Clusters as a probe of the equation-of-state of strongly interacting matter

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&
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SQM 2024
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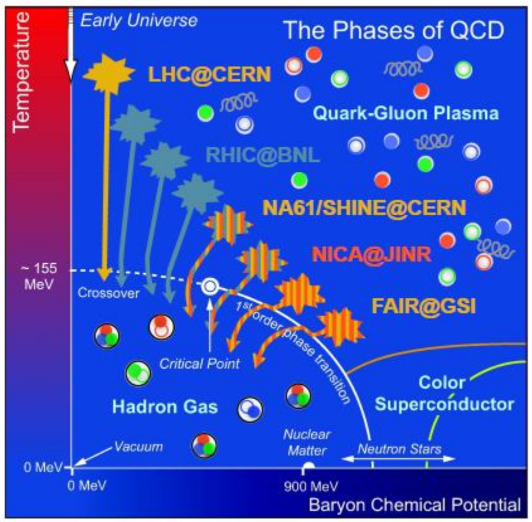
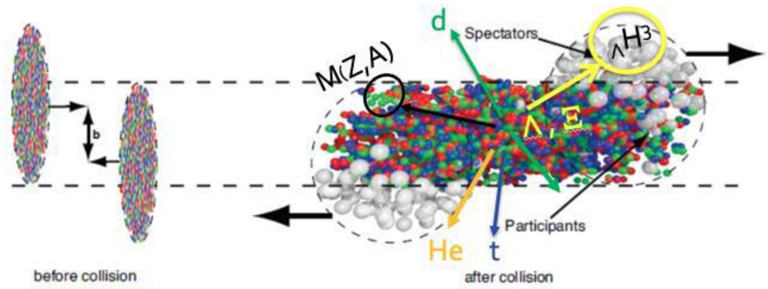
Cluster production in heavy-ion collisions

The phase diagram of QCD

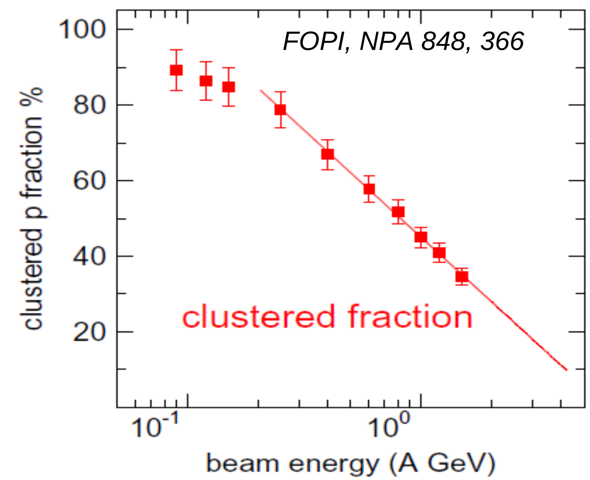


Clusters and (anti-) hypernuclei

are observed experimentally at all energies

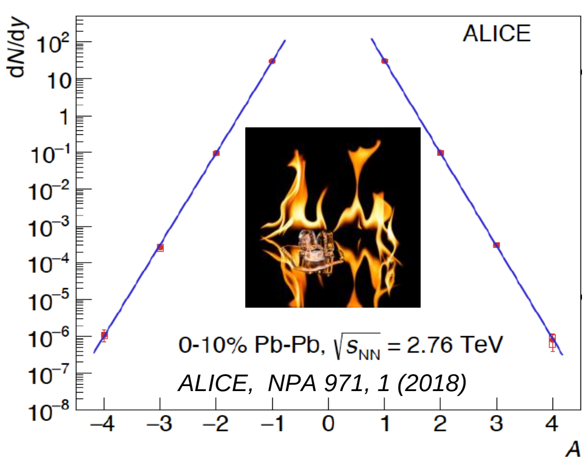


- **Clusters are very abundant at low energy**



- **High energy HIC:**
  ‚Ice in a fire' puzzle: how the weakly bound objects can be formed and survive in a hot enviroment?!

Au+Au, central. midrapidity



➔ **Mechanisms of cluster formation in strongly interacting matter are not well understood**

Modeling of cluster and hypernuclei formation

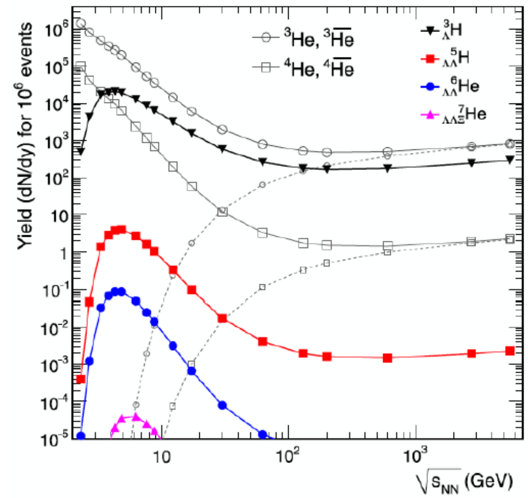
Existing models for cluster formation:

- ❑ **statistical model:**
  - **assumption of thermal equilibrium**

In order to understand the microscopic origin of cluster formation one needs a realistic model for the dynamical time evolution of the HIC

Dynamical Models:

A. Andronic et al., PLB 697, 203 (2011)



I. cluster formation by coalescence mechanism
at a freeze-out time by coalescence radii in coordinate and momentum space

II. dynamical modeling of cluster formation based on interactions within microscopic transport models:

– potential‘ mechanism - via potential NN (NY) interactions
(applied during the whole reaction time of HIC)

– ‘kinetic‘ mechanism - by hadronic scattering
(hadronic reactions as NNN → dN ; NNπ → dπ)

PHQMD

PHQMD

PHQMD: a unified n-body microscopic transport approach for the description of heavy-ion collisions and dynamical cluster formation from low to ultra-relativistic energies

Realization: combined model PHQMD = (PHSD & QMD) + (MST/SACA)

Parton-Hadron-Quantum-Molecular Dynamics

PHSD mode

Initialization → propagation of baryons:
QMD (Quantum-Molecular Dynamics)

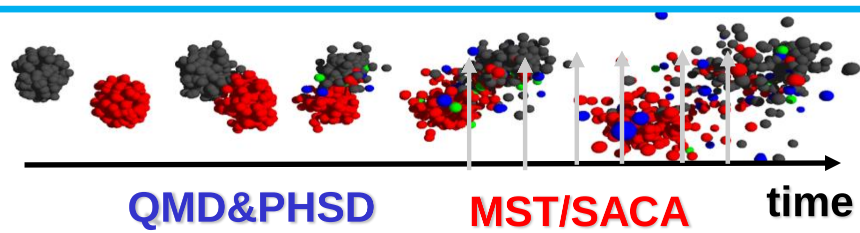
J.Aichelin, Phys. Rept. 202 (1991)

Propagation of partons (quarks, gluons) and mesons
+ collision integral = interactions of hadrons and partons (QGP)
from PHSD (Parton-Hadron-String Dynamics)

W. Cassing, E. Bratkovskaya, PRC 78 (2008) 034919; NPA831 (2009) 215

Cluster recognition:
MST (Minimum Spanning Tree)
or SACA (Simulated Annealing Clusterization Algorithm)
or coalescence mechanism

R. K. Puri, J. Aichelin, J.Comp. Phys. 162 (2000) 245-266



PHQMD:
J. Aichelin et al., PRC 101 (2020) 044905;
S. Gläßel et al., PRC 105 (2022) 1;
V. Kireyeu et al., PRC 105 (2022) 044909;
G. Coci et al., PRC 108 (2023) 1, 014902

QMD propagation (EoM)

- **Generalized Ritz variational principle:** $$\delta \int_{t_1}^{t_2} dt < \psi(t)| i\frac{d}{dt} - H|\psi(t) >= 0.$$

Many-body wave function:

$$\psi(t) = \prod_{i=1}^{N} \psi(\mathbf{r}_i, \mathbf{r}_{i0}, \mathbf{p}_{i0}, t)$$

Ansatz:
Gaussian trial wave function (with width L) centered at r_{i0}, p_{i0}

$$\psi(\mathbf{r}_i, \mathbf{r}_{i0}, \mathbf{p}_{i0}, t) = C\ \mathrm{e}^{-\frac{1}{4L}\left(\mathbf{r}_i - \mathbf{r}_{i0}(t) - \frac{\mathbf{p}_{i0}(t)}{m}t\right)^2} \cdot \mathrm{e}^{i\mathbf{p}_{i0}(t)(\mathbf{r}_i - \mathbf{r}_{i0}(t))} \cdot \mathrm{e}^{-i\frac{\mathbf{p}_{i0}^2(t)}{2m}t}$$

L=4.33 fm²

- **Equations-of-motion (EoM) for Gaussian centers in coordinate and momentum space:**

$$\boxed{\dot{r}_{i0} = \frac{\partial\langle H\rangle}{\partial p_{i0}} \qquad \dot{p}_{i0} = -\frac{\partial\langle H\rangle}{\partial r_{i0}}}$$

