Observation of hydrodynamic behavior with few strongly-interacting fermions:
A zero-temperature small system puzzle

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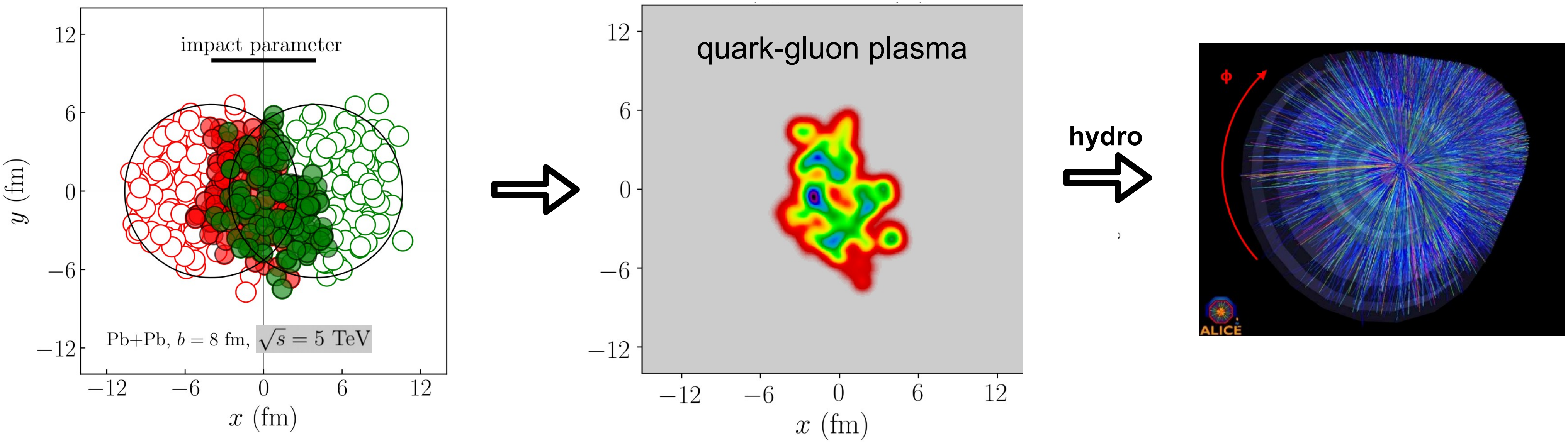


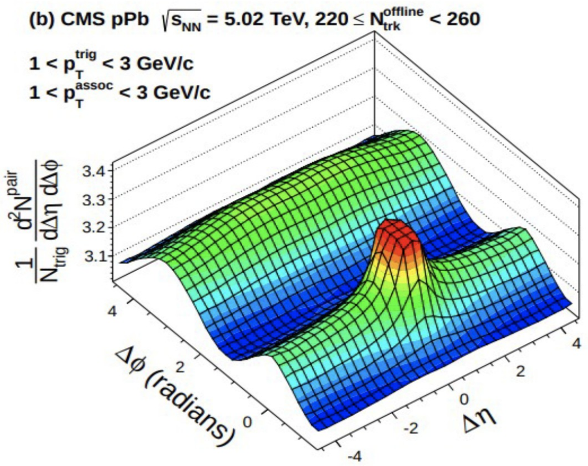
Effective fluid description: $T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu - P g^{\mu\nu}$ [Romatschke & Romatschke, arXiv:1712.05815]

Equation of state from lattice QCD. [HoTQCD collaboration, PRD **90** (2014) 094503]
[Gardim, Giacalone, Luzum, Ollitrault, Nature Phys. **16** (2020) 6, 615-619]

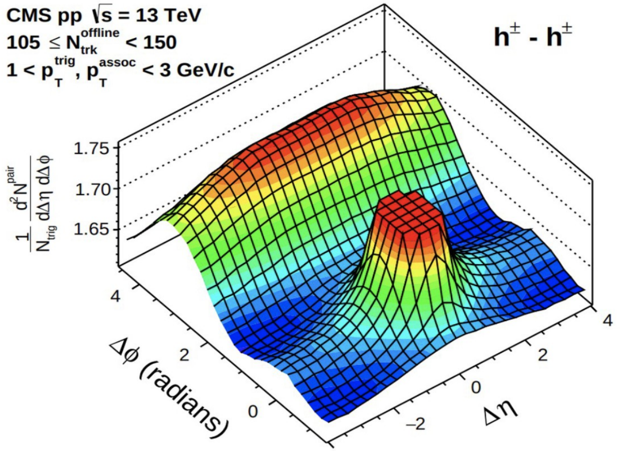
Fluid is viscous (η/s , ζ/s , …). [Bernhard, Moreland, Bass, Nature Phys. **15** (2019) 11, 1113-1117]

Collective flow in small-system collisions

p-Pb



p-p



[CMS collaboration, PLB **724** (2013) 213-240]
[CMS collaboration, PLB **765** (2017) 193-220]

1) How does collectivity emerge out of equilibrium?

[e.g. Soloviev, EPJC **82** (2022) 4, 319]

2) Can systems made of only tens of particles even develop collective behavior driven by interactions? **Absence of a separation of scales?**

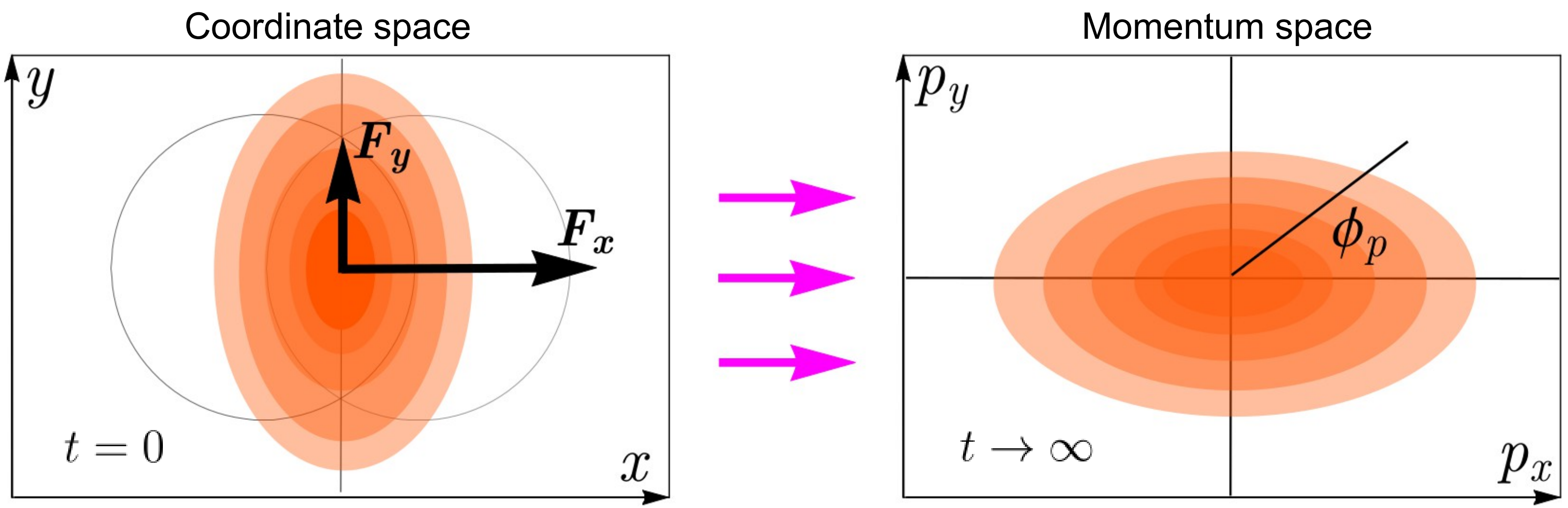
⬇

We address this question
by means of experiments
on ultra-cold atomic gases.

Key probe of hydrodynamic behavior – Elliptic flow

[Ollitrault, PRD **46** (1992) 229-245]

$$F = -\nabla P \quad \longrightarrow \quad V_2 = \langle e^{-2i\phi_p} \rangle$$



Coordinate space

Momentum space

Why ultracold atoms?

