CRITICAL POINT FLUCTUATIONS IN HEAVY-ION COLLISIONS WITHIN MOLECULAR DYNAMICS WITH EXPANSION

Volodymyr Kuznietsov

Based on: V.K., M.I. Gorenstein, V. Koch, V. Vovchenko, arXiv: 2404.00476 [nucl-th]

UNIVERSITY OF HOUSTON

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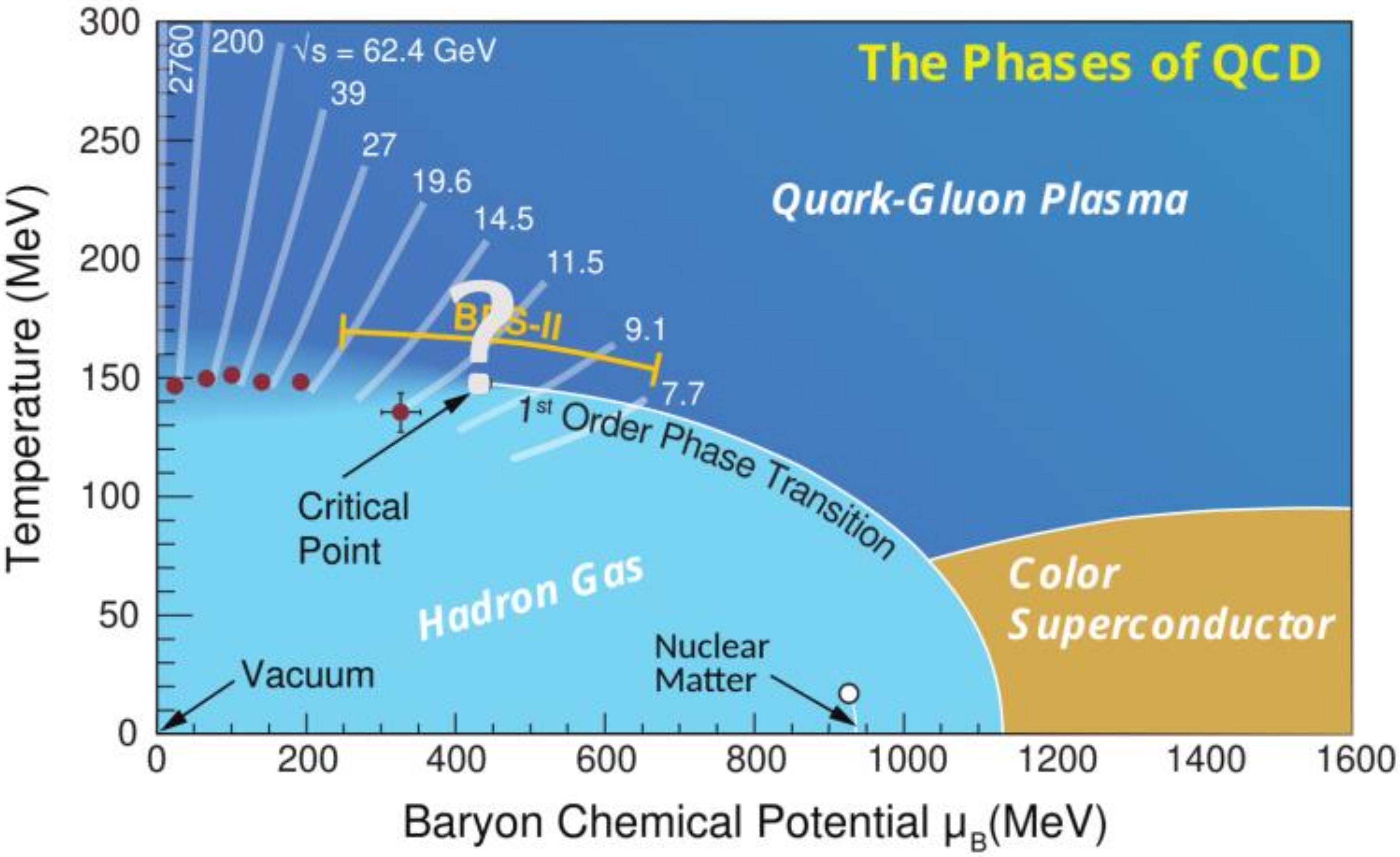
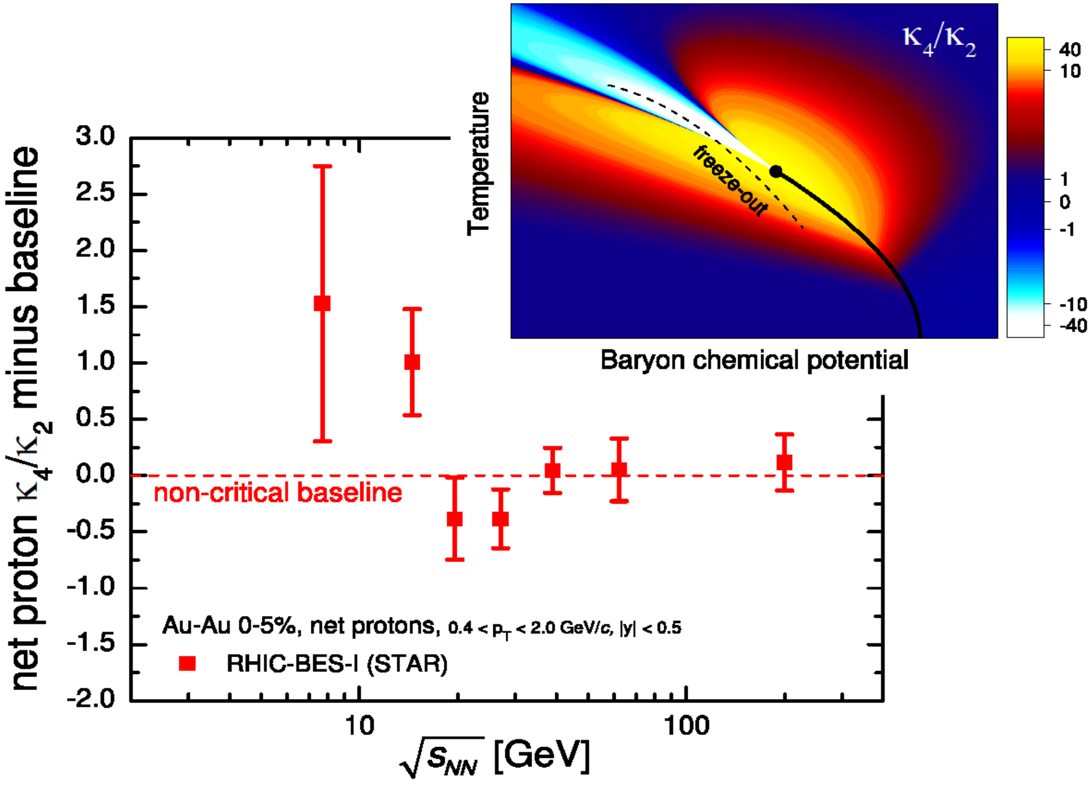
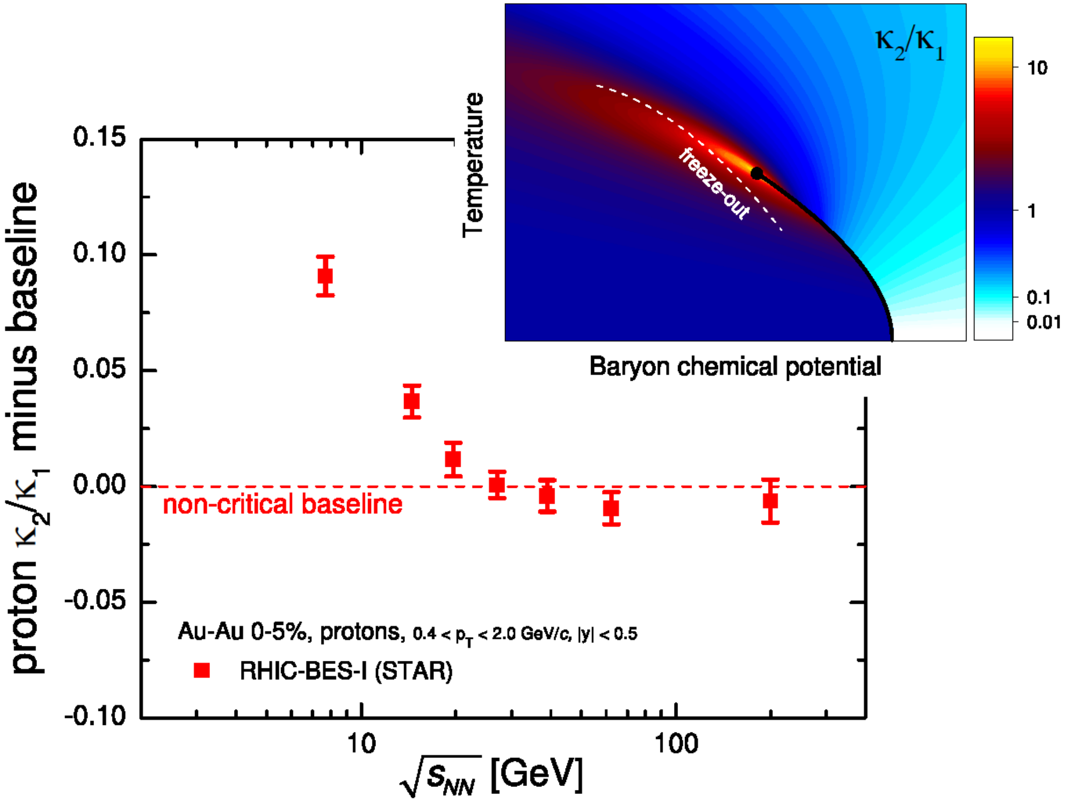
QCD PHASE DIAGRAM



