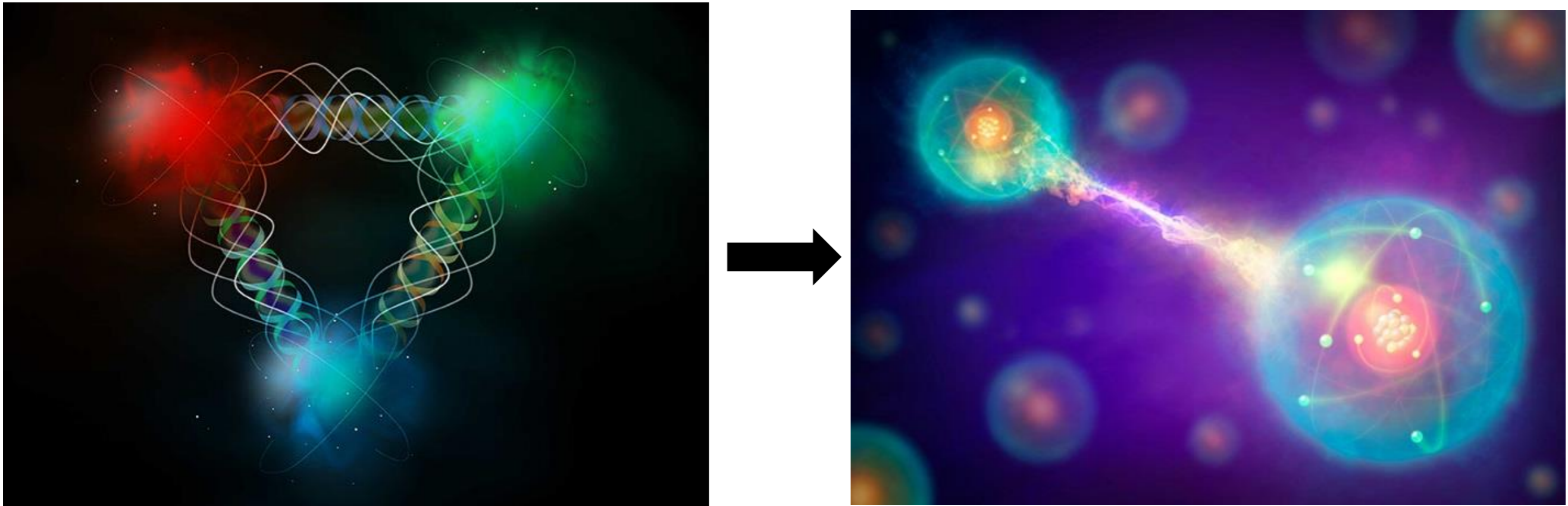


# Signal of initial state quantum entanglement in relativistic particle collisions

A. Hutson, R. Bellwied (University of Houston)



**References:** R. Bellwied (arXiv:1807.04589), Tu, Kharzeev, Ullrich (arXiv:1904.11974), and other papers by Kharzeev et al., Floerchinger et al. (arXiv:1702.03489, arXiv:1712.04558, arXiv:1707.05338, arXiv:1712.09362)



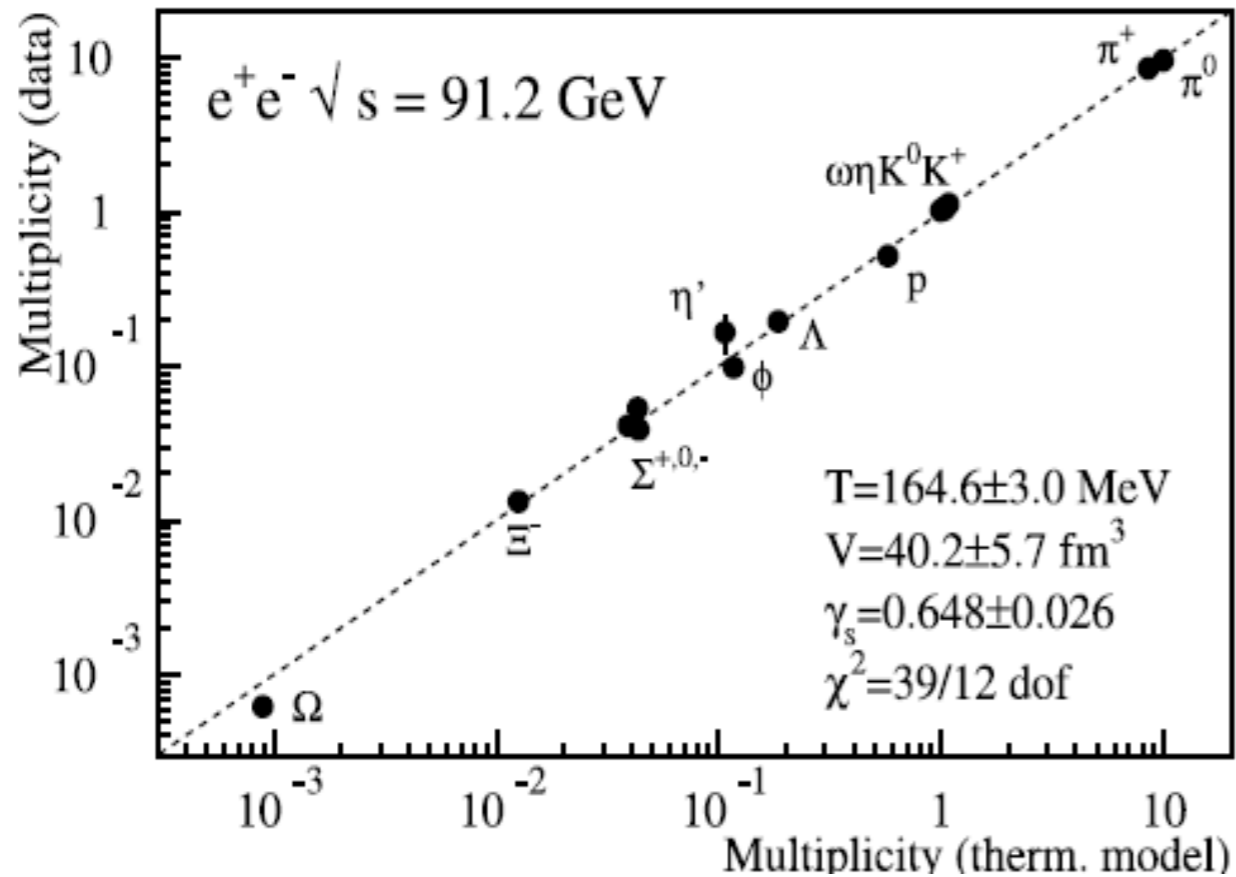
SOM 2024

The 21<sup>st</sup> International Conference on Strangeness in Quark Matter  
3-7 June 2024, Strasbourg, France

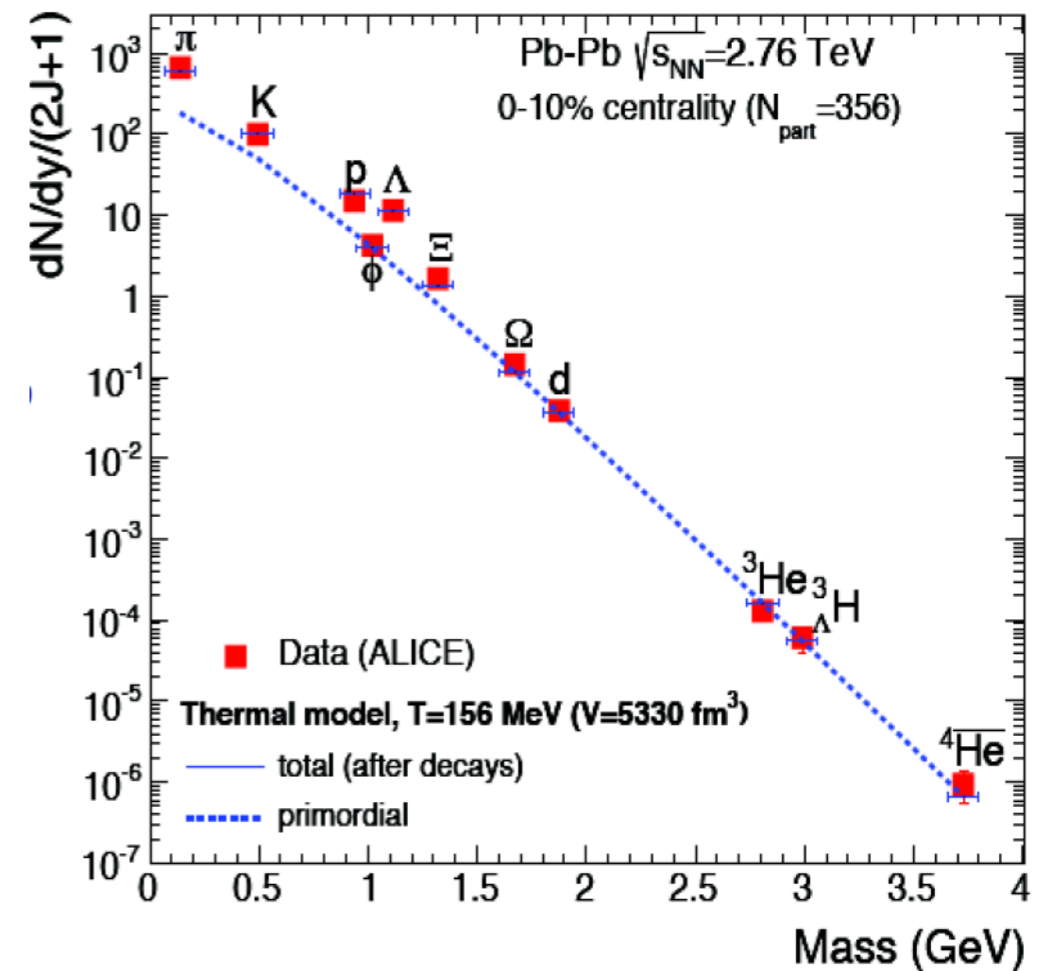
# Three basic questions for relativistic particle collisions

- How can the partons thermalize fast enough that hydrodynamical and statistical hadronization models are applicable ?
- How can the final state particles in elementary collisions be thermal ?
- How come that there seems to be a one to one relation between the initial parton density and the final state particle density (parton-hadron duality)

# 'Thermal behavior' in elementary relativistic collisions and in light nuclei production



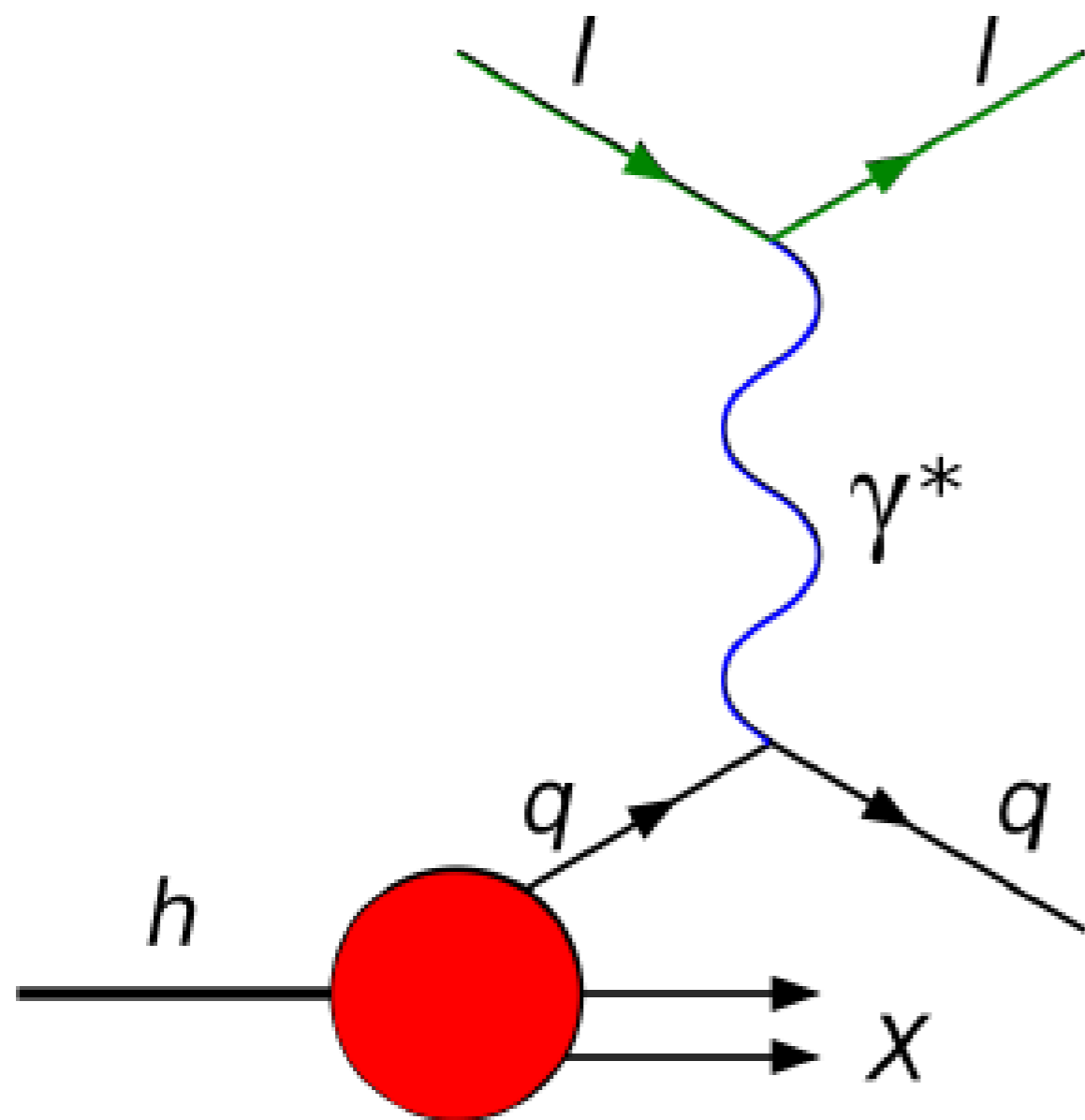
Becattini et al., EPJC 66, 377 (2010)



