Dependences of Directed Flow on Net Conserved Charges from Quark Coalescence

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Outline

Introduction

(1) Coalescence sum rule relations for Δv_1

(2) Extracting the Δq, ΔS, or ΔB dependences

(3) Demonstration with the STAR data

Summary

(1) and (2) are based on work with Kishora Nayak and Shusu Shi:
Phys. Lett. B 849 (2024) 138479 & Universe 10 (2024) 3, 112

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Introduction

- Directed flow v_1 depends on properties of the created matter in heavy ion collisions such as the equation of state and electromagnetic fields.

Sorge, Phys. Rev. Lett. 78 (1997) 2309; Herrmann, Wessels, Wienold, Annu. Rev. Nucl. Part. Sci. 49 (1999) 581; Das, Plumari, Chatterjee, Alam, Scardina, Greco, Phys. Lett. B 768 (2017) 260; STAR, Phys. Rev. X 14 (2024) 1, 011028

- Directed flow v_1, like elliptic flow v_2, are expected to follow the coalescence sum rule (CSR) or NCQ scaling when initial matter is in parton degrees of freedom and hadronizes via quark coalescence.

Greco, Ko, Levai, Phys. Rev. Lett. 90 (2003) 202302; Fries, Muller, Nonaka, Bass, *Phys. Rev. Lett.* 90 (2003) 202303; *Molnár, Voloshin, Phys. Rev. Lett. 91 (2003) 092301*

- CSR relates the primordial hadron v_n to the sum of v_n of its constituent quarks.

- If 2 combination of hadrons have the same constituent quarks, we expect their v_1 to be the same.

Introduction

To avoid complications from transported u/d quarks,
combinations consisting of 7 produced hadrons ($K^-, \phi, \bar{p}, \overline{\Lambda}, \bar{\Xi}^+, \overline{\Omega}, \Omega$) were constructed:
two sides in each combination have the same $N_{\bar{u}} + N_{\bar{d}}$ and the same $N_s + N_{\bar{s}}$.

Sheikh, Keane, Tribedy, Phys. Rev. C 105 (2022) 014912

Index	Quark mass	Charge	Strangeness	Δv_1 combination
1	$\Delta m = 0$	$\Delta q = 0$	$\Delta S = 0$	$[\bar{p}(\bar{u}\bar{u}\bar{d}) + \phi(s\bar{s})] - [K^-(\bar{u}s) + \bar{\Lambda}(\bar{u}\bar{d}\bar{s})]$
2	$\Delta m = 0$	$\Delta q = 0$	$\Delta S = 0$	$[\overline{\Xi}^+(\bar{d}\bar{s}\bar{s}) + K^-(\bar{u}s)] - [\bar{\Lambda}(\bar{u}\bar{d}\bar{s}) + \phi(s\bar{s})]$
3	$\Delta m \approx 0$	$\Delta q = \frac{1}{3}$	$\Delta S = 0$	$\frac{1}{3}[\Omega^-(sss) + \bar{p}(\bar{u}\bar{u}\bar{d})] - [K^-(\bar{u}s)]$
4	$\Delta m \approx 0$	$\Delta q = \frac{2}{3}$	$\Delta S = 1$	$[\bar{\Lambda}(\bar{u}\bar{d}\bar{s})] - [\frac{1}{2}\phi(s\bar{s}) + \frac{2}{3}\bar{p}(\bar{u}\bar{u}\bar{d})]$
5	$\Delta m \approx 0$	$\Delta q = 1$	$\Delta S = 2$	$[\bar{\Lambda}(\bar{u}\bar{d}\bar{s})] - [\frac{1}{3}\Omega^-(sss) + \frac{2}{3}\bar{p}(\bar{u}\bar{u}\bar{d})]$
6	$\Delta m \approx 0$	$\Delta q = \frac{4}{3}$	$\Delta S = 2$	$[\bar{\Lambda}(\bar{u}\bar{d}\bar{s})] - [K^-(\bar{u}s) + \frac{1}{3}\bar{p}(\bar{u}\bar{u}\bar{d})]$
7	$\Delta m \approx 0$	$\Delta q = \frac{4}{3}$	$\Delta S = 2$	$[\overline{\Xi}^+(\bar{d}\bar{s}\bar{s})] - [\phi(s\bar{s}) + \frac{1}{3}\bar{p}(\bar{u}\bar{u}\bar{d})]$
8	$\Delta m \approx 0$	$\Delta q = \frac{5}{3}$	$\Delta S = 2$	$[\overline{\Xi}^+(\bar{d}\bar{s}\bar{s})] - [K^-(\bar{u}s) + \frac{1}{3}\overline{\Omega}^+(\bar{s}\bar{s}\bar{s})]$
9	$\Delta m = 0$	$\Delta q = 2$	$\Delta S = 6$	$[\overline{\Omega}^+(\bar{s}\bar{s}\bar{s})] - [\Omega^-(sss)]$
10	$\Delta m \approx 0$	$\Delta q = \frac{7}{3}$	$\Delta S = 4$	$[\overline{\Xi}^+(\bar{d}\bar{s}\bar{s})] - [K^-(\bar{u}s) + \frac{1}{3}\Omega^-(sss)]$

Non-zero v_1 difference of the two sides in these hadron sets,
especially if Δv_1 depends on Δq from the data,
was proposed to be a consequence of electromagnetic fields.

STAR, arXiv:2304.02831

Introduction

There are only 5 independent hadron sets for such hadron combinations.

Sheikh, Keane, Tribedy, Phys. Rev. C 105 (2022) 014912

Nayak, Shi & ZWL, Phys. Lett. B 849 (2024) 138479

For example, our default choice = Sets 1, 2, 3, 4, 5A.

Set #	Δq_{ud}	ΔS	Δq	L (left side)	R (right side)
1	0	0	0	$v_1[K^-(\bar{u}s)] + v_1[\overline{\Lambda}(\bar{u}\bar{d}\bar{s})]$	$v_1[\bar{p}(\bar{u}\bar{u}\bar{d})] + v_1[\phi(s\bar{s})]$
2	0	0	0	$v_1\left[\overline{\Lambda}(\bar{u}\bar{d}\bar{s})\right]$	$\frac{1}{2}v_1[\bar{p}(\bar{u}\bar{u}\bar{d})] + \frac{1}{2}v_1[\overline{\Xi}^+(\bar{d}\bar{s}\bar{s})]$
3	0	0	0	$\frac{1}{3}v_1[\Omega^-(sss)] + \frac{1}{3}v_1[\overline{\Omega}^+(\bar{s}\bar{s}\bar{s})]$	$v_1[\phi(s\bar{s})]$
4	0	1	1/3	$\frac{1}{2}v_1[\phi(s\bar{s})]$	$\frac{1}{3}v_1[\Omega^-(sss)]$
5A	1/3	1	2/3	$\frac{1}{2}v_1[\phi(s\bar{s})] + \frac{1}{3}v_1[\bar{p}(\bar{u}\bar{u}\bar{d})]$	$v_1\left[K^-(\bar{u}s)\right]$
5B	1/3	1	2/3	$v_1\left[\overline{\Lambda}(\bar{u}\bar{d}\bar{s})\right]$	$\frac{1}{2}v_1[\phi(s\bar{s})] + \frac{2}{3}v_1[\bar{p}(\bar{u}\bar{u}\bar{d})]$

- One can choose other 5 independent hadron sets, e.g., Sets 1, 2, 3, 4, 5B,

- Any other hadron set will not be independent,

including a hadron set with the hadron weights *scaled* by a constant.