Bzdak et al., Phys. Rept. 2020 & 2015 Long Range plan

FLUCTUATIONS AS CP SIGNATURE





[1] Non-critical baseline: Vovchenko, Koch, Shen, PRC 105, 014904 (2022)
[2] STAR data (BES-I): J. Adam et al., PRL 126, 092301 (2021)

CONNECTION TO THE EXPERIMENT

Experiment

- Momentum space
- Expanding in vacuum
- Non-conserved particle numbers
- Inhomogenous
- Fluctuating volume

Theory

- Coordinate and/or momentum space
- In contact with the heat bath
- Conserved charges
- Uniform
- Fixed volume

Here we study critical fluctuations in a microscopic approach **(MD)**

LENNARD-JONES POTENTIAL

The Lennard-Jones potential reads

$$V_{LJ} = 4\varepsilon[(\sigma/r)^6 - (\sigma/r)^{12}]$$

In reduced dimensionless variables it can be rewritten as

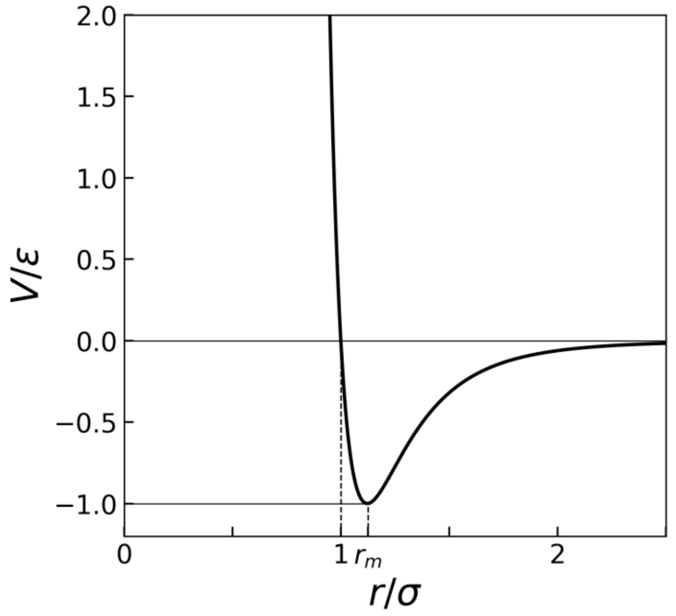
$$\tilde{V}_{LJ} = 4(\tilde{r}^{-6} - \tilde{r}^{-12})$$

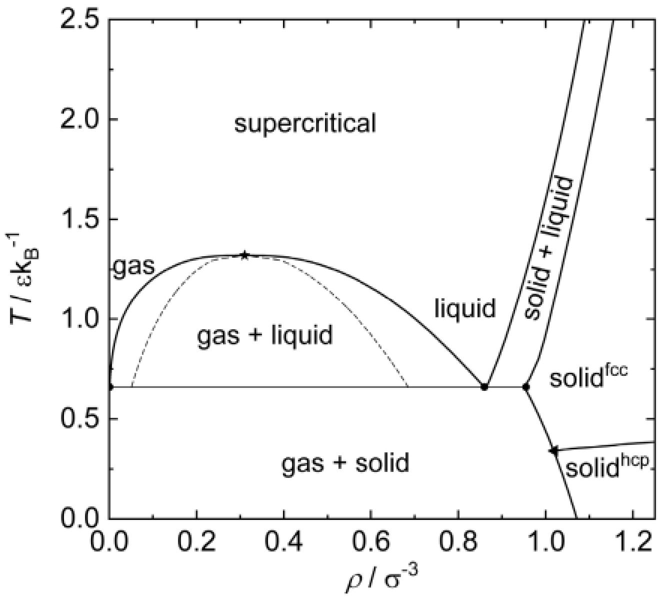
where the reduced variables are use:

$$\tilde{r} = r/\sigma, \quad \tilde{t} = t\sqrt{\varepsilon/(m\sigma^2)}, \quad \tilde{V}_{LJ} = V_{LJ}/\varepsilon$$

Advantages:

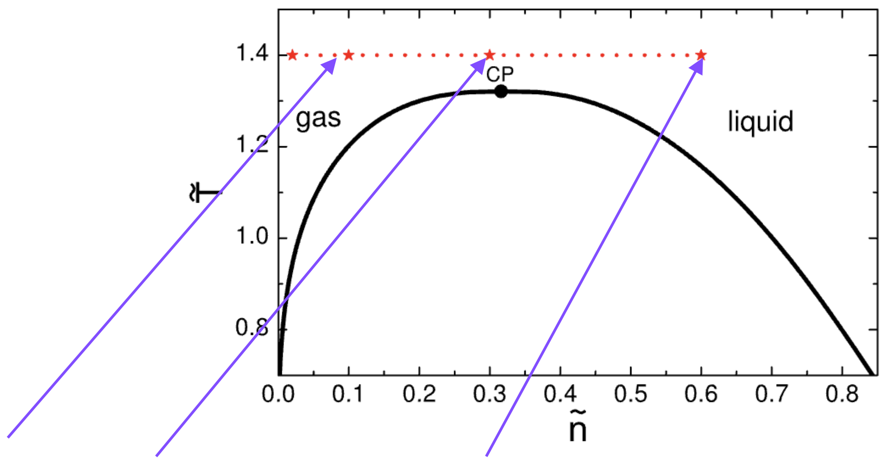
- Well-studied system
- The LJ fluid contains a critical point in the **3D-Ising universality class**, same as QCD critical point.
- Critical fluctuations are automatically projected on finite particle number statistics.





