Bayesian uncertainty quantification of pQCD input to the neutron-star equation of state

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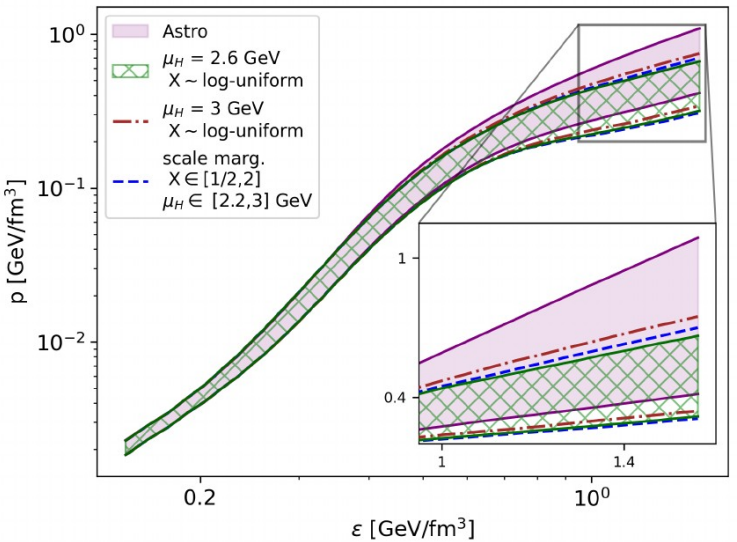

Tyler Gorda, Oleg Komoltsev, Aleksi Kurkela, AM, JHEP (2023), 2303.02175
Claude Duhr, Alexander Huss, AM, Robert Szafron, JHEP (2021), 2106.04585

Emmy Noether-Programm
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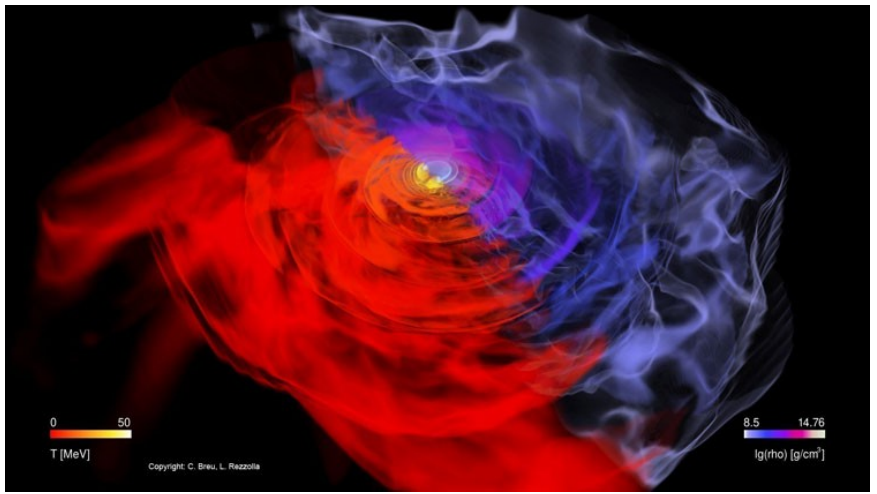
Neutron stars – corpses of dead stars

$$M \leq 2M_\odot \quad R \sim 10\,\mathrm{km} \quad T \leq \mathrm{keV} = 10^7\mathrm{K}$$

