SQM 2024

The 21st International Conference on Strangeness in Quark Matter
3-7 June 2024, Strasbourg, France

Flavor hierarchy of parton energy loss in quark-gluon plasma from a Bayesian analysis

Wen-Jing Xing (邢文静)

Shandong university

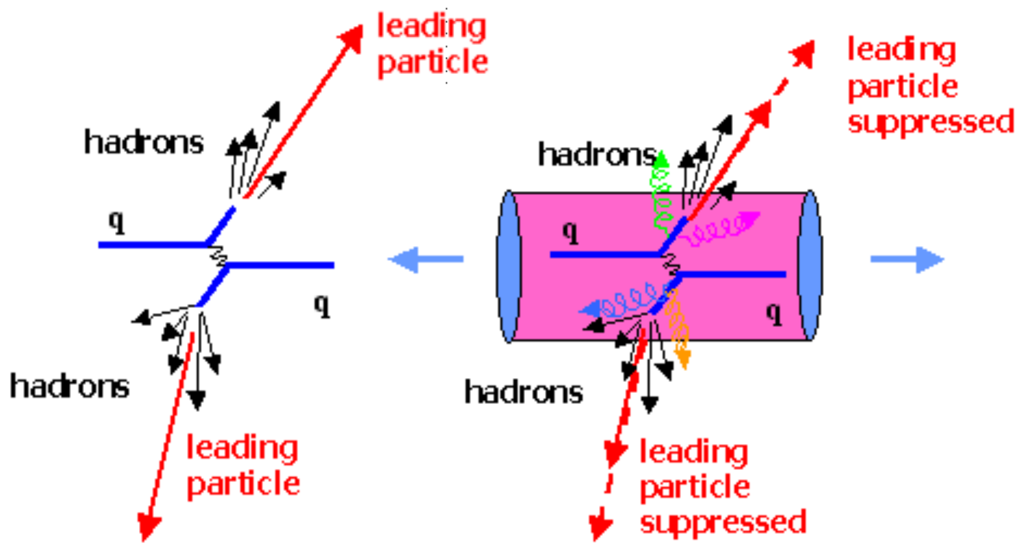
In collaboration with Prof. Shanshan Cao and Prof. Guang-You Qin

Outline of my talk

WJX, Cao, Qin, Phys.Lett.B 850 (2024) 138523

- Introduction
- Perturbative framework for high p_T hadron production and medium modification
- The extracted hierarchy of parton energy loss from Bayesian analysis
- Summary

Jet quenching: high-p_T hadron suppression

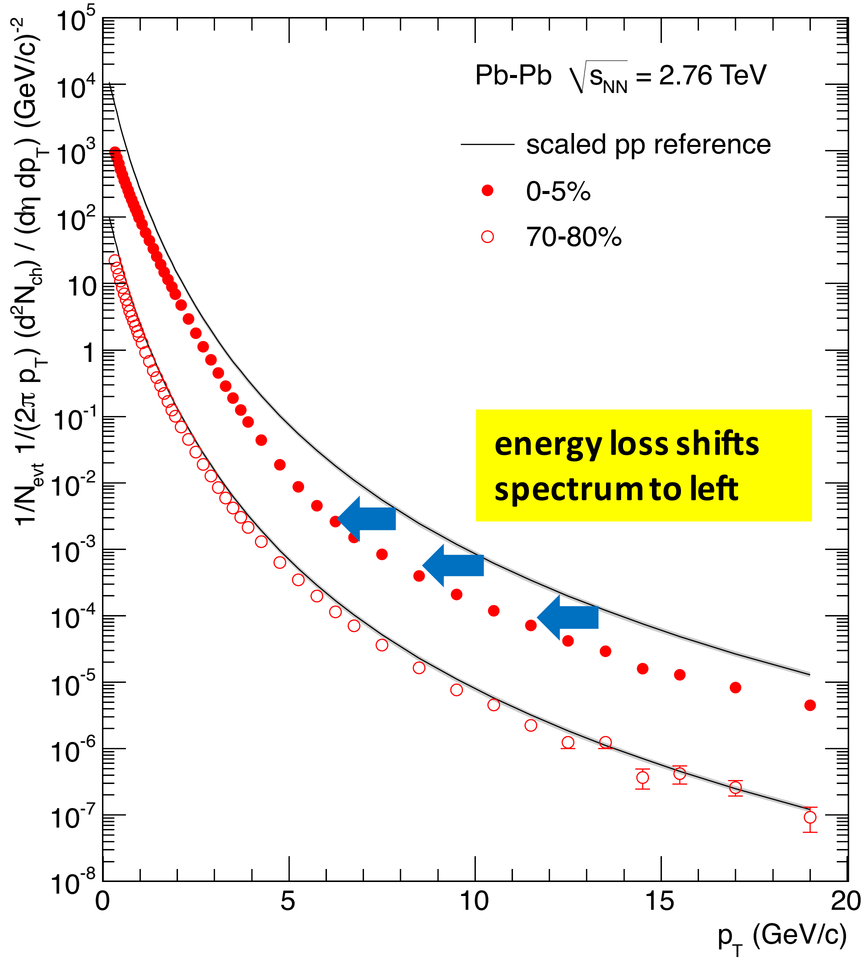


- **The nuclear modification factor R_{AA} :**

$$R_{AA} = \frac{1}{N_{\rm coll}} \frac{dN^{AA} / d^2 p_T dy}{dN^{pp} / d^2 p_T dy}$$

- **Expressing R_{AA} in terms of parton energy loss $\langle \Delta p_T \rangle$:**

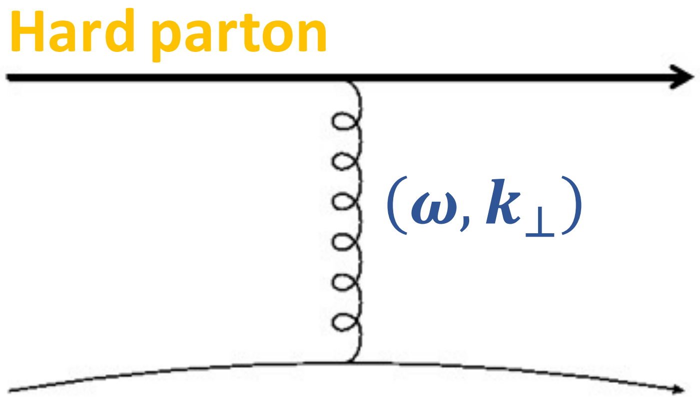
$$R_{\rm AA}(p_T) = \frac{P_{\rm pp}(p_T + \Delta p_T)}{P_{\rm pp}(p_T)}$$