Many-body Hamiltonian: $$H = \sum_i H_i = \sum_i (T_i + V_i) = \sum_i \left(T_i + \sum_{j\neq i} V_{i,j}\right)$$

[Aichelin, Phys. Rept. 202 (1991)]

- **Nucleon-nucleon local two-body potential:**

$$V_{ij} = V(\mathbf{r}_i, \mathbf{r}_j, \mathbf{r}_{i0}, \mathbf{r}_{j0}, \mathbf{p}_{i0}, \mathbf{p}_{j0}, t) = V_{\text{Skyrme loc}} + \boxed{V_{\text{mom}}} + V_{\text{Coul}}$$

momentum dependent potential

➔ **Single-particle potential <V> :**

1) Skyrme potential ('static') :

$$\langle V_{Skyrme}(\mathbf{r_{i0}}, t)\rangle = \alpha\left(\frac{\rho_{int}(\mathbf{r_{i0}}, t)}{\rho_0}\right) + \beta\left(\frac{\rho_{int}(\mathbf{r_{i0}}, t)}{\rho_0}\right)^{\gamma}$$

with relativistic extended interaction density:

$$\rho_{int}(\mathbf{r_{i0}}, t) \rightarrow C\sum_j \left(\frac{4}{\pi L}\right)^{3/2} \mathrm{e}^{-\frac{4}{L}(\mathbf{r}_{i0}^{\mathrm{T}}(t) - \mathbf{r}_{j0}^{\mathrm{T}}(t))^2} \times \mathrm{e}^{-\frac{4\gamma_{cm}^2}{L}(\mathbf{r}_{i0}^{\mathrm{L}}(t) - \mathbf{r}_{j0}^{\mathrm{L}}(t))^2},$$

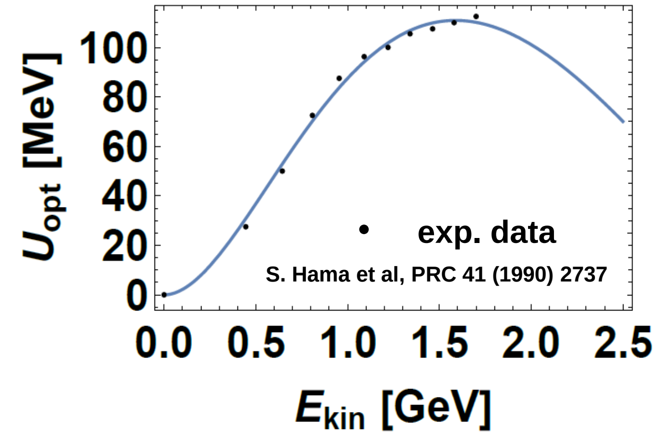
Momentum dependent potential → EoS in PHQMD

2) Momentum dependent potential :

$$\boxed{V(\mathbf{r}_1,\mathbf{r}_2,\mathbf{p}_{01},\mathbf{p}_{02}) = (a\Delta p + b\Delta p^2)\ exp[-c\sqrt{\Delta p}]\ \delta(\mathbf{r}_1-\mathbf{r}_2)}$$

$$\Delta p = \sqrt{(\mathbf{p}_{01}-\mathbf{p}_{02})^2}$$

Parameters a, b, c are fitted to the "optical" potential (Schrödinger equivalent potential U_{SEP}) extracted from elastic scattering data in pA: $U_{SEQ}(p) = \frac{\int^{p_F} V(\mathbf{p}-\mathbf{p}_1)dp_1^3}{\frac{4}{3}\pi p_F^3}$



❖ **In infinite matter a potential corresponds to the EoS:**

$$\boxed{E/A(\rho) = \frac{3}{5}E_F + V_{Skyrme\ stat}(\rho) + V_{mom}(\rho)}$$

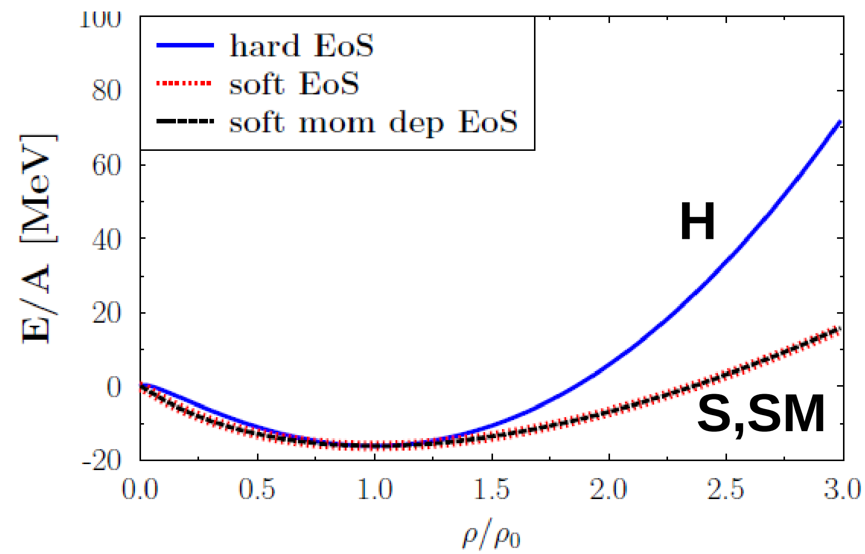
$$V_{Skyrme} = \alpha\frac{\rho}{\rho_0} + \beta\frac{\rho}{\rho_0}^{\gamma}$$

compression modulus K of nuclear matter:

$$K = -V\frac{dP}{dV} = 9\rho^2\frac{\partial^2(E/A(\rho))}{(\partial\rho)^2}|_{\rho=\rho_0}.$$

E.o.S.	$\alpha[MeV]$	$\beta[MeV]$	γ	K [MeV]
S	-383.5	329.5	1.15	200
H	-125.3	71.0	2.0	380
SM	-478.87	413.76	1.10	200
	a $[MeV^{-1}]$	b$[MeV^{-2}]$	c$[MeV^{-1}]$	
	236.326	-20.73	0.901	

EoS for infinite cold nuclear matter at rest



Mechanisms for cluster production in PHQMD:

I. potential interactions

(recongnized by MST)

&

II. kinetic reactions

III. Coalescence (to compare with I+II)

PHQMD

I. Cluster recognition: Minimum Spanning Tree (MST)

R. K. Puri, J. Aichelin, J.Comp. Phys. 162 (2000) 245-266

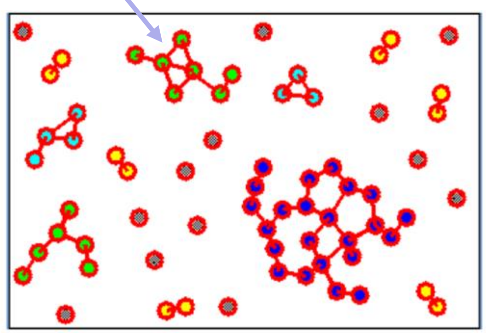
The **Minimum Spanning Tree** (MST) is a **cluster recognition** method applicable for the (asymptotic) **final states** where coordinate space correlations may only survive for bound states.

The MST algorithm searches for **accumulations of particles in coordinate space:**

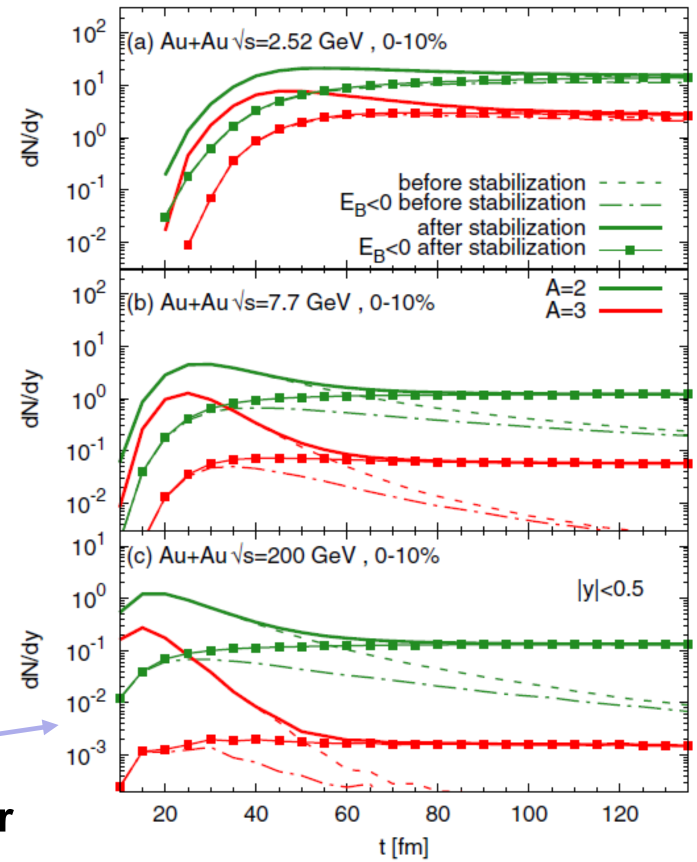
1. Two particles are **'bound'** if their **distance in the cluster rest frame** fulfills

$$|\vec{r_i} - \vec{r_j}| \leq 4 \text{ fm} \quad \text{(range of NN potential)}$$

