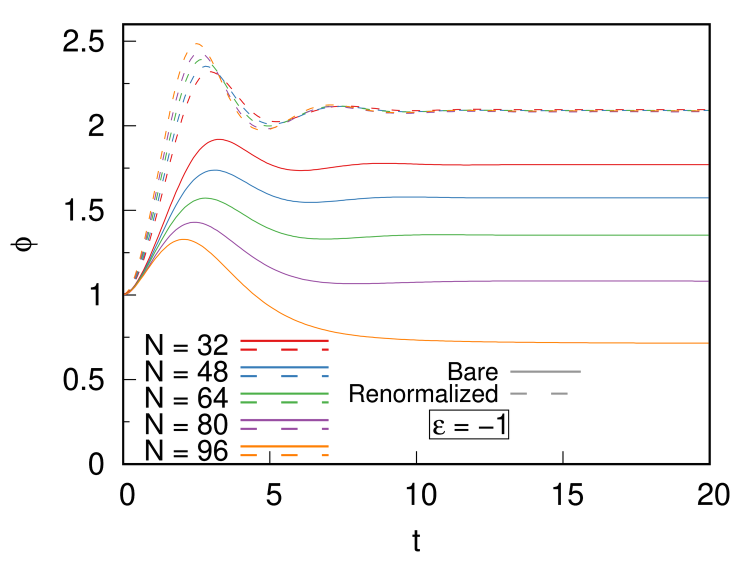


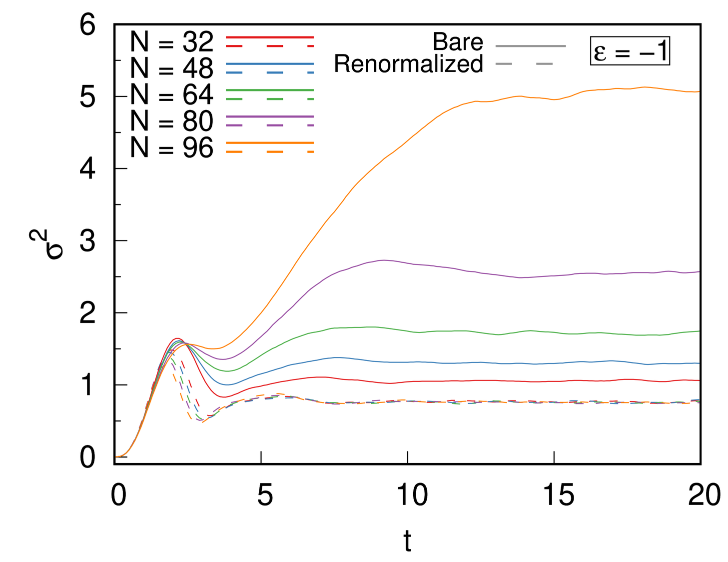
Chirally broken phase with 1st-order transition, $t_{\rm fin} = 60$. **Renormalization cures dx-dependence and restores expected equilibrium value of Φ.**



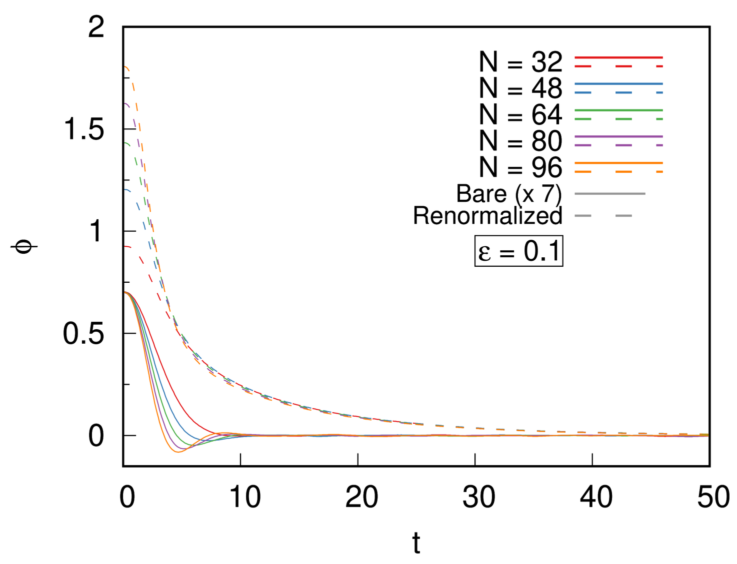
Close to critical point, $t_{\rm fin} = 60$. **Renormalization restores dx-independence.**

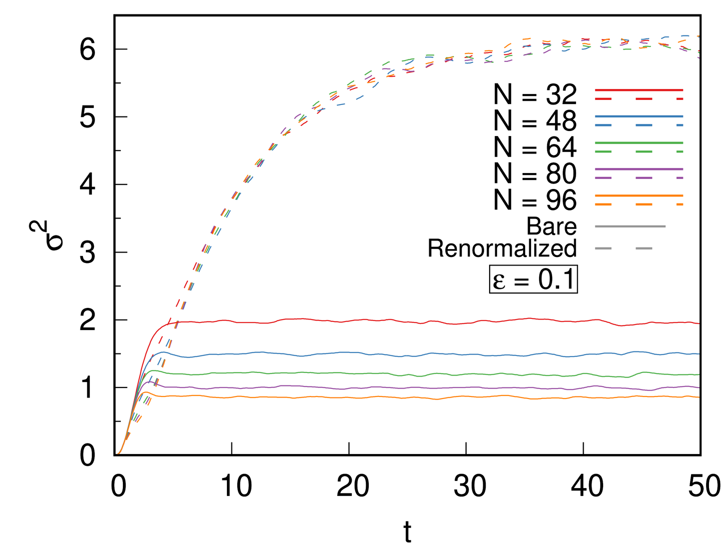
Dynamical relaxation of Φ and σ^2





Chirally broken phase with 1st-order transition, $t_{\rm fin} = 60$ and $\varphi_0 = 1$. **Addition of counterterm cures dx-dependence.**





Close to critical point, $t_{\rm fin} = 60$. $\varphi_0 = 0.1$ for bare system, $\varphi_0 = \varphi(dx)$ in renormalized case. **Renormalization cures dx-dependence.**

Summary and conclusion

- **Lattice renormalization derived counterterm cures dx-dependence in Φ and σ^2**, at equilibrium and during relaxation, both in chirally broken phase ($\epsilon = -1$) and close to critical point ($\epsilon = 0.1$);
- **The same mass counterterm restores non-Gaussian behavior of $\kappa\sigma^2$** closer to criticality ($\epsilon = 0.01$);
- Within available statistics for $\kappa\sigma^2$, robust conclusions for non-Gaussian behavior when $\epsilon = 0.1$ and for restoration of dx-independence when $\epsilon = 0.01$ were not obtainable.

SQM Strasbourg 2024

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