



# Heavy flavor production under a strong magnetic field

Shile Chen

in collaboration with Jiaxing Zhao and Pengfei Zhuang

Based on arXiv:2401.17559v1

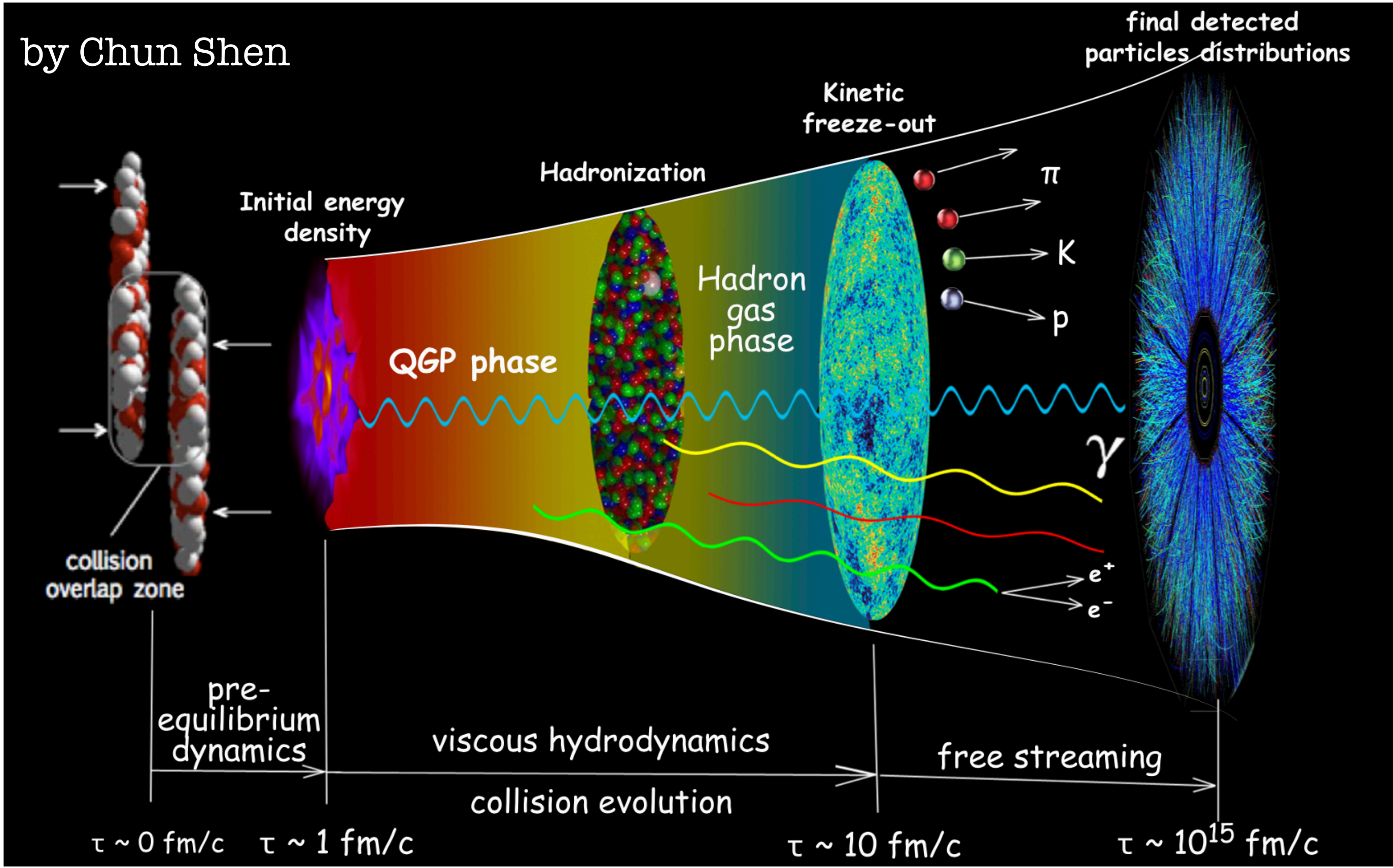


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# Heavy ion collision and magnetic field

by Chun Shen



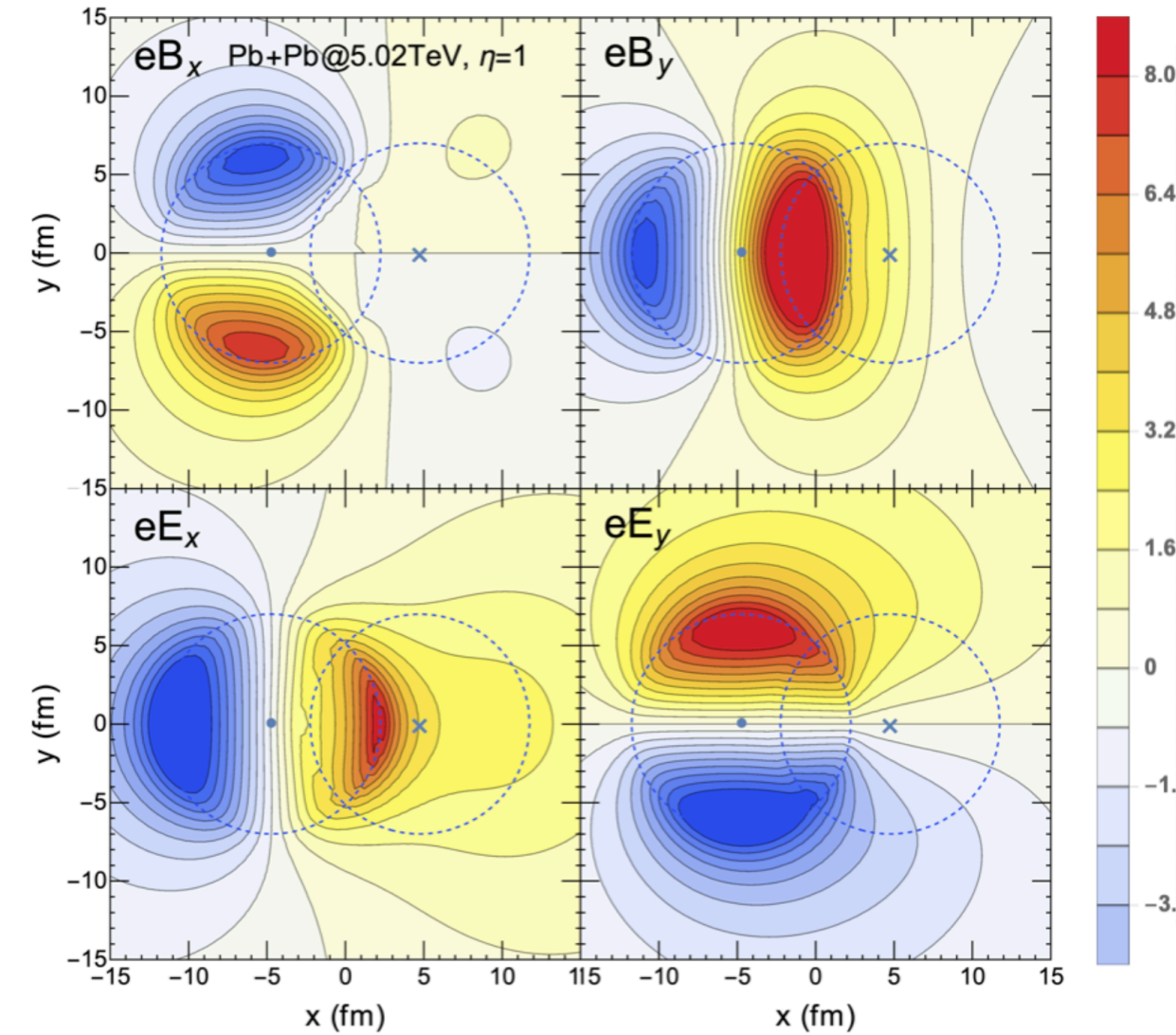
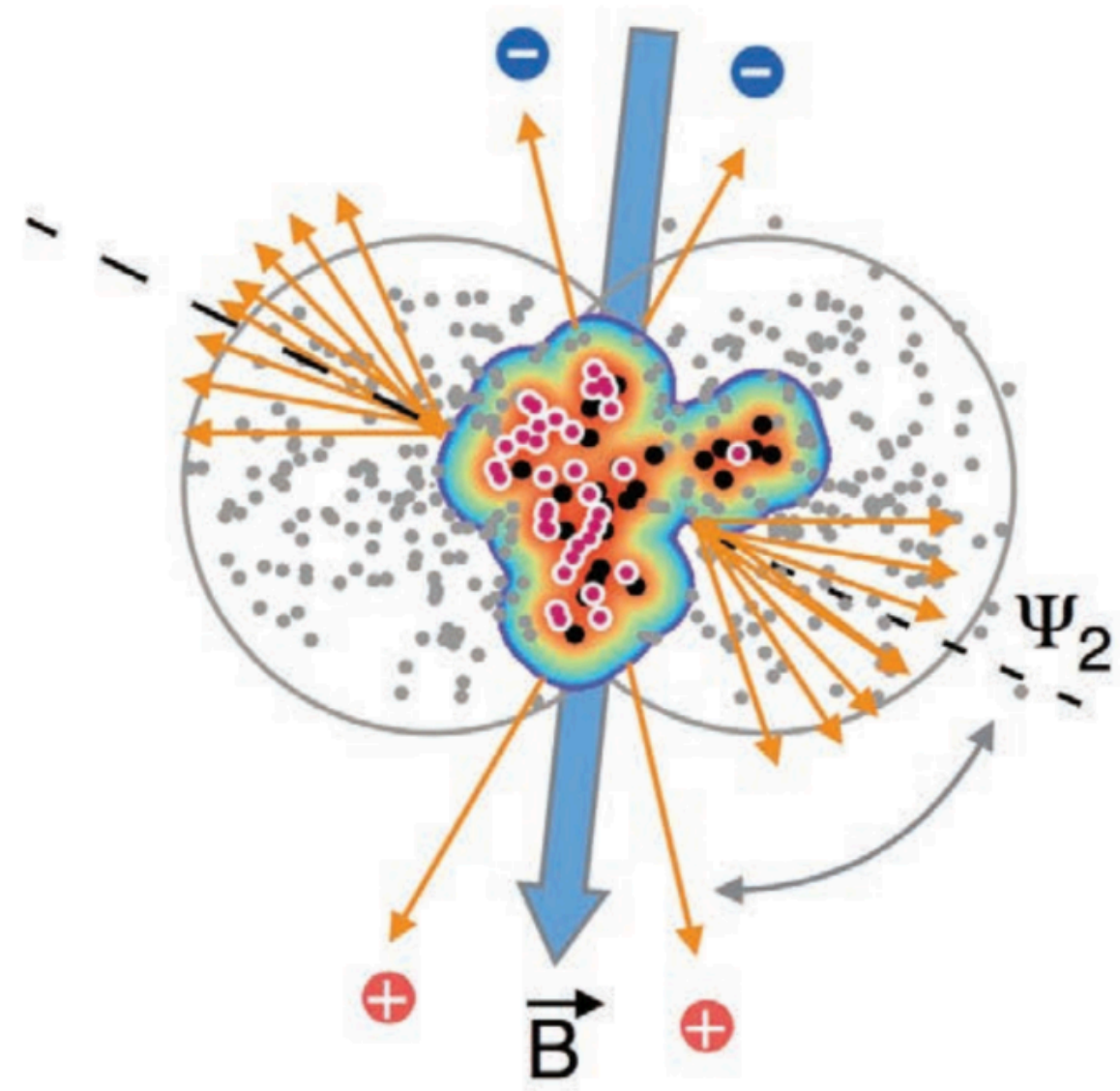
Initial state:  
 Hard process without hot medium  
 Strong magnetic field

