Study of hadron two-body and three-body interactions with femtoscopy

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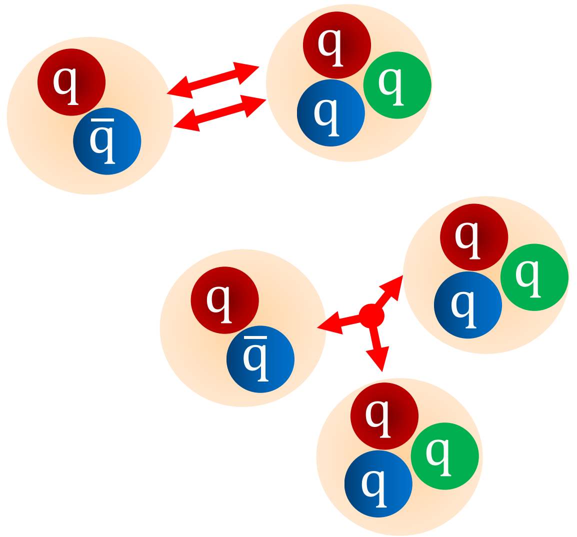
Why do we study hadronic interactions?

Understand how QCD evolves from high-energy to low-energy regime

How do hadrons interact?

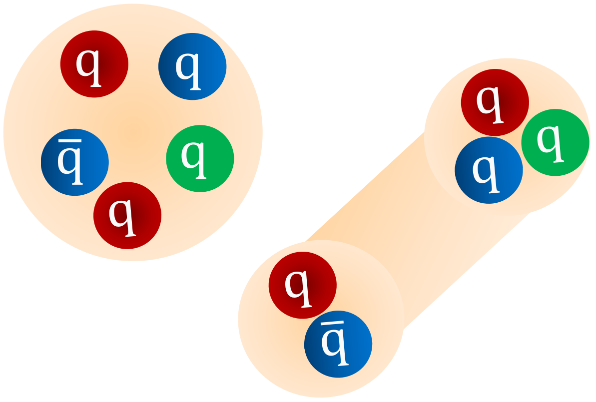
2-body and many-body interactions



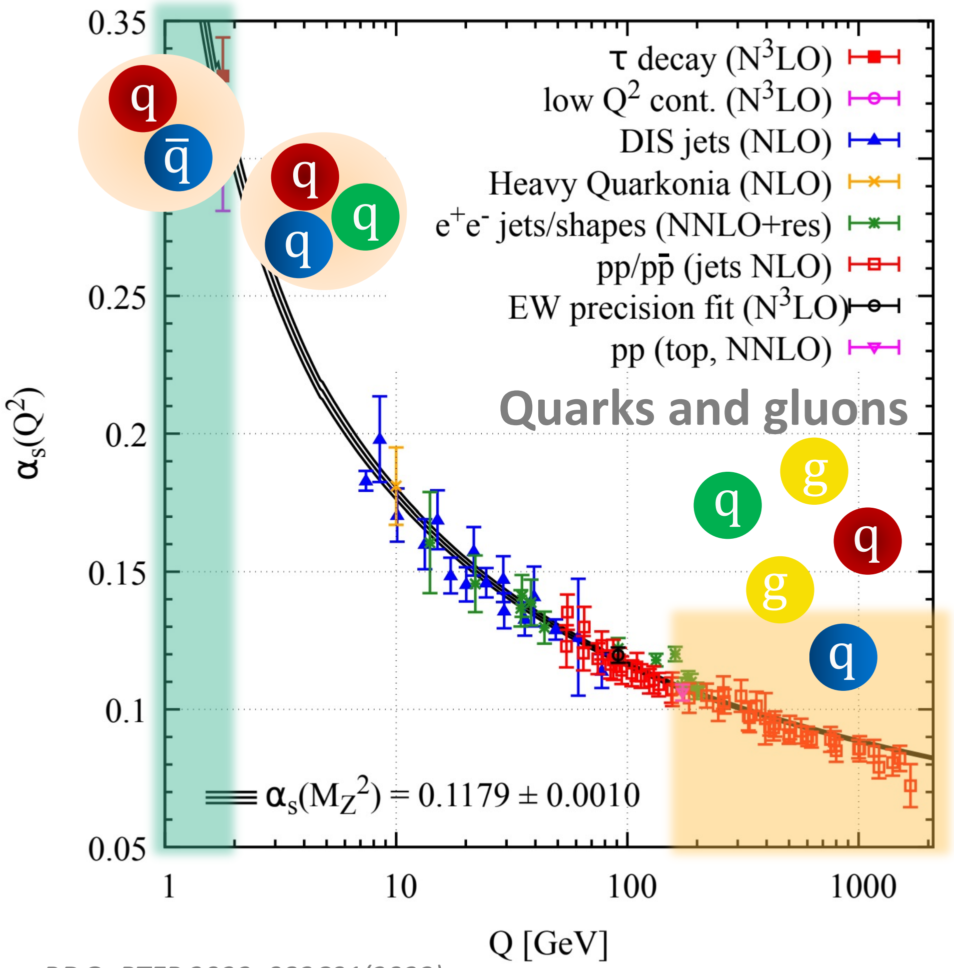
How do hadrons emerge?

Bound states/resonances

Conventional and exotic states



Need for experimental data

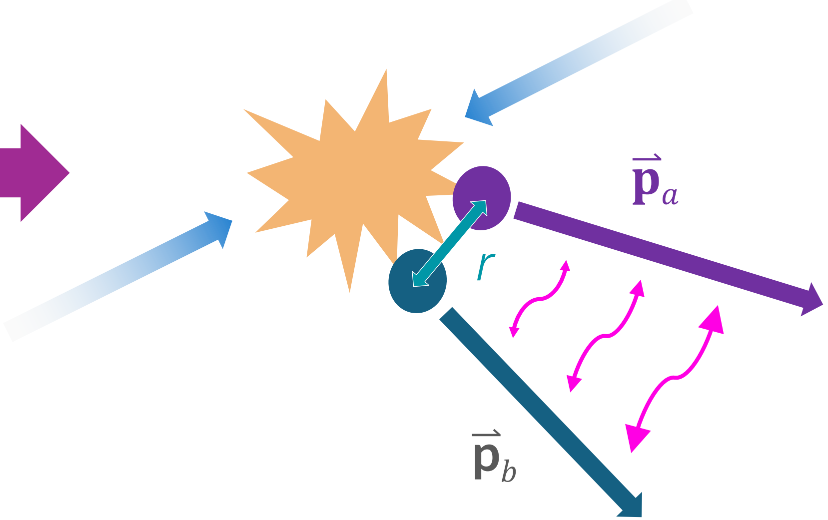


PDG, PTEP 2022, 083C01(2022)

Femtoscopy technique at the Large Hadron Collider

ALICE at the LHC

Hadronic interaction



Femtoscopy technique

$$C(\vec{\mathbf{p}}_a, \vec{\mathbf{p}}_b) \equiv \frac{P(\vec{\mathbf{p}}_a, \vec{\mathbf{p}}_b)}{P(\vec{\mathbf{p}}_a)\, P(\vec{\mathbf{p}}_b)}$$

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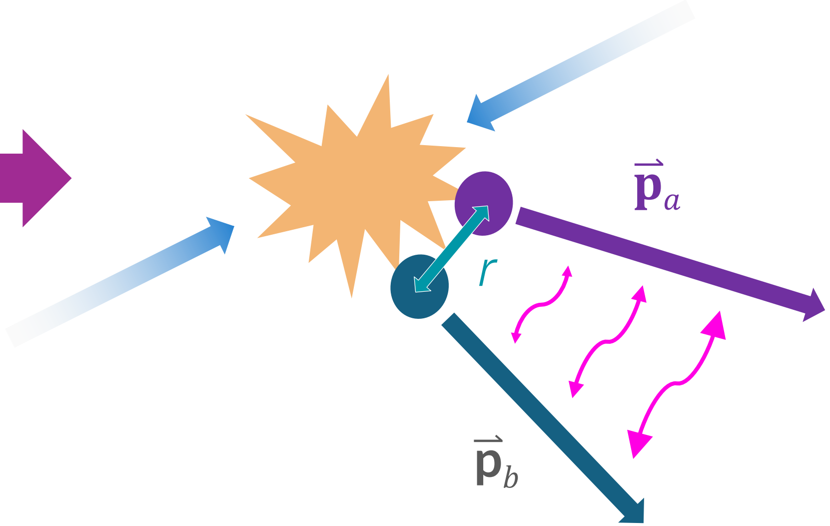

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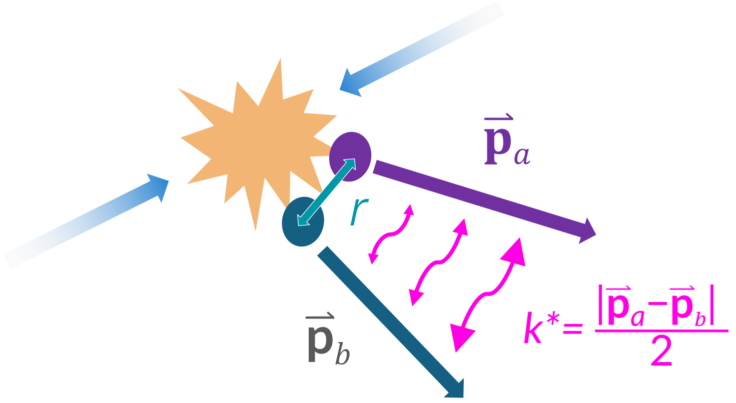
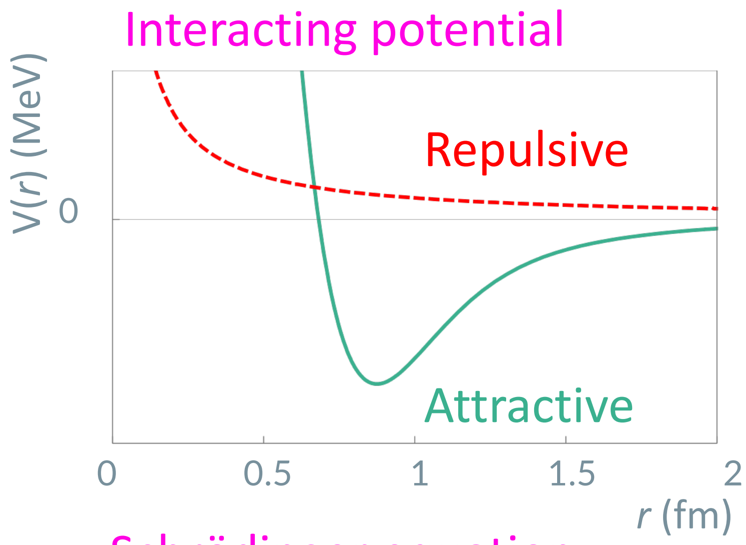
ALICE: *Thomas Humanic 4 Jun, 09:10*
Neelima Agrawal 4 Jun, 18:30
Anton Riedel 5 Jun, 09:30
Valentina Mantovani Sarti 5 Jun, 08:30
Laura Serksnyte/Anton Riedel 4 Jun, 17:30

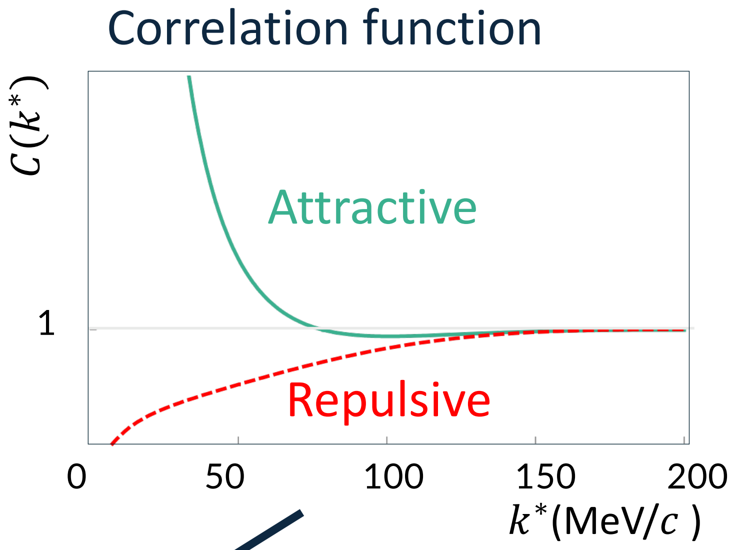
STAR: *Priyanka Roy 4 Jun, 17:10*
Bijun Fan 4 Jun, 17:30
Boyang Fu 5 Jun, 09:10

