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Subatech

Quarkonium dynamics in the quantum Brownian regime with non-abelian quantum master equations

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In collaboration with Pol-Bernard Gossiaux, Thierry Gousset, Jean-Paul Blaizot, Roland Katz and Aoumeur Daddi Hammou

Open quantum systems

System + environment

$$\rho(t) = \rho_{Q\bar{Q}} \otimes \rho_{QGP}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho = -i[H, \rho]$$

Evolution of the total system

$$\rho(t') = U(t', t)\left[\rho_{Q\bar{Q}} \otimes \rho_{QGP}\right] U(t', t)^\dagger$$

Trace out environment degrees of freedom

System ($Q\bar{Q}$ pair)

$$\rho_{Q\bar{Q}}(t)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_{Q\bar{Q}} = \mathcal{L}\rho_{Q\bar{Q}}$$

Evolution of the system

$$\rho_{Q\bar{Q}}(t') = Tr_{QGP}\left[U(t', t)\rho(t)U(t', t)^\dagger\right]$$

Quantum Master Equation (Quantum Brownian Regime)

singlet density operator

$$\frac{d}{dt}\begin{pmatrix}\mathcal{D}_s\\ \mathcal{D}_o\end{pmatrix} = \mathcal{L}\begin{pmatrix}\mathcal{D}_s(\mathbf{s},\mathbf{s}',t)\\ \mathcal{D}_o(\mathbf{s},\mathbf{s}',t)\end{pmatrix}$$

$$\mathcal{L} = \begin{pmatrix}\mathcal{L}_{ss} & \mathcal{L}_{so}\\ \mathcal{L}_{os} & \mathcal{L}_{oo}\end{pmatrix}$$

octet density operator

singlet-octet transitions

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 + \boxed{\mathcal{L}_4}$$

subleading but necessary for positivity

$\mathcal{L}_0$: Kinetic terms

$\mathcal{L}_1$: Static screening (V)

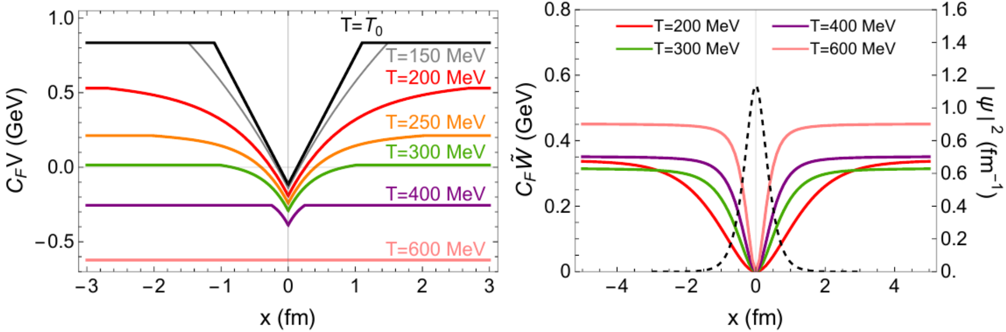
$\mathcal{L}_2$: Fluctuations (W)

$\mathcal{L}_3/\mathcal{L}_4$: Dissipation (W'/W"/W"')

Dynamical processes

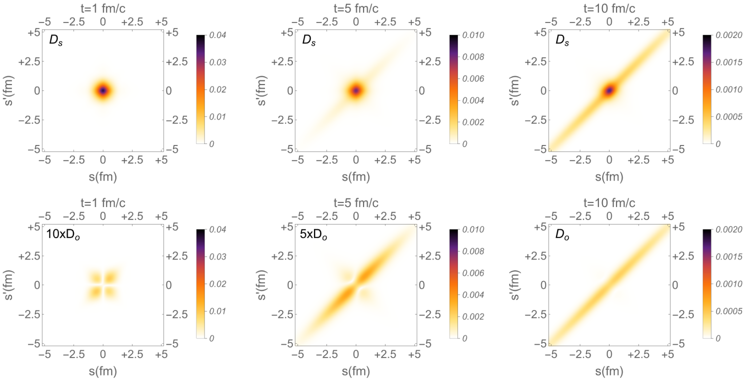
S.D, P-B. Gossiaux, T. Gousset, R. Katz, J-P. Blaizot, 2402.04488 (accepted for publication in JHEP)