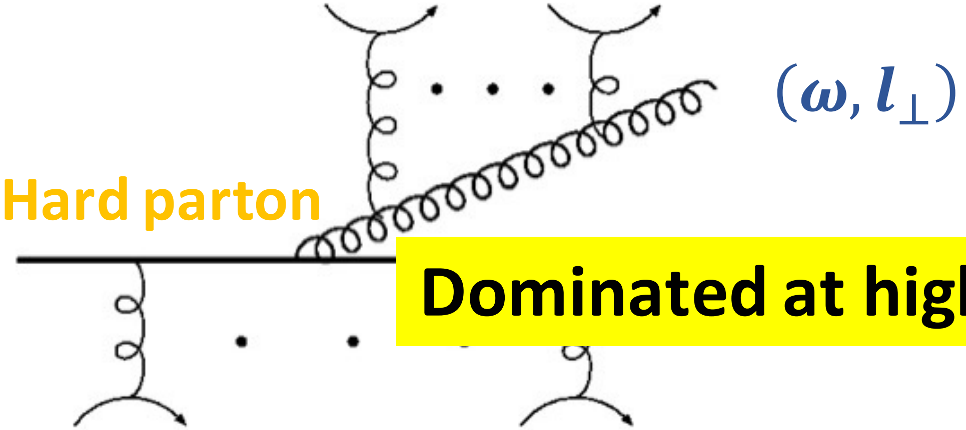
ALICE, PLB 2011

Parton-medium interaction

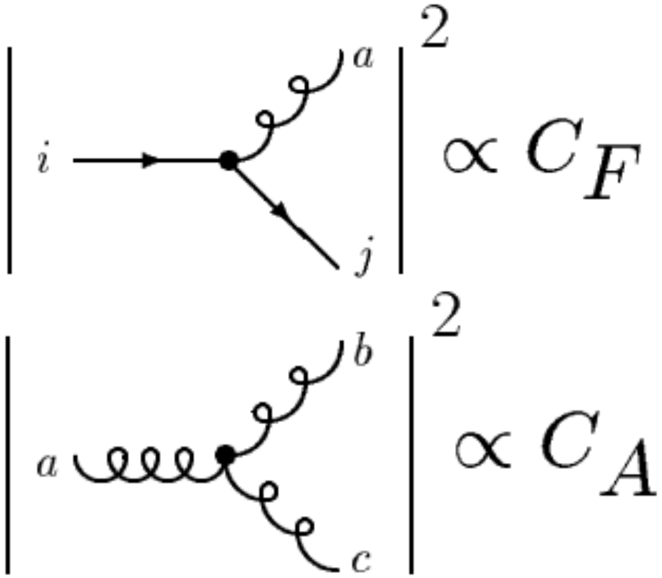
Elastic (collisional) process



Inelastic (radiative) process



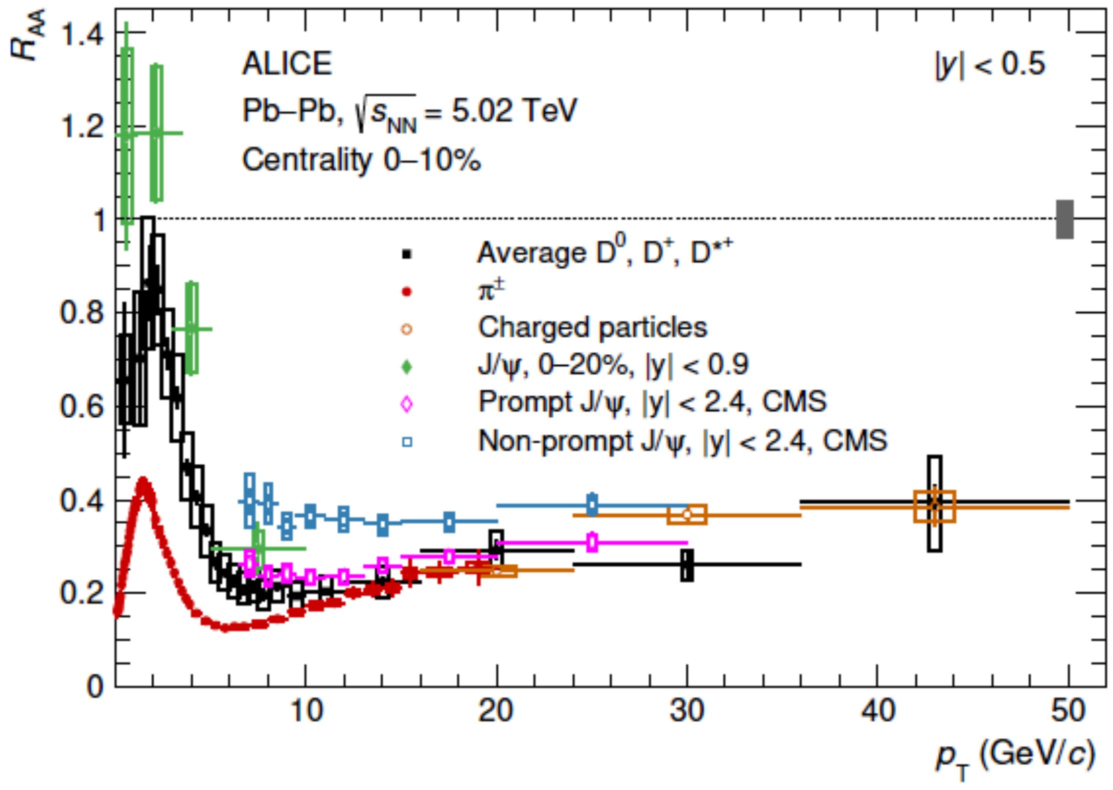
Dominated at high p_T

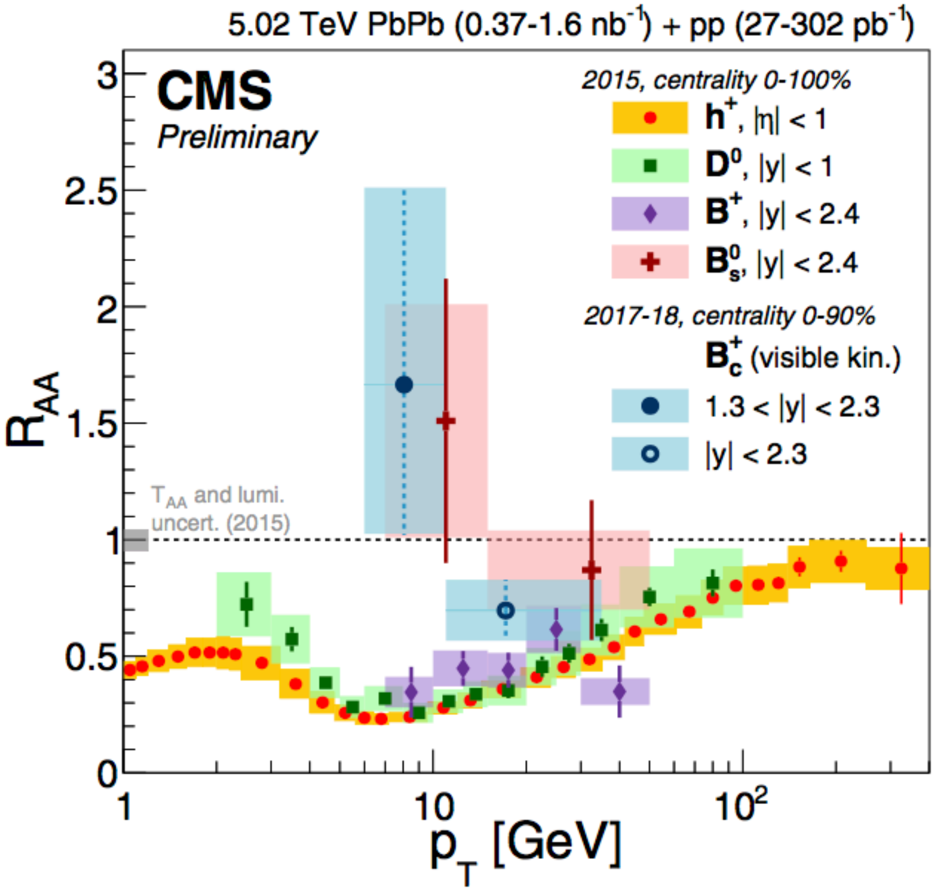


Perturbative QCD calculation gives:

- **Radiative energy loss:**
  $\langle \Delta E_{rad} \rangle \sim \alpha_s C_R \hat{q} L^{\beta}, \ (C_q = C_F, C_g = C_A)$
- $\langle \Delta E^g_{rad} \rangle / \langle \Delta E^q_{rad} \rangle \sim C_A / C_F.$

Flavor hierarchy of high-p_T hadron R_{AA}





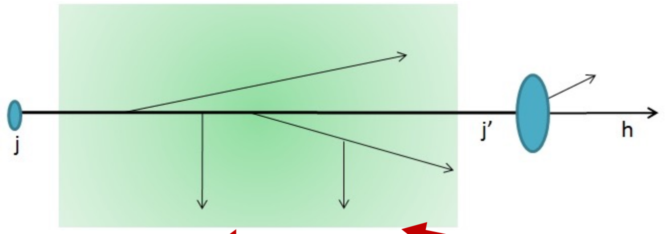
- **Constrain the flavor hierarchy of parton energy loss.**

