

p - ϕ femtoscopic correlation analysis using a dynamical model

SOPHIA U



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In collaboration with

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Hadron correlations in nuclear collisions
→ **Useful for studying hadron interactions**

Correlation Function (CF) at Pair Rest Frame ($\mathbf{P} = 0$)

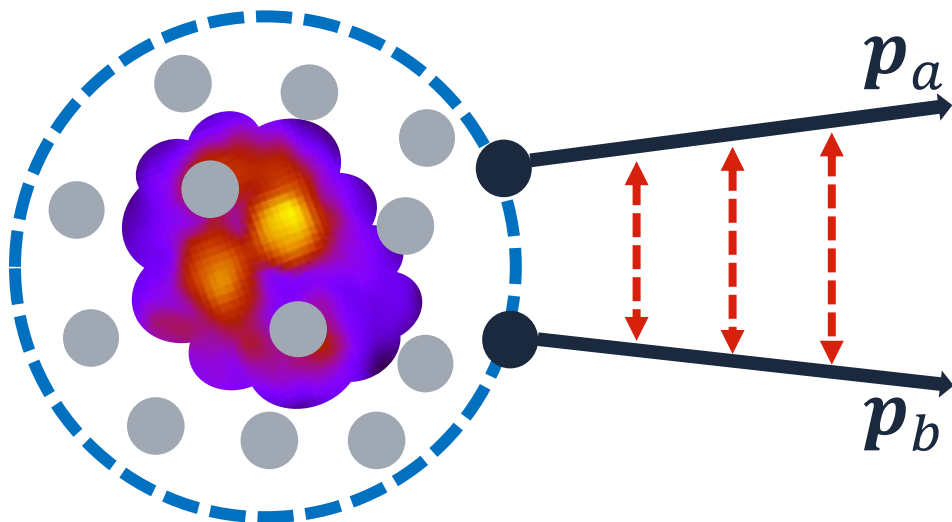
$$C(\mathbf{q}) := \frac{N_{\text{pair}}(\mathbf{p}_a, \mathbf{p}_b)}{N_a(\mathbf{p}_a) N_b(\mathbf{p}_b)}$$

Total momentum: $\mathbf{P} = \mathbf{p}_a + \mathbf{p}_b$

Relative momentum: $\mathbf{q} = \frac{m_b \mathbf{p}_a - m_a \mathbf{p}_b}{m_a + m_b}$

Two-particle momentum dist.: $N_{\text{pair}}(\mathbf{p}_a, \mathbf{p}_b)$

One-particle momentum dist.: $N_a(\mathbf{p}_a)$



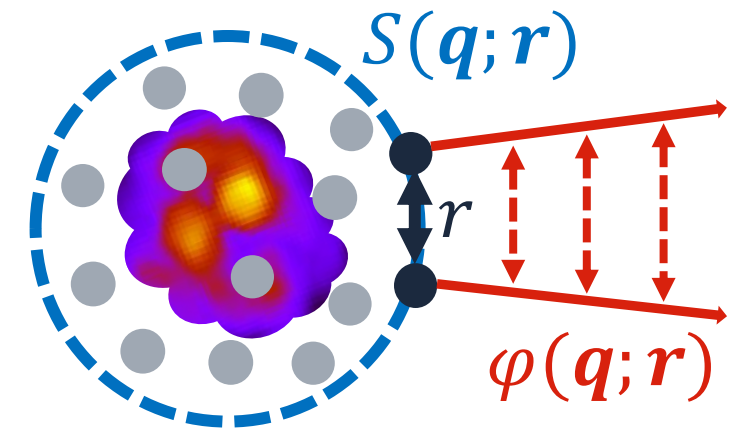
Hadron CF contains information on

- **Space-time structure of the matter**
- **Final state hadron interactions**

Koonin-Pratt (KP) formula

S. E. Koonin, PLB **70**, 43 (1977); S. Pratt, PRD **33**, 1314 (1986)

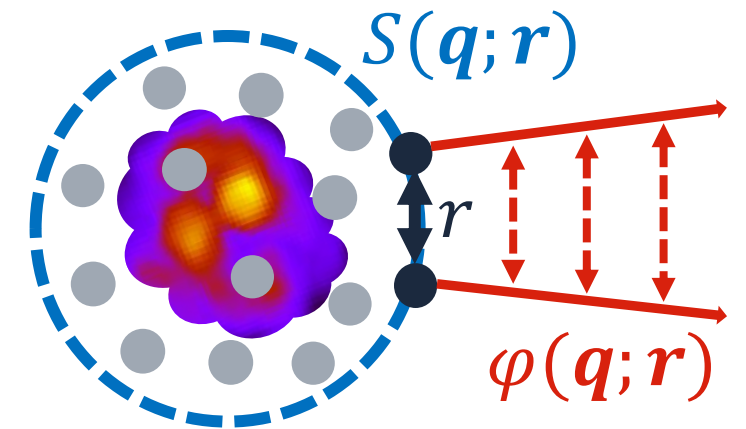
$$C(\mathbf{q}) = \int d^3r S(\mathbf{q}; \mathbf{r}) |\varphi(\mathbf{q}; \mathbf{r})|^2$$



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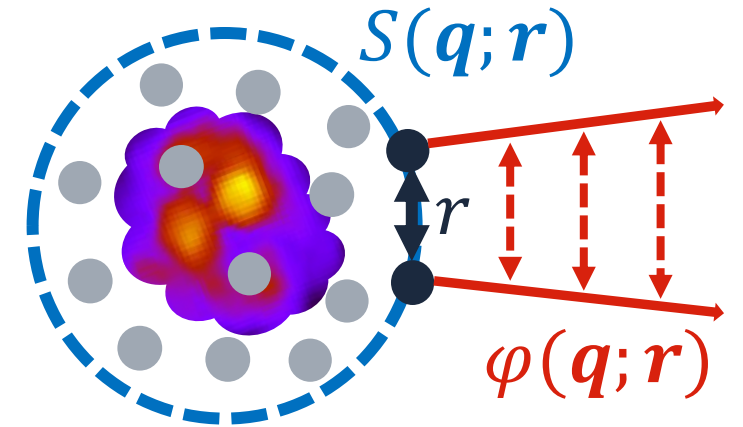
From measured **correlation function**

- **Input: hadron interaction** → **Output: source function**
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From measured **correlation function**

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Only *s*-wave scatt. with spherical SF

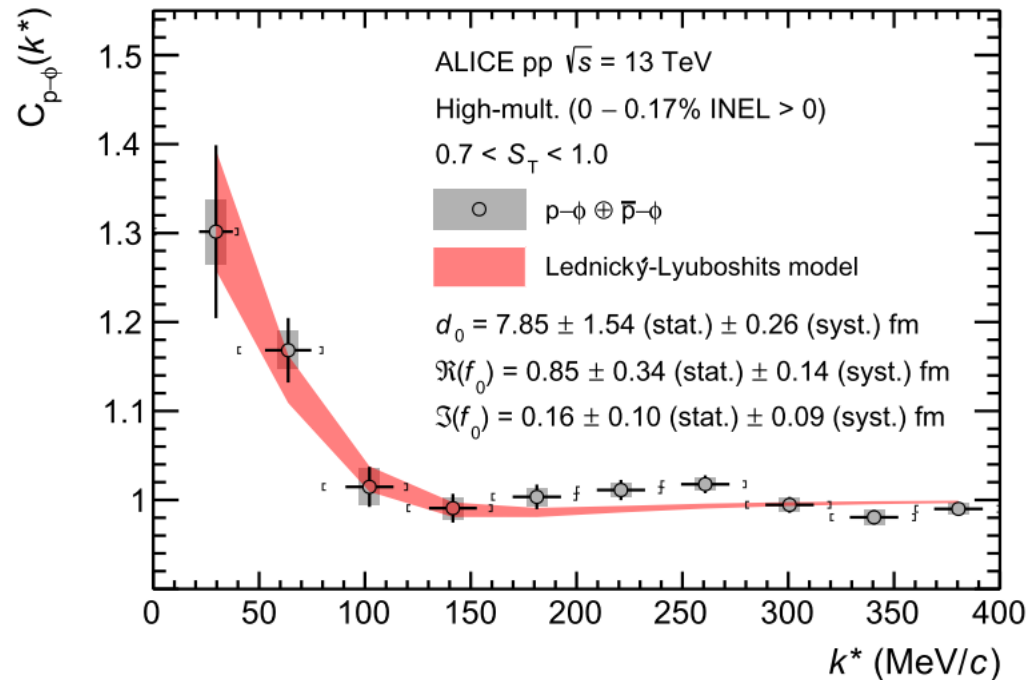
Weight Function

$$C(q) = 1 + \int_0^\infty dr 4\pi r^2 S(q; r) \{ |\varphi_0(q; r)|^2 - [j_0(qr)]^2 \}$$

Deviation of **CF** from 1 = How much **SF** “picks up” **WF**

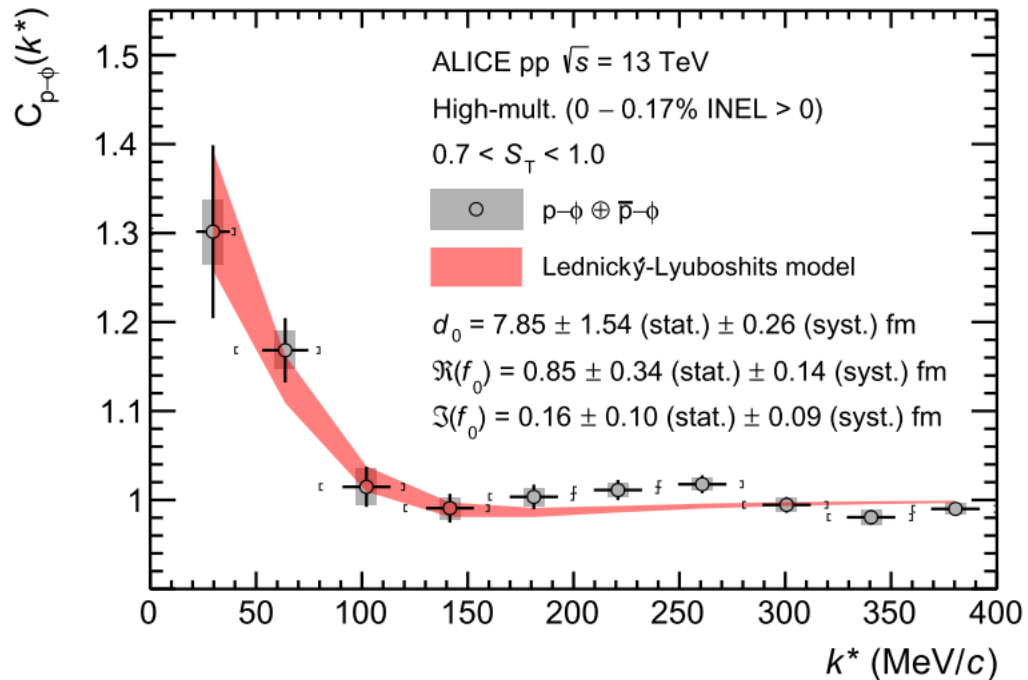
Experimental CF ALICE, PRL 127, 172301 (2021)

High-multiplicity (0–0.17%) p+p collisions at $\sqrt{s} = 13$ TeV



Experimental CF ALICE, PRL 127, 172301 (2021)

High-multiplicity (0–0.17%) p+p collisions at $\sqrt{s} = 13$ TeV



Lednický-Lyuboshits fit

R. Lednický and V. L. Lyuboshits, Yad. Fiz. **35**, 1316 (1981)

Gaussian source size: $r_0 = 1.08$ fm

Scattering length: $a_0 \cong -0.85 - 0.16i$ fm

Effective range: $r_{\text{eff}} \cong 7.85$ fm

Attractive p-φ interaction as a spin-average

Spin-channel-by-channel femtoscopy E. Chizzali *et al.*, PLB **848**, 138358 (2023)

Gaussian source size: $r_0 = 1.08$ fm

$^4S_{3/2}$: HAL QCD potential Y. Lyu *et al.*, PRD 106, 074507 (2022)

$$a_0^{(3/2)} \cong -1.43 \text{ fm}, \quad r_{\text{eff}}^{(3/2)} \cong 2.36 \text{ fm}$$

Attraction without bound states

$^2S_{1/2}$: Parameterised potential ← Constrain by **experimental CF**

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Attraction without bound states

$^2S_{1/2}$: Parameterised potential ← Constrain by **experimental CF**

$$a_0^{(1/2)} \cong 1.54 - i0.00 \text{ fm}, \quad r_{\text{eff}}^{(1/2)} \cong 0.39 + i0.00 \text{ fm}$$

■ Strong attraction

■ Small effects of channel-coupling

Indication of a p- ϕ bound state

SF should reflect the complex dynamics of nuclear collisions

This study: **Femtoscscopy using SF from a dynamical model**

SF should reflect the complex dynamics of nuclear collisions

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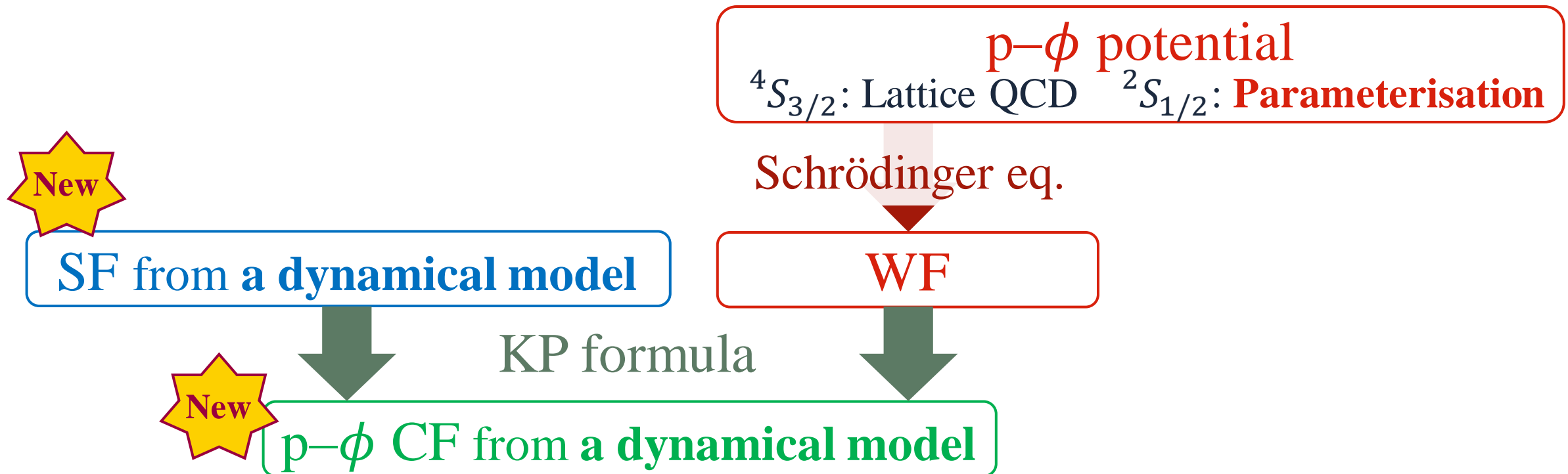
${}^4S_{3/2}$: Lattice QCD $p-\phi$ potential ${}^2S_{1/2}$: **Parameterisation**

Schrödinger eq.

WF

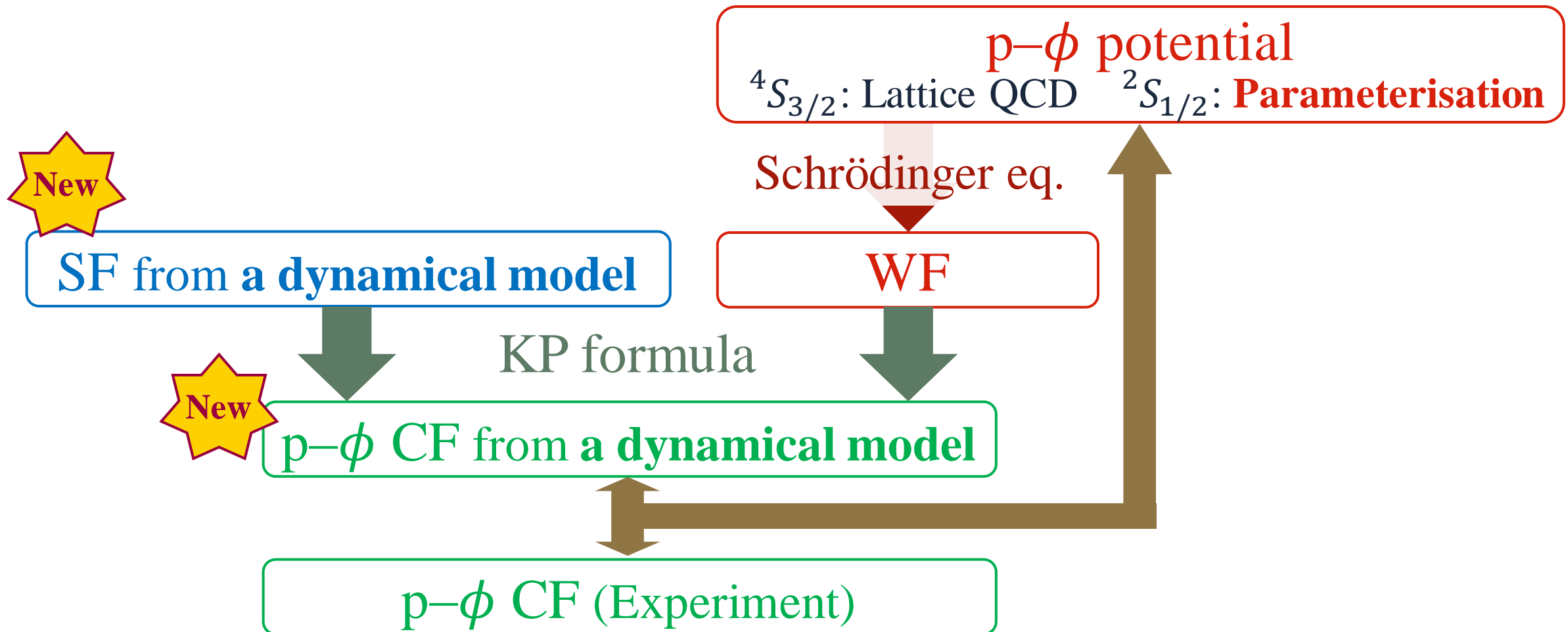
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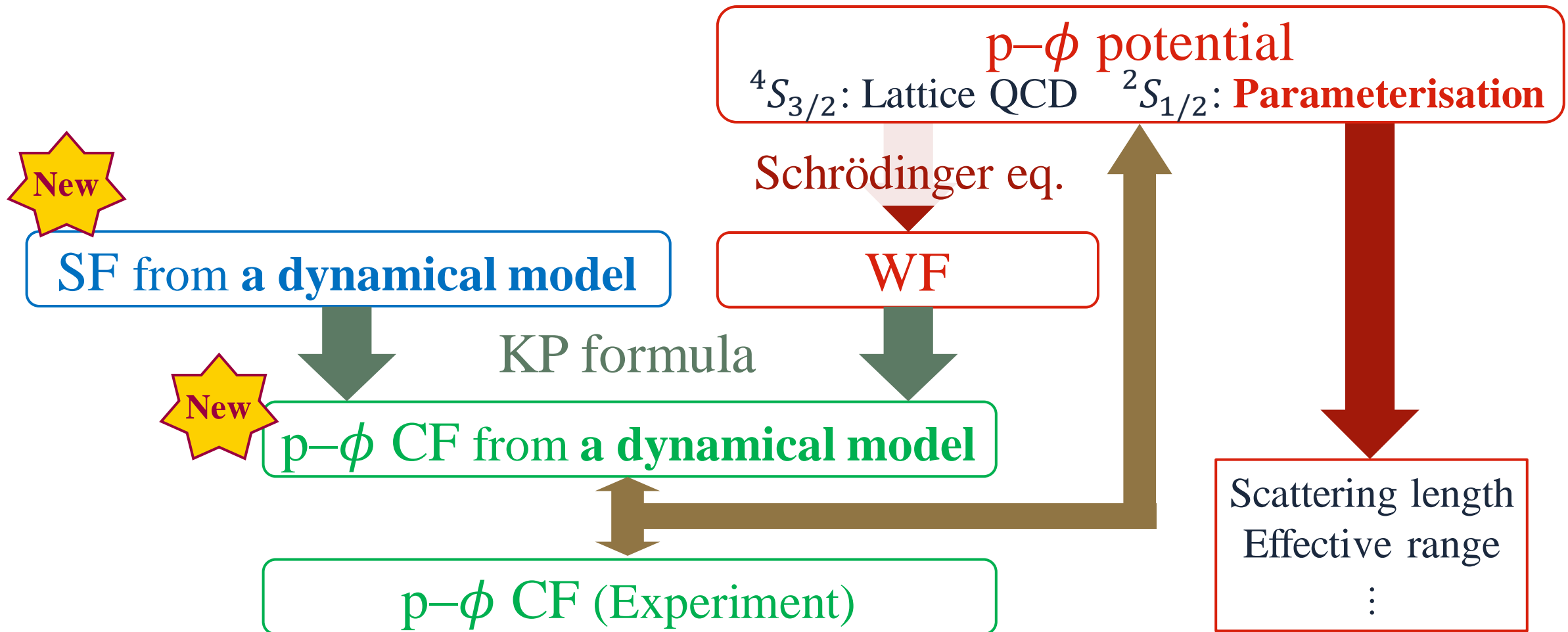
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HAL QCD potential Y. Lyu *et al.*, PRD **106**, 074507 (2022)