2. Particle is **bound to a cluster** if it **binds with at least one particle** of the cluster



* Remark: inclusion of an additional momentum cut (coalescence) leads to small changes: particles with large relative momentum are almost never at the same position (V. Kireyeu, Phys.Rev.C 103 (2021) 5)



G. Coci et al., Phys.Rev.*C* 108 (2023) 014902

Advanced MST (aMST)

- MST + extra condition: $E_B<0$
  negative binding energy for identified clusters
- **Stabilization procedure** – to correct artifacts of the semi-classical QMD:
  recombine the final "lost" nucleons back into cluster if they left the cluster without rescattering

II. Deuteron production by hadronic reactions

"Kinetic mechanism"

1) hadronic inelastic reactions NN $\leftrightarrow$ dπ , πNN $\leftrightarrow$ dπ , NNN $\leftrightarrow$ dN
2) hadronic elastic π+d, N+d reactions

SMASH: D. Oliinychenko et al., PRC 99 (2019) 044907;
J. Staudenmaier et al., PRC 104 (2021) 034908
AMPT: R.Q. Wang et al. PRC 108 (2023) 3

- **Collision rate** for hadron "*i*" is the number of reactions in the **covariant volume** $d^4x = dt*dV$
- With **test particle ansatz the transition rate for 3→2 reactions**:

W. Cassing, NPA 700 (2002) 618

$$\frac{\Delta N_{coll}[3+4+5 \to 1(d)+2]}{\Delta N_3 \Delta N_4 \Delta N_5} = P_{3,2}(\sqrt{s})$$

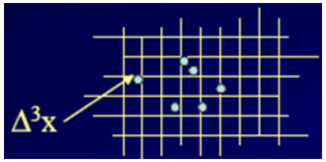
$$\boxed{P_{3,2}(\sqrt{s}) = F_{spin} F_{iso} P_{2,3}(\sqrt{s}) \frac{E_1^f E_2^f}{2E_3 E_4 E_5} \frac{R_2(\sqrt{s}, m_1, m_2)}{R_3(\sqrt{s}, m_3, m_4, m_5)} \frac{1}{\Delta V_{cell}}}$$

Energy and momentum
of final particles

2,3-body phase space
integrals
[Byckling, Kajantie]

$$\boxed{P_{2,3}(\sqrt{s}) = \sigma_{tot}^{2,3}(\sqrt{s}) v_{rel} \frac{\Delta t}{\Delta V_{cell}}}$$

➔ solved by **stochastic method**



- Numerically tested in **"static" box:** PHQMD provides a good agreement with analytic solutions from rate equations and with SMASH for the same selection of reactions
- **New in PHQMD: π+N+N⟷ d+π** inclusion of **all possible isospin channels** allowed by total isospin T conservation ➔ enhancement of the d production

$$\pi^{\pm,0} + p + n \leftrightarrow \pi^{\pm,0} + d$$
$$\pi^- + p + p \leftrightarrow \pi^0 + d$$
$$\pi^+ + n + n \leftrightarrow \pi^0 + d$$
$$\pi^0 + p + p \leftrightarrow \pi^+ + d$$
$$\pi^0 + n + n \leftrightarrow \pi^- + d$$

G. Coci et al., Phys.Rev.*C* 108 (2023) 014902

Modelling finite-size effects in kinetic mechanism

G. Coci et al., PRC 108 (2023) 014902

How to account for the **quantum nature of deuteron**, i.e. for

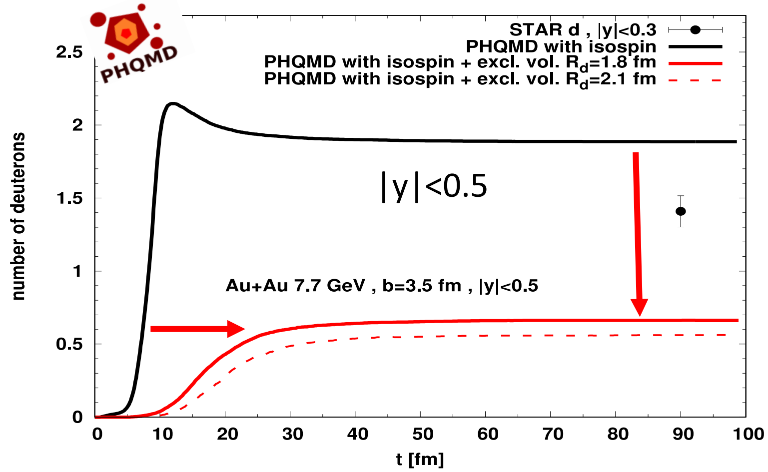
1) the **finite-size of *d* in coordinate space** (*d* is not a point-like particle) – for in-medium d production
2) the **momentum correlations of *p* and *n*** inside *d*

Realization:

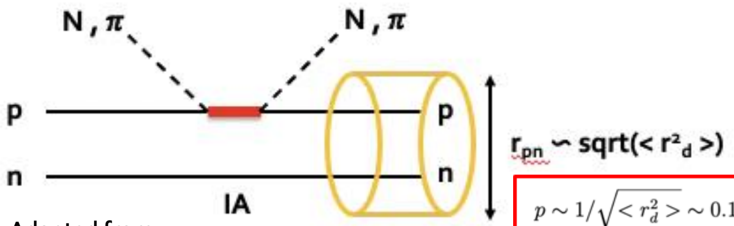
1) assume that a deuteron can not be formed in a high density region, i.e. if there are other particles (hadrons or partons) inside the 'excluded volume':

Excluded-Volume Condition: $\boxed{|\vec{r}(i)^* - \vec{r}(d)^*| < R_d}$

- ❑ **Strong reduction of d production**
- ❑ p_T slope is not affected by excluded volume condition

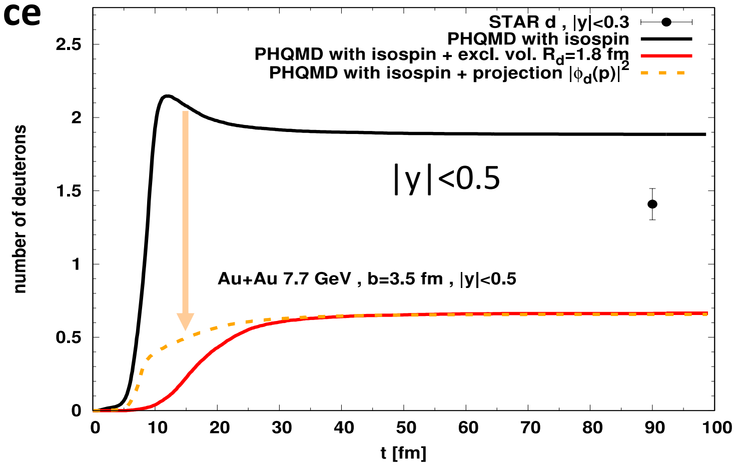


2) QM properties of deuteron must be also in momentum space
→ **momentum correlations of pn-pair**



$\boxed{p \sim 1/\sqrt{< r_d^2 >} \sim 0.1\,GeV}$

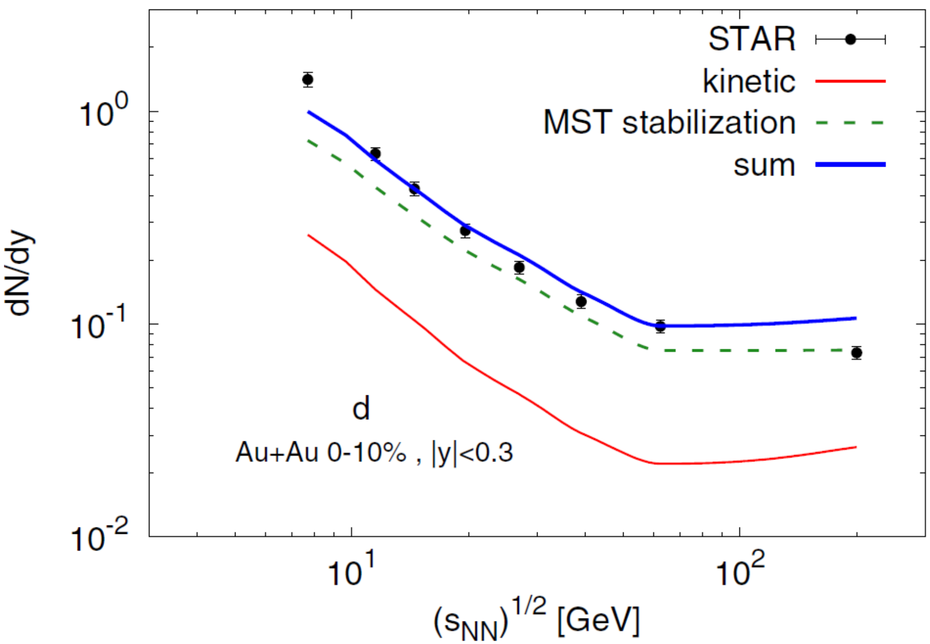
Adapted from
[Haidelbauer, Uzikov PLB 562(2003)]
[Hoftiezer et al. PRC23 (1981)]
Same spirit as AMPT [K.-J. Sun, R. Wang, C.-M. Ko et al., 2106.12742]