Theo: *Kenshi Kuroki 5 Jun, 08:50*
Juan Torres-Rincon 6 Jun, 18:00

Correlation function







Emission source $S(\vec{r}\,)$

Schrödinger equation

D. Mihaylov et al., EPJC 78 (2018), 5, 394

Two-particle wave function

$$C(k^*) = \int S(\vec{r}\,) \left|\psi(\vec{k}^*, \vec{r}\,)\right|^2 d^3\vec{r} \; = \mathcal{N}(k^*) \frac{N_{\text{same}}(k^*)}{N_{\text{mixed}}(k^*)}$$

Measuring $C(k^*)$, fixing the source $S(\vec{r}\,)$, study the interaction

M.Lisa, S. Pratt et al., ARNPS 55 (2005), 357-402
L. Fabbietti et al., ARNPS 71 (2021), 377-402

Source function in pp collisions at the LHC

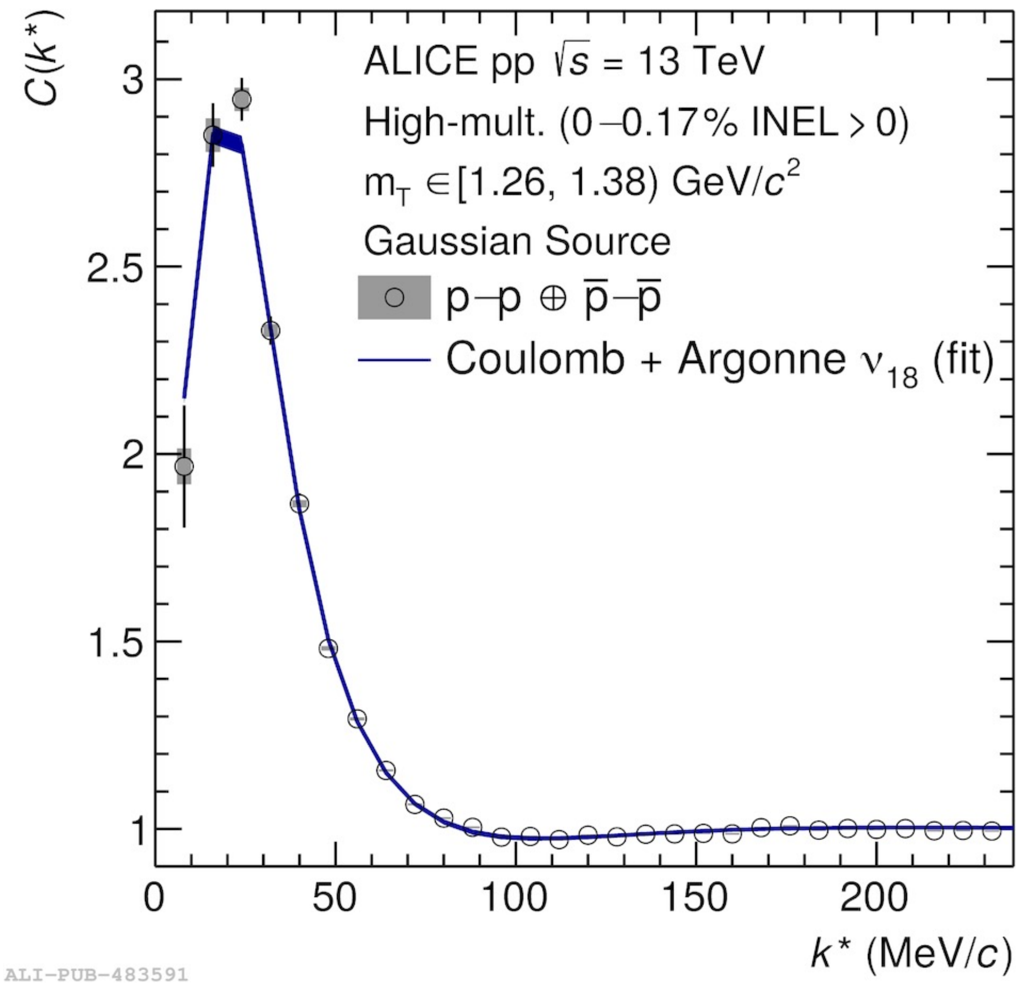
- Emitting source function anchored to p-p correlation function

$$\underbrace{C(k^*)}_{\text{measured}} = \int S(\vec{r}\,)\underbrace{\left|\psi(\vec{k}^*,\vec{r}\,)\right|^2}_{\text{known interaction}} d^3\vec{r}$$

- Gaussian parametrization

$$S(r) = \frac{1}{(4\pi r_{core}^2)^{3/2}} \exp\left(-\frac{r^2}{4 r_{core}^2}\right) \times$$ Effect of short lived resonances (cτ ~ 1 fm)

ALICE Coll., PLB, 811 (2020), 135849



ALICE Coll., PLB, 811 (2020)

Talk: *Anton Riedel 5 Jun, 09:30*
Poster: *Neelima Agrawal 4 Jun, 18:30*

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measured — known interaction

- Gaussian parametrization

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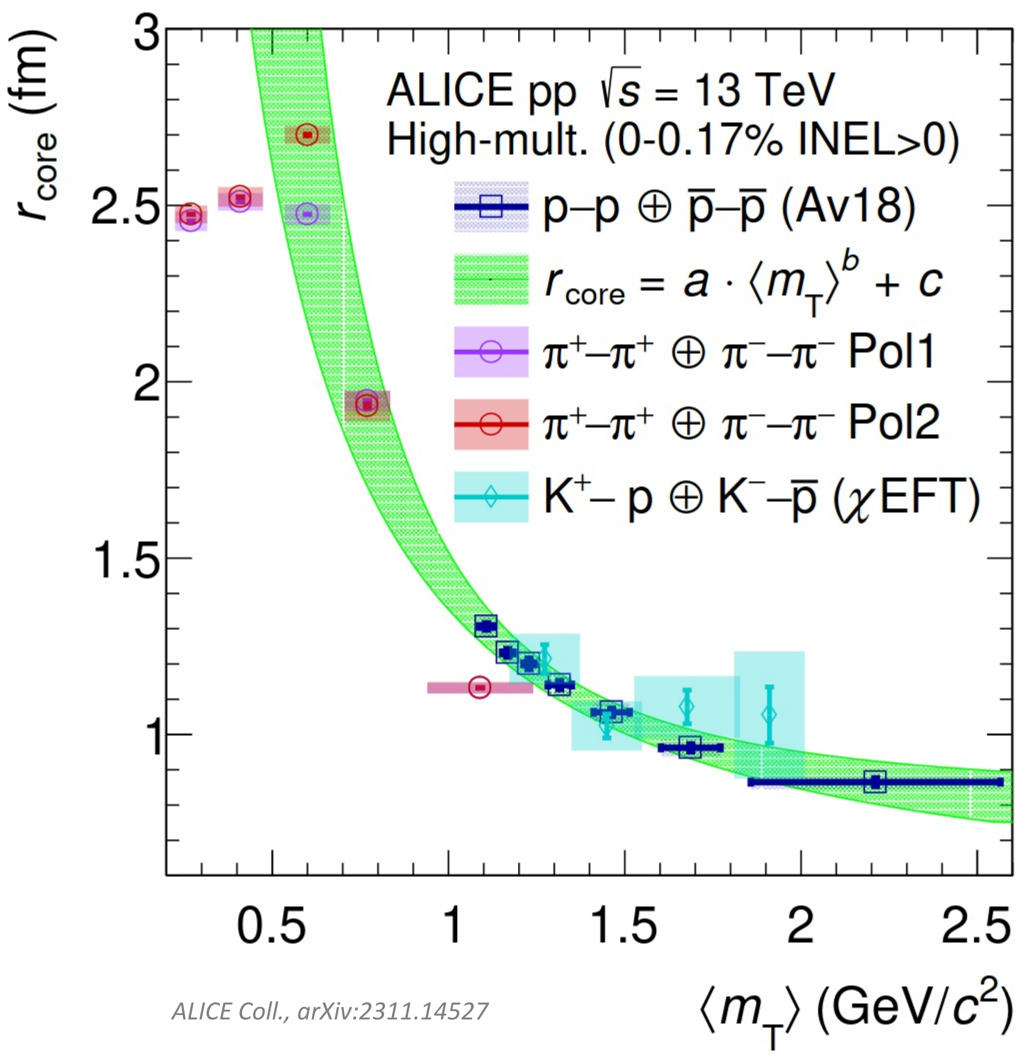
Effect of short lived resonances (cτ ~ 1 fm)

ALICE Coll., PLB, 811 (2020), 135849

- One universal source for all hadrons (cross-check with K^+-p, π-π, p-Λ, p-π)
- **Small particle-emitting source created in pp collisions at the LHC** *ALICE Coll., PLB, 811 (2020), 135849; ALICE Coll., arXiv:2311.14527*



ALICE Coll., arXiv:2311.14527

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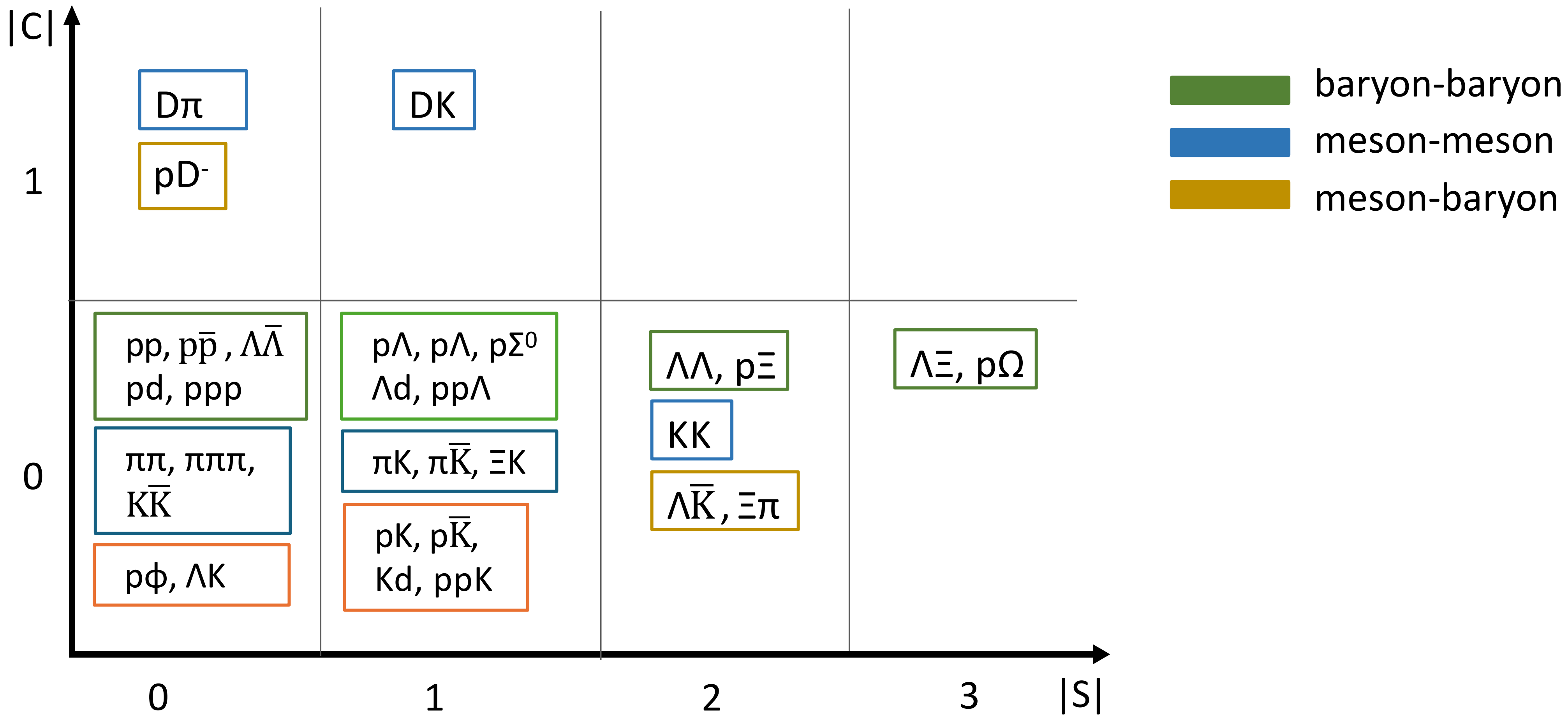
Femtoscopy measurements at the LHC