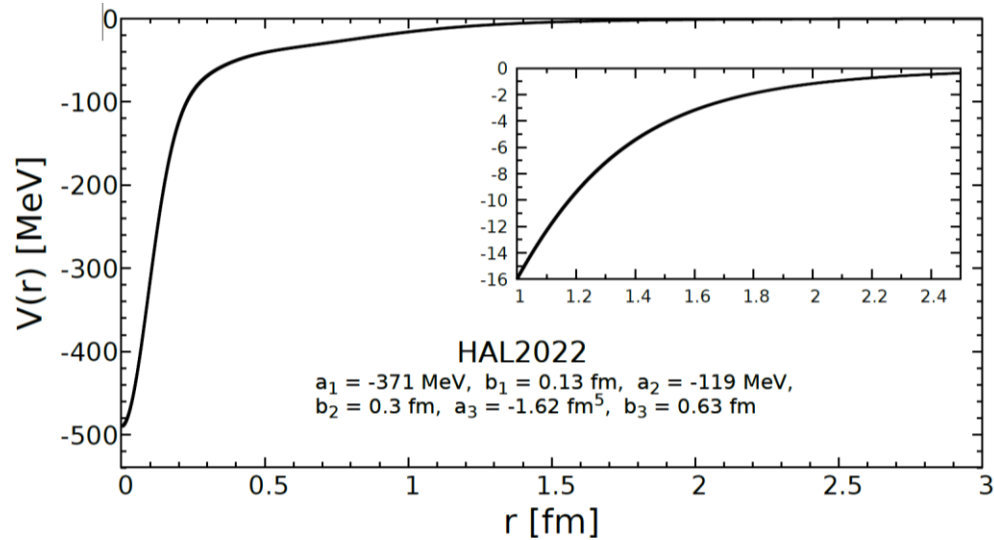
Lattice QCD at nearly physical point ($m_\pi = 146.4$ MeV)

$$V^{(3/2)}(r) = \underbrace{a_1 e^{-(r/b_1)^2} + a_2 e^{-(r/b_2)^2}}_{\text{Short-range attraction}} + \underbrace{a_3 m_\pi^4 f(r; b_3) \frac{e^{-2m_\pi r}}{r^2}}_{\text{TPE}}$$

Argonne-type form factor:

$$f(r; b_3) = [1 - e^{-(r/b_3)^2}]^2$$

Parameter	Fitted value
a_1 [MeV]	-371 ± 27
b_1 [fm]	0.13 ± 0.01
a_2 [MeV]	-119 ± 39
b_2 [fm]	0.30 ± 0.05
a_3 [fm ⁵]	-1.62 ± 0.23
b_3 [fm]	0.63 ± 0.04



No bound state

$^4S_{3/2}$ Channel

HAL QCD potential Y. Lyu *et al.*, PRD **106**, 074507 (2022)

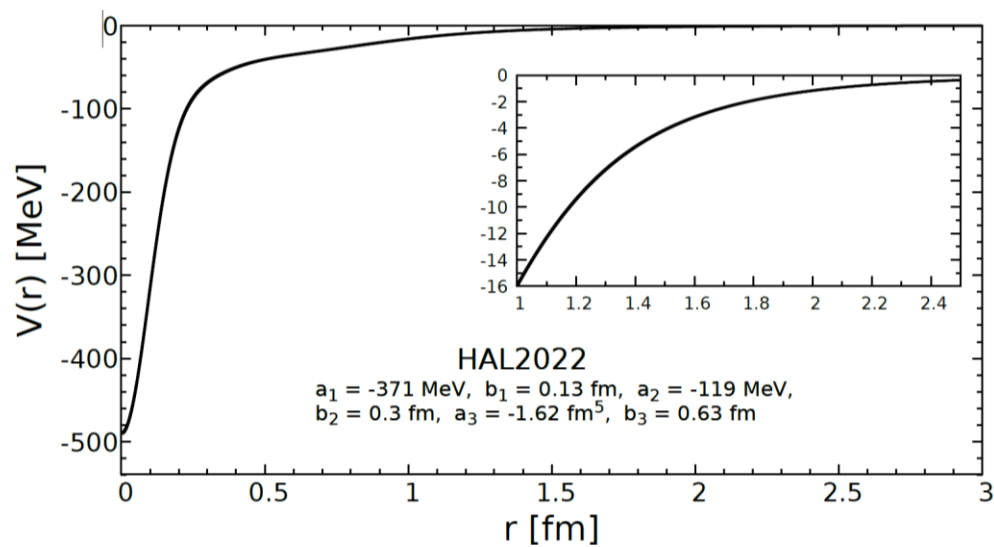
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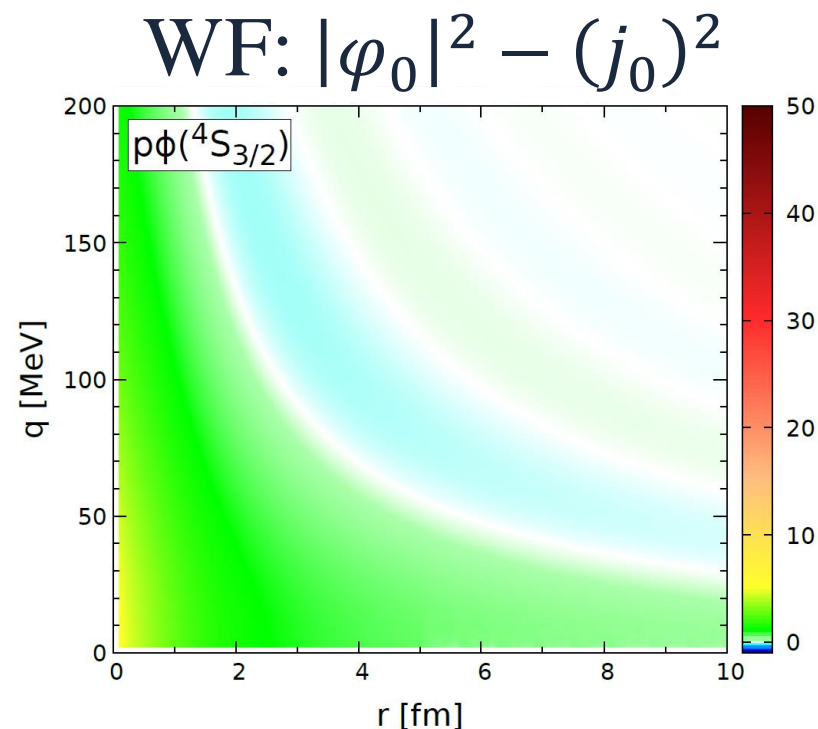
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No bound state



**Enhancement
at small qr
due to attraction**

Parameterised potential E. Chizzali *et al.*, PLB **848**, 138358 (2023)

Channel-couplings are neglected for simplicity

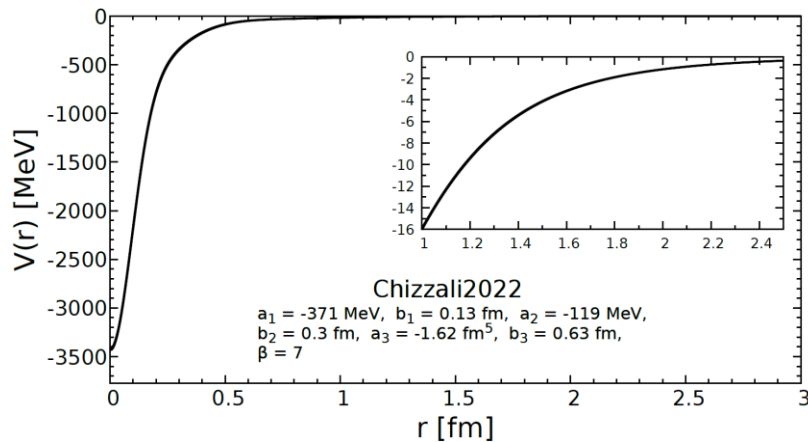
$$V^{(1/2)}(r) = \beta \left[a_1 e^{-(r/b_1)^2} + a_2 e^{-(r/b_2)^2} \right] + a_3 m_\pi^4 f(r; b_3) \frac{e^{-2m_\pi r}}{r^2}$$

Short-range interaction TPE

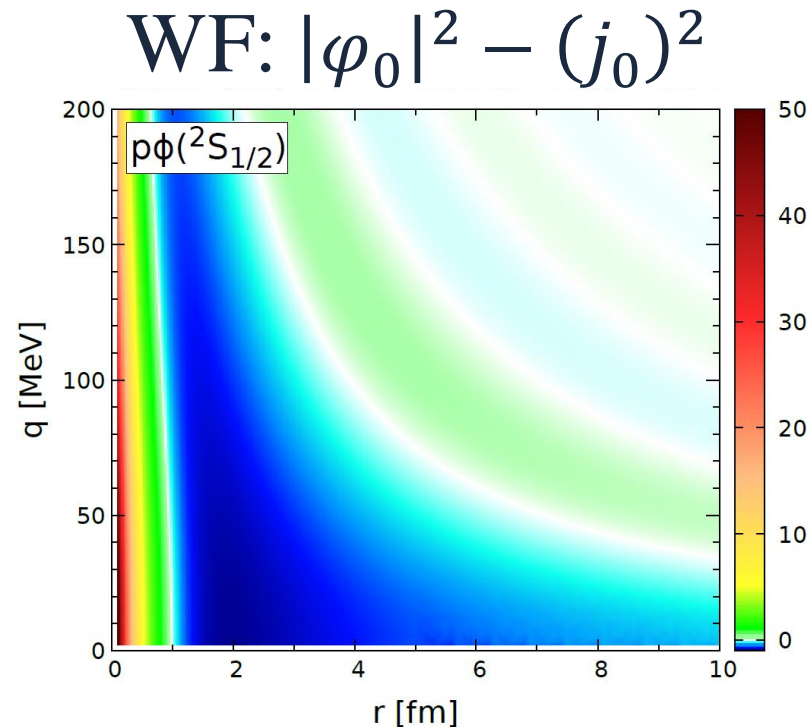
Only one adjustable parameter

β

default: $\beta = 7$



$a_0 = 1.99$ fm
 $r_{\text{eff}} = 0.46$ fm
A bound state



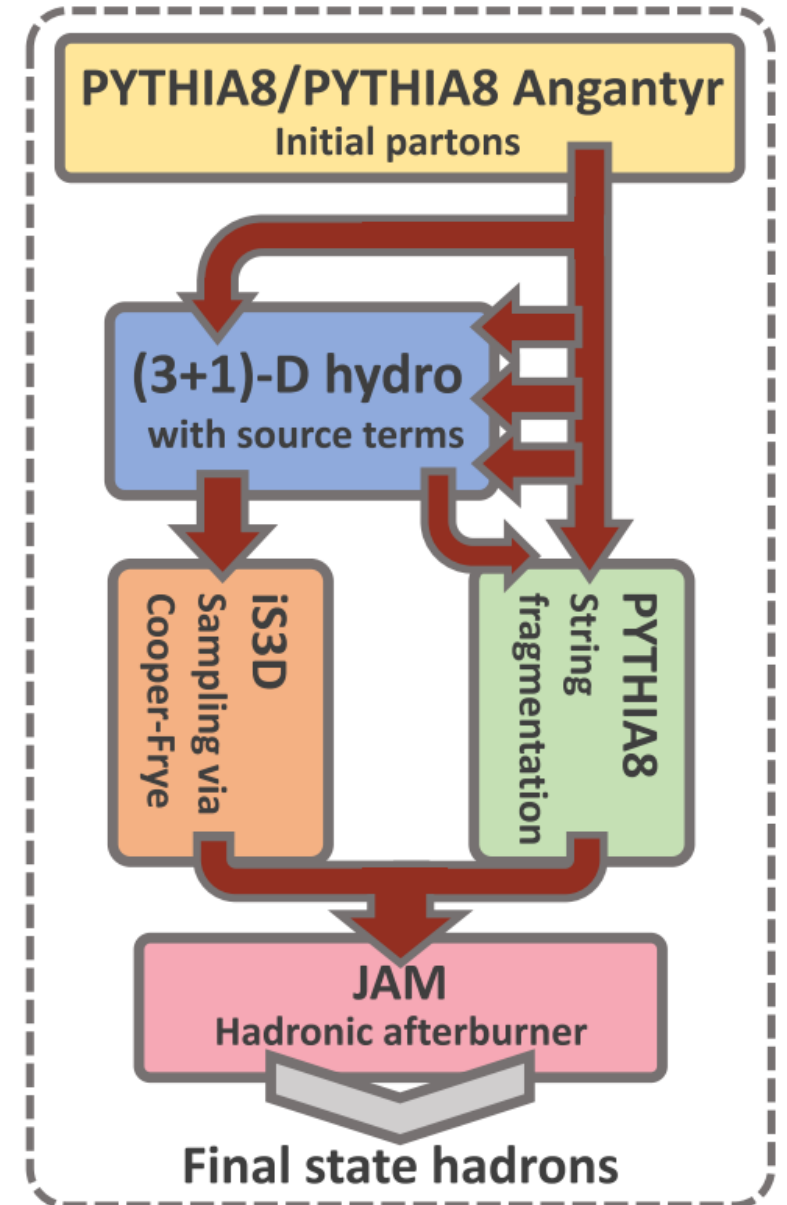
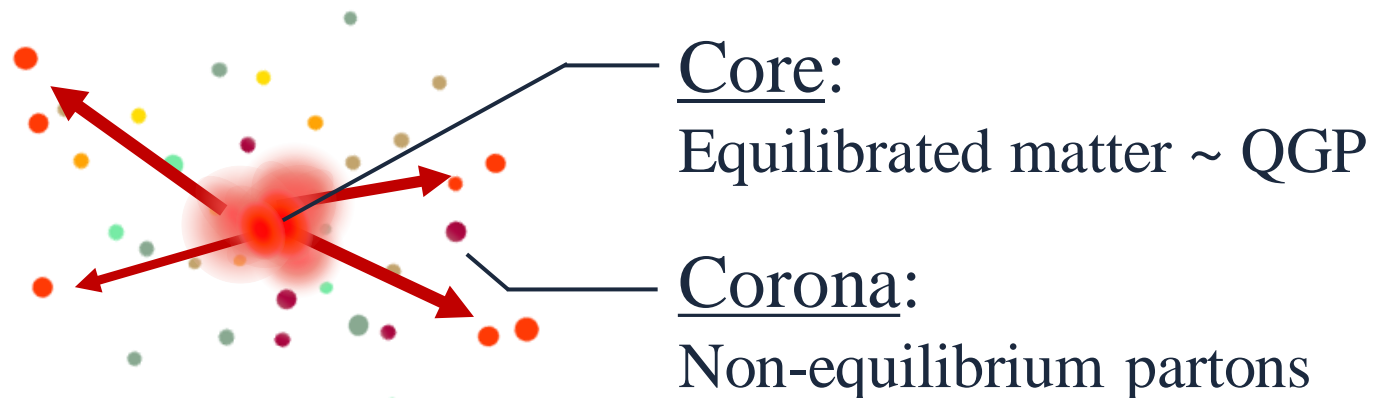
■ **Strong enhancement at small qr**

■ **“Negative valley” around a_0**

Dynamical Core–Corona Initialisation model (DCCI2)

Y. Kanakubo, Y. Tachibana, and T. Hirano, PRC 105, 024905 (2022)

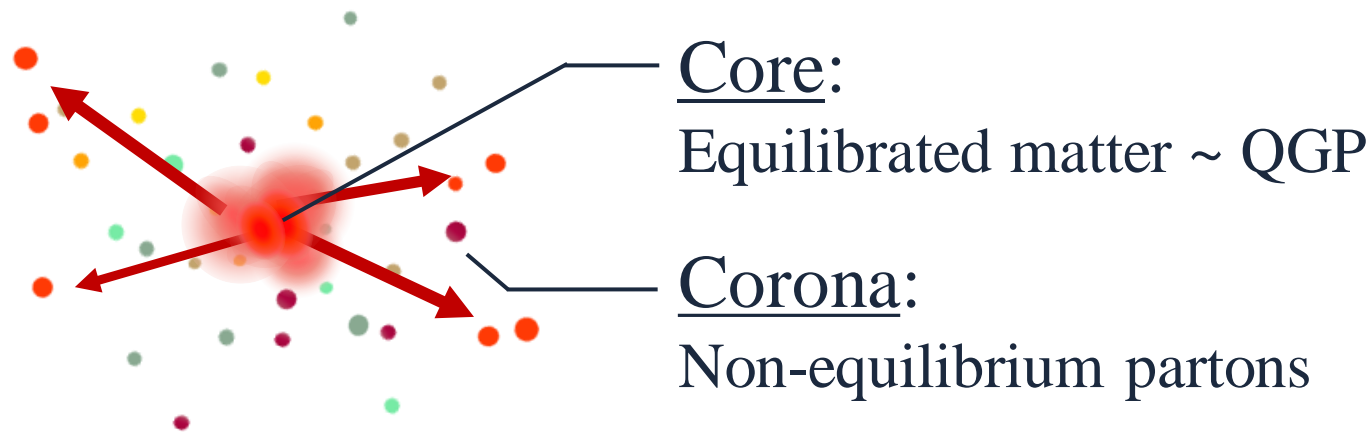
A state-of-the-art dynamical model
based on **core–corona picture**



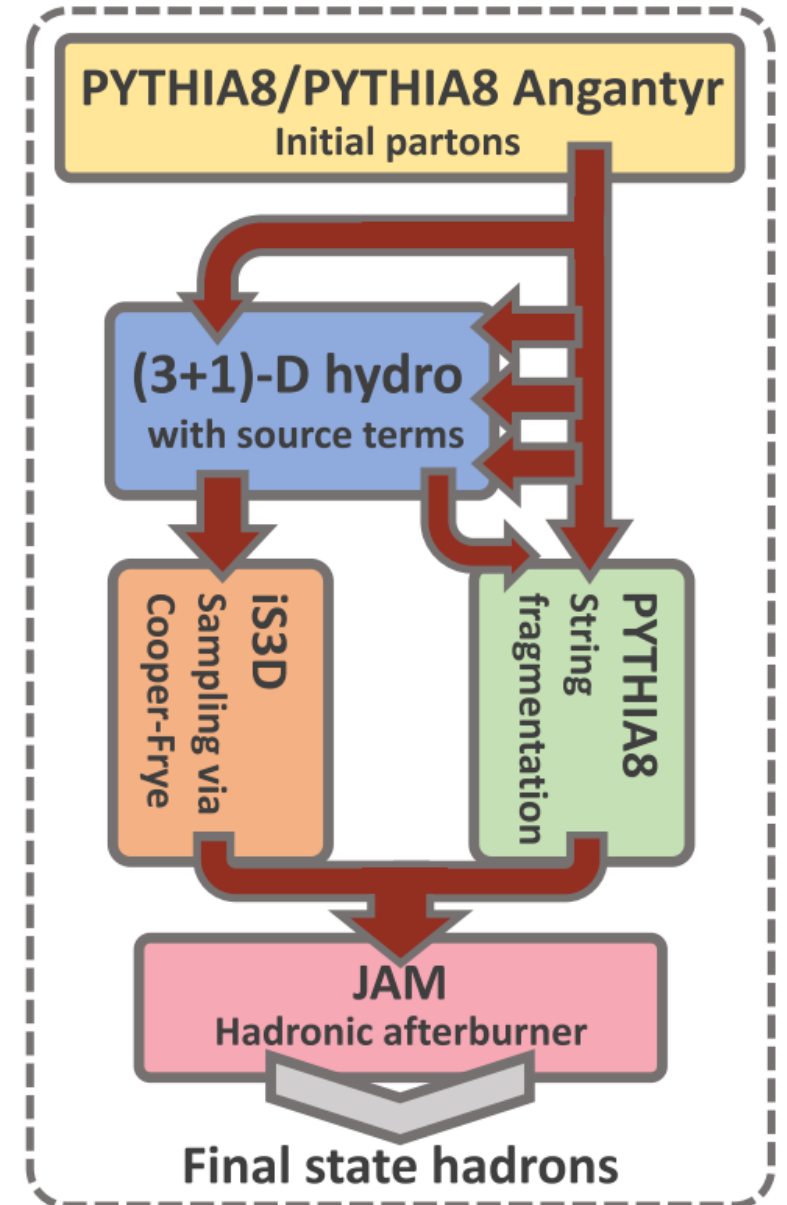
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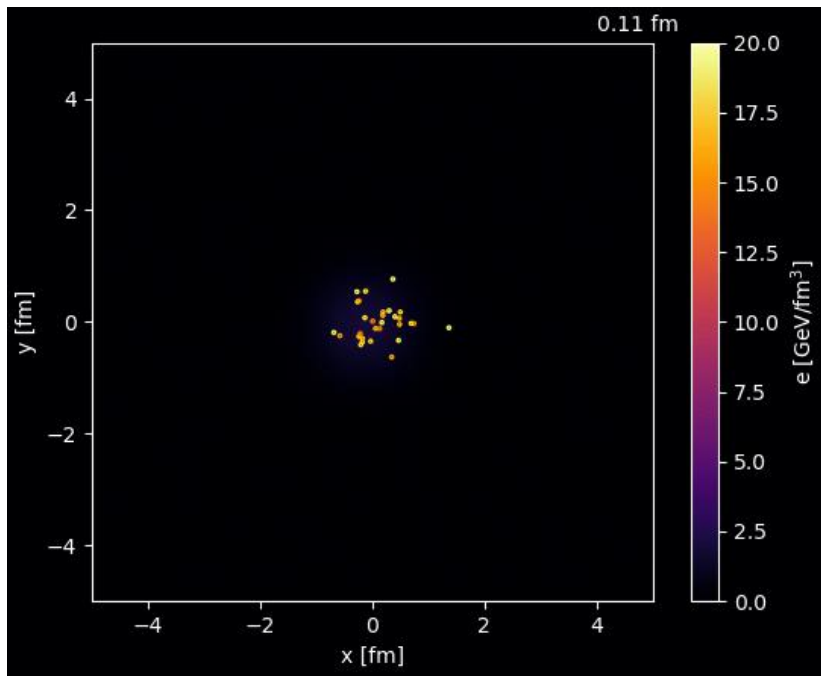
Applicable to HM p+p collisions