Exquisite experimental control

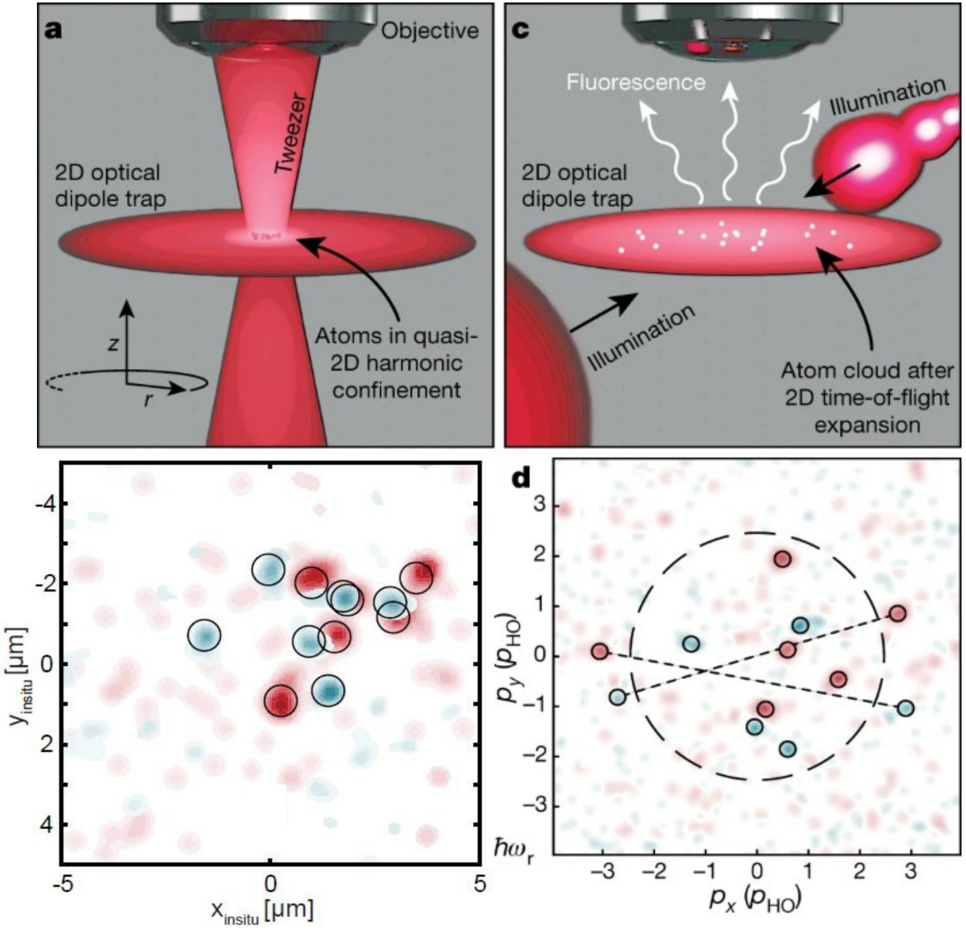
$$\mathcal{H} = -\sum_{i=1}^{\overset{\text{particle number}}{\overset{\downarrow}{N}}} \frac{\hbar^2}{2m}\boldsymbol{\nabla}_i^2 + \sum_{i<j}\frac{\hbar^2}{2m}\underset{\underset{\text{interaction strength}}{\uparrow}}{g_0}\,\delta^{(d)}\left(\boldsymbol{r}_i - \boldsymbol{r}_j\right) + \sum_{i=1}^{N}\overset{\overset{\text{trapping potential}}{\downarrow}}{\mathcal{V}_{\text{ext}}}\left(\boldsymbol{r}_i\right)$$

Interactions are tunable via Feshbach resonance, particle number via spilling technique [Serwane et al. arXiv:1101.2124]

We can probe transition **from non-interacting to strongly-interacting systems,**
from few to many particles
with known initial geometry

Precise imaging techniques

[Holten *et al.*, Nature **606**, 287-291 (2022)]



Method to image atoms in “free space”.

[Bergschneider *et al.*, PRA **97**, 063613 (2018)]

The measurement can be carried out in either real or momentum space.

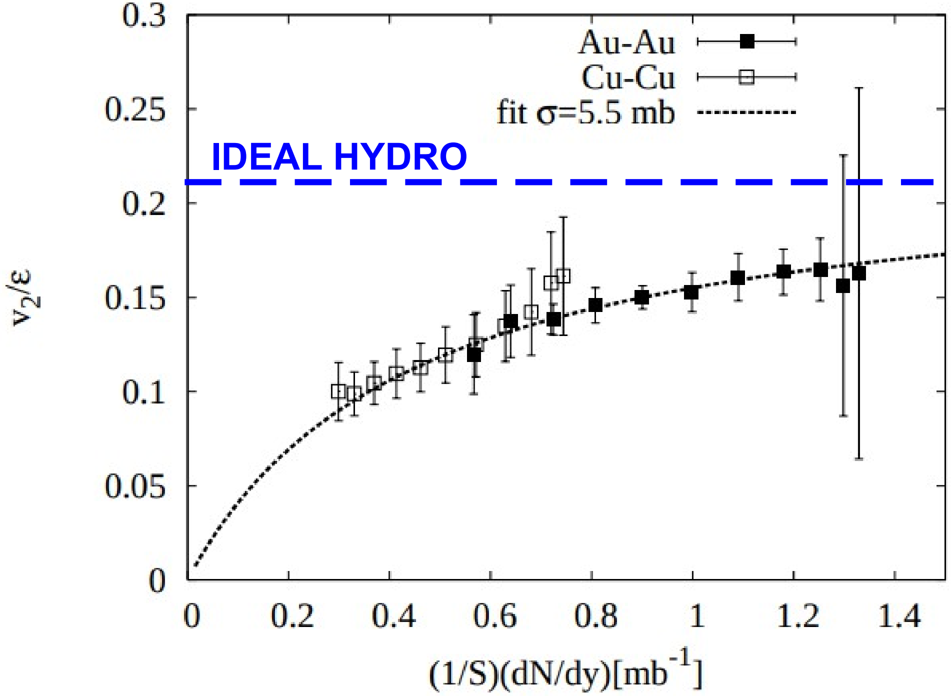
Emergence of “hydrodynamic response” as the system becomes more dense/macroscopic.

$\frac{v_2}{\varepsilon_2}$

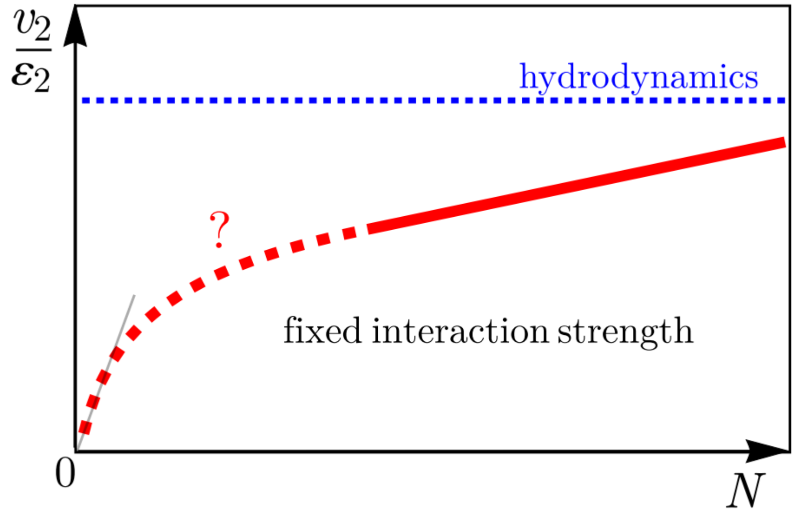
v_2 → **final state anisotropy (momentum space)**

ε_2 → **initial state anisotropy (real space)**

[Drescher *et al.*, PRC **76** (2007) 024905]