neutron star merger



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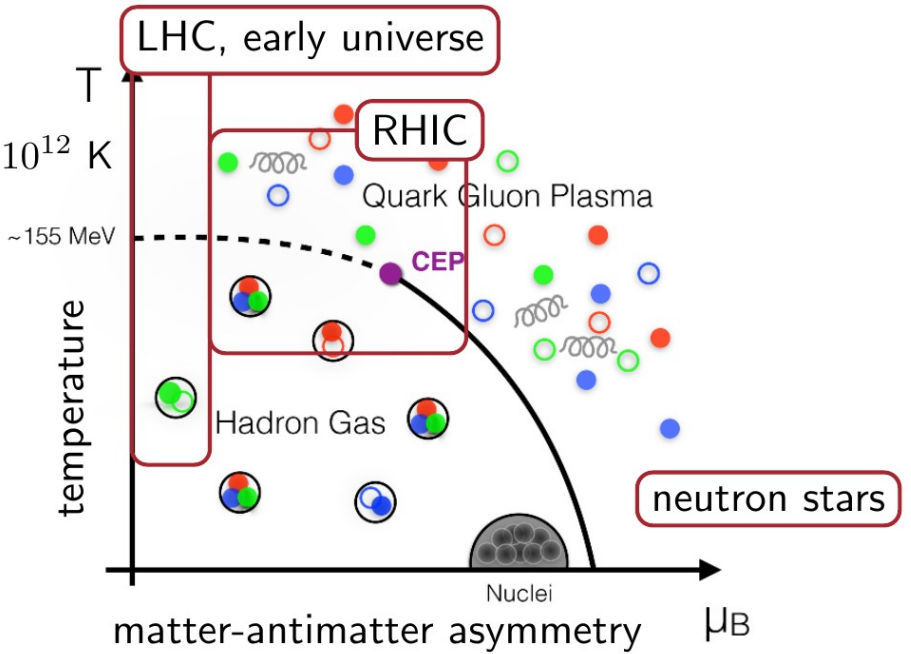


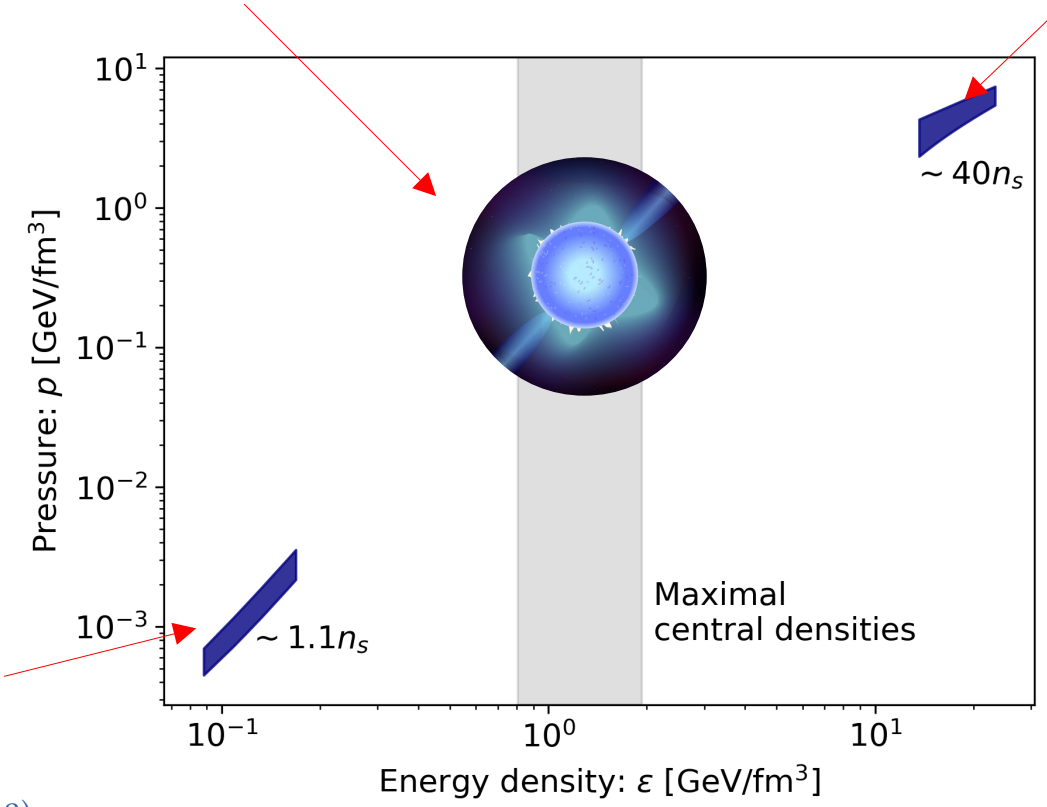
Neutron stars probe cold nuclear matter equation of state: $p(\mu), n(\mu), \varepsilon(\mu)$

