Finite density QCD equation of state: critical point and lattice-based T′-expansion (Ising-TExS)

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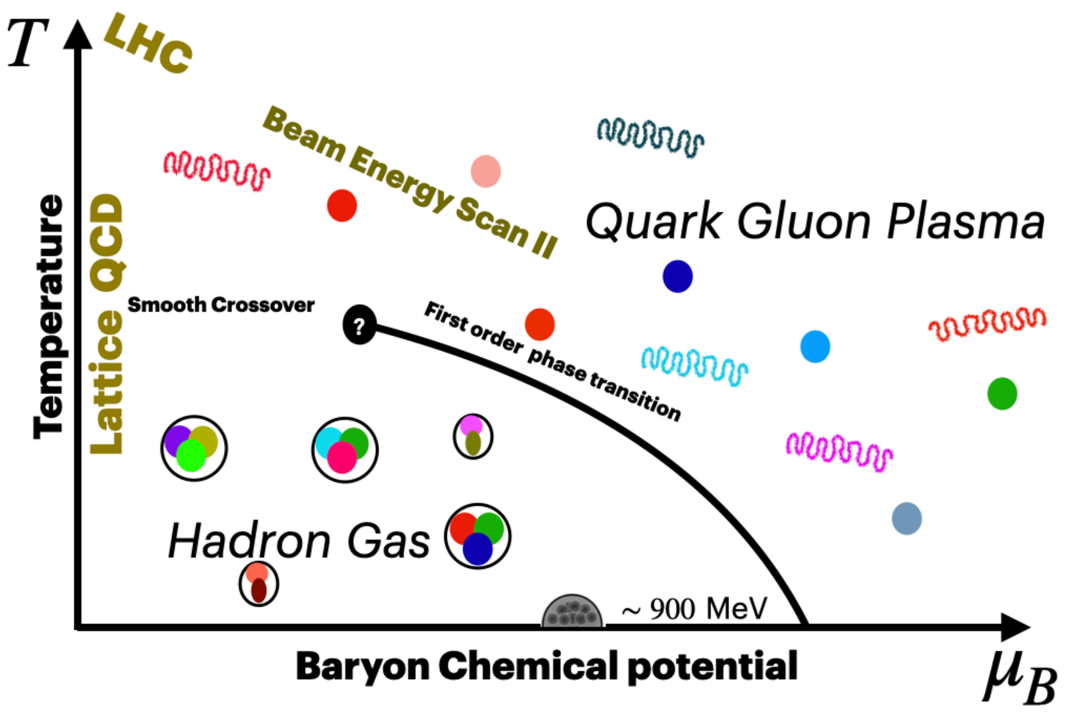
Phase diagram



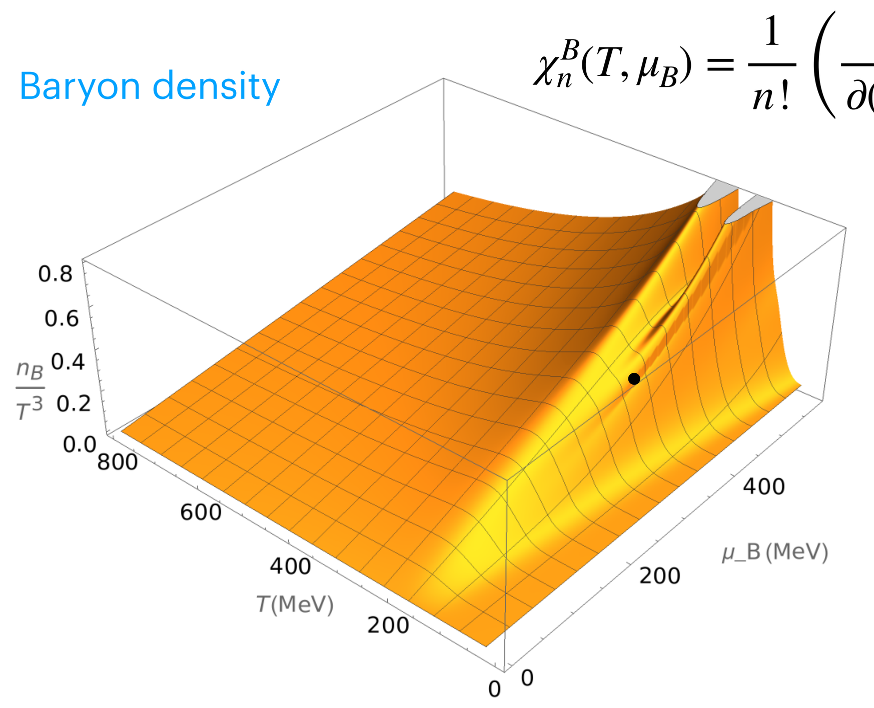
At $\mu_B = 0$ Quantum chromodynamics (QCD) phase diagram is well established.