**Heavy quark:**

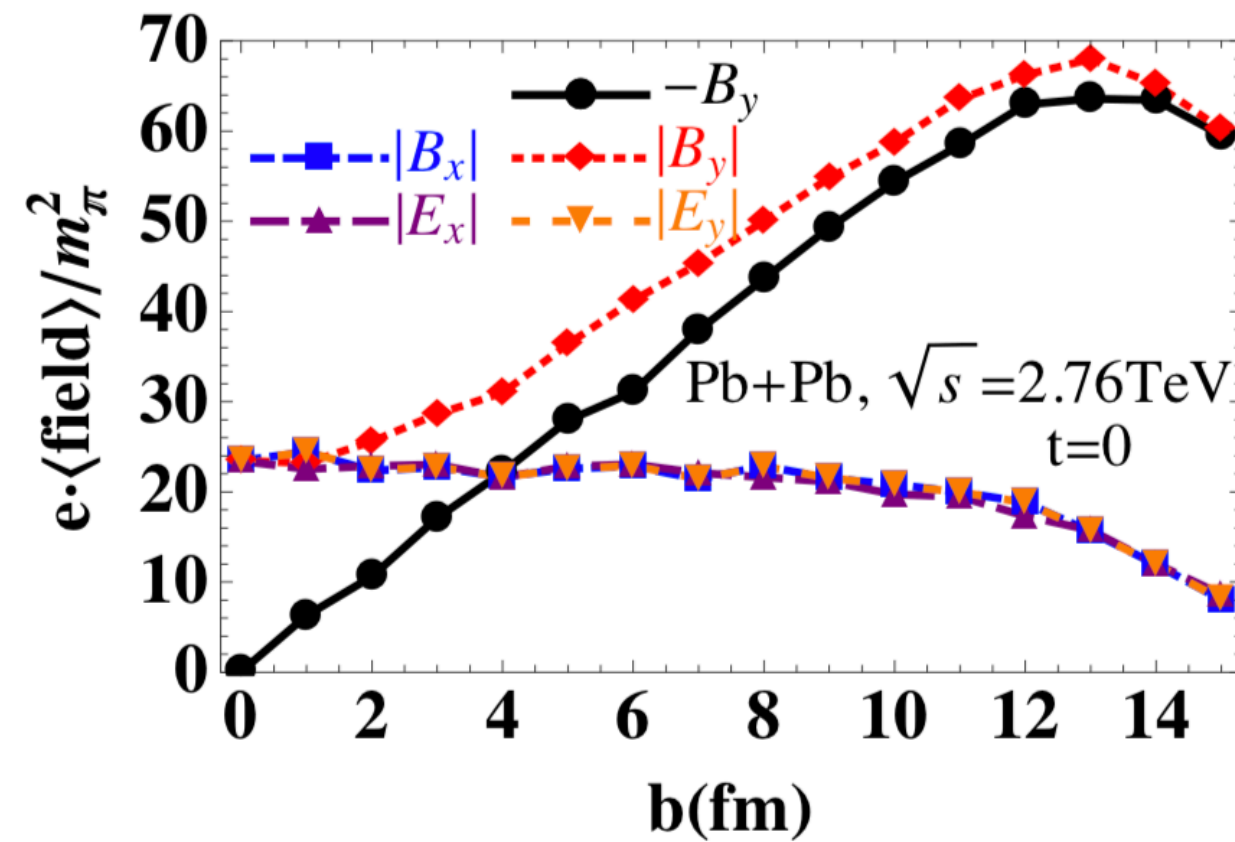
1. Number considered to be conserved after initial state
2. Remember more about initial stage

**A probe of Magnetic field!**

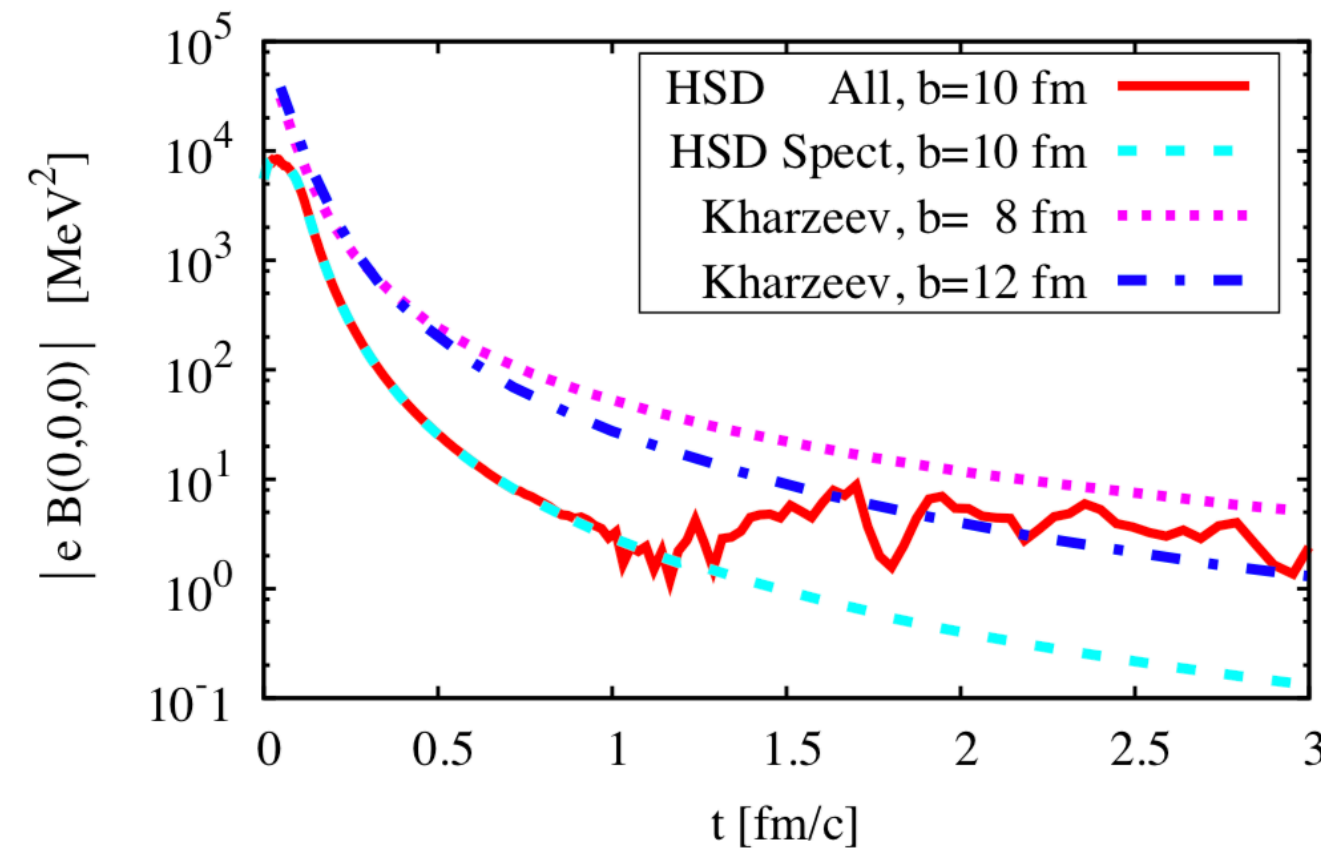
# Heavy ion collision and magnetic field



AuAu,  $\sqrt{s_{NN}} = 200$  GeV



W-T Deng, X-G Huang. Phys. Rev. C 2012



Voronyuk V, Toneev V, Cassing W, et al. Phys.Rev. C, 2011

## Strong Magnetic field :

$$\text{RHIC } eB \sim 5m_\pi^2 \quad \text{LHC } eB \sim 70m_\pi^2$$

## Magnetic Catalysis & Inverse Magnetic Catalysis

Igor A. Shovkovy, Magnetic Catalysis: A Review, Lect.Notes Phys. 871 (2013) 13-49

Falk Bruckmann, Gergely Endrodi, Tamas G. Kovacs, Inverse magnetic catalysis and the Polyakov loop, JHEP, 2013, 04:112.