QCD equation of state $p(\varepsilon)$ at $T=0$

Astrophysical observations

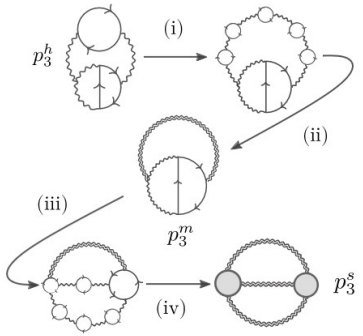
- masses
- deformability
- radii

See talk by Laura Tolós on Mon



Perturbative QCD

Gorda et al. PRL (2021, 2023)



Chiral effective field theory (CET)

Hebeler et.al. Astrophys.J. (2013)

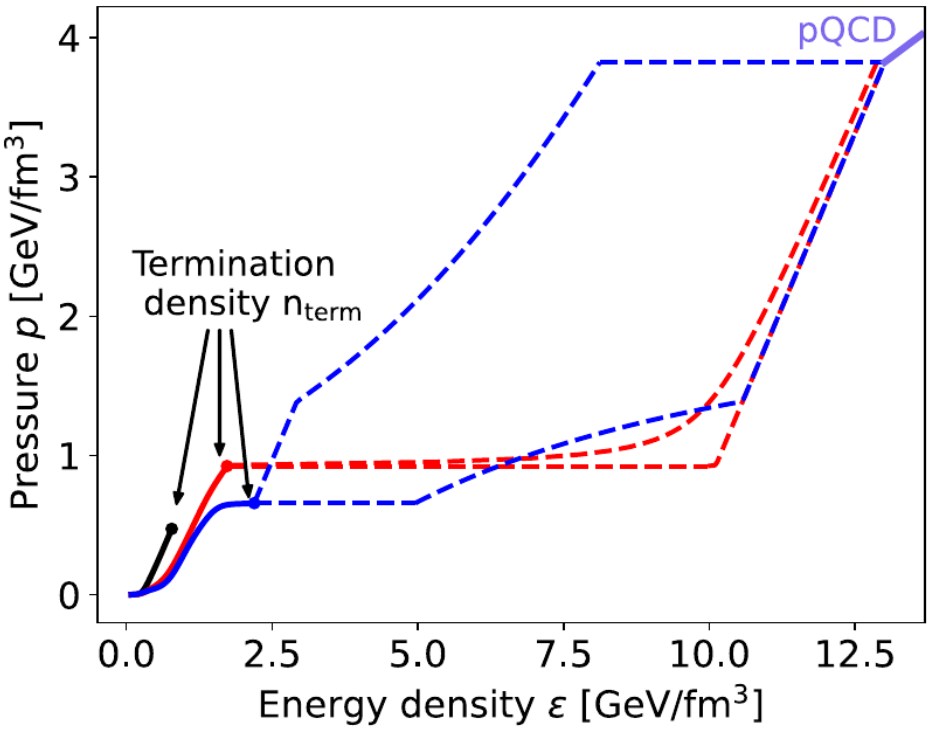
Connecting low and high density regions

Komoltsev, Kurkela, PRL (2022)

- Stability $\partial_\mu n(\mu) \geq 0$
- Causality $c_s^2 = \dfrac{n/\mu}{\partial_\mu n} \leq 1$
- Consistency

$$\int n(\mu) d\mu = p_{\mathrm{pQCD}} - p_{\mathrm{CET}}$$

Komoltsev et al., PRD (2024)



pQCD results at high densities constrain neutron-star equation of state

pQCD equation of state at high densities

$$\frac{p}{p_0} = 1 + a_1\alpha_s(\bar{\Lambda}) + a_2\alpha_s^2(\bar{\Lambda}) + a_3\alpha_s^3(\bar{\Lambda}) + \dots$$

NLO N2LO N3LO* MHO

$p_0 \propto \mu^4$

Freedman McLerran PRD (1977)