At finite μ_B a critical point is expected but has not yet been observed.

Taylor Expansion

$$n_B(T,\mu_B) = T^3 \sum_{n=1}^{2} \frac{1}{(2n-1)!} \chi_{2n}^{\text{non-Ising}}(T) \left(\frac{\mu_B}{T}\right)^{2n-1} + \frac{T_C^4}{T} n_B^{\text{Ising}}(T,\mu_B)$$

$$\chi_n^B(T,\mu_B) = \frac{1}{n!}\left(\frac{\partial}{\partial(\mu_B/T)}\right)^n (P/T^4)$$



For $\mu_B > 450$ MeV, thermodynamics observables have unphysical oscillations due to limitations of truncated Taylor expansion, hindering critical point studies [1]

Motivation

The available Equation of State (EoS) with a critical point has limited coverage in baryon chemical potential μ_B due to the truncation of the Taylor expansion. [1]

Goal: To build an Equation of State with a critical point from a 3D Ising model that captures a large part of the phase diagram and matches lattice QCD results at low chemical potential μ_B.

Tools

- T′-expansion scheme [2]
- 3D-Ising Model

T′- Expansion Scheme

As a solution, the Wuppertal-Budapest lattice collaboration [2] developed a T'- expansion scheme that exhibits smooth behavior at high μ_B and copes well with the QCD transition temperature.