How can loosely bound objects 'survive' the fireball heat bath ?

- $\Lambda$  separation energy in hypertriton is 130 keV, i.e. a factor 1000 less than the chemical freeze-out temperature of the fireball
- Successful description of composite objects with a statistical hadronization model implies no entropy production after chemical freeze-out

# The proton in the basic parton model (PYTHIA etc.)



Any parton model describes the proton as a collection of point-like quasi-free partons frozen in the infinite momentum frame due to Lorentz dilation.

Cross-sections are given by the incoherent sum of cross sections of scattering off individual partons.

**These models ignore quantum mechanics**

Sometimes 'patched' through DGLAP, cluster (HERWIG), parton cascade (PCM) implementations, but e.g. DGLAP has to be applied on the energy dependent gluon saturation scale to take into account the high production of 'clusters' from soft processes in the initial state (see. T. Lappi, arXiv:1104.3725)

Maybe our picture of independent parton-parton interactions in proton-proton collisions is wrong

# Quantum entanglement in transverse and longitudinal direction

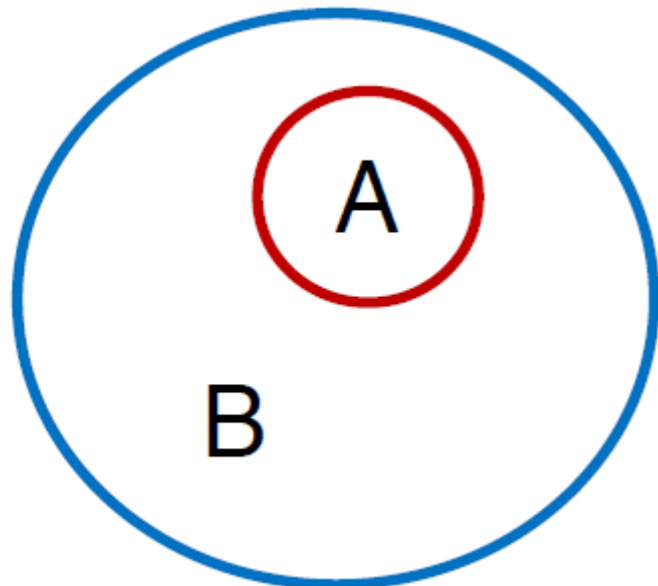
## Transverse:

DIS probes only part of the proton's wave function (region a), but we sum over all hadronic final states, which, in QM, corresponds to the density matrix of a mixed state:

$$\hat{\rho}_A = \text{tr}_B \hat{\rho}$$

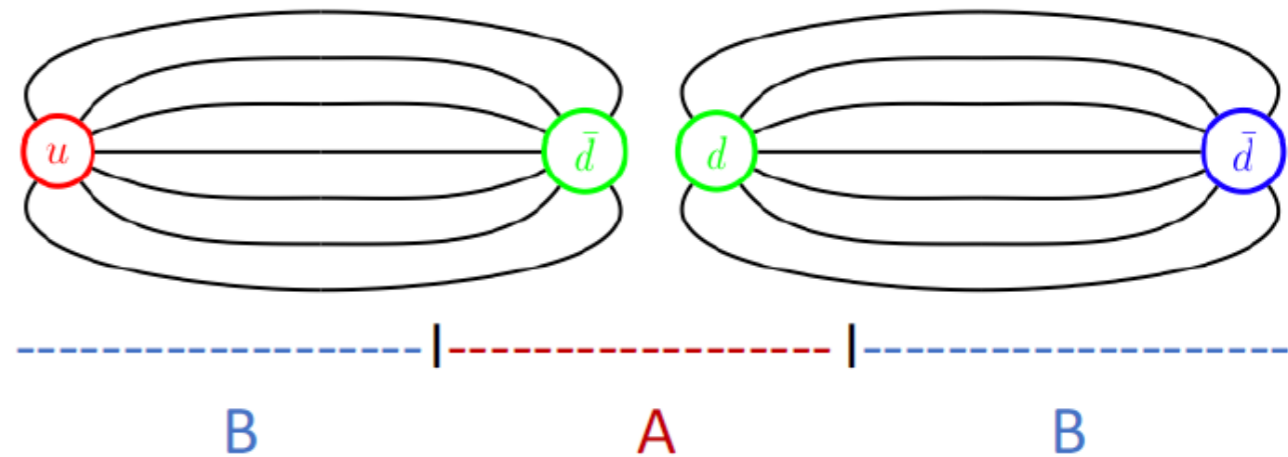
with a non-zero entanglement entropy:

$$S_A = -\text{tr} [\hat{\rho}_A \ln \hat{\rho}_A]$$



## Longitudinal:

Particle production in QCD strings:



Example: PYTHIA

Different regions in a string are entangled. Again A is described by a mixed state reduced density matrix. Could this lead to thermal-like behavior in the final state particles ?

**Conclusion:** Entanglement entropy is an extensive quantity (depends on volume)

# 'Thermalization' through quantum entanglement ?

Groundbreaking paper in condensed matter (experimental) (published in Science):

A.M. Kaufman et al., (Harvard), arXiv:1603.04409

*Quantum thermalization through entanglement in isolated many-body system, but cold and small (quantum quench in BE condensate of  $^{87}\text{Rb}$  atoms), effective  $T = 5-10 \text{ J}$ , study impact on neighboring atoms*

Even more relevant paper also in CM (experimental) (published in Nature Comm):

J. Kong et al., May 2020



ARTICLE

<https://doi.org/10.1038/s41467-020-15899-1>

OPEN

Measurement-induced, spatially-extended entanglement in a hot, strongly-interacting atomic system

Jia Kong<sup>1,2</sup>, Ricardo Jiménez-Martínez<sup>2</sup>, Charikleia Troullinou<sup>2</sup>, Vito Giovanni Lucivero<sup>2</sup>, Géza Tóth<sup>3,4,5,6</sup> & Morgan W. Mitchell<sup>2,7</sup>

Quantum technologies use entanglement to outperform classical technologies, and often employ strong cooling and isolation to protect entangled entities from decoherence by random interactions. Here we show that the opposite strategy—promoting random interactions—can help generate and preserve entanglement. We use optical quantum non-demolition measurement to produce entanglement in a hot alkali vapor, in a regime dominated by random spin-exchange collisions. We use Bayesian statistics and spin-squeezing inequalities to show that at least  $1.52(4) \times 10^{13}$  of the  $5.32(12) \times 10^{13}$  participating atoms enter into singlet-type entangled states, which persist for tens of spin-thermalization times and span thousands of times the nearest-neighbor distance. The results show that high temperatures and strong random interactions need not destroy many-body quantum coherence, that collective measurement can produce very complex entangled states, and that the hot, strongly-interacting media now in use for extreme atomic sensing are well suited for sensing beyond the standard quantum limit.

# Entanglement in QCD evolution



Erwin Schrödinger, 1952

“...we never experiment with just one electron or atom or (small) molecule. In thought experiments, we sometimes assume that we do; this invariably entails ridiculous consequences ... .”

**Idea:** initial state is entangled transversely (proton confinement) and longitudinally (string formation). Can we measure remnants of coherence? Are final state multiplicities due to initial state entanglement (all the way to light nuclei)?