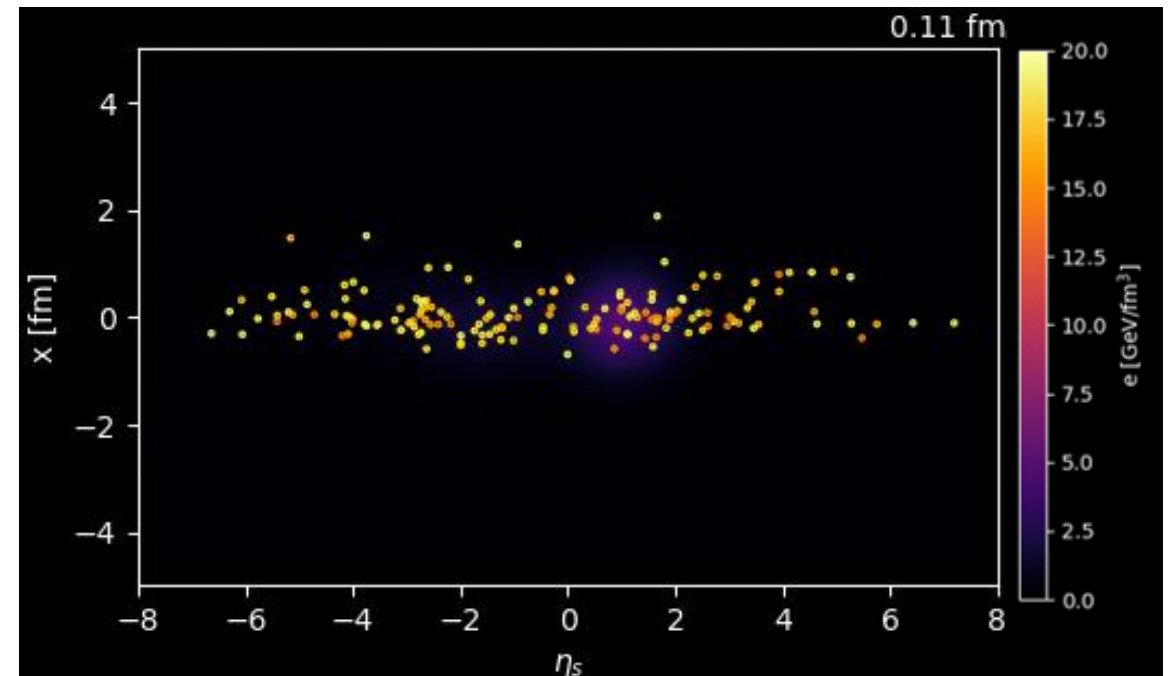
HM p+p collisions at $\sqrt{s} = 7$ TeV

Movies: Courtesy of Y. Kanakubo

Transverse plane



$x-\eta_s$ plane

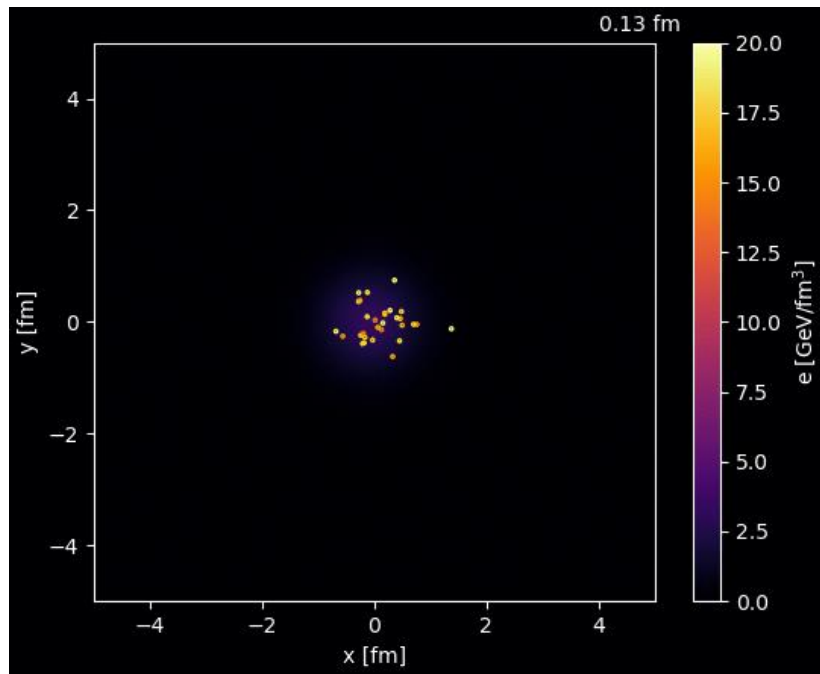


Describe the space-time evolution of nuclear collisions
→ SF that reflects collision dynamics

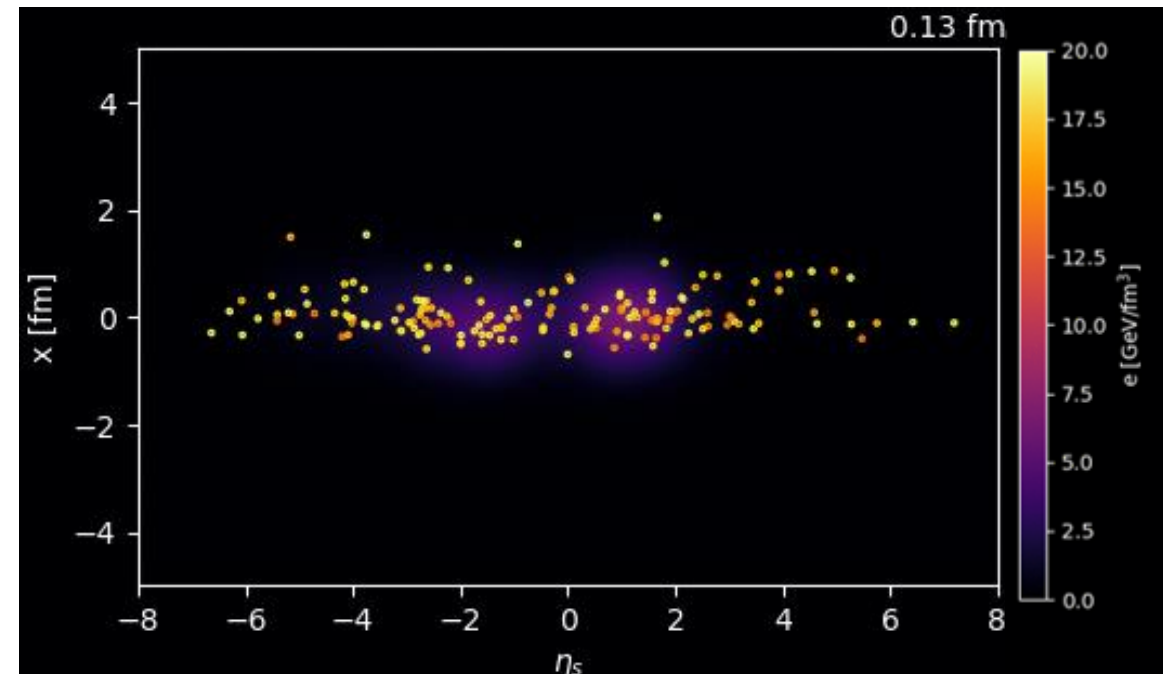
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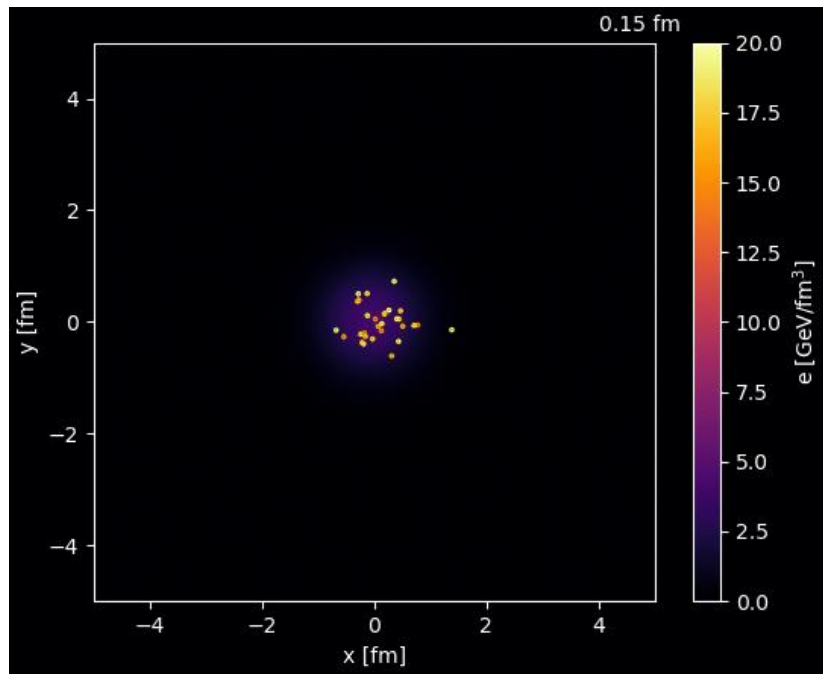


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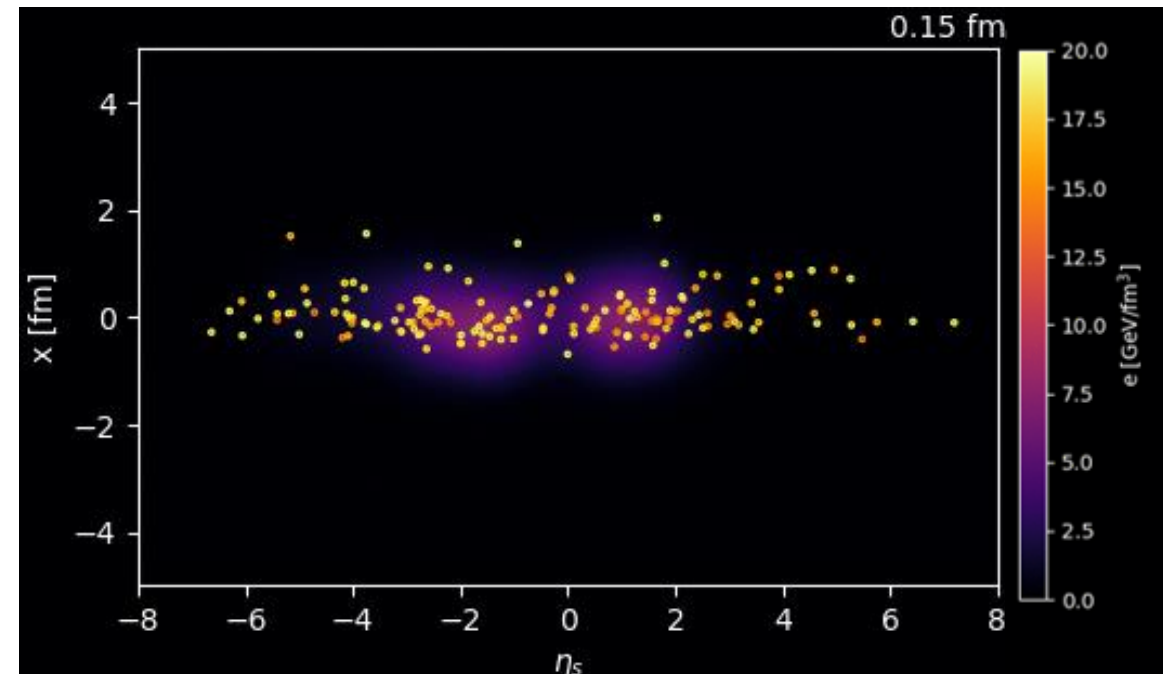
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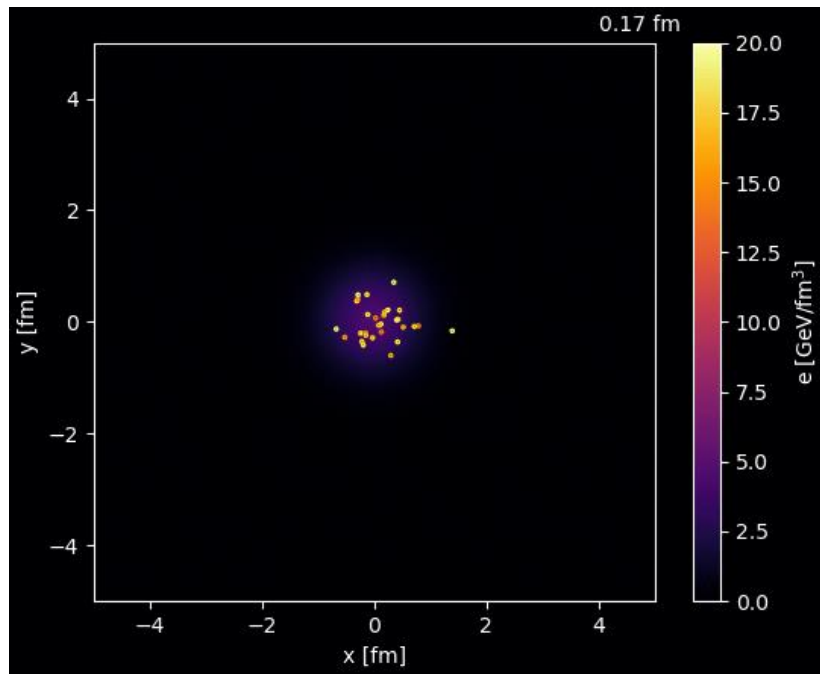


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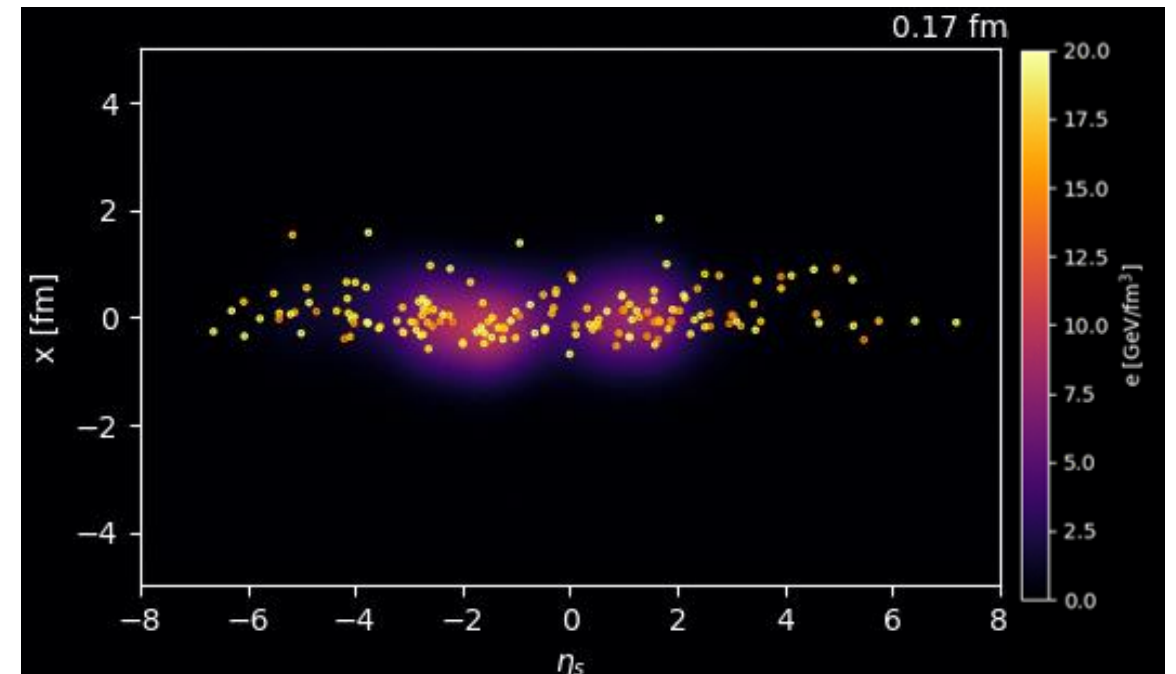
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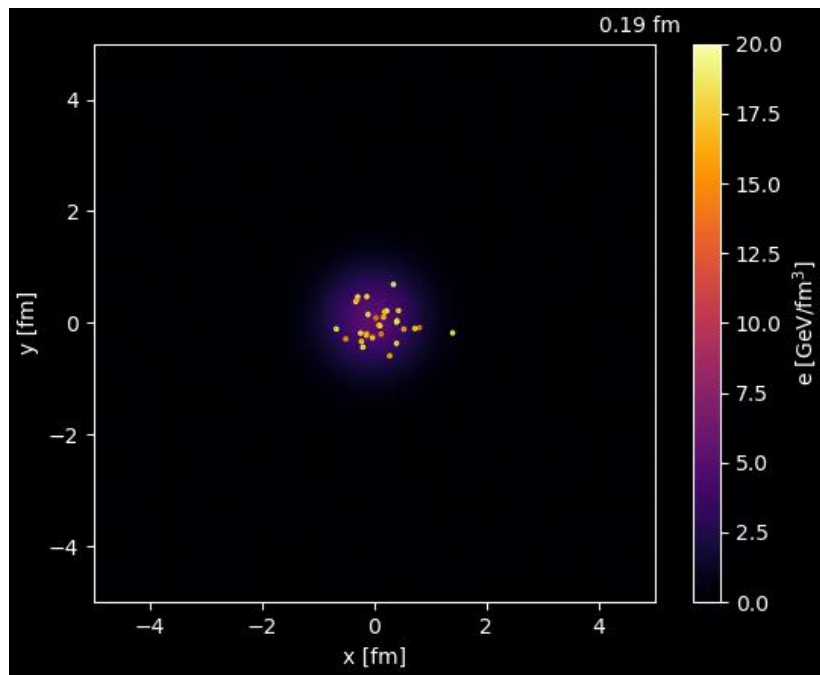