1D Potential



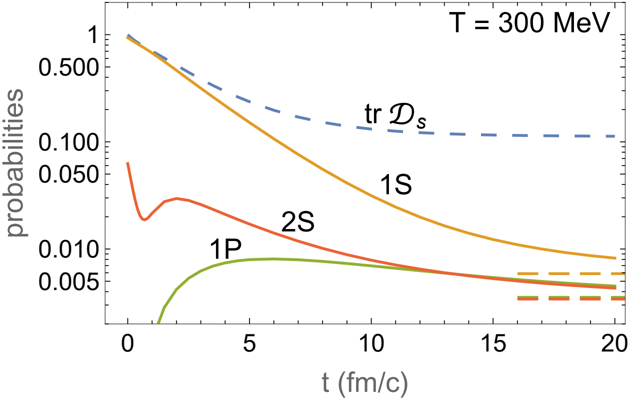
- Based on a 3D potential inspired from Lattice results D. Lafferty, A. Rothkopf (2020)
- Real part: parametrization to reproduce 3D mass spectra
- Imaginary part: separated in a coulombic and string part, aims at reproducing 3D decay widths R. Katz, S.D, P-B. Gossiaux (2022)

$c\bar{c}$ evolution at fixed temperature



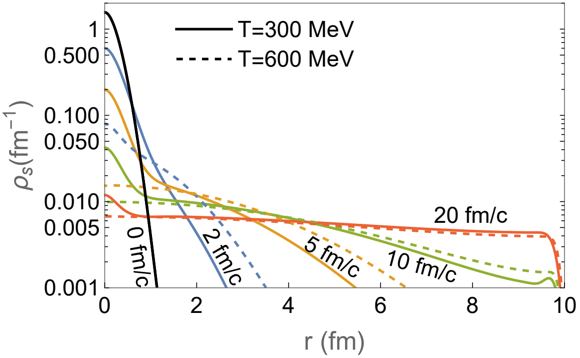
- Initial singlet in-medium 1S state at T = 300 MeV
- Octet populated via dipolar transitions
- Repulsive octet potential $\Rightarrow$ delocalization
- Delocalization in singlet channel via transitions
- Surviving central peak in singlet channel
- Non-diagonal elements (width equal to $\lambda_{th} = \frac{1}{\sqrt{MT}}$)

$c\bar{c}$ evolution at fixed temperature



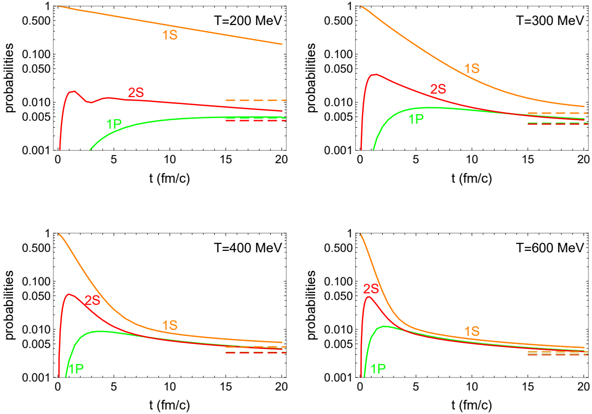
- Instantaneous projections on vacuum eigenstates
- In-medium 1S state very close to vacuum ($p_{1S,\mathrm{v}}(0) \approx 0.95$)
- Complex evolution of p_{2S} (coupling to other states + decay to continuum)
- Delayed appearence of 1P states (chain of transitions at 3rd order in perturbation theory)
- Global evolution towards asymptotic values (dashed horizontal lines)