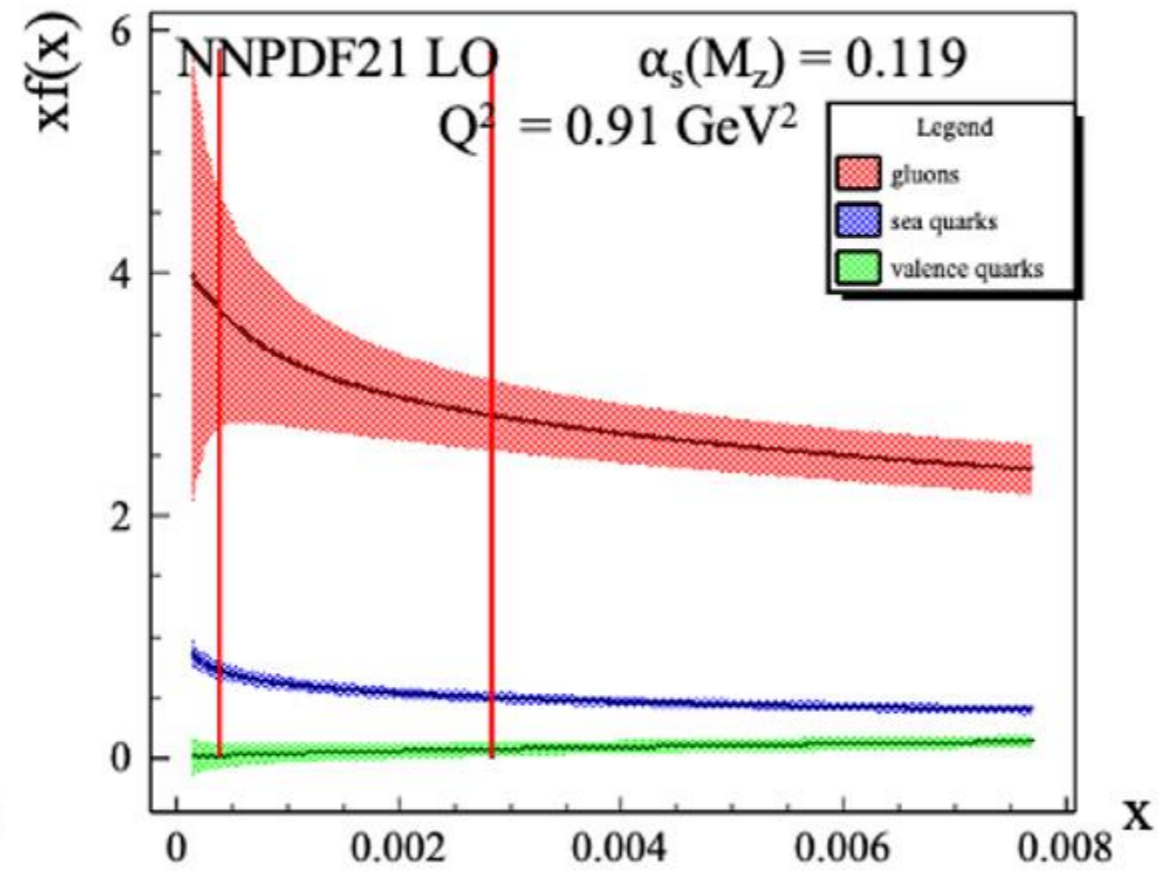
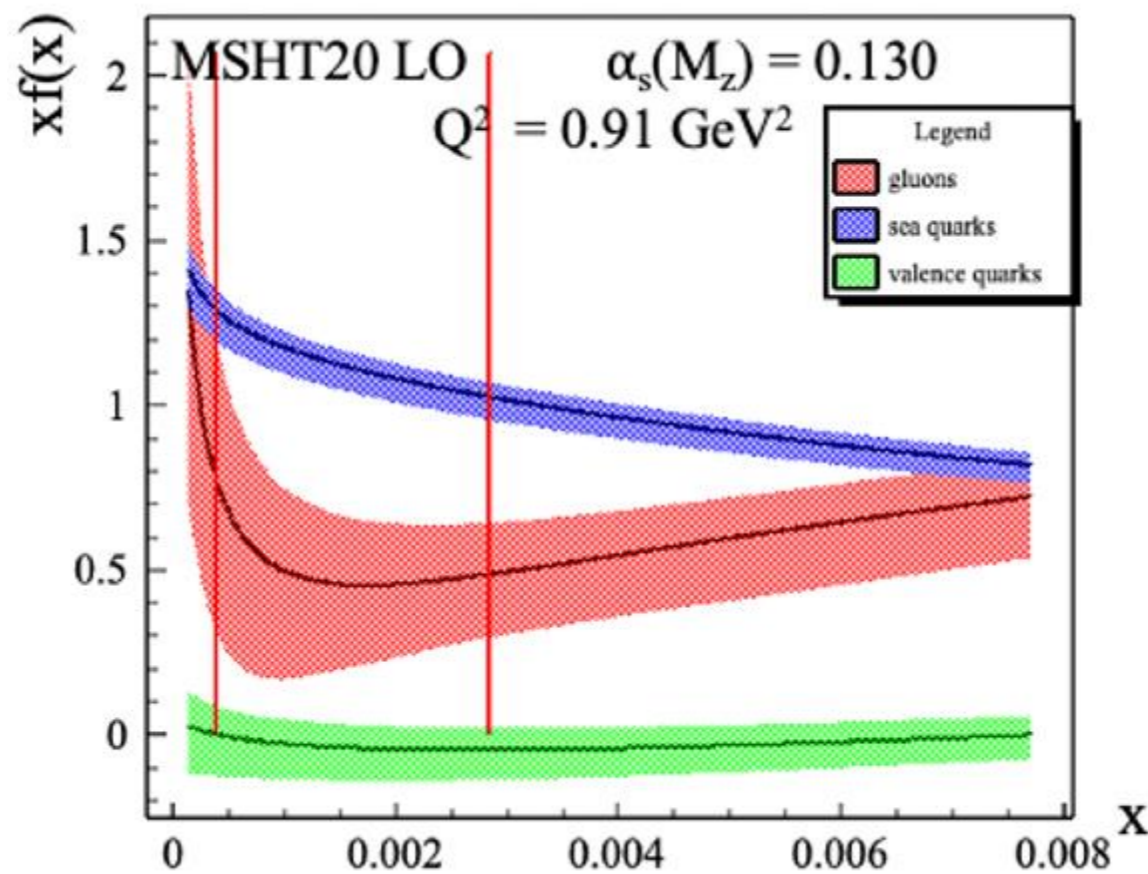
**Basis:** in an entangled proton the number of possible states is given by the parton distribution function which saturates at low  $x$ . **The entanglement entropy can then be calculated through the distribution functions.** All partonic states have about equal probability, which means the entanglement entropy is maximal and the proton is a maximally entangled state  $S = \ln[xG(x)]$

If the second law of thermodynamics applies to entanglement entropy then **the thermodynamic entropy of the hadronic final state reflects the entanglement entropy of the initial state deduced from the structure function (*parton-hadron duality*)**. Is the system not driven by thermalization but by initial coherence, which looks thermal?  $S_{\text{hadrons}} \simeq S_{EE}(x)$

**Measurements:** particle multiplicities as a function of  $x$ , particle multiplicities at hadronization trace back to initial parton entanglement (distribution of complex quark states based on string fragmentation?)

# Different Parton Distribution Functions

- Contributions from quarks might still be relevant at low  $x$





# How to map parton entanglement to parton distribution functions and experiment (from 1904.11974)

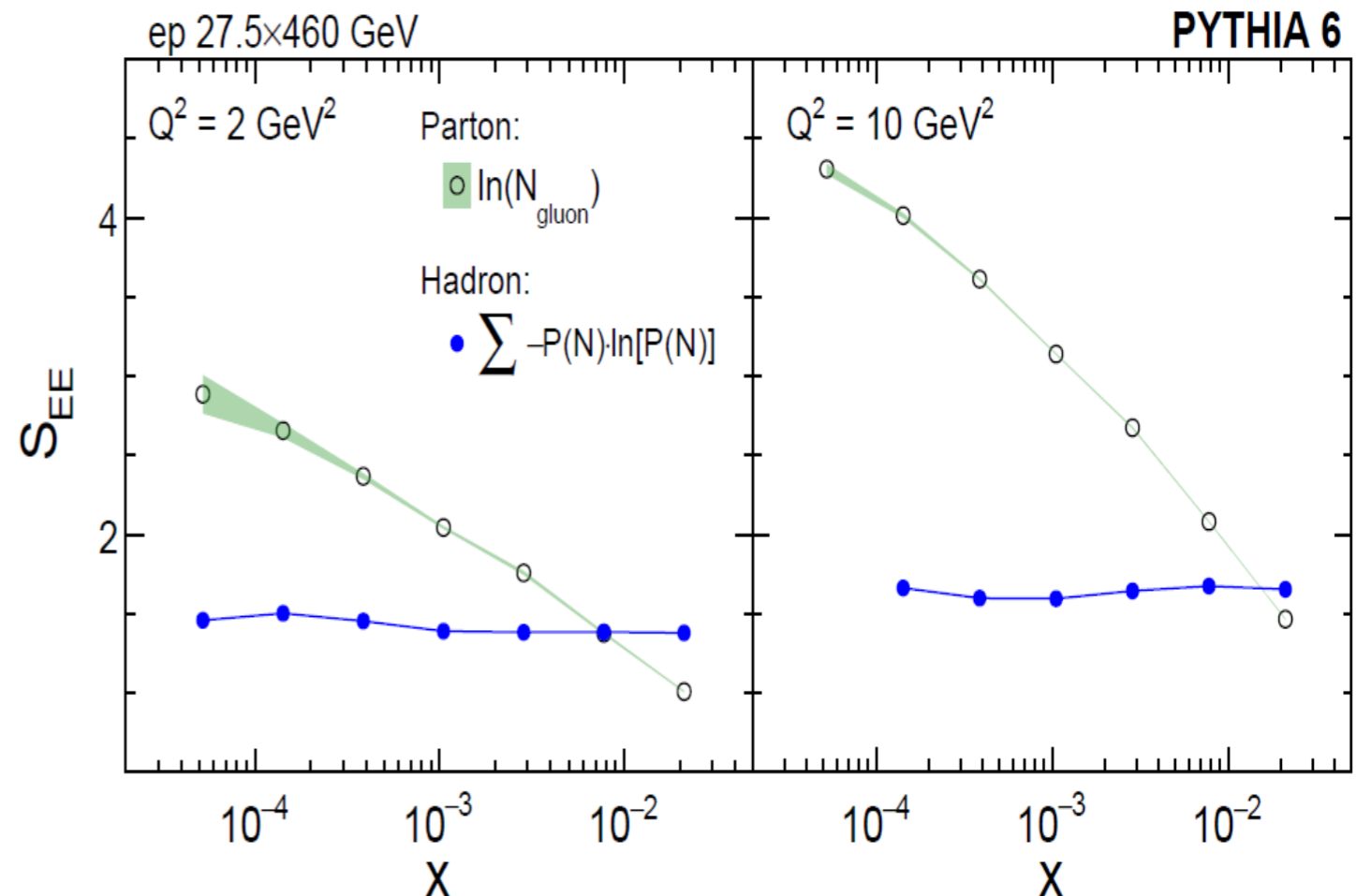
## Model Calculations

● First we obtain the number of gluons,  $N_{\text{gluon}}$ , by integrating the gluon distribution  $xG(x)$  over a given  $x$  range at a chosen scale  $Q^2$ . We use the leading order Parton Distribution Function (PDF) set MSTW at the 90% C.L. -> **Entanglement Entropy in green**

● The **Boltzmann entropy of the final-state hadrons is shown as blue filled circles**. It is calculated from the multiplicity distribution,  $P(N)$ , in a rapidity range determined by the  $x$  range used to derive  $N_{\text{gluon}}$ .  $P(N)$  is taken from ep DIS events created with the PYTHIA 6 or 8 event generator

● Since  $x$  and momentum transfer scale  $Q^2$  are not directly available in pp collisions, an alternative way of comparing the entropy at similar  $x$  and scales are used.

$$\ln(1/x) \sim y_{\text{proton}} - y_{\text{hadron}}$$



● **In ep collisions:**  $y_{\text{proton}}$  is the proton beam rapidity and  $y_{\text{hadron}}$  is the final-state hadron rapidity. For example, events with 27.5 GeV electrons scattering off 460 GeV protons with  $x$  between  $3 \times 10^{-5}$  and  $8 \times 10^{-5}$  correspond to a rapidity range of  $-3.5 < y < -2.5$ .

# This is slightly more complicated in pp

In pp collisions: two gluon distributions are involved, one from each proton, while we calculate the entanglement entropy from one distribution. Instead of altering the definition of the entanglement entropy, one can modify the  $P(N)$  distributions by extrapolating the  $P(N)$  distribution to reflect a single proton similar to that in ep collisions, by fitting a generalized Negative Binomial Distribution (NBD) to the  $P(N)$  distributions. The final  $P(N)$  is then taken as the same NBD function but with only half of the average multiplicity. This approach relies on the assumption that the final-state hadrons are produced coherently by the two colliding protons instead by incoherent and independent fragmentation.

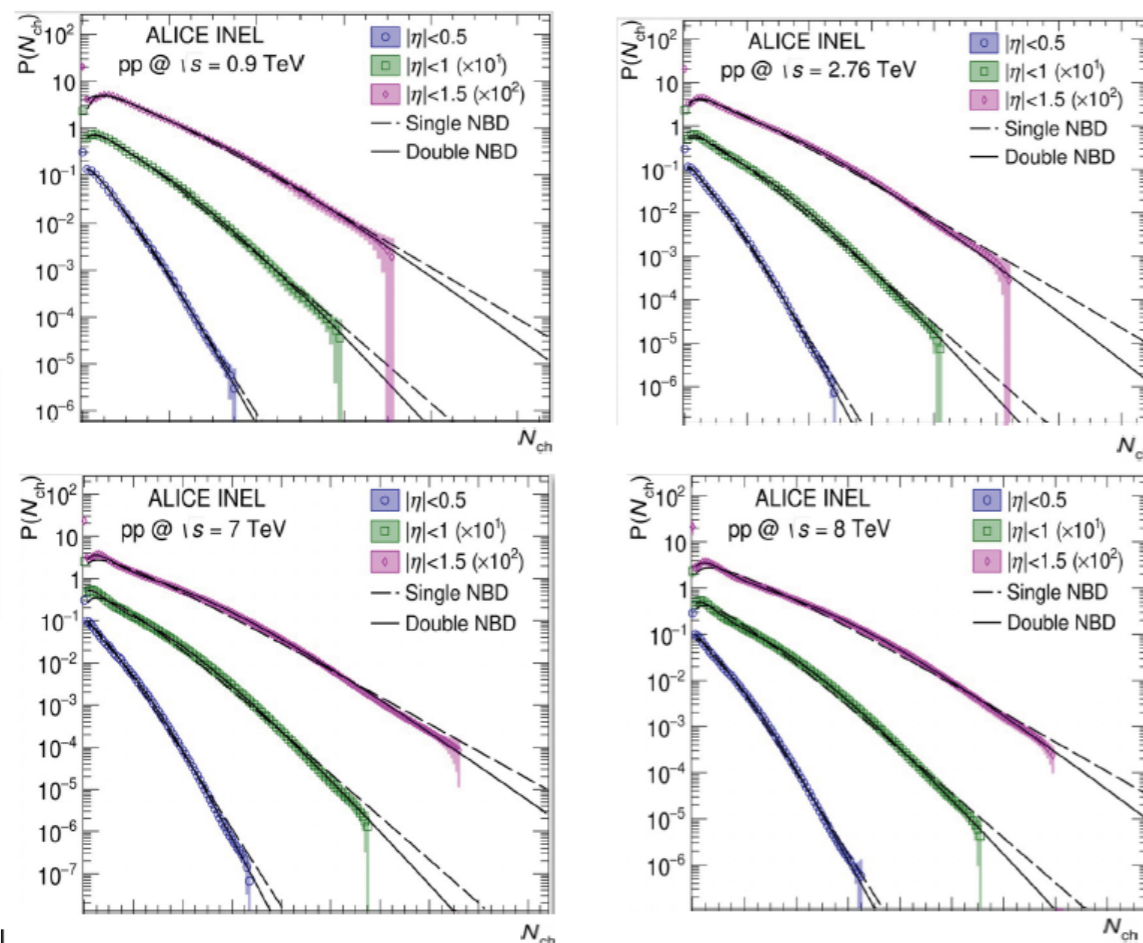
Now that we understand how to calculate the initial state entropy we would like to compare this to the entropy of the final state hadrons.