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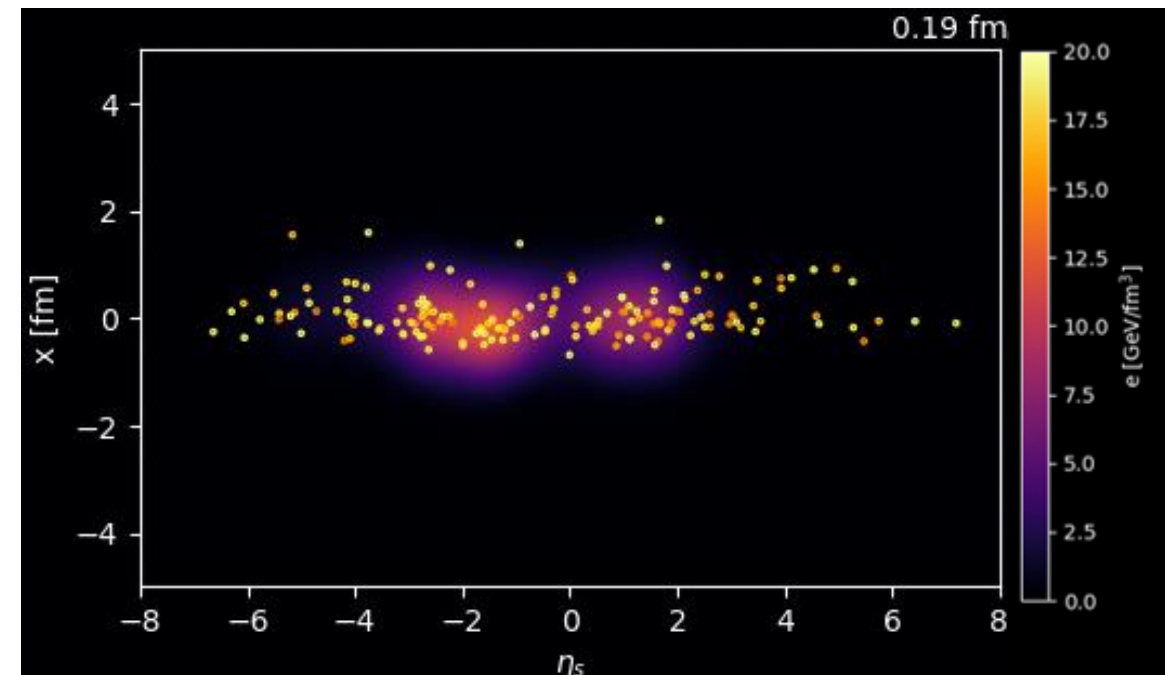
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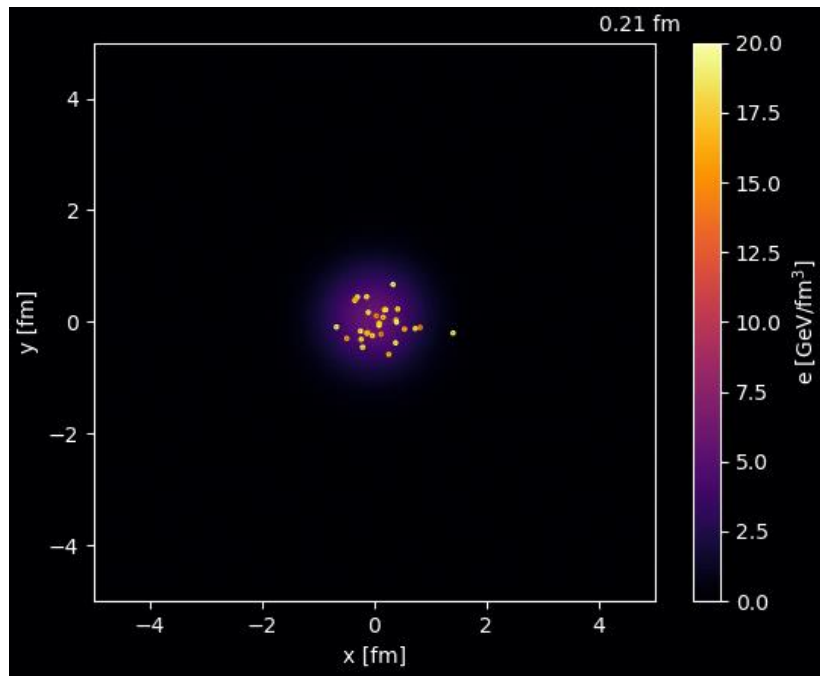


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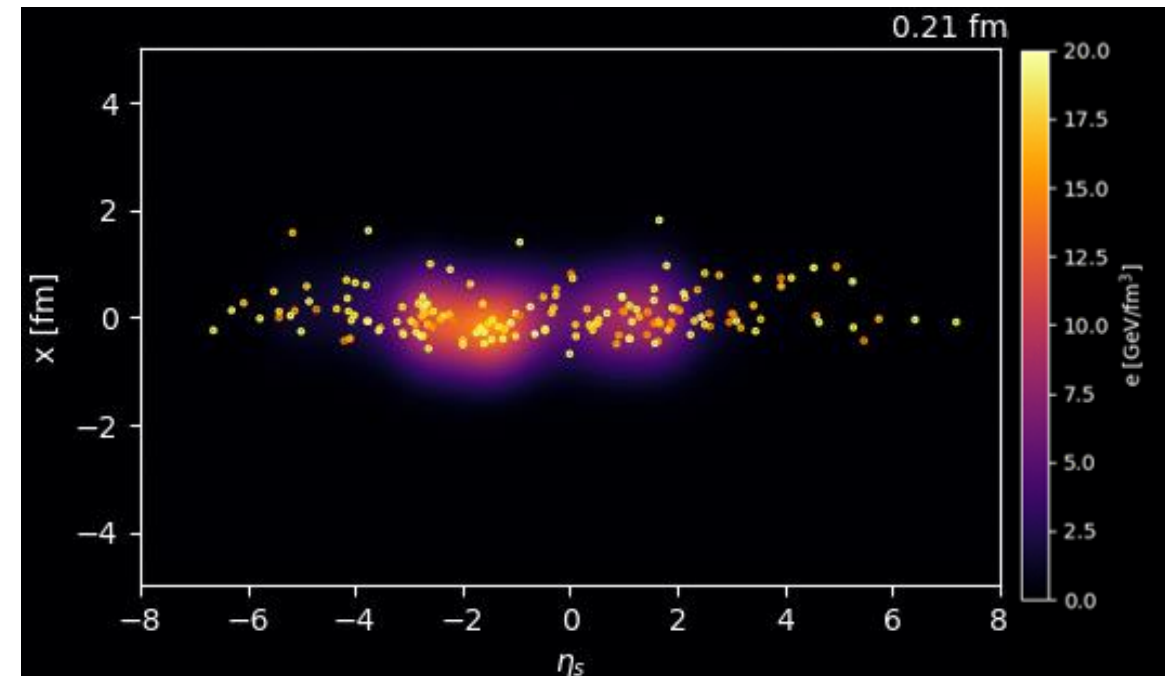
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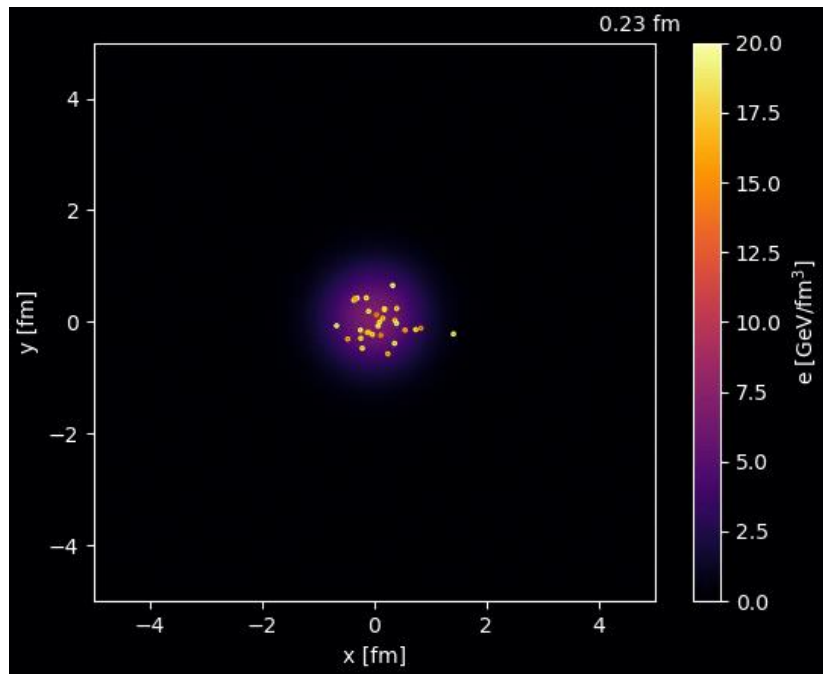


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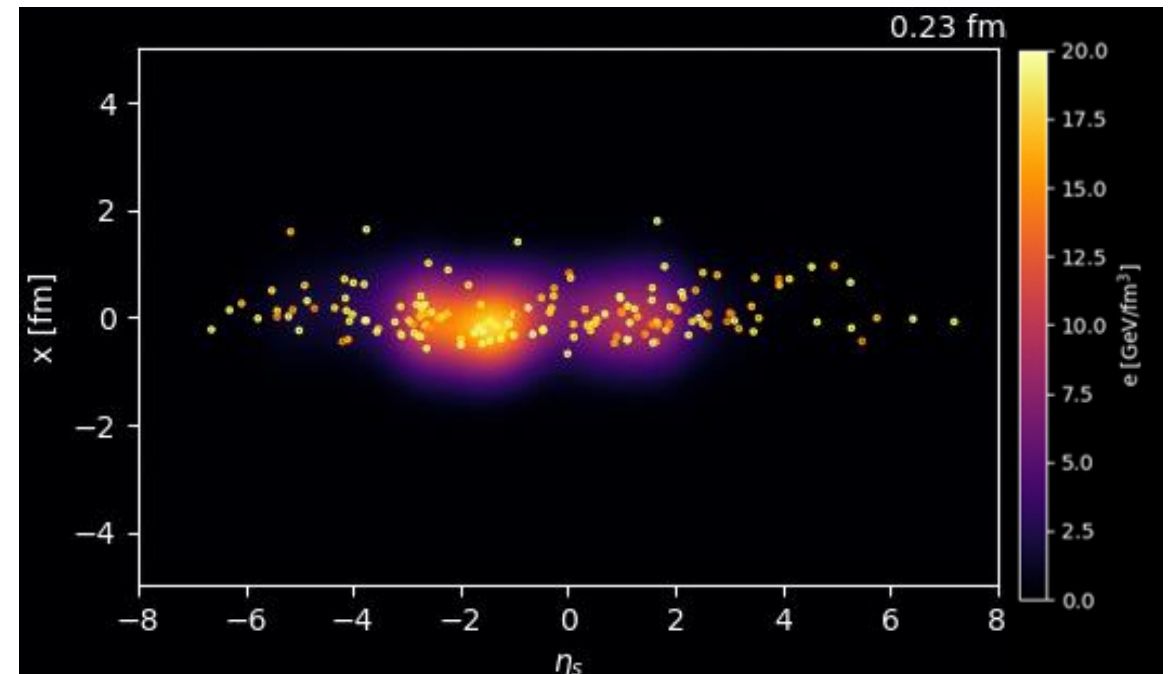
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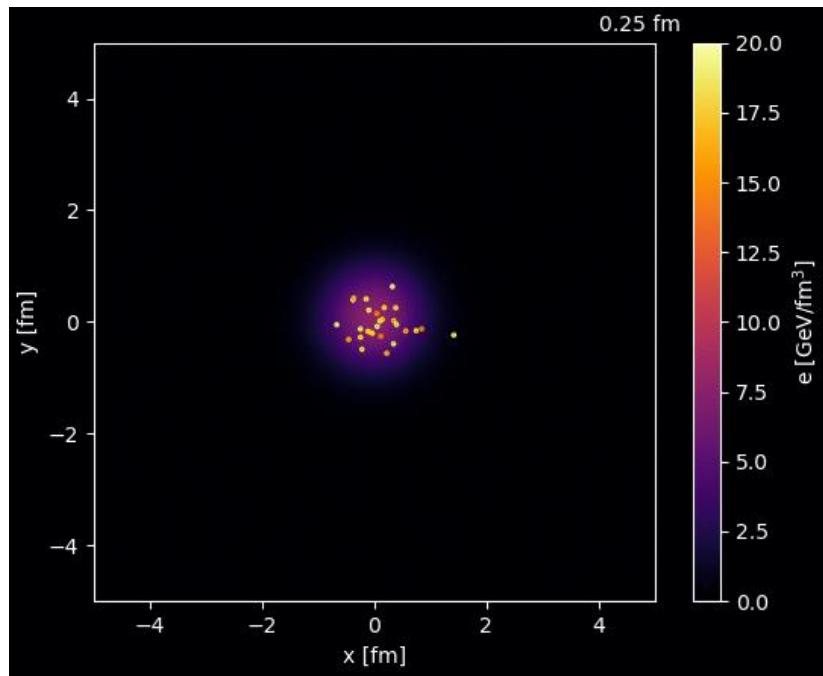


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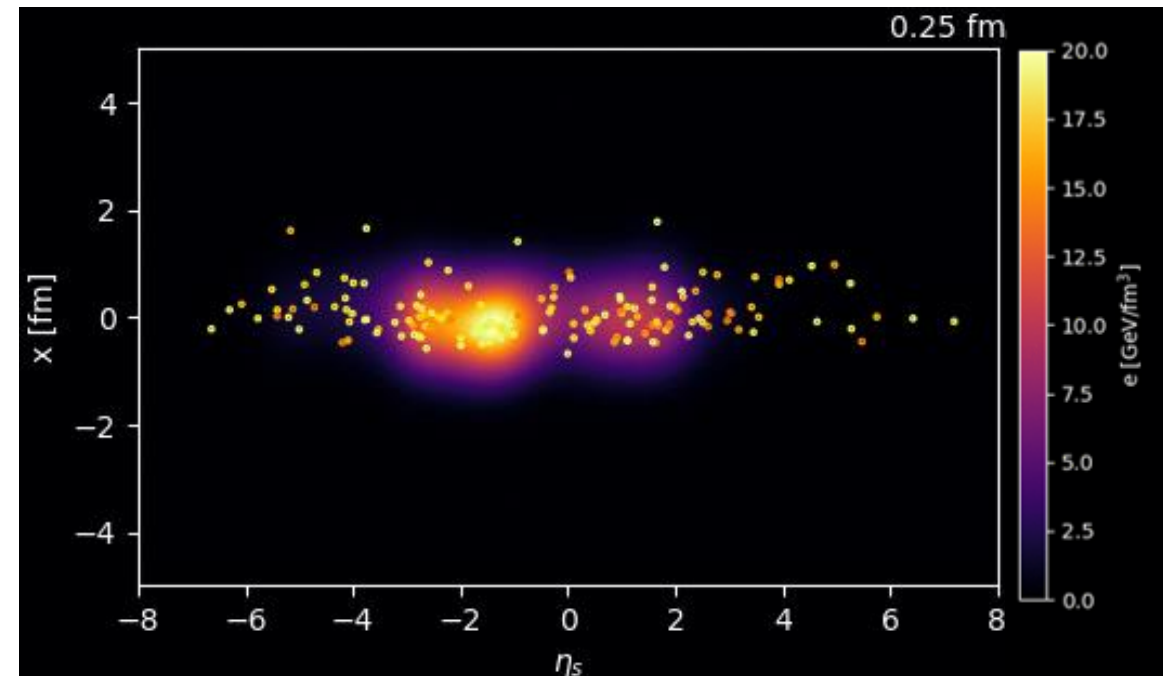
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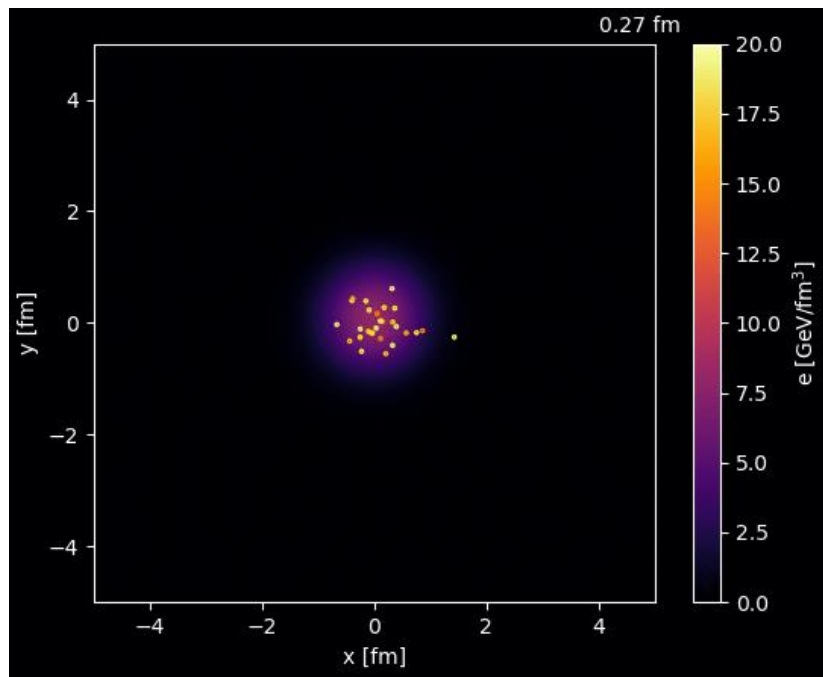


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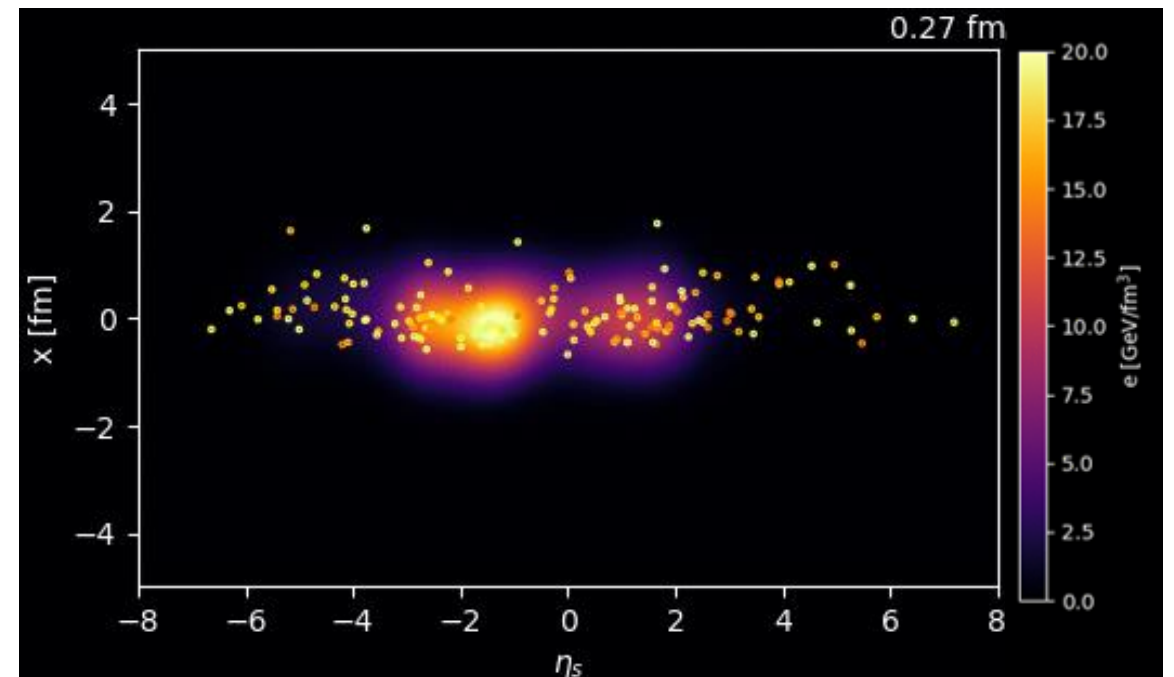
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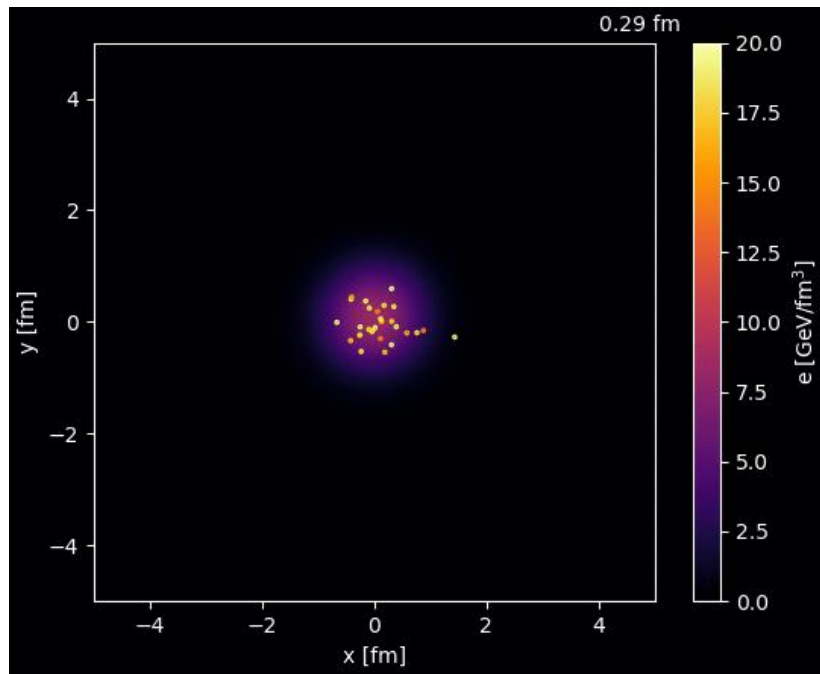