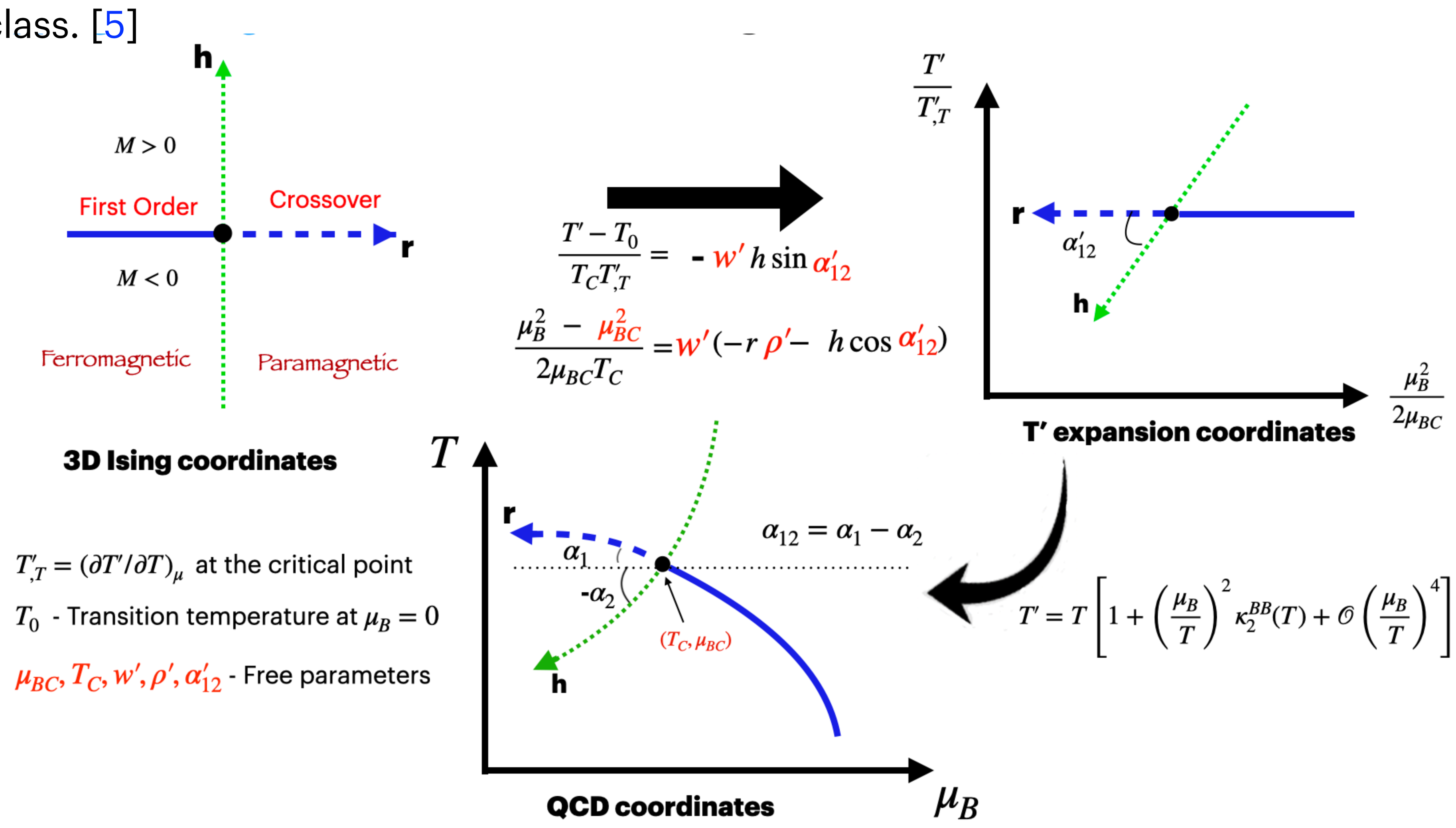
$$T\frac{\chi_1^B(T,\mu_B)}{\mu_B} = \chi_2^B(T',0)$$

$$T'(T,\mu_B) = T\left[1 + \kappa_2^{BB}(T)\left(\frac{\mu_B}{T}\right)^2 + \kappa_4^{BB}(T)\left(\frac{\mu_B}{T}\right)^4 + \mathcal{O}\left(\frac{\mu_B}{T}\right)^6\right]$$

From the T′-expansion scheme, as long as $T\chi_1^B/\mu_B$ is smooth, then finite density physics, such as the critical point can be encoded in T'.

Mapping 3D-Ising to QCD

If the critical point in QCD exists, then it must be in the 3D-Ising model universality class. [5]