We measure the hadron entropy using Gibbs entropy formula and summing over the probability distribution  $P(N)$ .

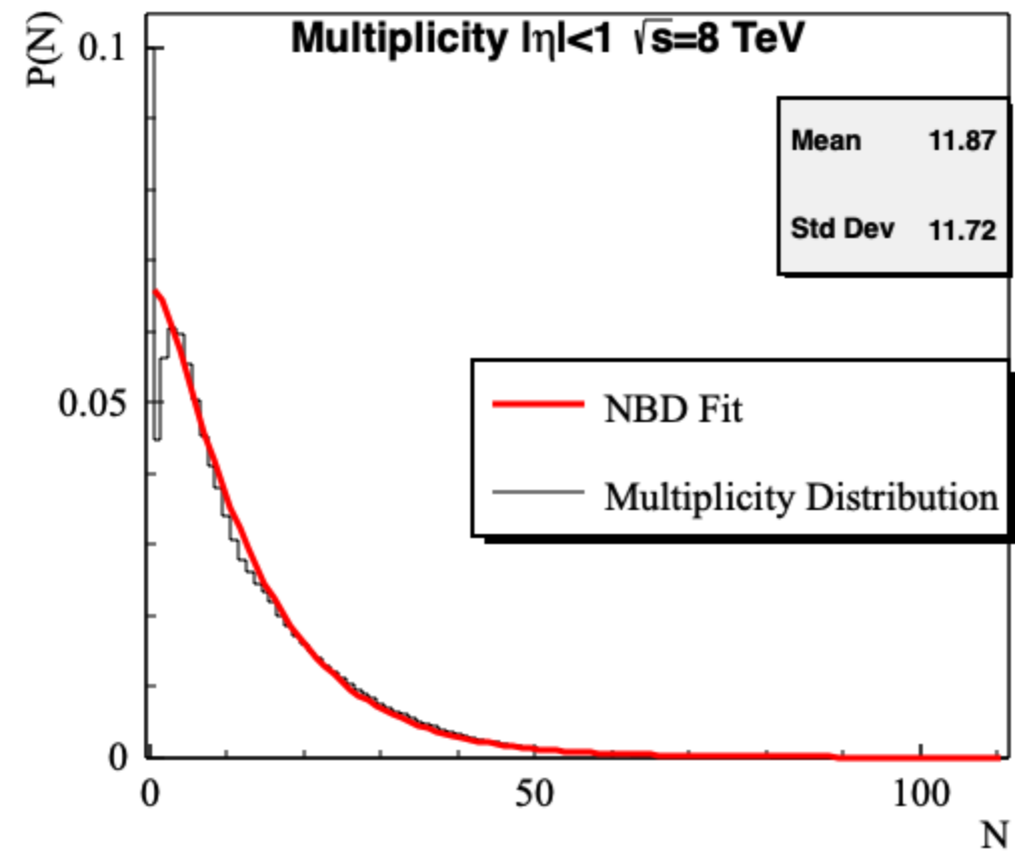
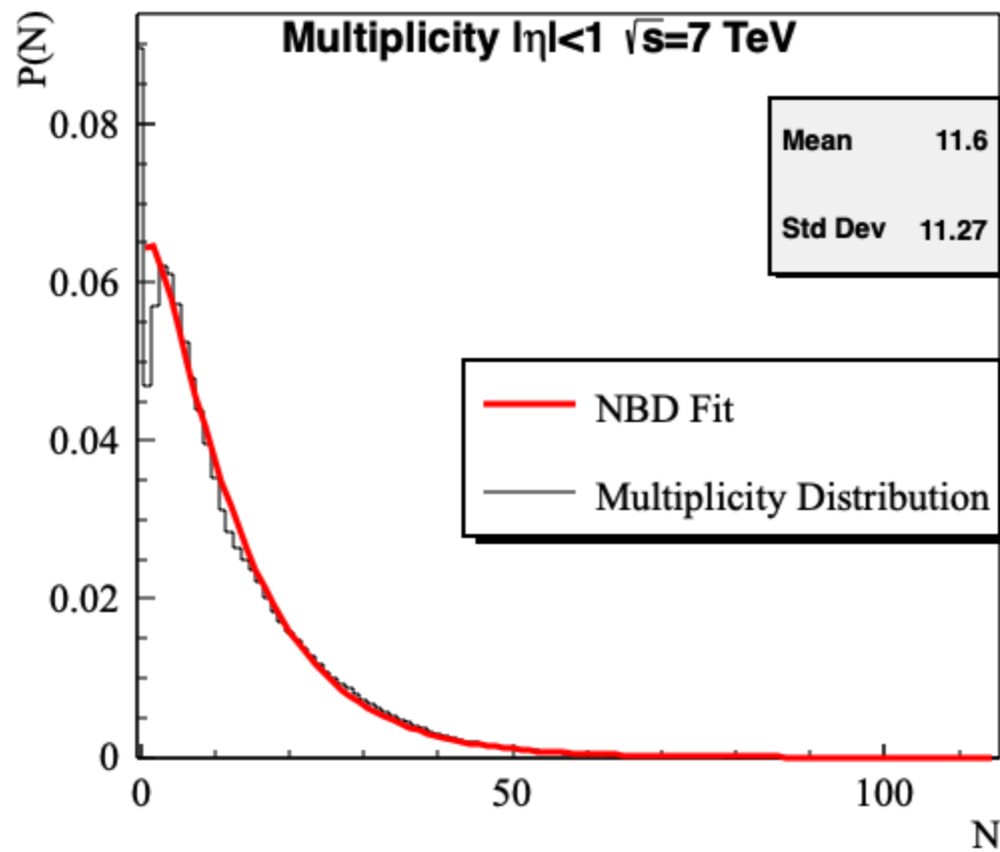
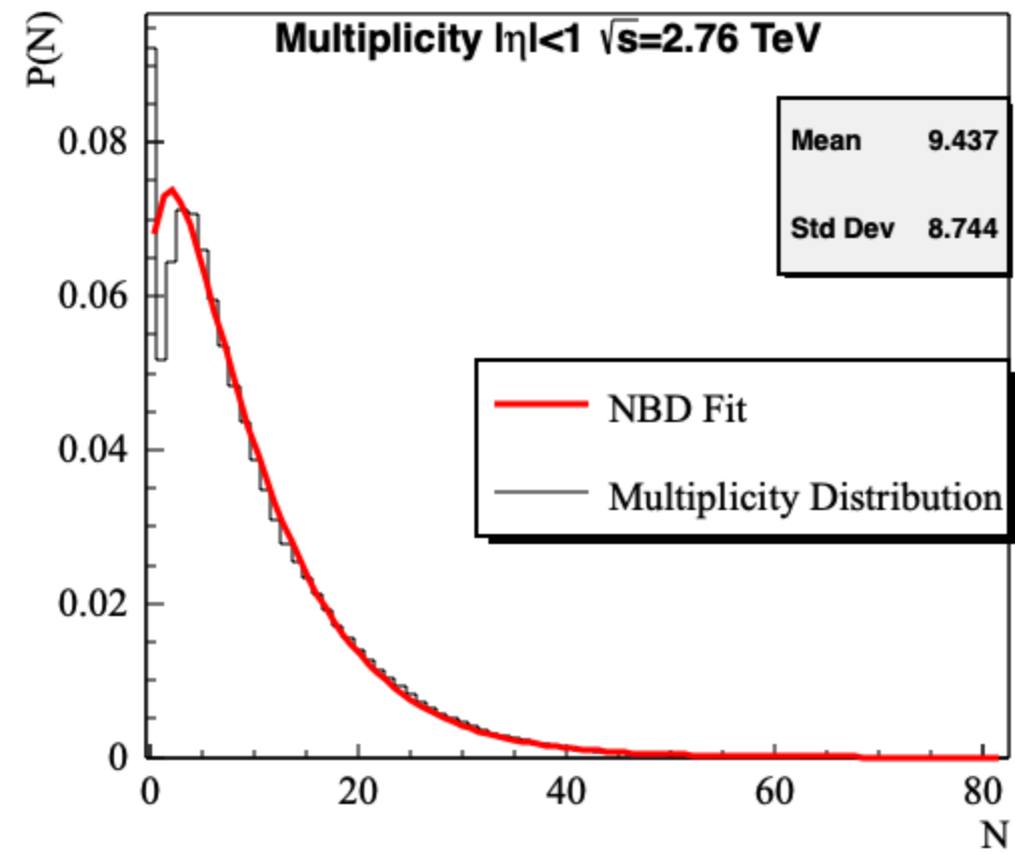
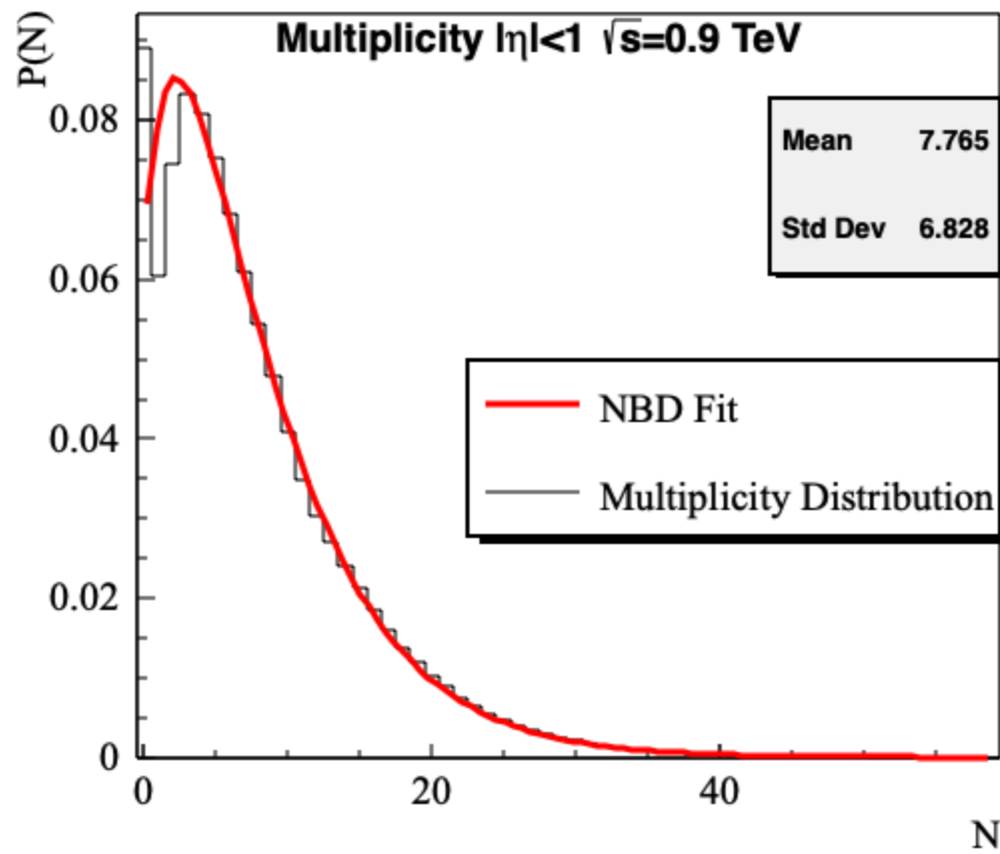
$$S_{final} \propto \sum P(N_h) \ln(P(N_h))$$

