Bayesian constraints on the high density QCD EoS from Heavy-ion collision data

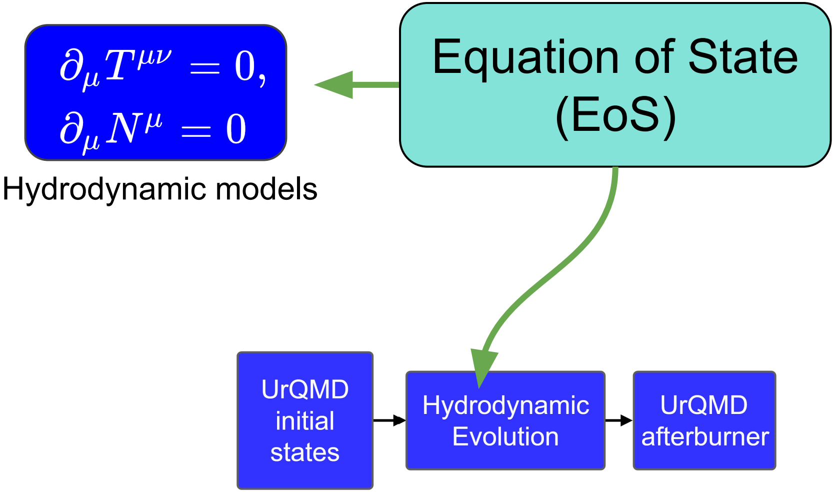
Phys Rev Lett 131 (20), 202303

Manjunath Omana Kuttan, Jan Steinheimer, Kai Zhou, Horst Stöcker

FIAS Frankfurt Institute for Advanced Studies

KISS

Equation of State in heavy-ion collisions



Hydrodynamic models

Hybrid models

Justified only at very high beam energies !

- $\sqrt{s_{\rm NN}} \lesssim 15$GeV: large interpenetration time
- No clear separation of compression and expansion phases
- Dynamics in the initial non-equilibrium phase will influence the Observables

How can we consistently describe the entire evolution by a single EoS?

UrQMD with potentials

Eur. Phys. J. C **82**, 427 (2022),
Eur. Phys. J. C **82**, 911 (2022)

UrQMD cascade

- Microscopic non-equilibrium description
  - hadrons on classical trajectories
  - stochastic binary scatterings
  - color string formation
  - resonance excitation and decays
- interactions based on scattering cross sections
- default setup
  - effective EoS: Hadron Resonance Gas
- Non-trivial interactions can be added through QMD approach

Density dependent EoS in UrQMD

- density dependent potential enters QMD equations:

$$\dot{\mathbf{r}}_i = \frac{\partial \mathbf{H}}{\partial \mathbf{p}_i}, \quad \dot{\mathbf{p}}_i = -\frac{\partial \mathbf{H}}{\partial \mathbf{r}_i}.$$

  - density dependent potential energy term
- potential energy $V(n_B)$ is related to the pressure as:

$$P(n_B) = P_{\rm id}(n_B) + \int_0^{n_B} n' \frac{\partial U(n')}{\partial n'} dn', \; U(n_B) = \frac{\partial (n_B \cdot V(n_B))}{\partial n_B}$$

$P_{id}(n_B) \Rightarrow$ pressure of an ideal Fermi gas of baryons

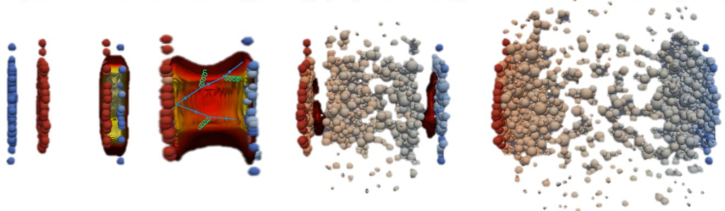
$U(n_B) \Rightarrow$ single particle potential,

Constraining the potential energy $\Longrightarrow$ constraining the EoS

Recovering the physics in final state data

Transport as discovery tool:

•The goal of constructing a transport model is to test its underlying assumptions via a comparison to data and gaining physics insight from such a comparison



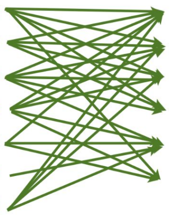
physics of interest ← transport model → final state data

Transport overview talk , S. A. Bass

Model to Data Comparison: Parametric Nightmare

Transport Models have multiple parameters encoding its underlying physics that are sensitive to experimental data

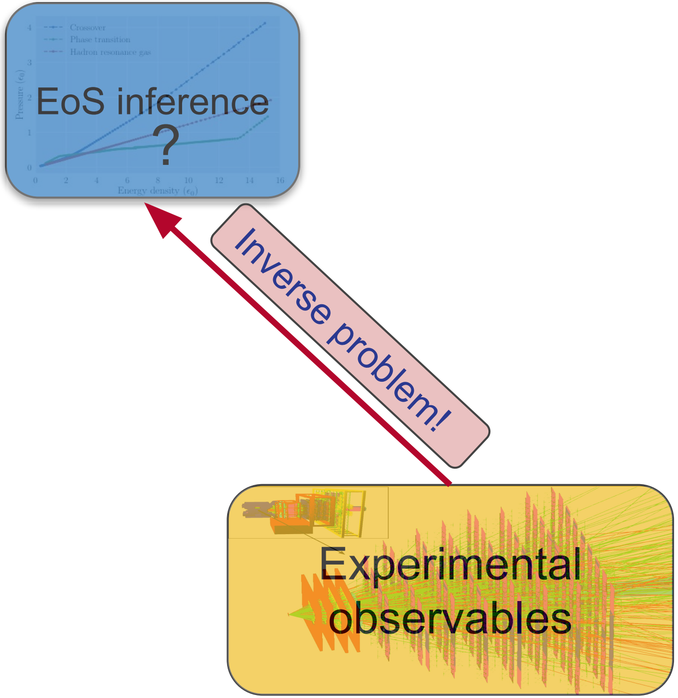
Model Parameter:
- eqn. of state
- shear viscosity
- initial state
- pre-equilibrium dynamics
- thermalization time
- quark/hadron chemistry
- particlization/freeze-out



experimental data:
- π/K/P spectra
- yields vs. centrality & beam
- elliptic flow
- HBT
- charge correlations & BFs
- density correlations

- We now have a model that consistently uses a single EoS for the entire evolution!
- Employ modern statistical tools and learn what the data tell about the EoS!