Temperature dependence



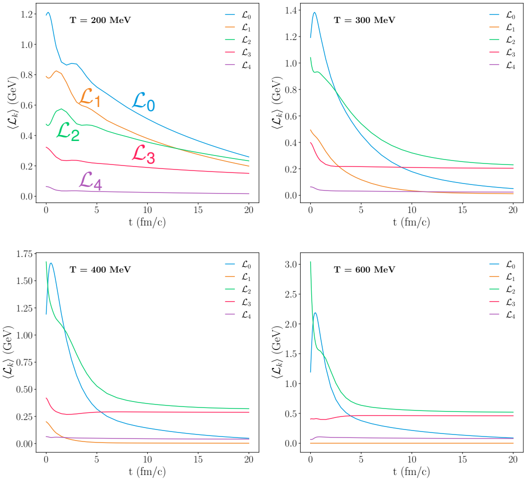
- $\rho_s(r,t) = \mathcal{D}_s(r,r,t)$
- Density reaching box boundaries between 5 and 10 fm/c
  $\Rightarrow$ Level off to asymptotic value
- At 20 fm/c: stationnary pedestal at large distances with trace of bound state at small distances
- $T = 600$ MeV: harder and more frequent collisions $\Rightarrow$ faster increase of relative distance
- Central peak disappears faster than at 300 MeV, no peak after 5 fm/c (potential not binding at 600 MeV)

Temperature dependence



- Initial singlet vacuum 1S state
- Exponential decay of p_{1S}
- Growth of p_{1P}/p_{2S} followed by global decay
- Faster evolution with increasing T
- Close asymptotic values as T increases ($\mathcal{D}_s$ nearly diagonal)
- Oscillations of p_{2S} disappear for $T \geq 300$ MeV (overdamped regime)

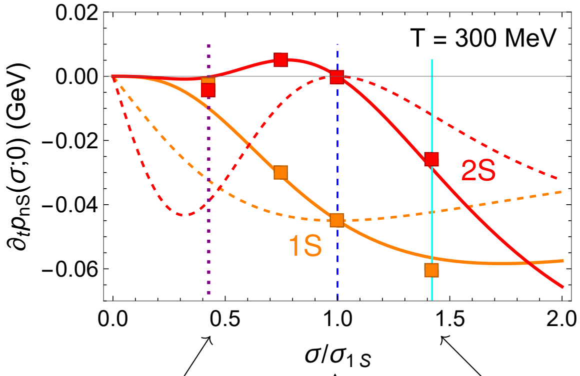
Hierarchy of operators in different regimes



$$\langle \mathcal{L}_k \rangle = \frac{\sum_{i,i'} \left|\mathcal{L}_k^{\mathrm{ss}}[\mathcal{D}_{\mathrm{s}}] + \mathcal{L}_k^{\mathrm{so}}[\mathcal{D}_{\mathrm{o}}]\right|_{i,i'} + \left(N_c^2 - 1\right)\left|\mathcal{L}_k^{\mathrm{os}}[\mathcal{D}_{\mathrm{s}}] + \mathcal{L}_k^{\mathrm{oo}}[\mathcal{D}_{\mathrm{o}}]\right|_{i,i'}}{\sum_{i,i'} \left|\mathcal{D}_{\mathrm{s}}\right|_{i,i'} + \left(N_c^2 - 1\right)\left|\mathcal{D}_{\mathrm{o}}\right|_{i,i'}}$$

- Clear hierarchy at $T = 200$ MeV
- $\langle \mathcal{L}_{0,1} \rangle \geq \langle \mathcal{L}_2 \rangle$ until ≈ 12 fm/c
  $\Rightarrow$ Quantum mechanical evolution
- Effect of $\mathcal{L}_1$ less and less impactful
- $\langle \mathcal{L}_2 \rangle \sim \langle \mathcal{L}_3 \rangle \Rightarrow$ Equilibrium
- Marginal $\mathcal{L}_4$ contribution