Theoretical framework for high-p_T hadron production

pp:

$$\frac{d\sigma_{\mathrm{pp}\to hX}}{dp_{\mathrm{T}}^{h}} = \sum_{j}\int dp_{\mathrm{T}}^{j} dz \frac{d\hat{\sigma}_{\mathrm{pp}\to jX}}{dp_{\mathrm{T}}^{j}}(p_{\mathrm{T}}^{j}) D_{j\to h}(z)\delta\left(p_{\mathrm{T}}^{h} - z p_{\mathrm{T}}^{j}\right)$$

WJX, Cao, Qin, Xing, PLB 2020



AA:

$$\frac{1}{\langle N_{\mathrm{coll}}\rangle}\frac{d\sigma_{\mathrm{AA}\to hX}}{dp_{\mathrm{T}}^{h}} = \sum_{j}\int dp_{\mathrm{T}}^{j} dx dz \frac{d\hat{\sigma}_{\mathrm{p'p'}\to jX}}{dp_{\mathrm{T}}^{j}}(p_{\mathrm{T}}^{j}) W_{\mathrm{AA}}(x) D_{j\to h}(z)\delta\left(p_{\mathrm{T}}^{h} - z(p_{\mathrm{T}}^{j} - x\langle\Delta p_{\mathrm{T}}^{j}\rangle)\right)$$

- $\left\langle \Delta p_T^j \right\rangle$ is the averaged parton energy loss for hard parton j.
- $W_{AA}(x)$ is the parton energy loss distribution with $x = \Delta p_T^j / \left\langle \Delta p_T^j \right\rangle$.

Different works to extract $\langle \Delta p_T \rangle$ and $W_{AA}(x)$

$$\frac{1}{\langle N_{\rm coll}\rangle}\frac{d\sigma_{\rm AA\to hX}}{dp_{\rm T}^h} = \sum_j \int dp_{\rm T}^j dx dz \frac{d\hat{\sigma}_{\rm p'p'\to jX}}{dp_{\rm T}^j}(p_{\rm T}^j) \underline{W_{\rm AA}(x)} D_{j\to h}(z)\delta\left(p_{\rm T}^h - z(p_{\rm T}^j - x\underline{\langle \Delta p_{\rm T}^j}\rangle)\right)$$

- Taking $W_{AA}(x)$ from BDMPS medium-induced gluon spectrum, the $\langle \Delta p_T \rangle$ is extracted from R_{AA} data of hadrons ($h^{\pm}$, D, J/ψ).
  - F. Arleo, PRL 2017.
- Using a general ansatz of the jet $W_{AA}(x)$, the flavor-averaged jet $W_{AA}(x)$ and $\langle \Delta P_T \rangle$ is extracted from R_{AA} data of single inclusive and γ-triggered jets.
  - He, Pang and Wang, PRL 2019.
- Parton $W_{AA}(x)$ and $\langle \Delta p_T \rangle$ of both gluons and charm quarks is extracted from R_{AA} data of J/ψ.
  - Zhang, Liao, Qin, Xing, Science Bulletin 2023
- **A simultaneous data-driven analysis on $\langle \Delta p_T \rangle$ of all parton species (g, q, c, b) is still absent.**

Parametric form of $\langle \Delta p_T \rangle$ and W_{AA} in QGP

- The $\langle N_{coll} \rangle$-rescaled cross section of hadron production in AA collision :

$$\frac{1}{\langle N_{\rm coll}\rangle}\frac{d\sigma_{\rm AA\to hX}}{dp_{\rm T}^h} = \sum_j \int dp_{\rm T}^j dx dz \frac{d\hat{\sigma}_{\rm p'p'\to jX}}{dp_{\rm T}^j}(p_{\rm T}^j) \underline{W_{\rm AA}(x)} D_{j\to h}(z)\delta\left(p_{\rm T}^h - z(p_{\rm T}^j - x\underline{\langle\Delta p_{\rm T}^j\rangle})\right)$$

- The parametric p_T-dependence of $\langle \Delta p_T \rangle$ for gluons (g), light quarks (q), charm quarks (c) and bottom quarks (b) is :

$$\left\langle \Delta p_{\rm T}^j \right\rangle = C_j \beta_g p_{\rm T}^\gamma \log(p_{\rm T})$$

Notice that $C_g = 1$ and C_q, C_c, C_b represents the $\langle \Delta p_T \rangle$ ratio relative to gluon's.

- The parton energy loss distribution is:

$$W_{\rm AA}(x) = \frac{\alpha^\alpha x^{\alpha-1} e^{-\alpha x}}{\Gamma(\alpha)}$$

- $\theta = (\beta_g, C_q, C_c, C_b, \gamma, \alpha)$ **is to be calibrated in Bayesian analysis.**

Bayesian analysis

Bayes' theorem

$$P(\boldsymbol{\theta}|\text{data}) \propto P(\boldsymbol{\theta})\, P(\text{data}|\boldsymbol{\theta})$$

- **Prior: uniform in the region of**

$$[\beta_g, C_q, C_c, C_b, \gamma, \alpha] \subset [(0,10), (0,1), (0,1), (0,1), (-0.15, 0.5), (0,15)]$$

- **Likelihood: Gaussian form**

$$P(\text{data}|\boldsymbol{\theta}) = \prod_i \frac{1}{\sqrt{2\pi}\sigma_i} e^{-\left[y_i(\boldsymbol{\theta}) - y_i^{\text{exp}}\right]^2 / (2\sigma_i^2)}$$

Bayesian analysis

Physics model ($y_i(\boldsymbol{\theta})$)

Bayes' theorem

$$P(\boldsymbol{\theta}|\text{data}) \propto P(\boldsymbol{\theta})\, P(\text{data}|\boldsymbol{\theta})$$

- **Prior: uniform in the region of**

$$[\beta_g, C_q, C_c, C_b, \gamma, \alpha] \subset [(0,10),(0,1),(0,1),(0,1),(-0.15,0.5),(0,15)]$$

- **Likelihood: Gaussian form**

$$P(\text{data}|\boldsymbol{\theta}) = \prod_i \frac{1}{\sqrt{2\pi}\sigma_i} e^{-\left[y_i(\boldsymbol{\theta})-y_i^{\exp}\right]^2/(2\sigma_i^2)}$$

Bayesian analysis

Experimental data (y_i^{exp}, σ_i)

- **CMS, PbPb@5.02TeV 0-10%, R_{AA}**
  $h^{\pm}, D^0, b \to J/\Psi$

Physics model ($y_i\left(\boldsymbol{\theta}\right)$)

Bayes' theorem

$$P\left(\boldsymbol{\theta}|\mathrm{data}\right) \propto P\left(\boldsymbol{\theta}\right) P\left(\mathrm{data}|\boldsymbol{\theta}\right)$$

- **Prior: uniform in the region of**

$$[\beta_g, C_q, C_c, C_b, \gamma, \alpha] \subset [(0,10), (0,1), (0,1), (0,1), (-0.15, 0.5), (0,15)]$$

- **Likelihood: Gaussian form**

$$P\left(\mathrm{data}|\boldsymbol{\theta}\right) = \prod_i \frac{1}{\sqrt{2\pi}\sigma_i} e^{-\left[y_i(\boldsymbol{\theta}) - y_i^{\mathrm{exp}}\right]^2 / (2\sigma_i^2)}$$

Bayesian analysis

Experimental data (y_i^{exp}, σ_i)

- **CMS, PbPb@5.02TeV 0-10%, R_{AA}**
  $h^{\pm}, D^0, b \to J/\Psi$

Physics model ($y_i(\boldsymbol{\theta})$)

Bayes' theorem

$$P(\boldsymbol{\theta}|\text{data}) \propto P(\boldsymbol{\theta})\, P(\text{data}|\boldsymbol{\theta})$$

- **Prior: uniform in the region of**

$$[\beta_g, C_q, C_c, C_b, \gamma, \alpha] \subset [(0,10), (0,1), (0,1), (0,1), (-0.15, 0.5), (0,15)]$$