Gorda et al. PRL (2018, 2021, 2023) - ongoing

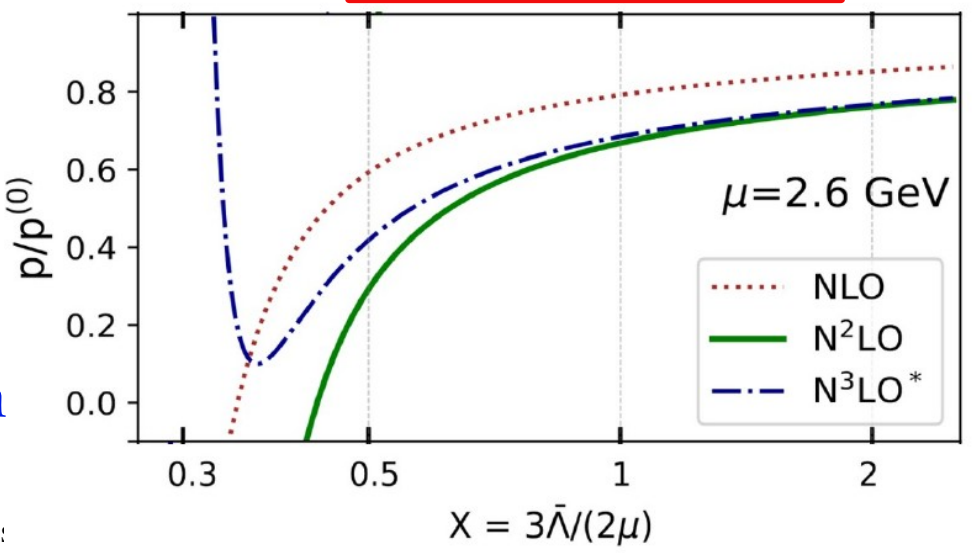
Uncertainties:
1) Missing higher order terms (MHO)
2) Unphysical renormalisation scale

$$\bar{\Lambda} = X \times \frac{2}{3}\mu$$

Bayesian inference for error estimation

Duhr, Huss, AM, Szafron, JHEP (2021), 2106.04585

scale dependence



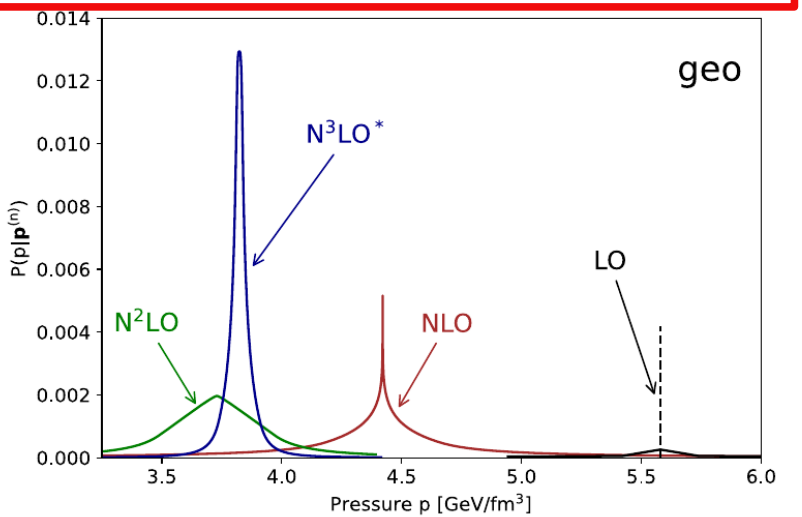
Bayesian model for MHO

Duhr, Huss, AM, Szafron, JHEP (2021)
Bonvini, EPJC (2020)

$$\Sigma_n = \Sigma_0 \times \Big(1 + \underbrace{\delta_1}_{\mathcal{O}(\alpha_s^1)} + \underbrace{\delta_2}_{\mathcal{O}(\alpha_s^2)} + \ldots + \underbrace{\delta_n}_{\mathcal{O}(\alpha_s^n)}\Big)$$