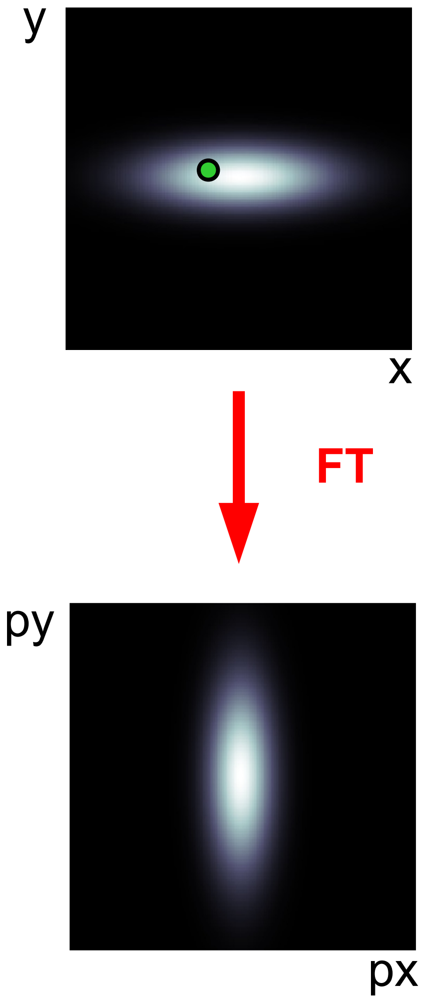


Our Proposal



(see also arXiv:2405.02847 for experimental results with a thermal gas)

“Background” elliptic flow from position-momentum relation



One atom in a harmonic potential:

$$\boxed{\lambda = \frac{\omega_y}{\omega_x}}$$

$$\begin{aligned}\langle \cos(2\phi_p) \rangle_{\psi_{0,0}} &= \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle_{\psi_{0,0}} \\ &= \int \mathrm{d}p_x \mathrm{d}p_y \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} |\psi_{0,0}(p_x, p_y)|^2 \\ &= \frac{1 - \sqrt{\lambda}}{1 + \sqrt{\lambda}} = v_2\end{aligned}$$

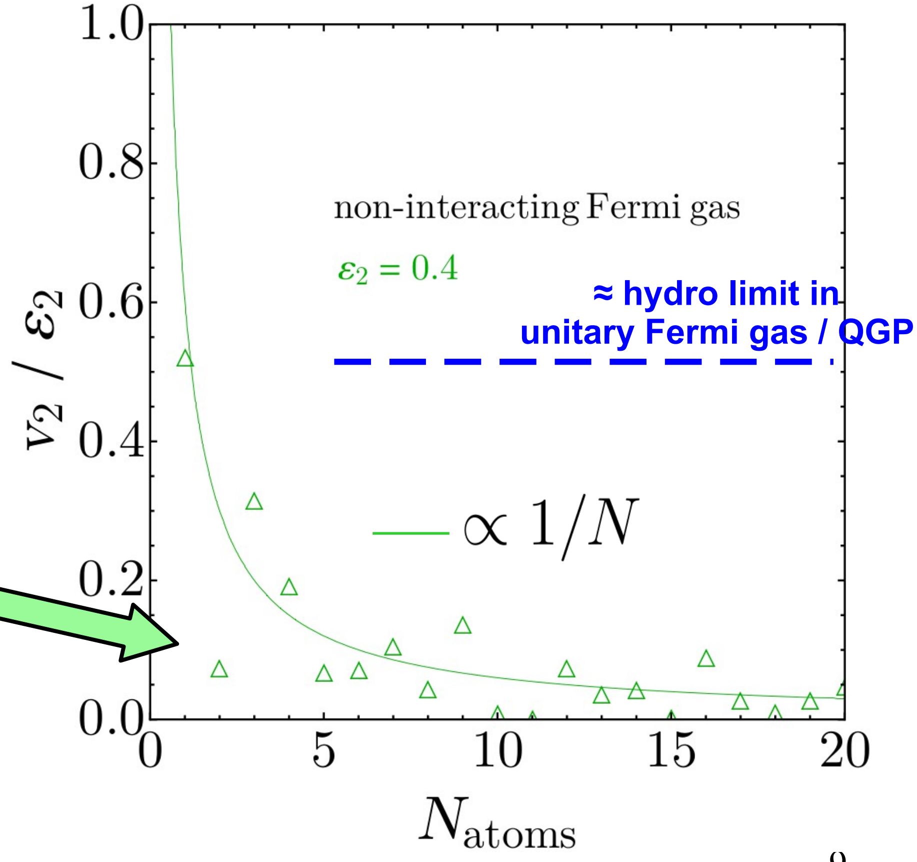
Shape inversion with only a single particle
→ definitely not hydrodynamics

Dependence of background elliptic flow on particle number

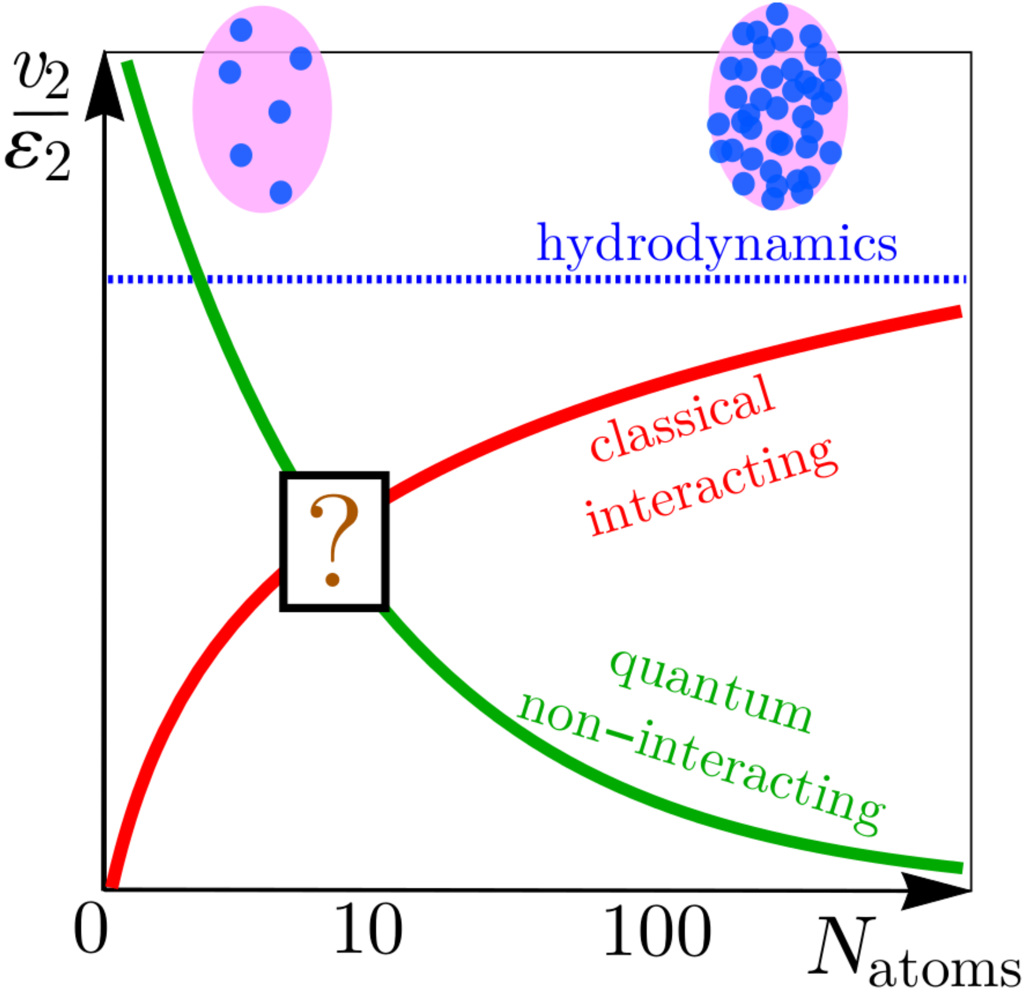
$$v_2 = \langle \cos(2\phi_p) \rangle_\Psi$$

N trapped
non-interacting
fermions

Elliptic flow for N non-interacting fermions disappears quickly, like 1/N.