SIMULATION SETUP

$$m\frac{d^2\vec{r}_{i,j}}{d\tilde{t}^2} = -\vec{\nabla}V_{LJ}(\vec{r}_{i,j})$$



- Three points on the phase diagram, $\tilde{n} \approx 0.3n_c$, $\tilde{n} \approx 0.95n_c$, $\tilde{n} \approx 1.9n_c$ ($\tilde{T} \approx 1.9T_c$)
- $N_{ev} = 32\,000$ events at each density
- Initialize each event with random initial coordinates and momenta
- Run each event for long time ($\tilde{t} = 100$) write snapshots to file at regular time intervals
- Calculate observables as event-by-event (ensemble) or time average

The simulations are performed on PhysGPU cluster at UH. Code is available at:

https://github.com/vlvovch/lennard-jones-cuda

OBSERVABLES AND ERGODICITY

Time average

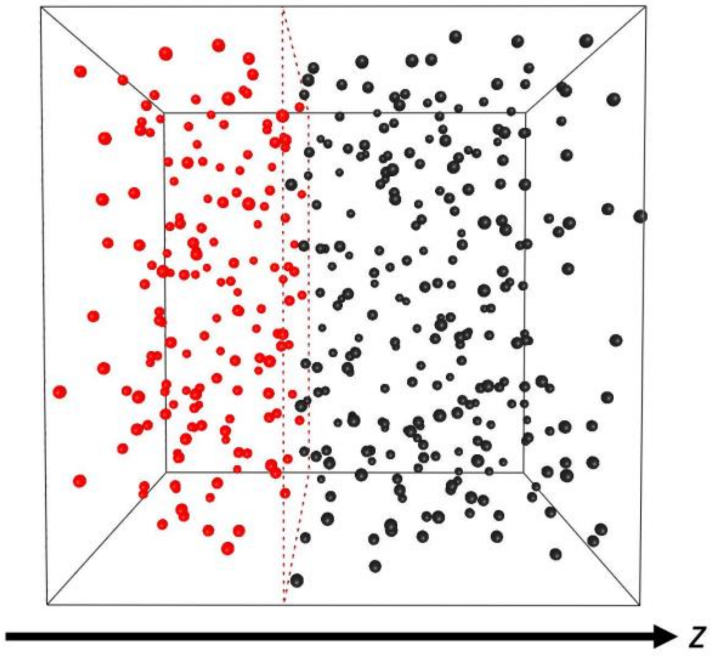
$$\langle A \rangle_\tau = \frac{1}{\tau} \int_{\tilde{t}_{eq}}^{\tilde{t}_{eq}+\tau} A(t)\, dt$$

versus ensemble average:

$$\langle A \rangle_M = \frac{1}{M} \sum_{i=0}^{M} A_i$$

Ergodic hypothesis:

$$\lim_{M\to\infty} \langle A \rangle_M = \lim_{\tau\to\infty} \langle A \rangle_\tau$$



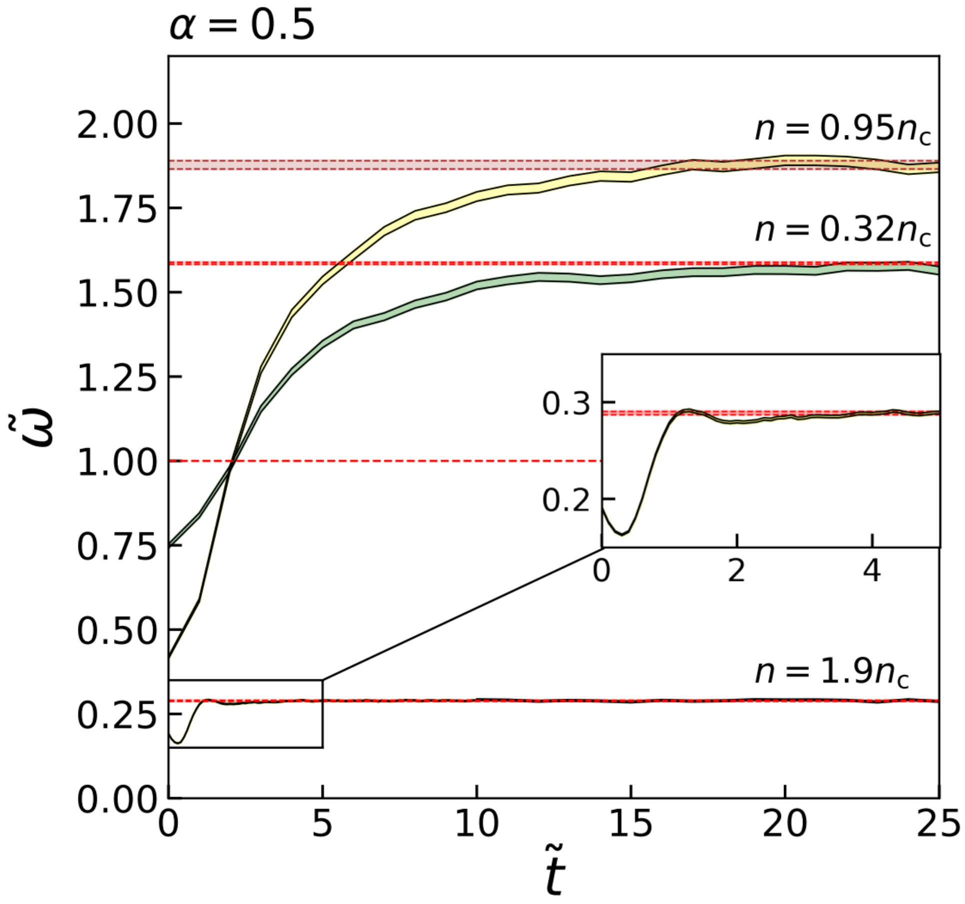
Namely:

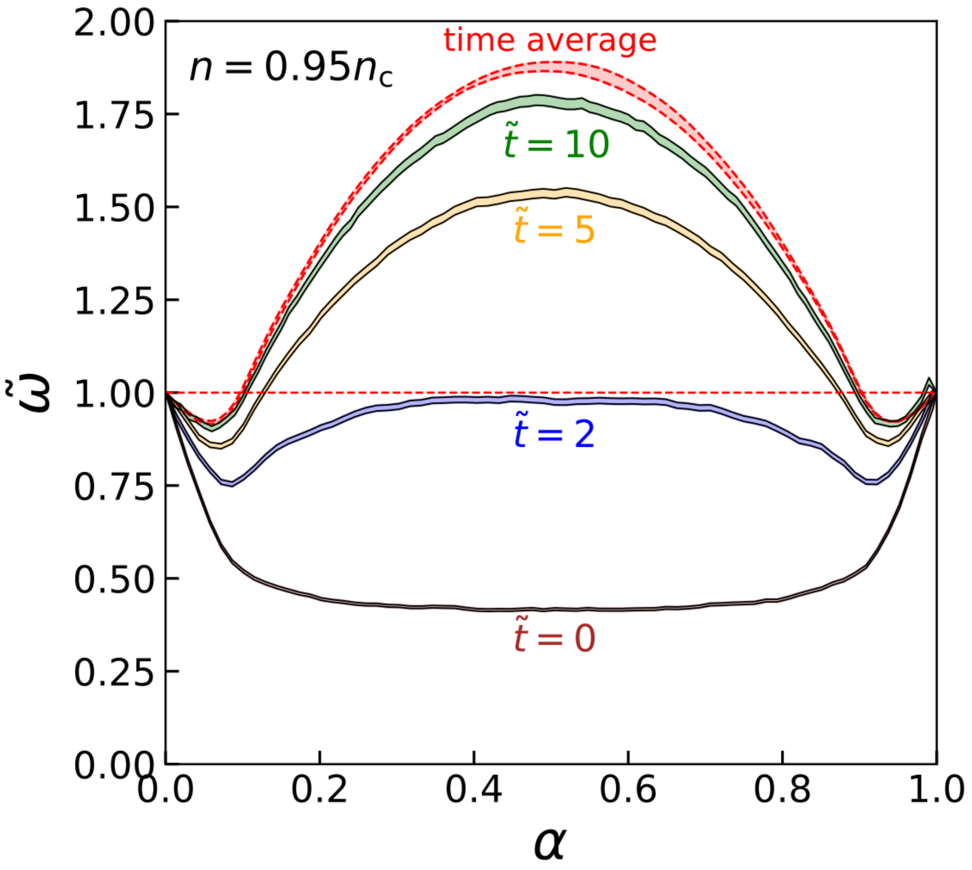
$$\langle N \rangle = \frac{1}{M} \sum_{i=0}^{M} N_i(\alpha)\,, \qquad \tilde{\omega} = \frac{1}{1-\alpha} \frac{\langle N \rangle^2 - \langle N^2 \rangle}{\langle N \rangle}^{*}, \qquad \alpha = \frac{\langle N \rangle}{N_{tot}}$$