Geometric model: $|\delta_k| \leq c a^k$

probability distribution for δ_{k+1}



$$P(ac|\vec{\delta}_k) = \frac{P(\vec{\delta}_k|ac)P_0(ac)}{P(\vec{\delta}_k)}$$

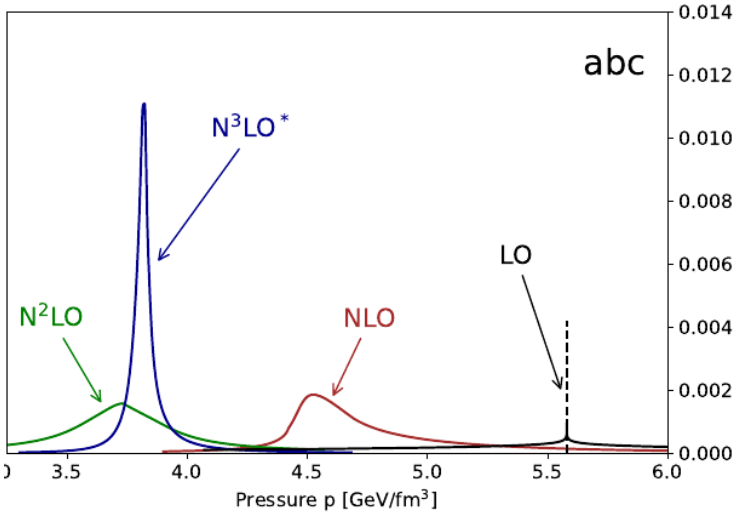
likelihoods prior

marginal likelihood

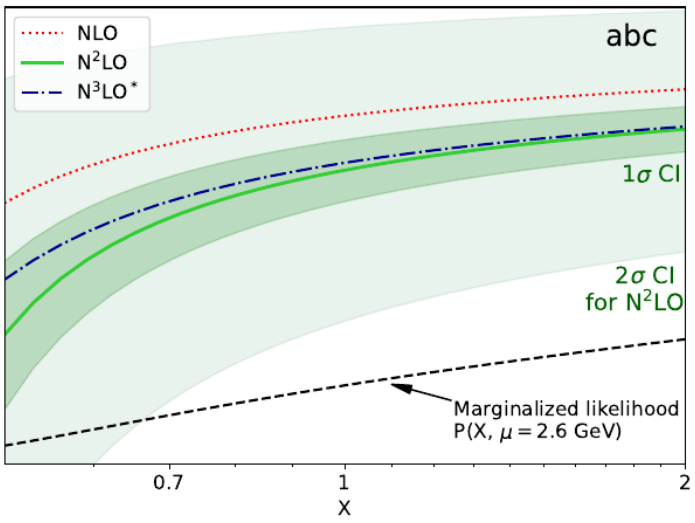
Asymmetric geometric (ABC) model

Extension to asymmetric and alternating series: $b - c \leq \dfrac{\delta_n}{a^n} \leq b + c$