## Chiral Magnetic Effect

Fukushima K, Kharzeev D E, Warringa H J. The Chiral Magnetic Effect Phys. Rev., 2008, D78:074033

D.E. Kharzeev, J. Liao, Isobar Collisions at RHIC to Test Local Parity Violation in Strong Interactions Nucl.Phys.News 29 (2019) 1, 26-31

## Isobar testing

Shi, Shuzhe and Zhang, Hui and Hou, Defu and Liao, Jinfeng, Signatures of Chiral Magnetic Effect in the Collisions of Isobars, Phys. Rev. Lett., 125, 242301 (2020)

Sergei A. Voloshin, Testing the Chiral Magnetic Effect with Central U + U collisions, Phys. Rev. Lett. 105, 172301 (2010)

# Magnetic field to heavy flavor

## To Static properties

### Heavy quarkonium mass

J. Alford and M. Strickland, Charmonia and Bottomonia in a Magnetic Field, Phys. Rev. D 88 (2013) 105017

### Heavy quarkonium dissociation

K. Marasinghe and K. Tuchin, Quarkonium dissociation in quark-gluon plasma via ionization in magnetic field, Phys. Rev. C 84 (2011) 044908

### Magnetically Induced Mixing between $\eta_c$ and $J/\psi$

S. Cho, K. Hattori, S. H. Lee, K. Morita, and S. Ozaki, QCD sum rules for magnetically induced mixing between  $\eta_c$  and  $J/\psi$ , Phys. Rev. Lett. 113 (2014) 172301

### Heavy quark potential in magnetized hot QGP

B. Singh, L. Thakur, and H. Mishra, Heavy quark complex potential in a strongly magnetized hot QGP medium, Phys. Rev. D 97 (2018) 096011

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## To Dynamical process

### Charmonium dissociation hair structure

Hu, Jin and Shi, Shuzhe and Xu, Zhe and Zhao, Jiaying and Zhuang, Pengfei, Phys. Rev. D, 105, 9, 094013(2022)

### Magnetic Induced Charmonium Collective Behavior

X. Guo, S. Shi, N. Xu, Z. Xu, and P. Zhuang, Magnetic Field Effect on Charmonium Production in High Energy Nuclear Collisions, Phys. Lett. B 751 (2015) 215-219

### Magnetic field induced open charm direct flow

S.K. Das, S. Plumari, S. Chatterjee, J. Alam, F. Scardina, V. Greco, Directed Flow of Charm Quarks as a Witness of the Initial Strong Magnetic Field in Ultra-Relativistic Heavy Ion Collisions, Phys. Lett. B 768, 260 (2017)

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How does strong magnetic field influence the heavy quark production in the initial stage?

# Heavy quark pair production elementary process $gg \rightarrow Q\bar{Q}$

Since 1. the color degree of freedom of gluon is much larger than light quark

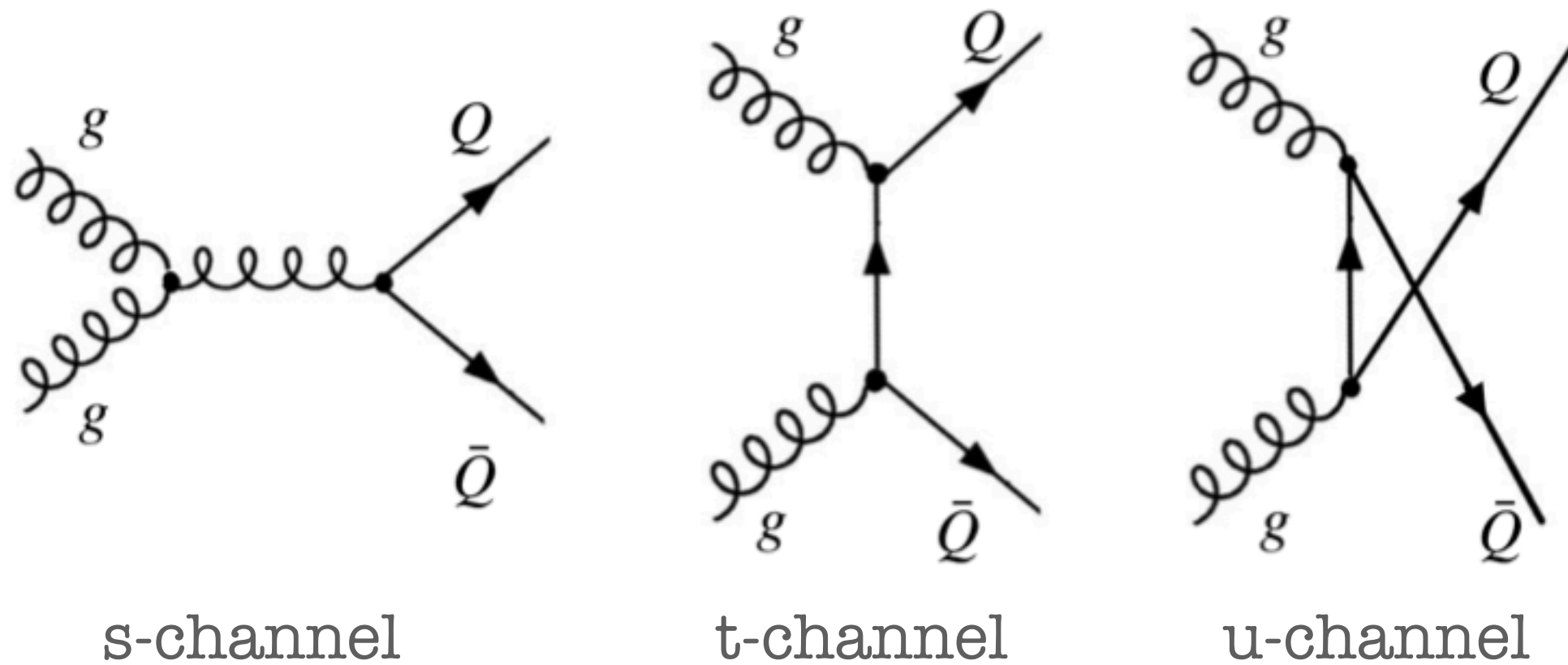
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Leading order tree diagram