V. A. Kuznietsov et al., PRC 105, 044903 (2022)

$^{*}1/(1-\alpha)$ correction is related to the V. Vovchenko et al., Phys. Let. B, 2020

TIME VS ENSEMBLE AVERAGE, EQUILIBRATION





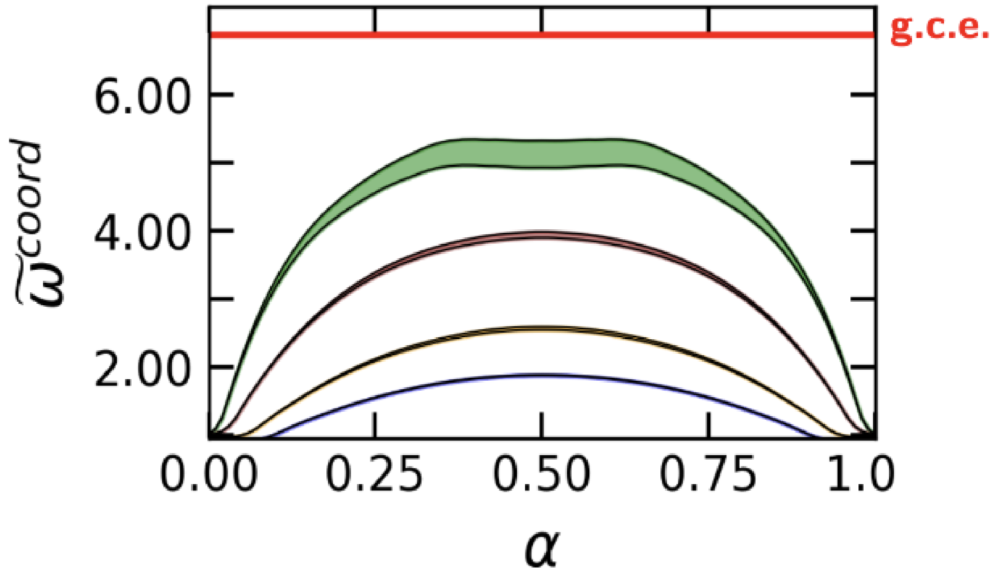
Ergodic hypothesis works

V.K., Gorenstein, Koch, Vovchenko, arXiv: 2404.00476 [nucl-th]

PREVIOUS RESULTS: TIME AVERAGE IN BOX

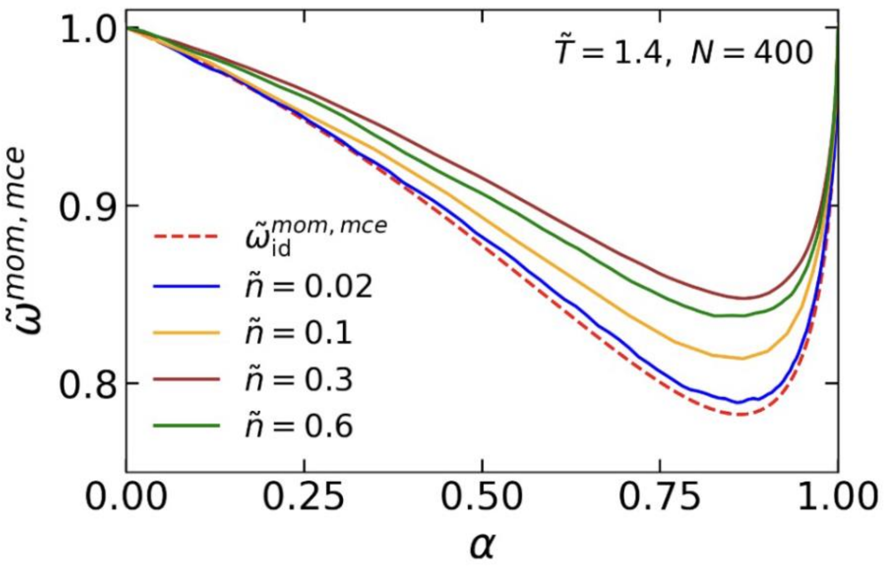
$$\widetilde{\omega} = \frac{1}{1-\alpha}\frac{\langle N\rangle^2 - \langle N^2\rangle}{\langle N\rangle}$$