Procedure:

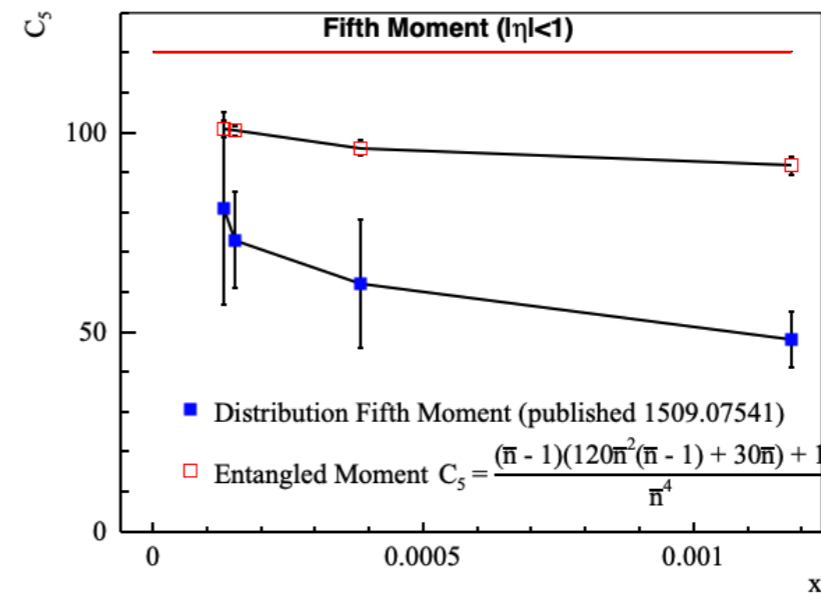
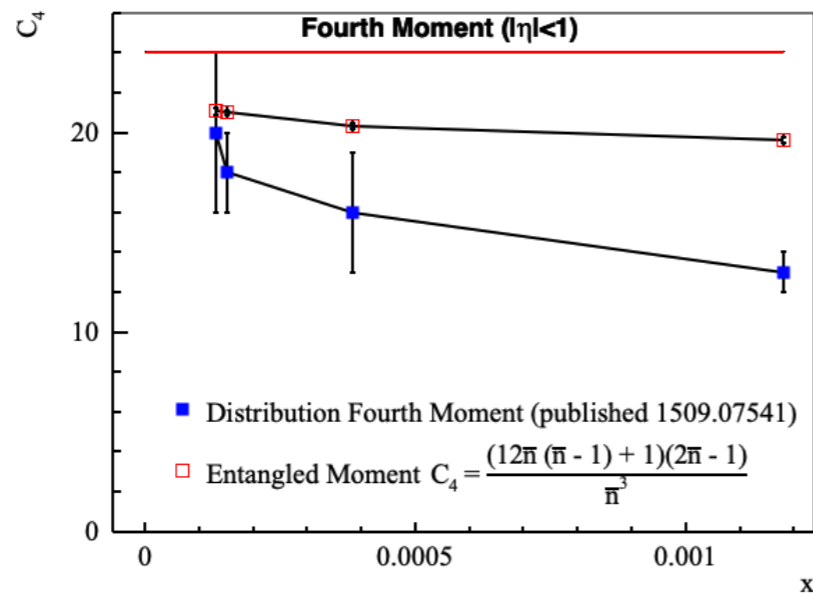
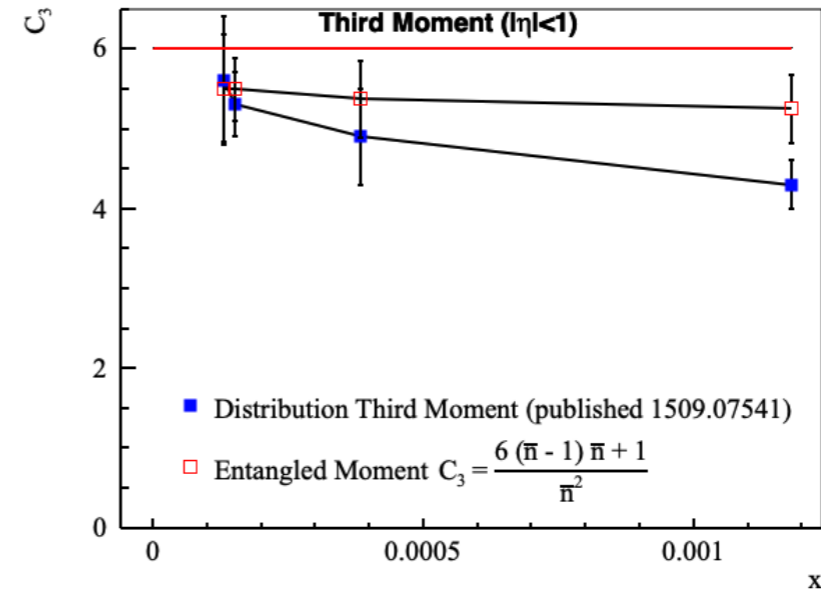
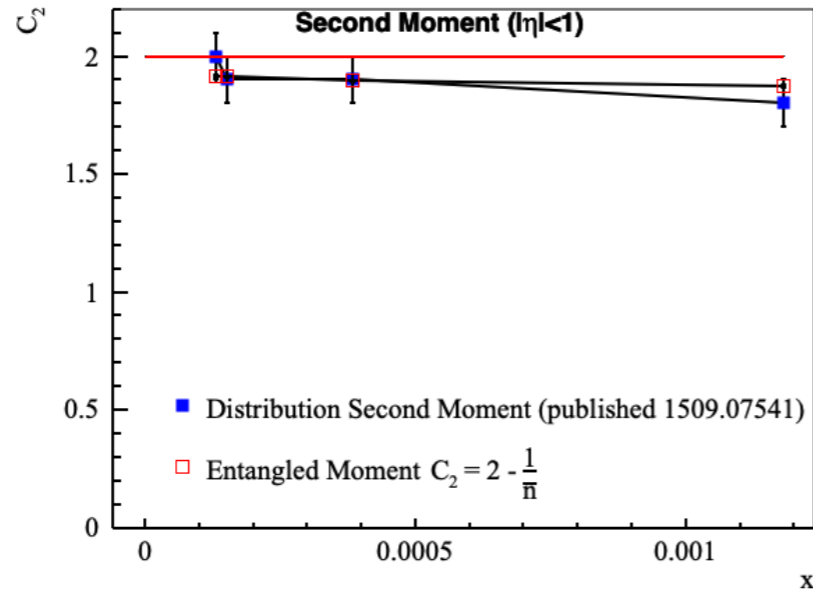
- 1.) measure multiplicity distributions  
In a fixed rapidity range
- 2.) calculate x-value distribution
- 3.) calculate entropy distribution