ALICE provided unprecedented precision input in the study of the hadronic interactions



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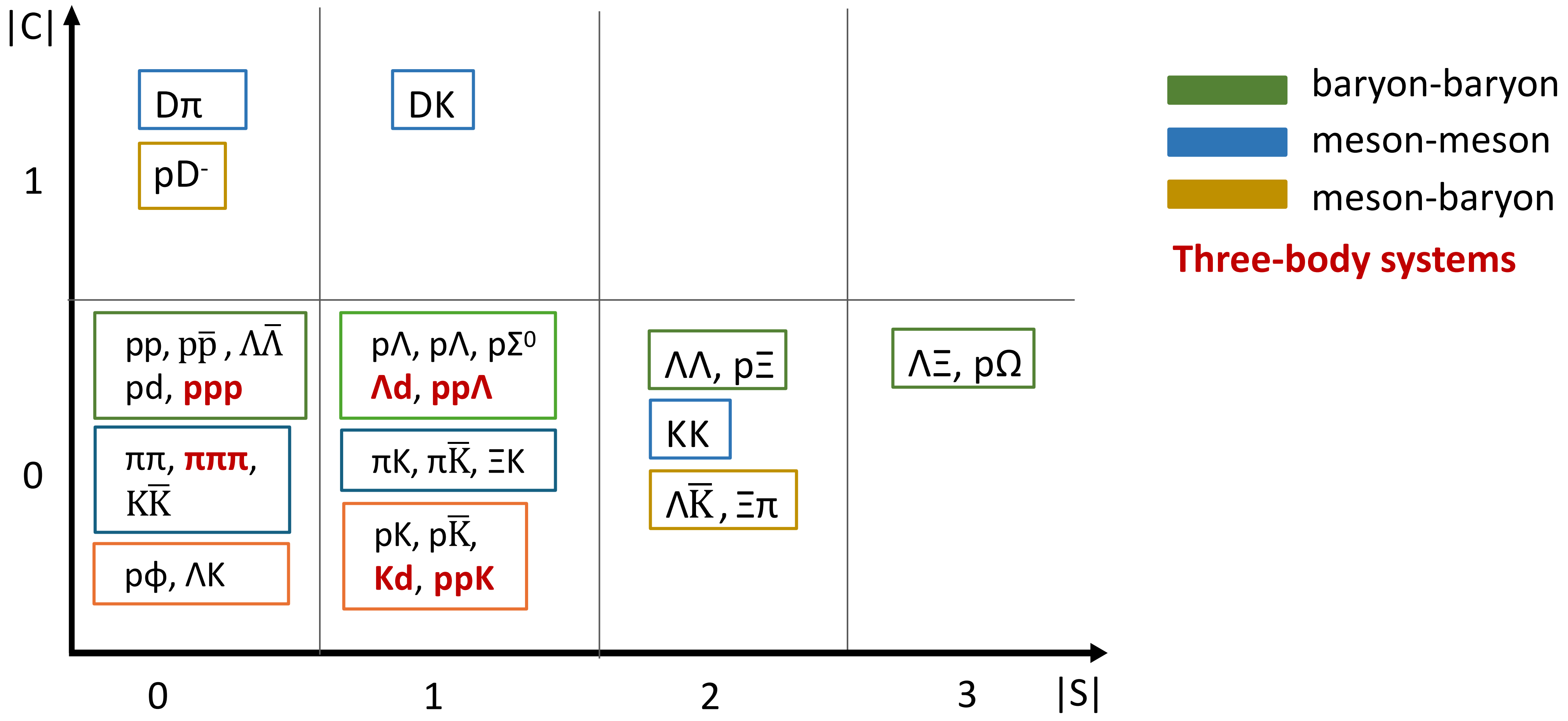
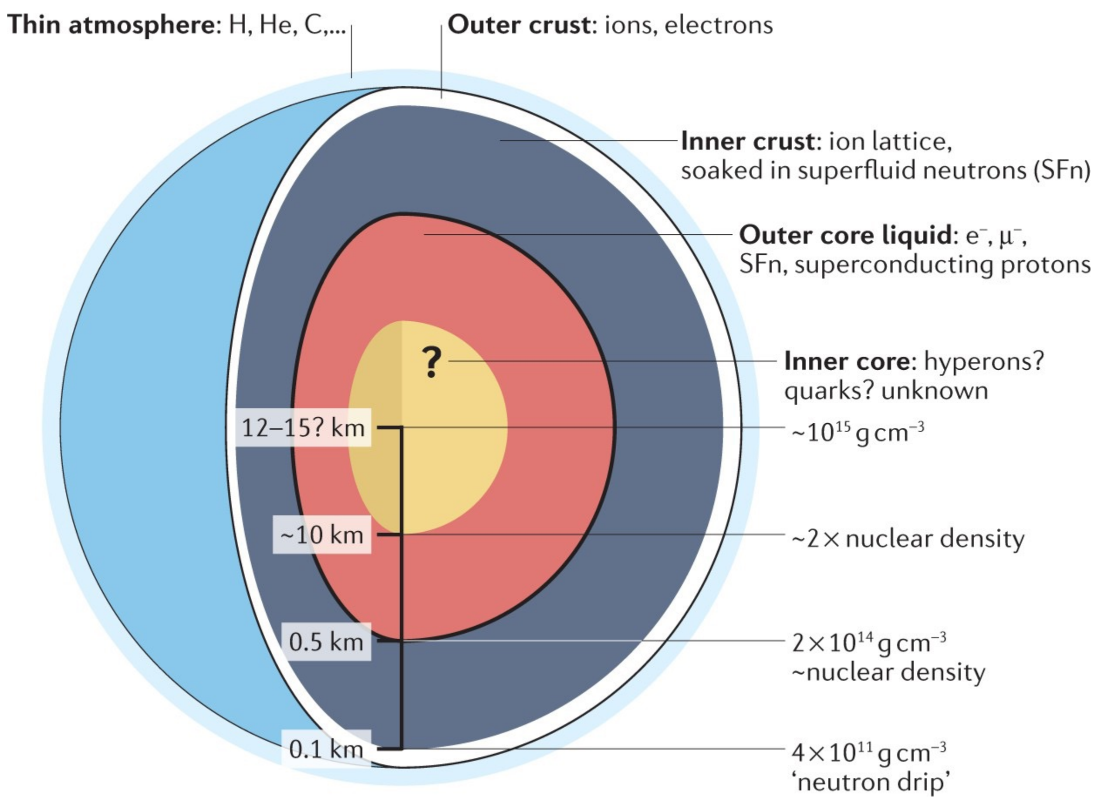


The NΛ and NNΛ interactions in neutron stars

- High density in the core of neutron stars
  → Production of hyperons as Λ at $\rho = 2\text{-}3\rho_0$ and softening of the equation of state
  → Incompatibility with astrophysical measurements of $M_{NS} \gtrsim 2\ M_\odot$
  → Long-standing hyperon puzzle



Nature Reviews Physics 4 (2022)
Figure adapted from NICER

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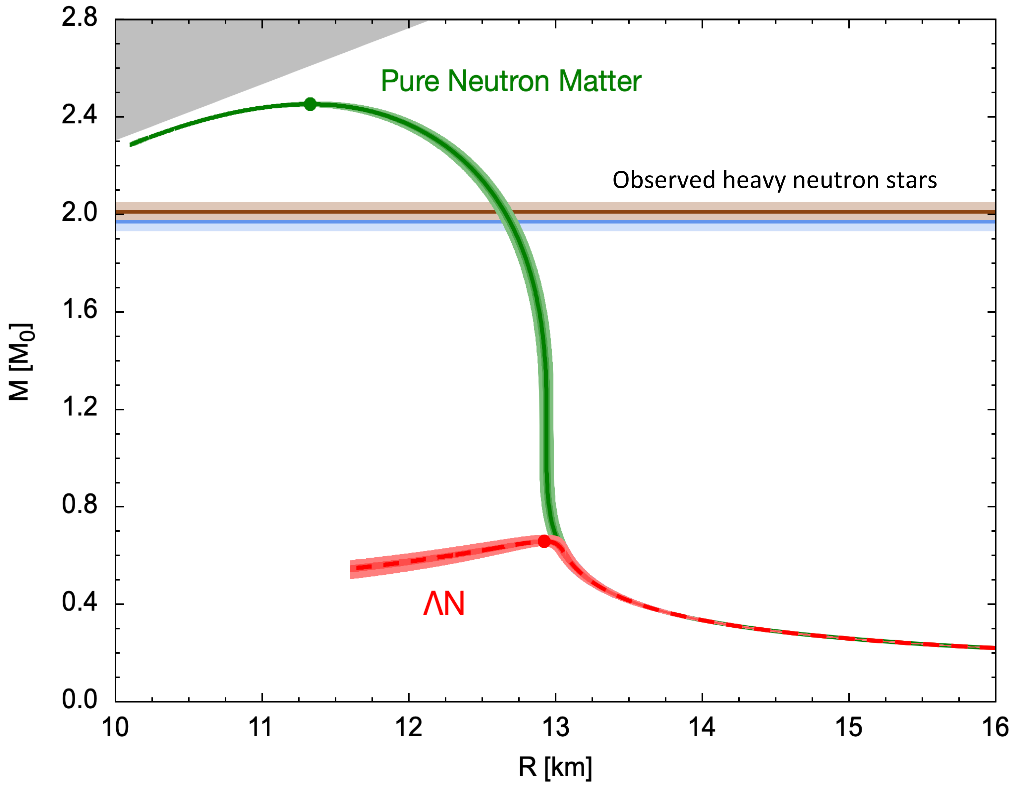

D. Lonardoni et al., PRL 114 (2019)

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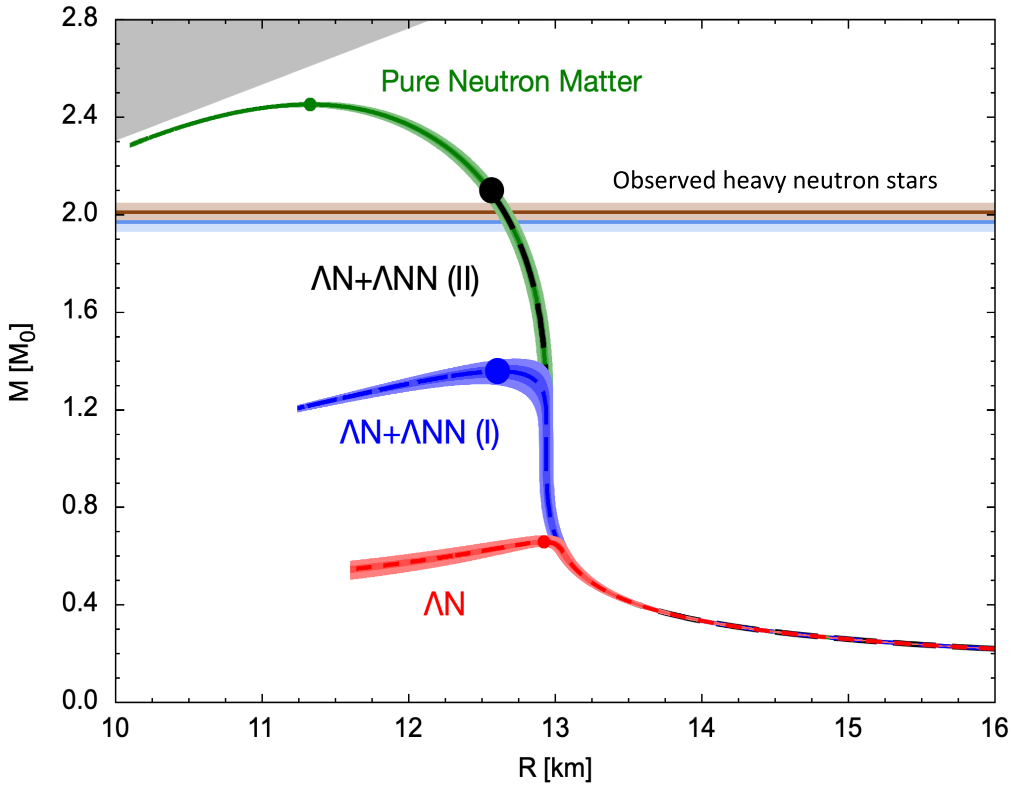
- Repulsive 3-body ΛNN interaction can stiffen the EoS but:
  → Effect on EoS largely model dependent
  → too repulsive YNN leads to no hyperons in the NSs