Bayesian approach to constrain the EoS



θ : EoS parameters $\quad D$: data

Probability of observed data given the EoS

Posterior $\propto$ likelihood $\times$ prior

$$P(\boldsymbol{\theta}|\mathbf{D}) \propto P(\mathbf{D}|\boldsymbol{\theta})P(\boldsymbol{\theta})$$

Updated belief about EoS (after considering the data)

Initial belief about EoS (before seeing the data)

Parameterizing the EoS

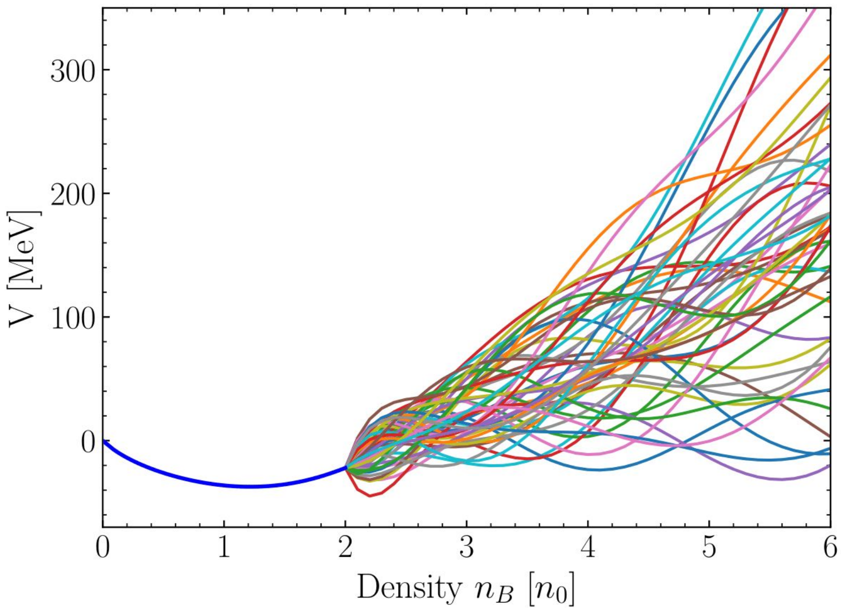
$$P(\boldsymbol{\theta}|\mathbf{D}) \propto P(\mathbf{D}|\boldsymbol{\theta})P(\boldsymbol{\theta})$$

CMF | $V(n_B) = \sum_{i=1}^{7} \theta_i \left(\frac{n_B}{n_0} - 2\right)^i + h$

$0 - 2n_0$ | $> 2n_0$

- $< 2n_0$, CMF model-fit A. Motornenko et al., PRC 103.5 (2021)
  - reproduces nuclear matter properties
  - *E_0~-15.2 MeV, K_0~267 MeV, S_0~31.9 MeV*
- $>2n_0$, 7^{th} degree polynomial
  - h=-22.07 *MeV* to match CMF at $2n_0$
- reasonable constraints exist upto $2n_0$
  - flow data,
  - incompressibility data
  - bayesian analysis

P. Danielewicz, Et al Science 298, 1592 (2002),
H. Kruse, Et al. Phys. Rev. Lett. 54, 289 (1985),
Y. Wang, Et al. PLB 778, 207 (2018),
S. Huth et al., Nature 606, 276 (2022)

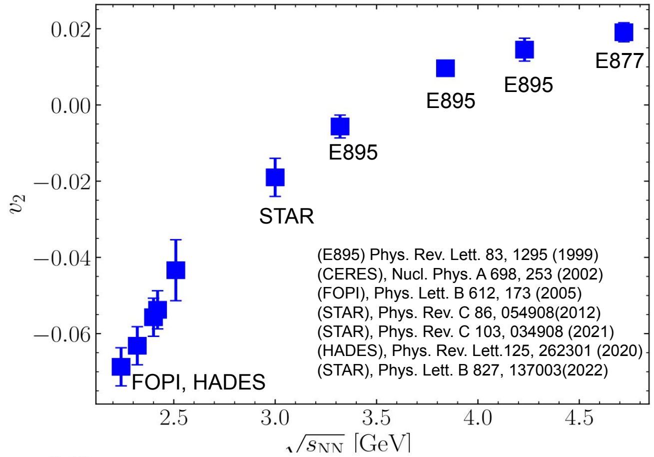


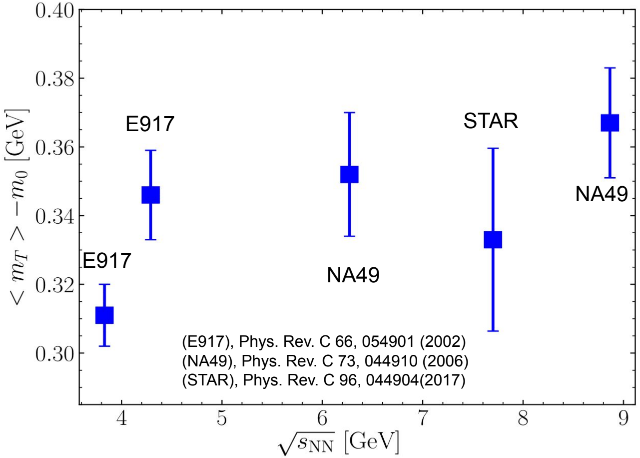
Potentials generated using the polynomial parameterization

We constrain $\boldsymbol{\theta} = \{\theta_1, \theta_2, ..., \theta_7\}$

* a c_s^2 parameterization used in D. Oliinychenko, Et al *PRC* 108 (2023) 3, 034908

Data to constrain the EoS $P(\boldsymbol{\theta}|\mathbf{D}) \propto P(\mathbf{D}|\boldsymbol{\theta})P(\boldsymbol{\theta})$

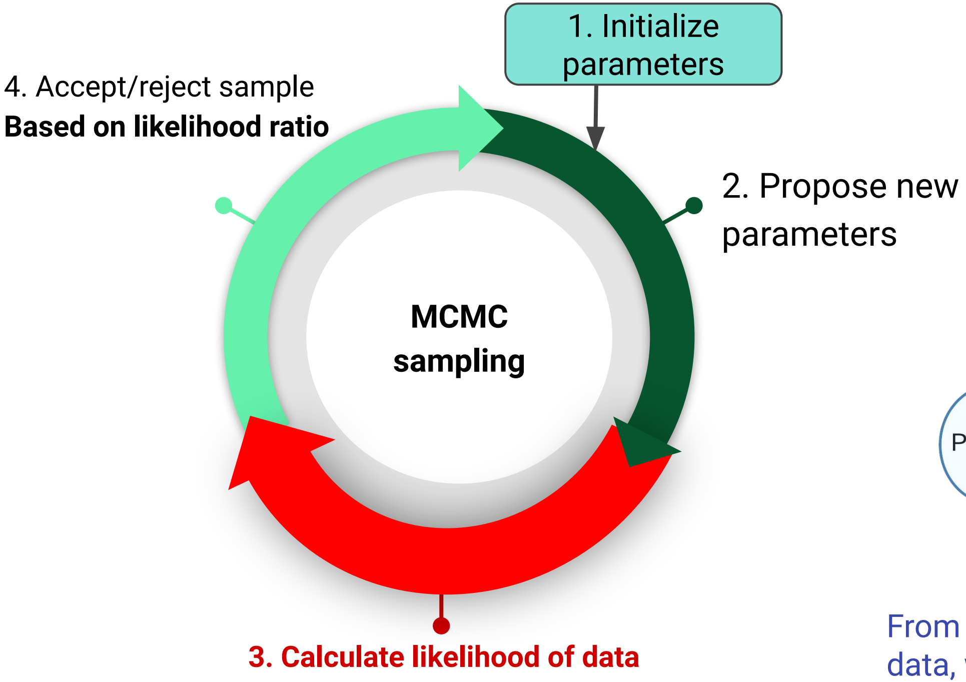




- Proton observables (mid rapidity)
- Elliptic flow : 10 data points
  - E895, CERES, FOPI, STAR, HADES
  - Mid-central collisions
- Transverse kinetic energy: 5 data points
  - E917, NA49, STAR
  - Central collisions