Coordinate space



Large correlations in coordinate space

Momentum space

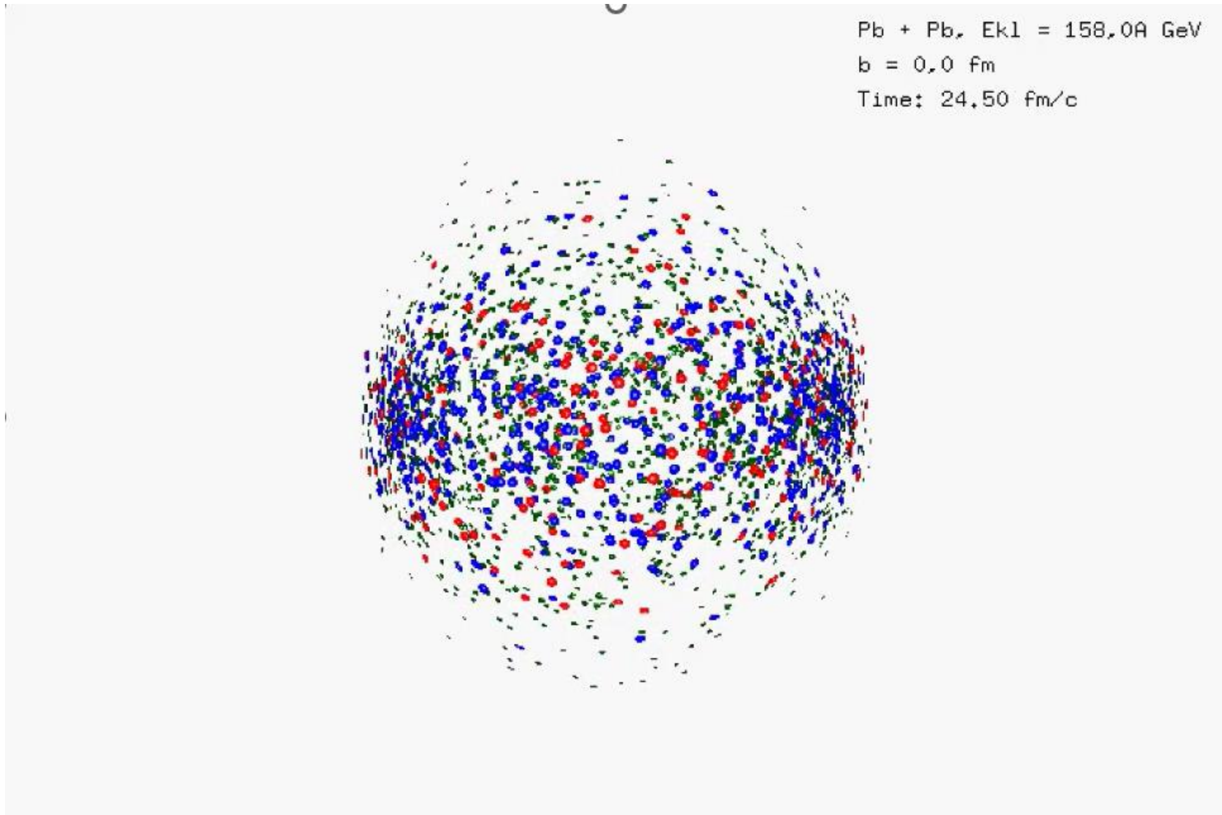


Signal disappears in momentum space

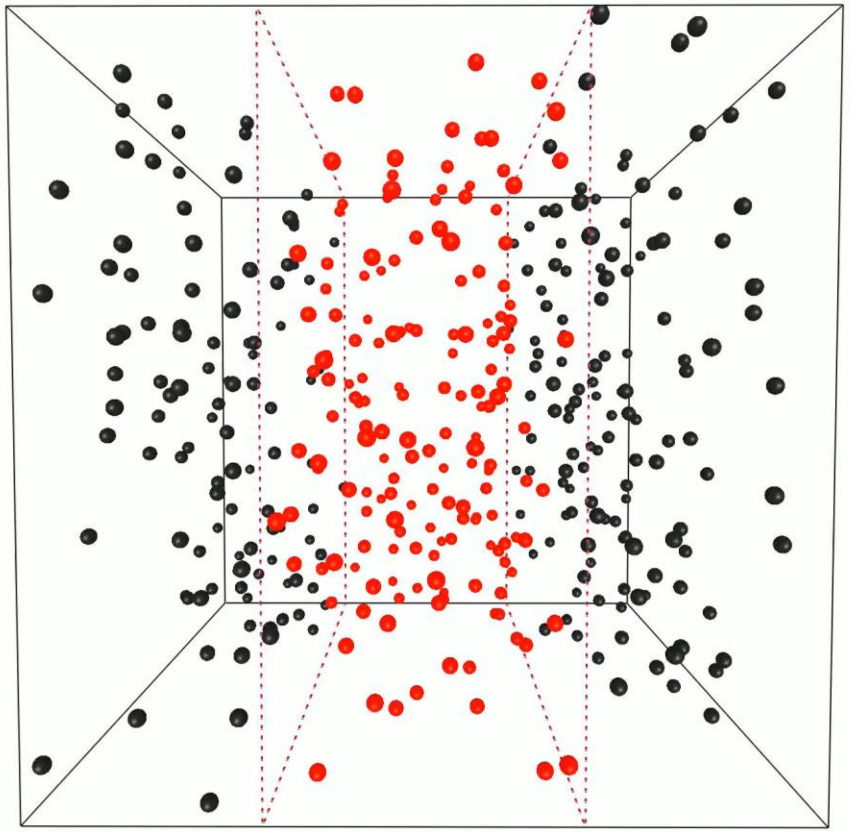
V.K., Savchuk, Gorenstein, Koch, Vovchenko, PRC 105, 044903 (2022)

EXPERIMENT VS BOX

Heavy-ion simulation (UrQMD)



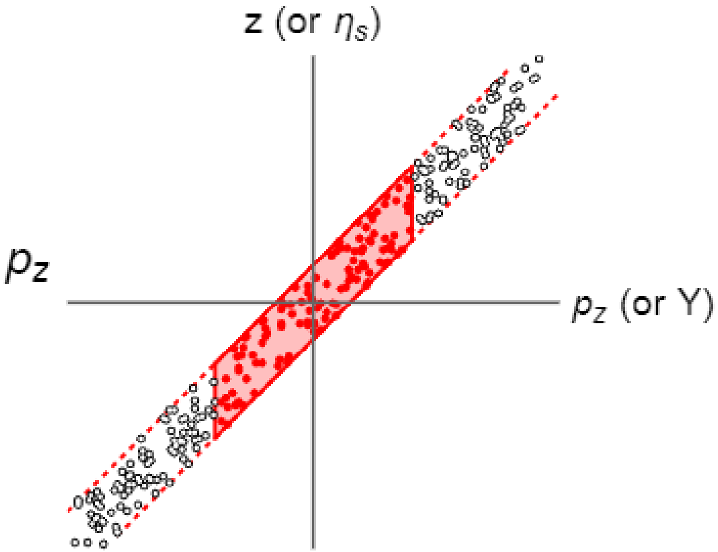
MD box simulation (LJ)



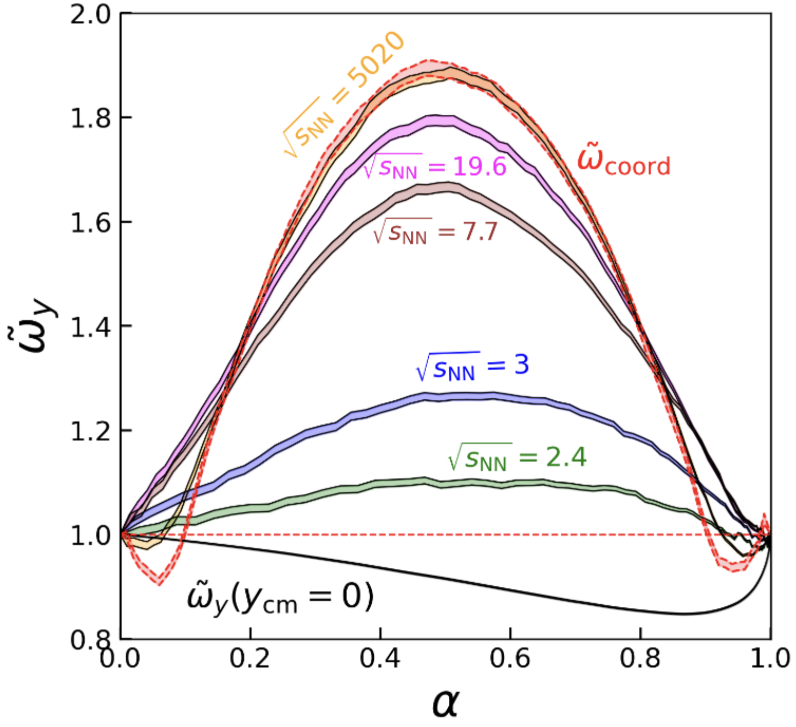
ADDING COLLECTIVE FLOW

We now add longitudinal flow (Bjorken-like) by boosting particles according to the z-coordinates

$$\tilde{y} = \tilde{y}^{LJ} + \sqrt{\frac{T_{frz}}{m\tilde{T}}}\,\kappa\left(\frac{\tilde{z}}{\tilde{L}} - \frac{1}{2}\right), \qquad \kappa = 2y_{cm}\sqrt{\frac{m\tilde{T}}{T_{frz}}}$$



Moment space (with flow)