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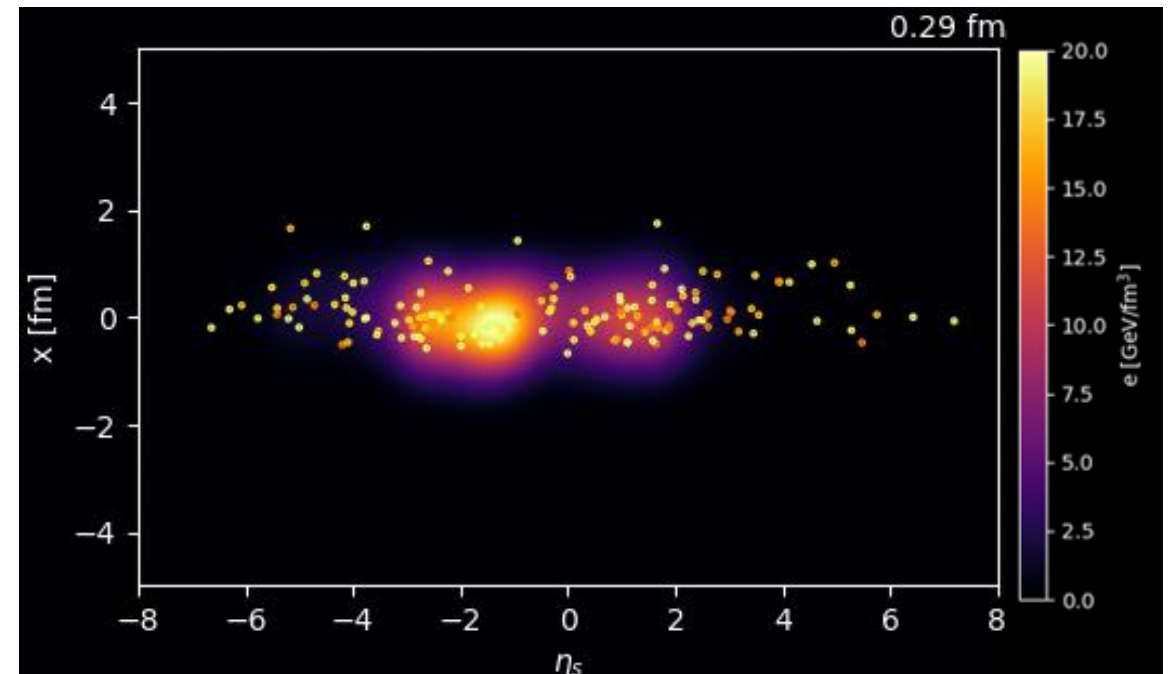
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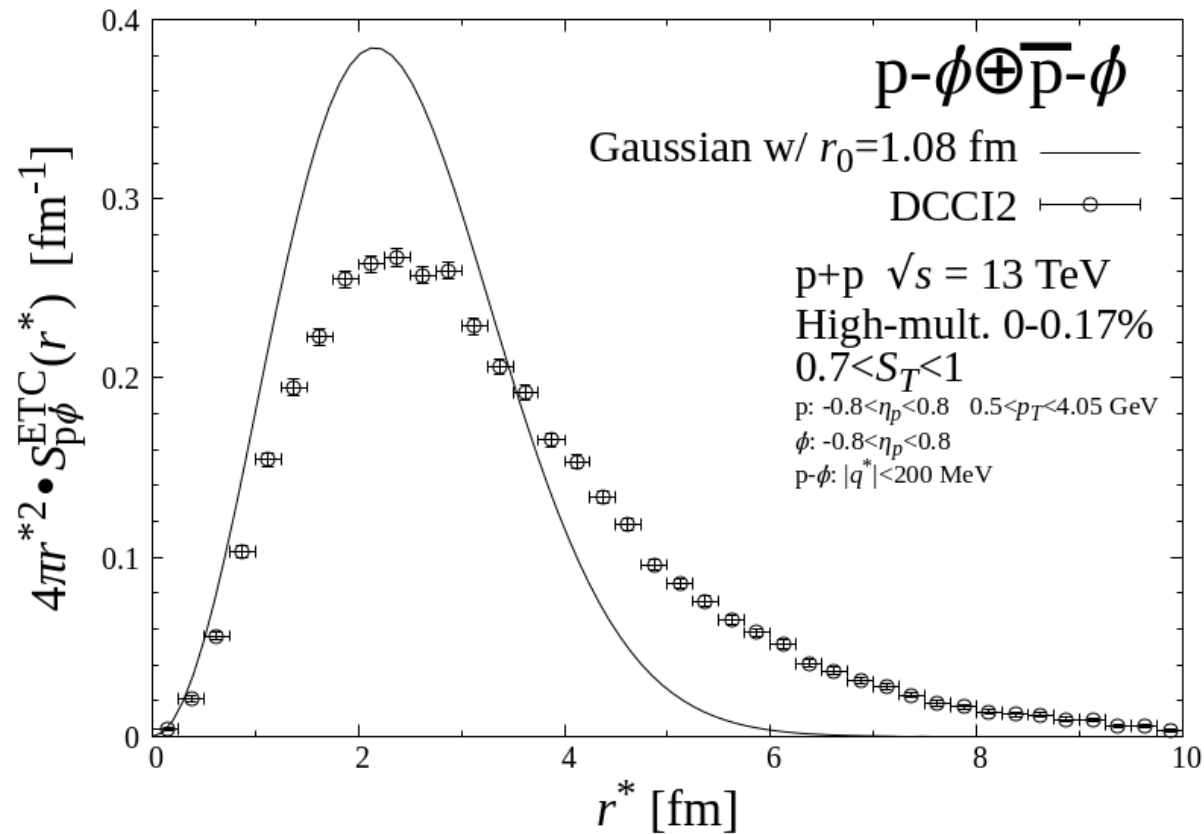


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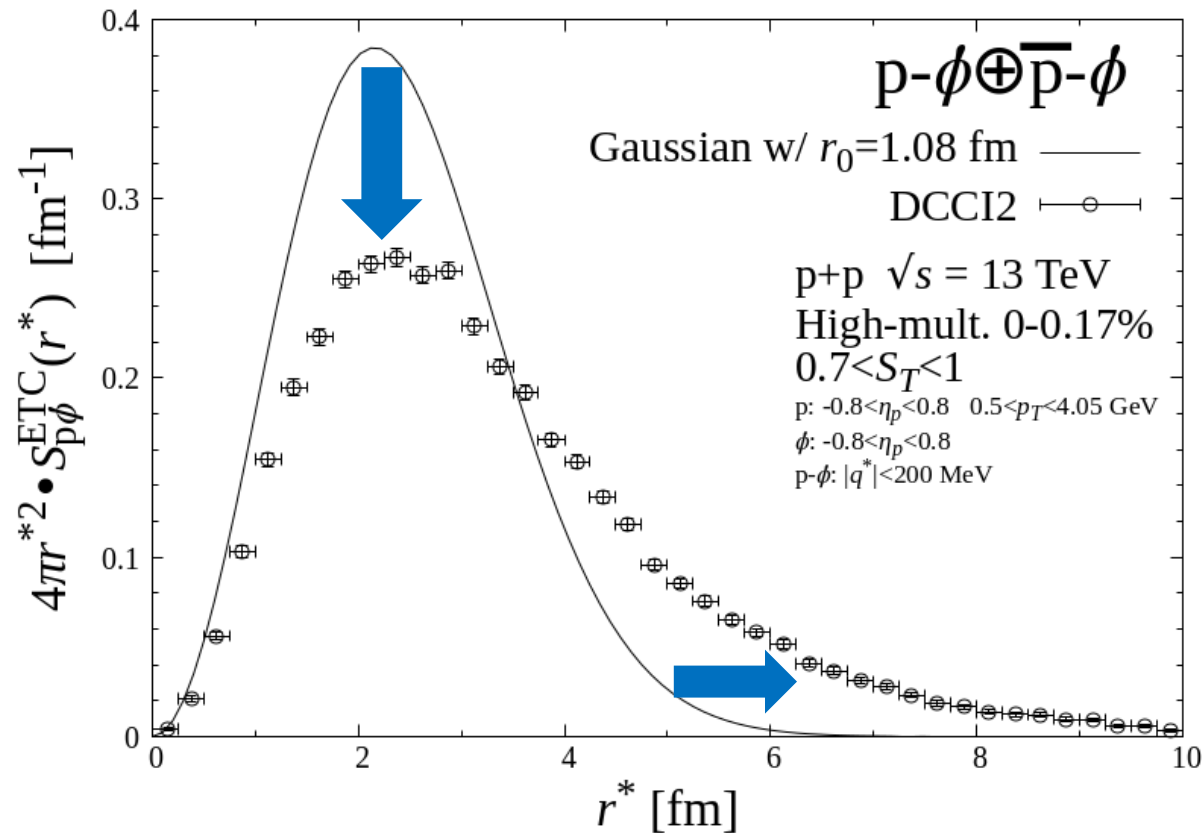


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q -integrated SF

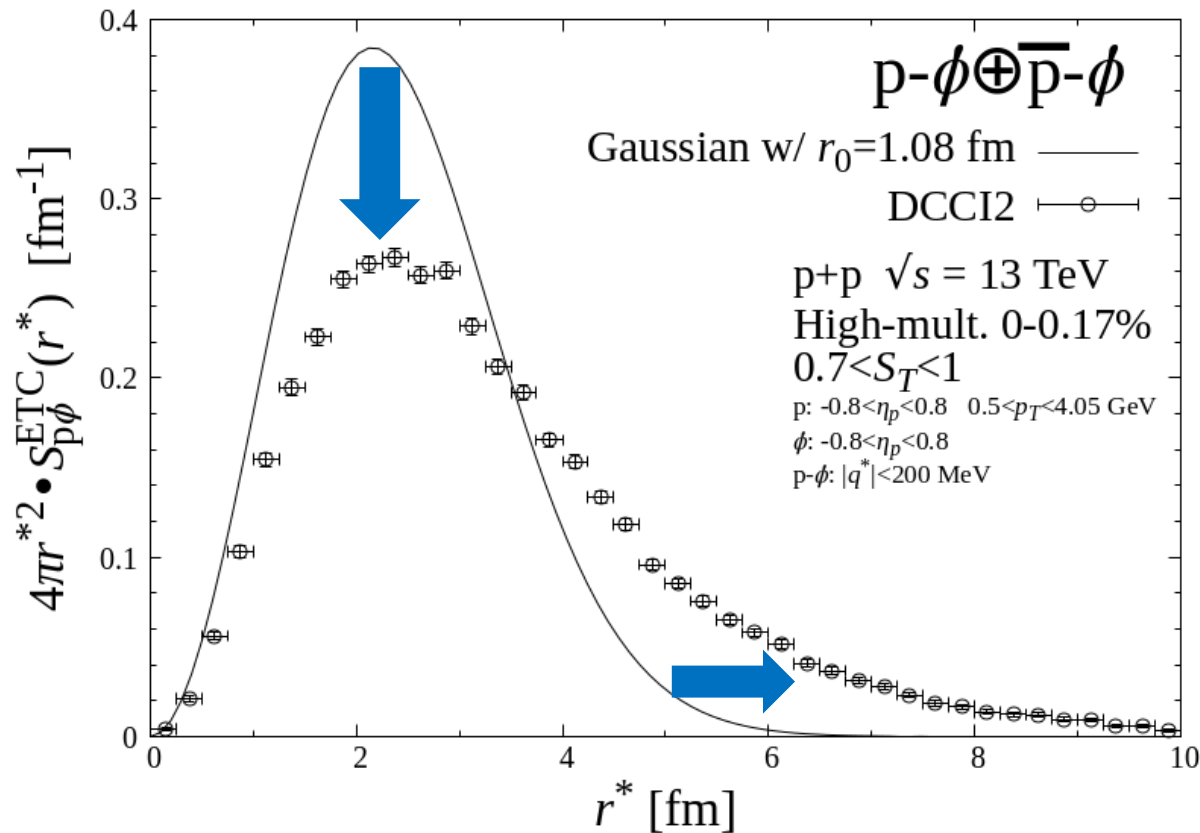


q-integrated SF



Larger source size $\langle r^2 \rangle$
 mainly from hadronic rescatterings

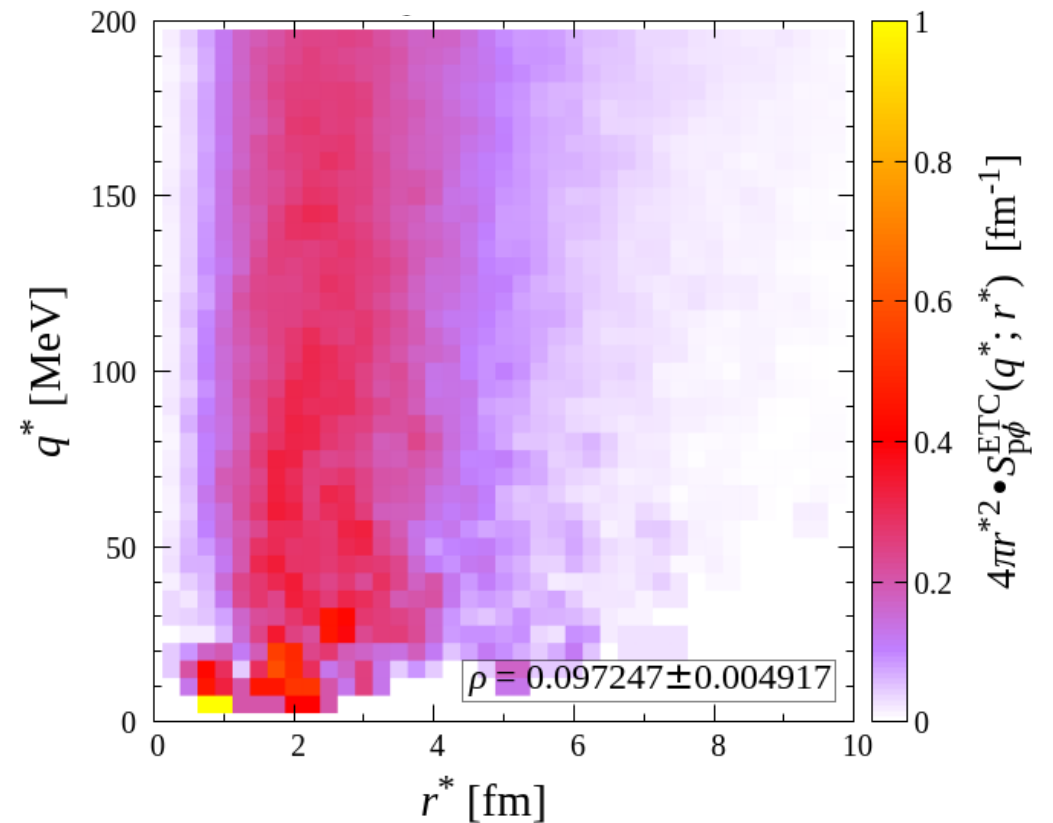
q-integrated SF



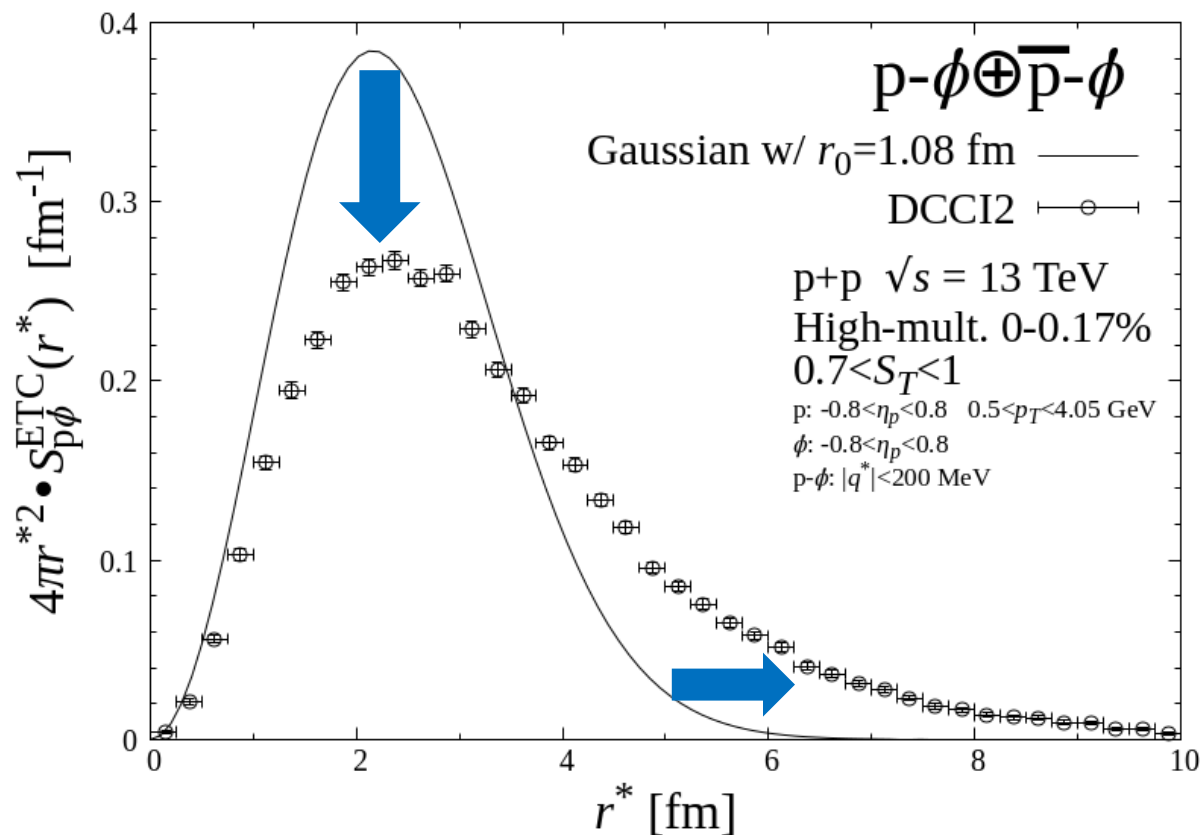
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q-differential SF