For example, Set5B = Set1+Set5A and is not independent of Choice 1;
hadron set $(3\phi - 2\Omega)$ is equivalent to Set4 $(\phi/2 - \Omega/3)$ (*scaled* by 1/6)

Introduction

Relations between our 5 hadron sets and those used by STAR:

Index1= -Set1, Index2 = Set1+Set4+Set5A, Index3 = Set1+2*Set5A,
Index4 = 3*Set3+6*Set4, Index5 = 2*Set1-2*Set2+Set4+3*Set5A. (1)

So they are equivalent.

Nayak, Shi & ZWL, Phys. Lett. B 849 (2024) 138479

Set #	Δq_{ud}	ΔS	Δq	L (left side)	R (right side)
1	0	0	0	$v_1[K^-(\bar{u}s)] + v_1[\overline{\Lambda}(\bar{u}\bar{d}\bar{s})]$	$v_1[\bar{p}(\bar{u}\bar{u}\bar{d})] + v_1[\phi(s\bar{s})]$
2	0	0	0	$v_1[\overline{\Lambda}(\bar{u}\bar{d}\bar{s})]$	$\frac{1}{2}v_1[\bar{p}(\bar{u}\bar{u}\bar{d})] + \frac{1}{2}v_1[\overline{\Xi}^+(\bar{d}\bar{s}\bar{s})]$
3	0	0	0	$\frac{1}{3}v_1[\Omega^-(sss)] + \frac{1}{3}v_1[\overline{\Omega}^+(\bar{s}\bar{s}\bar{s})]$	$v_1[\phi(s\bar{s})]$
4	0	1	1/3	$\frac{1}{2}v_1[\phi(s\bar{s})]$	$\frac{1}{3}v_1[\Omega^-(sss)]$
5A	1/3	1	2/3	$\frac{1}{2}v_1[\phi(s\bar{s})] + \frac{1}{3}v_1[\bar{p}(\bar{u}\bar{u}\bar{d})]$	$v_1[K^-(\bar{u}s)]$
5B	1/3	1	2/3	$v_1[\overline{\Lambda}(\bar{u}\bar{d}\bar{s})]$	$\frac{1}{2}v_1[\phi(s\bar{s})] + \frac{2}{3}v_1[\bar{p}(\bar{u}\bar{u}\bar{d})]$

STAR, arXiv:2304.02831

Index	Quark mass	Δq	ΔS	Δv_1 combination
1	$\Delta m = 0$	0	0	$[\bar{p}(\bar{u}\bar{u}\bar{d}) + \phi(s\bar{s})] - [K^-(\bar{u}s) + \bar{\Lambda}(\bar{u}\bar{d}\bar{s})]$
2	$\Delta m \approx 0$	1	2	$[\bar{\Lambda}(\bar{u}\bar{d}\bar{s})] - [\frac{1}{3}\Omega^-(sss) + \frac{2}{3}\bar{p}(\bar{u}\bar{u}\bar{d})]$
3	$\Delta m \approx 0$	$\frac{4}{3}$	2	$[\bar{\Lambda}(\bar{u}\bar{d}\bar{s})] - [K^-(\bar{u}s) + \frac{1}{3}\bar{p}(\bar{u}\bar{u}\bar{d})]$
4	$\Delta m = 0$	2	6	$[\overline{\Omega}^+(\bar{s}\bar{s}\bar{s})] - [\Omega^-(sss)]$
5	$\Delta m \approx 0$	$\frac{7}{3}$	4	$[\overline{\Xi}^+(\bar{d}\bar{s}\bar{s})] - [K^-(\bar{u}s) + \frac{1}{3}\Omega^-(sss)]$

Coalescence sum rule relations for Δv_1

Hadrons formed via quark coalescence are expected to follow CSR:

$$v_n^H(p_T^H) = \sum_j v_n^j(p_T^j)$$

Molnár, Voloshin, Phys. Rev. Lett. 91 (2003) 092301
ZWL, Molnár, Phys. Rev. C 68 (2003) 044901

We neglect the mass difference of *u, d* and *s* constituent quarks, then every constituent quark in a hadron has the same p_T:

$$v_n^H(p_T^H) = \sum_j v_n^j(p_T) \qquad (2)$$

When v_n of different quark species are the same, Eq(2) reduces to the NCQ scaling relation:

$$v_2^H(p_T^H) = N_{cq} v_n(p_T)$$
$$\text{with} \quad p_T^H = N_{cq} p_T$$

Coalescence sum rule relations for Δv_1

Index	Quark mass	Charge	Strangeness	Δv_1 combination
1	$\Delta m = 0$	$\Delta q = 0$	$\Delta S = 0$	$[\bar{p}(\bar{u}\bar{u}\bar{d}) + \phi(s\bar{s})] - [K^-(\bar{u}s) + \bar{\Lambda}(\bar{u}\bar{d}\bar{s})]$
2	$\Delta m = 0$	$\Delta q = 0$	$\Delta S = 0$	$[\overline{\Xi}^+(\bar{d}\bar{s}\bar{s}) + K^-(\bar{u}s)] - [\bar{\Lambda}(\bar{u}\bar{d}\bar{s}) + \phi(s\bar{s})]$
3	$\Delta m \approx 0$	$\Delta q = \frac{1}{3}$	$\Delta S = 0$	$\frac{1}{3}[\Omega^-(sss) + \bar{p}(\bar{u}\bar{u}\bar{d})] - [K^-(\bar{u}s)]$
4	$\Delta m \approx 0$	$\Delta q = \frac{2}{3}$	$\Delta S = 1$	$[\bar{\Lambda}(\bar{u}\bar{d}\bar{s})] - [\frac{1}{2}\phi(s\bar{s}) + \frac{2}{3}\bar{p}(\bar{u}\bar{u}\bar{d})]$
5	$\Delta m \approx 0$	$\Delta q = 1$	$\Delta S = 2$	$[\bar{\Lambda}(\bar{u}\bar{d}\bar{s})] - [\frac{1}{3}\Omega^-(sss) + \frac{2}{3}\bar{p}(\bar{u}\bar{u}\bar{d})]$
6	$\Delta m \approx 0$	$\Delta q = \frac{4}{3}$	$\Delta S = 2$	$[\bar{\Lambda}(\bar{u}\bar{d}\bar{s})] - [K^-(\bar{u}s) + \frac{1}{3}\bar{p}(\bar{u}\bar{u}\bar{d})]$
7	$\Delta m \approx 0$	$\Delta q = \frac{4}{3}$	$\Delta S = 2$	$[\overline{\Xi}^+(\bar{d}\bar{s}\bar{s})] - [\phi(s\bar{s}) + \frac{1}{3}\bar{p}(\bar{u}\bar{u}\bar{d})]$
8	$\Delta m \approx 0$	$\Delta q = \frac{5}{3}$	$\Delta S = 2$	$[\overline{\Xi}^+(\bar{d}\bar{s}\bar{s})] - [K^-(\bar{u}s) + \frac{1}{3}\overline{\Omega}^+(\bar{s}\bar{s}\bar{s})]$
9	$\Delta m = 0$	$\Delta q = 2$	$\Delta S = 6$	$[\overline{\Omega}^+(\bar{s}\bar{s}\bar{s})] - [\Omega^-(sss)]$
10	$\Delta m \approx 0$	$\Delta q = \frac{7}{3}$	$\Delta S = 4$	$[\overline{\Xi}^+(\bar{d}\bar{s}\bar{s})] - [K^-(\bar{u}s) + \frac{1}{3}\Omega^-(sss)]$

Index	Quark mass	Δq	ΔS	Δv_1 combination
1	$\Delta m = 0$	0	0	$[\bar{p}(\bar{u}\bar{u}\bar{d}) + \phi(s\bar{s})] - [K^-(\bar{u}s) + \bar{\Lambda}(\bar{u}\bar{d}\bar{s})]$
2	$\Delta m \approx 0$	1	2	$[\bar{\Lambda}(\bar{u}\bar{d}\bar{s})] - [\frac{1}{3}\Omega^-(sss) + \frac{2}{3}\bar{p}(\bar{u}\bar{u}\bar{d})]$
3	$\Delta m \approx 0$	$\frac{4}{3}$	2	$[\bar{\Lambda}(\bar{u}\bar{d}\bar{s})] - [K^-(\bar{u}s) + \frac{1}{3}\bar{p}(\bar{u}\bar{u}\bar{d})]$
4	$\Delta m = 0$	2	6	$[\overline{\Omega}^+(\bar{s}\bar{s}\bar{s})] - [\Omega^-(sss)]$
5	$\Delta m \approx 0$	$\frac{7}{3}$	4	$[\overline{\Xi}^+(\bar{d}\bar{s}\bar{s})] - [K^-(\bar{u}s) + \frac{1}{3}\Omega^-(sss)]$

Set #	Δq_{ud}	ΔS	Δq	L (left side)	R (right side)
1	0	0	0	$v_1[K^-(\bar{u}s)] + v_1[\overline{\Lambda}(\bar{u}\bar{d}\bar{s})]$	$v_1[\bar{p}(\bar{u}\bar{u}\bar{d})] + v_1[\phi(s\bar{s})]$
2	0	0	0	$v_1\left[\overline{\Lambda}(\bar{u}\bar{d}\bar{s})\right]$	$\frac{1}{2}v_1[\bar{p}(\bar{u}\bar{u}\bar{d})] + \frac{1}{2}v_1[\overline{\Xi}^+(\bar{d}\bar{s}\bar{s})]$
3	0	0	0	$\frac{1}{3}v_1[\Omega^-(sss)] + \frac{1}{3}v_1[\overline{\Omega}^+(\bar{s}\bar{s}\bar{s})]$	$v_1[\phi(s\bar{s})]$
4	0	1	1/3	$\frac{1}{2}v_1[\phi(s\bar{s})]$	$\frac{1}{3}v_1[\Omega^-(sss)]$
5A	1/3	1	2/3	$\frac{1}{2}v_1[\phi(s\bar{s})] + \frac{1}{3}v_1[\bar{p}(\bar{u}\bar{u}\bar{d})]$	$v_1\left[K^-(\bar{u}s)\right]$
5B	1/3	1	2/3	$v_1\left[\overline{\Lambda}(\bar{u}\bar{d}\bar{s})\right]$	$\frac{1}{2}v_1[\phi(s\bar{s})] + \frac{2}{3}v_1[\bar{p}(\bar{u}\bar{u}\bar{d})]$

For every such hadron set,
the two sides have the same $N_{\bar{u}} + N_{\bar{d}}$ and the same $N_s + N_{\bar{s}}$.
So the differences between the two sides
in electric charge and strangeness are given by

$$\Delta q = \Delta N_{\bar{d}} + \frac{2}{3}\Delta N_{\bar{s}}\,, \qquad \Delta S = 2\Delta N_{\bar{s}}$$

CSR of Eq(2) then gives $\Delta v_1 \equiv v_1^L - v_1^R = \sum_{i=\bar{u},\bar{d},s,\bar{s}} \Delta N_i \, v_{1,i}$

$$= \left(v_{1,\bar{d}} - v_{1,\bar{u}}\right)\Delta q + \left(\frac{v_{1,\bar{s}} - v_{1,s}}{2} - \frac{v_{1,\bar{d}} - v_{1,\bar{u}}}{3}\right)\Delta S \quad (3)$$

Coalescence sum rule relations for Δv_1

The relation gets simpler
when we use the electric charge difference from $\bar{u}$ and $\bar{d}$:

$$\Delta q_{ud} \equiv q^L_{ud} - q^R_{ud} = \Delta q - \frac{1}{3}\Delta S$$

Then Eq(3) becomes

$$\Delta v_1 = \left(v_{1,\bar{d}} - v_{1,\bar{u}}\right)\Delta q_{ud} + \left(\frac{v_{1,\bar{s}} - v_{1,s}}{2}\right)\Delta S \qquad (4)$$

So CSR predicts:

- Δv_1 depends linearly on both Δq and ΔS
- the coefficients reflect quark-level v_1 differences
  for quarks of different electric charges,
  so they should be affected by electromagnetic fields

- $\Delta v_1 = 0$ if $\Delta q = \Delta S = 0$, while $\Delta v_1 \neq 0$ if Δq or $\Delta S \neq 0$

Extracting the Δq, ΔS, or ΔB dependences

The same applies to the v_1 slope differences:

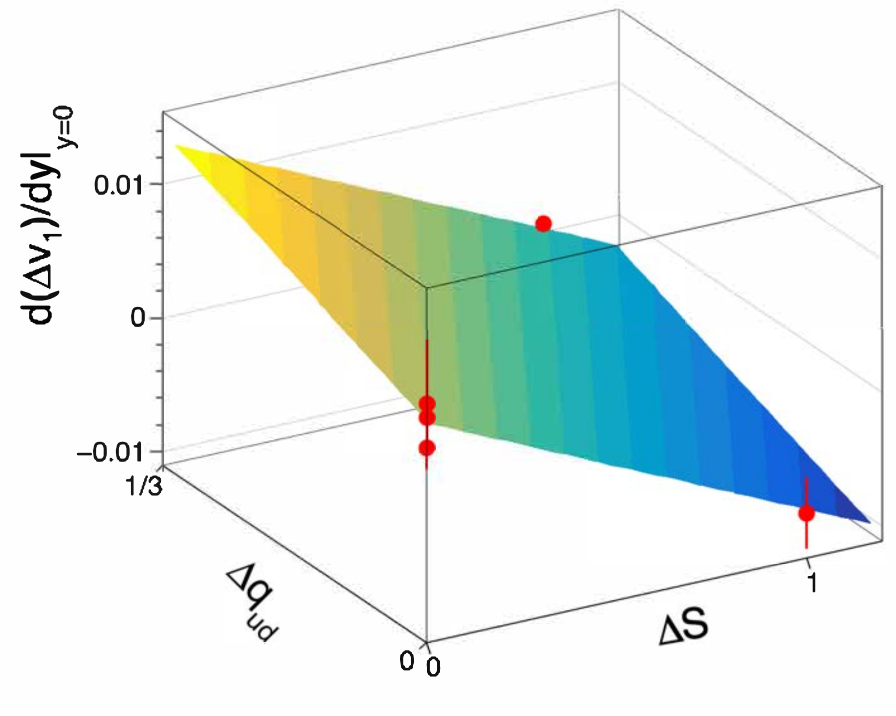
$$\Delta v_1' = (v_{1,\bar{d}}' - v_{1,\bar{u}}')\Delta q_{ud} + \left(\frac{v_{1,\bar{s}}' - v_{1,s}'}{2}\right)\Delta S$$

$$= (v_{1,\bar{d}}' - v_{1,\bar{u}}')\Delta q + \left(\frac{v_{1,\bar{s}}' - v_{1,s}'}{2} - \frac{v_{1,\bar{d}}' - v_{1,\bar{u}}'}{3}\right)\Delta S$$

To extract the dependences,
a 2-variable linear function (*a plane over* Δq *and* ΔS) should be used
to fit the five independent data points (we call this the 5-set method):

$$\Delta v_1' = c_0 + c_q \Delta q_{ud} + c_S \Delta S \quad \text{or} \quad c_0^* + c_q^* \Delta q + c_S^* \Delta S$$

Nayak, Shi & ZWL, Phys. Lett. B 849 (2024) 138479



- Although CSR expects $c_0 = 0$ and $c_0^* = 0$, we add them in the fit function, because a non-zero c_0 or c_0^* value extracted from data indicates the breaking of CSR.
- A linear combination or scaling of hadron sets changes the value of non-zero c_0 or c_0^*, so dependence of coefficients on the choice of hadron sets also indicates the breaking of CSR.

Extracting the Δq, ΔS, or ΔB dependences

Set #	Δq_{ud}	ΔS	Δq	L (left side)	R (right side)
1	0	0	0	$v_1[K^-(\bar{u}s)] + v_1[\overline{\Lambda}(\bar{u}\bar{d}\bar{s})]$	$v_1[\bar{p}(\bar{u}\bar{u}\bar{d})] + v_1[\phi(s\bar{s})]$
2	0	0	0	$v_1\left[\overline{\Lambda}(\bar{u}\bar{d}\bar{s})\right]$	$\frac{1}{2}v_1[\bar{p}(\bar{u}\bar{u}\bar{d})] + \frac{1}{2}v_1[\overline{\Xi}^+(\bar{d}\bar{s}\bar{s})]$
3	0	0	0	$\frac{1}{3}v_1[\Omega^-(sss)] + \frac{1}{3}v_1[\overline{\Omega}^+(\bar{s}\bar{s}\bar{s})]$	$v_1[\phi(s\bar{s})]$
4	0	1	1/3	$\frac{1}{2}v_1[\phi(s\bar{s})]$	$\frac{1}{3}v_1[\Omega^-(sss)]$
5A	1/3	1	2/3	$\frac{1}{2}v_1[\phi(s\bar{s})] + \frac{1}{3}v_1[\bar{p}(\bar{u}\bar{u}\bar{d})]$	$v_1\left[K^-(\bar{u}s)\right]$
5B	1/3	1	2/3	$v_1\left[\overline{\Lambda}(\bar{u}\bar{d}\bar{s})\right]$	$\frac{1}{2}v_1[\phi(s\bar{s})] + \frac{2}{3}v_1[\bar{p}(\bar{u}\bar{u}\bar{d})]$

Nayak, Shi & ZWL, Phys. Lett. B 849 (2024) 138479

Since our Sets 1, 2, 3 all have $\Delta q_{ud} = \Delta S = \Delta q = 0$, we can combine their data into one.
Then we will have 3 data points: $\Delta v'_{1,1-3}$, $\Delta v'_{1,4}$ & $\Delta v'_{1,5}$.

For $\Delta v'_1 = c_0 + c_q \Delta q_{ud} + c_S \Delta S$ we can analytically extract

$$c_0 = \Delta v'_{1,1-3}$$
$$c_q = -3(\Delta v'_{1,4} - \Delta v'_{1,5})$$
$$c_S = -\Delta v'_{1,1-3} + \Delta v'_{1,4}$$

We call this the 3-set method.

Extracting the Δq, ΔS, or ΔB dependences

CSR gives
$$\Delta v_1' = c_0 + c_q \Delta q_{ud} + c_S \Delta S$$

with $c_0 = 0$, $c_q = v'_{1,\bar{d}} - v'_{1,\bar{u}}$, $c_S = \frac{1}{2}(v'_{1,\bar{s}} - v'_{1,s})$

For these hadron combinations, $\Delta B = -\Delta S/3$,
so $\Delta v_1'$ can also be written as

$$\Delta v_1' = c_0 + c_q \Delta q_{ud} + c_B \Delta B$$

with $c_B = -3c_S = \frac{3}{2}(v'_{1,s} - v'_{1,\bar{s}})$

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Demonstration with the STAR data