Comparison of states populations early time evolution



Initial compact gaussian

Initial vacuum 1S

Initial in-medium 1S

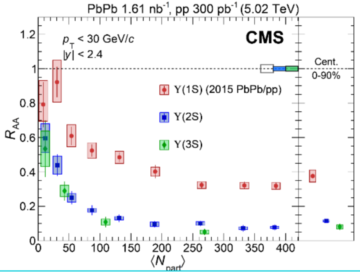
- Dashed lines: $\dot{p}_{nS}(\sigma, 0) = -\Gamma_{nS} p_{nS}(\sigma, 0)$
- Full lines: analytical solution with $\mathcal{L}_2$ only
- Points: Full QME results
- Good agreement between QME and "$\mathcal{L}_2$-only" evolution around $\sigma = \sigma_{1S}$
- Very different results between dashed and full curves: quantum effects
- Positive derivative for 2S completely absent from dashed curves

Bottomonium system

- 3 different initial states:
  - $\Upsilon(1S)$-like initial state
  - $\Upsilon(2S)$-like initial state
  - Mixture of S and P states:

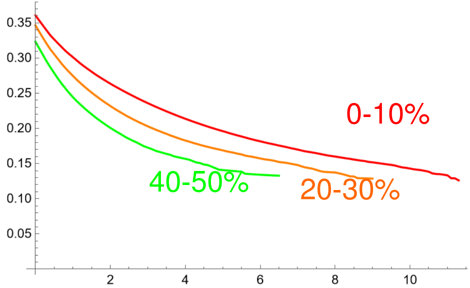
    $\Psi(x) \propto e^{-\frac{x^2}{2\sigma^2}} \left(1 + a_{\rm odd}\frac{x}{\sigma}\right)$

    $\sigma = 0.045\ \mathrm{fm} \quad a_{\rm odd} = 3.5$

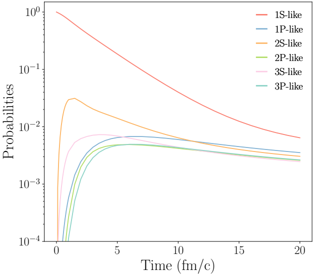


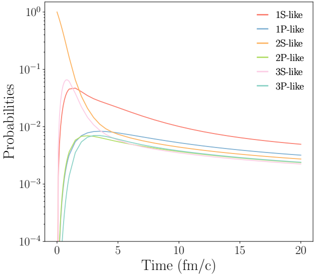
2303.17026

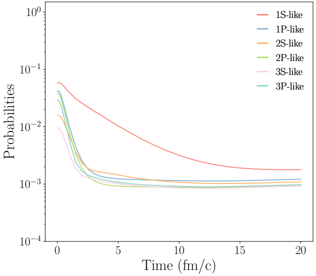
- 4 different medium settings
  - Fixed temperature T = 400 MeV
  - Average temperature profiles obtained from EPOS4 for three different centrality classes: 0-10%, 20-30% and 40-50% with $|y| < 2.4$ (CMS conditions)



Bottomonium dynamics at fixed temperature





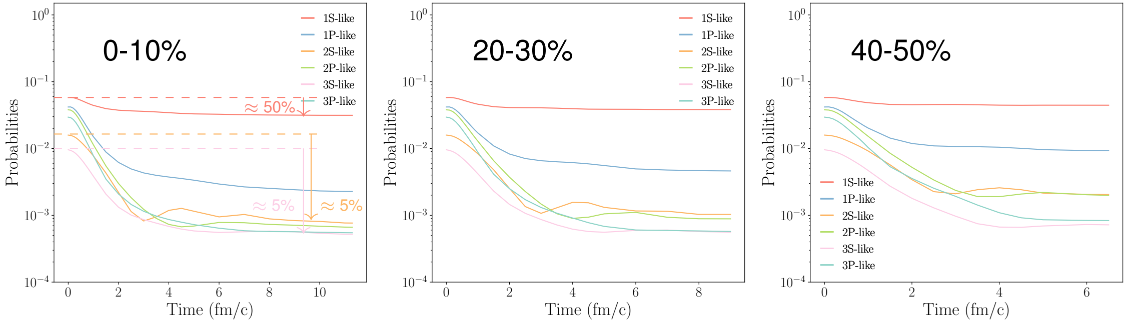


- Similar evolution to charmonium
- 1S-like reduced by a factor 100
- Factor 2 between 1S and 2S