# NBD describes published ALICE data very well



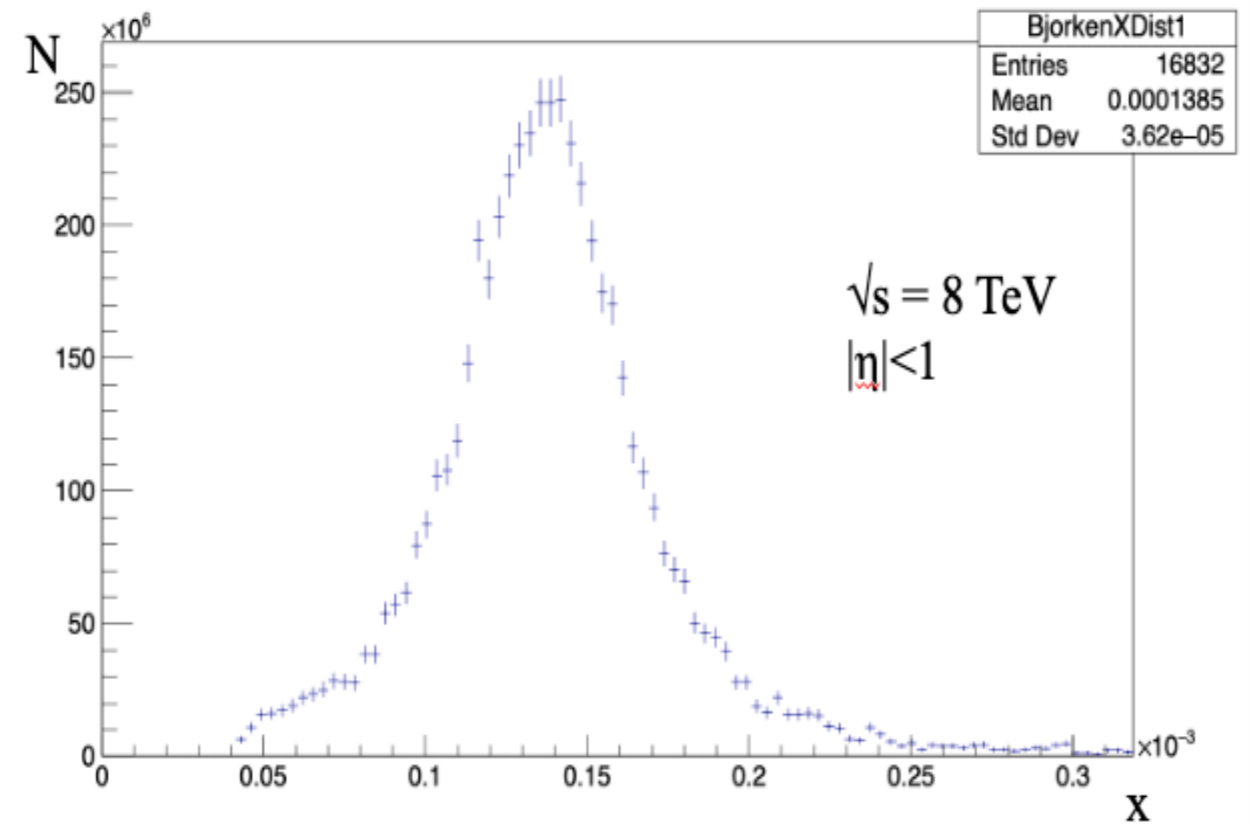
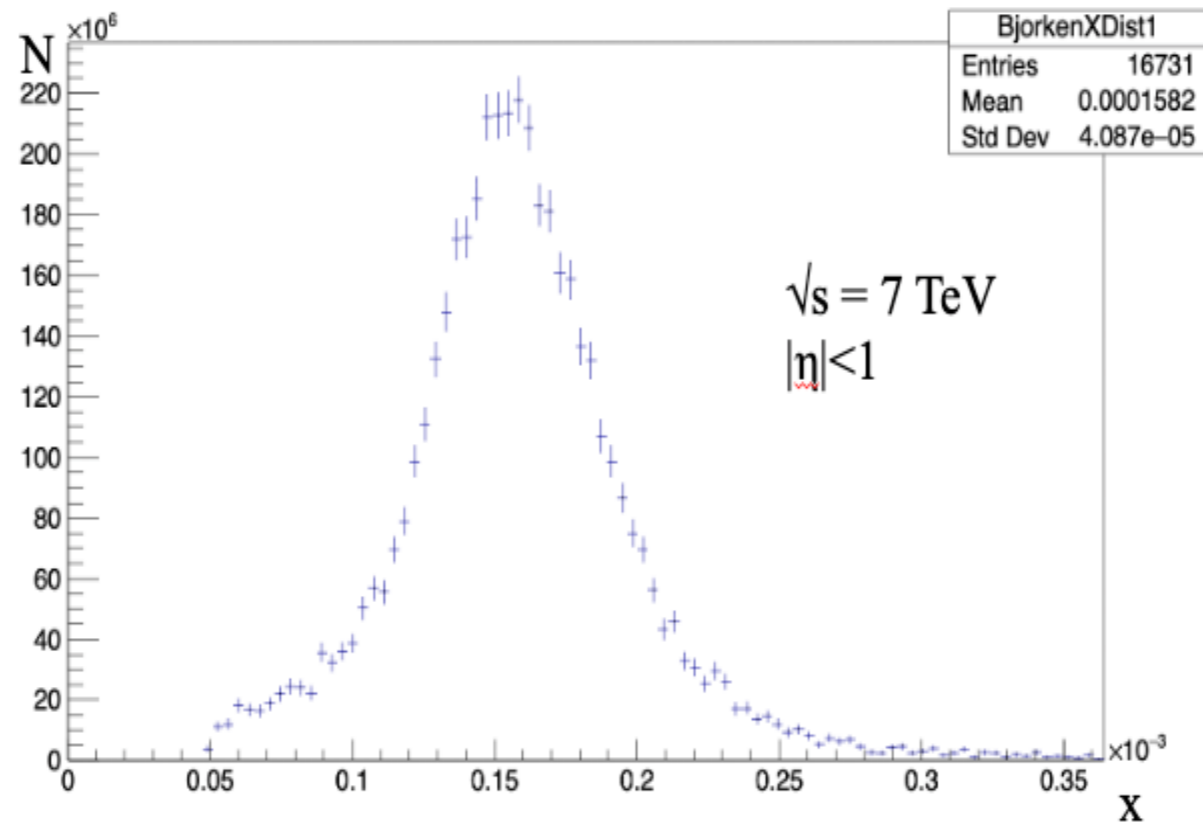
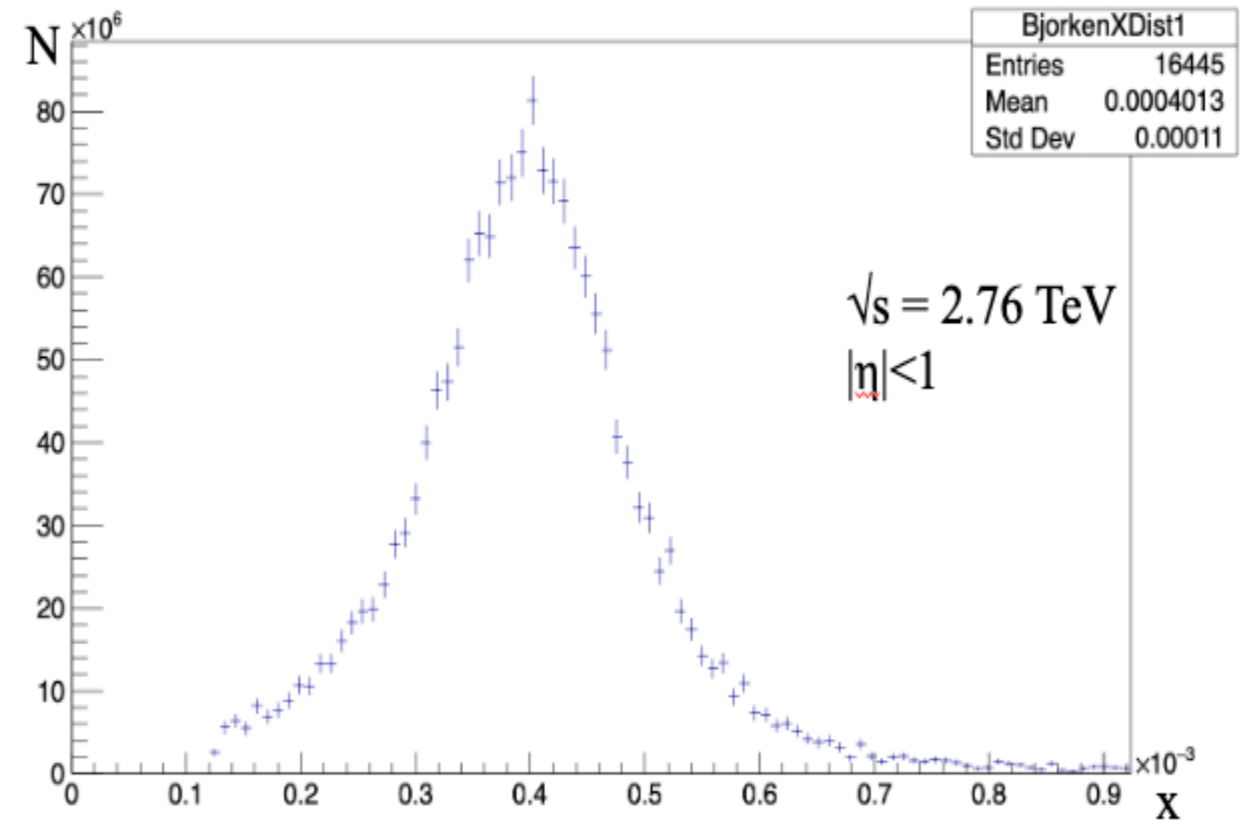
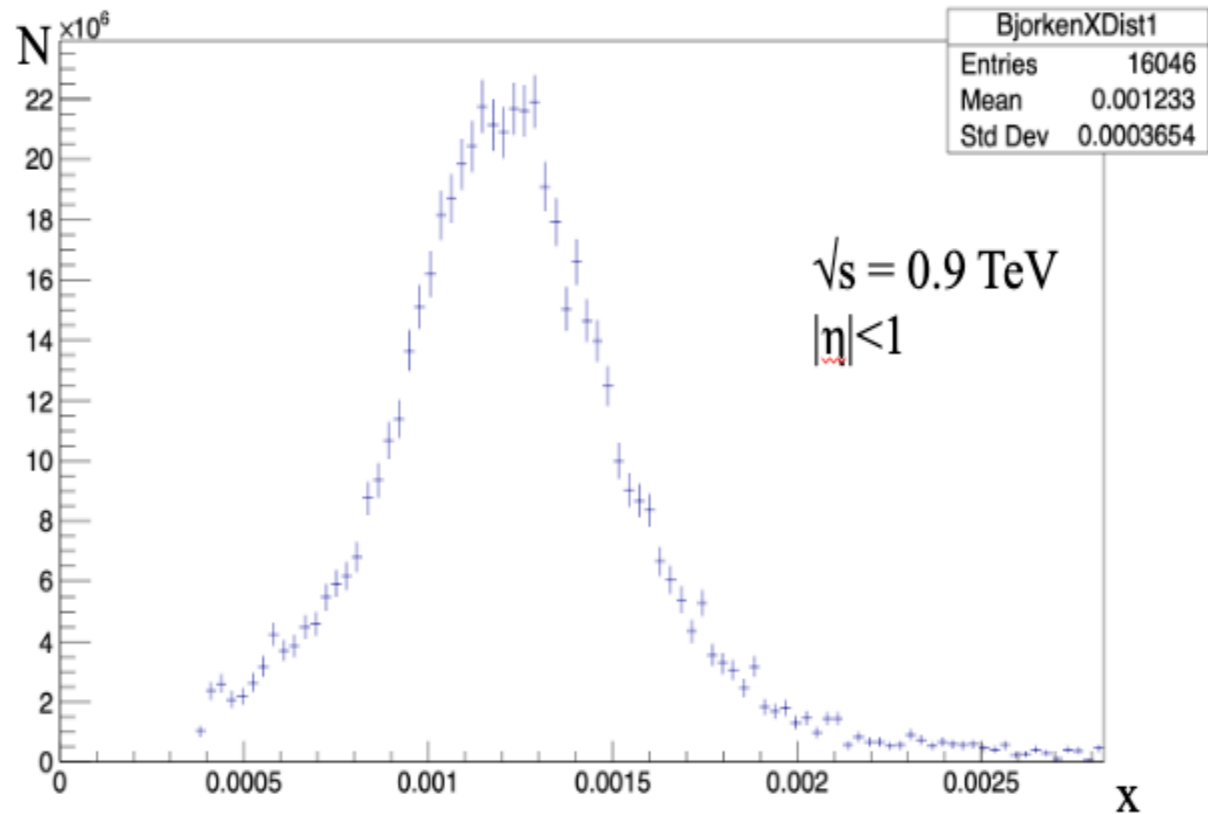
# Comparison of data to non-linear QCD model (Kharzeev, Levin (2017)) using a BK generating function of interacting color dipoles (parton cascade)



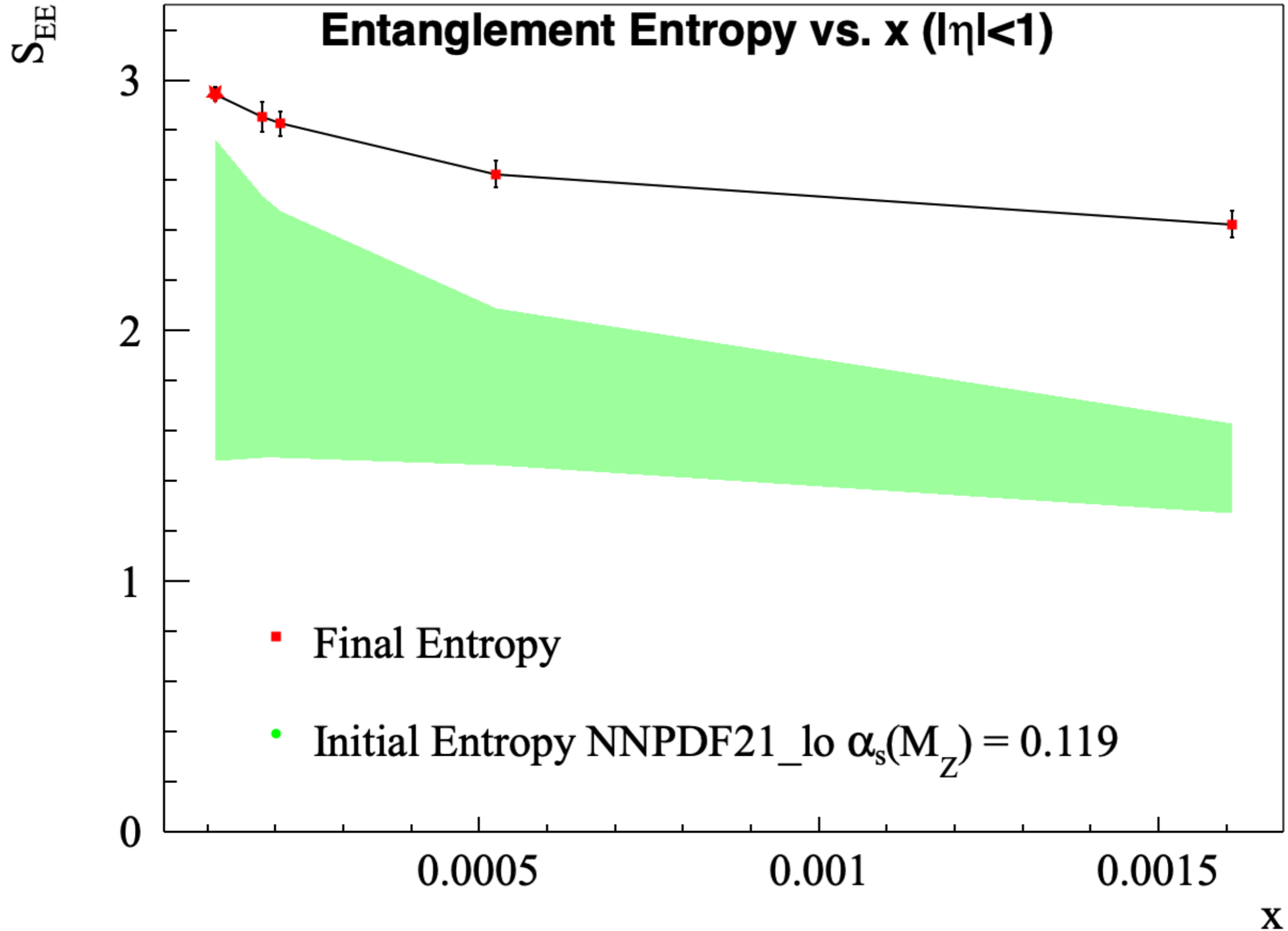
Agreement between data and model indicates that the multiplicity distribution of the produced hadrons is very close to the distribution in the number of partons that determines the entanglement entropy.