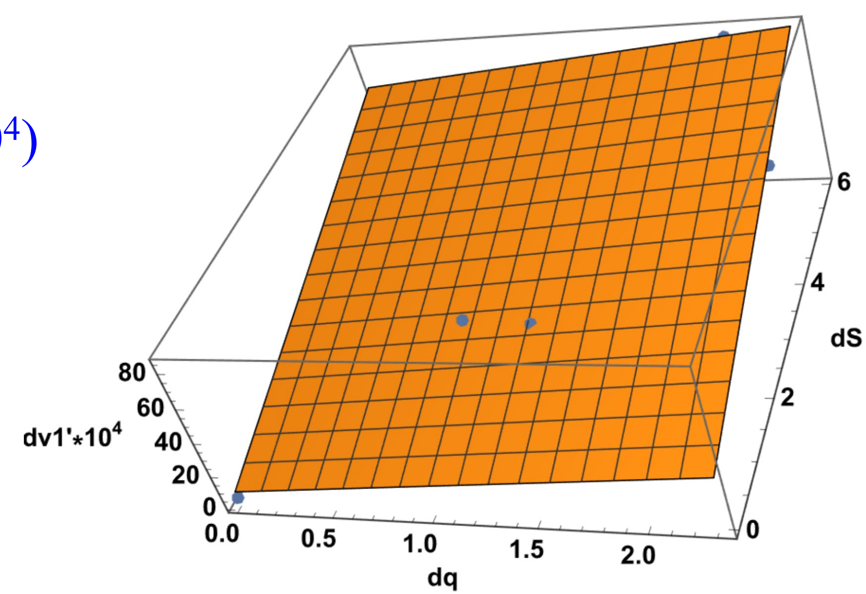
We now demonstrate a correct way of extracting the Δq and ΔS dependences by using the STAR 10-40% Au+Au data at 27A GeV as example (*central values only for demonstration, without considering the error bars*).

Index	Quark mass	Δq	ΔS	Δv_1 combination	$F_\Delta \times 10^4$ (27 GeV) $= \Delta v_1' * 10^4$
1	$\Delta m = 0$	0	0	$[\bar{p}(\bar{u}\bar{u}\bar{d}) + \phi(s\bar{s})] - [K^{-}(\bar{u}s) + \bar{\Lambda}(\bar{u}\bar{d}\bar{s})]$	$03 \pm 43 \pm 13$
2	$\Delta m \approx 0$	1	2	$[\bar{\Lambda}(\bar{u}\bar{d}\bar{s})] - [\frac{1}{3}\Omega^{-}(sss) + \frac{2}{3}\bar{p}(\bar{u}\bar{u}\bar{d})]$	$41 \pm 25 \pm 16$
3	$\Delta m \approx 0$	$\frac{4}{3}$	2	$[\bar{\Lambda}(\bar{u}\bar{d}\bar{s})] - [K^{-}(\bar{u}s) + \frac{1}{3}\bar{p}(\bar{u}\bar{u}\bar{d})]$	$39 \pm 07 \pm 03$
4	$\Delta m = 0$	2	6	$[\overline{\Omega}^{+}(\bar{s}\bar{s}\bar{s})] - [\Omega^{-}(sss)]$	$83 \pm 130 \pm 25$
5	$\Delta m \approx 0$	$\frac{7}{3}$	4	$[\overline{\Xi}^{+}(\bar{d}\bar{s}\bar{s})] - [K^{-}(\bar{u}s) + \frac{1}{3}\Omega^{-}(sss)]$	$64 \pm 36 \pm 19$

STAR, arXiv:2304.02831

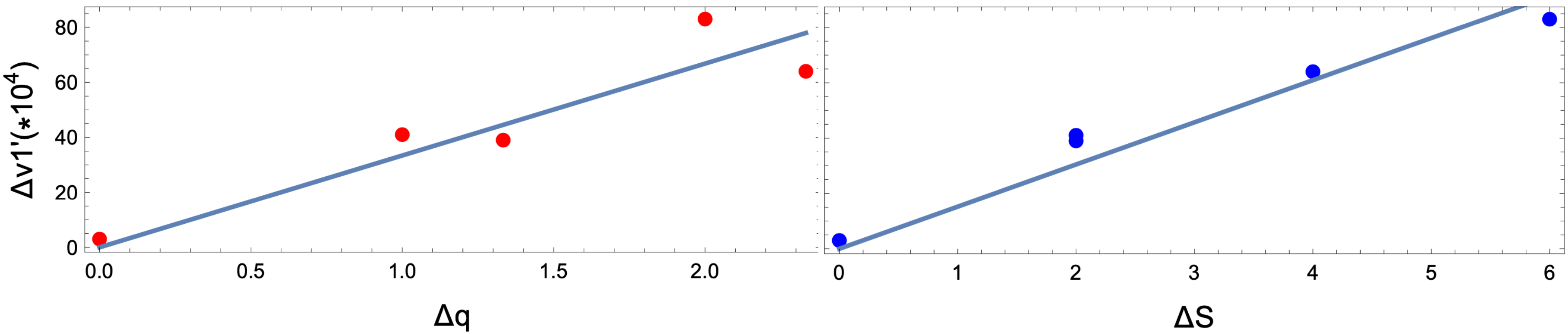
Demo1) Using the 5-set method,
we perform 2-D fit of the five data points (after $*10^4$)
with $\Delta v_1' = c_0^* + c_q^* \Delta q + c_S^* \Delta S$

We get $\Delta v_1' = 6.66 + 8.66\,\Delta q + 9.93\,\Delta S$
or $12.8\,\Delta q + 9.67\,\Delta S$ (without intercept c_0^*)



Demonstration with the STAR data

When we follow the STAR method in *STAR arXiv:2304.02831* and perform 1-D fits
with $\Delta v_1' = K_{\Delta q}\, \Delta q$ & $\Delta v_1' = K_{\Delta S}\, \Delta S$ (without intercept),
we get $\Delta v_1' = 33.4\, \Delta q$ & $\Delta v_1' = 15.2\, \Delta S$



They are close to the STAR values in *STAR arXiv:2304.02831* :
$K_{\Delta q}(\mathrm{x}10^4) = 29 \pm 4.2 \pm 3.7$, $K_{\Delta S}(\mathrm{x}10^4) = 19 \pm 2.8 \pm 2.5$.

However, some of them are **far** from values from the 2-D fit:

$$\Delta v_1' = 6.66 + 8.66\, \Delta q + 9.93\, \Delta S$$

or $\mathbf{12.8}\, \Delta q + 9.67\, \Delta S$ (without intercept c_0^*)

Since $\Delta v_1'$ is a linear function of **both** Δq and ΔS, one should use 2-D fit.
One cannot use 1-D fit, e.g., simply fit data as a 1-D function of Δq without correcting for the different ΔS values of the data points.