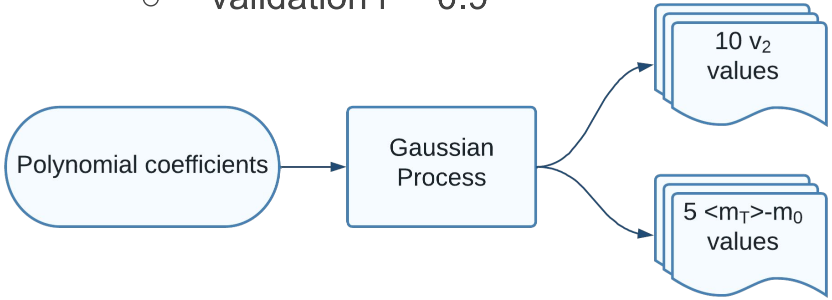
The data $\mathbf{D} = \{v_2^{exp}, \langle m_T \rangle^{exp} - m_0\}$ is used to constrain the parameters $\boldsymbol{\theta} = \{\theta_1, \theta_2, ..., \theta_7\}$

Calculating the likelihood: the bottleneck $P(\boldsymbol{\theta}|\mathbf{D}) \propto P(\mathbf{D}|\boldsymbol{\theta})P(\boldsymbol{\theta})$



- **Expensive!**
- UrQMD ~ 80 s/event
- Each sample~125K events* 80 s/event =**~2700 hrs**

- Gaussian Process (GP) models: fast emulators
  - trained on 200 random EoSs
  - tested on 50 EoSs
  - validation r^2 ~0.9



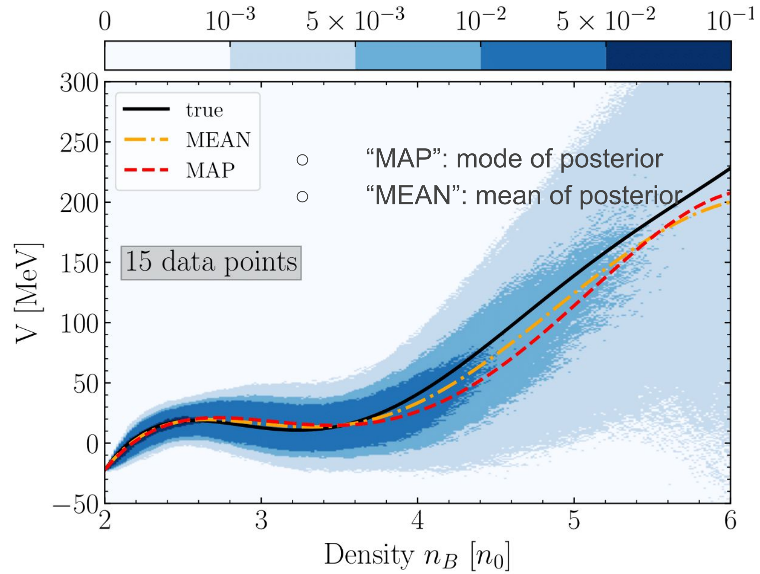
From GP predictions and experimental data, we can calculate the likelihood

$$\ln P(\mathbf{D}|\boldsymbol{\theta}) = -\frac{1}{2}\sum_i \left[\frac{(x_i^{\boldsymbol{\theta}} - d_i)^2}{\sigma_i^2} + (\ln(2\pi\sigma_i^2))\right]$$

Includes uncertainty from experiment & GP model

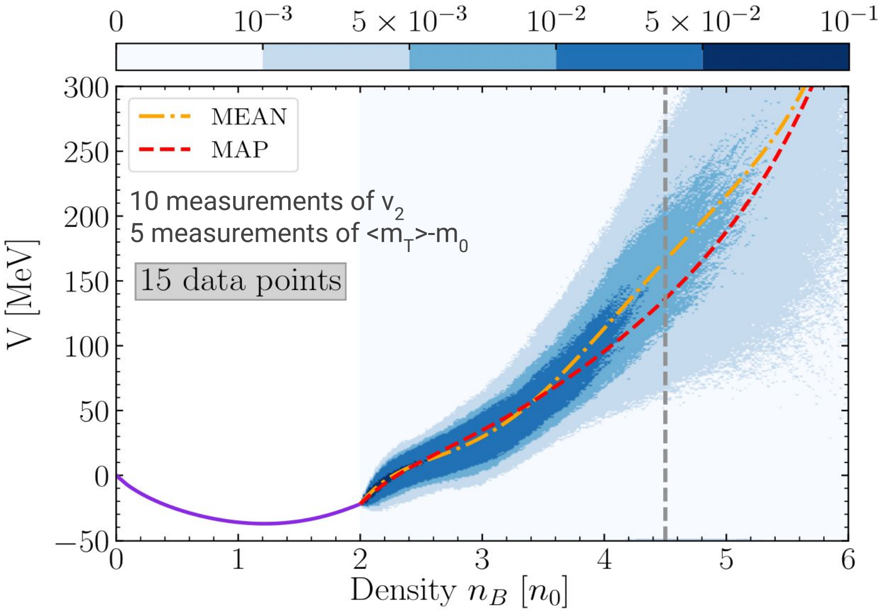
Testing the pipeline

1. “ground truth” :a random EoS
2. “data” : v2, <m_T>-m_0 from UrQMD
3. Construct posterior
   - Use error bars of experiment
4. Compare to “ground-truth”



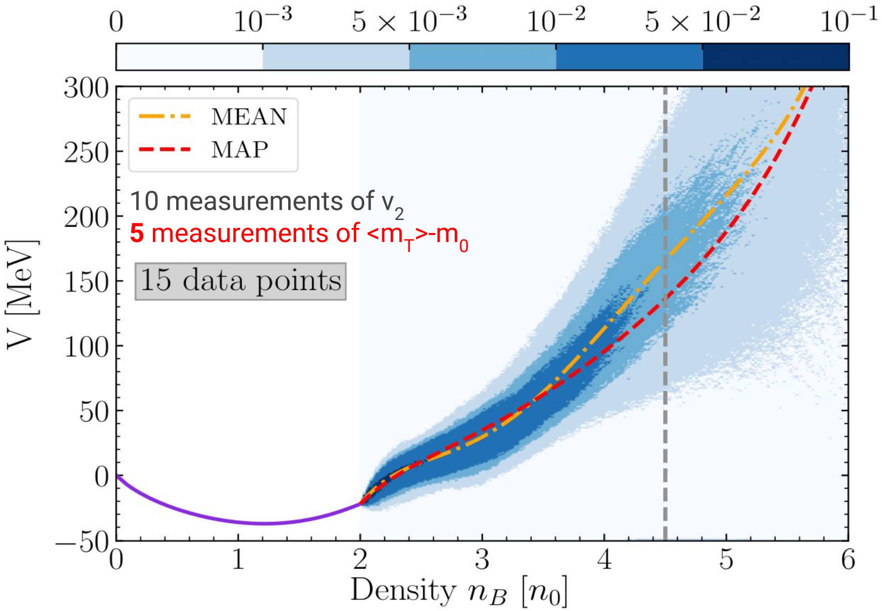
- Tight constraints up to $4n_0$
  - mean closely follows “ground-truth”
- MEAN and MAP closely follows “ground-truth” upto $6n_0$