- Similar final state
- Similar 2S/1S ratio

- Lower initial populations
- 1S (2S) evolution similar to the evolution with the 1S (2S) initial state

Bottomonium dynamics in a dynamical medium



- Reduction of suppression for more peripheral profiles
- 3S not much more suppressed than 2S for more central collisions

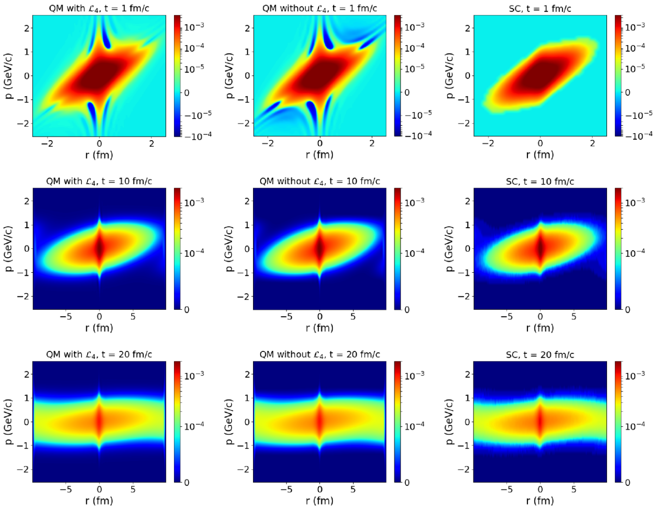
Semi-classical approach

- Semi-classical approaches can treat multiple pairs

  J.-P. Blaizot and M.-A Escobedo, JHEP 06 (2018) 034
- Focus on $c\bar{c}$ (more affected by recombination)

  A. Daddi Hammou, J.-P. Blaizot, S. D, P.-B. Gossiaux and T. Gousset (in preparation)
- Case of abelian dynamics ("scalar QCD") as first step towards QCD case as QCD requires semi-classical treatment of color
- Full QME as reference for comparison $\frac{d\mathcal{D}_{sQCD}}{dt} = \left.\frac{d\mathcal{D}_s}{dt}\right|_{\mathcal{D}_s=\mathcal{D}_o}$
- Semi-classical treatment not including $\mathcal{L}_4$
  $\Rightarrow$ "Uncertainty band" for QME results by turning on and off $\mathcal{L}_4$ contribution
- Real potential regularized for SC approach as the original potential is sharp at the origin

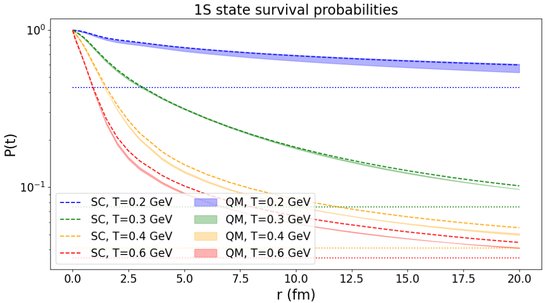
Comparison with semi-classical results

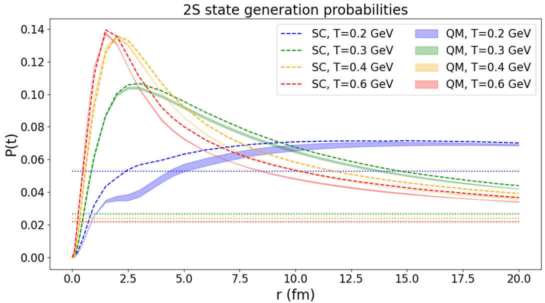


$$W(r,p) = \frac{1}{2\pi\hbar} \int \mathrm{d}y \, e^{-\frac{ipy}{\hbar}} \left\langle r + \frac{y}{2} \middle| \hat{\mathcal{D}} \middle| r - \frac{y}{2} \right\rangle$$

- Initial vacuum 1S state, T = 300 MeV
- Differences at early times, due to the presence of quantum effects absent in semi-classical approach
- Very good agreement later on