Demonstration with the STAR data

Demo2) We can convert the STAR hadron sets into our hadron sets:

Index	Quark mass	Δq	ΔS	Δv_1 combination	$F_\Delta \times 10^4$ (27 GeV)
1	$\Delta m = 0$	0	0	$[\bar{p}(\bar{u}\bar{u}\bar{d}) + \phi(s\bar{s})] - [K^-(\bar{u}s) + \bar{\Lambda}(\bar{u}\bar{d}\bar{s})]$	03 $\pm 43 \pm 13$
2	$\Delta m \approx 0$	1	2	$[\bar{\Lambda}(\bar{u}\bar{d}\bar{s})] - [\frac{1}{3}\Omega^-(sss) + \frac{2}{3}\bar{p}(\bar{u}\bar{u}\bar{d})]$	41 $\pm 25 \pm 16$
3	$\Delta m \approx 0$	$\frac{4}{3}$	2	$[\bar{\Lambda}(\bar{u}\bar{d}\bar{s})] - [K^-(\bar{u}s) + \frac{1}{3}\bar{p}(\bar{u}\bar{u}\bar{d})]$	39 $\pm 07 \pm 03$
4	$\Delta m = 0$	2	6	$[\overline{\Omega}^+(\bar{s}\bar{s}\bar{s})] - [\Omega^-(sss)]$	83 $\pm 130 \pm 25$
5	$\Delta m \approx 0$	$\frac{7}{3}$	4	$[\overline{\Xi}^+(\bar{d}\bar{s}\bar{s})] - [K^-(\bar{u}s) + \frac{1}{3}\Omega^-(sss)]$	64 $\pm 36 \pm 19$

Eq(1) ⇒

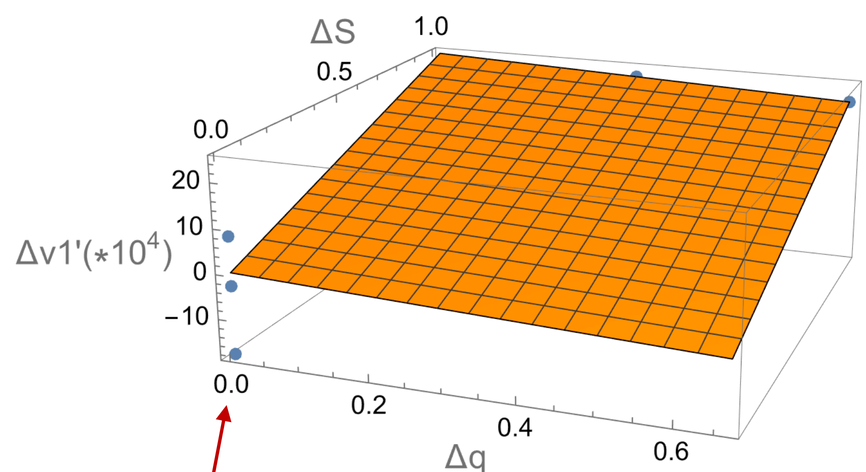
	Set #	Δq_{ud}	ΔS	Δq	L (left side)	R (right side)
-3	1	0	0	0	$v_1[K^-(\bar{u}s)] + v_1[\overline{\Lambda}(\bar{u}\bar{d}\bar{s})]$	$v_1[\bar{p}(\bar{u}\bar{u}\bar{d})] + v_1[\phi(s\bar{s})]$
8	2	0	0	0	$v_1[\overline{\Lambda}(\bar{u}\bar{d}\bar{s})]$	$\frac{1}{2}v_1[\bar{p}(\bar{u}\bar{u}\bar{d})] + \frac{1}{2}v_1[\overline{\Xi}^+(\bar{d}\bar{s}\bar{s})]$
-55/3	3	0	0	0	$\frac{1}{3}v_1[\Omega^-(sss)] + \frac{1}{3}v_1[\overline{\Omega}^+(\bar{s}\bar{s}\bar{s})]$	$v_1[\phi(s\bar{s})]$
23	4	0	1	1/3	$\frac{1}{2}v_1[\phi(s\bar{s})]$	$\frac{1}{3}v_1[\Omega^-(sss)]$
21	5A	1/3	1	2/3	$\frac{1}{2}v_1[\phi(s\bar{s})] + \frac{1}{3}v_1[\bar{p}(\bar{u}\bar{u}\bar{d})]$	$v_1[K^-(\bar{u}s)]$

From 2-D fit of our five data points

with $\Delta v_1' = c_0^* + c_q^*\Delta q + c_S^*\Delta S,$

we get $\Delta v_1' = -4.44 - 6.00\,\Delta q + 29.4\,\Delta S$

or -6.00 Δq + 25.0 ΔS (without intercept c_0^*)



They are very different from values from the 2-D fit of the STAR hadron sets:

$\Delta v_1' = 6.66 + 8.66\,\Delta q + 9.93\,\Delta S$

or 12.8 Δq + 9.67 ΔS (without intercept c_0^*)

This difference is due to the nonzero value at $\Delta q = \Delta S = 0$ (or nonzero c_0^*), which indicates the breaking of CSR.

Demonstration with the STAR data

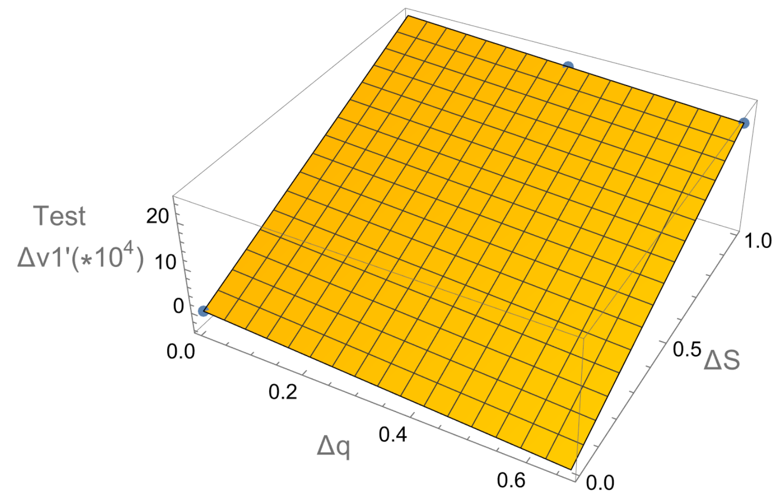
To further demonstrate this, we set $\Delta v_1'$ data=0 at $\Delta q = \Delta S = 0$ (to test the ideal CSR case) and then convert to the STAR hadron sets:

Index	Quark mass	Δq	ΔS	Δv_1 combination	
1	$\Delta m = 0$	0	0	$[\bar{p}(\bar{u}\bar{u}\bar{d}) + \phi(s\bar{s})] - [K^-(\bar{u}s) + \bar{\Lambda}(\bar{u}\bar{d}\bar{s})]$	0
2	$\Delta m \approx 0$	1	2	$[\bar{\Lambda}(\bar{u}\bar{d}\bar{s})] - [\frac{1}{3}\Omega^-(sss) + \frac{2}{3}\bar{p}(\bar{u}\bar{u}\bar{d})]$	44
3	$\Delta m \approx 0$	$\frac{4}{3}$	2	$[\bar{\Lambda}(\bar{u}\bar{d}\bar{s})] - [K^-(\bar{u}s) + \frac{1}{3}\bar{p}(\bar{u}\bar{u}\bar{d})]$	42
4	$\Delta m = 0$	2	6	$[\bar{\Omega}^+(\bar{s}\bar{s}\bar{s})] - [\Omega^-(sss)]$	138
5	$\Delta m \approx 0$	$\frac{7}{3}$	4	$[\bar{\Xi}^+(\bar{d}\bar{s}\bar{s})] - [K^-(\bar{u}s) + \frac{1}{3}\Omega^-(sss)]$	86

Eq(1)

	Set #	Δq_{ud}	ΔS	Δq	L (left side)	R (right side)
0	1	0	0	0	$v_1[K^-(\bar{u}s)] + v_1[\bar{\Lambda}(\bar{u}\bar{d}\bar{s})]$	$v_1[\bar{p}(\bar{u}\bar{u}\bar{d})] + v_1[\phi(s\bar{s})]$
0	2	0	0	0	$v_1[\bar{\Lambda}(\bar{u}\bar{d}\bar{s})]$	$\frac{1}{2}v_1[\bar{p}(\bar{u}\bar{u}\bar{d})] + \frac{1}{2}v_1[\bar{\Xi}^+(\bar{d}\bar{s}\bar{s})]$
0	3	0	0	0	$\frac{1}{3}v_1[\Omega^-(sss)] + \frac{1}{3}v_1[\bar{\Omega}^+(\bar{s}\bar{s}\bar{s})]$	$v_1[\phi(s\bar{s})]$
23	4	0	1	1/3	$\frac{1}{2}v_1[\phi(s\bar{s})]$	$\frac{1}{3}v_1[\Omega^-(sss)]$
21	5A	1/3	1	2/3	$\frac{1}{2}v_1[\phi(s\bar{s})] + \frac{1}{3}v_1[\bar{p}(\bar{u}\bar{u}\bar{d})]$	$v_1[K^-(\bar{u}s)]$

- From 2-D fit of the STAR sets test data, we get $\Delta v_1' = 0.00 - \mathbf{6.00}\,\Delta q + 25.0\,\Delta S$ same as fit w/o intercept on previous slide (as expected).
- 2-D fit of our hadron sets gives exactly the same result (as expected).
- So STAR and our hadron sets are equivalent *(for the ideal case where CSR is satisfied).*

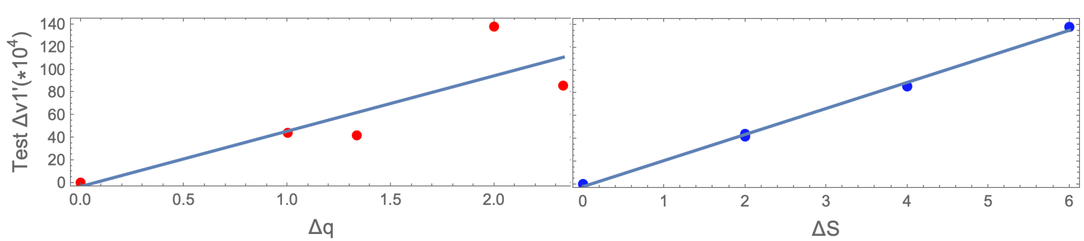


When we follow *STAR arXiv:2304.02831*, 1-D fits of the STAR sets test data give

$$\Delta v_1' = -3.33 + 49.0\,\Delta q \quad \text{or} \quad 47.2\,\Delta q \text{ (without intercept)}$$

& $$\Delta v_1' = -2.08 + 22.9\,\Delta S \quad \text{or} \quad 22.4\,\Delta S \text{ (without intercept)}$$

Some of them can be **far** from values of the 2-D fit and are thus incorrect.



Demonstration with the STAR data

Demo3) we scale the STAR Index4 set by ½ (*still the same ideal case v_1 test data*):

Index	Quark mass	Δq	ΔS	Δv_1 combination	
1	$\Delta m = 0$	0	0	$[\bar{p}(\bar{u}\bar{u}\bar{d}) + \phi(s\bar{s})] - [K^{-}(\bar{u}s) + \bar{\Lambda}(\bar{u}\bar{d}\bar{s})]$	0
2	$\Delta m \approx 0$	1	2	$[\bar{\Lambda}(\bar{u}\bar{d}\bar{s})] - [\frac{1}{3}\Omega^{-}(sss) + \frac{2}{3}\bar{p}(\bar{u}\bar{u}\bar{d})]$	44
3	$\Delta m \approx 0$	$\frac{4}{3}$	2	$[\bar{\Lambda}(\bar{u}\bar{d}\bar{s})] - [K^{-}(\bar{u}s) + \frac{1}{3}\bar{p}(\bar{u}\bar{u}\bar{d})]$	42
4	$\Delta m = 0$	2	6	$[\bar{\Omega}^{+}(\bar{s}\bar{s}\bar{s})] - [\Omega^{-}(sss)]$	138
5	$\Delta m \approx 0$	$\frac{7}{3}$	4	$[\bar{\Xi}^{+}(\bar{d}\bar{s}\bar{s})] - [K^{-}(\bar{u}s) + \frac{1}{3}\Omega^{-}(sss)]$	86

→

	Index	Quark mass	Δq	ΔS	Δv_1 combination
0	1	$\Delta m = 0$	0	0	$[\bar{p}(\bar{u}\bar{u}\bar{d}) + \phi(s\bar{s})] - [K^{-}(\bar{u}s) + \bar{\Lambda}(\bar{u}\bar{d}\bar{s})]$
44	2	$\Delta m \approx 0$	1	2	$[\bar{\Lambda}(\bar{u}\bar{d}\bar{s})] - [\frac{1}{3}\Omega^{-}(sss) + \frac{2}{3}\bar{p}(\bar{u}\bar{u}\bar{d})]$
42	3	$\Delta m \approx 0$	$\frac{4}{3}$	2	$[\bar{\Lambda}(\bar{u}\bar{d}\bar{s})] - [K^{-}(\bar{u}s) + \frac{1}{3}\bar{p}(\bar{u}\bar{u}\bar{d})]$
69	4	$\Delta m = 0$	2 **/2**	6**/2**	**(**$[\bar{\Omega}^{+}(\bar{s}\bar{s}\bar{s})] - [\Omega^{-}(sss)]$**)/2**
86	5	$\Delta m \approx 0$	$\frac{7}{3}$	4	$[\bar{\Xi}^{+}(\bar{d}\bar{s}\bar{s})] - [K^{-}(\bar{u}s) + \frac{1}{3}\Omega^{-}(sss)]$

2-D fits of unscaled & scaled test data give exactly same result (as expected):

$$\Delta v_1' = 0.00 - 6.00\Delta q + 25.0\Delta S.$$

From 1-D fits of the scaled test data, we get

$\Delta v_1' = 9.13 + 34.5\ \Delta q$ or $40.1\ \Delta q$ (w/o intercept)

& $\Delta v_1' = 21.9\ \Delta S$ (with or w/o intercept).