- **Likelihood: Gaussian form**

$$P(\text{data}|\boldsymbol{\theta}) = \prod_i \frac{1}{\sqrt{2\pi}\sigma_i} e^{-\left[y_i(\boldsymbol{\theta}) - y_i^{\text{exp}}\right]^2 / (2\sigma_i^2)}$$

Markov-Chain Monte-Carlo (MCMC)

- **Random walk through parameter space weighted by posterior distribution** $P(\boldsymbol{\theta}|\text{data})$.
- **Generate a parameter array** $X = [\theta_0, \theta_1, \theta_2, ..., \theta_i, \theta_{i+1}, ..., \theta_n]$ **from which we draw the posterior distribution.**

Bayesian analysis

Experimental data (y_i^{exp}, σ_i)

- **CMS, PbPb@5.02TeV 0-10%, R_{AA}**
  $h^{\pm}, D^0, b \to J/\Psi$

Gaussian Process Emulator

- **Interpolates the physics model to generate output $y_i(\boldsymbol{\theta})$ at arbitrary point.**
- **Fast surrogate to full Physics model.**

Bayes' theorem

$$P(\boldsymbol{\theta}|\text{data}) \propto P(\boldsymbol{\theta})\, P(\text{data}|\boldsymbol{\theta})$$

- **Prior: uniform in the region of**

$$[\beta_g, C_q, C_c, C_b, \gamma, \alpha] \subset [(0,10),(0,1),(0,1),(0,1),(-0.15,0.5),(0,15)]$$

- **Likelihood: Gaussian form**

$$P(\text{data}|\boldsymbol{\theta}) = \prod_i \frac{1}{\sqrt{2\pi}\sigma_i} e^{-\left[y_i(\boldsymbol{\theta})-y_i^{\text{exp}}\right]^2 / (2\sigma_i^2)}$$

Markov-Chain Monte-Carlo (MCMC)

- **Random walk through parameter space weighted by posterior distribution $P(\boldsymbol{\theta}|\text{data})$.**
- **Generate a parameter array $X = [\theta_0, \theta_1, \theta_2, ..., \theta_i, \theta_{i+1}, ..., \theta_n]$ from which we draw the posterior distribution.**

Bayesian analysis

Experimental data (y_i^{exp}, σ_i)

- **CMS, PbPb@5.02TeV 0-10%, R_{AA}**
  $h^{\pm}, D^0, b \to J/\Psi$

Gaussian Process Emulator

- **Interpolates the physics model to generate output $y_i(\boldsymbol{\theta})$ at arbitrary point.**
- **Fast surrogate to full Physics model.**

Model parameters

$\theta = [\beta_g, C_q, C_c, C_b, \gamma, \alpha]$

- **Sample 600 design points uniformly distributed in the prior range.**

Bayes' theorem

$$P(\boldsymbol{\theta}|\text{data}) \propto P(\boldsymbol{\theta})\, P(\text{data}|\boldsymbol{\theta})$$

- **Prior: uniform in the region of**

$$[\beta_g, C_q, C_c, C_b, \gamma, \alpha] \subset [(0,10), (0,1), (0,1), (0,1), (-0.15, 0.5), (0,15)]$$

- **Likelihood: Gaussian form**

$$P(\text{data}|\boldsymbol{\theta}) = \prod_i \frac{1}{\sqrt{2\pi}\sigma_i} e^{-[y_i(\boldsymbol{\theta}) - y_i^{\text{exp}}]^2 / (2\sigma_i^2)}$$

Markov-Chain Monte-Carlo (MCMC)

- **Random walk through parameter space weighted by posterior distribution** $P(\boldsymbol{\theta}|\text{data})$**.**
- **Generate a parameter array** $X = [\theta_0, \theta_1, \theta_2, ..., \theta_i, \theta_{i+1}, ..., \theta_n]$ **from which we draw the posterior distribution.**

Bayesian analysis

Experimental data (y_i^{exp}, σ_i)

- **CMS, PbPb@5.02TeV 0-10%, R_{AA}** $h^{\pm}, D^0, b \to J/\Psi$

Gaussian Process Emulator

- **Interpolates the physics model to generate output $y_i(\theta)$ at arbitrary point.**
- **Fast surrogate to full Physics model.**

Physics model ($y_i(\boldsymbol{\theta})$)

Model parameters

$\theta = [\beta_g, C_q, C_c, C_b, \gamma, \alpha]$

- **Sample 600 design points uniformly distributed in the prior range.**

Bayes' theorem

$$P(\boldsymbol{\theta}|\text{data}) \propto P(\boldsymbol{\theta}) P(\text{data}|\boldsymbol{\theta})$$

- **Prior: uniform in the region of**

$$[\beta_g, C_q, C_c, C_b, \gamma, \alpha] \subset [(0,10),(0,1),(0,1),(0,1),(-0.15,0.5),(0,15)]$$

- **Likelihood: Gaussian form**

$$P(\text{data}|\boldsymbol{\theta}) = \prod_i \frac{1}{\sqrt{2\pi}\sigma_i} e^{-\left[y_i(\boldsymbol{\theta}) - y_i^{\text{exp}}\right]^2 / (2\sigma_i^2)}$$

Markov-Chain Monte-Carlo (MCMC)

- **Random walk through parameter space weighted by posterior distribution** $P(\boldsymbol{\theta}|\text{data})$.
- **Generate a parameter array** $X = [\theta_0, \theta_1, \theta_2, ..., \theta_i, \theta_{i+1}, ..., \theta_n]$ **from which we draw the posterior distribution.**

Bayesian analysis

Experimental data (y_i^{exp}, σ_i)

- **CMS, PbPb@5.02TeV 0-10%, R_{AA}**
  $h^{\pm}, D^0, b \to J/\Psi$

Gaussian Process Emulator

- **Interpolates the physics model to generate output $y_i(\theta)$ at arbitrary point.**
- **Fast surrogate to full Physics model.**

Physics model ($y_i(\theta)$)

Model parameters

$\theta = [\beta_g, C_q, C_c, C_b, \gamma, \alpha]$

- **Sample 600 design points uniformly distributed in the prior range.**

Bayes' theorem

$$P(\boldsymbol{\theta}|\text{data}) \propto P(\boldsymbol{\theta})\, P(\text{data}|\boldsymbol{\theta})$$

- **Prior: uniform in the region of**

$$[\beta_g, C_q, C_c, C_b, \gamma, \alpha] \subset [(0,10), (0,1), (0,1), (0,1), (-0.15, 0.5), (0,15)]$$

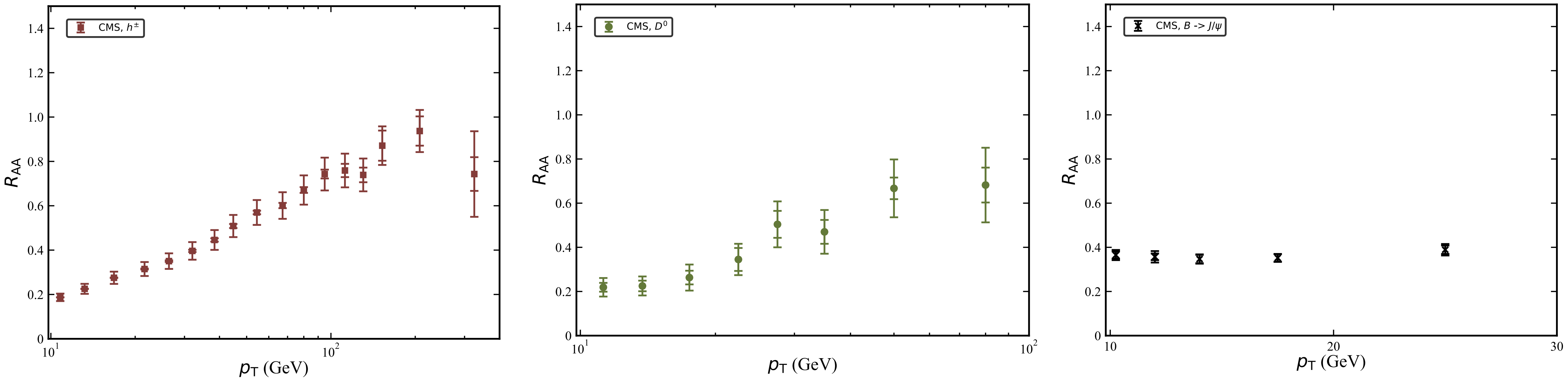
- **Likelihood: Gaussian form**

$$P(\text{data}|\boldsymbol{\theta}) = \prod_i \frac{1}{\sqrt{2\pi}\sigma_i} e^{-[y_i(\boldsymbol{\theta}) - y_i^{\text{exp}}]^2 / (2\sigma_i^2)}$$