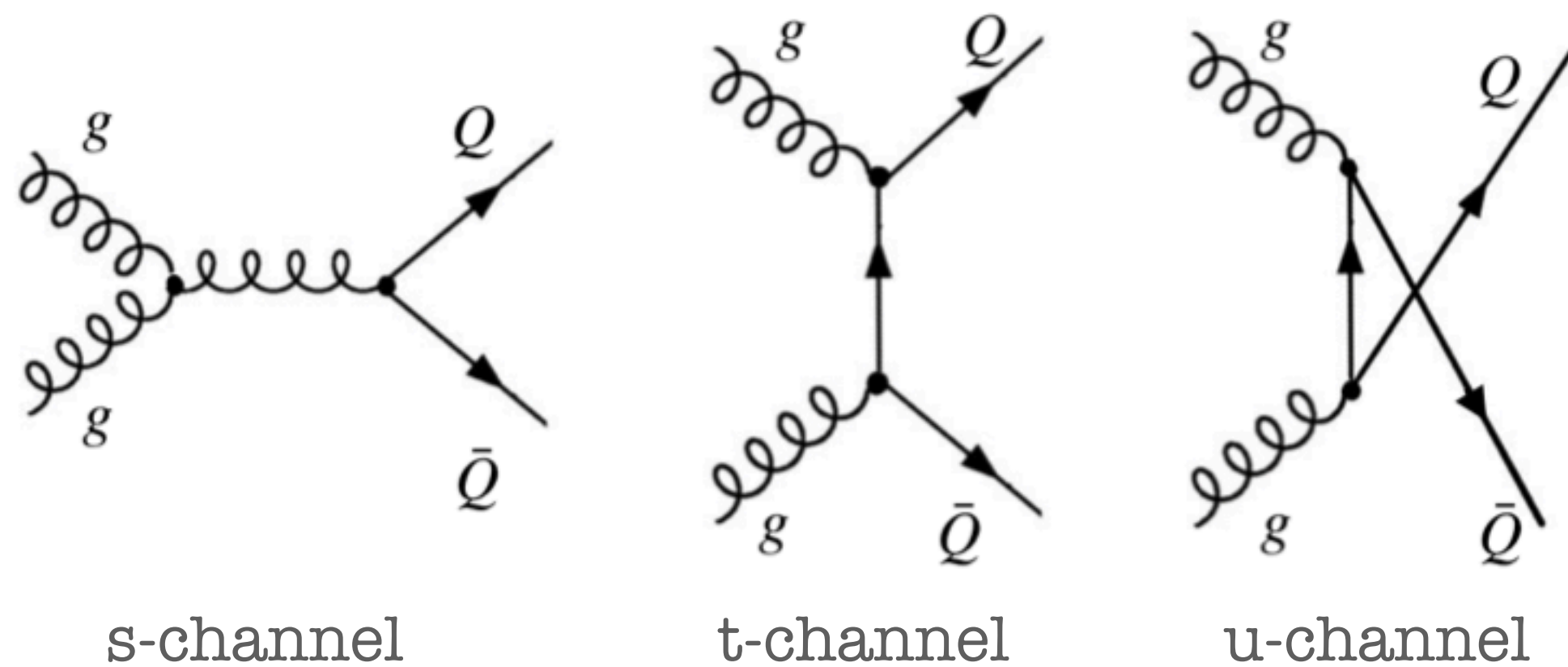
in external magnetic field

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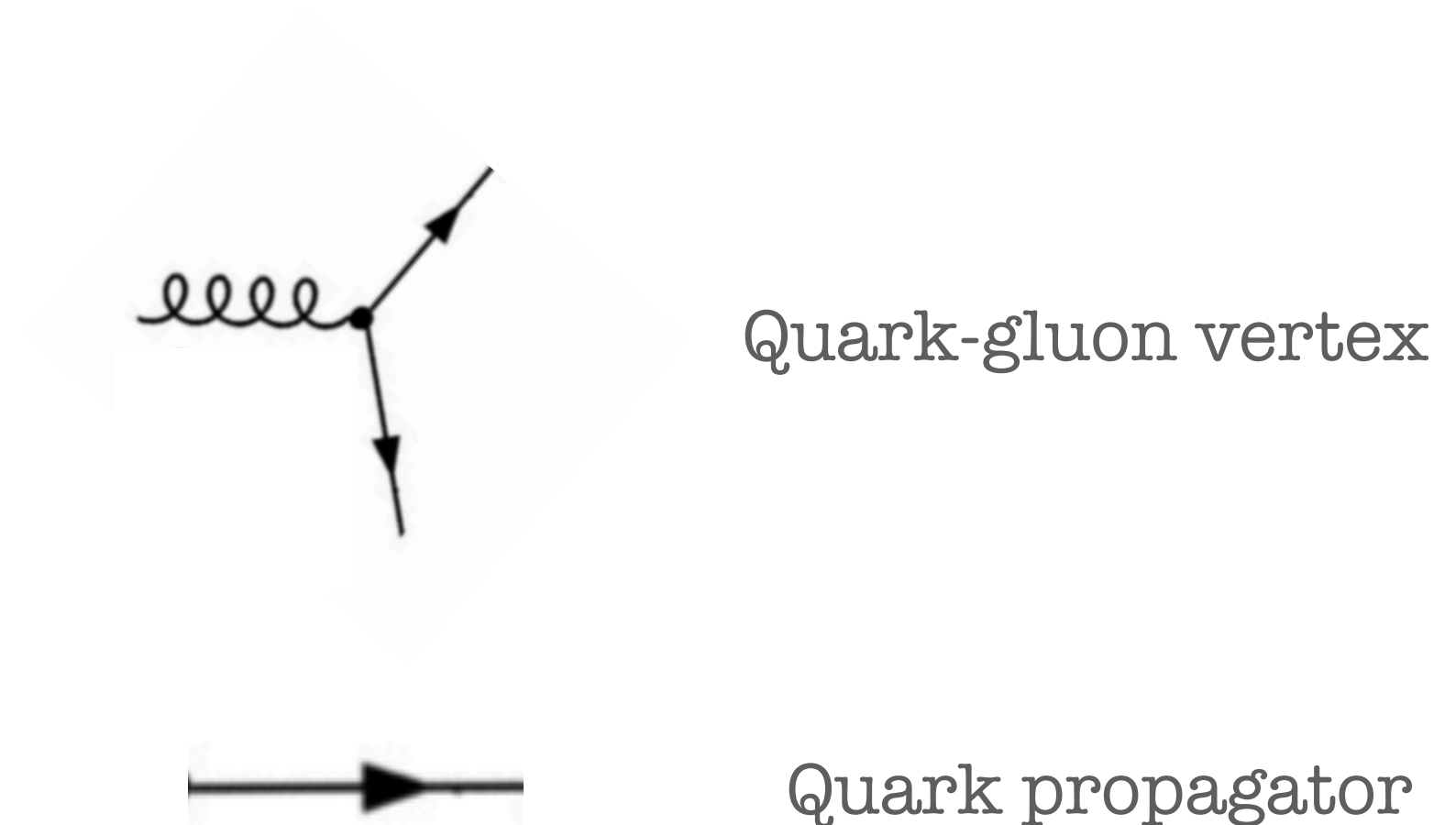
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Leading order tree diagram



in external magnetic field

Feynman rules in external magnetic field



Dirac equation under magnetic field

$$[i\gamma^\mu(\partial_\mu + iqA_\mu) - m]\psi = 0$$



## Dirac equation under magnetic field

$$[i\gamma^\mu(\partial_\mu + iqA_\mu) - m]\psi = 0$$

In Landau gauge

$$A_0 = 0 \quad \mathbf{A} = Bx\mathbf{e}_y$$

Landau energy levels for a fermion moving in an external magnetic field

$$\varepsilon^2 = p_z^2 + \varepsilon_n^2 \quad \varepsilon_n^2 = m^2 + p_n^2 \quad p_n^2 = 2n|qB|$$

**$n$  Landau level**

Stationary solution of the Dirac spinor

$$\psi_{n,\sigma}^-(x,p) = e^{-ip \cdot x} u_{n,\sigma}(\mathbf{x},p)$$

$$\psi_{n,\sigma}^+(x,p) = e^{ip \cdot x} v_{n,\sigma}(\mathbf{x},p)$$

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$$u_{n,-}(\mathbf{x}, p) = \frac{1}{f_n} \begin{bmatrix} -ip_z p_n \phi_{n-1} \\ (\varepsilon + \varepsilon_n)(\varepsilon_n + m)\phi_n \\ -ip_n(\varepsilon + \varepsilon_n)\phi_{n-1} \\ -p_z(\varepsilon_n + m)\phi_n \end{bmatrix}$$

$$u_{n,+}(\mathbf{x}, p) = \frac{1}{f_n} \begin{bmatrix} (\varepsilon + \varepsilon_n)(\varepsilon_n + m)\phi_{n-1} \\ -ip_z p_n \phi_n \\ p_z(\varepsilon_n + m)\phi_{n-1} \\ ip_n(\varepsilon + \varepsilon_n)\phi_n \end{bmatrix}$$

$\sigma = \pm$  labels spin states

$p_\mu = (\varepsilon, 0, p_y, p_z)$  labels four momentum

$p_y = aqB$  with  $a$  the center of gyration

$$f_n = 2\sqrt{\varepsilon\varepsilon_n(\varepsilon_n + m)(\varepsilon_n + \varepsilon)}$$

$$v_{n,+}(\mathbf{x}, p) = \frac{1}{f_n} \begin{bmatrix} -p_n(\varepsilon + \varepsilon_n)\phi_{n-1} \\ -ip_z(\varepsilon_n + m)\phi_n \\ -p_z p_n \phi_{n-1} \\ i(\varepsilon_n + m)(\varepsilon + \varepsilon_n)\phi_n \end{bmatrix}$$

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$$\phi_n(x - a) = \sqrt{\sqrt{\frac{|qB|}{\pi}} \frac{1}{L^2 2^n n!}} H_n(\sqrt{|qB|}(x - a)) e^{-|qB|(x-a)^2/2}$$

is harmonic oscillator

# Dirac equation under magnetic field

Reconstruct quark-gluon vertex

$$-ig \int d^4x \bar{\psi}_{n,\sigma}^-(x, p) \gamma_\mu t^c A_c^\mu(x, k) \psi_{n',\sigma'}^-(x, p') = \frac{-ig}{\sqrt{2\omega L^3}} \int d^4x e^{-i(p' \pm k - p) \cdot x} \bar{u}_{n,\sigma}(\mathbf{x}, p) \gamma_\mu \epsilon^\mu u_{n',\sigma'}(\mathbf{x}, p')$$

Reconstruct quark propagator

$$G(x' - x) = -i \left( \frac{\sqrt{|qB|} L}{2\pi} \right)^2 \int dp_z da \sum_{\sigma,n} [\theta(t' - t) u_{n,\sigma}(\mathbf{x}', p) \bar{u}_{n,\sigma}(\mathbf{x}, p) e^{-ip \cdot (x' - x)} - \theta(t - t') v_{n,\sigma}(\mathbf{x}', p) \bar{v}_{n,\sigma}(\mathbf{x}, p) e^{ip \cdot (x' - x)}]$$

Feynman rule in external uniform magnetic field ✓

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Feynman rule in external uniform magnetic field ✓