One can also calculate the upper bounds for these cumulants achieved at asymptotically high collision energy, when the average multiplicity  $n$  becomes very large (solid line)

# Resulting x-value distributions

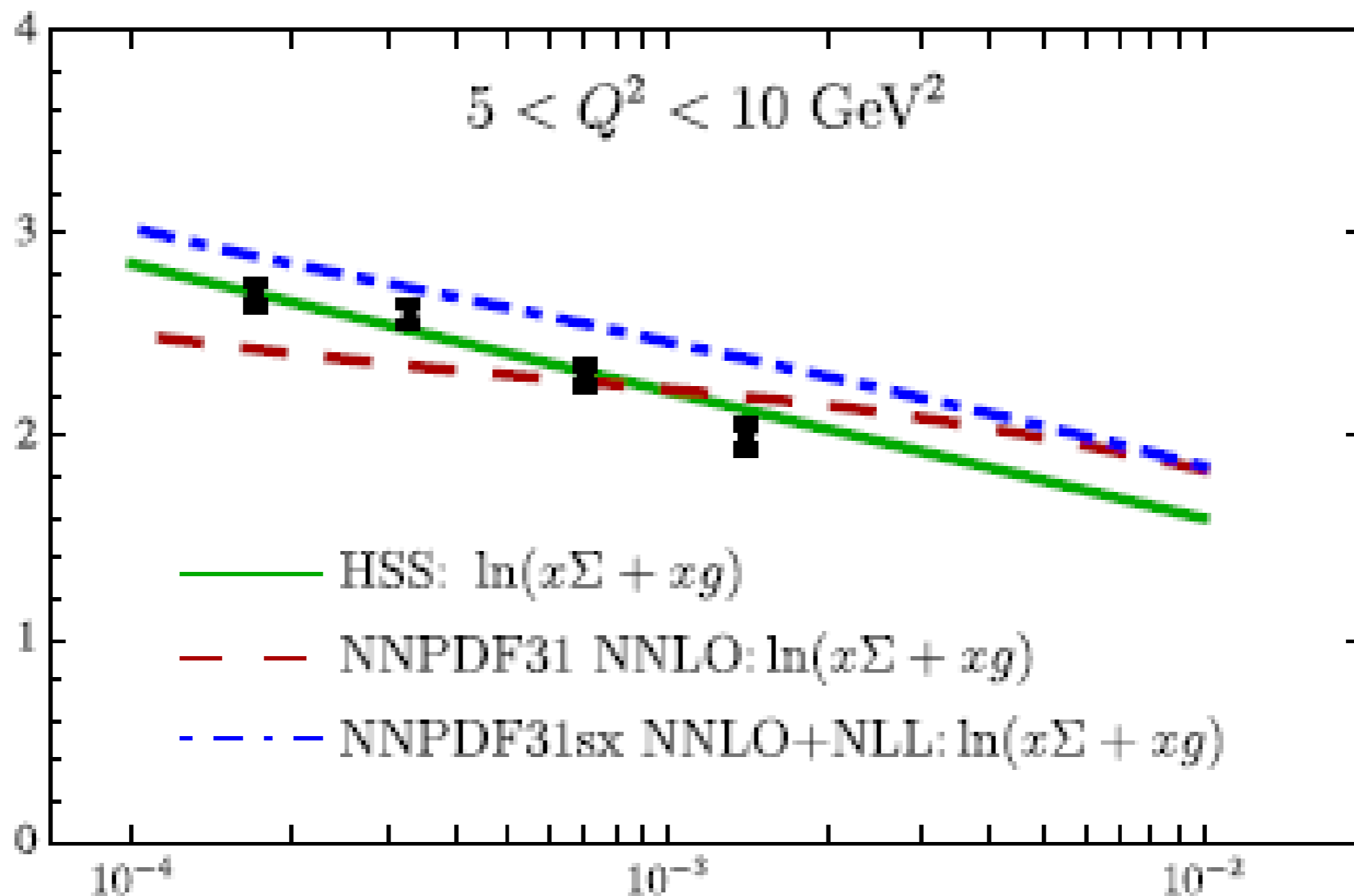


# NNPDF21\_leading order with gluons only

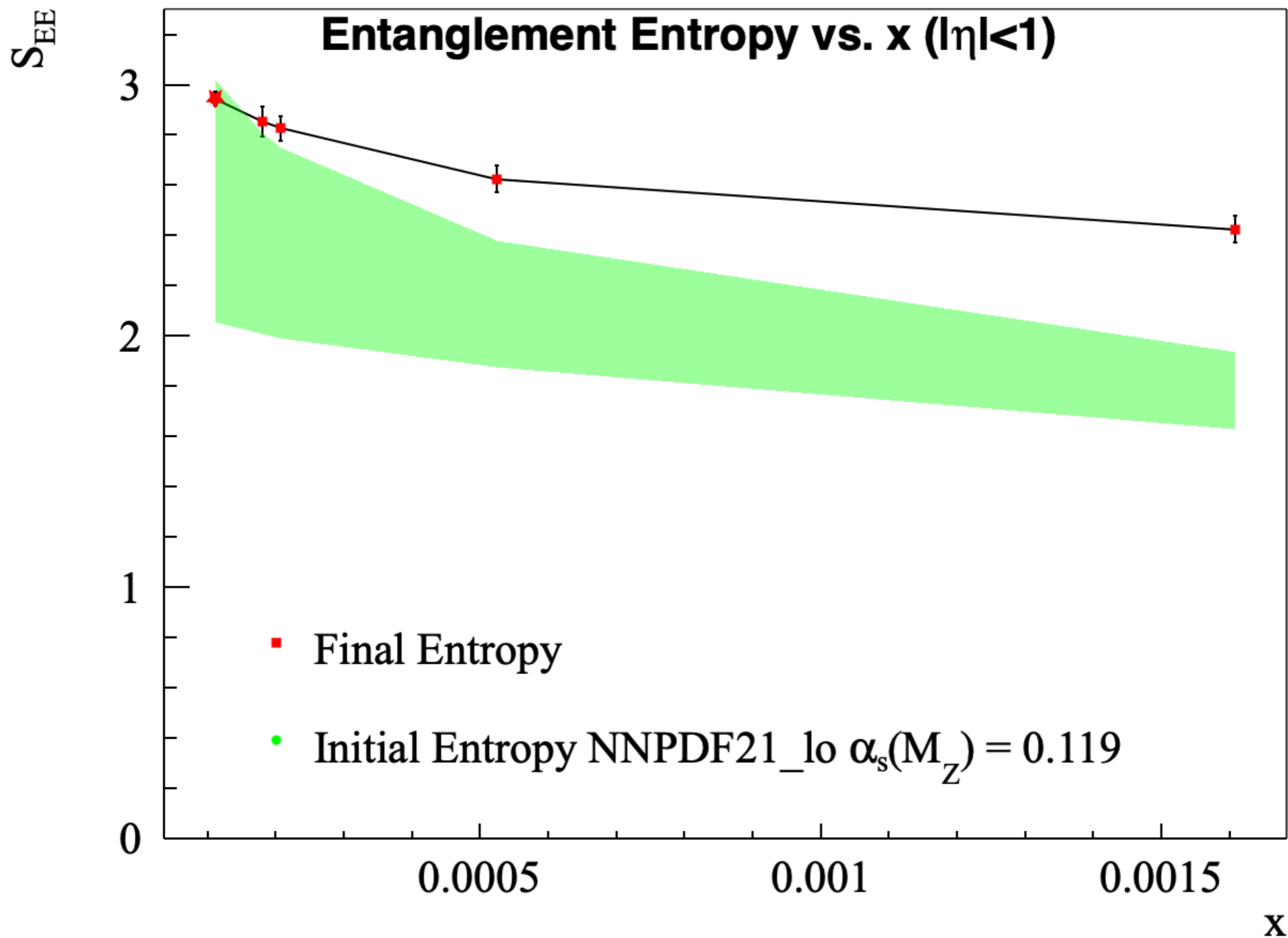


# The impact of quark contributions

Hentschinski & Kutak (2021): Disagreement at higher  $x$  could be due to significant sea-quark contributions (shown here in comparison to H1 data)