Merging 3D-Ising with T′-Expansion

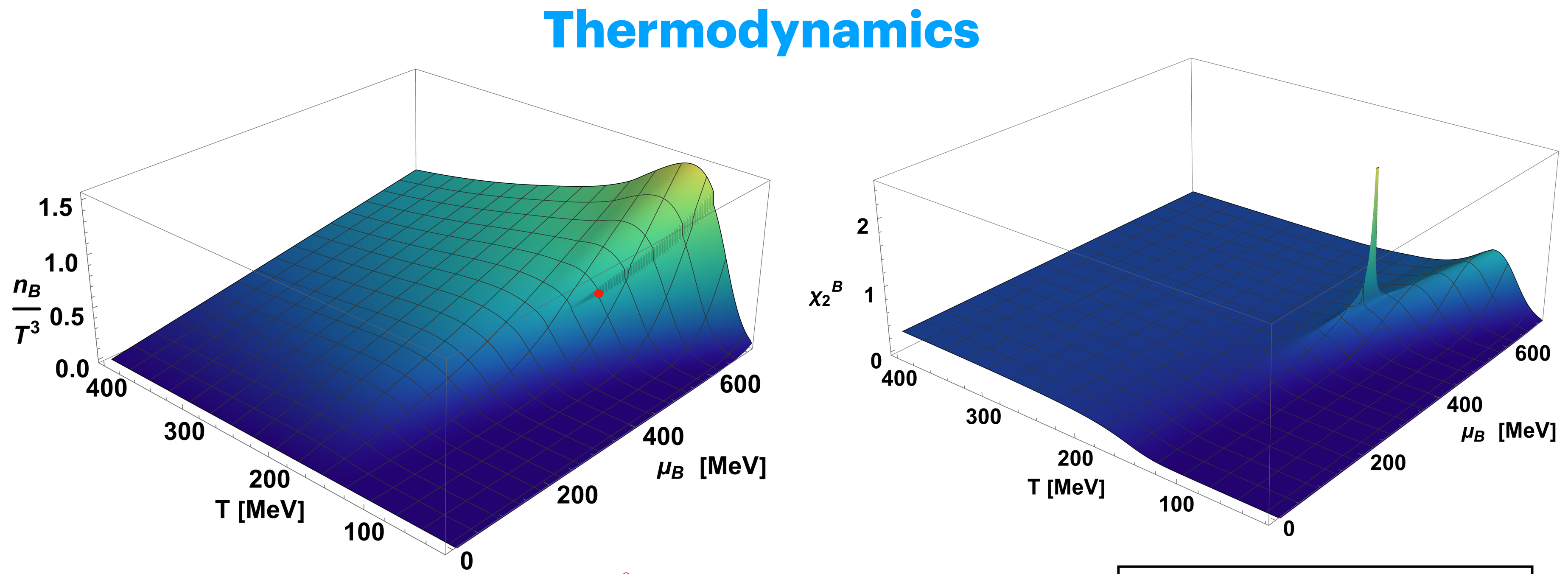
$$\frac{n_B(T,\mu_B)}{T^3} = \chi_1^B(T,\mu_B) = \left(\frac{\mu_B}{T}\right)\chi_{2,lat}^B(T',0)$$

We introduce the critical point in T' by separating into the critical part T'_{crit} and the non-critical parts [4,5]

$$T' = \underbrace{T'_{lat}(T,\mu_B)}_{\text{lower order in } (\frac{\mu_B}{T})} + \underbrace{T'_{crit}(T,\mu_B) - \text{Taylor}[T'_{crit}(T,\mu_B)]}_{\text{higher orders in } (\frac{\mu_B}{T})}$$

$$T'_{crit}(T,\mu_B) \approx \left(\frac{\partial \chi_{2,lat}^B(T,0)}{\partial T}\bigg|_{T=T_0}\right)^{-1} \frac{n_B^{crit}(T,\mu_B)/T^3}{(\mu_B/T)}$$