posterior at fixed scale



scale dependent confidence interval



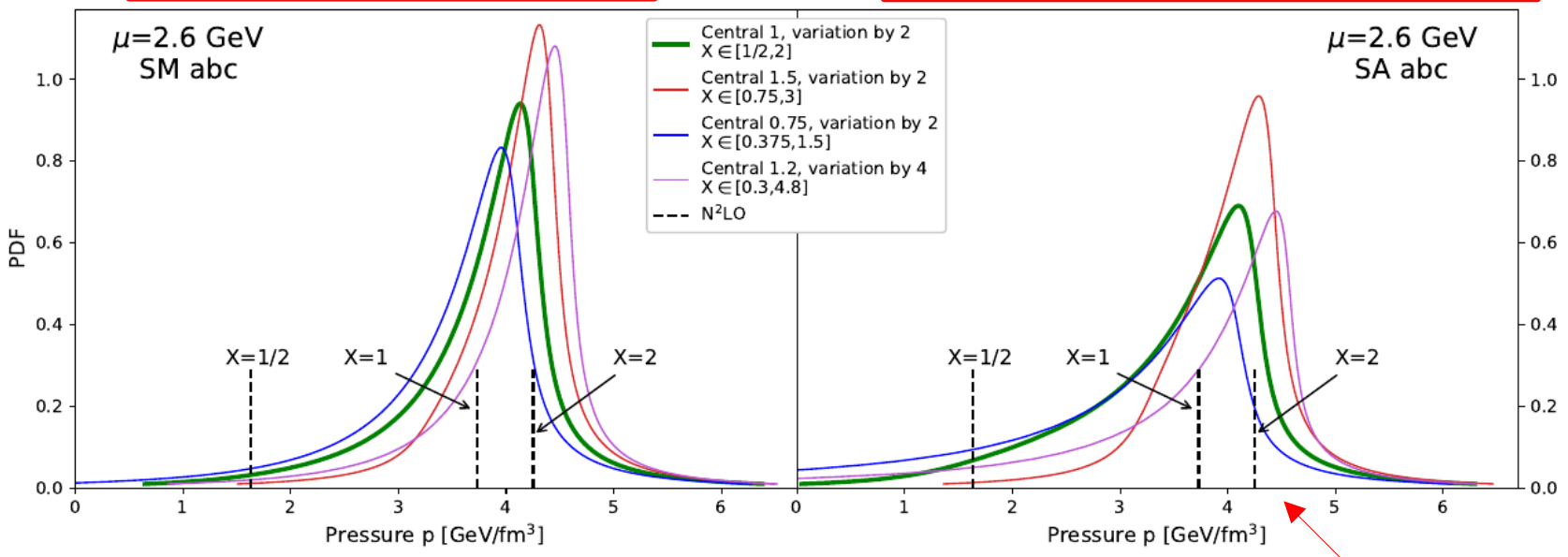
Need prescription to combine error estimates at different scales.

Scale marginalisation vs averaging

Integrate distributions at different scales weighted with

likelihood $P(X|\vec{\delta}_k)$

uniform weight in log X



Mild dependence on the range of scales

preference for larger scale

Master formula for EoS

Using public MiHO code: github.com/aykhuss/miho

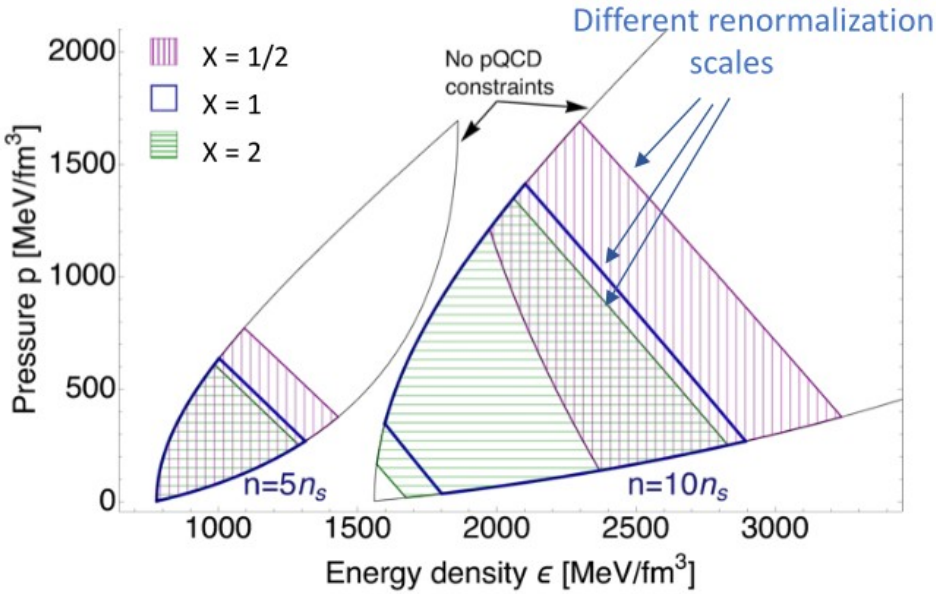
$$P(\epsilon_L, p_L | n_L, p^{(k)}) = \int d\mu_H dp_H dn_H dX$$