Consistence with Schwinger propagator

J. Schwinger, 1951

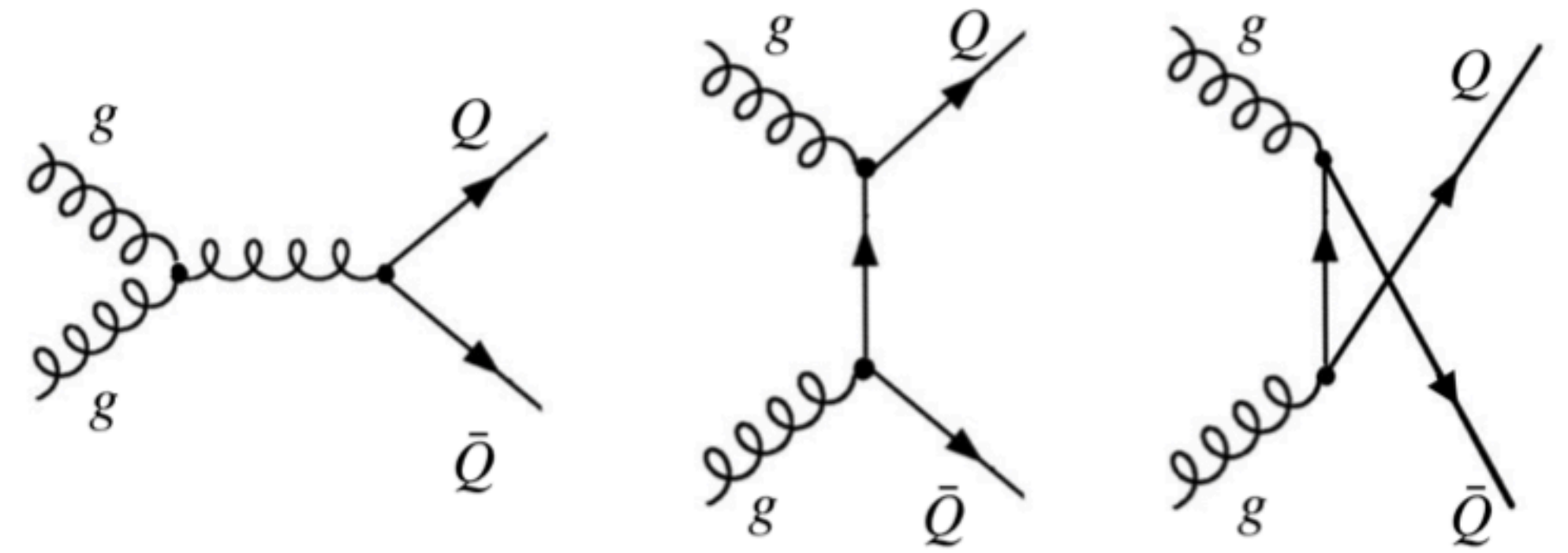
LLL Schwinger propagator  $G(x - x') = e^{-i(y-y')(x+x')|qB|/2} \int \frac{d^4p}{(2\pi)^4} e^{-ip \cdot (x-x')} \left[ i e^{-p_\perp^2} \left( \frac{\not{p}_\parallel + m}{p_\parallel^2 - m^2} (1 - i\gamma^1 \gamma^2) \right) \right]$

Exactly the same with the reconstructed quark propagator when  $n = 0$

# Cross section for elementary process

$$gg \rightarrow Q\bar{Q}$$

## S-matrix



s-channel

t-channel

u-channel

$$S_s = -\frac{g^2 t_c f^{abc}}{(2\pi)^4 2\sqrt{\omega'\omega''} L^3} \int d^4x d^4x' d^4k \frac{1}{k^2} \bar{u}_{n',\sigma'}(\mathbf{x}, p') \gamma_\mu \nu_{n'',\sigma}(\mathbf{x}, p'') \epsilon''_\rho \epsilon''_\lambda \left[ g^{\mu\rho} (k' + k)^\lambda - g^{\mu\lambda} (k + k'')^\rho + g^{\lambda\rho} (k'' - k')^\mu \right] e^{i[(p'+p'')\cdot x - k\cdot(x-x') - (k'+k'')\cdot x']}$$

$$S_t = ig^2 t^a t^b \left( \frac{\sqrt{|qB|} L}{2\pi} \right)^2 \frac{1}{\sqrt{\omega'\omega''} L^3} \sum_{n,\sigma} \int d^4x d^4x' da dp_z \bar{u}_{n',\sigma'}(\mathbf{x}, p') \gamma^\nu \epsilon''_\nu \left[ \theta(t' - t) u_{n,\sigma} \bar{u}_{n,\sigma}(\mathbf{x}, p) e^{-ip\cdot(x'-x)} - \theta(t - t') v_{n,\sigma} \bar{v}_{n,\sigma}(\mathbf{x}, p) e^{ip\cdot(x'-x)} \right] \gamma^\mu \epsilon'_\mu \nu_{n'',\sigma}(\mathbf{x}, p'') e^{-i(k''\cdot x' + k'\cdot x - p''\cdot x - p'\cdot x')}$$

$$S_u = S_t(a \leftrightarrow b, p \leftrightarrow p'')$$

## Cross section

$$\begin{aligned} \sigma &= \frac{L^3}{v_{rel} T} \int \frac{L^2 dp'_y dp'_z}{(2\pi)^2} \int \frac{L^2 dp''_y dp''_z}{(2\pi)^2} \sum_{\sigma', \sigma''} |S_s + S_t + S_u|^2 \\ &= \frac{L^5}{v_{rel}} \left( \frac{L\sqrt{|qB|}}{2\pi} \right)^4 \int da' dp'_z \int da'' dp''_z \sum_{\sigma', \sigma''} |\mathcal{M}_s + \mathcal{M}_t + \mathcal{M}_u|^2 (2\pi)^3 \delta_x^3(k' + k'' - p' - p'') \\ &= \frac{L^{10}}{16\pi} \frac{\sqrt{s} |qB|}{p'_z} \sum_{\sigma', \sigma''} |\mathcal{M}_s + \mathcal{M}_t + \mathcal{M}_u|^2 \end{aligned}$$

# Cross section under Lowest Landau Level

## Lowest Landau level (LLL)

The external magnetic field is strong enough, the fermion would be suppressed into  $n = 0$  state

Spin state:  $\sigma = -1$  for quarks  $\sigma = +1$  for antiquarks

## Cross section under LLL in center of mass frame

$$\sigma(s, B, \theta) = \frac{\pi m^2 \alpha_s^2 |qB|}{s^3 \chi} \left\{ \frac{3}{2} \cos^2 \theta \left[ \frac{1}{2} - \frac{\sin^3 \theta}{1 + \sqrt{4m^2/s}} \frac{1 + \cos^2 \theta - 4\chi^2}{\sin^4 \theta + 16m^2/s \cos^2 \theta} e^{-\frac{s \sin^2 \theta}{8|qB|}} \right] \right.$$

s-channel involved  
Non-Abelian contribution

$$\left. + \frac{2}{3} \sin^4 \theta \left[ \left( \frac{\cos \theta + 2\chi}{(\chi + \cos \theta)^2 + 4m^2/s} \right)^2 + \left( \frac{-\cos \theta + 2\chi}{(\chi - \cos \theta)^2 + 4m^2/s} \right)^2 - \frac{1}{4} \frac{4\chi^2 - \cos^2 \theta}{\sin^4 \theta + 16m^2/s \cos^2 \theta} \right] e^{-\frac{s \sin^2 \theta}{4|qB|}} \right\}$$

only t- & u- channel just QED-like process

$s$  labels invariant mass

$\chi = \sqrt{1 - 4m^2/s}$  gives the threshold of  $s$

$\theta$  labels gluon polarization angle

# Cross section under Lowest Landau Level

## LLL in magnetic field v.s. vacuum

$$\sigma(s, B, \theta) = \frac{\pi m^2 \alpha_s^2 |qB|}{s^3 \chi} \left\{ \frac{3}{2} \cos^2 \theta \left[ \frac{1}{2} - \frac{\sin^3 \theta}{1 + \sqrt{4m^2/s}} \frac{1 + \cos^2 \theta - 4\chi^2}{\sin^4 \theta + 16m^2/s \cos^2 \theta} e^{-\frac{s \sin^2 \theta}{8|qB|}} \right] \right.$$

s-channel involved  
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---


$$\sigma_0(s) = \frac{\pi m^2 \alpha_s^2}{3s} \left[ \left( 1 + \frac{4m^2}{s} + \frac{m^4}{s^2} \right) \log \left( \frac{1 + \chi}{1 - \chi} \right) - \left( \frac{7}{4} + \frac{31m^2}{4s} \right) \chi \right]$$