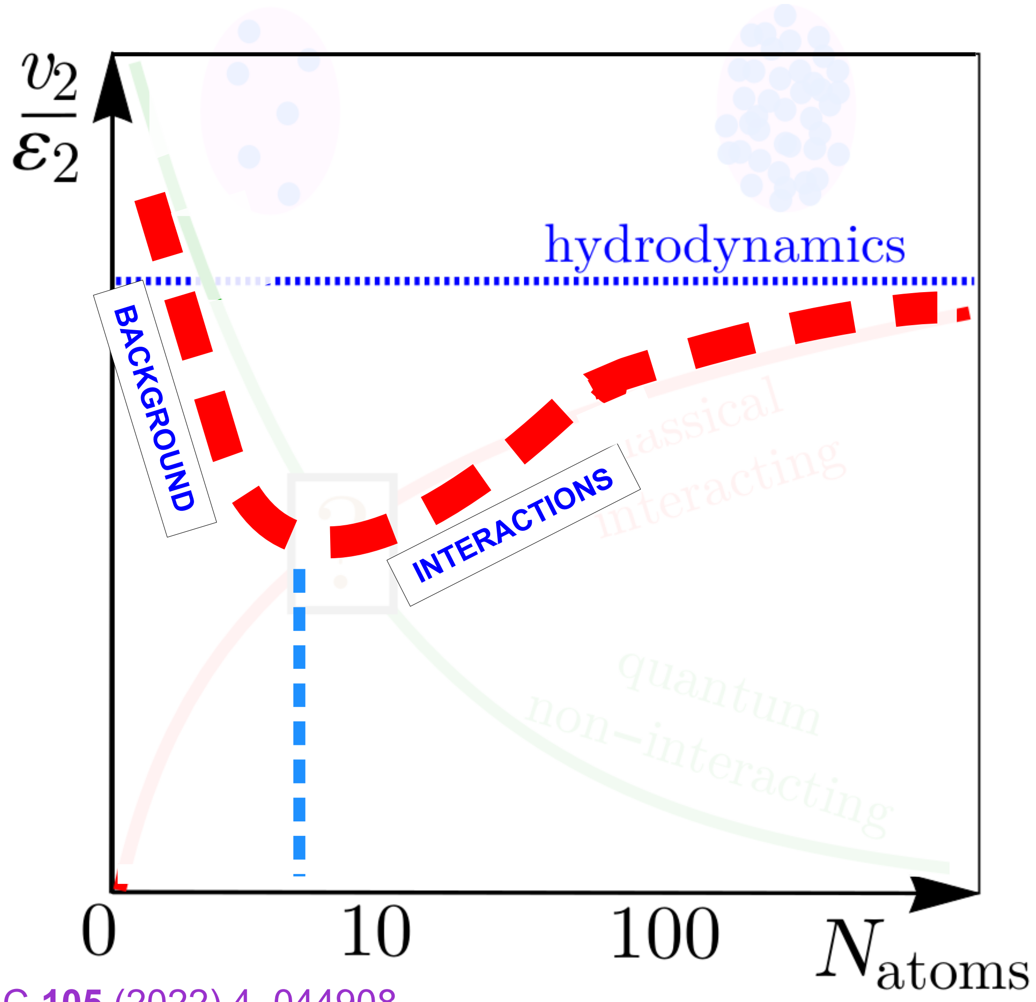


Qualitative expectations



Combining the curves…

We predict non-monotonic behavior.

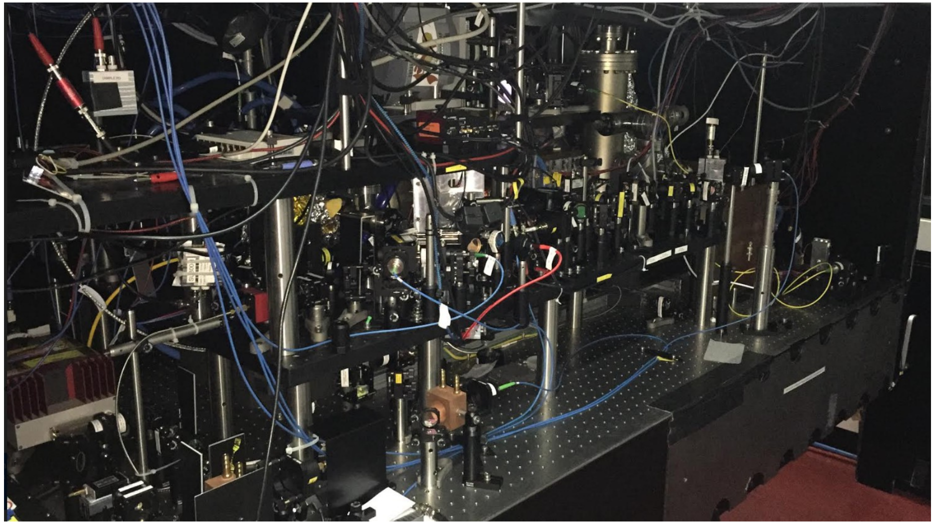


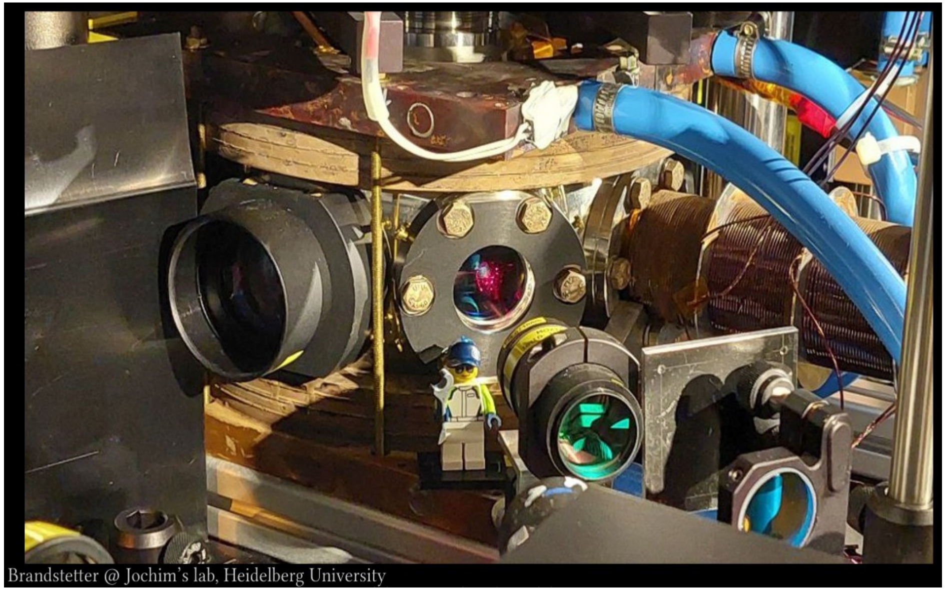
Flörchinger, Giacalone, Heyen, Tharwat, PRC **105** (2022) 4, 044908

A zero-temperature small system puzzle

Numerical and Experimental Results

Brandstetter, Lunt, Heintze, Giacalone, Heyen, Gałka, Subramanian, Holten, Preiss, Floerchinger, Jochim
arXiv:2308.09699, accepted for publication in Nature Physics (2024)