D. Logoteta et al., EPJA 55 (2019); D. Lonardoni et al., PRL 114 (2019)



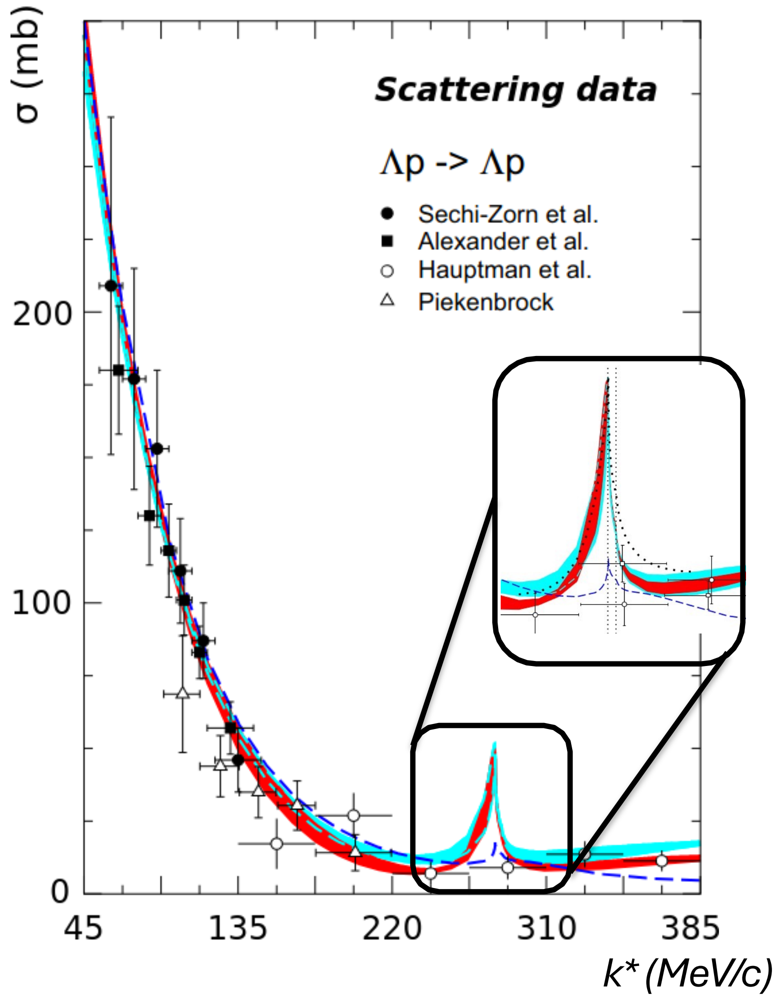
D. Lonardoni et al., PRL 114 (2019)

Can we exploit femtoscopy measurements?

The pΛ interaction so far…

- Mainly investigated with scattering data
  → High-precision results by CLAS at large momenta
  CLAS coll.PRL 127 (2021), 27, 27230
  → Large uncertainties at low momenta and not available down to threshold

- Cusp structure at ΣN opening
  → Coupling ΛN-ΣN driving the behaviour of Λ at finite ρ
  D. Gerstung et al. Eur.Phys.J.A 56 (2020), 6, 175; J.Haidenbauer, U. Meißner, EPJA 56 (2020), 3, 91
  → State-of-art chiral potentials with different ΛN-ΣN strength

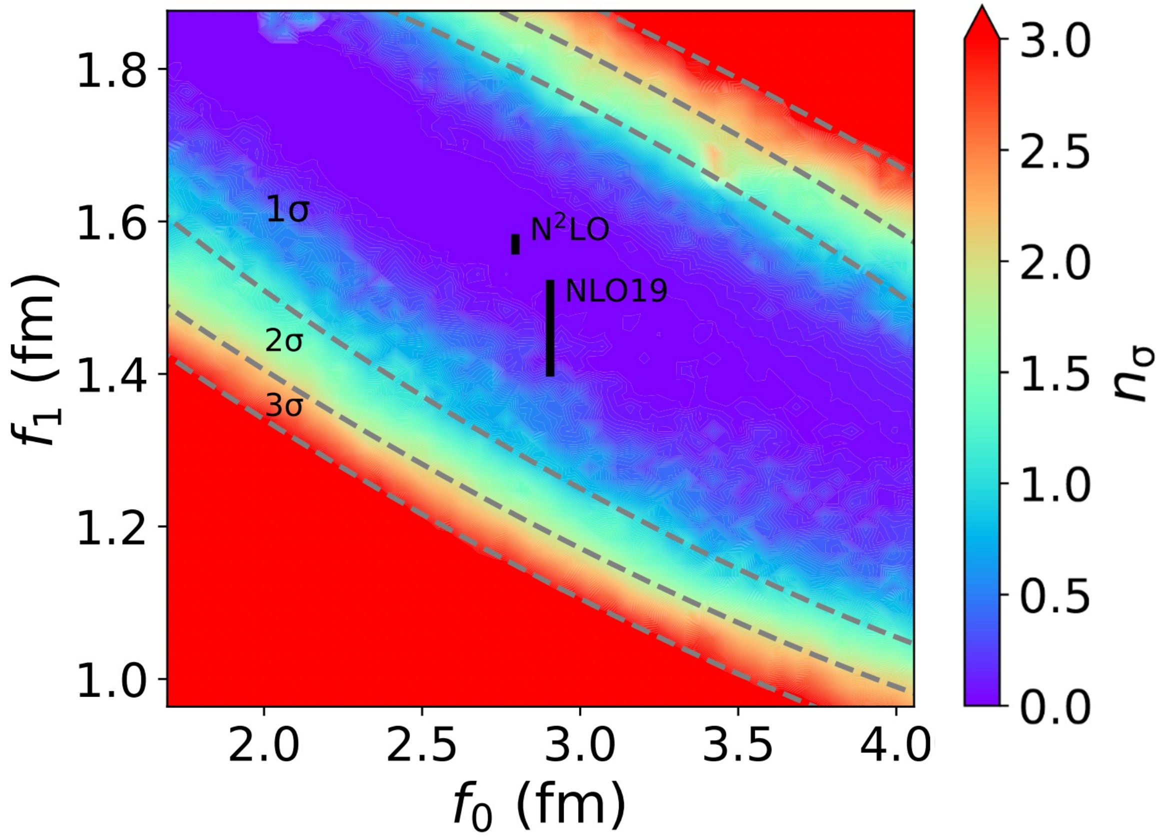


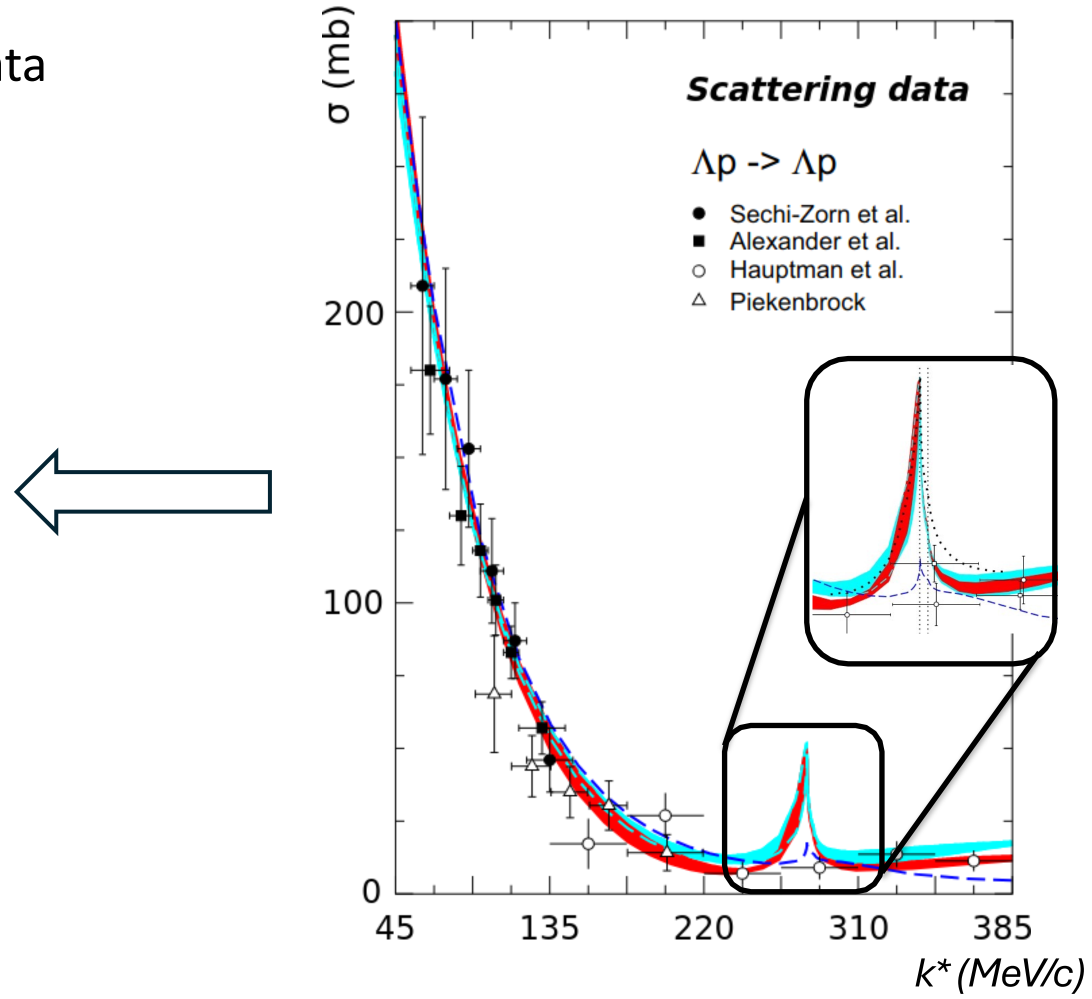
NLO19: J.Haidenbauer, U. Meißner, EPJA 56 (2020), 3, 91
NLO13: J.Haidenbauer, N.Kaiser et al., NPA 915, 24 (2013)

The pΛ interaction before femtoscopy

- Spin-0 and Spin-1 scattering length from scattering data
- Agreement with N2LO and NLO19