- Anisotropy: dependence on gluon polarization angle

unique QCD process

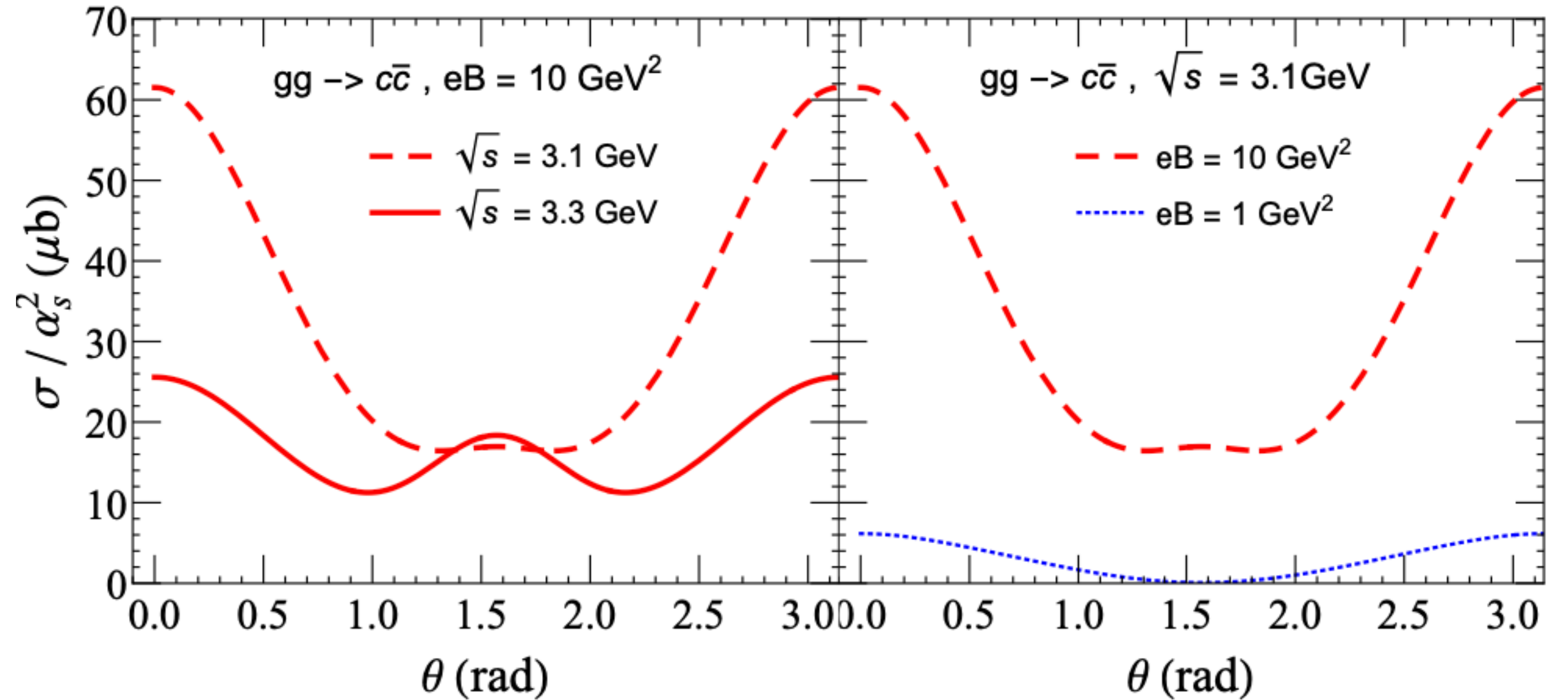
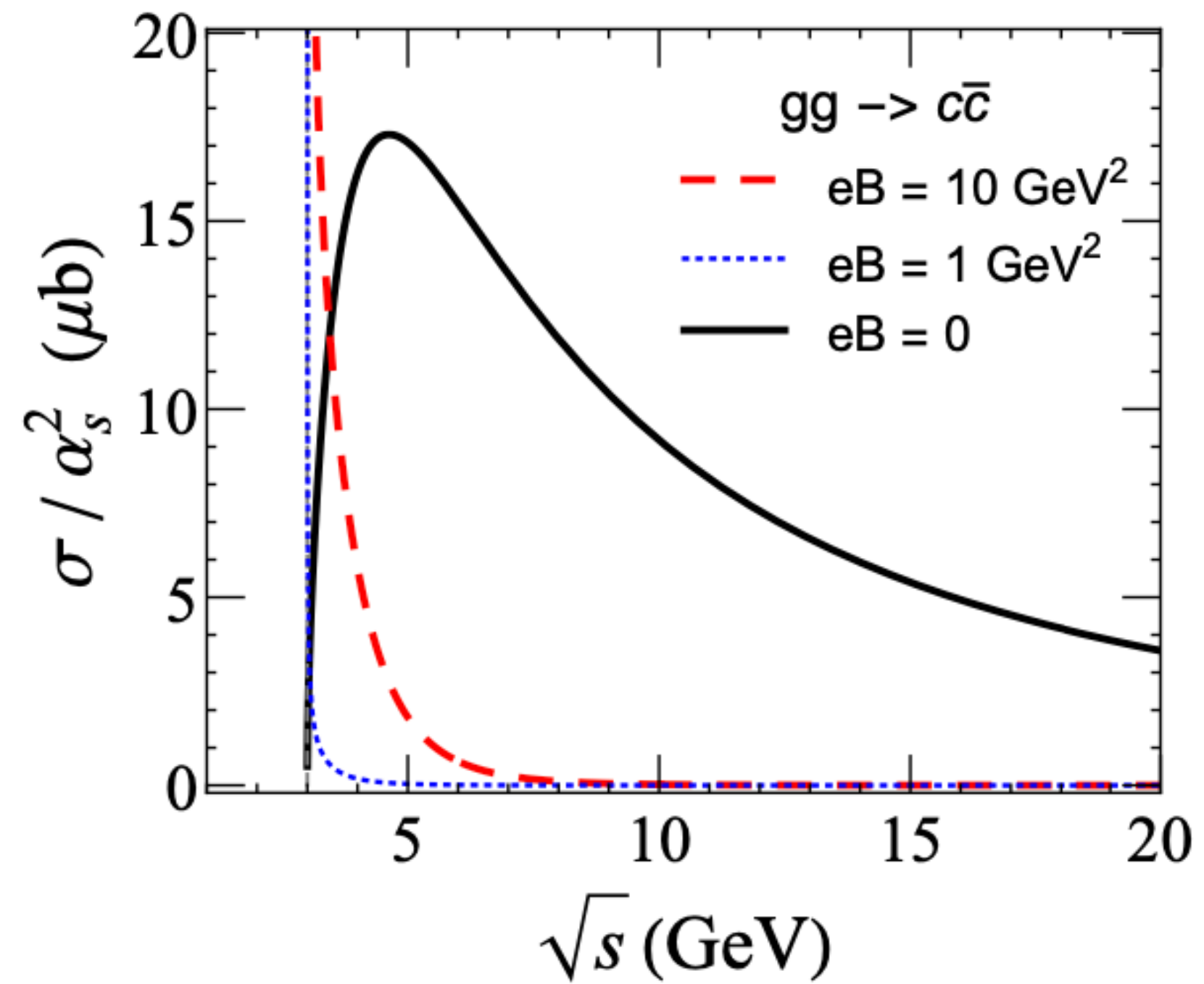
$$\sigma(s, B, 0) = \sigma(s, B, \pi) = \frac{3\pi m^2 \alpha_s^2 |qB|}{4s^3 \chi}$$

QED like process

$$\sigma(s, B, \frac{\pi}{2}) = \frac{14\pi m^2 \alpha_s^2 |qB| \chi}{3s^3} e^{-\frac{s}{4|qB|}}$$

- Divergence at threshold: phase space dimension reduction under strong magnetic field.

# Cross section under Lowest Landau Level



- Increase with magnetic field
- A narrow  $\sqrt{s}$  region
- Stronger magnetic field, wider energy range

- Maximum at  $\theta = 0, \pi$  **unique QCD process**

the gluon self interaction plays the dominant role



# Transverse momentum spectrum under Lowest Landau Level

Neglecting shadowing effect the differential cross section in heavy-ion collision

$$\frac{d^3\sigma_{gg\rightarrow c\bar{c}}^{AB}}{dp_T^2 dy_c dy_{\bar{c}}} = \int T_A(\mathbf{x}_T - \mathbf{b}/2) T_B(\mathbf{x}_T + \mathbf{b}/2) d\mathbf{x}_T^2 \frac{d^3\sigma_{gg\rightarrow c\bar{c}}^{pp}}{dp_T^2 dy_c dy_{\bar{c}}}$$

$T_A, T_B$  labels nuclear thickness function with  $b$  the impact parameter

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In pp collision

$$\frac{d^3\sigma_{gg\rightarrow c\bar{c}}^{pp}}{dp_T^2 dy_c dy_{\bar{c}}} = x_1 x_2 f_g(x_1, Q^2) f_g(x_2, Q^2) \frac{d\sigma_{gg\rightarrow c\bar{c}}}{d\hat{t}}(\hat{s}, \hat{t}, \hat{u})$$

$\{\hat{s}, \hat{t}, \hat{u}\}$  Mandelstam variables

$f_g(x, Q^2)$  parton distribution function of gluon

$x_1, x_2$  momentum fractions of initial gluons

$$\frac{d^3\sigma_{gg\rightarrow c\bar{c}}}{d\hat{t}} = \frac{|\mathcal{M}_{gg\rightarrow c\bar{c}}|^2}{16\pi\hat{s}^2}$$