Heavy-ion collision inspired parameters: $T_{frz} = 150$ MeV, $m = 938$ MeV

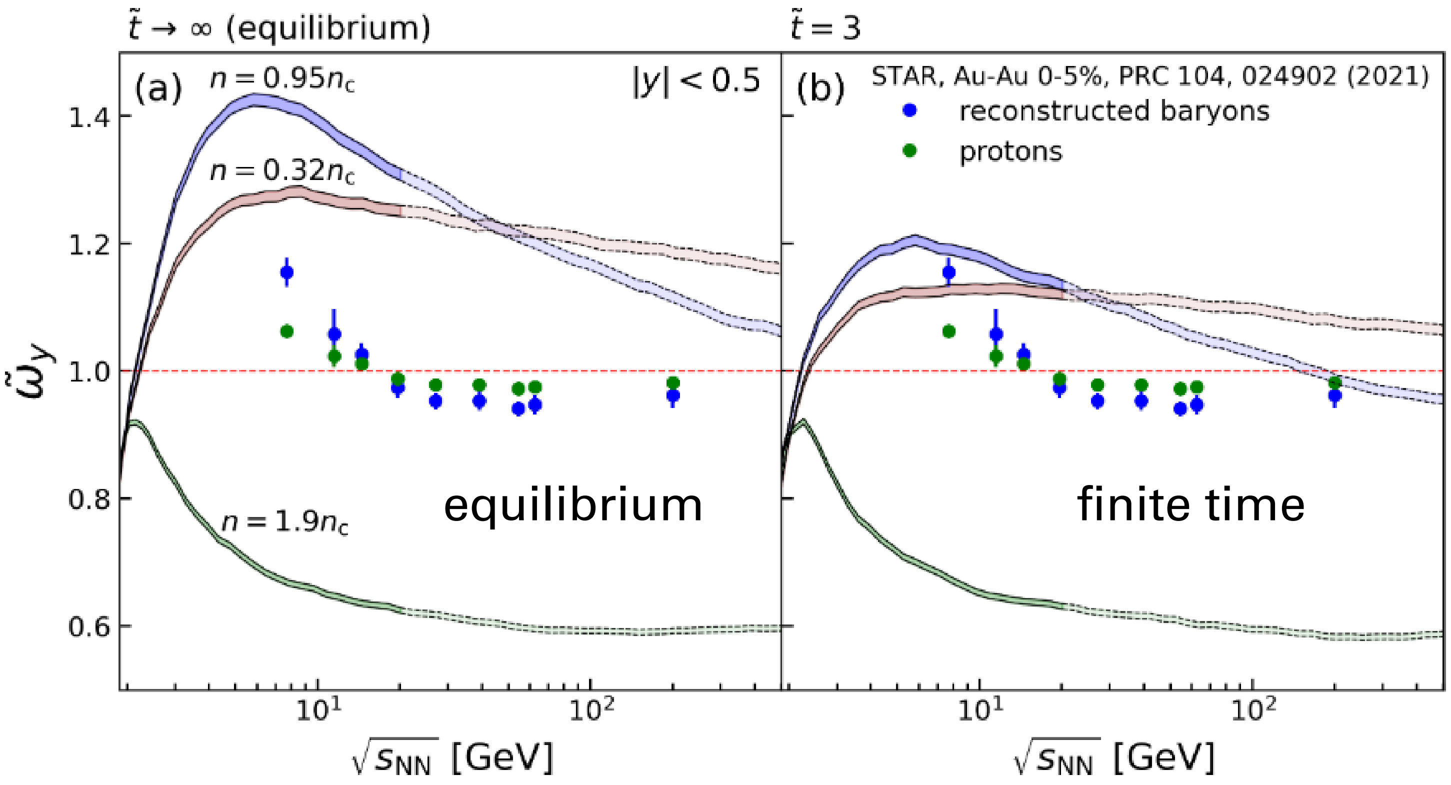
Collective flow correlates momenta to coordinates and recovers correlations

FLUCTUATIONS FOR CONSTANT RAPIDITY CUT

$$\alpha = \alpha(\sqrt{s_{NN}}) = \frac{\langle N \rangle}{N_{tot}}, \qquad y_{cut} = 0.5 \text{ (experiment)}$$

(Reconstructed STAR BES-I data was used, BES-II in future)

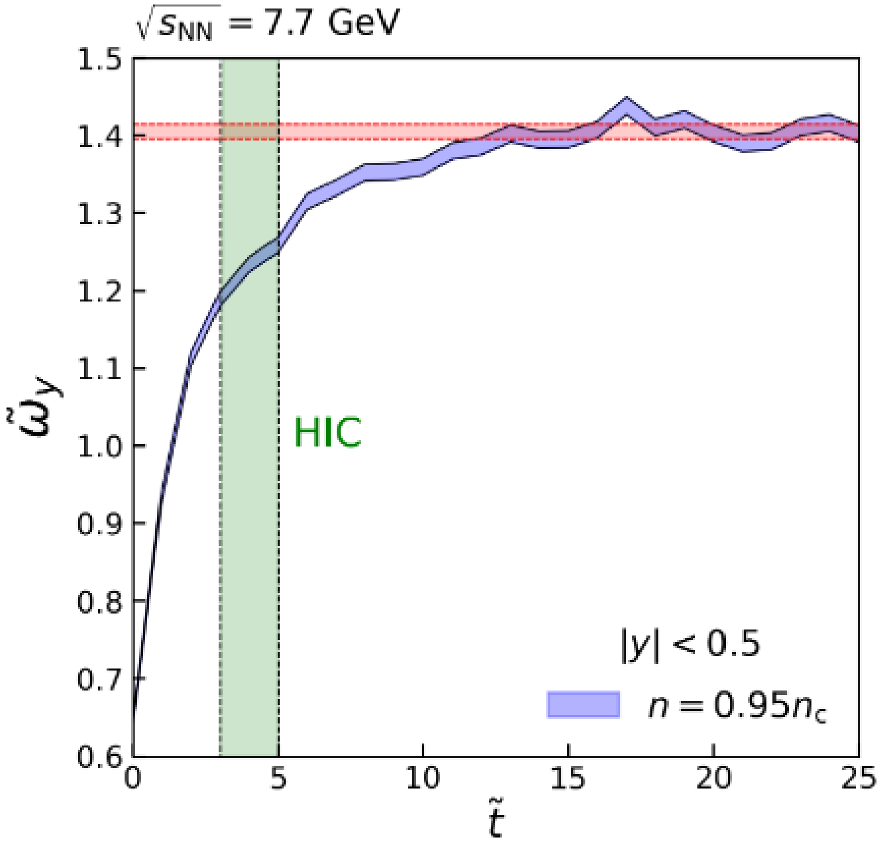
STAR data provided for the qualitative comparison

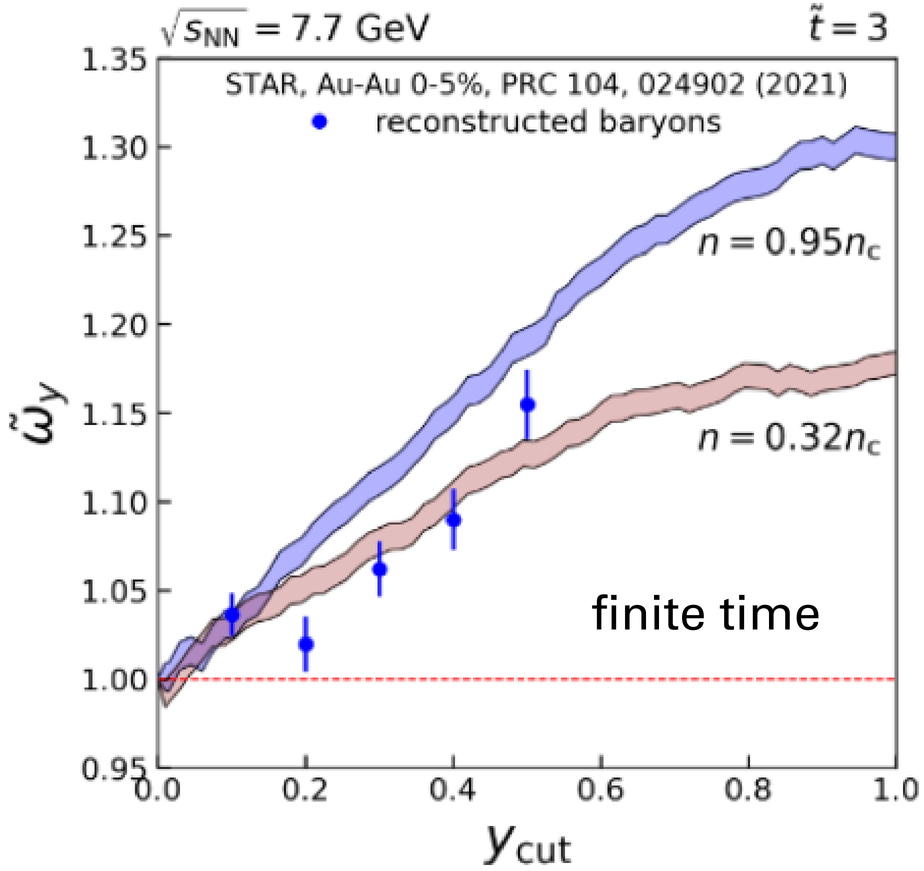


Data: M. S. Abdallah *et al.* (STAR Collaboration), PRC 104, 024902 (2021)
1-α correction based on hydro: Vovchenko, Koch, Shen, PRC105, 014904 (2022)

FLUCTUATIONS FOR CONSTANT RAPIDITY CUT

τ: 7 − 10 fm → 3 − 5 at units of $\tilde{t}$





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SUMMARY

- Critical fluctuations are studied in a microscopic setup
- Ergodic hypothesis is shown to work for 2nd-order fluctuations along the $\tilde{T} = 1.4 \sim 1.06 T_c$ isotherm, including the vicinity of the critical point
- The collective flow effect allow us to see the enhancement of fluctuations in the momentum space
- Fluctuations in experimental rapidity acceptance $|y| < 0.5$ are studied. If critical point is close to freeze-out, the largest signal is observed at $\sqrt{s_{NN}} \sim 5$ GeV as interplay between longitudinal flow and number of protons in acceptance (system size).