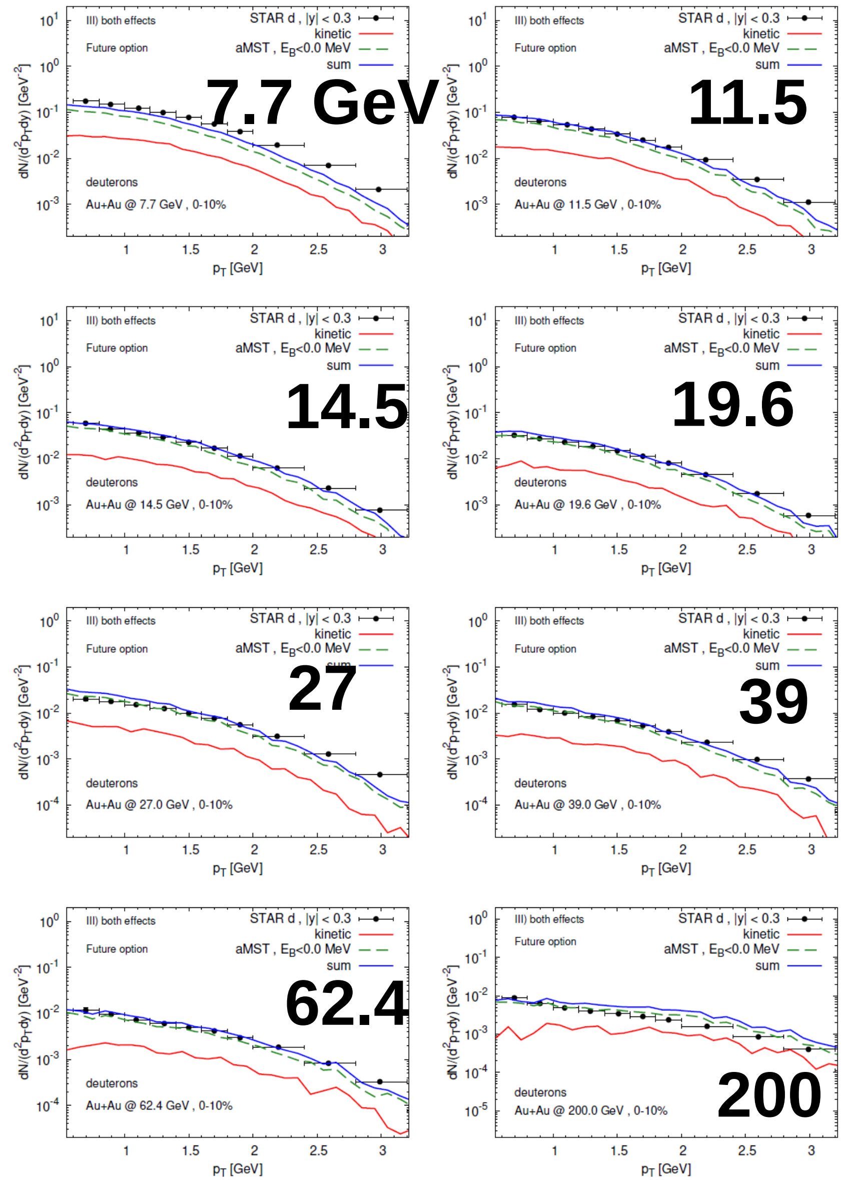
- ❑ **Strong reduction of d production** by projection on DWF $|\phi_d(p)|^2$

Kinetic vs. potential deuteron production

Excitation function dN/dy of deuterons at midrapidity



p_T – spectra (BES RHIC)

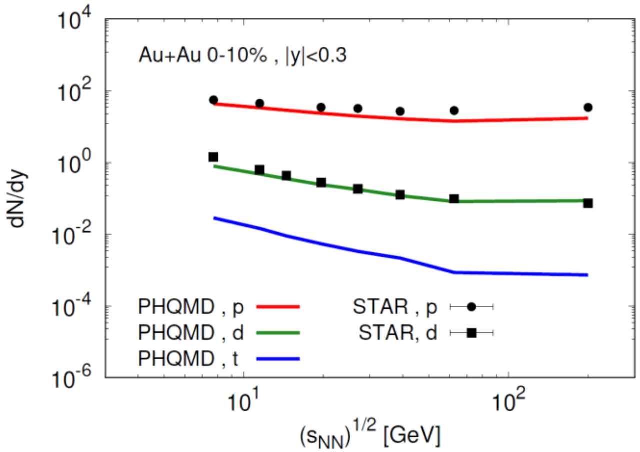


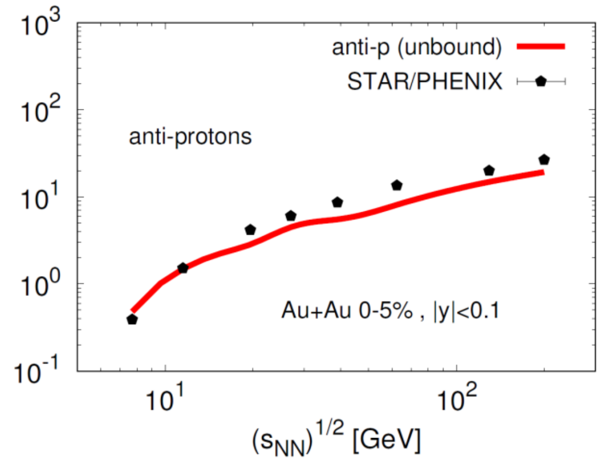
- **PHQMD provides a good description of STAR data**
- **Functional forms of y- spectra are similar for kinetic and potential deuterons**
- **The potential mechanism is dominant for d production at all energies!**

G. Coci et al., Phys.Rev.*C* 108 (2023) 014902

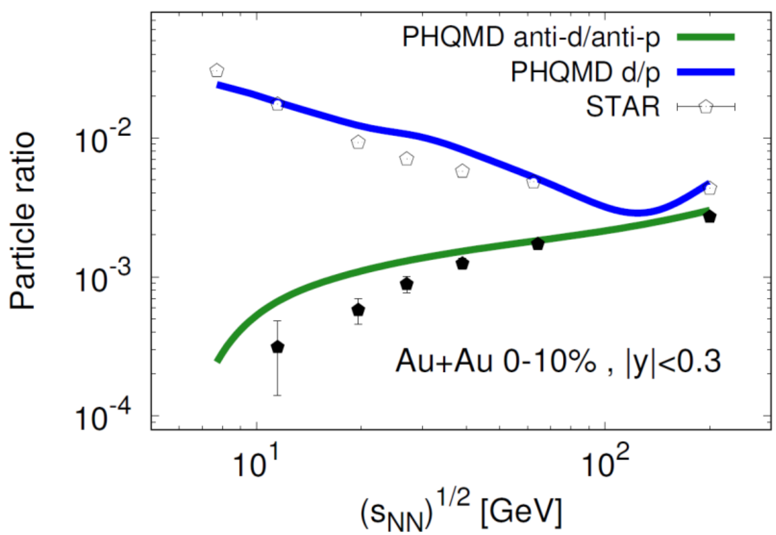
Anti-deuteron versus deuteron production

Excitation function dN/dy of p, d, anti-d at midrapidity

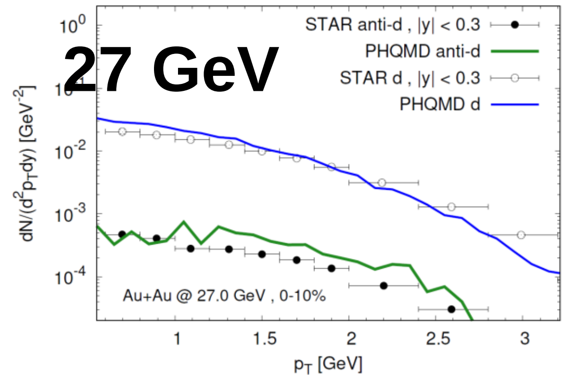


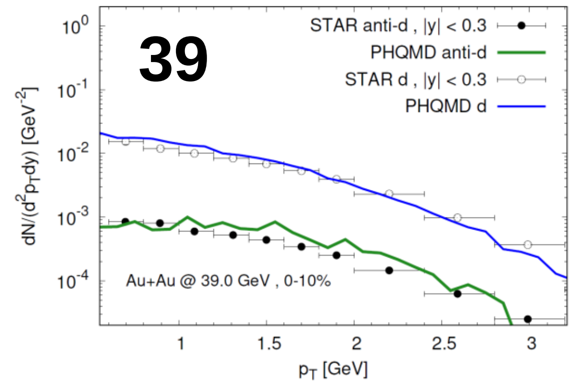


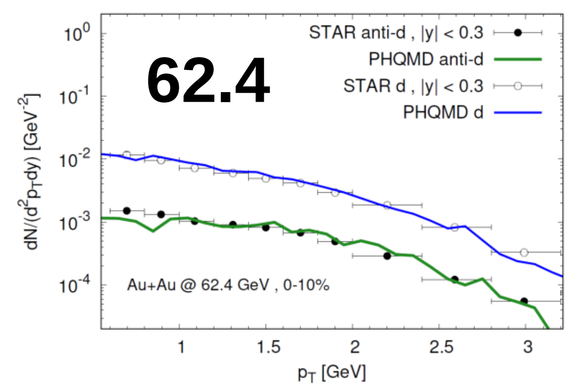
Excitation function of d/p and anti-d/p ratio at y=0



p_T – spectra (BES RHIC)