# Transverse momentum spectrum under Lowest Landau Level

**LLL**

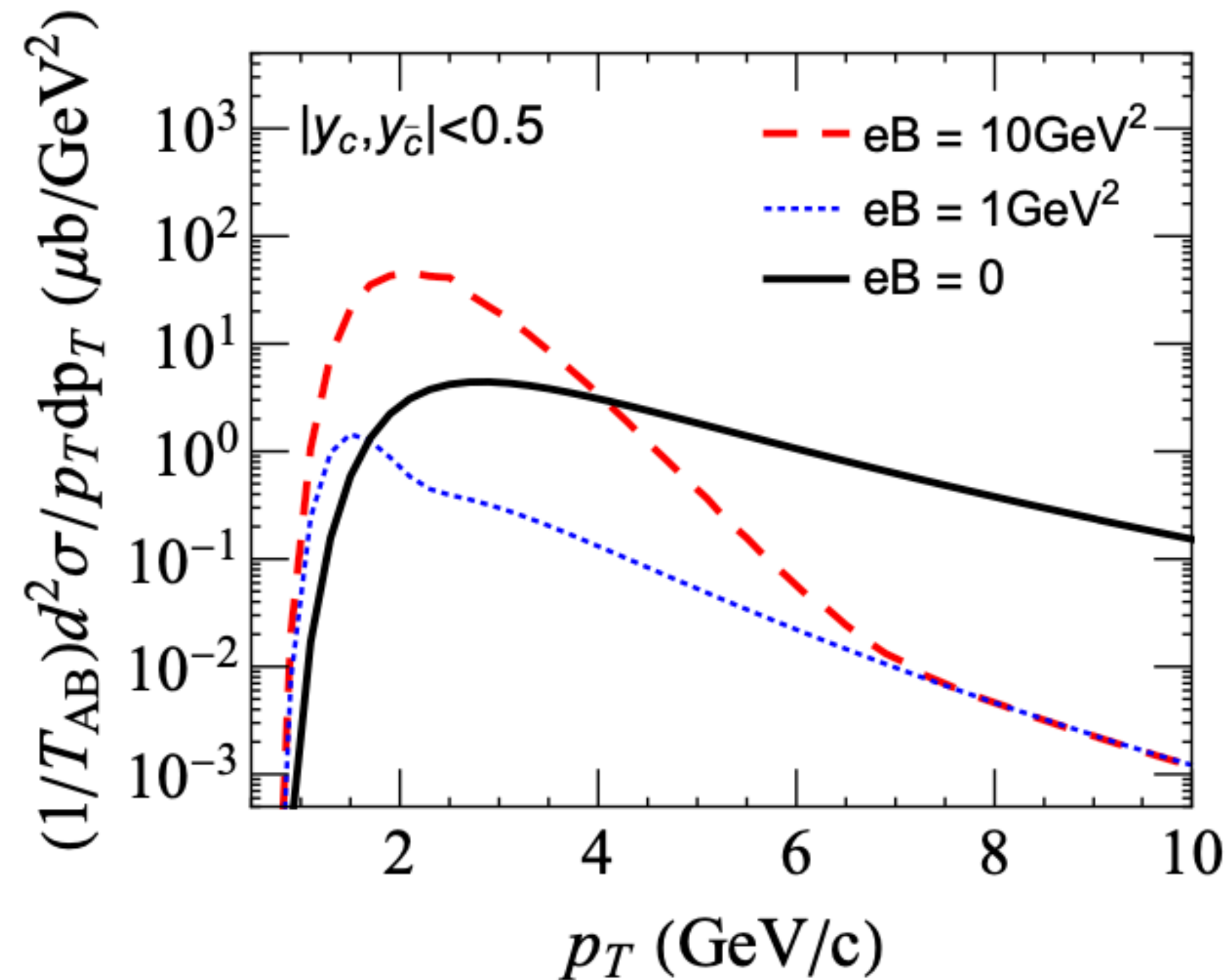
$$\begin{aligned}
 \frac{|\mathcal{M}_{gg \rightarrow c\bar{c}}|^2}{\pi^2 \hat{s}^2} = & 24m^2 \cos^2 \theta \left[ \frac{1}{\hat{s}} + \frac{\sin^2 \theta (\hat{s} \sin^2 \theta / 4 + (\hat{t} + \hat{u}) / 2 + (\hat{t} - \hat{u})^2 / (\hat{s} \cos^2 \theta))}{(\hat{s} \sin^2 \theta / 2 + \hat{t} + \hat{u})^2 - ((\hat{t}^2 - \hat{u}^2) / (\hat{s} \cos \theta))^2} e^{-\frac{\hat{s} \sin^2 \theta}{8|qB|}} \right] \\
 & + \frac{64}{3} m^2 \sin^4 \theta \left[ \frac{(-\sqrt{\hat{s}} \cos \theta / 2 + (\hat{t} - \hat{u}) / (\sqrt{\hat{s}} \cos \theta))^2 + (\sqrt{\hat{s}} \cos \theta / 2 + (\hat{t} - \hat{u}) / (\sqrt{\hat{s}} \cos \theta))^2}{(\hat{s} \sin^2 \theta / 2 + \hat{t} + \hat{u} - (\hat{t}^2 - \hat{u}^2) / (\hat{s} \cos \theta))^2} e^{-\frac{\hat{s} \sin^2 \theta}{4|qB|}} \right] \\
 & - \frac{16}{3} m^2 \sin^4 \theta \left[ \frac{(\hat{t} - \hat{u})^2 / (\hat{s} \cos^2 \theta) - \hat{s} \cos^2 \theta / 4}{(\hat{s} \sin^2 \theta / 2 + \hat{t} + \hat{u})^2 - ((\hat{t}^2 \hat{u}^2) / (\hat{s} \cos \theta))^2} e^{-\frac{\hat{s} \sin^2 \theta}{4|qB|}} \right]
 \end{aligned}$$

$$\frac{d^3 \sigma_{gg \rightarrow c\bar{c}}}{d\hat{t}} = \frac{|\mathcal{M}_{gg \rightarrow c\bar{c}}|^2}{16\pi \hat{s}^2}$$

# Transverse momentum spectrum under Lowest Landau Level

Integration over gluon polarization angle & collision energy at  $\sqrt{s} = 5.02 \text{ TeV}$

within central rapidity  $-0.5 < y_c, y_{\bar{c}} < 0.5$



- Low  $p_T$  region, **enhanced**
- High  $p_T$  region, **suppressed**

which could also be seen in the element process cross section in the narrow window of incoming energy

◦  $eB = 0$  using the vacuum solution

## Beyond Lowest Landau Level

Whether LLL is a good approximation of fermion under external strong magnetic field?

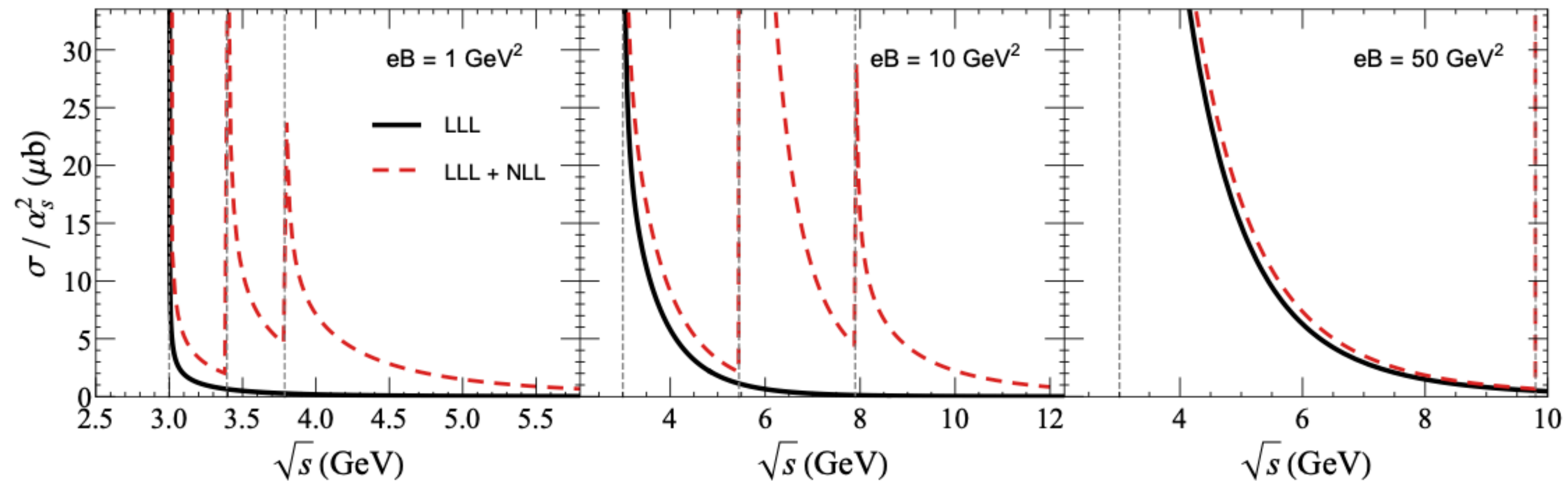
Even though we consider the early stage of high energy nuclear collisions, where the system is not yet thermalized

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**Next Landau Level (NLL)** quantum number  $n = 0, 1$



New divergence

$$0+0 \quad \sqrt{s_{th}^{(1)}} = 2m$$

$$0+1 \quad \sqrt{s_{th}^{(2)}} = m + \sqrt{m^2 + 2|qB|}$$

$$1+1 \quad \sqrt{s_{th}^{(3)}} = 2\sqrt{m^2 + 2|qB|}$$

Competition between energy scales of  $B$  &  $\sqrt{s}$

**LLL is a good approximation with**

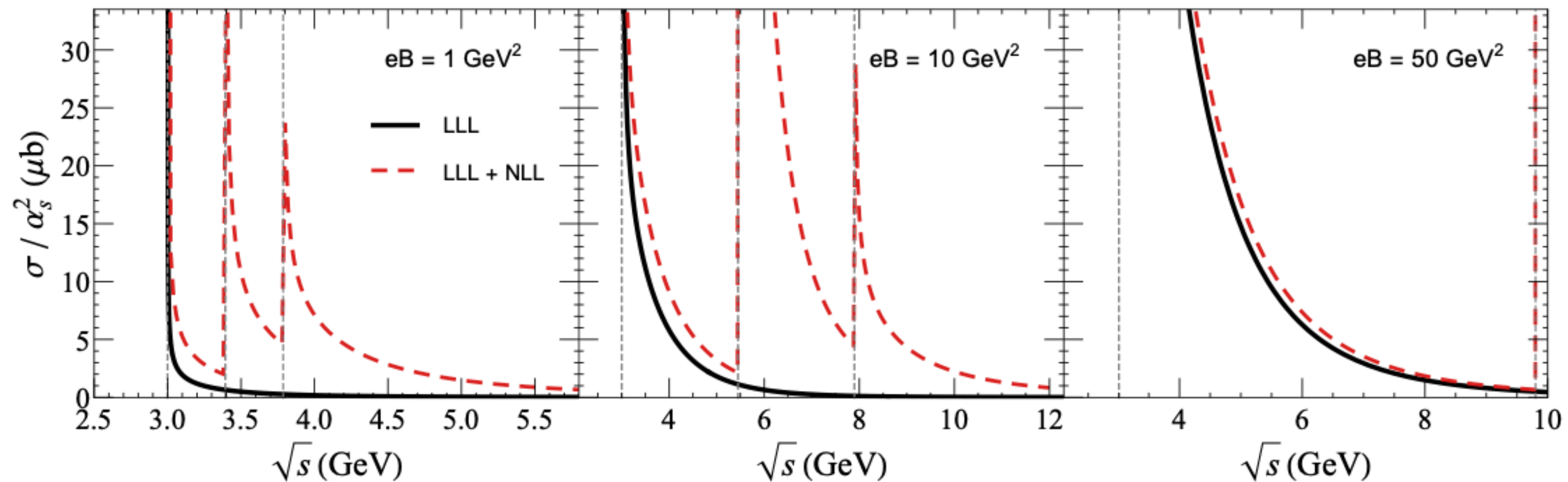
- Stronger magnetic field
- Smaller collision energy