They are different from 1-D fits of the unscaled test data:

$\Delta v_1' = -3.33 + \mathbf{49.0}\ \Delta q$ or $\mathbf{47.2}\ \Delta q$ (w/o intercept)

& $\Delta v_1' = -2.08 + 22.9\ \Delta S.$ or $22.4\ \Delta S$ (w/o intercept).

→ 1-D slope parameters depend on the **arbitrary** scaling factor of hadron sets, this is unphysical.

Demonstration with the STAR data

Demo3) we scale the STAR Index4 set by ½ (*still the same ideal case v_1 test data*):

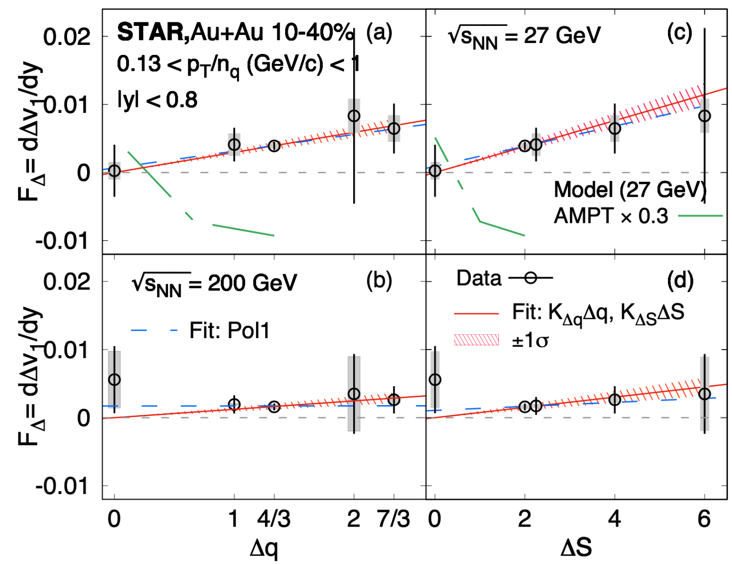
Index	Quark mass	Δq	ΔS	Δv_1 combination	
1	$\Delta m = 0$	0	0	$[\bar{p}(\bar{u}\bar{u}\bar{d}) + \phi(s\bar{s})] - [K^-(\bar{u}s) + \bar{\Lambda}(\bar{u}\bar{d}\bar{s})]$	0
2	$\Delta m \approx 0$	1	2	$[\bar{\Lambda}(\bar{u}\bar{d}\bar{s})] - [\frac{1}{3}\Omega^-(sss) + \frac{2}{3}\bar{p}(\bar{u}\bar{u}\bar{d})]$	44
3	$\Delta m \approx 0$	$\frac{4}{3}$	2	$[\bar{\Lambda}(\bar{u}\bar{d}\bar{s})] - [K^-(\bar{u}s) + \frac{1}{3}\bar{p}(\bar{u}\bar{u}\bar{d})]$	42
4	$\Delta m = 0$	2	6	$[\bar{\Omega}^+(\bar{s}\bar{s}\bar{s})] - [\Omega^-(sss)]$	138
5	$\Delta m \approx 0$	$\frac{7}{3}$	4	$[\bar{\Xi}^+(\bar{d}\bar{s}\bar{s})] - [K^-(\bar{u}s) + \frac{1}{3}\Omega^-(sss)]$	86

→

0
44
42
69
86

Index	Quark mass	Δq	ΔS	Δv_1 combination
1	$\Delta m = 0$	0	0	$[\bar{p}(\bar{u}\bar{u}\bar{d}) + \phi(s\bar{s})] - [K^-(\bar{u}s) + \bar{\Lambda}(\bar{u}\bar{d}\bar{s})]$
2	$\Delta m \approx 0$	1	2	$[\bar{\Lambda}(\bar{u}\bar{d}\bar{s})] - [\frac{1}{3}\Omega^-(sss) + \frac{2}{3}\bar{p}(\bar{u}\bar{u}\bar{d})]$
3	$\Delta m \approx 0$	$\frac{4}{3}$	2	$[\bar{\Lambda}(\bar{u}\bar{d}\bar{s})] - [K^-(\bar{u}s) + \frac{1}{3}\bar{p}(\bar{u}\bar{u}\bar{d})]$
4	$\Delta m = 0$	2 **/2**	6**/2**	**(**$[\bar{\Omega}^+(\bar{s}\bar{s}\bar{s})] - [\Omega^-(sss)]$**)/2**
5	$\Delta m \approx 0$	$\frac{7}{3}$	4	$[\bar{\Xi}^+(\bar{d}\bar{s}\bar{s})] - [K^-(\bar{u}s) + \frac{1}{3}\Omega^-(sss)]$

Slope parameters $K_{\Delta q}$ & $K_{\Delta S}$ from 1-D fits depend on the **arbitrary** scaling factor of hadron sets in the ideal CSR case and are thus ill-defined;
so one cannot do simple 1-D fit of the data points.



STAR, arXiv:2304.02831

Note: the numerical differences between 1-D and 2-D coefficients depend on uncertainties of the actual data; the difference could be small when data error bars are large.
Our purpose here is to show the 1-D fit method to be mathematically incorrect.

Summary

Relations for v_1 differences of combinations of produced hadrons
are derived from the quark coalescence sum rule (CSR).

Δv_1 or $\Delta v_1'$ is a linear function of both Δq and ΔS, or of both Δq and ΔB,
and the coefficients reflect quark-level v_1 differences.

To extract the dependences from the five independent data points,
one cannot do simple 1-D fit because those coefficients are ill-defined.
One should do 2-D fit with $\Delta v_1' = c_0^* + c_q^* \Delta q + c_S^* \Delta S$ or $c_0 + c_q \Delta q_{ud} + c_S \Delta S$;
nonzero c_0 or c_0^* (or dependence of coefficients on choice of hadron sets)
indicates the breaking or CSR.

The coefficients can be obtained analytically
after we combine the five independent data points into three.

Electromagnetic fields do not affect these CSR relations,
but they should affect the value of the coefficients.

Thanks for your attention!