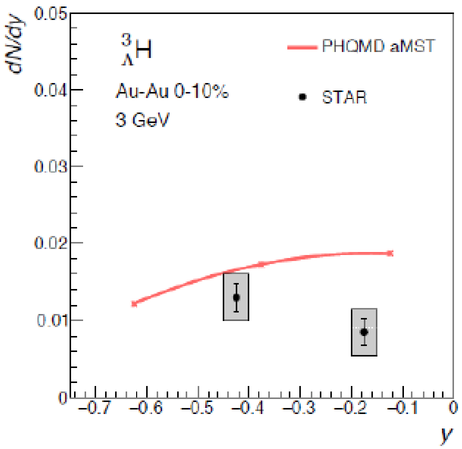
➔ **Exp. data on anti-d are well reproduced by the PHQMD**

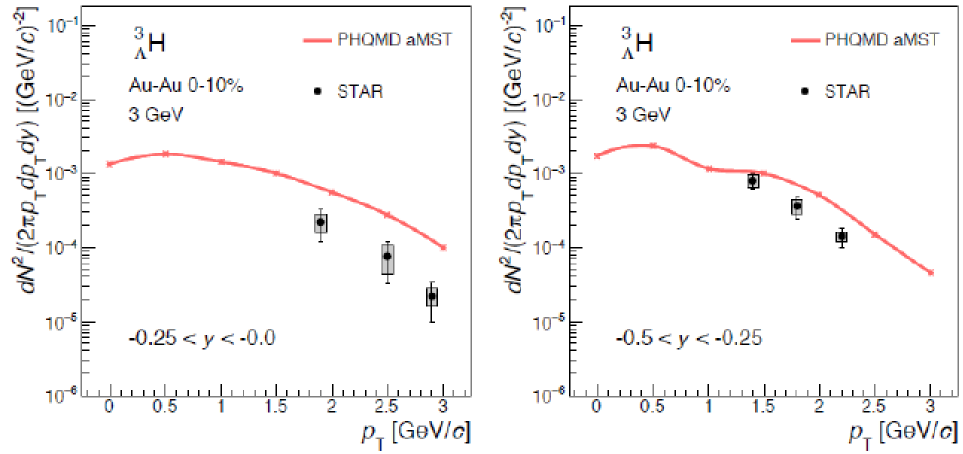
Hypernuclei production at STAR : $s^{1/2}$=3 GeV

y – spectra (extrapolated)

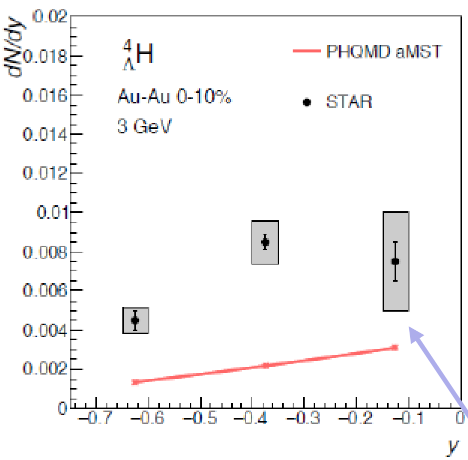
p_T – spectra (measured)

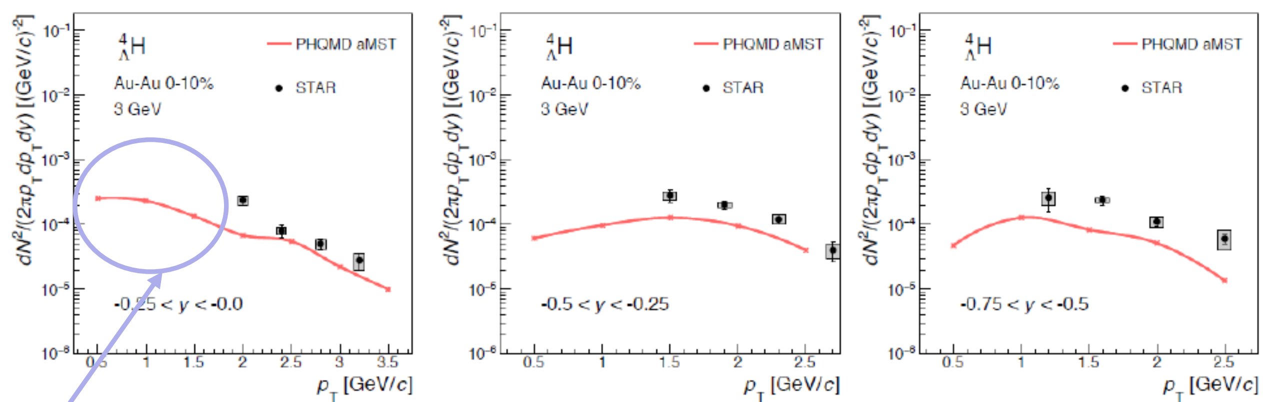
$^{3}{}_{\Lambda}H$





$^{4}{}_{\Lambda}H$





➔ **Low p_T – exp. data are needed for reliable estimation of y-spectra**

PHQMD: S. Gläßel et al., Phys. Rev. C 105 (2022) 1

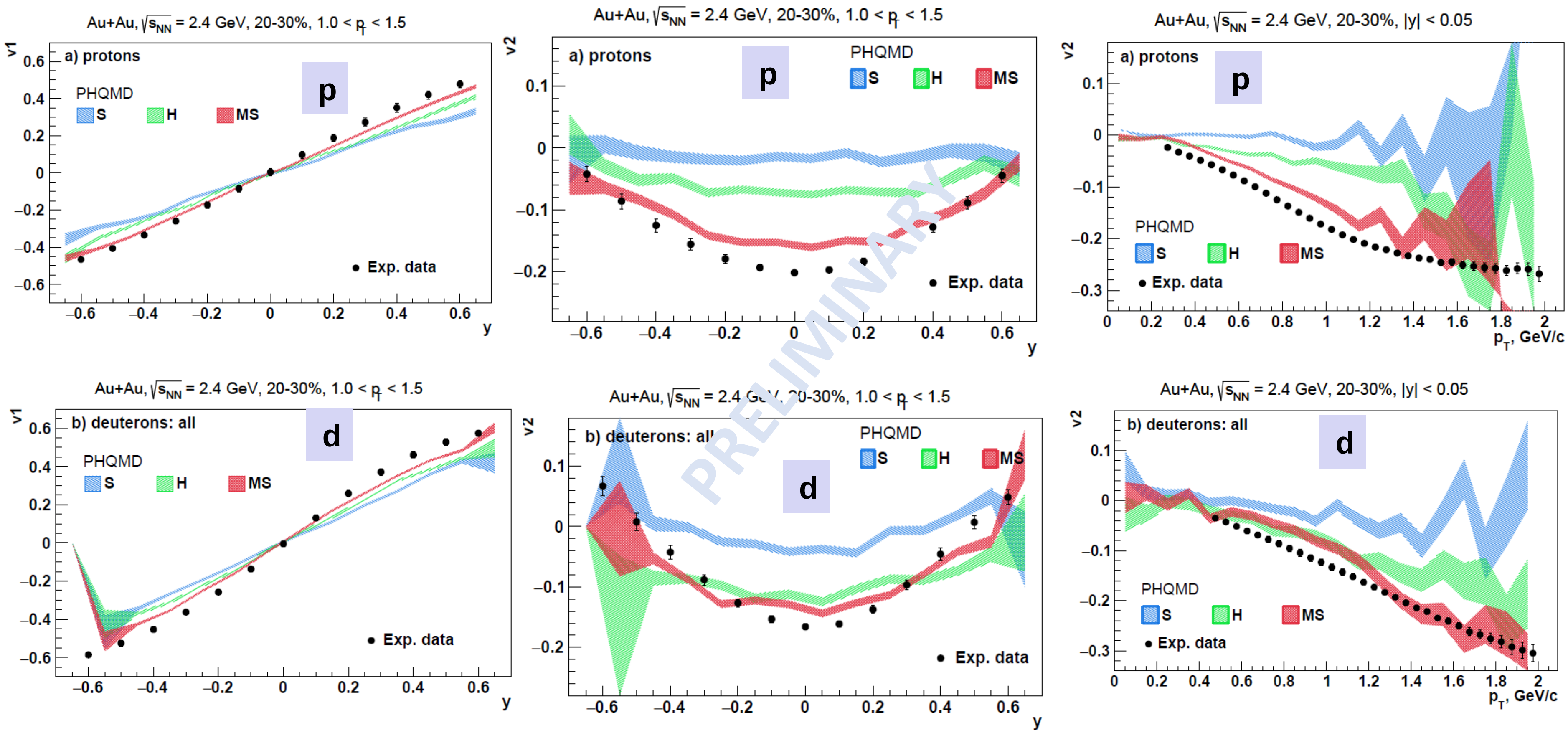
See the talk by Yuanjing Ji, Thu, 11:00AM