Brandstetter @ Jochim's lab, Heidelberg University

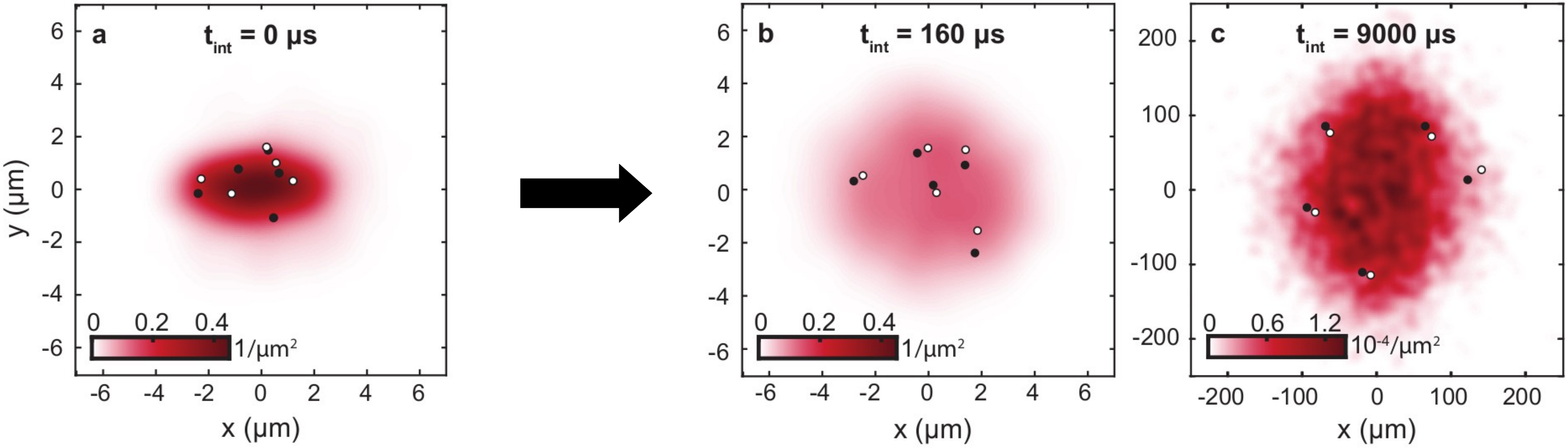
Few-body experiments run at the Physics Institute of Heidelberg University.

http://ultracold.physi.uni-heidelberg.de/

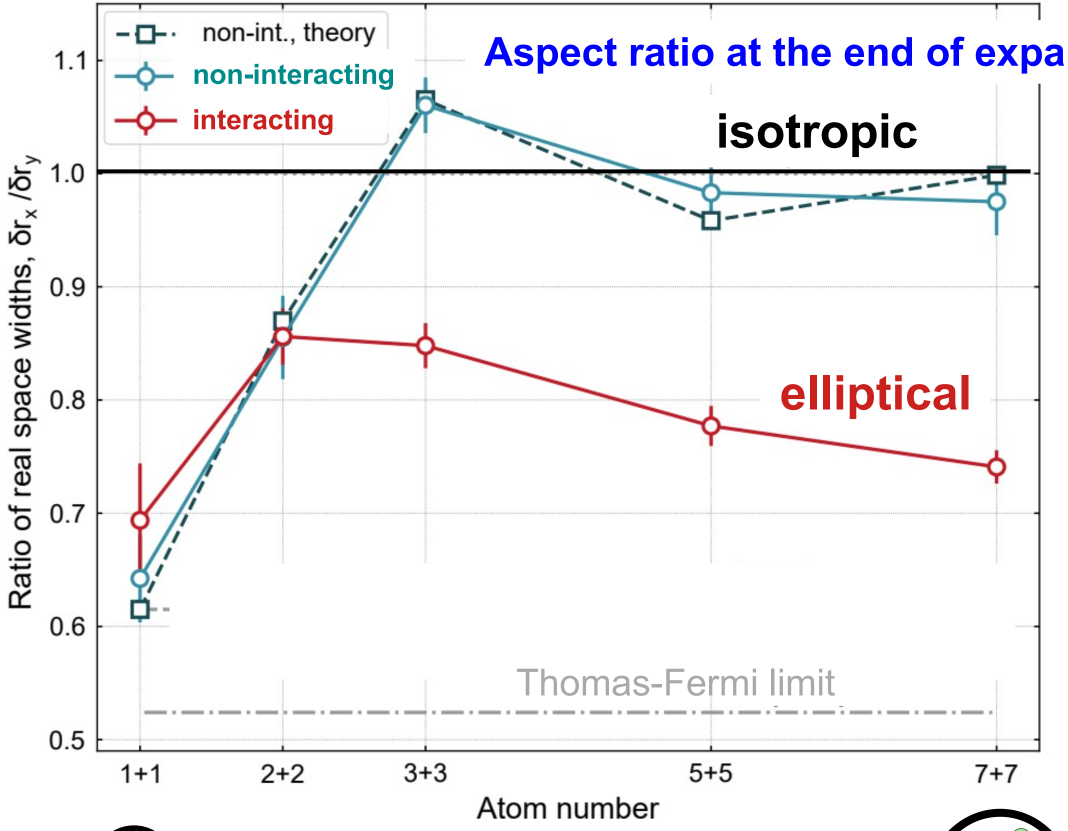
Main collaborators:
Selim Jochim (PI)
Sandra Brandstetter (PhD student)
Carl Heintze (PhD student)
Philipp Lunt (PhD student)

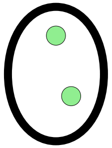
No separation of scales in the initial system
System size = interparticle spacing = scattering length

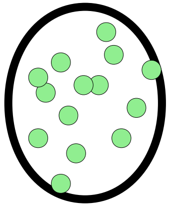
Fluid behavior of 10 particles!

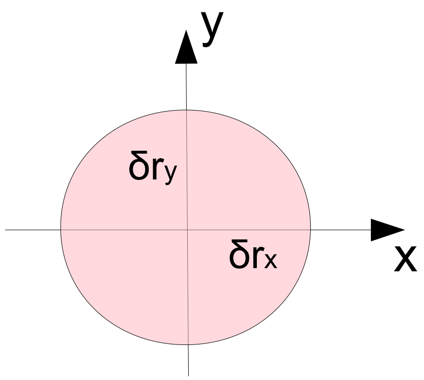


Aspect ratio at the end of expansion









Predicted non-monotonic behavior is observed!

Beyond N_{atoms}=6, elliptic flow is determined by inter-atom interactions.

↓

Collective behavior

Hydrodynamic model – ideal fluid in 2D

Use ideal fluid dynamics at T=0
→ need only pressure as a function of density

↓

Fit experimental results for pressure:

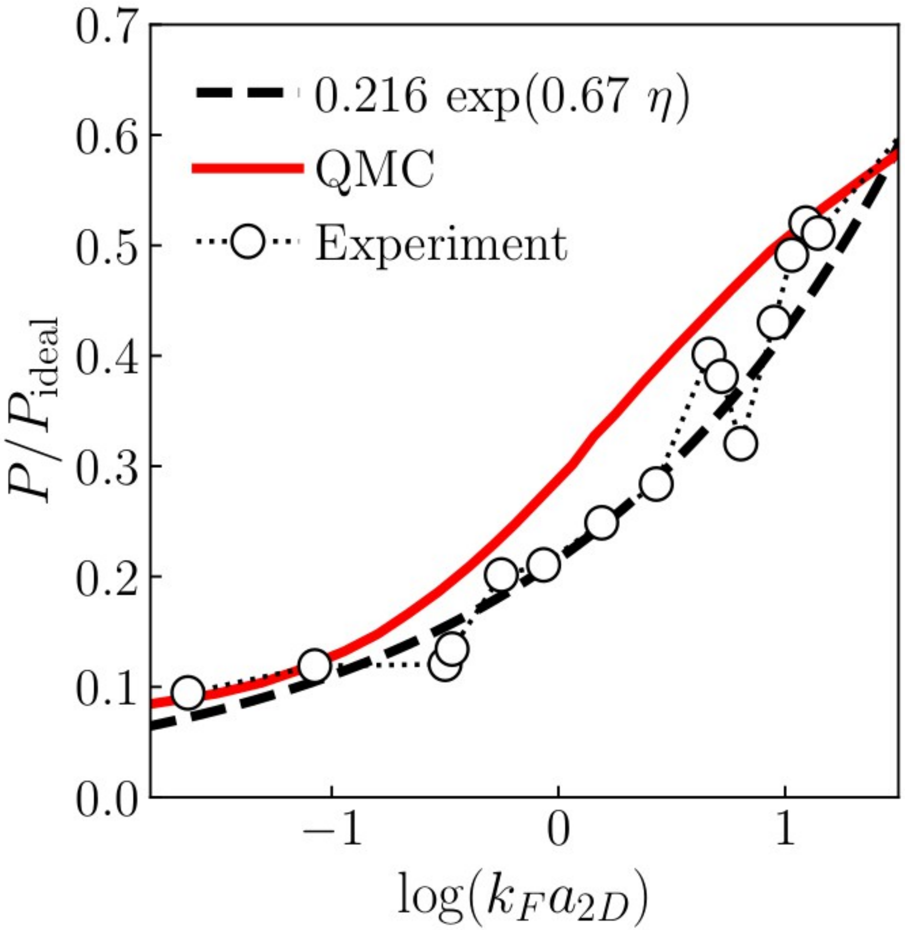
$$P_{\text{ideal}} = \frac{\pi\hbar^2}{2M^3}\rho^2\,, \quad P_{\text{fit}} = g\rho^{\kappa}$$