SF generally depends on q
 due to e.g., collectivity



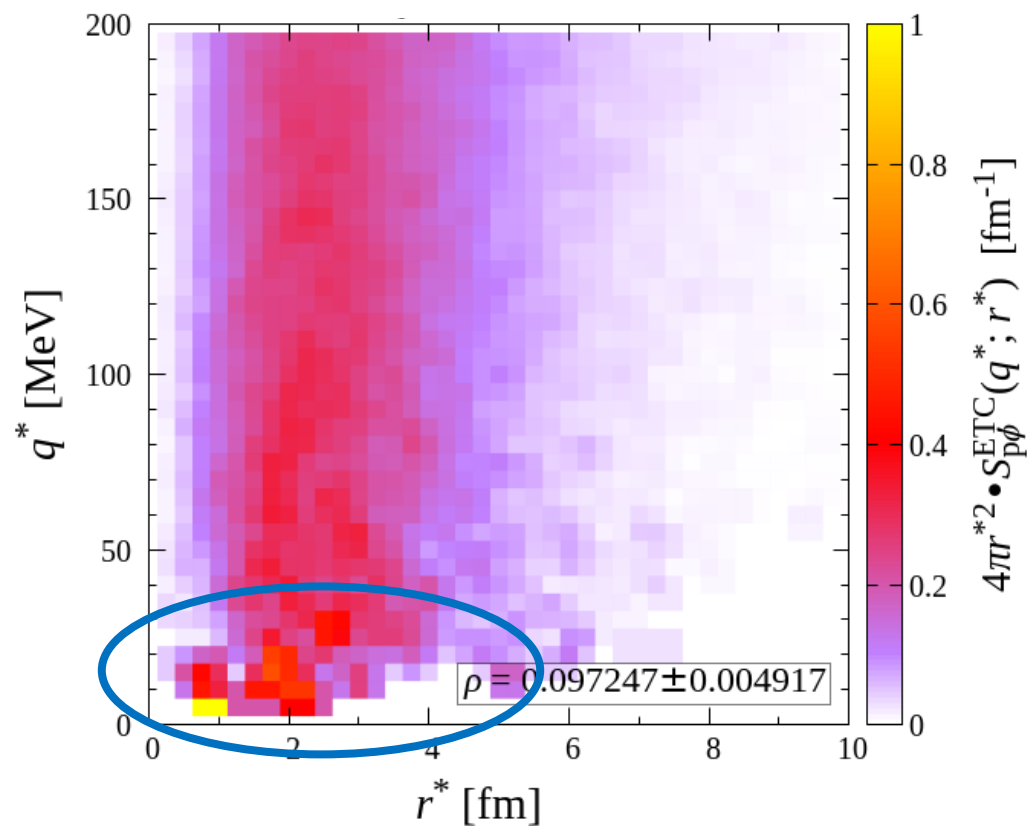
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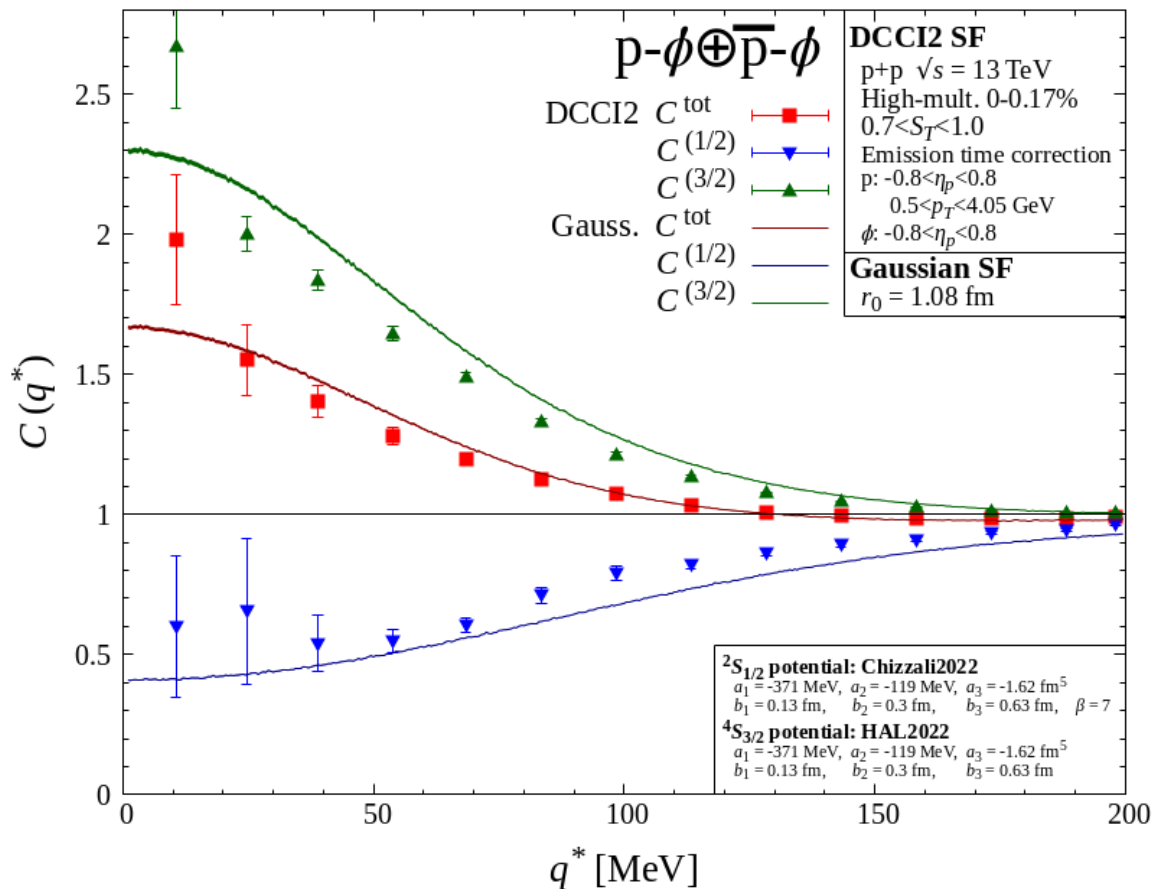
SF generally depends on q
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Noticeably smaller source

Green: $C^{(3/2)}$, Blue: $C^{(1/2)}$, Red: $C^{\text{tot}} = \frac{2}{3}C^{(3/2)} + \frac{1}{3}C^{(1/2)}$

Plots: **DCCI2 SF**, Lines: Gaussian SF ($r_0 = 1.08$ fm)



DCCI2 vs. Gaussian

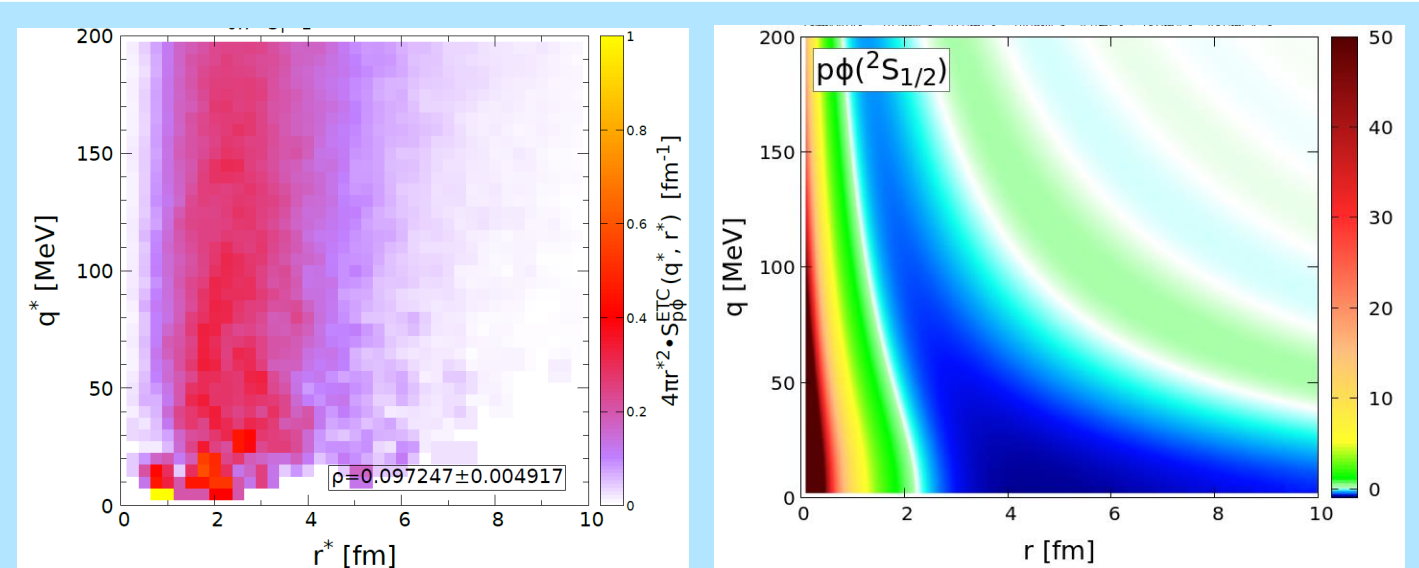
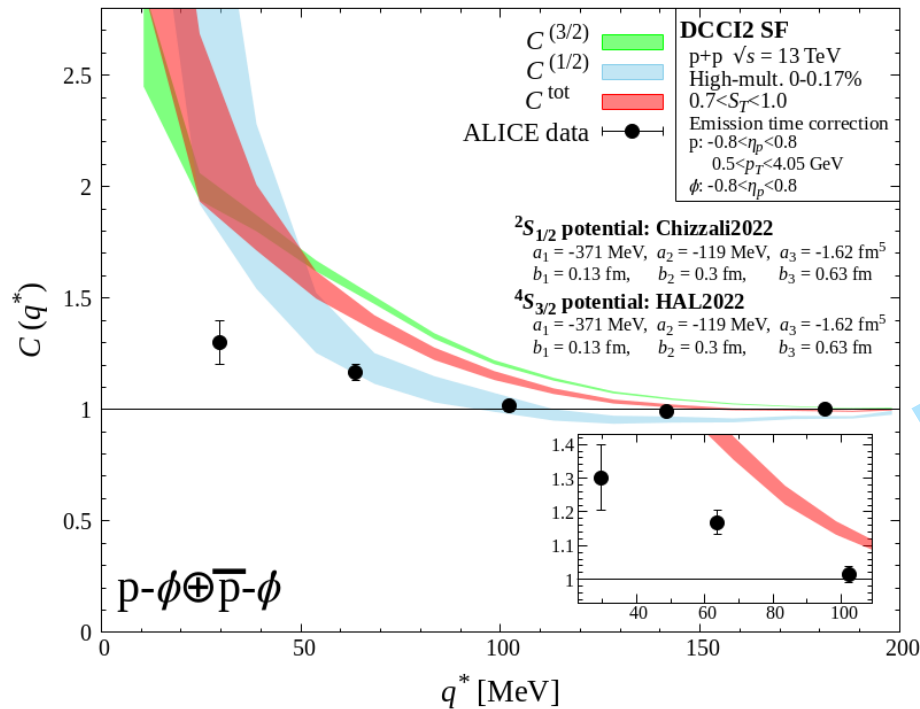
- Slightly weaker correlation
- Non-trivial behaviour at small q

Small but statistically significant effects of collision dynamics

$C^{(3/2)}$: Fixed, $C^{(1/2)}$: Change with β
Compare $C^{\text{tot}} = \frac{2}{3} C^{(3/2)} + \frac{1}{3} C^{(1/2)}$ with ALICE data

$C^{(3/2)}$: Fixed, $C^{(1/2)}$: Change with β
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$\beta = 6$



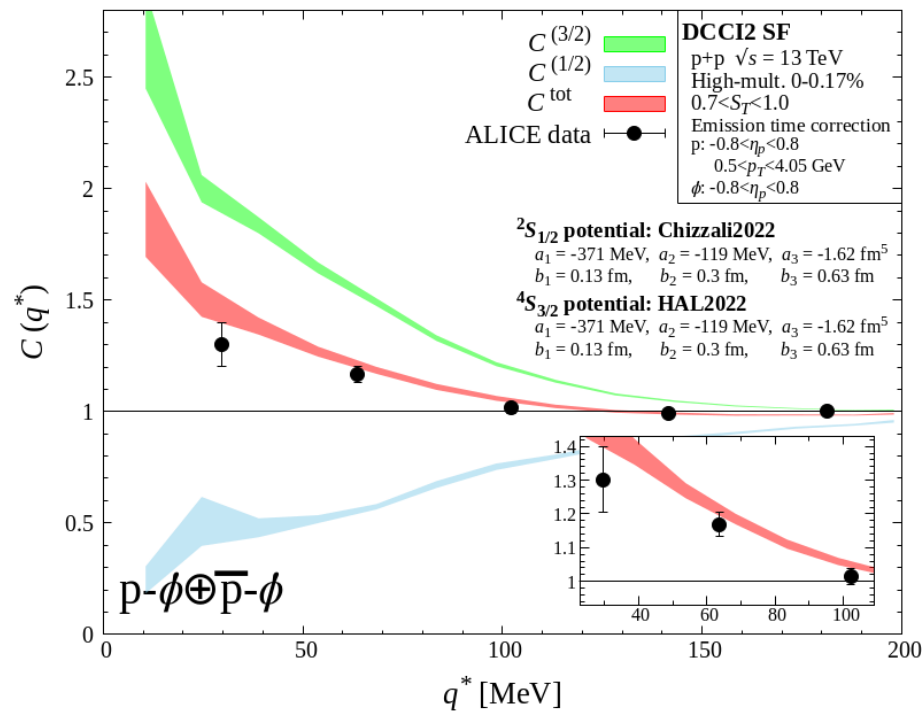
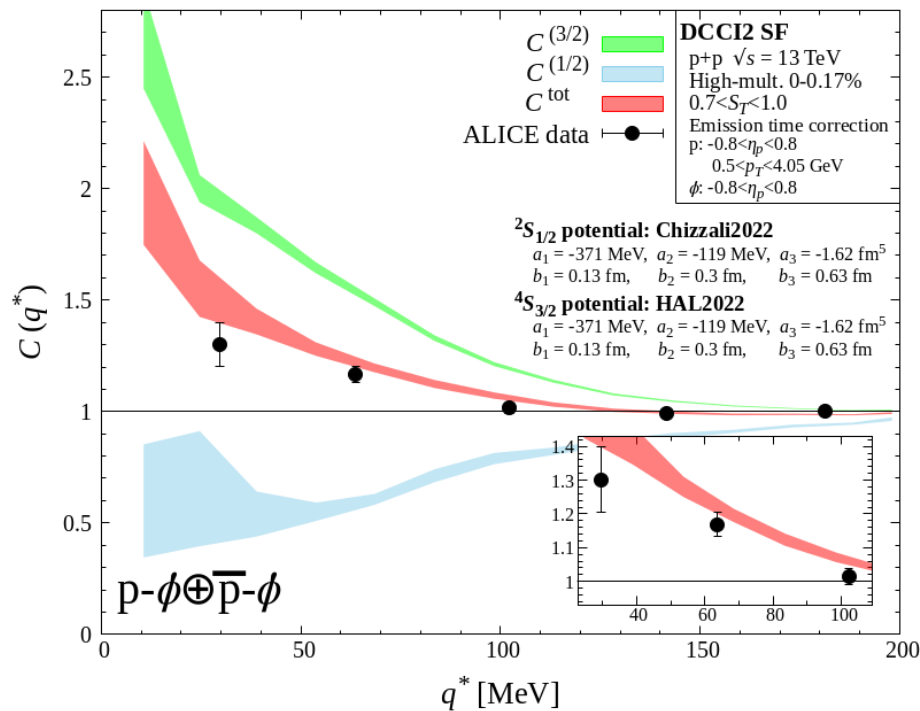
SF picks up strong positive region of WF

$C^{\text{tot}} > C^{\text{exp}}$

$C^{(3/2)}$: Fixed, $C^{(1/2)}$: Change with β
 Compare $C^{\text{tot}} = \frac{2}{3} C^{(3/2)} + \frac{1}{3} C^{(1/2)}$ with ALICE data

$\beta = 7$

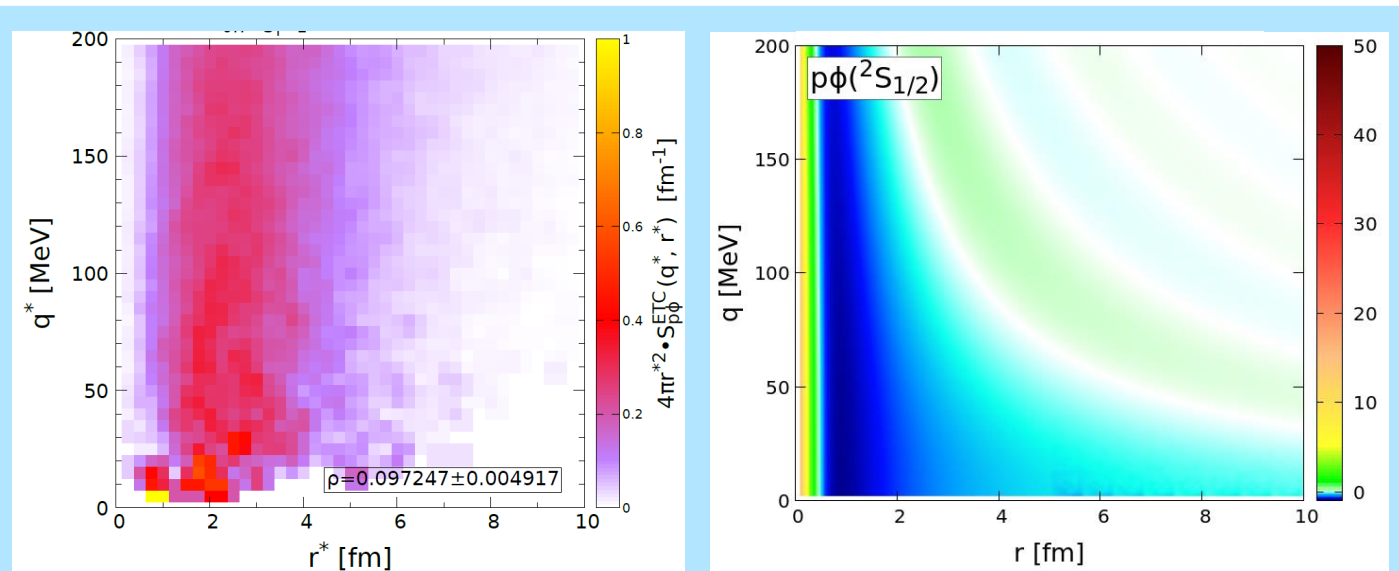
$\beta = 8$



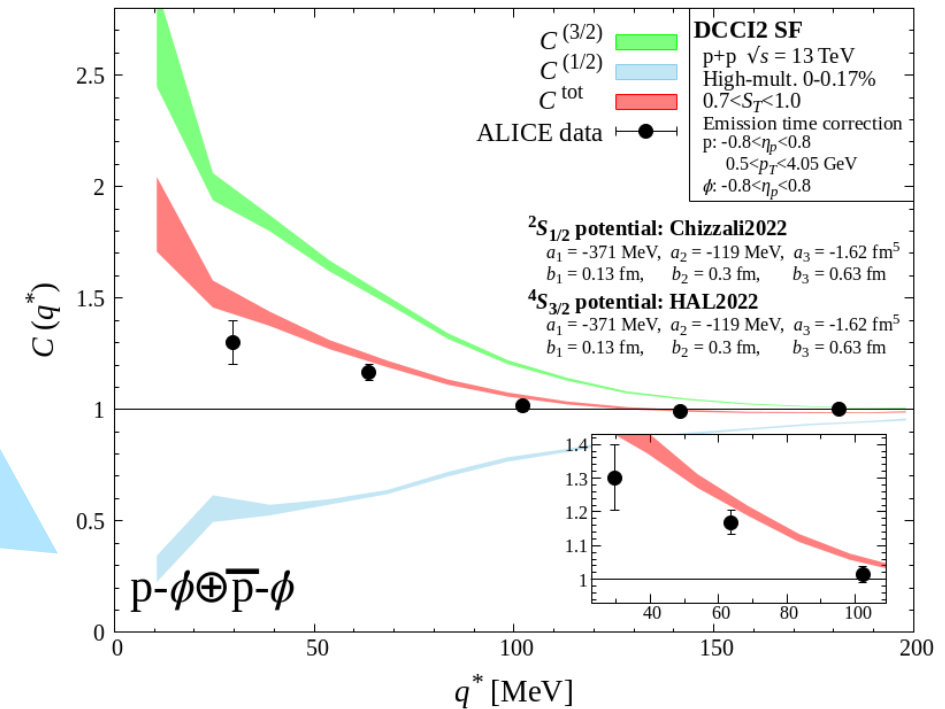
$$C^{\text{tot}} \approx C^{\text{exp}}$$

$C^{(3/2)}$: Fixed, $C^{(1/2)}$: Change with β
 Compare $C^{\text{tot}} = \frac{2}{3} C^{(3/2)} + \frac{1}{3} C^{(1/2)}$ with ALICE data

$\beta = 9$

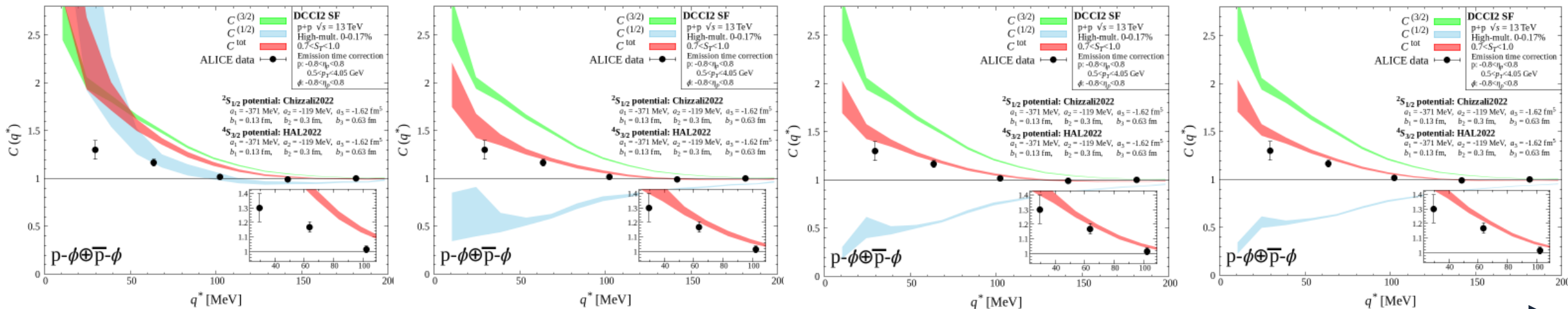


SF cannot pick up negative valley efficiently



$C^{\text{tot}} > C^{\text{exp}}$

$C^{(3/2)}$: Fixed, $C^{(1/2)}$: Change with β
 Compare $C^{\text{tot}} = \frac{2}{3} C^{(3/2)} + \frac{1}{3} C^{(1/2)}$ with ALICE data