# NNPDF21\_lo with gluons, sea quarks, valence quarks

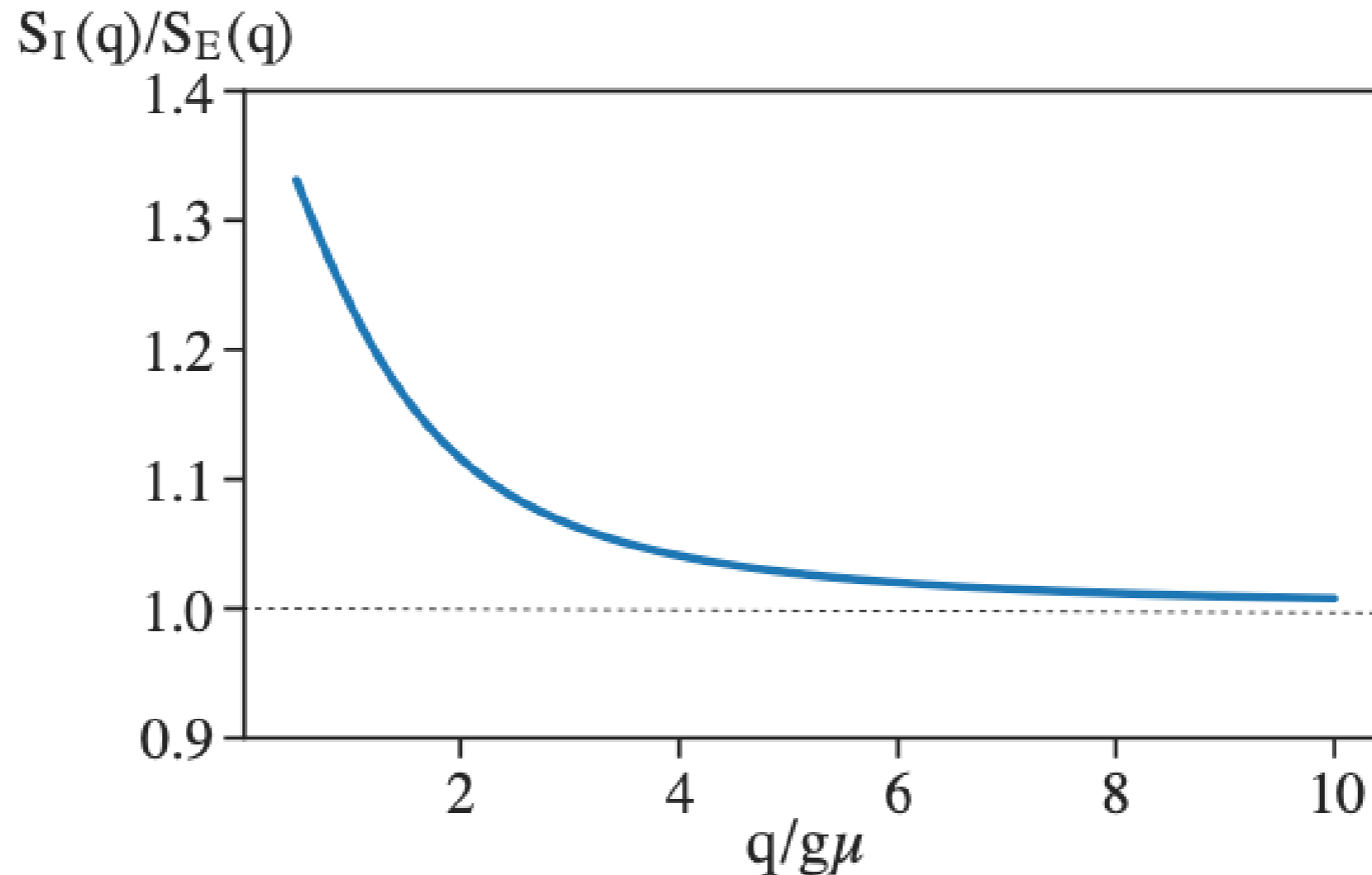




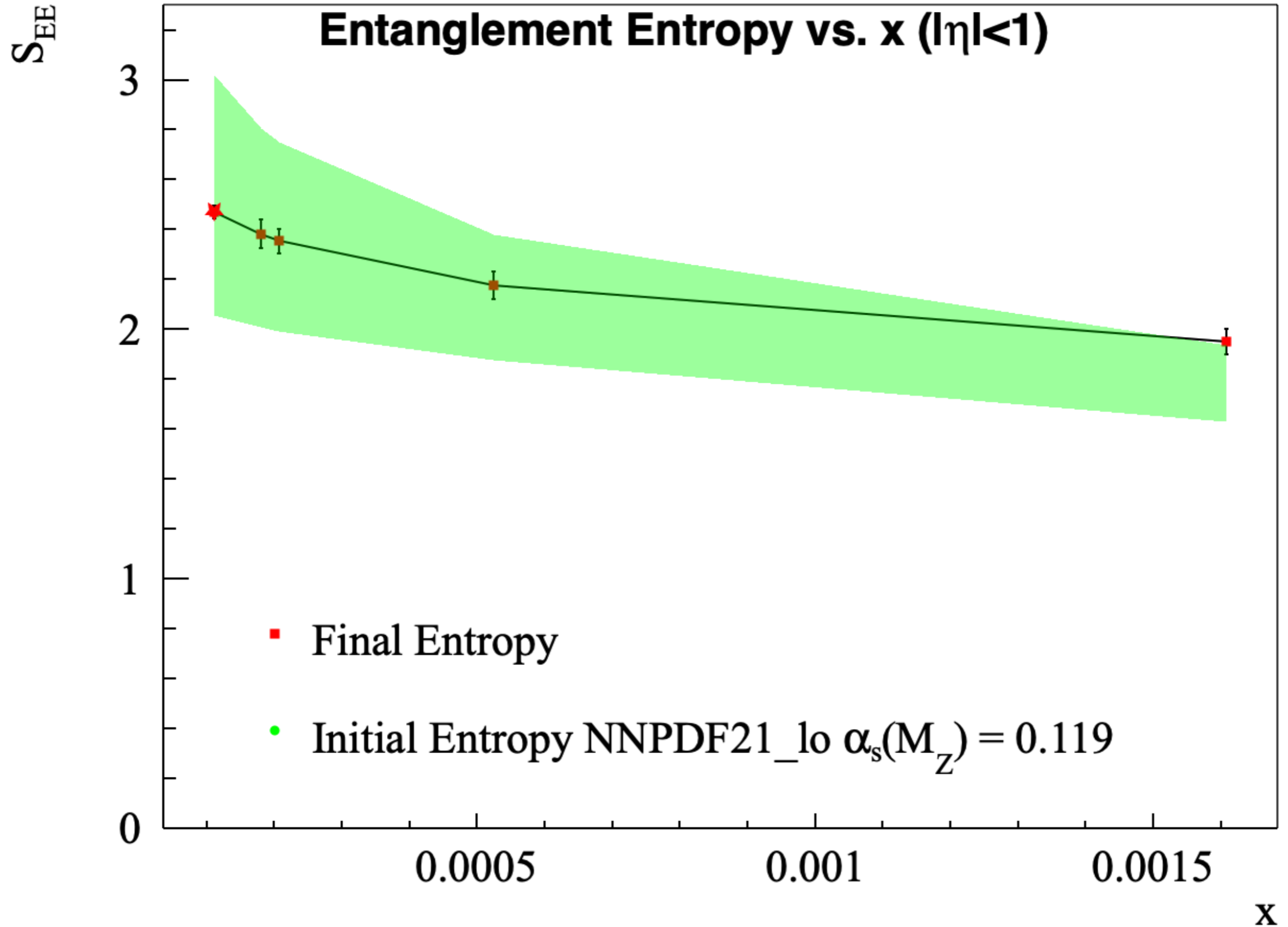
# 'Ignorance' scaling

A calculation by Duan, Akkaya, Kvoner, Skokov (arXiv:2001.01726)  
based on the Page curve of limited acceptance (Mueller, Schaefer (arXiv: 2211.16265))

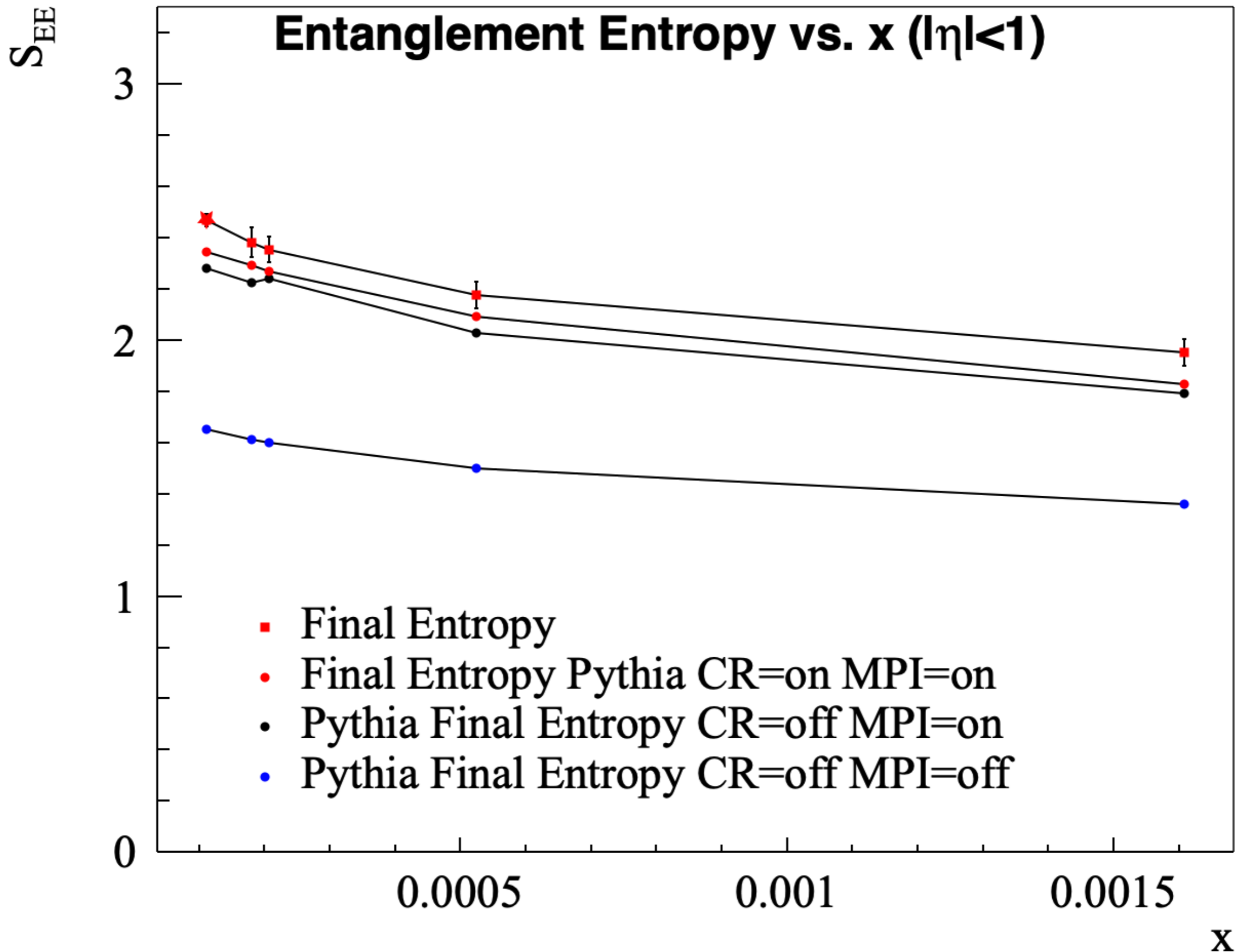
$S_E$  is based on the set of observables (only sensitive to the diagonal matrix elements of the density matrix).  $S_I$  takes into account off-diagonal elements



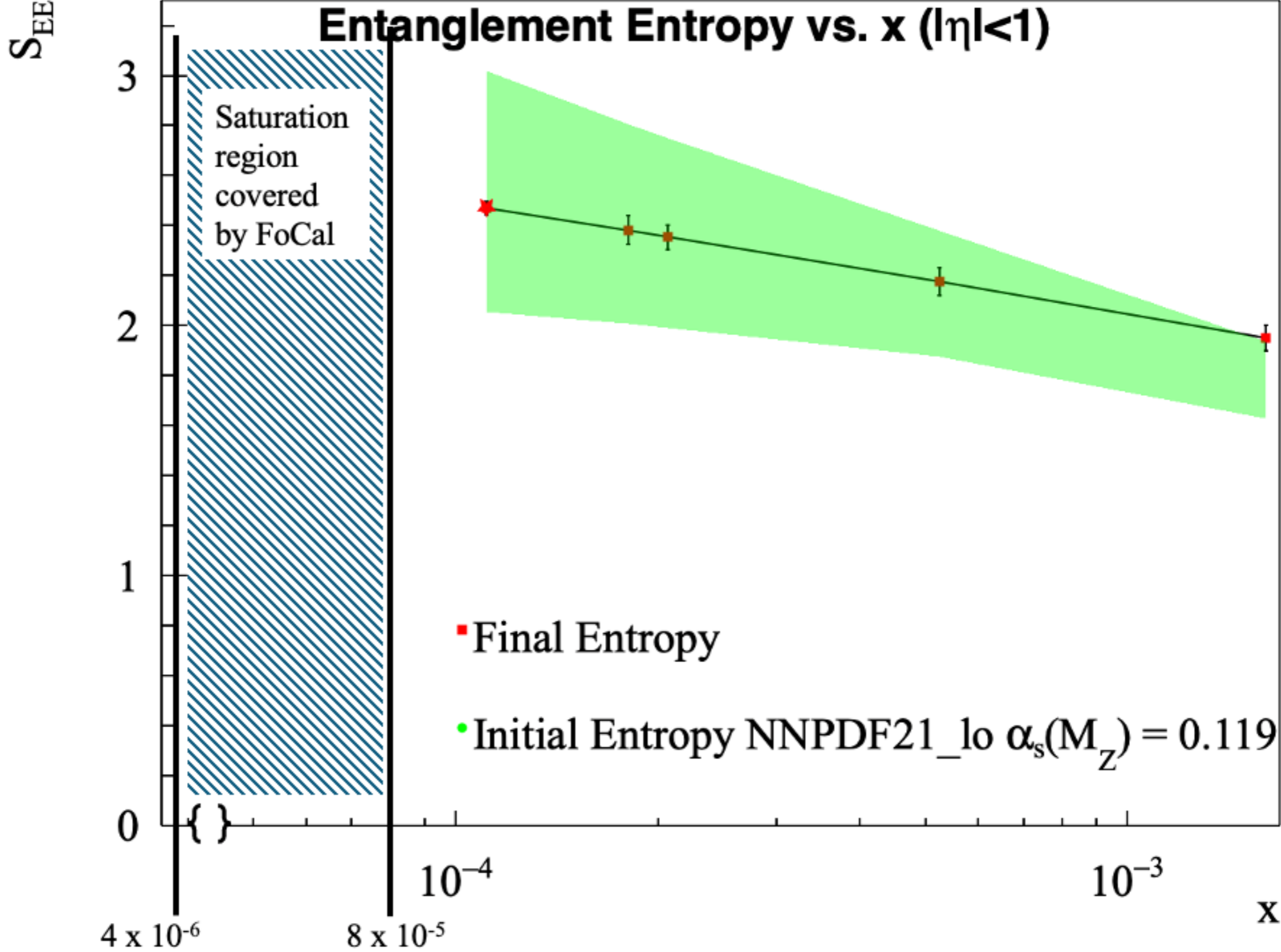
# NNPDF21\_lo with gluons, quarks and 'ignorance scaling'



# The alternative (PYTHIA Monash Tune)



# The future



# Conclusions and outlook

- Partons in proton collisions are entangled transversely and longitudinally during the expansion of the QCD.
- Entanglement entropy is extensive (volume dependent), just like thermodynamic entropy.
- The reduced density matrix for a conformal field theory is locally thermal.

## Entanglement generates 'thermalization'

- If the system looks 'thermal' due to entanglement, but actually never thermalizes through interactions, then there is no decoherence effect and hadronic re-interaction effects are negligible. The entanglement entropy translates one to one into the final hadronic entropy and stays constant throughout the system evolution.
- Particle production looks thermal, but is driven by parton-hadron duality, which also means that composite hadronic objects might be formed from a single multi-quark QCD string.
- All light quark hadron yields are frozen in during the initial state at a common 'temperature'. Entanglement entropy is calculated over an extended volume at QCD crossover. Temperature should then relate to Hagedorn temperature (e.g. Pajares et al., arXiv:1805.12444)

In pp: Hadron multiplicities as a  $f(x)$  in elementary collisions show already intriguing patterns that point at entanglement. String fragmentation models mimic same effect through interactions (CR, MPI). The ultimate test should be given at the Froissart bound (gluon saturation).

In AA: If there is no decoherence phase (global equilibration), then the 'temperature' from the entangled phase will drive the multiplicity of all states from pion to light nuclei and even hypernuclei and rare multi quark clusters. Measure identified particles as a function of  $\eta$ .