Comparison with semi-classical results





- Overall good agreement
- Differences at high temperatures due to overheating from QME

- Good agreement at high temperatures
- Differences at 200 MeV due to quantum effects. Agreement from $\approx$ 12 fm/c

Starting temperature from a realistic AA scenario would be high anyways
Small systems (lower T) $\Rightarrow$ Fewer pairs $\Rightarrow$ Full quantum treatment possible

Conclusions and perspectives

- Resolution of a quantum master equation in the quantum brownian regime
- Study of various temperature regimes, highlighting distinctive features of the $Q\overline{Q}$ evolution
- Direct application to $b\overline{b}$: Quantum evolution captures the general trends
- Benchmark of semi-classical approach in "scalar QCD" case: overall very good agreement

- Improvements of the QME (treatment of energy gaps)
- Study related to the potential (screening for example)
- Revisit the phenomenological study of $b\overline{b}$ with more statistics
- Extension of the semi-classical approach to QCD

Back-up

Quantum Master Equation

$$\mathcal{L}_0\mathcal{D} = -i\,[H_Q, \mathcal{D}]$$

$$\mathcal{L}_1\mathcal{D} = -\frac{i}{2}\int_{xx'} V(x-x')\,[n_x^a n_{x'}^a, \mathcal{D}]$$

$$\mathcal{L}_2\mathcal{D} = \frac{1}{2}\int_{xx'} W(x-x')\,(\{n_x^a n_{x'}^a, \mathcal{D}\} - 2n_x^a\mathcal{D}n_{x'}^a)$$

$$\mathcal{L}_3\mathcal{D} = -\frac{i}{4T}\int_{xx'} W(x-x')\left(\dot{n}_x^a\mathcal{D}n_{x'}^a - n_x^a\mathcal{D}\dot{n}_{x'}^a + \frac{1}{2}\{\mathcal{D}, [\dot{n}_x^a, n_{x'}^a]\}\right)$$

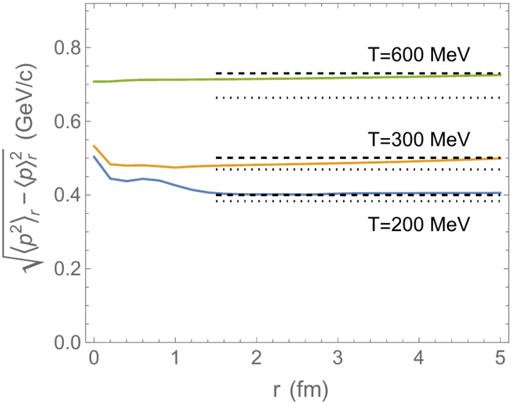
- n_x^a: color charge density
  $n_x^a = \delta(x-r)\,t^a \otimes \mathbb{I} - \mathbb{I} \otimes \delta(x-r)\,\tilde{t}^a$

- Can recover $\mathcal{L}_3$ from $\mathcal{L}_2$ by performing:

$$(\{n_x^a n_{x'}^a, \mathcal{D}\} - 2n_x^a\mathcal{D}n_{x'}^a) \longrightarrow \left\{\left(n_x^a - \frac{i}{4T}\dot{n}_x^a\right)\left(n_{x'}^a + \frac{i}{4T}\dot{n}_{x'}^a\right), \mathcal{D}\right\} - 2\left(n_x^a + \frac{i}{4T}\dot{n}_x^a\right)\mathcal{D}\left(n_{x'}^a - \frac{i}{4T}\dot{n}_{x'}^a\right)$$

- Additionnal terms $\Rightarrow \mathcal{L}_4$

Asymptotic Wigner distribution



- $\sqrt{< p^2 >}$ does not scale as $\sqrt{\frac{MT}{2}}$ (dotted lines)
- Equilibrium limit modified by $\mathcal{L}_4$
- At large distances, scaling as $\sqrt{\frac{1}{1+\frac{\gamma}{2}}\frac{MT}{2}}$ with $\gamma = \frac{\tilde{W}^{(4)}(0)}{16MT\tilde{W}''(0)}$ (dashed lines)