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## Application range

- Partons in a nucleon or a nucleus distribute mainly in the small  $x$  region
- Heavy quark production at low  $P_T$  region
- Magnetic field  $eB \sim 10 \text{ GeV}^2$

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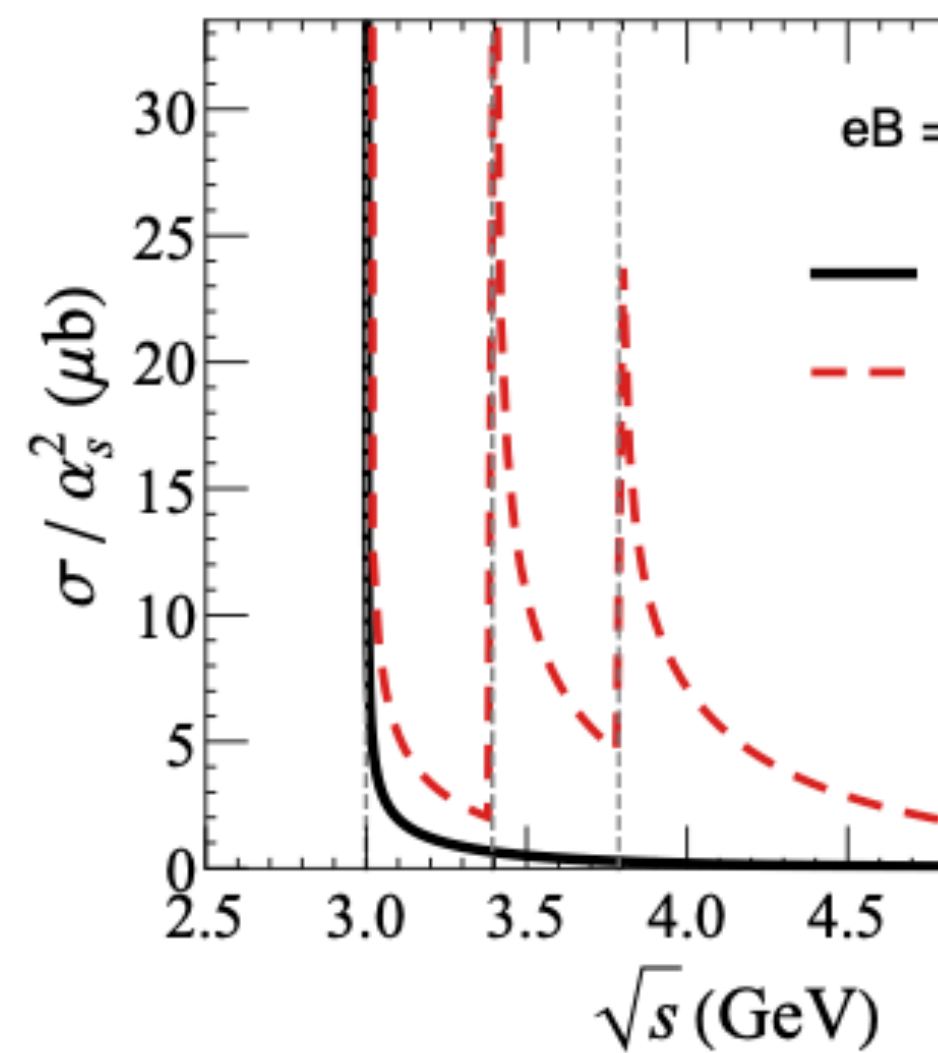
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## Next Landau Level

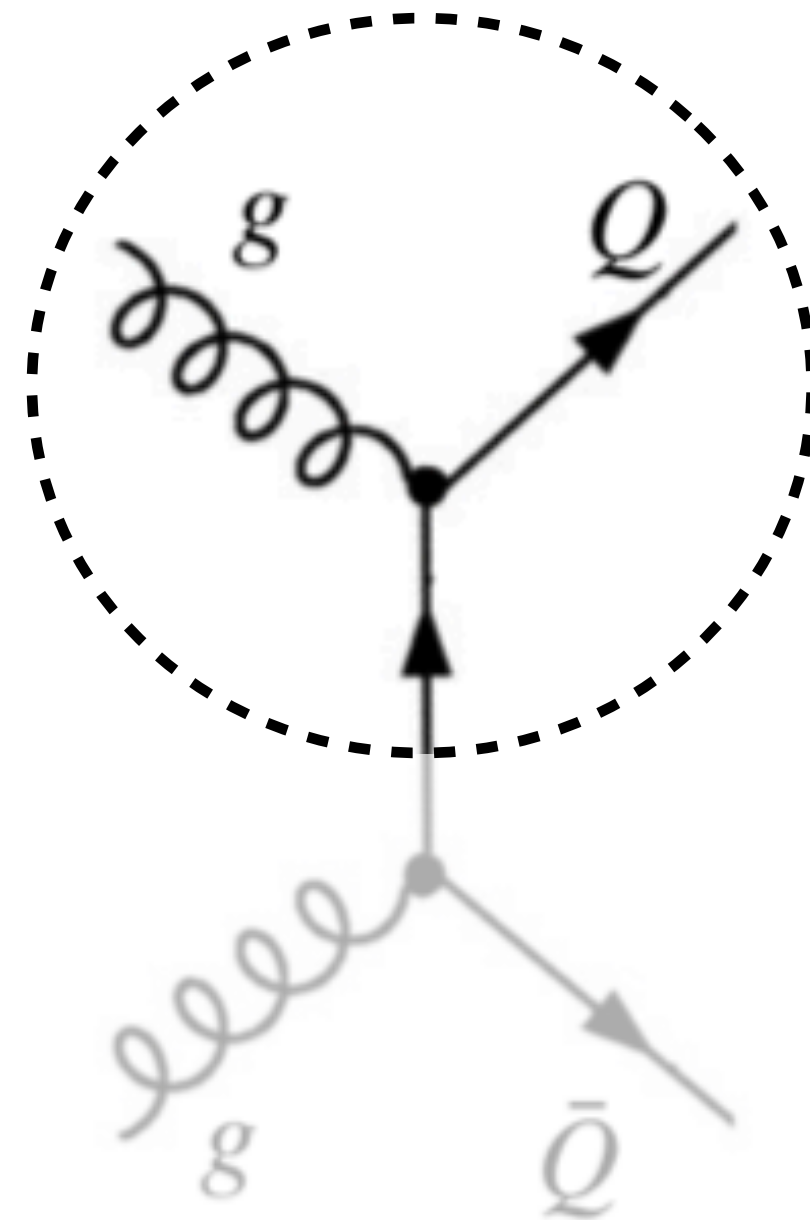


New divergence

$$0+0 \quad \sqrt{s_{th}^{(1)}} = 2m$$

$$0+1 \quad \sqrt{s_{th}^{(2)}} = m + \sqrt{m^2 + 2|qB|}$$

$$1+1 \quad \sqrt{s_{th}^{(3)}} = 2\sqrt{m^2 + 2|qB|}$$



## On shell gluon decay

- When magnetic field is strong enough, the internal quark could be **on-shell**.
- The sub-process  $g \rightarrow Q\bar{Q}$  may take place when the quark and antiquark are at different Landau levels.

Magnetic field which can guarantee the on-shell internal quark

$$|qB| = 2eB/3 \geq 4m^2 \quad \text{for NLL}$$

Competition between energy scales of  $B$  &  $\sqrt{s}$

## LLL is a good approximation with

- Stronger magnetic field
- Smaller collision energy



## Summary and outlook

We calculate the cross section of elementary process  $gg \rightarrow Q\bar{Q}$  under strong magnetic field to NLL, which qualitatively describes the heavy quark production at leading order in the initial stage of heavy ion collisions.

- **Anisotropy** of the system. Unique QCD process dominant the elementary process especially when the gluon incoming direction parallel to magnetic field.
- The **dimension reduction** in phase space leads to divergences of the cross section at the discrete Landau energy levels.
- The heavy quark pair production is **enhanced** at low  $p_T$  region and **suppressed** at high  $p_T$  region.

In the future work

- The process  $q\bar{q} \rightarrow Q\bar{Q}$  will also be included to reproduce full transverse momentum region
- Elementary process in weak field limit which can also be included to compare with heavy-ion collision.

Thank you for listening!