D. Mihaylov, J. Haidenbauer and V. Mantovani Sarti, PLB 850 (2024) 138550

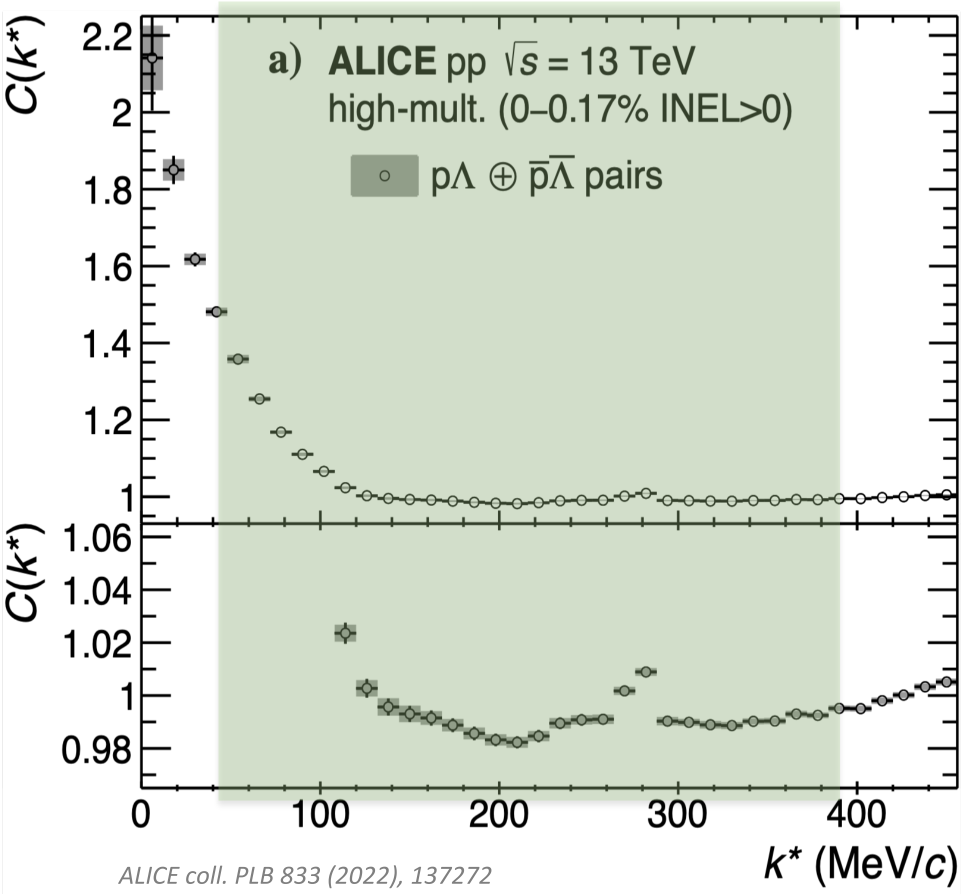




NLO19: J.Haidenbauer, U. Meißner, EPJA 56 (2020), 3, 91

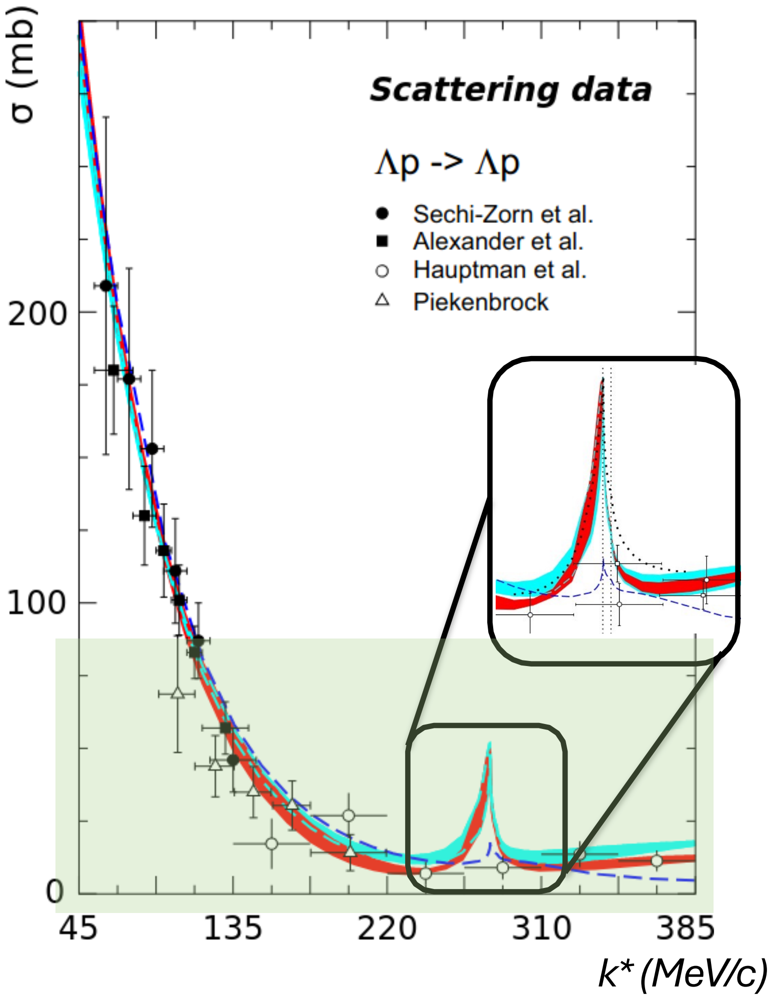
NLO13: J.Haidenbauer, N.Kaiser et al., NPA 915, 24 (2013)

The pΛ interaction in the femtoscopy era



ALICE coll. PLB 833 (2022), 137272

- Measurement down to zero momentum
- Factor 20 improved precision (<1%)
- First experimental evidence of ΛN-ΣN opening in 2-body channel



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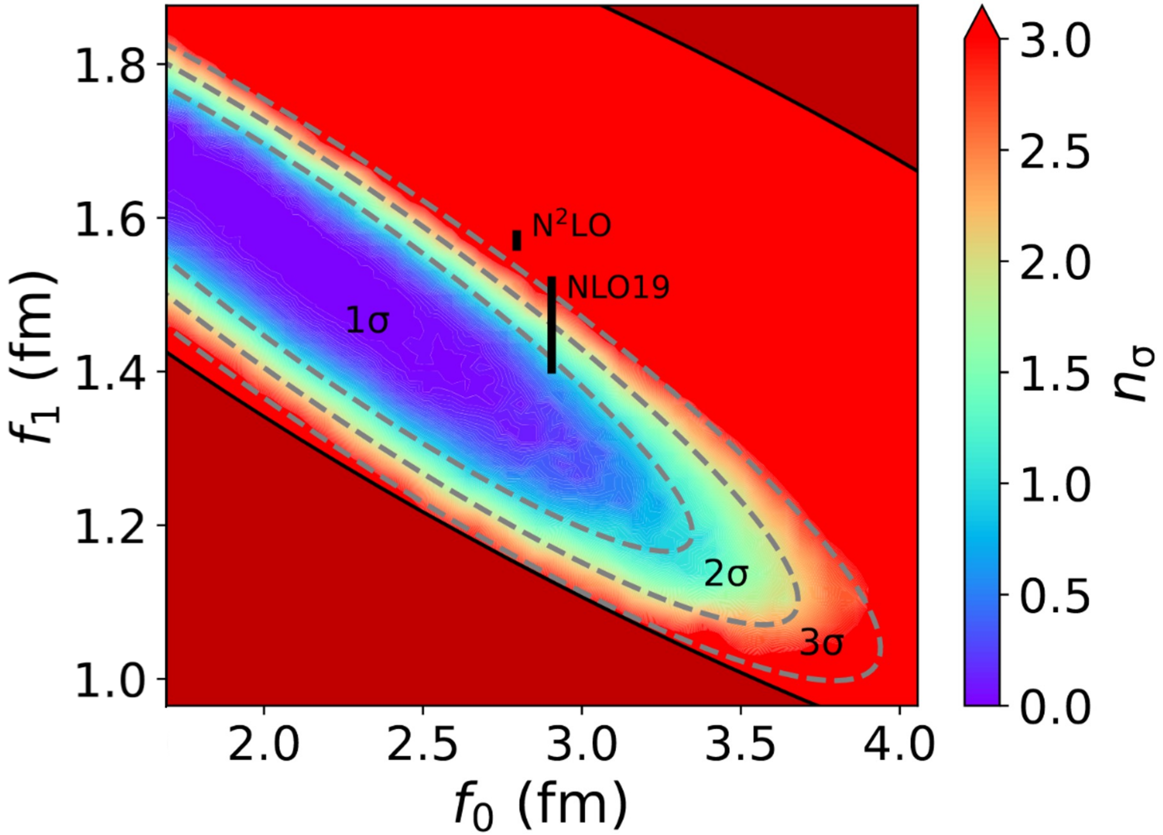
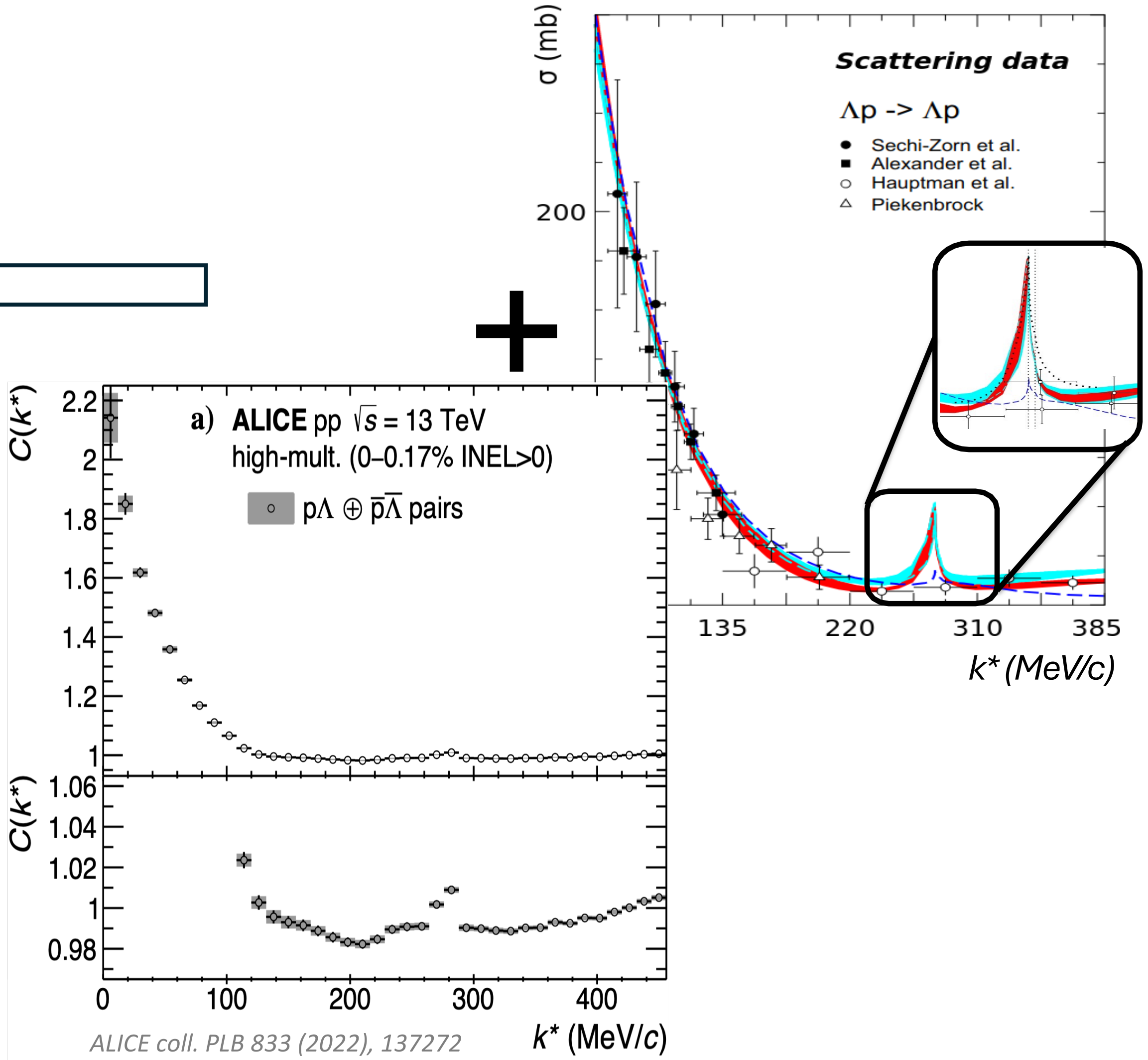
NLO13: J.Haidenbauer, N.Kaiser et al., NPA 915, 24 (2013)