OUTLOOK

- Putting our study in the context of the BES II
- Higher-order cumulants (need bigger statistics)
- Study of the mixed phase
- Implementing CP dynamics into the transport theory (UrQMD/SMASH)

Thank you for your time!

Questions?

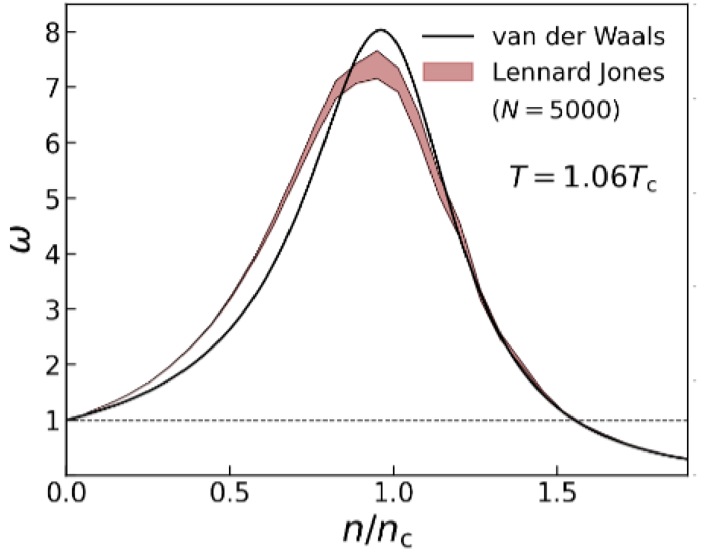
FLUCTUATIONS AS CP SIGNATURE

In GCE density cumulants shows singularity behavior in the critical point.

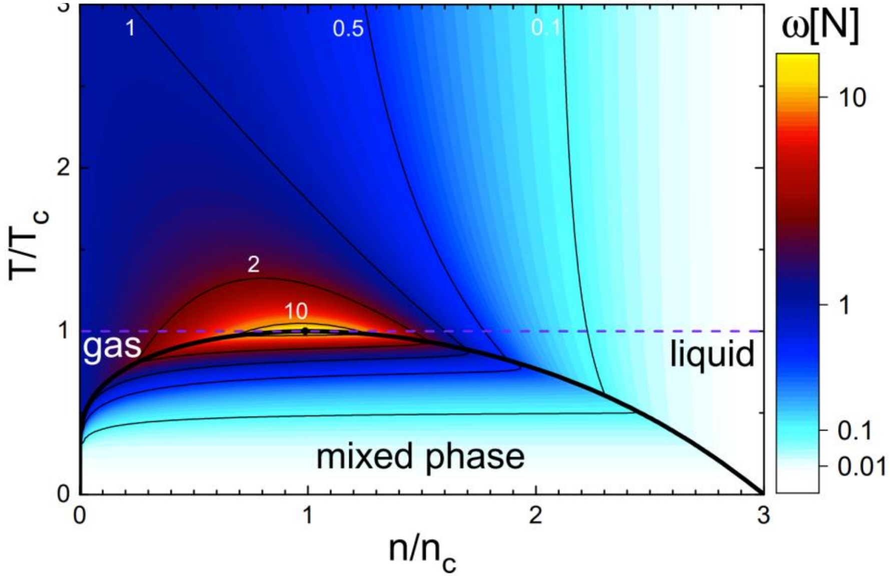
$$\ln(Z^{gce}) = \ln\left(\sum_{N=0}^{\infty} e^{\mu N} Z^{ce}(T,V,N)\right)$$

$$\kappa_n = \frac{\partial^n(\ln(Z^{ce}))}{\partial(\mu_N)^n}$$

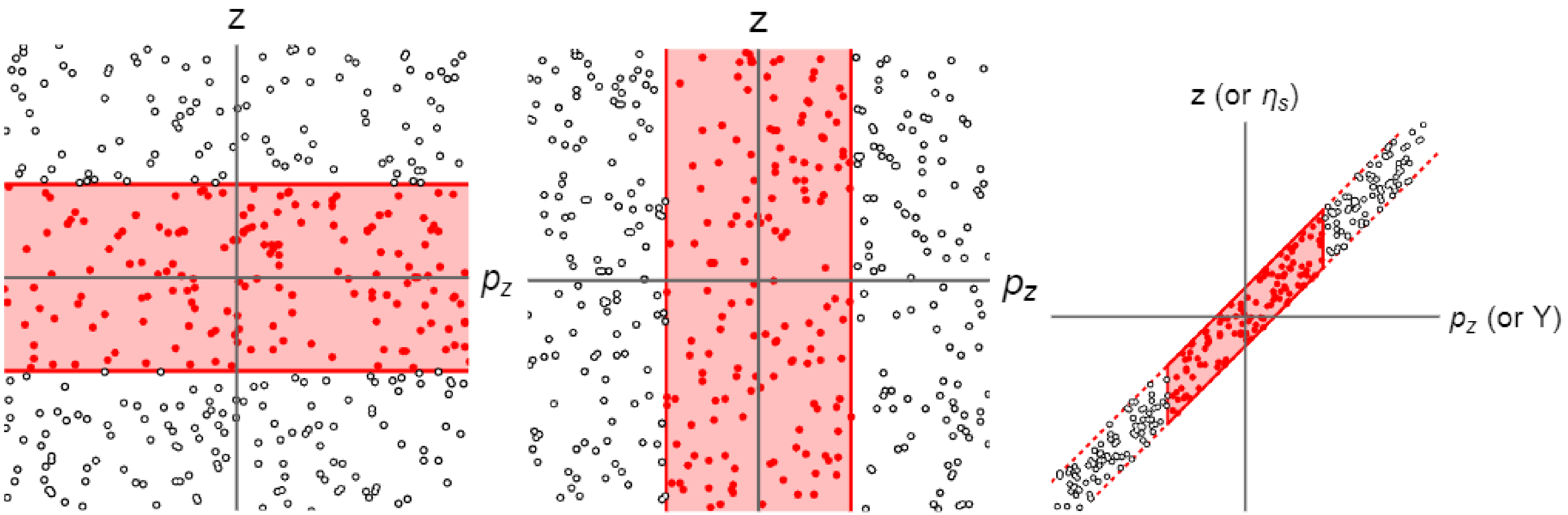
The real expression for Z^{gce} is unknown in QCD matter.