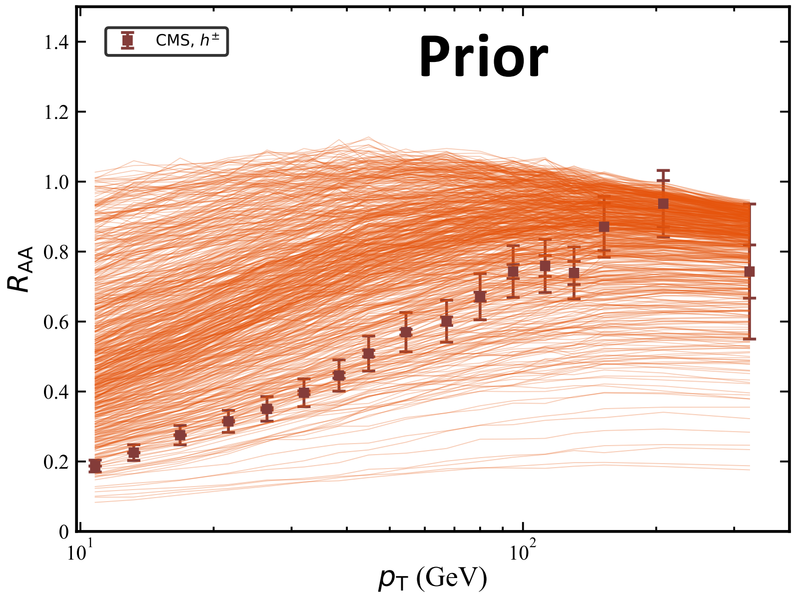
Markov-Chain Monte-Carlo (MCMC)

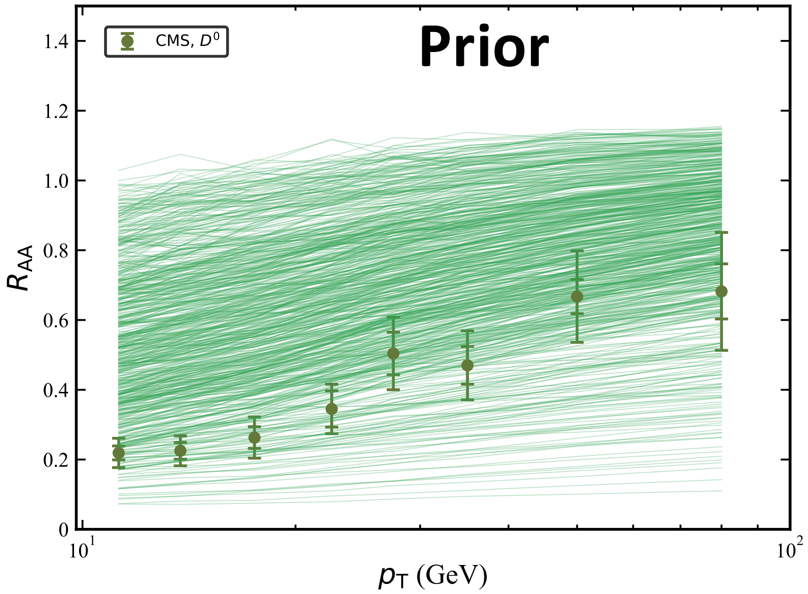
- **Random walk through parameter space weighted by posterior distribution $P(\boldsymbol{\theta}|\text{data})$.**
- **Generate a parameter array $X = [\theta_0, \theta_1, \theta_2, ..., \theta_i, \theta_{i+1}, ..., \theta_n]$ from which we draw the posterior distribution.**