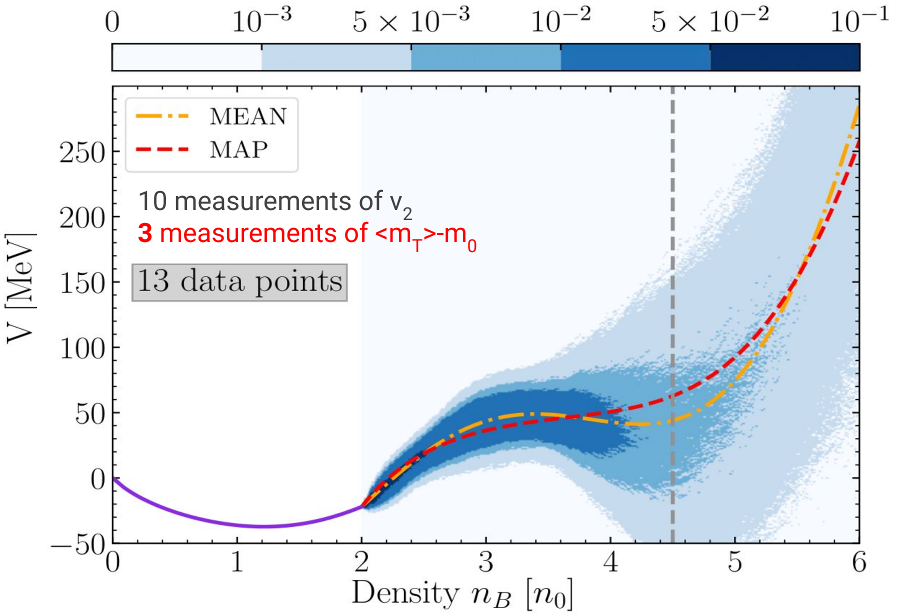
Results from experimental data



- Tight constraints upto $4n_0$
- MEAN, MAP suggests stiff EoS
- No phase transition

Results from experimental data

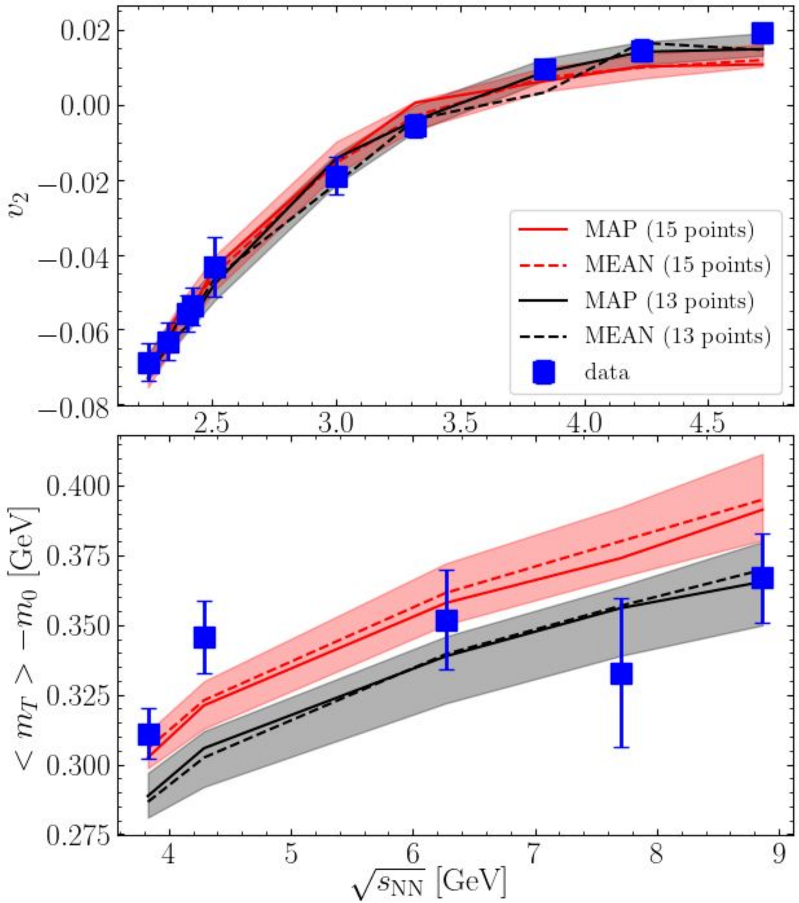




- Tight constraints upto $4n_0$
- MEAN, MAP suggests stiff EoS
- No phase transition

- <m_T>-m_0 at 3.83, 4.29 GeV not used
- softening at 3- $5n_0$
- phase transition ?
- >$3n_0$: strong dependence to choice of observables

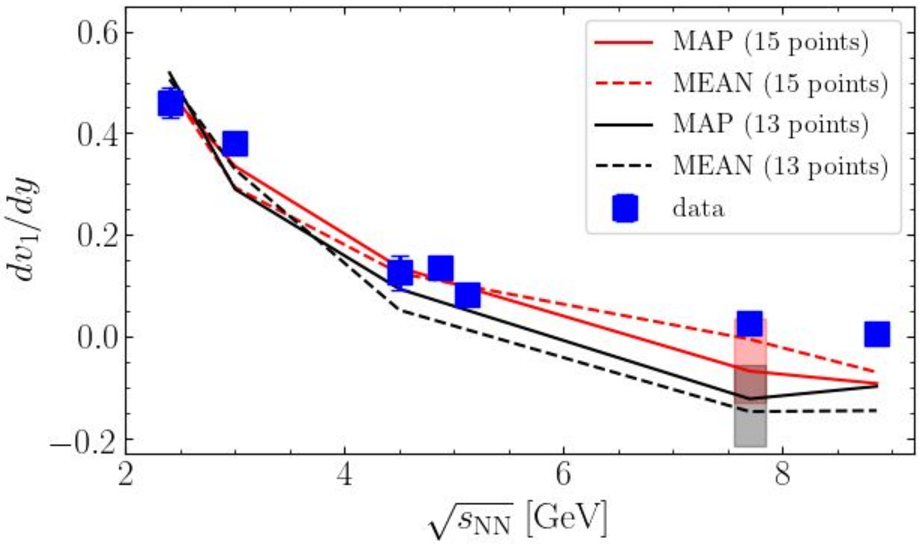
The extracted EoSs

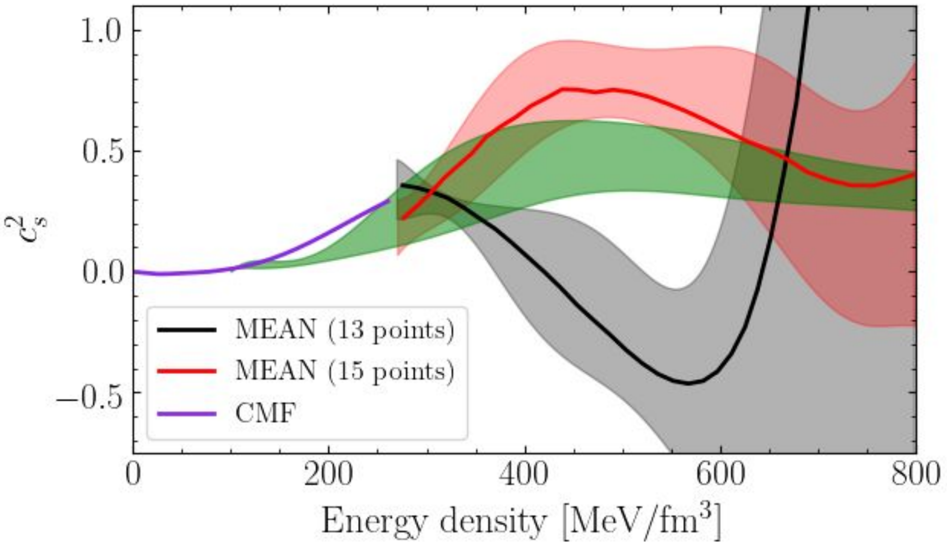


- ❑ better v_2 predictions at high energies (without 2 data points)
- ❑ but also results in lower $<m_T>-m_0$
- ❑ large $<m_T>-m_0$ values for the stiff EoS (extracted using all data points)
- ❑ possible tension in data at ~4 GeV!