Spectra and v_1, v_2 of light clusters with different EoS in PHQMD: hard, soft, momentum dependent potential

PHQMD

EoS dependence of $v_1(y)$, $v_2(y)$, $v_2(p_T)$ at SIS energies: p,d

$v_1(y)$ $v_2(y)$ $v_2(p_T)$



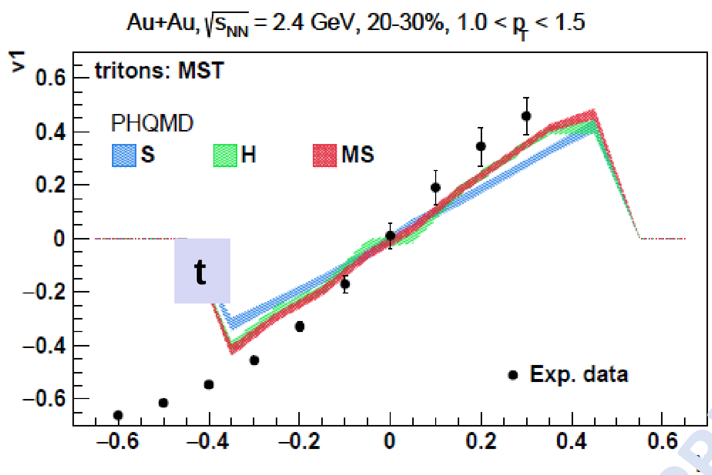
PHQMD5.2W: S= soft EoS, H=hard EoS, MS = soft momentum dependent EoS

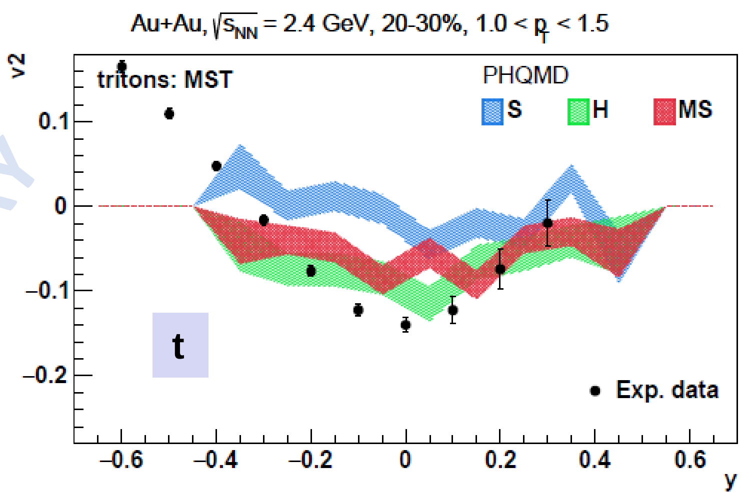
HADES data: of v_1, v_2 at high p_T: 1.0 < p_T < 1.5 GeV/c [HADES: Eur. Phys. J. A59 (2023) 80]

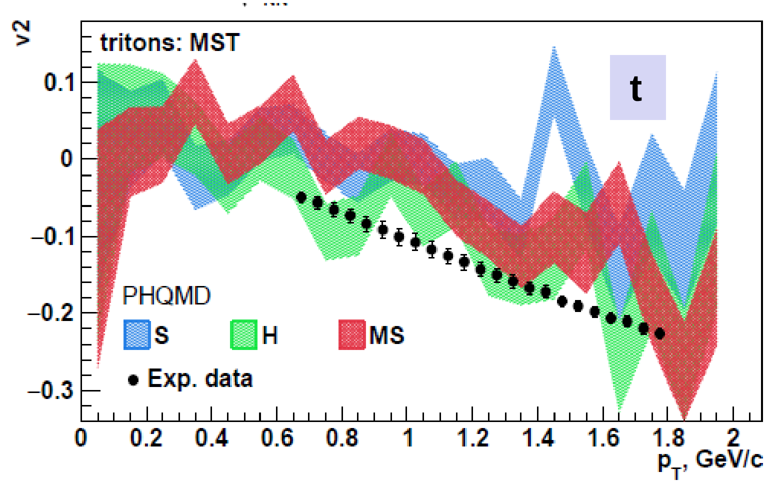
- **Strong EoS dependence of $v_1(y)$, $v_2(y)$ and $v_2(p_T)$ of protons and deuterons**
- **HADES data favor a soft momentum dependent potential (MS)**

Viktar Kireyeu, in progress

EoS dependence of $v_1(y)$, $v_2(y)$, $v_2(p_T)$ at SIS energies: triton







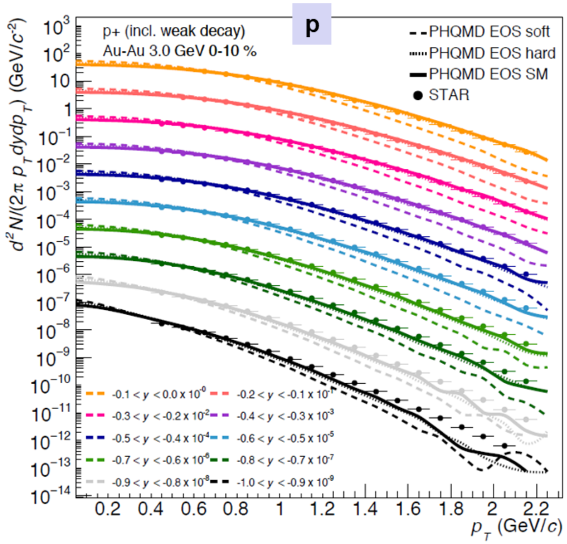
PHQMD5.2W: S= soft EoS, H=hard EoS,
MS = soft momentum dependent EoS
HADES data: of v_1, v_2 at high p_T:
1.0 < p_T < 1.5 GeV/c [HADES: Eur. Phys. J. A59 (2023) 80]

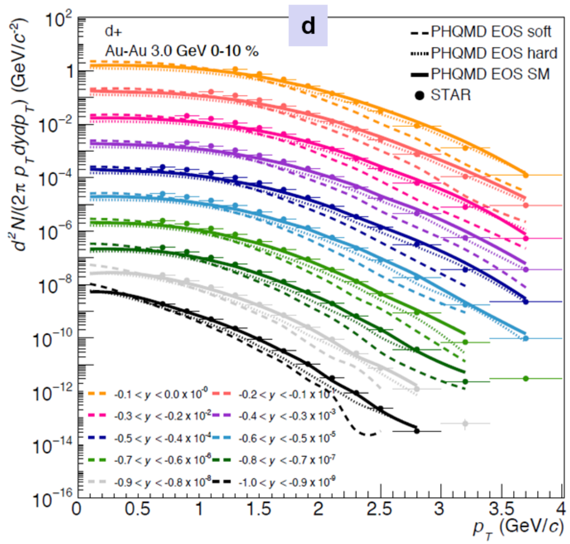
- **Strong EoS dependence of $v_1(y)$, $v_2(y)$ and $v_2(p_T)$ of tritons**
- **HADES data favor a soft momentum dependent potential (MS)**