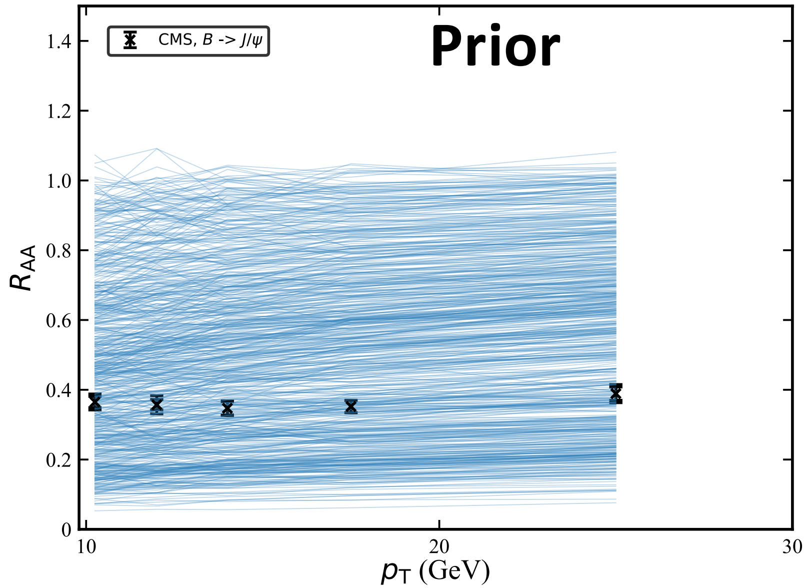
Hadron R_{AA}: Prior vs. Posterior



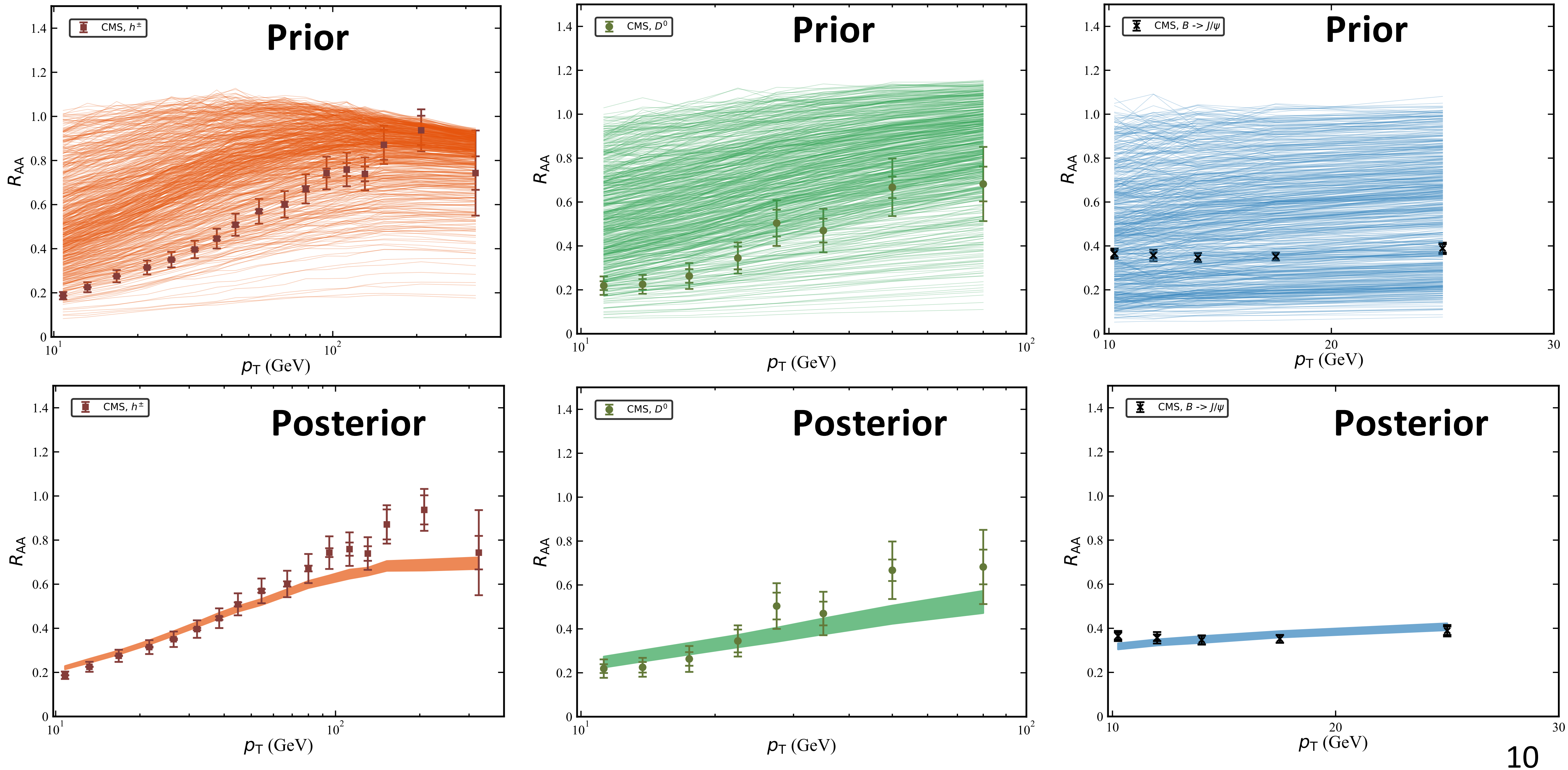
Hadron R_{AA}: Prior vs. Posterior



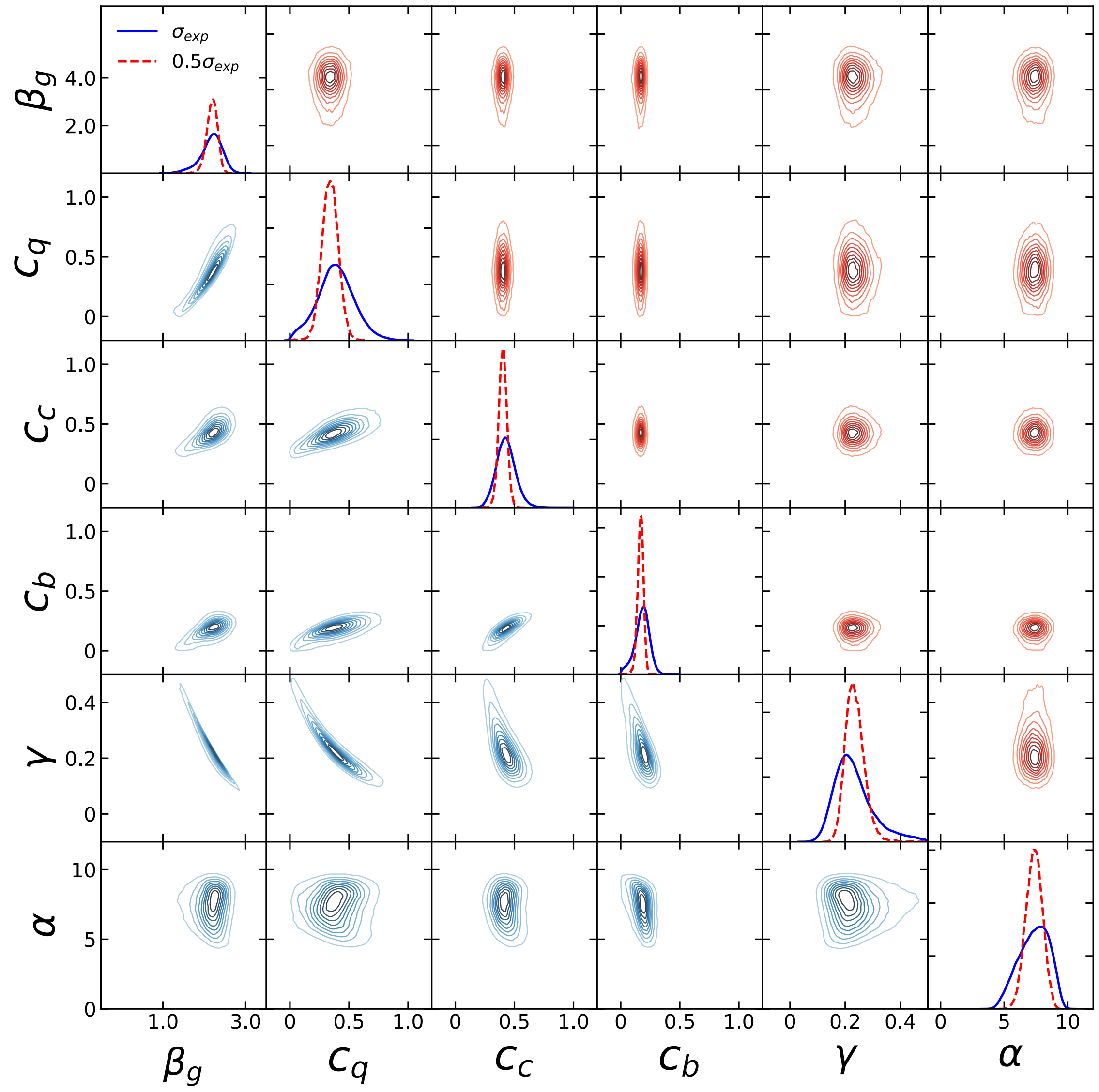




Hadron R_{AA}: Prior vs. Posterior



Posterior distribution of model parameters



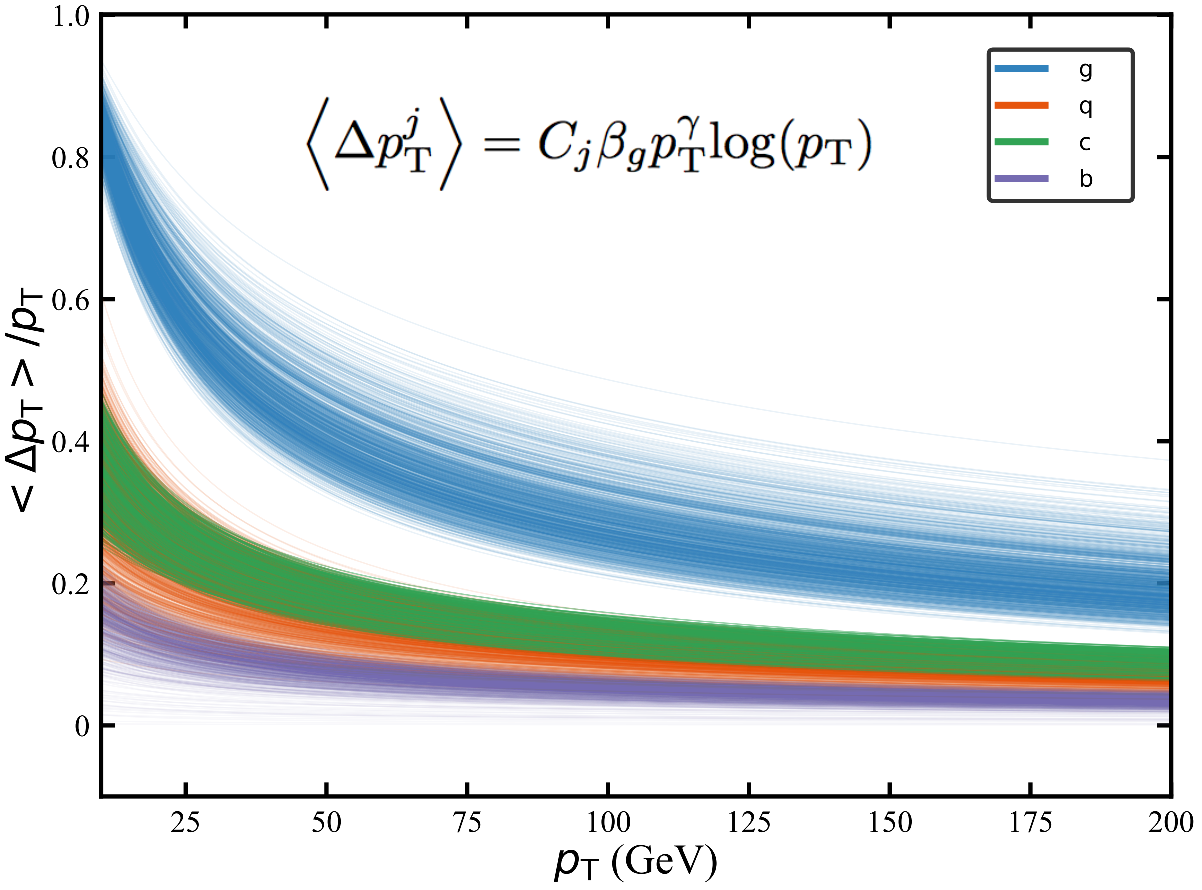
	with $\sigma_{\rm exp}$	with $0.5\sigma_{\rm exp}$
β_g	(1.646, 2.56)	(1.96, 2.39)
C_q	(0.129, 0.65)	(0.226, 0.454)
C_c	(0.3, 0.567)	(0.344, 0.459)
C_b	(0.065, 0.277)	(0.124, 0.207)
γ	(0.137, 0.378)	(0.184, 0.295)
α	(5.287, 9.061)	(6.266, 8.401)