$\beta = 6$

$a_0 = 4.54 \text{ fm}$
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 $E_B = 37.5 \text{ MeV}$

$\beta = 9$

$a_0 = 0.85 \text{ fm}$
 $E_B = 93.1 \text{ MeV}$

Overestimate

Agree within errors

Overestimate

p- ϕ femtoscopy using SF from a dynamical model (DCCI2)

Effects of collision dynamics

Small but statistically significant

- Slightly larger source size due to hadronic rescatterings
- SF depends on the relative momentum due to e.g. collectivity

Constraint on interaction

Indication of a bound state in $^2S_{1/2}$ channel ($E_B \cong 10-70$ MeV)

Importance of using SF that reflects collision dynamics

for precision hadron interaction study via femtoscopy

Backup



Assumptions

- **Chaotic source** ~ thermal equilibrium
- Same time approximation
- On-shell approximation
- **Closed system after emission** ~ In vacuum propagation

$$C(\mathbf{q}, \mathbf{P}) = \frac{\int d^4x_a d^4x_b S_a(\mathbf{p}_a; x_a) S_b(\mathbf{p}_b; x_b) |\varphi(\mathbf{q}; \mathbf{r})|^2}{\int d^4x_a S_a(\mathbf{p}_a; x_a) \int d^4x_b S_b(\mathbf{p}_b; x_b)}$$

Pair Rest Frame

Integrate out CM

$$C(\mathbf{q}) = \int d^3r S(\mathbf{q}; \mathbf{r}) |\varphi(\mathbf{q}; \mathbf{r})|^2$$

Spherical SF
 $S(q; r)$

Only s-wave scattering

$$\varphi(\mathbf{q}; \mathbf{r}) = \exp(i\mathbf{q} \cdot \mathbf{r}) - j_0(qr) + \varphi_0(q; r)$$

Plane-wave
Plane-wave
w.f.
(s-wave)
(s-wave)
(s-wave)

$$C(\mathbf{q}) = \int d^3r S(\mathbf{q}; \mathbf{r}) |\varphi(\mathbf{q}; \mathbf{r})|^2$$

$$= 1 + \int_0^\infty dr \underbrace{4\pi r^2 S(q; r)}_{\text{SF}} \underbrace{\{|\varphi_0(q; r)|^2 - [j_0(qr)]^2\}}_{\text{WF}}$$

SF

w/ Jacobian

WF

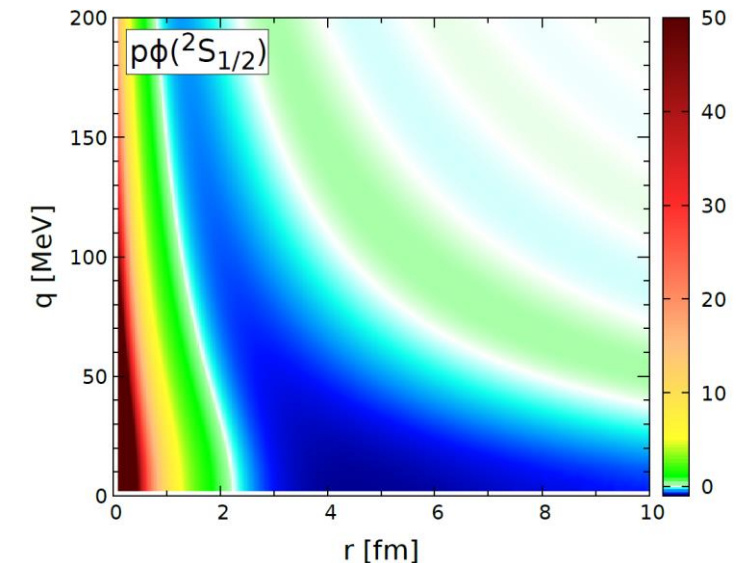
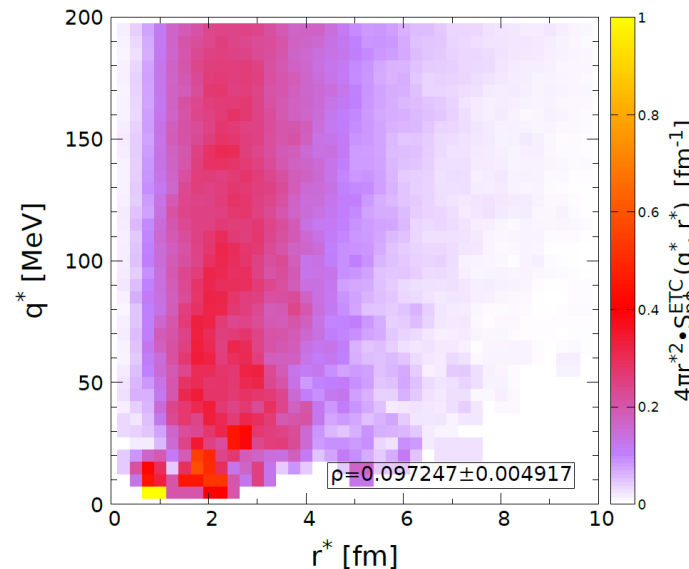
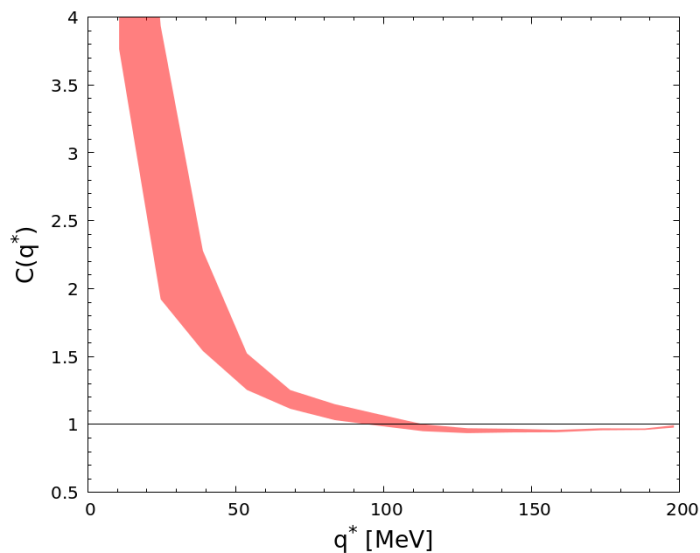
Increase/Decrease of w.f. by **FSI**

Interpretation of correlation

Considering only *s-wave scattering* together with spherical SF

$$C(q) = 1 + \int_0^{\infty} dr \quad \underbrace{4\pi r^2 S(q; r)}_{\substack{\text{SF} \\ \text{w/ Jacobian}}} \quad \underbrace{\{|\varphi_0(q; r)|^2 - [j_0(qr)]^2\}}_{\substack{\text{Weight Function} \\ \text{Increase/Decrease in w.f. by interaction}}}$$

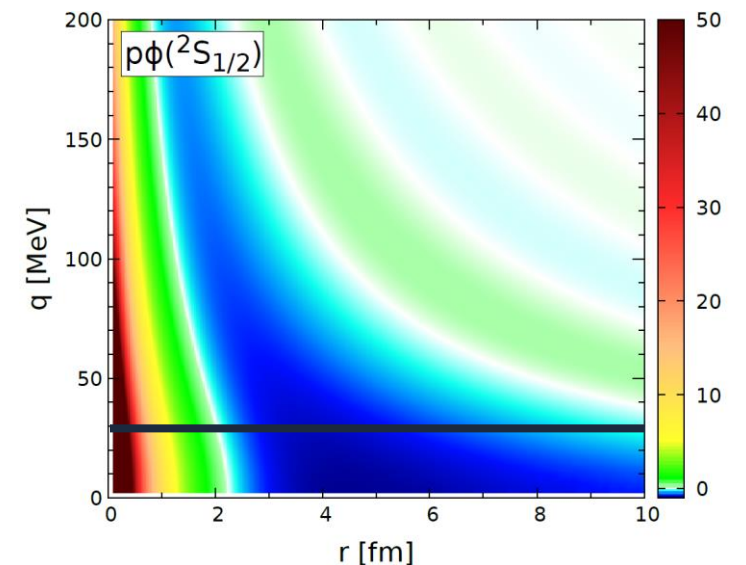
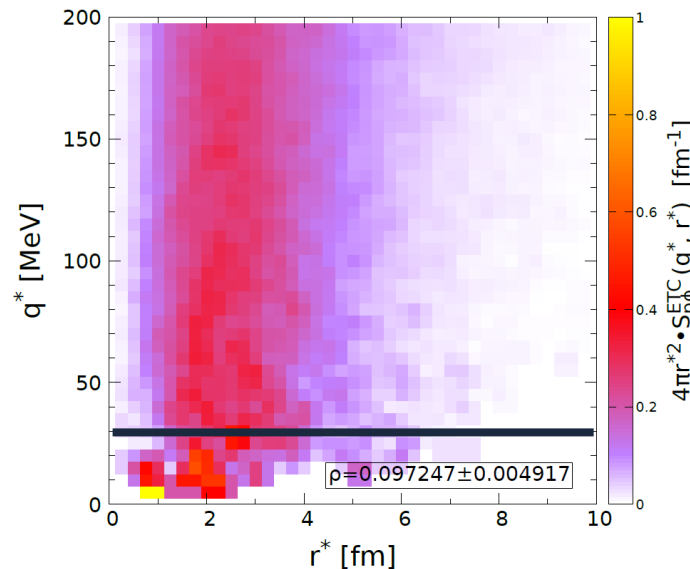
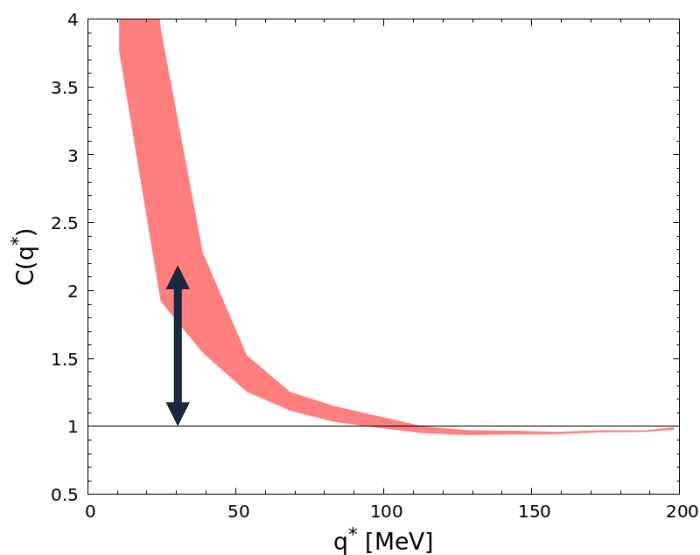
Deviation of $C(q)$ from unity = How much SF “picks up” WF



Considering only *s-wave scattering* together with *spherical SF*

$$C(q) = 1 + \int_0^\infty dr \underbrace{4\pi r^2 S(q; r)}_{\substack{\text{SF} \\ \text{w/ Jacobian}}} \underbrace{\{|\varphi_0(q; r)|^2 - [j_0(qr)]^2\}}_{\substack{\text{Weight Function} \\ \text{Increase/Decrease in w.f. by interaction}}}$$

Deviation of $C(q)$ from unity = How much **SF** “picks up” **WF**

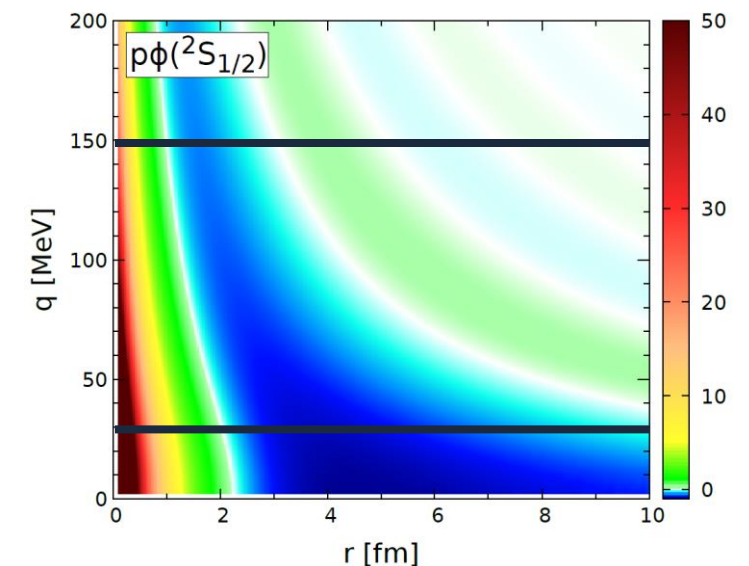
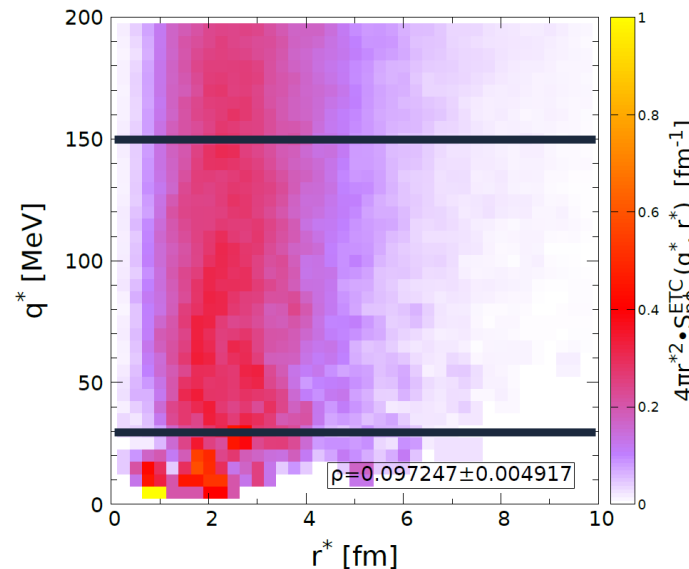
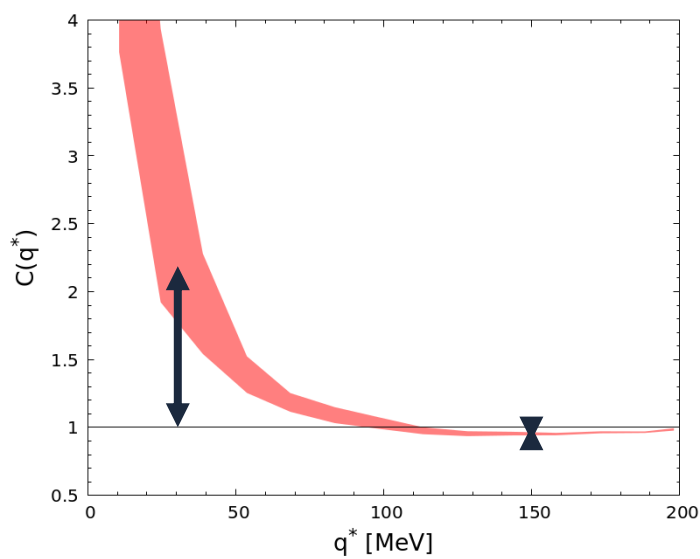


Interpretation of correlation

Considering only *s-wave scattering* together with *spherical SF*

$$C(q) = 1 + \int_0^{\infty} dr \quad \underbrace{4\pi r^2 S(q; r)}_{\substack{\text{SF} \\ \text{w/ Jacobian}}} \quad \underbrace{\{|\varphi_0(q; r)|^2 - [j_0(qr)]^2\}}_{\substack{\text{Weight Function} \\ \text{Increase/Decrease in w.f. by interaction}}}$$

Deviation of $C(q)$ from unity = How much SF “picks up” WF



R. Lednický and V. L. Lyuboshits, Yad. Fiz. **35**, 1316 (1981)

$$C(q) = 1 + \int_0^\infty dr 4\pi r^2 S(q; r) \{ |\varphi_0(q; r)|^2 - [j_0(qr)]^2 \}$$



Assumptions

- **Gaussian:** $S(q; r) \approx S(r) \propto \exp\left(-\frac{r^2}{4r_0^2}\right)$
- **Asymptotic w.f.** (+ correction)

$$C(q) = 1 + \frac{|f_0(q)|^2}{2r_0^2} F_3\left(\frac{r_{\text{eff}}}{r_0}\right) + \frac{2\text{Re}f_0(q)}{\sqrt{\pi}r_0} F_1(2qr_0) - \frac{\text{Im}f_0(q)}{r_0} F_2(2qr_0)$$

$$F_1, \dots, F_3: \text{Known functions, } f_0(q) = \frac{1}{q \cot \delta_0(q) - iq} \approx \frac{1}{-\frac{1}{a_0} + \frac{1}{2} r_{\text{eff}} q^2 - iq}$$

CF becomes a function of a_0 , r_{eff} , and r_0

Focusing on low- q region with chaotic source and closed system assumptions
→ **Steady-state Schrödinger eq. w/ central force**

Partial-wave expansion

$$\varphi(\mathbf{q}; \mathbf{r}) = \sum_{l=0}^{\infty} (2l + 1) i^l \varphi_l(q; r) P_l(\cos\theta)$$

For each $^{2S+1}L_J$ channel,

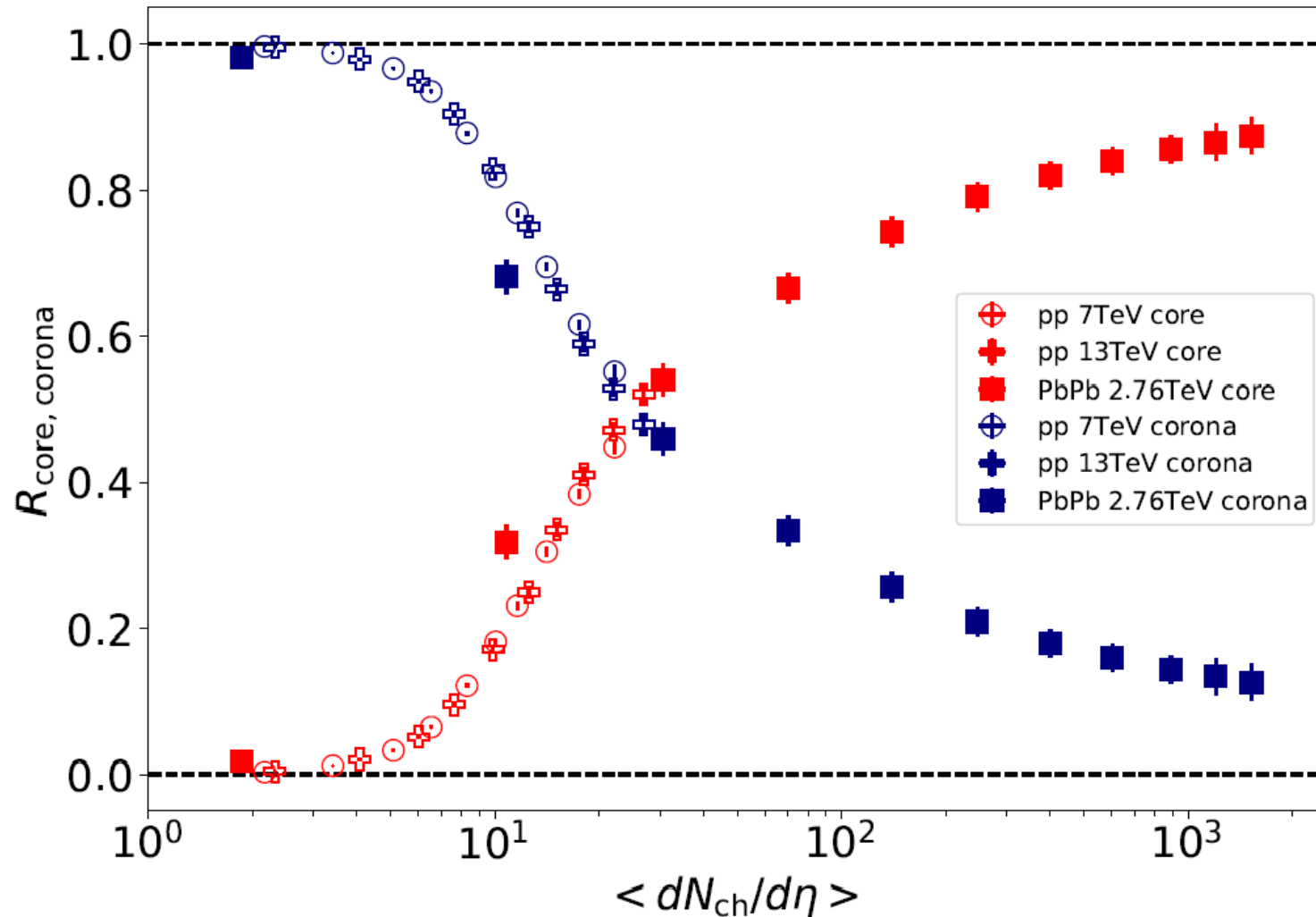
$$\left[-\frac{1}{2\mu} \frac{d^2}{dr^2} + V(r) + \frac{1}{2\mu} \frac{l(l+1)}{r^2} \right] u_l(q; r) = \frac{q^2}{2\mu} u_l(q; r)$$