Viktar Kireyeu, in progress

EoS dependence of p_T-spectra at STAR : $s^{1/2}$=3 GeV

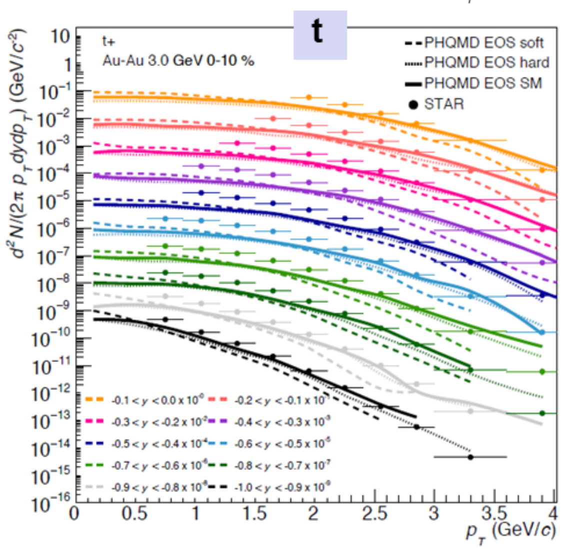
Susanne Gläßel, in progress

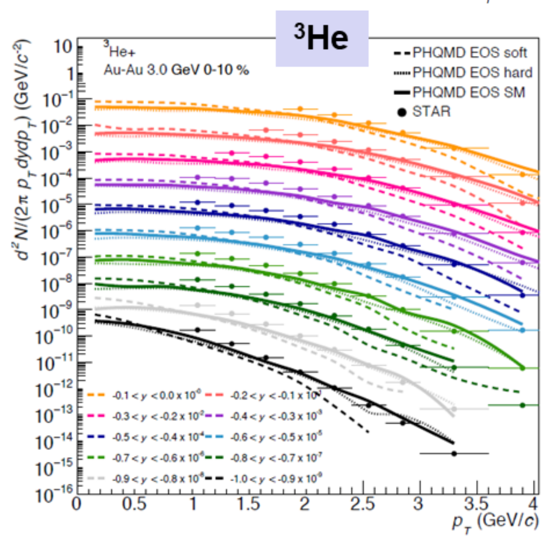


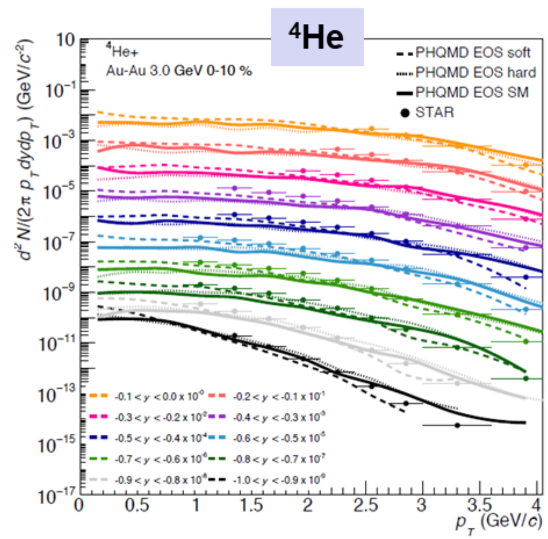


S= soft EoS,
H=hard EoS,
SM = soft momentum dependent EoS

PRELIMINARY







- Visible dependence of p_T spectra of p, d, t, ^{3}He, ^{4}He on EoS
- STAR p_T data favor a hard or soft-momentum dependent potential (H/SM)

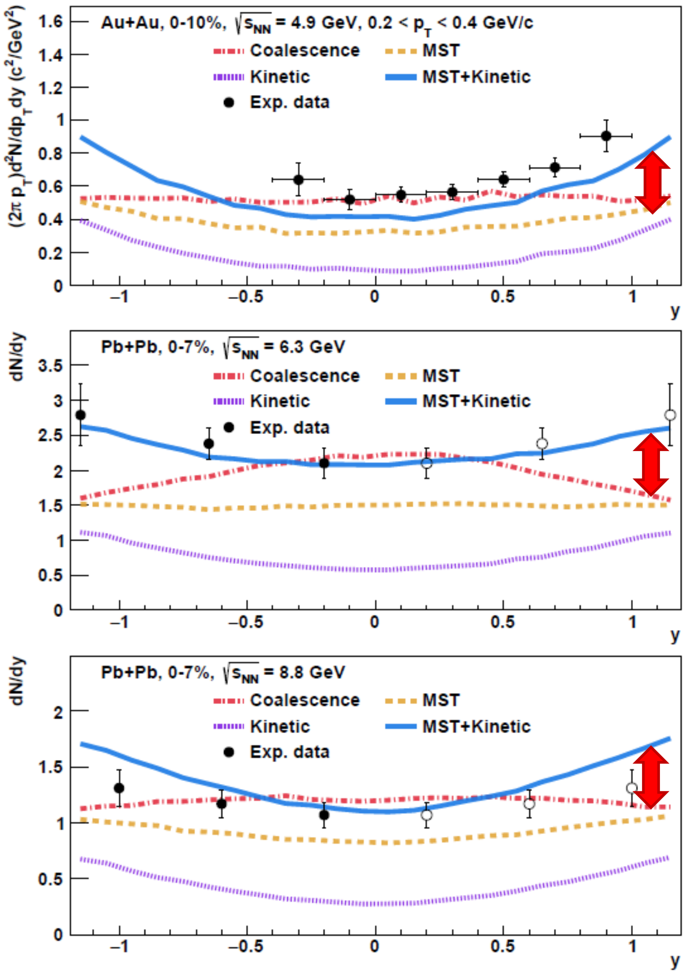
STAR:H. Liu et al., 2208.04650, Acta Phys.Polon.Supp. 16 (2023) 1, 148

Can the production mechanisms be identified experimentally?

Where the clusters are formed?

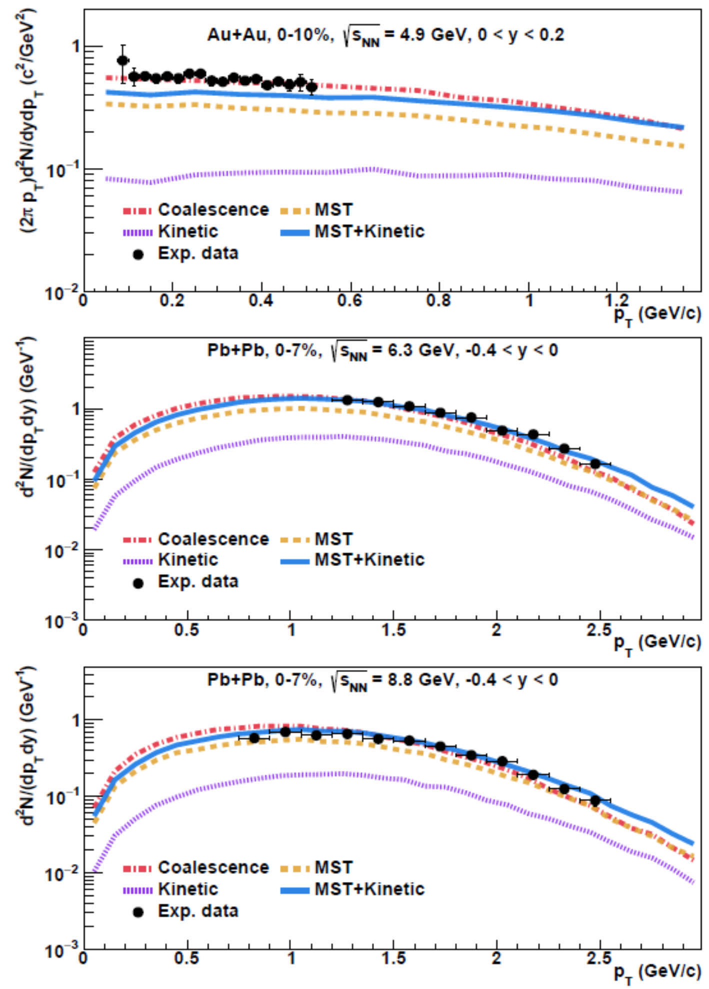
PHQMD

Mechanism for deuteron production: coalescence and MST+kinetic ⟺ experimental data



p_T -distributions have a different slope for coalescence/ MST+kinetic mechanisms

➔ y- distributions show differences

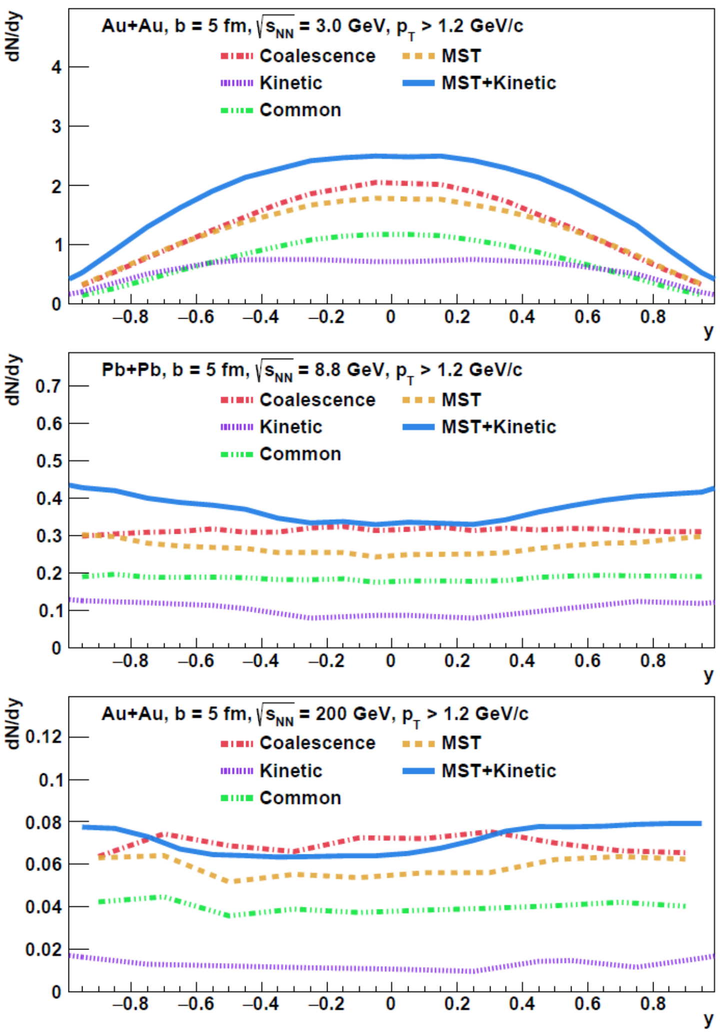


➔ The analysis of the presently available data **points tentatively to the MST + kinetic scenario** but further experimental data are necessary to establish the claster production mechanism.