Measurement uncertainty? or model limitation?

Further tests





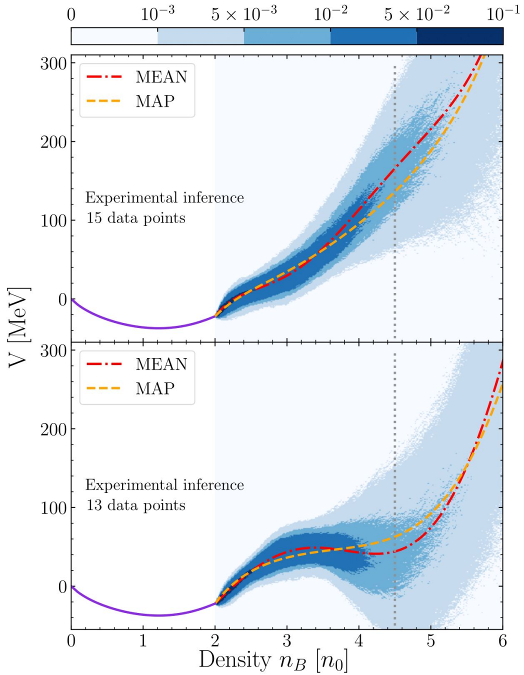
- V_1 data not used in inference
- 15 data points provide best fit to v_1 data
  - stiff EoS

- 15 points, predicts a stiff EoS
  - consistent with astrophysical constraints
  - broad peak structure
- 13 points, drastic drop in c_s^2
  - first order phase transition

Outlook

- ❑ Explored bayesian constraints on the high density QCD EoS
- ❑ observables provide tight constraints upto $3n_0$
- ❑ strong dependence on choice of observables for > $3n_0$
  - ❑ tension in data at ~4 GeV
- ❑ measurement uncertainty or model limitation?

Phys Rev Lett 131 (20), 202303

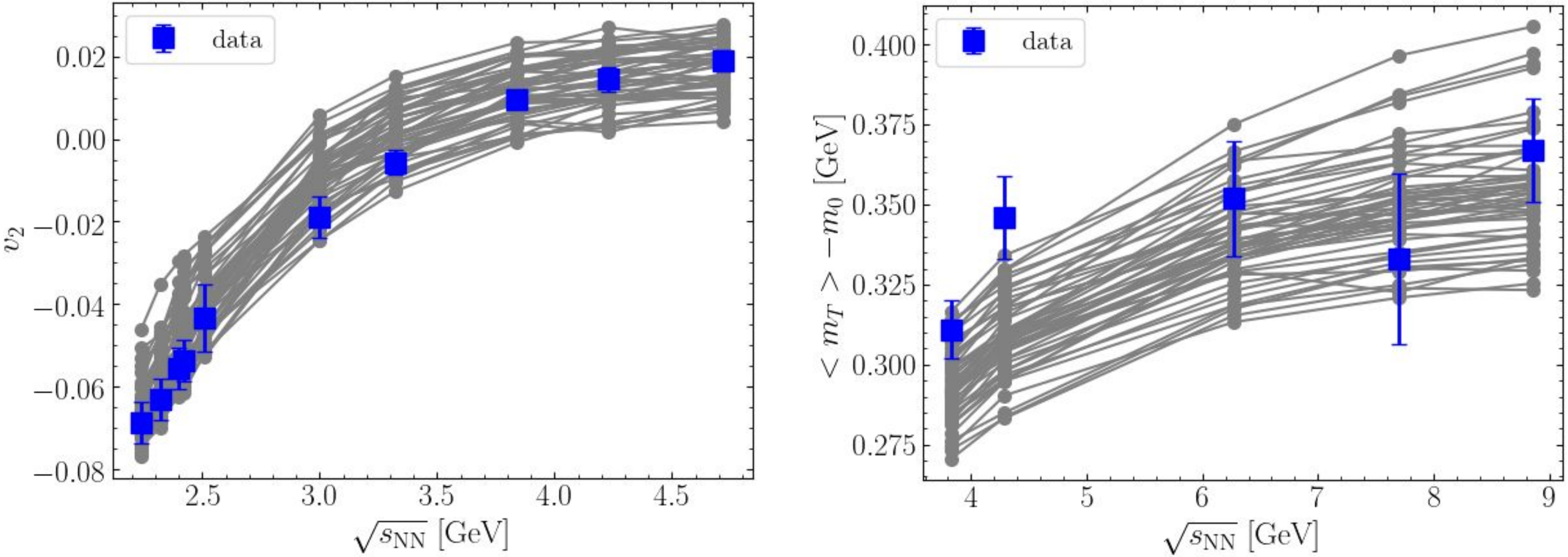


For stricter, robust constraints on the EoS below $4n_0$, significant improvements and consistency in flow measurements are necessary for E_{lab} = 2-15 *A*GeV

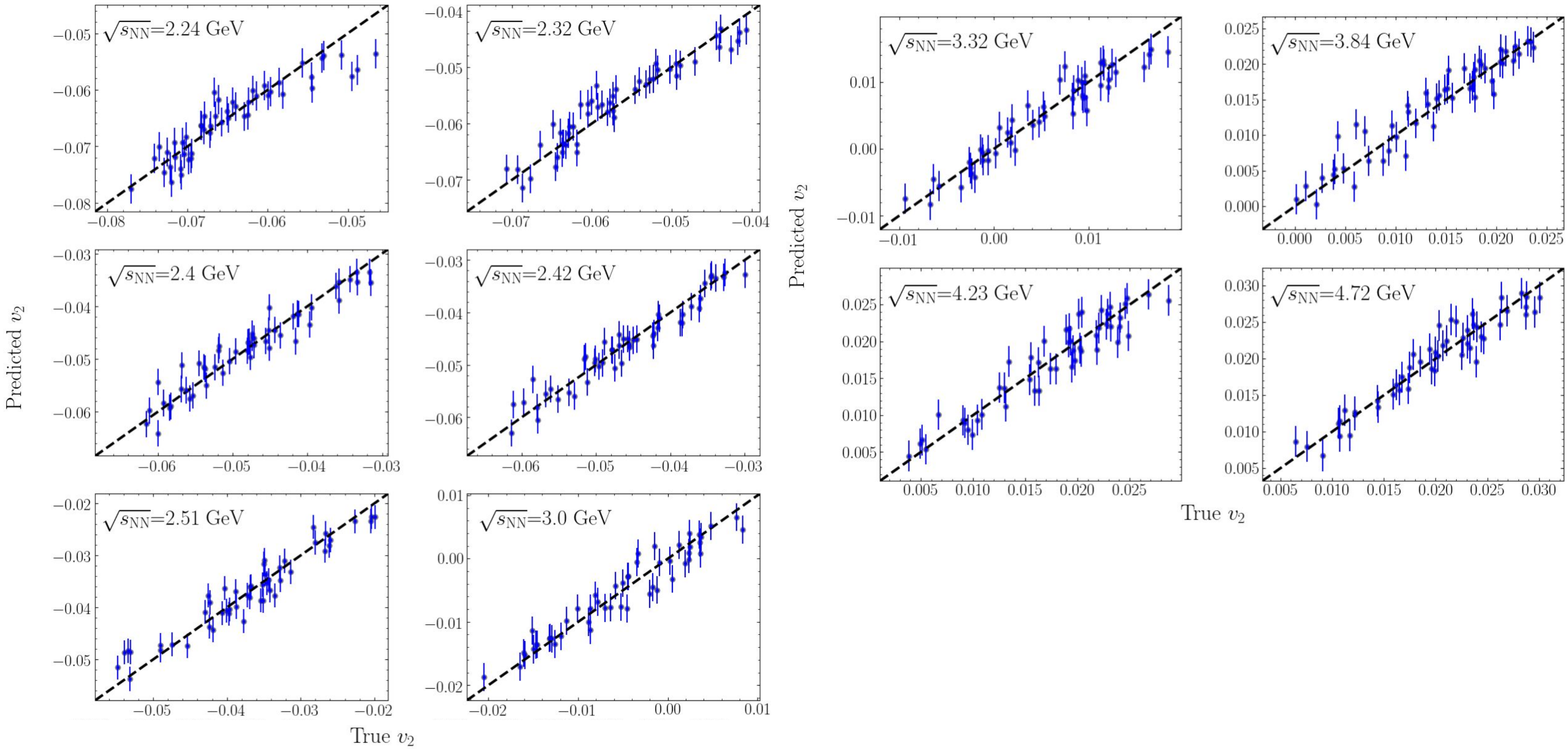
BES-II, fxt data ! Future CBM @FAIR measurements?