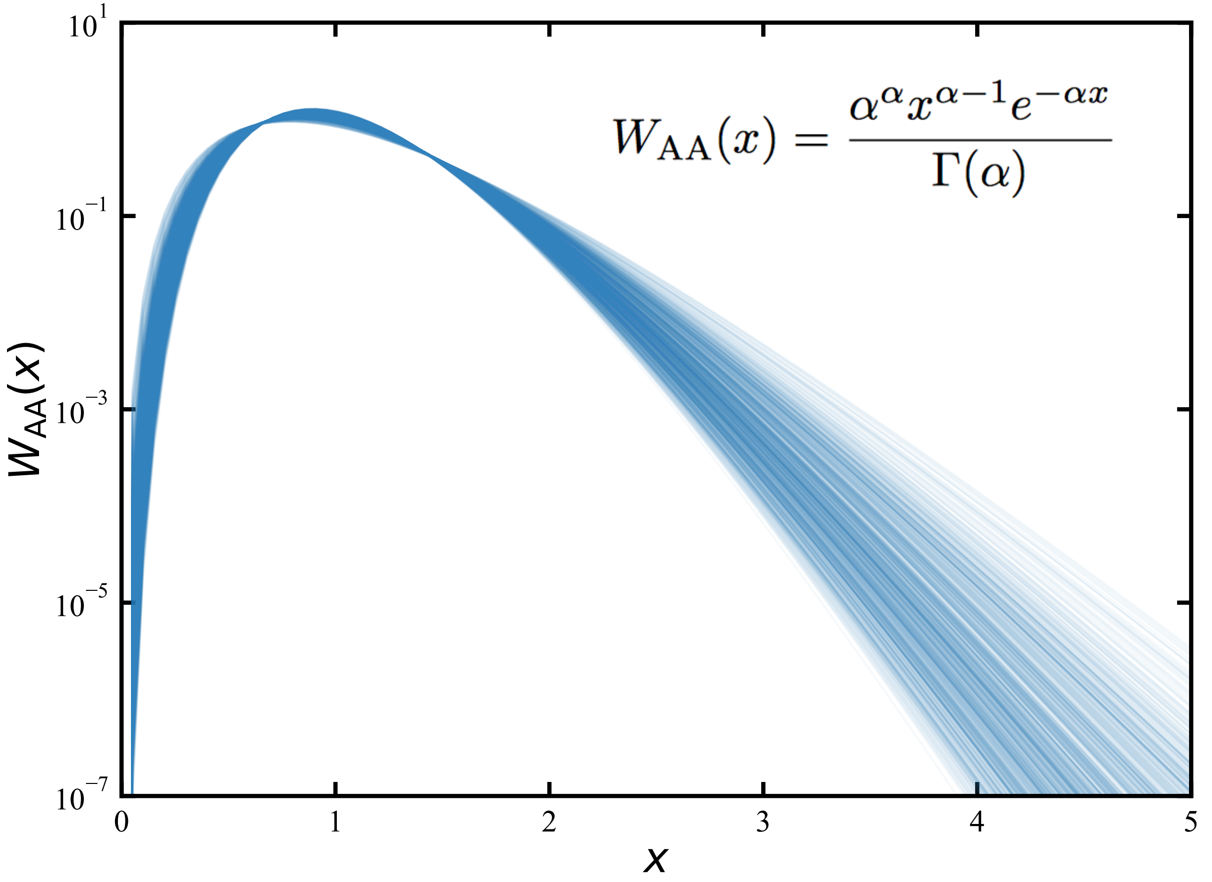
$$\langle \Delta p_{\rm T}^{g} \rangle = \beta_g p_{\rm T}^{\gamma} \log(p_{\rm T})$$

$$\langle \Delta p_{\rm T}^{q} \rangle = C_q \beta_g p_{\rm T}^{\gamma} \log(p_{\rm T})$$

- **Bayesian calibration gives $1/C_q \sim (1.53, 7.75)$ with σ_{exp} and $1/C_q \sim (2.20, 4.42)$ with $0.5\sigma_{exp}$, which are both consistent with the Casimir scaling factor $C_A/C_F = 2.25$.**

Bayesian extraction of $\langle \Delta p_T \rangle$ for g, q, c and b





- The extracted parton energy loss has a clear flavor hierarchy, $\langle \Delta E_g \rangle > \langle \Delta E_q \rangle \sim \langle \Delta E_c \rangle > \langle \Delta E_b \rangle$.
- Direct extraction of flavor dependence of parton energy loss in QGP from data.
- Provides a stringent test on *p*QCD calculation of parton-medium interaction.

Summary

WJX, Cao, Qin, Phys.Lett.B 850 (2024) 138523

- **By combining a NLO order perturbative QCD calculation of the parton production, a general ansatz of the parton energy loss function and parton fragmentation functions, we can calculate the nuclear modification of both heavy and light flavor hadrons over a wide p_T regime.**

- **By applying a Bayesian model-to-data analysis, we have performed a first simultaneous extraction of energy loss of gluons, light quarks, charm quarks and bottom quarks inside the QGP.**

- **The constrained parton energy loss exhibit a clear flavor hierarchy, $\langle \Delta E_g \rangle > \langle \Delta E_q \rangle \sim \langle \Delta E_c \rangle > \langle \Delta E_b \rangle$ inside the QGP, and is consistent with perturbative QCD expectation.**

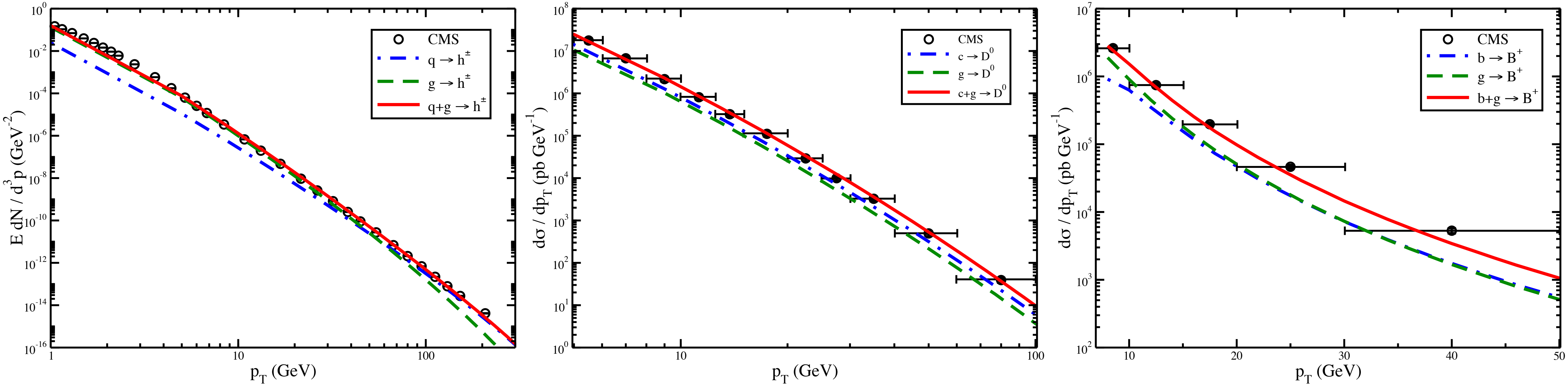
- **We find that a reduction of the data uncertainties can significantly improve the precision of the extracted parton energy loss.**

Thank You !