$\kappa_2 \sim \varepsilon^2$



MOMENTUM VS COORDINATE CUTS



SUB-ENSEMBLE METHOD: CORRECTION FOR GLOBAL CONSERVATION

In the case of interacting system one can find

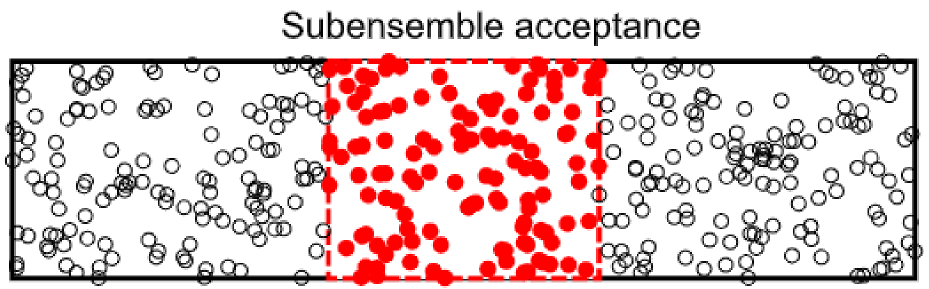
$$\kappa_1 = \alpha V T^3 \chi_1, \qquad \kappa_2 = \alpha(1-\alpha)\, V T^3 \chi_2$$

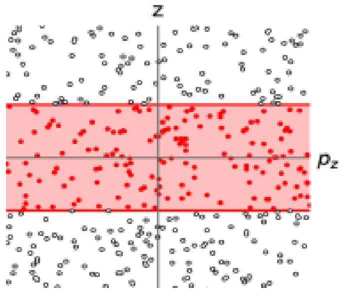
By definition in cumulants formalism

$$\omega^{coord} = \omega^{coord}/(1-\alpha)$$

Finally, one can introduce the new measure

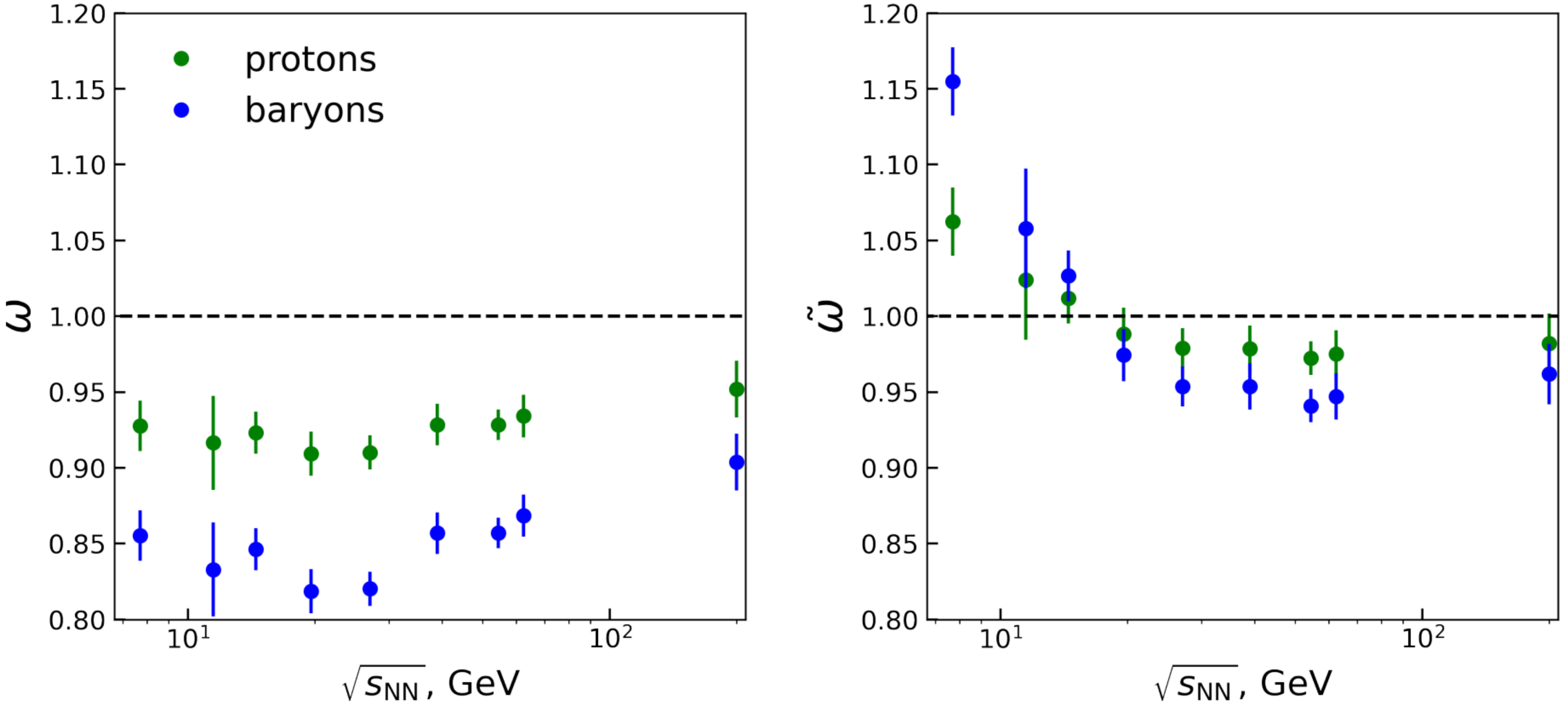
$$\widetilde{\omega}^{coord} = \frac{\kappa_2}{\kappa_1} = (1-\alpha)\frac{\chi_2}{\chi_1} = (1-\alpha)\omega$$





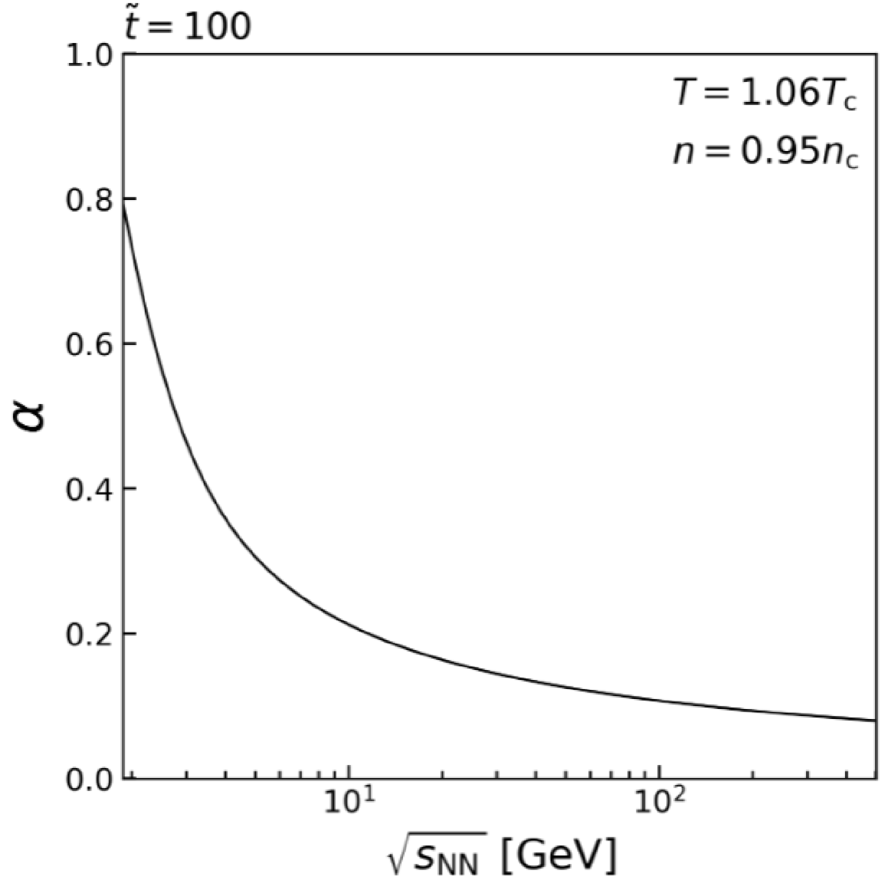
V. Vovchenko et al., Phys. Let. B, 2020

RECONSTRUCTED STAR DATA



ALPHA DEPENDENCE

$$\alpha = \alpha(\sqrt{s_{NN}}) = \frac{\langle N \rangle}{N}, \qquad y_{cut} = 0.5 \text{ (experiment)}$$



FLUCTUATIONS FOR CONSTANT ALPHA

$$\alpha = const = \frac{\langle N \rangle^{acc}}{N}, \qquad y_{cut} = y_{cut}(\sqrt{s_{NN}})$$