Backup slides

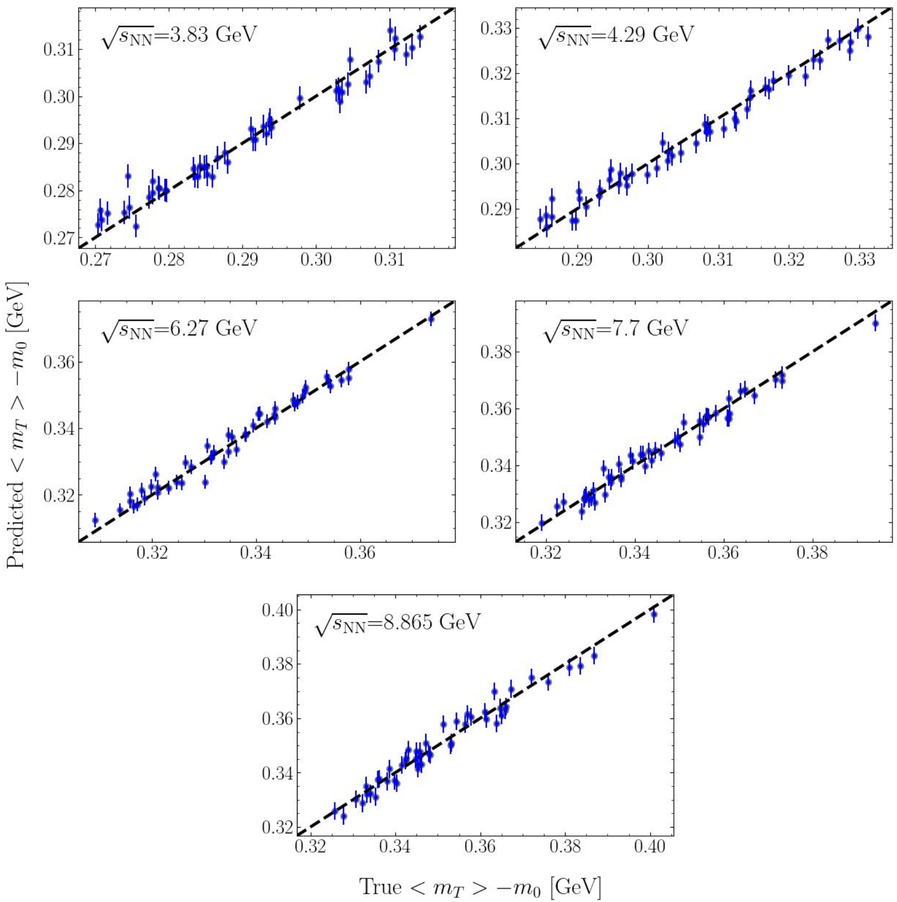
Potentials for training GP models



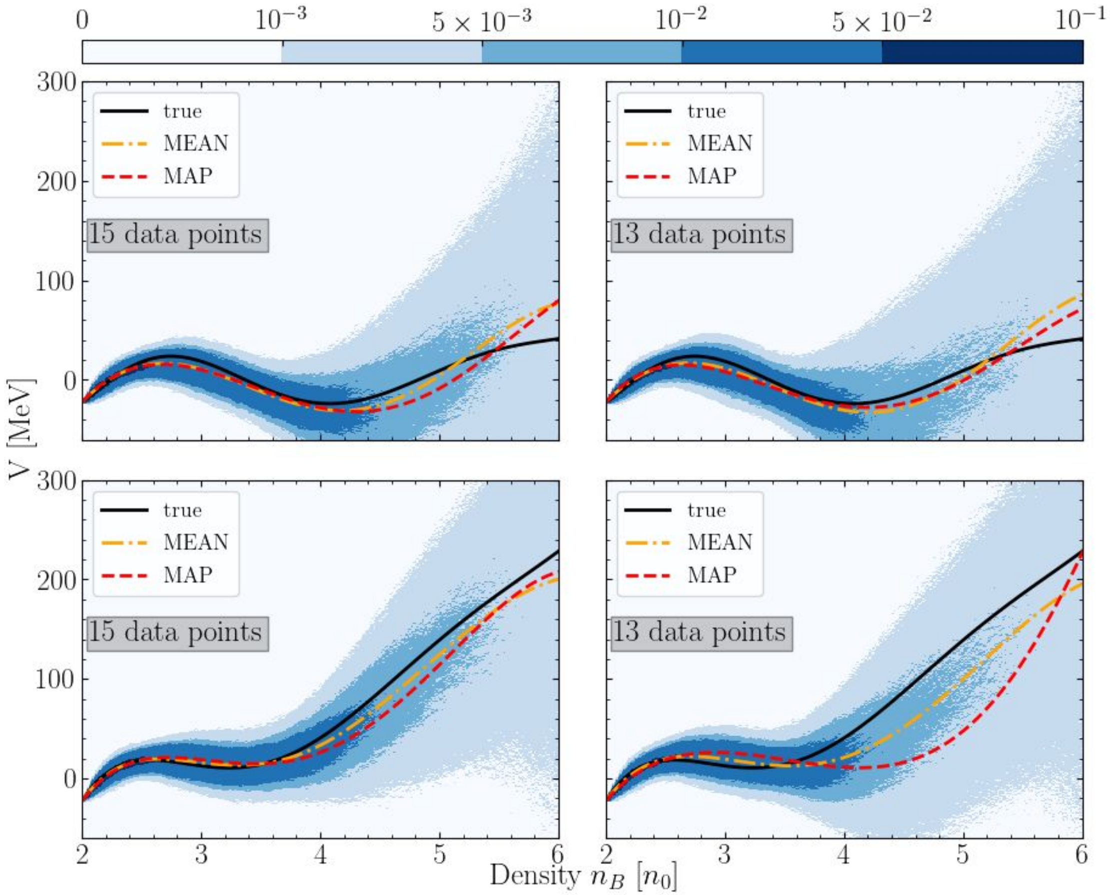
GP models: performance



GP models: performance



Closure tests



Experimental data

80 Page 14 of 21 Eur. Phys. J. A (2023) 59:80

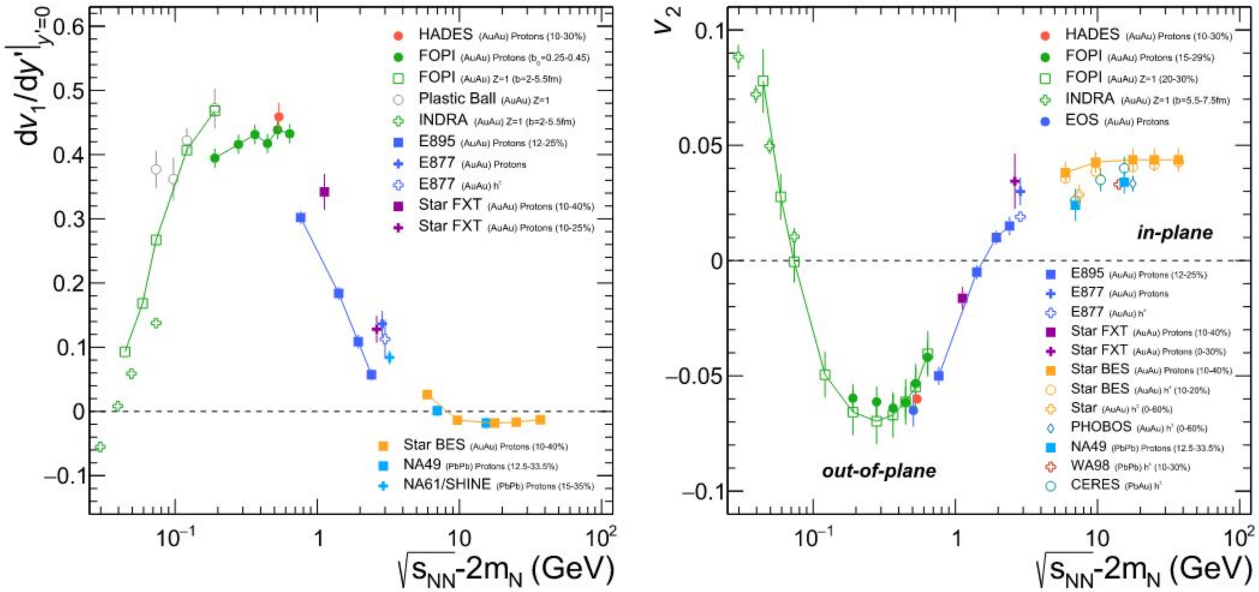
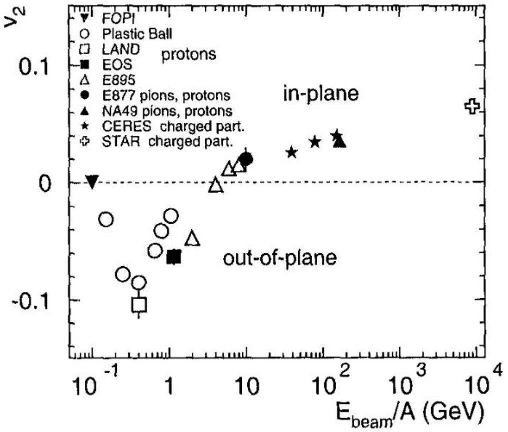


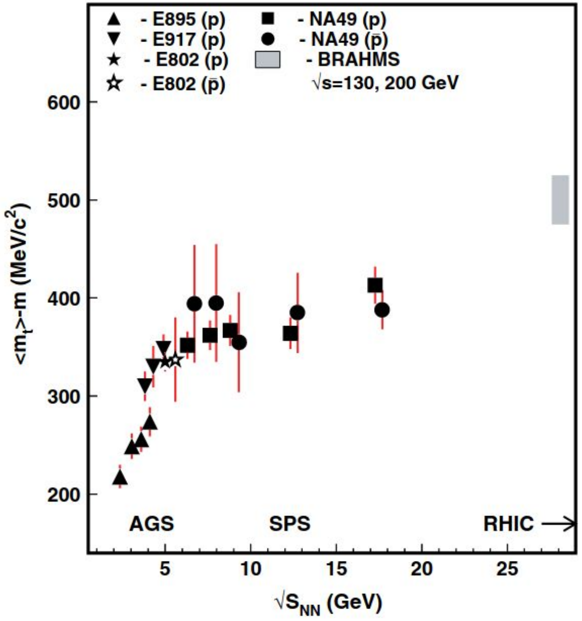
Fig. 11 Compilation of directed and elliptic flow measurements as a function of the subtracted centre-of-mass energy $\sqrt{s_{\mathrm{NN}}} - 2m_N$. Shown as red points are the slope of v_1 at mid-rapidity (left panel), $dv_1/dy'|_{y'=0}$, and the p_t integrated v_2 at mid-rapidity (right panel) for protons in Au+Au collisions at $\sqrt{s_{\mathrm{NN}}} = 2.4$ GeV ($10-30$ % centrality). These results are compared to data in the same or similar centrality ranges in Au+Au or Pb+Pb collisions for nuclei with $Z = 1$ (INDRA [7], FOPI [7,37,38] Plastic Ball [39,40]), for protons (FOPI [38,41], EOS/E895 [42,43], E877 [44], NA49 [45], STAR [46–48], NA61/SHINE [49]) and for inclusive charged particles (E877 [21,50], CERES [51], WA98 [52], STAR [53,54], PHOBOS [55])

Experimental data

- low energy v_2 : FOPi, HADES (<3 GeV)
- 3 GeV: STAR
- higher energies E895 (AGS)

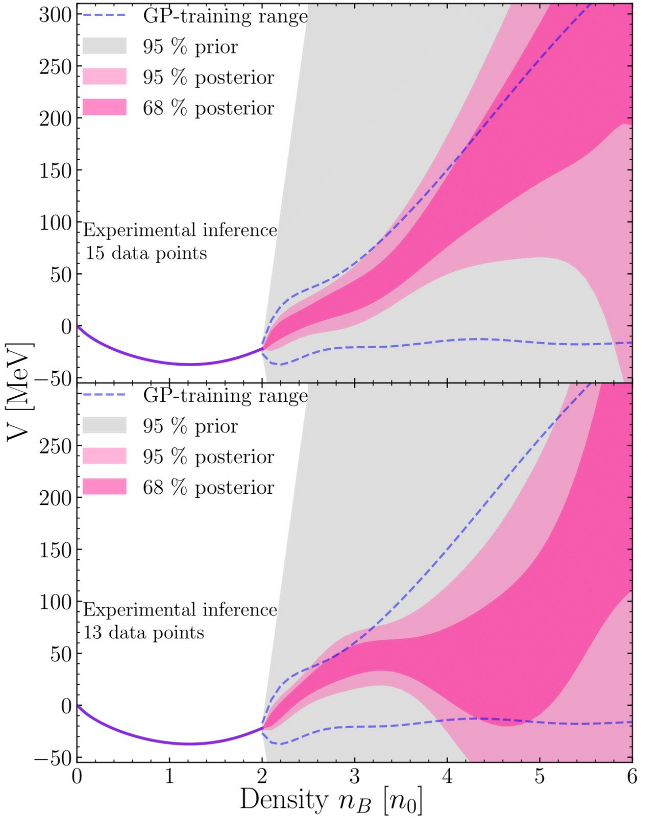
- $\langle m_T \rangle$ low energies: E917 (AGS)
- 7.7 GeV STAR
- remaining: NA49





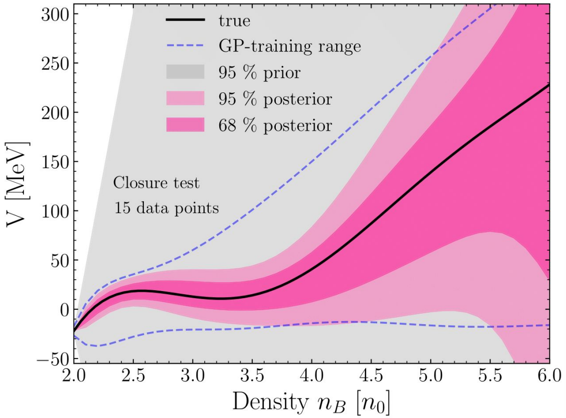
(NA49), Phys. Rev. C 73, 044910 (2006).