The pΛ interaction in the femtoscopy era

- **NEW**: combined analysis of femtoscopic and scattering data

D. Mihaylov, J. Haidenbauer and V. Mantovani Sarti, PLB 850 (2024) 138550





ALICE coll. PLB 833 (2022), 137272

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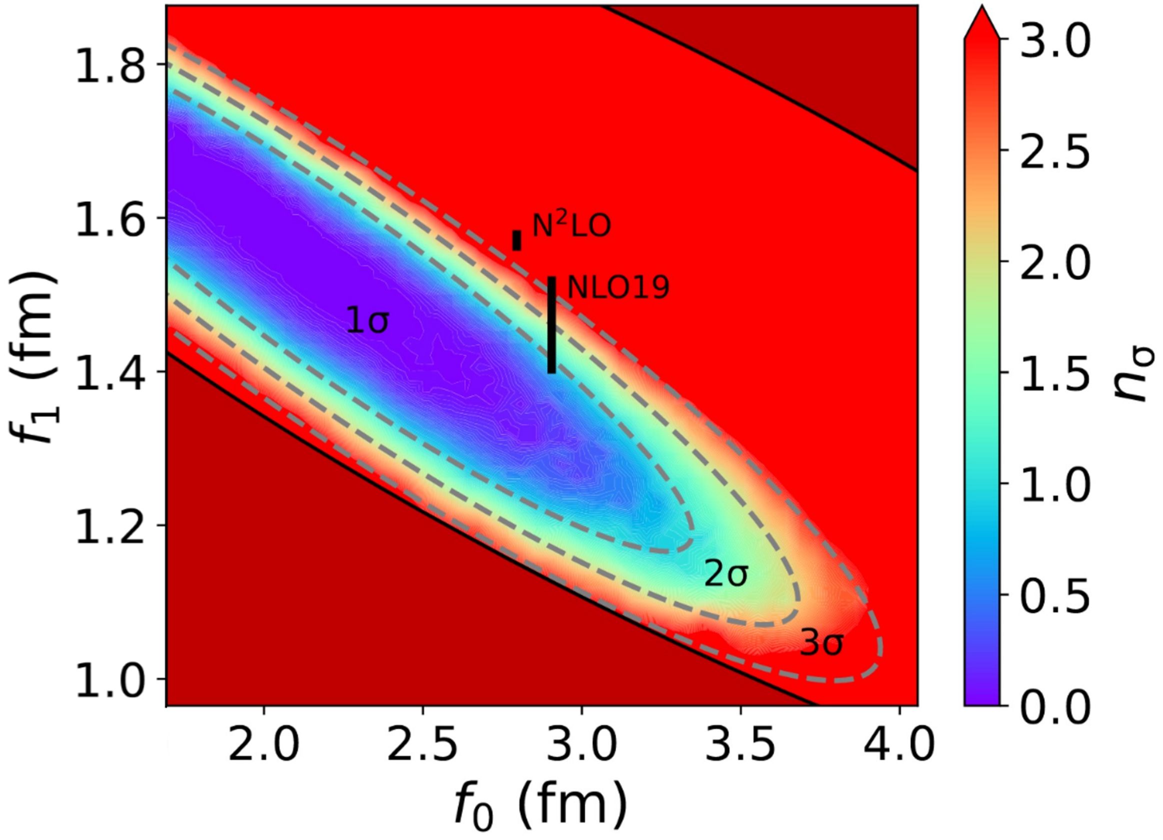
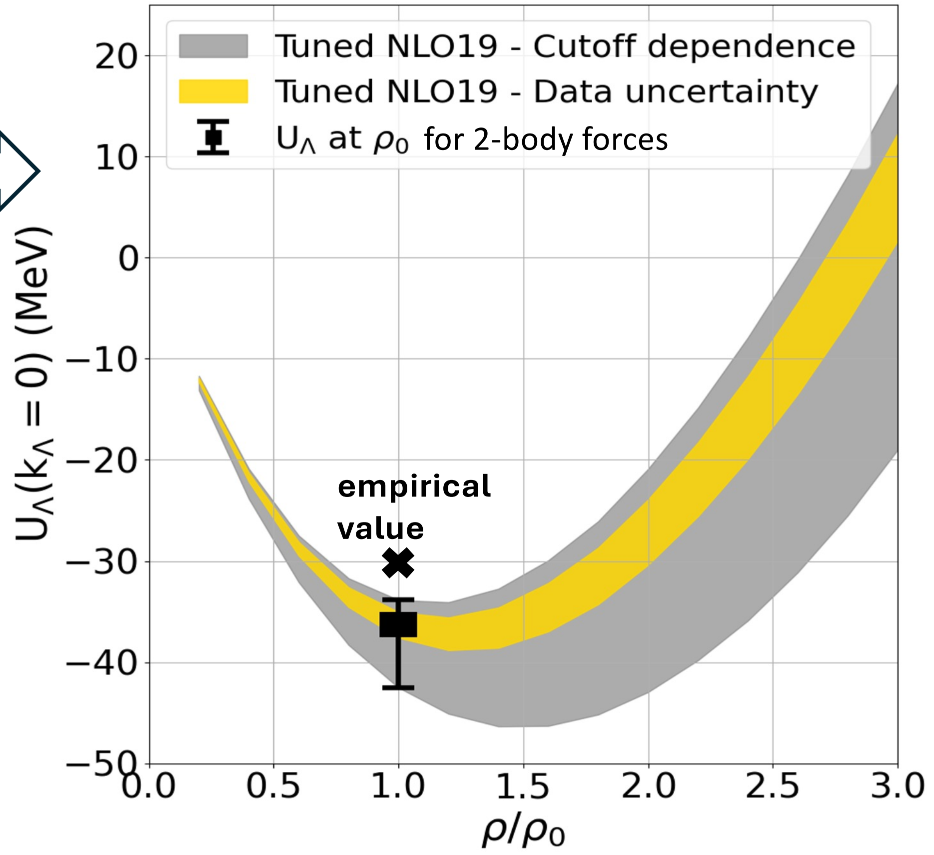


New parameterizations of the χEFT
Compatible with repulsive 3-body forces

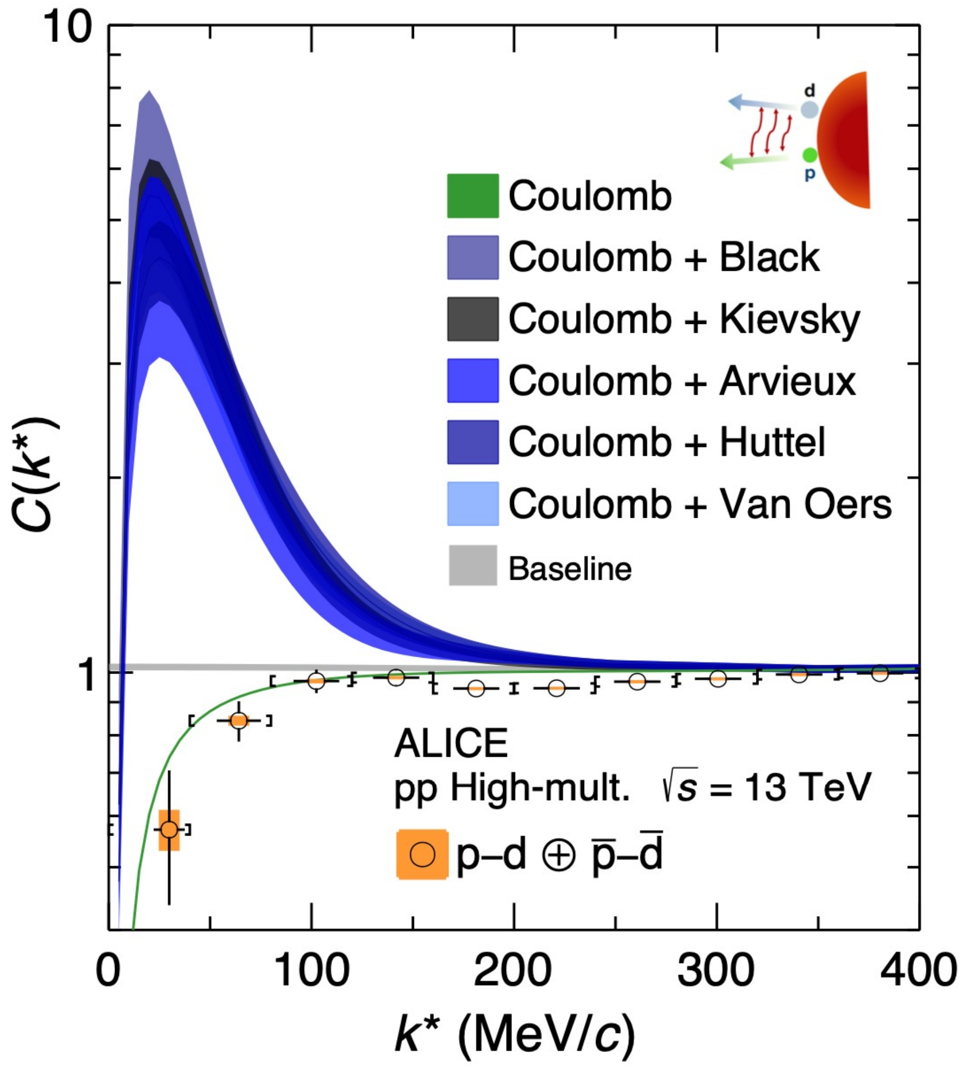


D. Mihaylov, J. Haidenbauer and V. Mantovani Sarti, PLB 850 (2024) 138550

NNN using proton-deuteron correlations

- Point-like particle models anchored to scattering experiments
  W. T. H. Van Oers et al., NPA 561 (1967);
  J. Arvieux et al., NPA 221 (1973); E. Huttel et al., NPA 406 (1983);
  A. Kievsky et al., PLB 406 (1997); T. C. Black et al., PLB 471 (1999);
- Coulomb + strong interaction using Lednický model
  Lednický, R. Phys. Part. Nuclei 40, 307–352 (2009)
- Only s-wave interaction
- Source radius evaluated using the universal m_T scaling

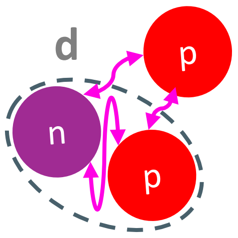
Point-like particle description doesn't work for p-d

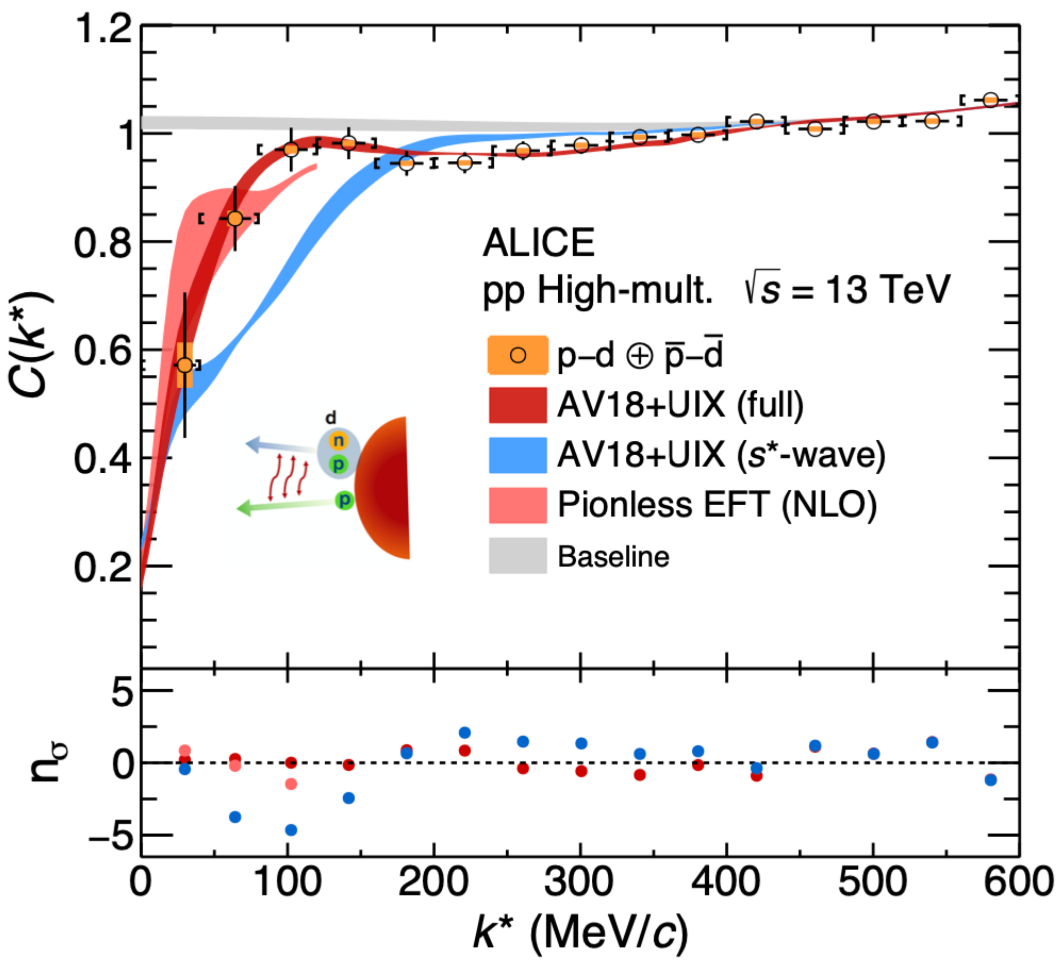


ALICE Coll. arXiv:2308.16120 (2023)

NNN using proton-deuteron correlations

- Full three-body calculations are required (NN + NNN + Quantum Statistics)
- Hadron-nuclei correlations at the LHC can be used to study many-body dynamics