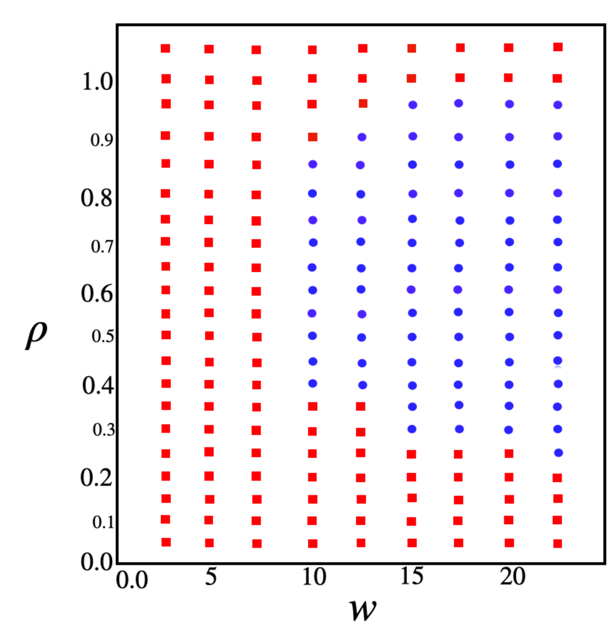
Thermodynamics



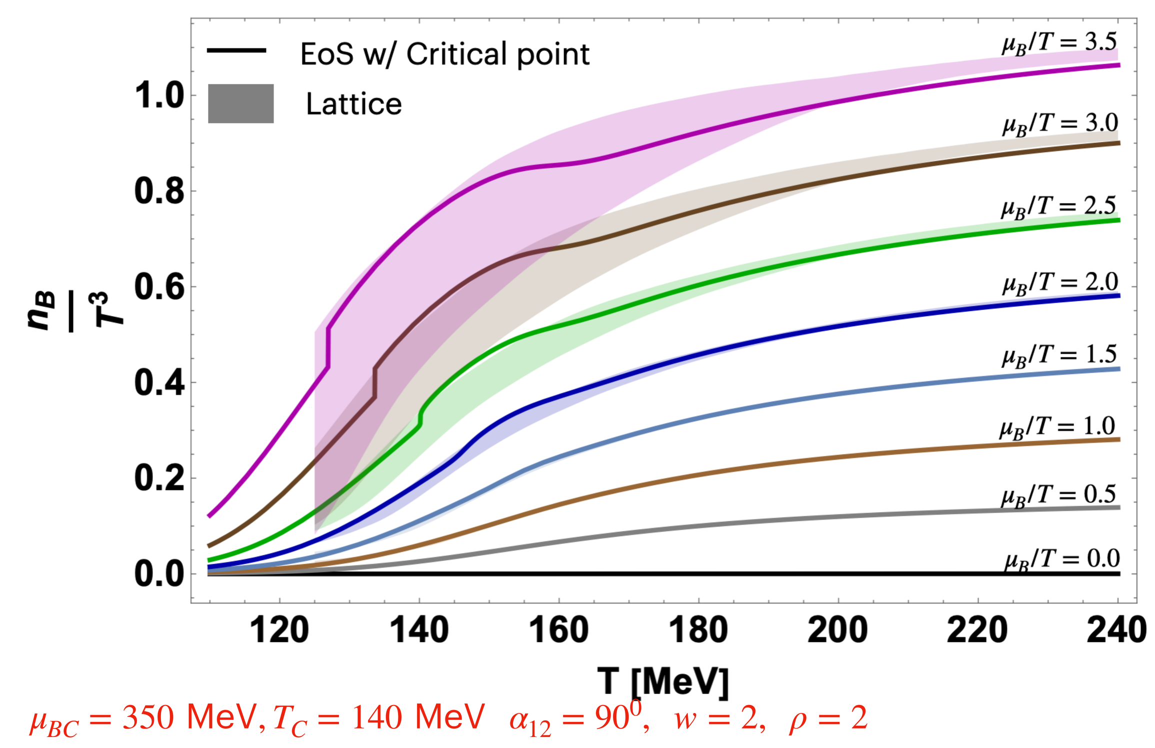
$\mu_{BC} = 600$ MeV, $T_C = 94$ MeV $\alpha_{12} = \alpha_1 = 14^0$, $w = 15$, $\rho = 0.3$

Known constraints on the EoS

- Lattice QCD: disfavors $\mu_{BC} < 300$ MeV
- Transition line: Choosing μ_{BC} fixes T_C and α_1 [3]
- Physical quark masses: $\alpha_{12} = \alpha_1$
- Stability and causality: fix w and ρ



Results



$\mu_{BC} = 350$ MeV, $T_C = 140$ MeV $\alpha_{12} = 90^0$, $w = 2$, $\rho = 2$

We check that we match lattice QCD results at $\mu_B = 0$, and our EoS with a critical point is within the error band of extrapolated lattice QCD results for certain parameter choices. [4,5]

Summary

Disclaimer! : We don't predict the location of the critical point

We provide an **enhanced coverage** for family of EoS with a 3D Ising critical point up to $\mu_B = 700$ MeV and matching lattice at low μ_B with adjustable parameters. [4,5]

References

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[3] Pradeep, M. S. & Stephanov, M., **Phys.Rev.D** 100(5), 056003 (2019)
[4] The code for this work is found at: DOI=10.5281/zenodo.10652327
[5] Kahangirwe M., Johannes J. *et al.*, **Phys.Rev.D** 109 094046(2024)

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