↓

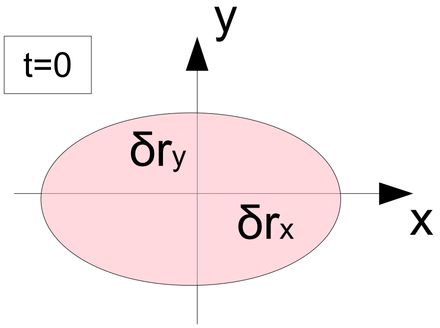
Solve continuity and Euler equations:

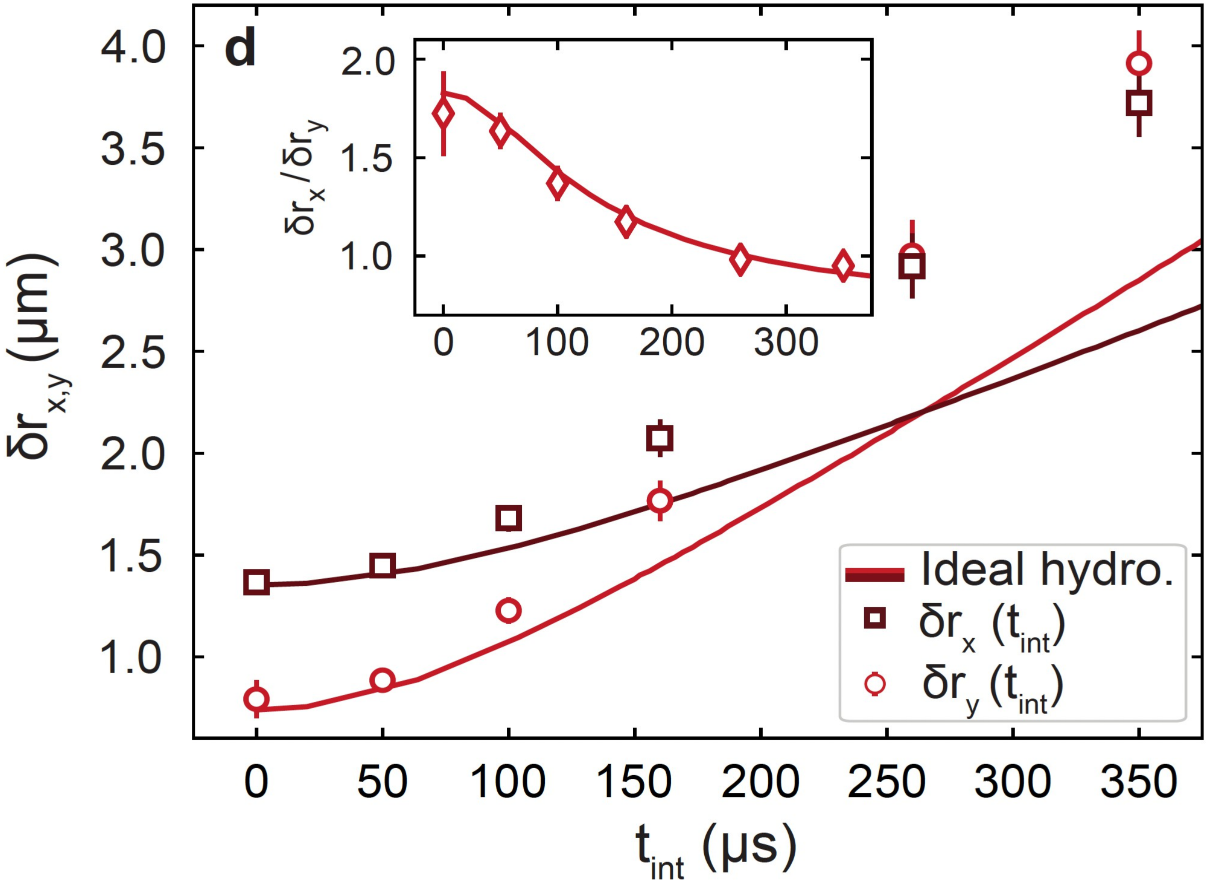
$$\partial_t\rho + \nabla\cdot(\rho\mathbf{v}) = 0$$

$$\rho(\partial_t + \mathbf{v}\cdot\nabla)\mathbf{v} = -\nabla P$$



Direct comparison in real space – Ideal hydrodynamic simulations





Hydro solver from Stony Brook:

https://python-hydro.github.io/pyro2/index.html

evolution of aspect ratio well-described, absolute magnitudes of sizes are a bit off

Momentum space prediction

Absolute magnitude of momenta affected by interaction quench. Hard to predict in theory.

Difference of momenta should mitigate experimental effects.

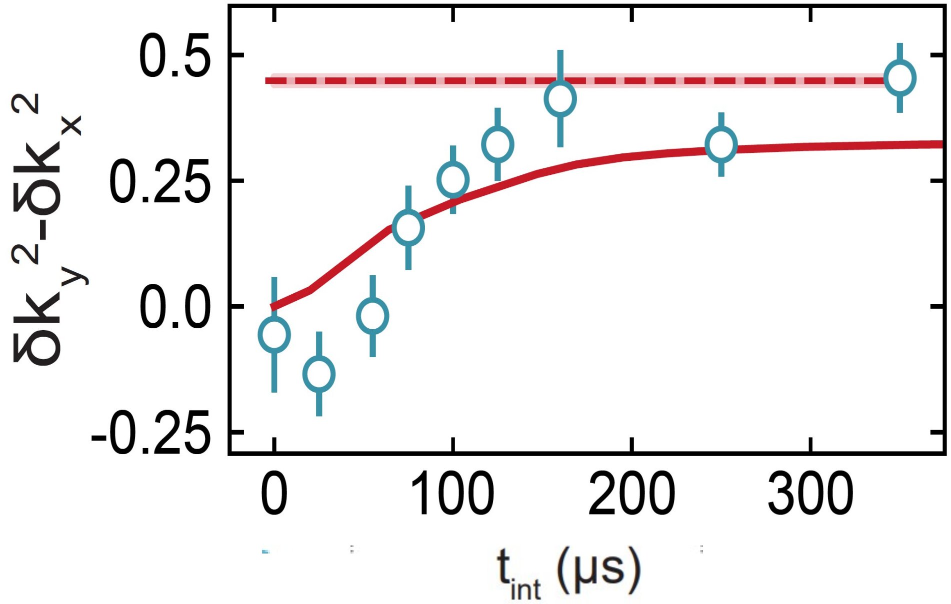
MOMENT OF DISTRIBUTION FUNCTION

MOMENTUM FLUX TENSOR

$$\mathscr{P}_{jk}(t,\mathbf{x}) = \int d^2p \left\{ \frac{p_j p_k}{m} f(t,\mathbf{x},\mathbf{p}) \right\}$$

$$(\delta p_y)^2 - (\delta p_x)^2 =$$

$$\frac{m}{2N} \int_{\mathbf{x}} \rho(t,\mathbf{x}) \left[v_y^2(t,\mathbf{x}) - v_x^2(t,\mathbf{x}) \right]$$



Conclusions

A system of 10 strongly interacting ^{6}Li atoms at T=0 behaves qualitatively like a fluid → "small systems question in cold atoms"

The fluidlike behavior emerges only if there are enough particles (and the interaction is sufficiently strong)

Ideal hydrodynamics does not capture all relevant features

Outlook

- **Add second order corrections**
- **Investigate pairing → Freeze-out to bound states?**
- **Hydrodynamic attractor in ultracold atoms** [Fujii, Enss, **arXiv:2404.12921**]

Theory postdoc position opening soon.
Send inquires to a.mazeliauskas@thphys.uni-heidelberg.de

Thank you!