$$u_l := r \varphi_l$$

Reduced mass:

$$\mu = \frac{m_a m_b}{m_a + m_b}$$

Y. Kanakubo, Y. Tachibana, and T. Hirano, PRC **105**, 024905 (2022)

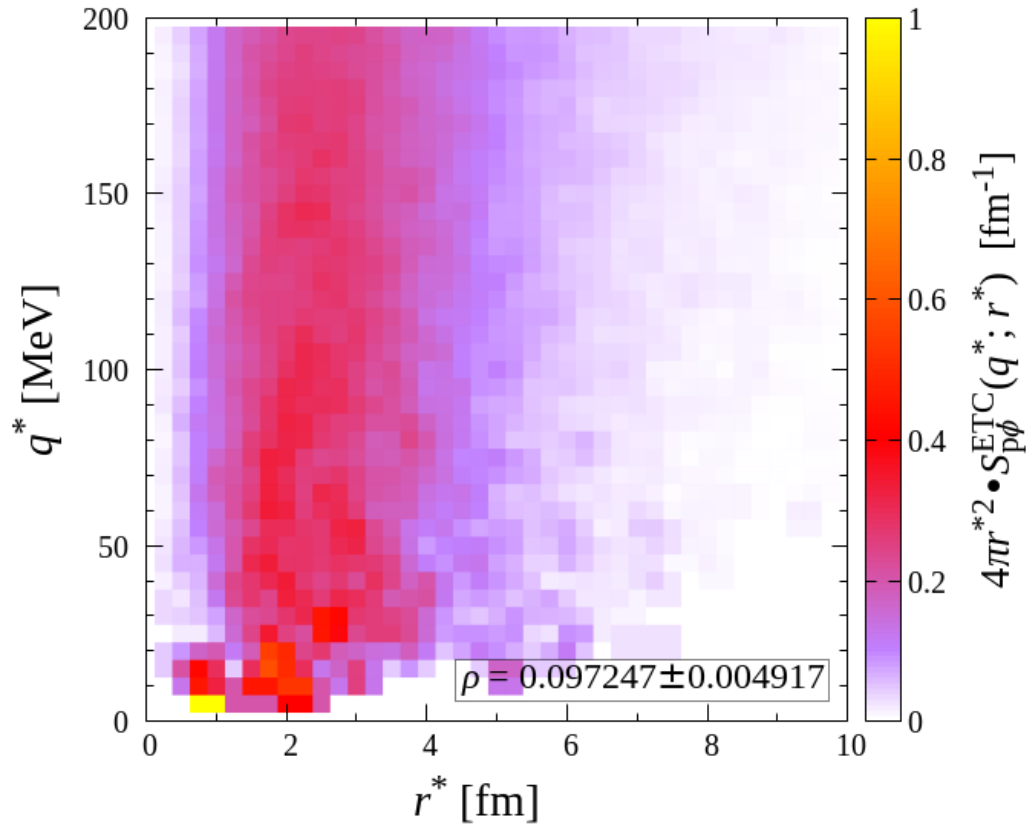


According to DCCI2

$$R_{\text{core}} \sim 0.5$$

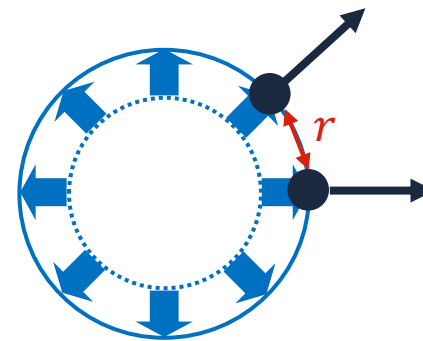
in high-multiplicity
p+p collisions
at $\sqrt{s} = 13$ TeV

q -differential SF



Positive $q-r$ correlation

Collectivity of generated matter



Close in position space



Close in momentum space

Relative w.f. = Weighted average of w.f. in each $^{2S+1}L_J$ channel

$$|\varphi|^2 = \sum_{\text{states}(S,L,J)} \omega_{(S,L,J)} |\varphi^{(S,L,J)}|^2$$

$$\omega_{(S,L,J)} = \frac{2S + 1}{(2s_a + 1)(2s_b + 1)} \frac{2J + 1}{(2L + 1)(2S + 1)}$$

KP formula

Spin-independent SF ← chaotic source

Spin-averaged CF

$$C^{\text{tot}}(\mathbf{q}) = \sum_{\text{states}(S,L,J)} \omega_{(S,L,J)} C^{(S,L,J)}(\mathbf{q})$$



Comparable
w/ exp.

Weak

Attractive potential w/ a bound state

Strong

$$\beta = 6$$

$$a_0 = 4.54 \text{ fm}$$

$$E_B = 2.3 \text{ MeV}$$

$$\beta = 7$$

$$a_0 = 1.99 \text{ fm}$$

$$E_B = 13.3 \text{ MeV}$$

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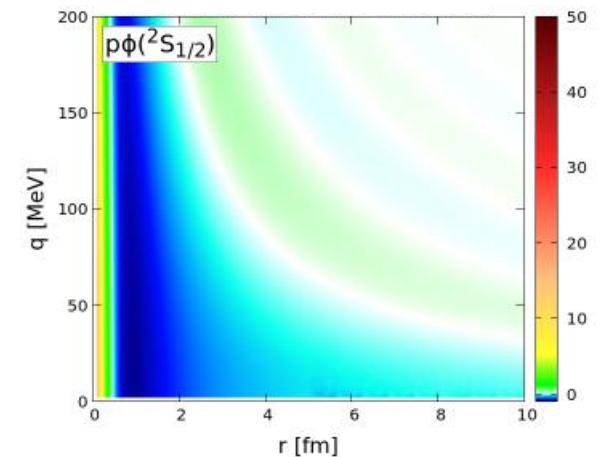
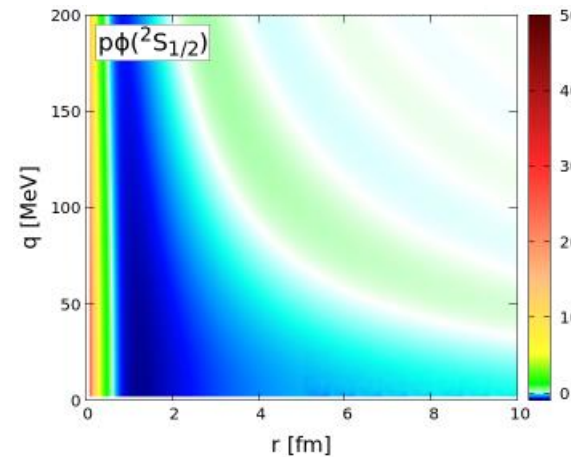
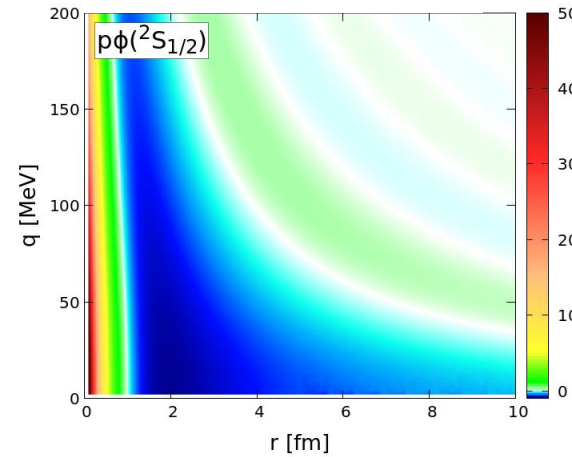
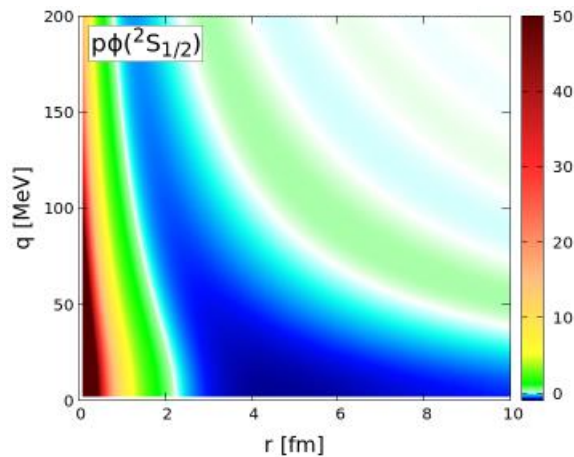
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The negative valley moves towards the small r region

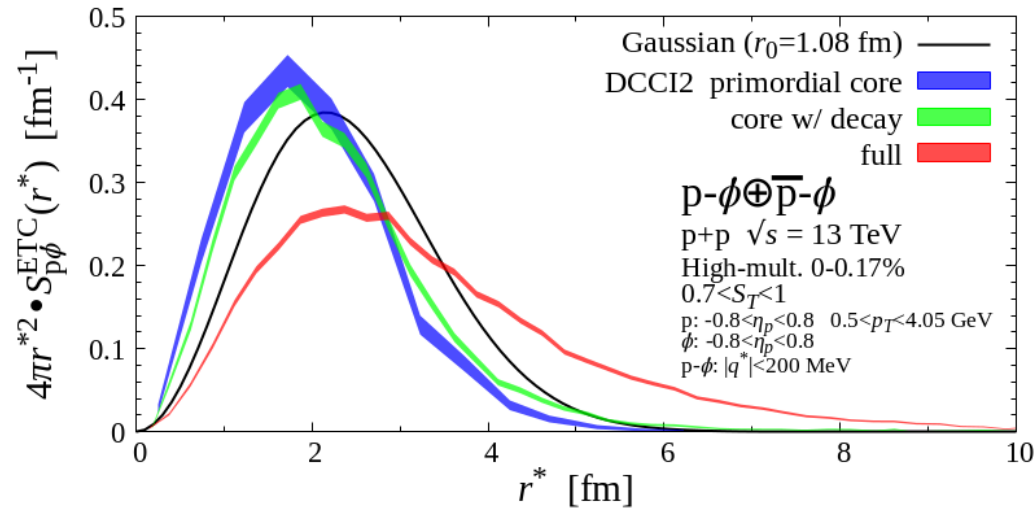
Primordial core
Direct p and ϕ
from core (hydro)

+ Decay

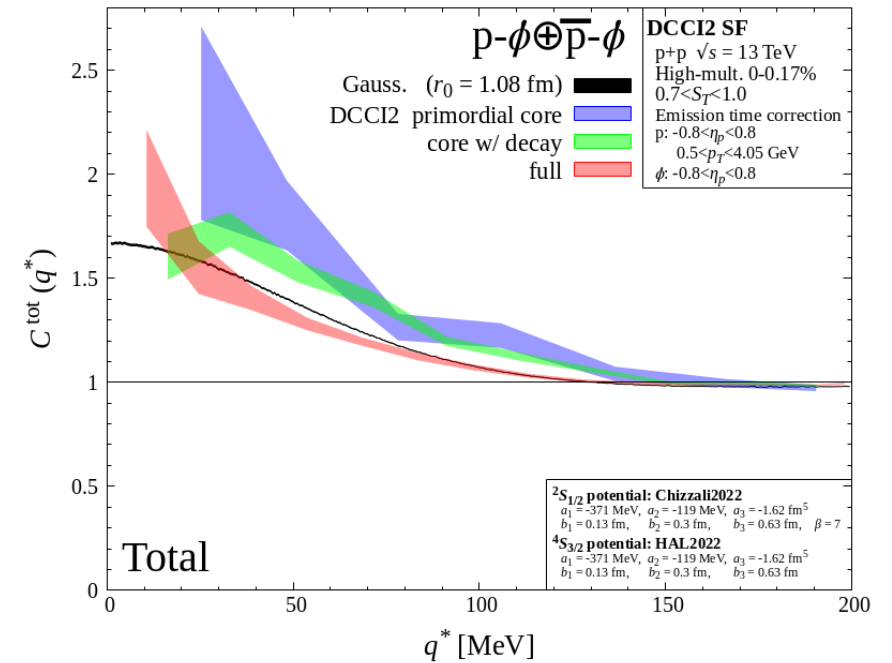
+ Corona + Decay + Rescatterings

Core w/ decay
Effects of feed-down

Full: Comparable with experiment
Effects of feed-down & rescatterings (& corona)



- ~Gaussian w/ smaller source size
- Decay → A little long-tail
- Rescatterings
→ Long-tail and larger source size



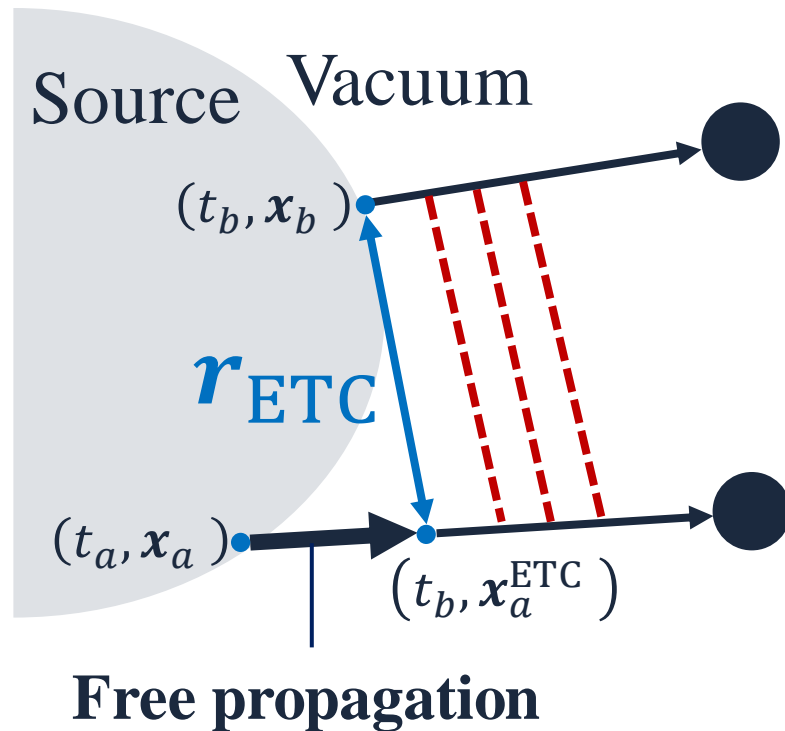
Larger effects of **rescatterings**
than **resonance decay** on SF & CF

Problem

Dynamical model \rightarrow Emission time difference: $S(\mathbf{q}; r^0 \neq 0, \mathbf{r})$

Violates **equal-time approximation** in KP formula

Free propagation until the other's emission



$$S(\mathbf{q}; r^0 \neq 0, \mathbf{r})$$

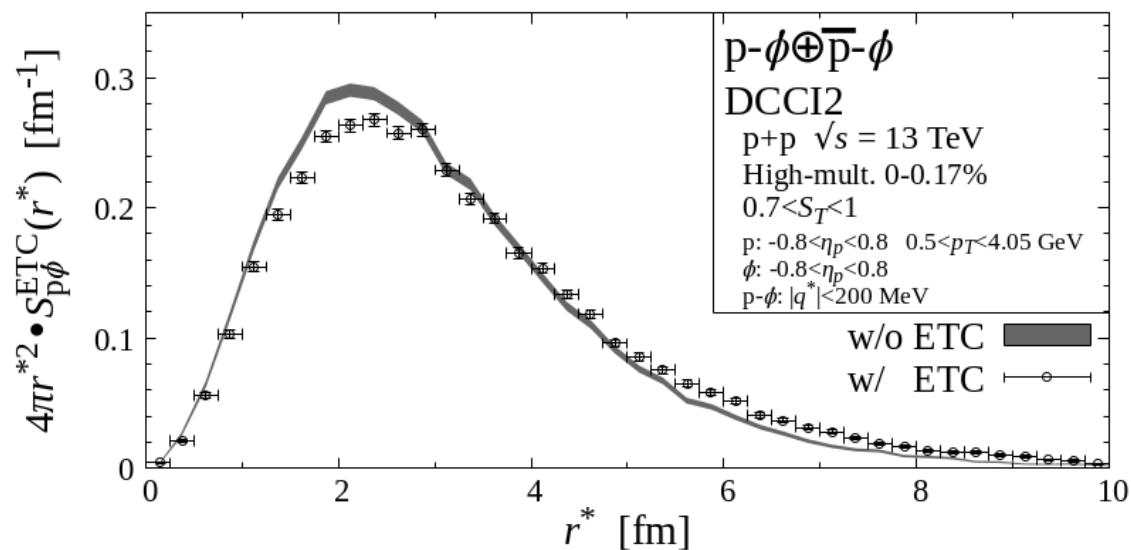
ETC \rightarrow

$$S^{\text{ETC}}(\mathbf{q}; \mathbf{r}_{\text{ETC}}) \delta(r^0)$$

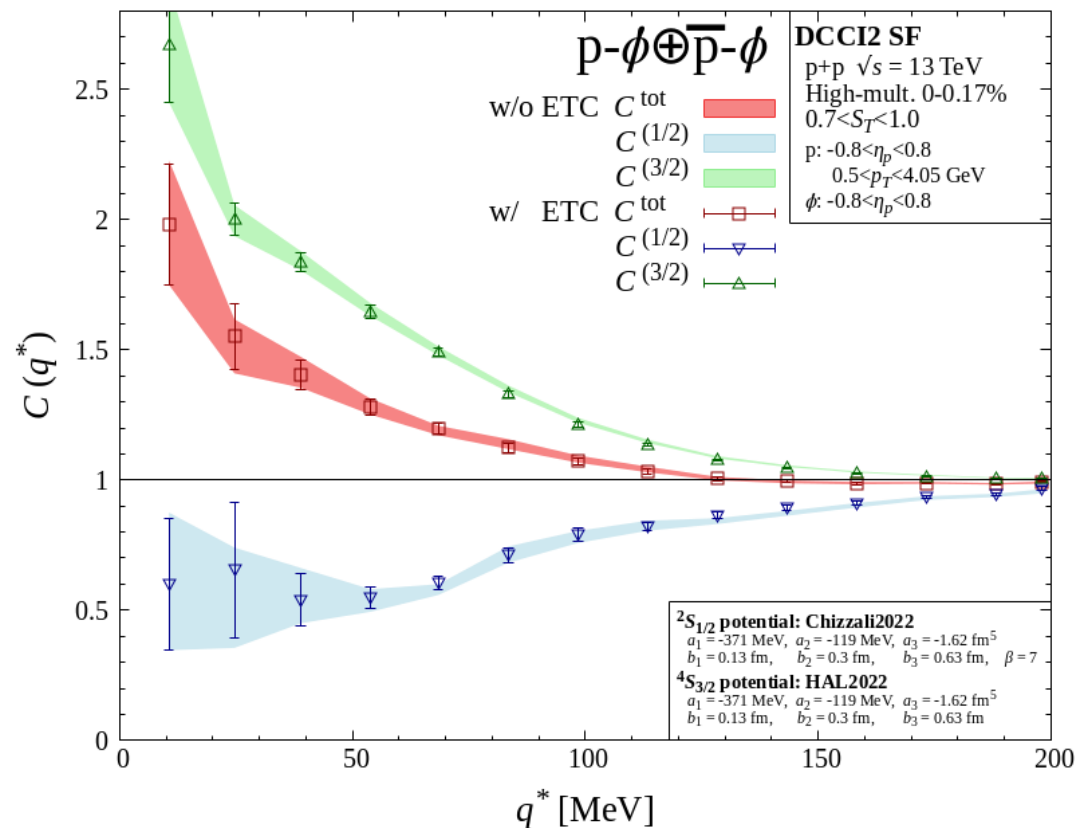
$$\mathbf{r}_{\text{ETC}} = \mathbf{r} + \frac{\mathbf{p}_a}{E_a} (t_b - t_a) \theta(t_b - t_a) - \frac{\mathbf{p}_b}{E_b} (t_a - t_b) \theta(t_a - t_b)$$

Correction

Plots: w/ ETC, Bands: w/o ETC



ETC slightly enlarge source size



No statistically significant effects on CF in this particular case