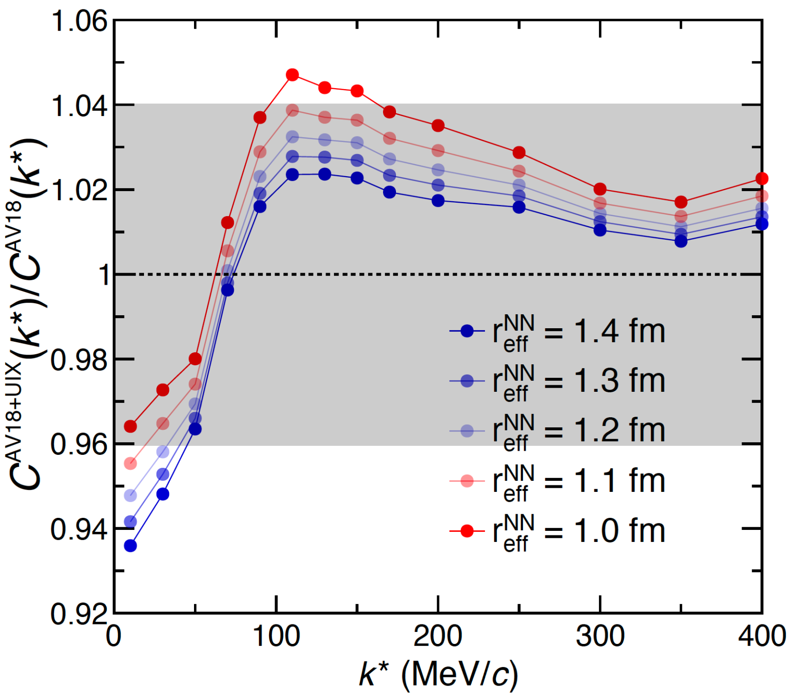


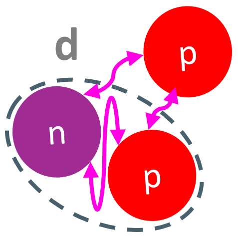


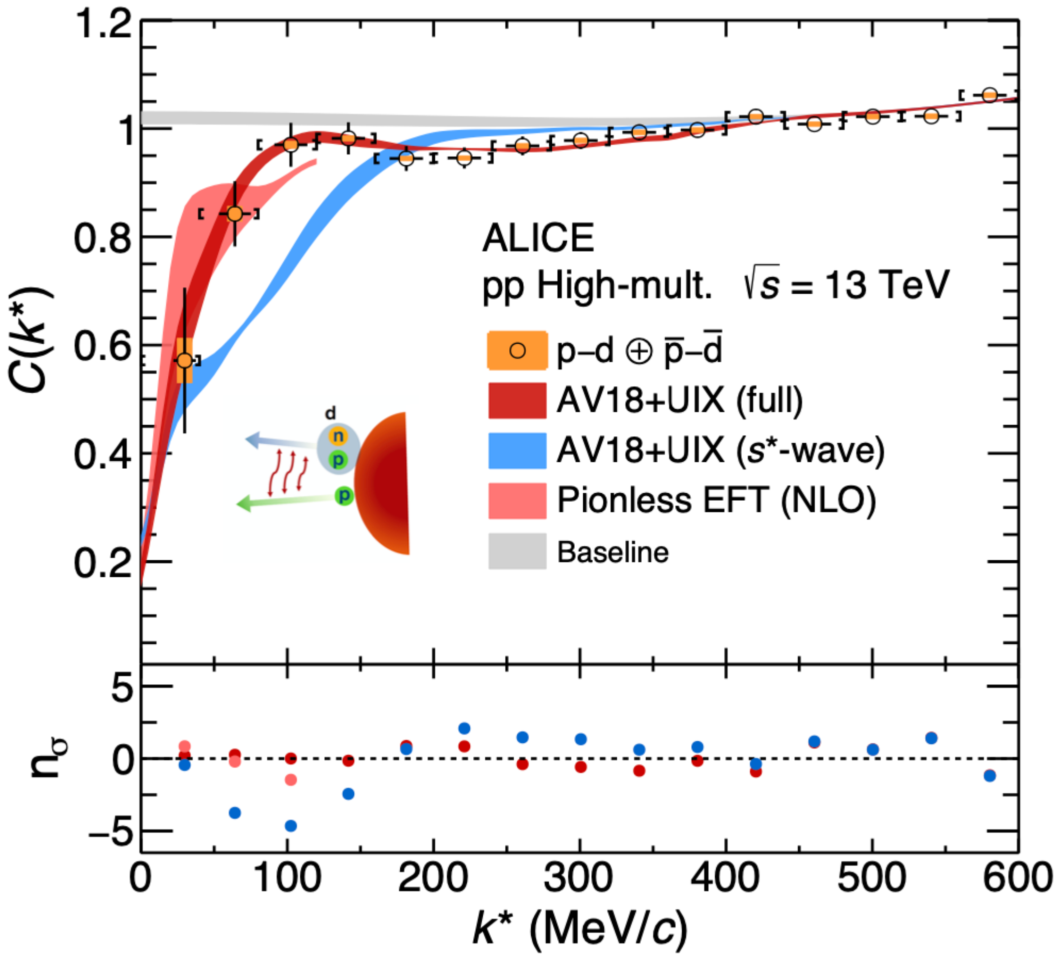
ALICE Coll., arXiv:2308.16120 (2023)
M. Viviani et al, Phys.Rev.C 108 (2023) 6, 064002

NNN using proton-deuteron correlations

- Full three-body calculations are required (NN + NNN + Quantum Statistics)
- Hadron-nuclei correlations at the LHC can be used to study many-body dynamics
- Sensitivity to three-body forces up to 5%



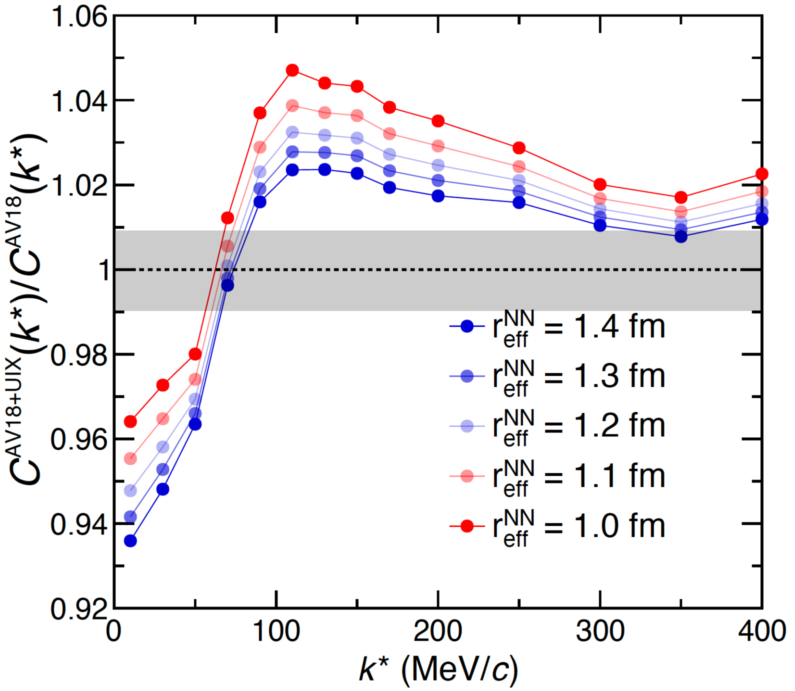


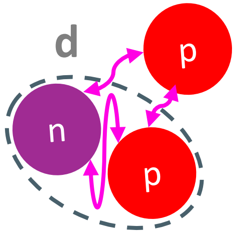


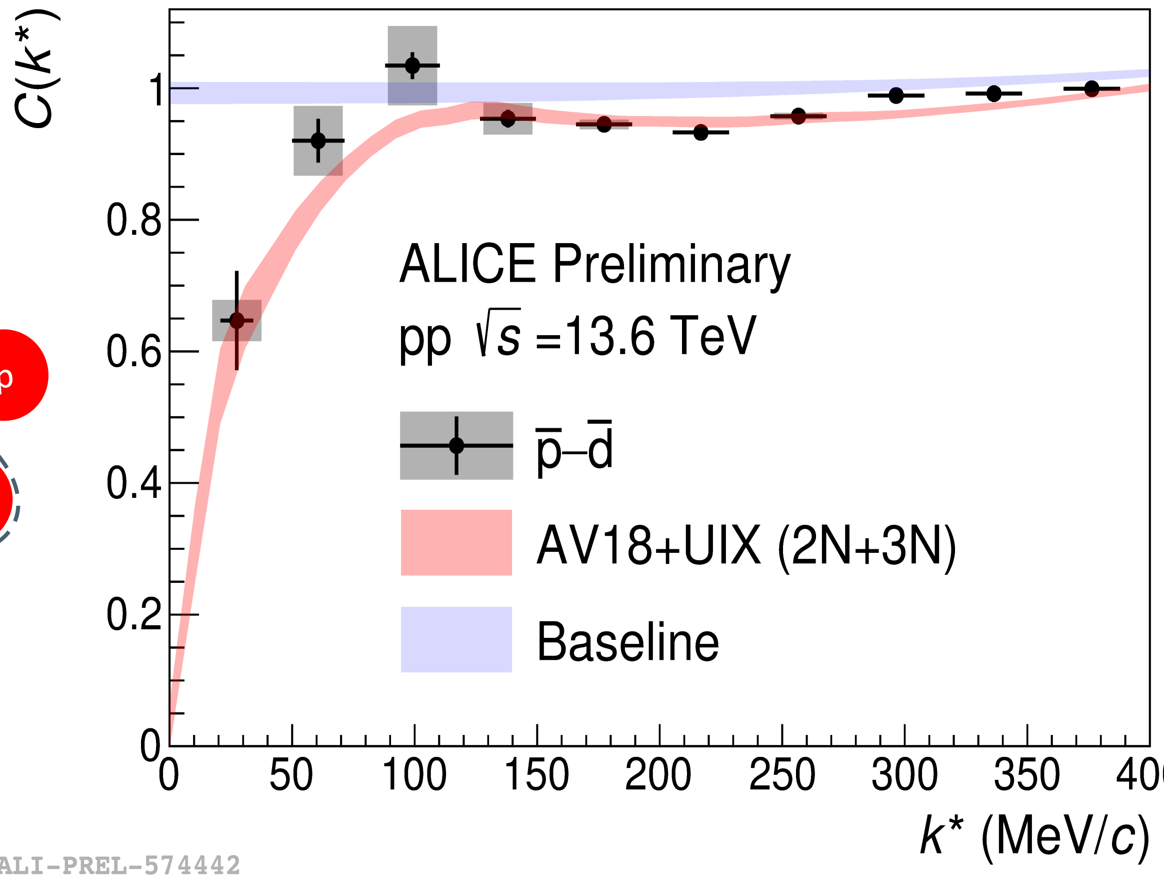
ALICE Coll., arXiv:2308.16120 (2023)
M. Viviani et al, Phys.Rev.C 108 (2023) 6, 064002

NNN using proton-deuteron correlations

- Full three-body calculations are required (NN + NNN + Quantum Statistics)
- Run 3 data from 2022 already analysed and results are promising!
- In Run 3 expected uncertainty of 1%