Conservation laws for mass, momentum and energy

mass density fluid velocity

$$\partial_t \rho + \partial_j(\rho v_j) = 0$$

$$\partial_t(\rho v_i) + \partial_j T_{ji} = 0$$

$$\partial_t \varepsilon + \partial_j(\varepsilon v_j + q_j) + (T_{jk} - \rho v_j v_k)(\partial_j v_k) = 0$$

energy density

with the stress tensor

$$T_{jk} = \rho v_j v_j + (P + \Pi)\delta_{jk} + \pi_{jk}$$

But now we have **only 5 equations for 14 unknowns**

Solution: derivative expansion

Zeroth order: ideal fluid (→ Euler equation)

$$\Pi = 0, \pi_{jk} = 0, q_j = 0$$

Pressure from thermodynamic equation of state

First order: viscous fluid (→ Navier-Stokes equations)

$$\Pi = -\zeta \partial_j v_j, \pi_{jk} = -\eta \sigma_{jk}, q_j = -\kappa \partial_i T$$

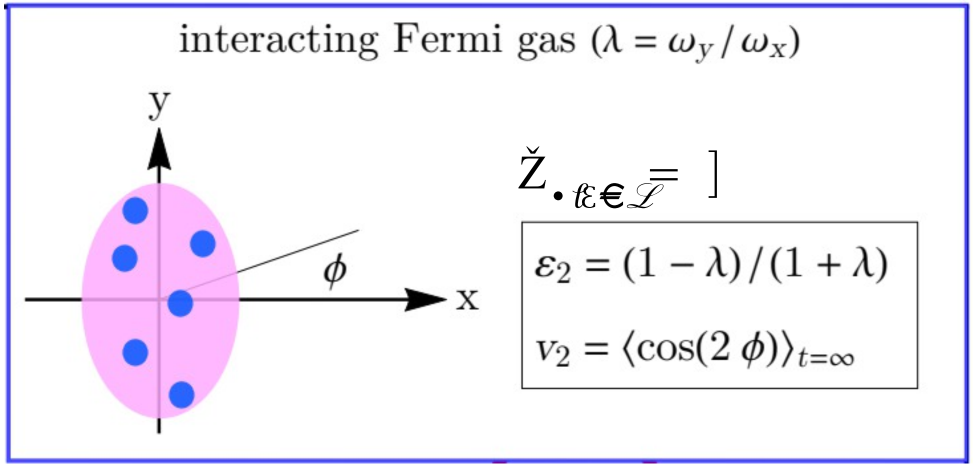
Transport coefficients from e.g. linear response

Guidelines for mesoscopic samples

1 – Statistical description, repeat the experiment many times like in heavy-ion collisions.

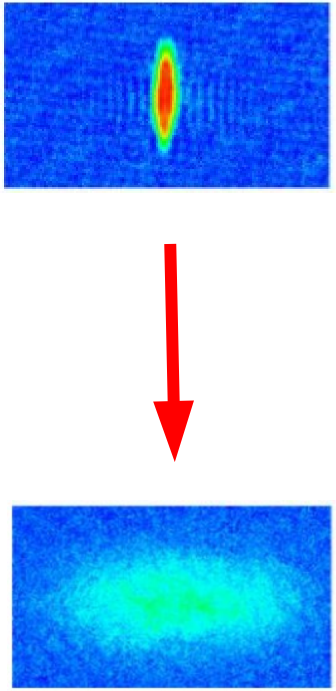
2 – Unlike heavy-ion collisions, orientation and ellipticity, ε_2, can be chosen.

3 – Measure elliptic flow as $\langle \cos 2\Phi_p \rangle$ with respect to the fixed axis.



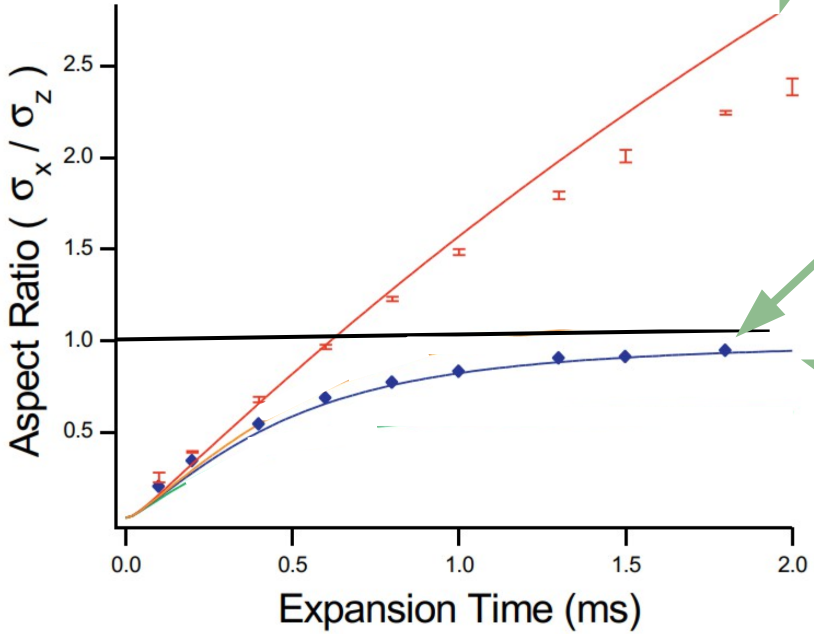
Inversion of shape in the many-body limit

$N \approx 10^5\ {}^6\mathrm{Li}$ atoms



100 μs

2000 μs



ideal hydrodynamic prediction

strongly-interacting data

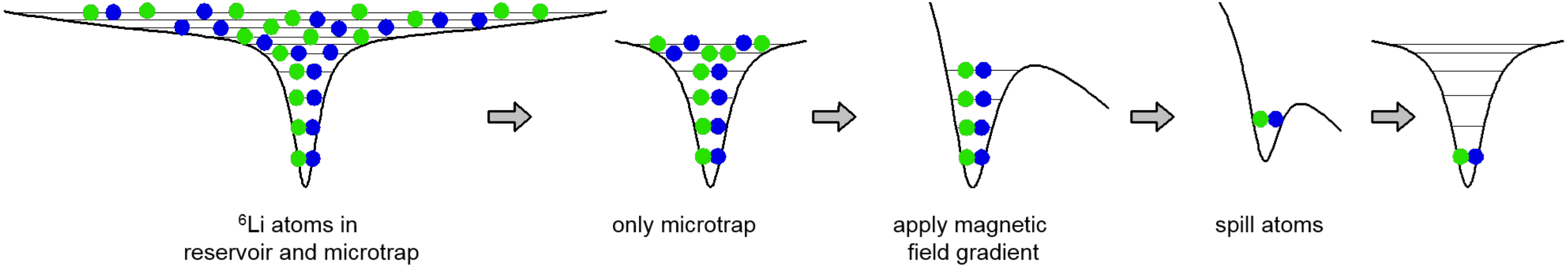
non-interacting data

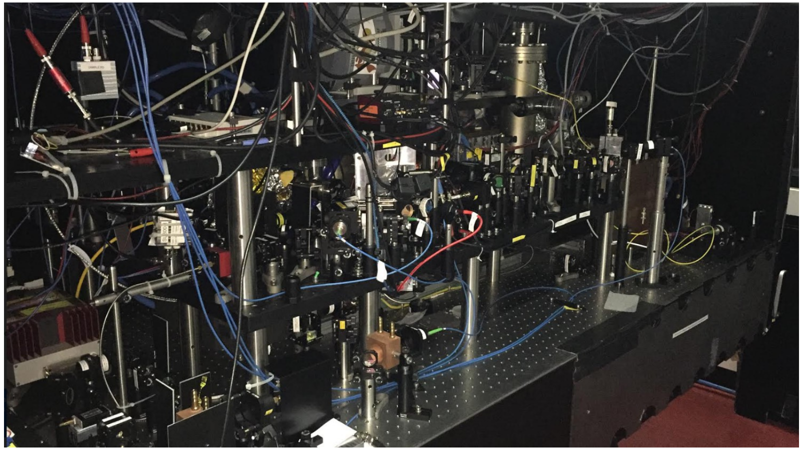
free streaming prediction

[O'Hara et al., Science **298** (2002) 2179-2182]
[Menotti, Pedri, Stringari, PRL **89**, 250402 (2002)]