The following are Back-up pages

High p_T hadron productions in pp @ NLO

WJX, Cao, Qin, Xing, PLB 2020



- **Gluon fragmentation dominates $h^{\pm}$ production at p_T < 50 GeV.**
- **Gluon fragmentation contributes to over 40% D^0 and 50% B production up to p_T > 50 GeV.**

Bayesian calibration to pseudo-data

Experimental data (y_i^{exp}, σ_i)

- y_i^{exp} **is generated by physics model at a design point -** $\boldsymbol{\theta}_0 = (2.35, 0.55, 0.5, 0.2, 0.15, 7)$
- $h^{\pm}, D^0, b \to J/\Psi$

Gaussian Process Emulator

- **Interpolates the physics model to generate output** $y_i(\boldsymbol{\theta})$ **at arbitrary point.**
- **Fast surrogate to full Physics model.**

Physics model ($y_i(\boldsymbol{\theta})$ **)**

Model parameters

$$\theta = [\beta_g, C_q, C_c, C_b, \gamma, \alpha]$$

- **Sample 600 design points uniformly distributed in the prior range.**

Bayes' theorem

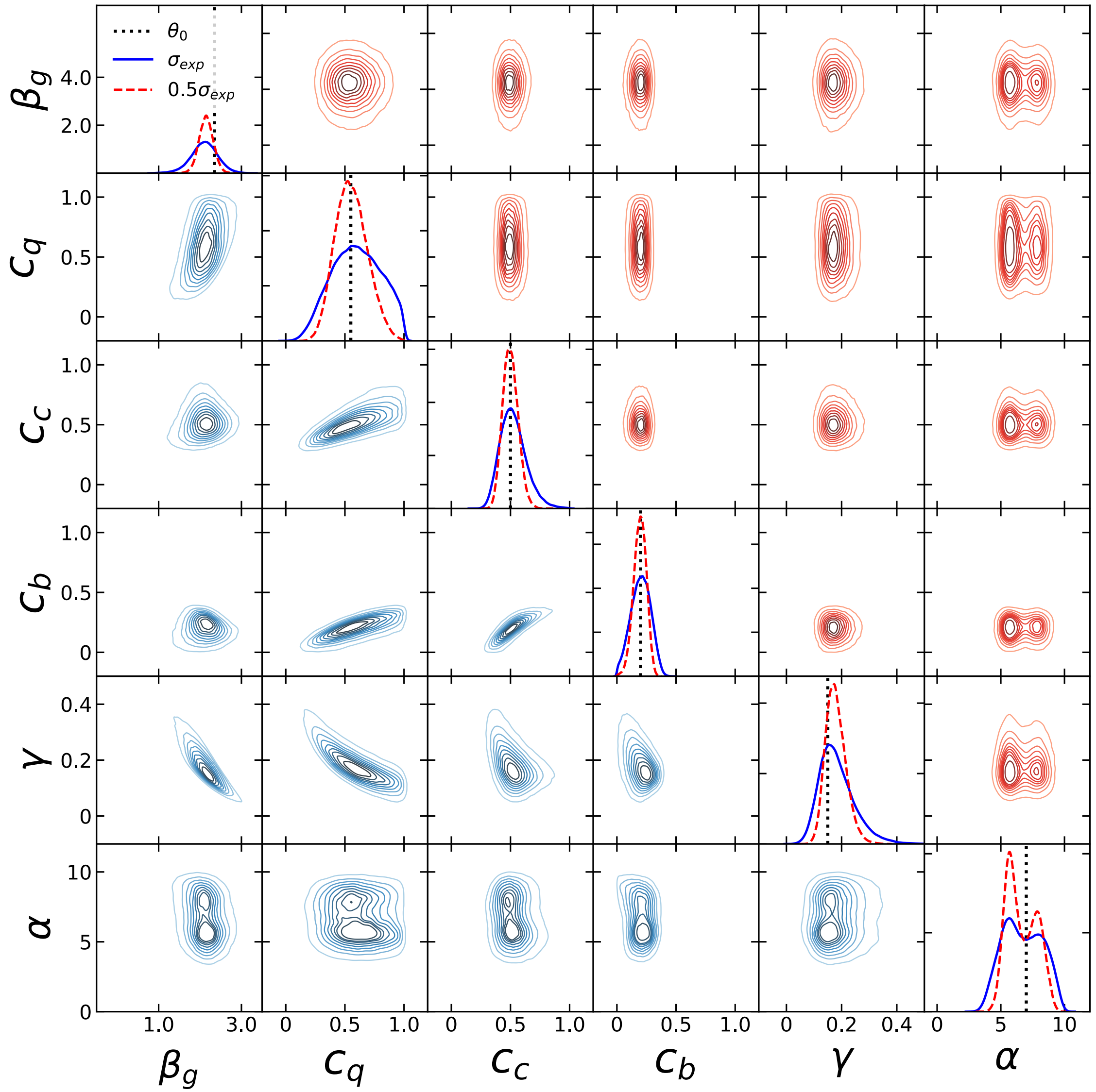
$$P(\boldsymbol{\theta}|\text{data}) \propto P(\boldsymbol{\theta}) P(\text{data}|\boldsymbol{\theta})$$

- **Prior: uniform in the region of**
  $[\beta_g, C_q, C_c, C_b, \gamma, \alpha] \subset [(0,10),(0,1),(0,1),(0,1),(-0.15,0.5),(0,15)]$
- **Likelihood: Gaussian form**
  $$P(\text{data}|\boldsymbol{\theta}) = \prod_i \frac{1}{\sqrt{2\pi}\sigma_i} e^{-\left[y_i(\boldsymbol{\theta}) - y_i^{\text{exp}}\right]^2 / (2\sigma_i^2)}$$

Markov-Chain Monte-Carlo (MCMC)

- **Random walk through parameter space weighted by posterior distribution** $P(\boldsymbol{\theta}|\text{data})$.
- **Generate a parameter array** $X = [\theta_0, \theta_1, \theta_2, ..., \theta_i, \theta_{i+1}, ..., \theta_n]$ **from which we draw the posterior distribution.**

Bayesian calibration to pseudo-data



	$\boldsymbol{\theta}_0$	with σ_{exp}	with $0.5\sigma_{\text{exp}}$
β_g	2.35	(1.565, 2.614)	(1.862, 2.49)
C_q	0.55	(0.266, 0.928)	(0.344, 0.789)
C_c	0.5	(0.362, 0.725)	(0.398, 0.61)
C_b	0.2	(0.063, 0.331)	(0.102, 0.278)
γ	0.15	(0.095, 0.303)	(0.125, 0.245)
α	7.0	(4.349, 9.146)	(5.01, 8.561)

- **Bayesian analysis to pseudo-data can recover the “true” values of model parameters.**
- **Halving the erro bars of pseudo-data can improve the precision of extracted parameters.**