SCC constraint $\times\ P(\epsilon_L, p_L | n_L, \mu_H, p_H, n_H)$

scale prescription $\times\ P_{\mathrm{sa/sm}}(\mu_H, X | p^{(k)})$

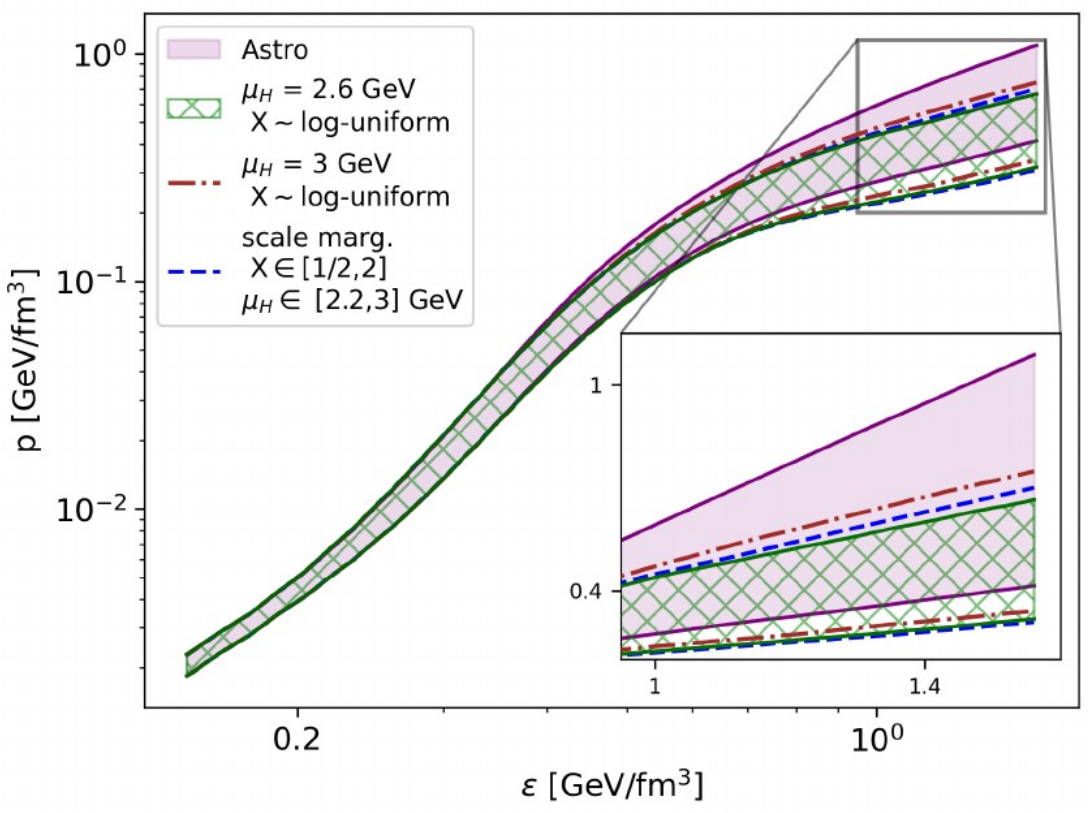
MHO probability $\times\ P_{\mathrm{MHO}}(p_H | p^{(k)}(\mu_H, X))$

convergence in n $\times\ \delta(n^{(k)}(\mu_H, X) - n_H)$



Different choices leads to different exclusion ranges ⟶ systematic scan

EoS inference at NS densities



Robust prediction: pQCD favours softer equation of state

Conclusions

- pQCD at high density + stability, causality and consistency
  $\longrightarrow$ constraints of neutron-star EoS
- Robust uncertainty estimation with Bayesian methods

  $\longrightarrow$ softening of neutron-star EoS

github.com/aykhuss/miho

github.com/OKomoltsev/QCD-likelihood-function

Open positions in my group:

- PhD, see Grossi, Tue 15:20, inspirehep.net/jobs/2786994,
- Postdoc, see Heyen, Tue 09:50, opening soon, send inquires to a.mazeliauskas@thphys.uni-heidelberg.de

www.isoquant-heidelberg.de

CE Poster: Minijet quenching in non-equilibrium QGP by Fabian Zhou

Pressure vs chemical potential