Control over particle number

[Serwane et al., Science **332** (2011) 6027]





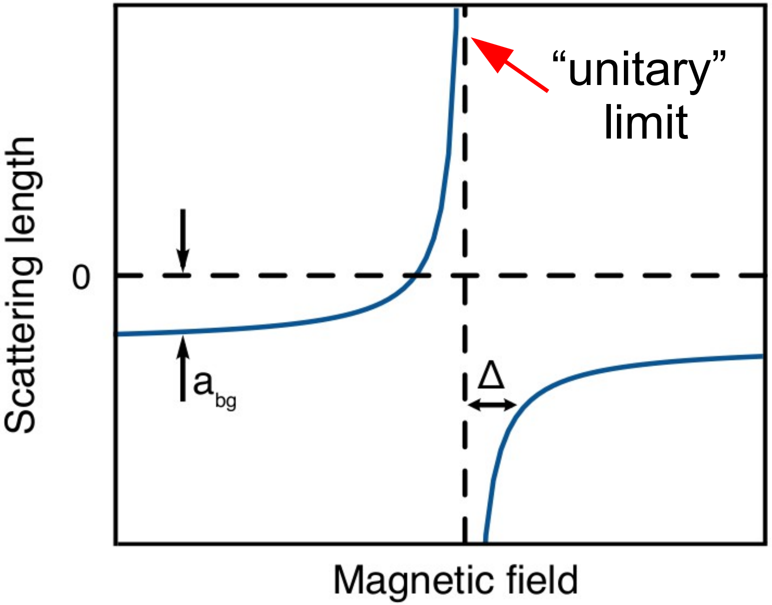
http://ultracold.physi.uni-heidelberg.de/

Exquisite experimental control – Feshbach resonance

$$\mathcal{H} = -\sum_{i=1}^{N} \frac{\hbar^2}{2m} \boldsymbol{\nabla}_i^2 + \sum_{i<j} \frac{\hbar^2}{2m} g_0 \, \delta^{(d)} \left(\boldsymbol{r}_i - \boldsymbol{r}_j\right) + \sum_{i=1}^{N} \mathcal{V}_{\text{ext}} \left(\boldsymbol{r}_i\right)$$

[e.g. Marvin Holten, PhD thesis, Heidelberg University (2022)]

FESHBACH RESONANCE



Low-energy scattering characterized by s-wave **scattering length** parameter, a.

Tune it with an external B field.

$$a_{3D} = a_{bg} \left(1 + \frac{\Delta}{B - B_0}\right)$$

lithium-6

$a_{bg} = -2100\, a_{\text{Bohr}}$

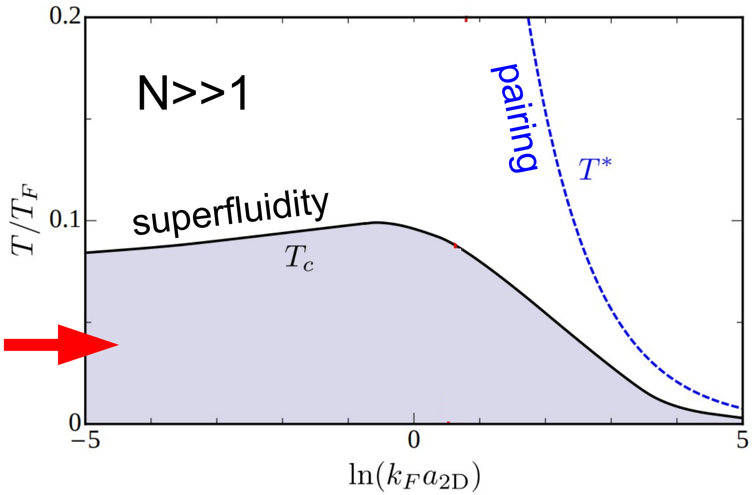
$B_0 = 690\,\text{G}$

$\Delta = 200\,\text{G}$

Transition from non-interacting to strongly-interacting systems.

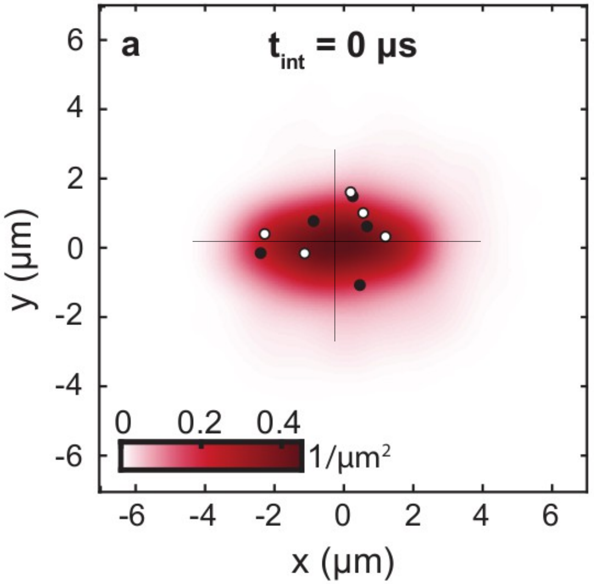
Superfluidity in interacting 2d fermion gases

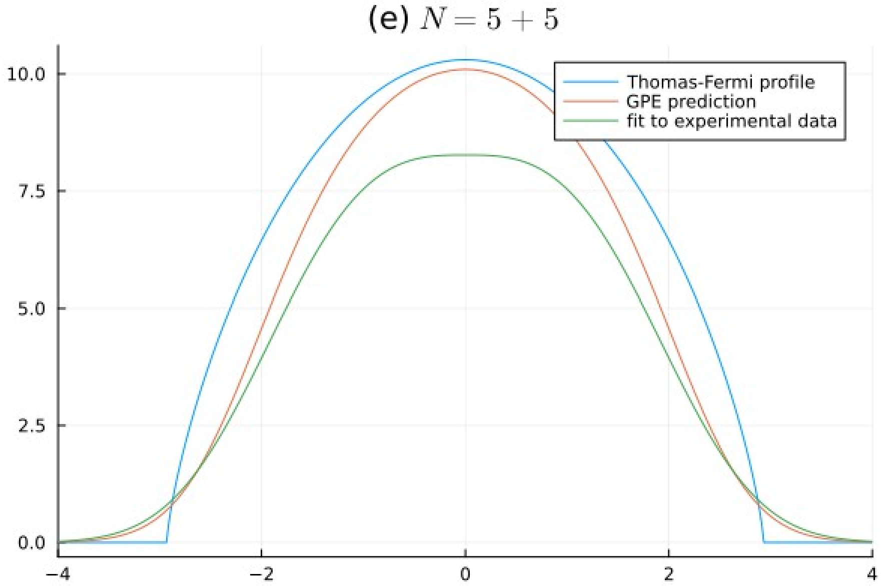
[Levinsen, Parish, arXiv:1408.2737]



The system we consider is at very low temperatures and interaction parameter ~1 → macroscopic limit is a superfluid

Matching to mass density at t=0 to the measured initial condition





Elliptic flow requires sufficiently strong interactions

strong interactions

weak interactions