Inference

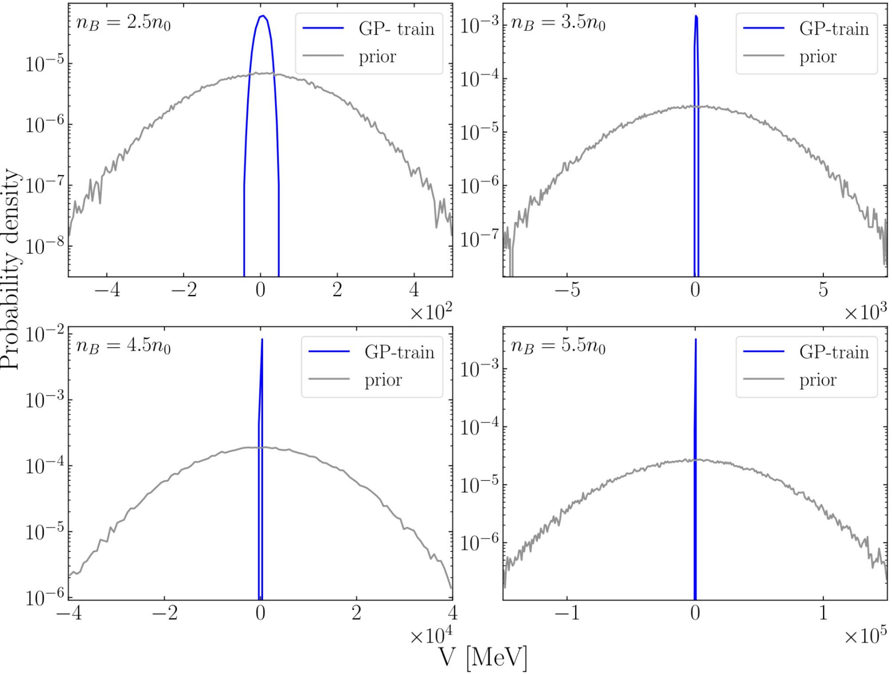


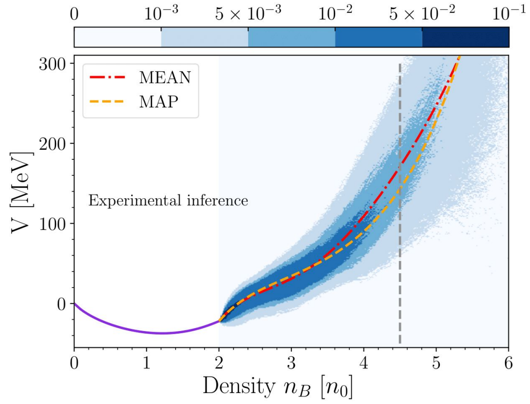
- ❖ Proton observables (mid rapidity)
  - ➢ Elliptic flow : 10 data points
    - ■ Mid-central collisions
  - ➢ Transverse kinetic energy: 5 data points

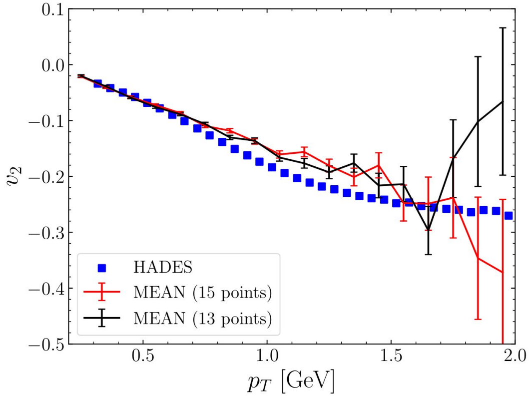
Central collisions



Prior, further tests







Microscopic transport with density dependent potential

- Non-equilibrium MD part of UrQMD is used
- UrQMD:
  - Propagation of hadrons on classic trajectories
    - stochastic binary scattering , color string formation, resonance excitation and decays
  - Imaginary part of interactions:
    - geometric interpretation of cross section
      - Experiment, detailed balance
  - Hadronic cascade
    - effective EoS of HRG with respective dof
- Real part of interactions in UrQMD
  - QMD + density dependent potential
    - Unlike other mean field models, QMD is an n-body theory of interactions between n nucleons

Microscopic transport with density dependent potential

A density dependent potential enters QMD equations

$$\dot{\mathbf{r}}_i = \frac{\partial \mathbf{H}}{\partial \mathbf{p}_i}, \quad \dot{\mathbf{p}}_i = -\frac{\partial \mathbf{H}}{\partial \mathbf{r}_i}.$$

The total hamiltonian function is sum over all hamiltonians of the i baryons

$$\mathbf{H} = \sum_i H_i, \quad H_i = E_i^{kin} + V_i$$

This include KE and total potential energy **V**

$$\mathbf{V} = \sum_i V_i \equiv \sum_i V(n_B(r_i))$$

The local interaction density n_B at r_k is calc by assuming each particle as gaussian wave packet

$$n_B(r_k) = n_k = \sum_{j,j\neq k} n_{j,k}$$

$$= \left(\frac{\alpha}{\pi}\right)^{3/2} \sum_{j,j\neq k} B_j \exp\left(-\alpha(\mathbf{r_k} - \mathbf{r_j})^2\right)$$

α=1/2L, L= 2 fm^2

The change in momentum for baryon 'i' is then

$$\dot{\mathbf{p}}_i = -\frac{\partial \mathbf{H}}{\partial \mathbf{r}_i} = -\frac{\partial \mathbf{V}}{\partial \mathbf{r_i}} \qquad n_{\{i,j\}} \equiv n_B(r_{\{i,j\}})$$

$$= -\left(\frac{\partial V_i}{\partial n_i} \cdot \frac{\partial n_i}{\partial \mathbf{r_i}}\right) - \left(\sum_{j\neq i} \frac{\partial V_j}{\partial n_j} \cdot \frac{\partial n_j}{\partial r_i}\right)$$

Force on ith baryon depends on change in potential energy at point r_i due to local gradient of $n_B(r_i)$ and change in potential at positions r_j of all baryons j due to change in r_i

-solved in timestep 0.2fm/c

$$P(n_B) = P_{\rm id}(n_B) + \int_0^{n_B} n' \frac{\partial U(n')}{\partial n'} dn' \,,\; U(n_B) = \frac{\partial\big(n_B \cdot V(n_B)\big)}{\partial n_B}$$

$$\mu'_B(n_B) = \mu_B^{id}(n_B) + U(n_B)$$

$$\epsilon(n_B) = -P(n_B) + \mu'_B n_B + sT$$