V. Kireyeu et al., PRC109, 044906

Is the difference big enough for experimental decision?

p_T > 1.2 GeV/c (STAR experimental acceptance)



Difference between coalescence and MST is mostly at low p_T

In the measured p_T range signal is gone for $\sqrt{s}$ = 3 GeV

But: there seems to be a 'sweet spot' around $\sqrt{s} = [6-8]$ GeV to identify the reaction mechanism

➔ More precise experimental data on rapidity distributions are needed

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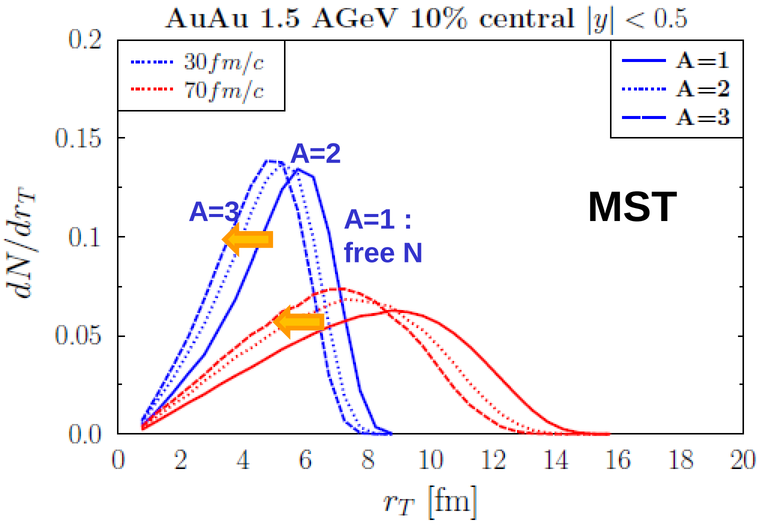
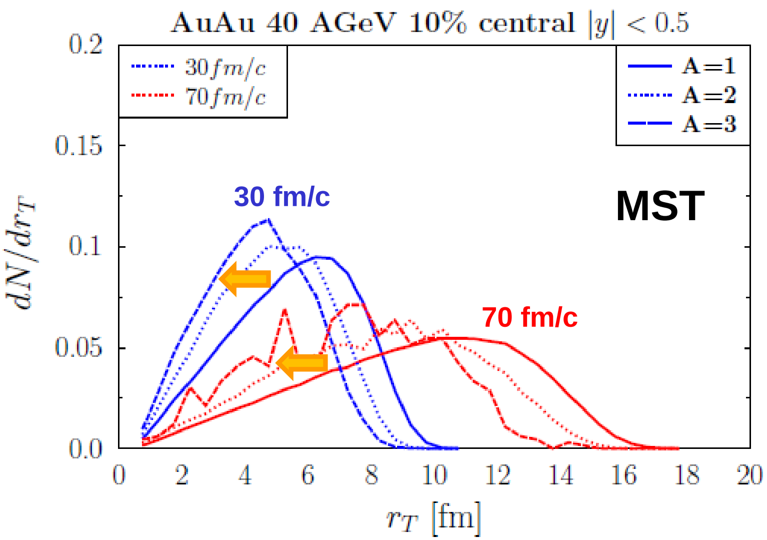
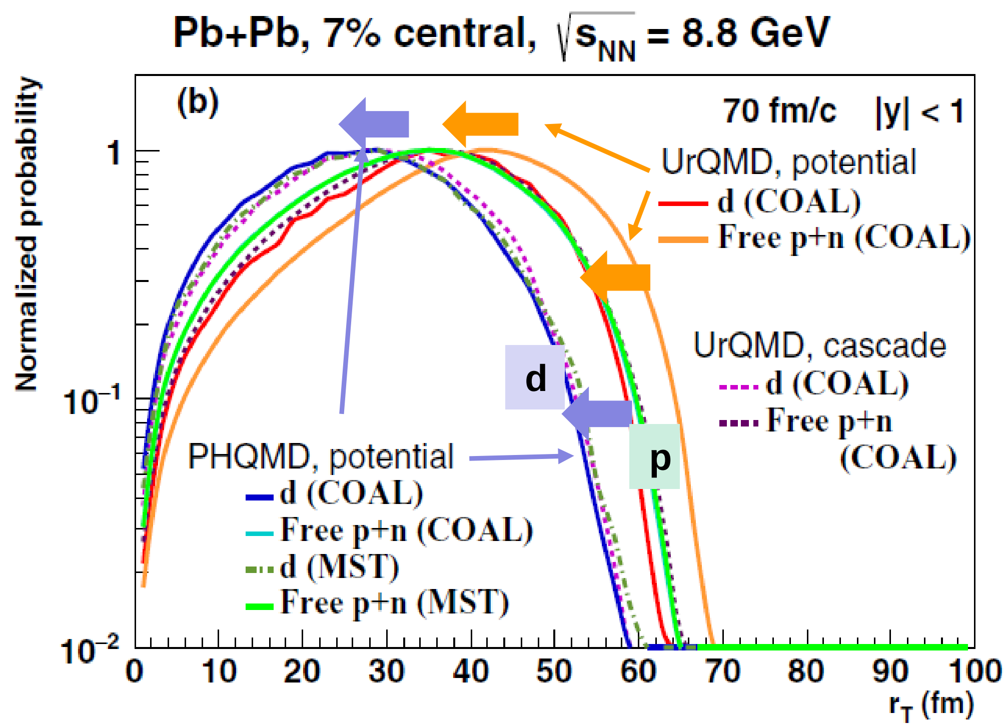
PHQMD and UrQMD: Where clusters are formed?







➔ **Coalescence (COAL) as well as the MST procedure show that the deuterons remain in transverse direction closer to the center of the heavy-ion collision than free nucleons**

➔ **deuterons are behind the fast nucleons (and pion wind)**

V. Kireyeu, J. Steinheimer, M. Bleicher, J. Aichelin, E.B., Phys. Rev. C 105 (2022) 044909

Summary

The PHQMD is a microscopic n-body transport approach for the description of heavy-ion dynamics and cluster and hypernuclei formation identified by Minimum Spanning Tree model

combined model PHQMD = (PHSD & QMD) & (MST | SACA)

Clusters are formed dynamically

1) by potential interactions among nucleons and hyperons
   Novel development: momentum dependent potential with soft EoS
2) by kinetic mechanism for d : hadronic inelastic reactions NN $\leftrightarrow$ dπ , πNN $\leftrightarrow$ dπ , NNN $\leftrightarrow$ dN
   with inclusion of all possible isospin channels which enhance d production
   + accounting of quantum properties of d, modelled by the finite-size excluded volume effect in coordinate space and projection of relative momentum of p+n pair on d wave-function in momentum space which leads to a strong reduction of d production

- The PHQMD reproduces cluster and hypernuclei data on dN/dy and dN/dp_T as well as ratios d/p and $\bar{d}/\bar{p}$ for heavy-ion collisions from SIS to top RHIC energies.
- Measurement of dN/dy beyond mid-rapidity will allow to distinguish the mechanisms for cluster production: coalescence versus dynamical cluster production recognized by MST + kinetic mechanism for deuterons

- Strong dependencee of y- and p_T-spectra and v_1,v_2 on EoS - soft, hard, soft-mom. dependent - at SIS energies
- The influence of U(p) decreases with increasing collision energy since the modelled U_{SEP}(p) has a maximum at energy 1.5 GeV and decreases for large p $\leftarrow$ no exp. data for extrapolation of U_{SEP}(p) to large p!
- HADES data data on v_1,v_2 favour a soft momentum dependent potential (SM)
- STAR data at 3 GeV favour a hard EoS or SM

- Stable clusters are formed shortly after elastic and inelastic collisions have ceased and behind the front of the expanding energetic hadrons (similar results within PHQMD and UrQMD)
  $\rightarrow$ since the 'fire' is not at the same place as the 'ice', cluster can survive