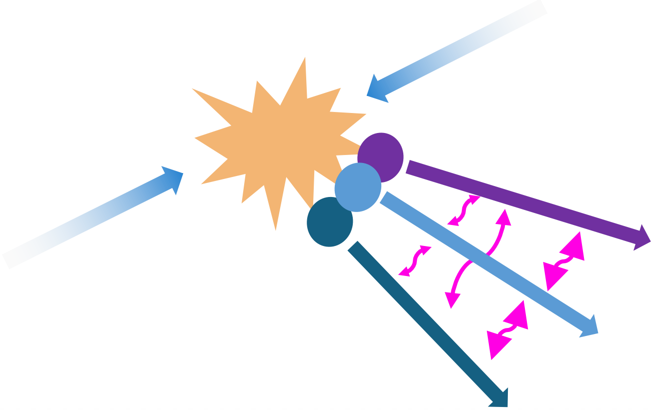






Talk by *Laura Serksnyte/Anton Riedel 4 Jun, 17:30*

p-p-p correlation function



- First ever full three-body correlation function calculations

three-proton wave function

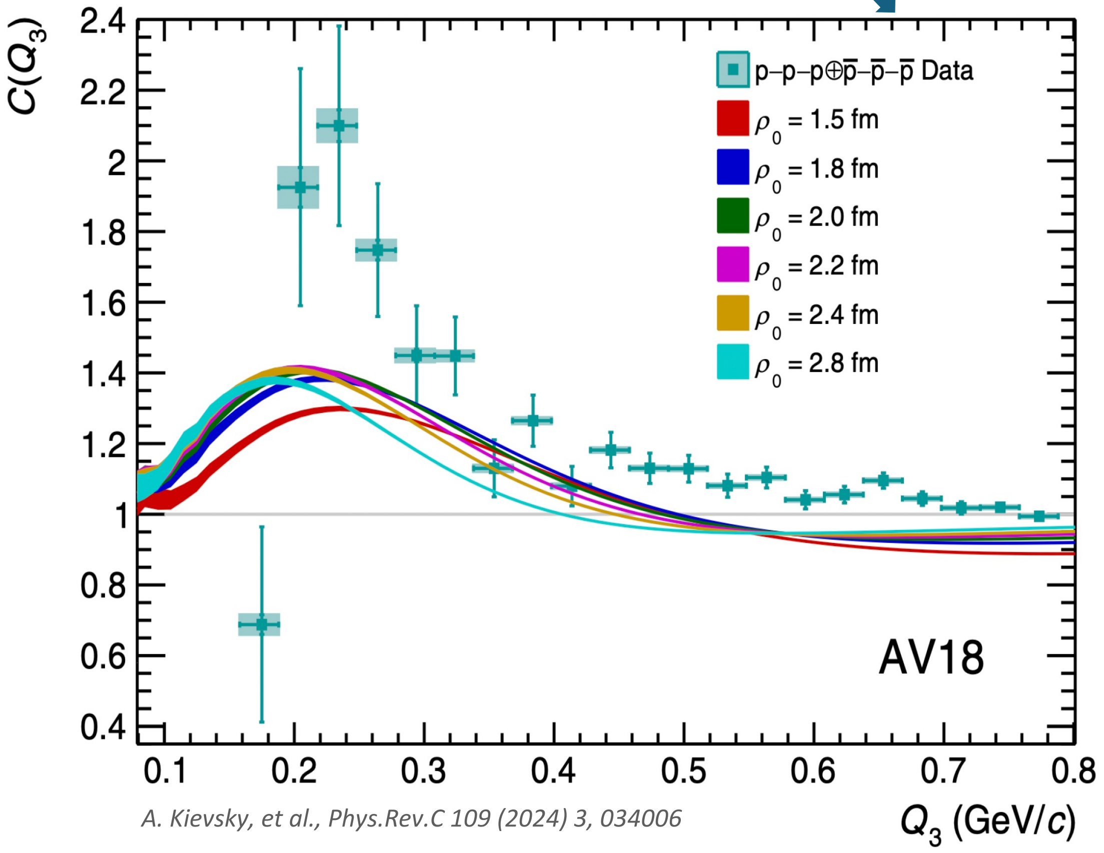
$$C(Q_3) = \int \rho^5 d\rho \, S(\rho, \rho_0) |\Psi(\rho, Q_3)|^2$$

hyperradius

- Wave function via HH:
  - AV18
  - Three-body Coulomb interaction
  - Quantum statistics

A. Kievsky, et al., Phys.Rev.C 109 (2024) 3, 034006

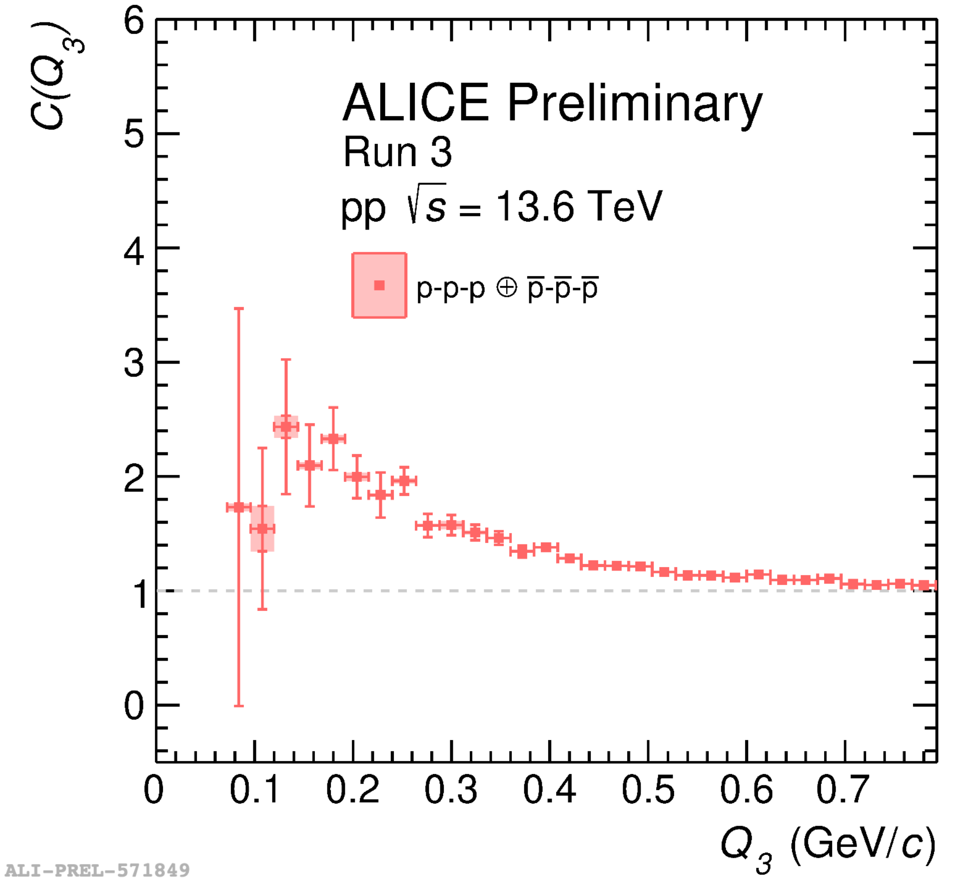
- Negligible contribution from UIX
- Utilise to study three-body source
- Only shape of the theory and data should be compared



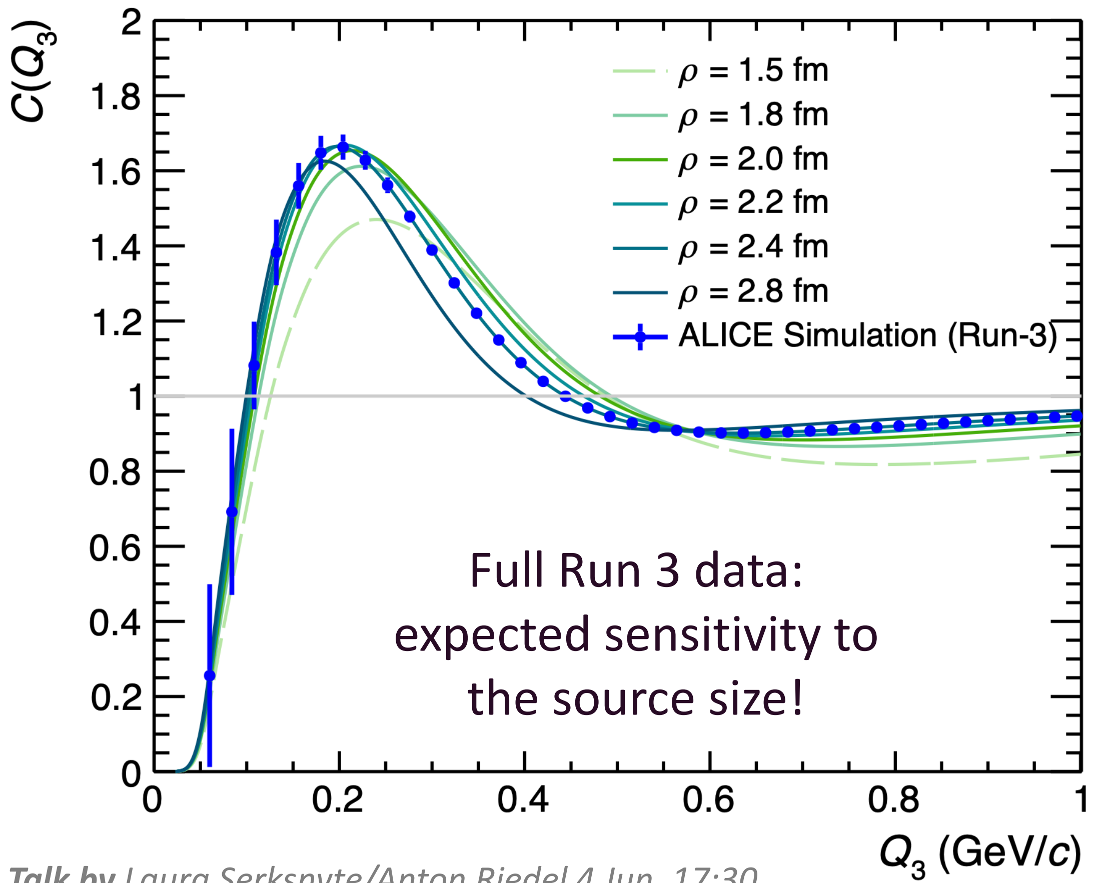
A. Kievsky, et al., Phys.Rev.C 109 (2024) 3, 034006

p-p-p correlation function

- ALICE Run 3 data from 2022 already analysed and results are promising!



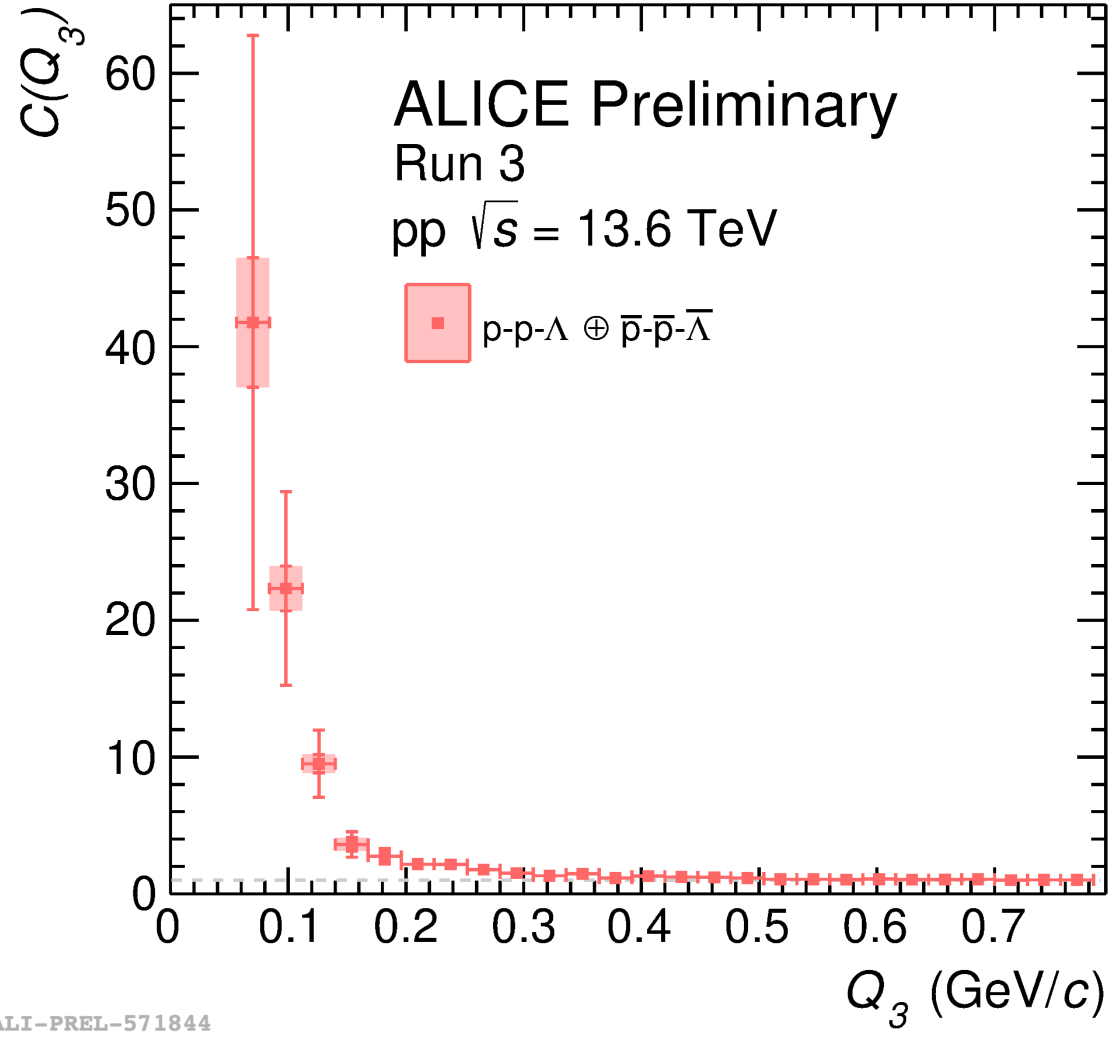
- At the end of Run 3: 25 times larger statistical sample than 2022 alone



Talk by Laura Serksnyte/Anton Riedel 4 Jun, 17:30

p-p-Λ correlation function

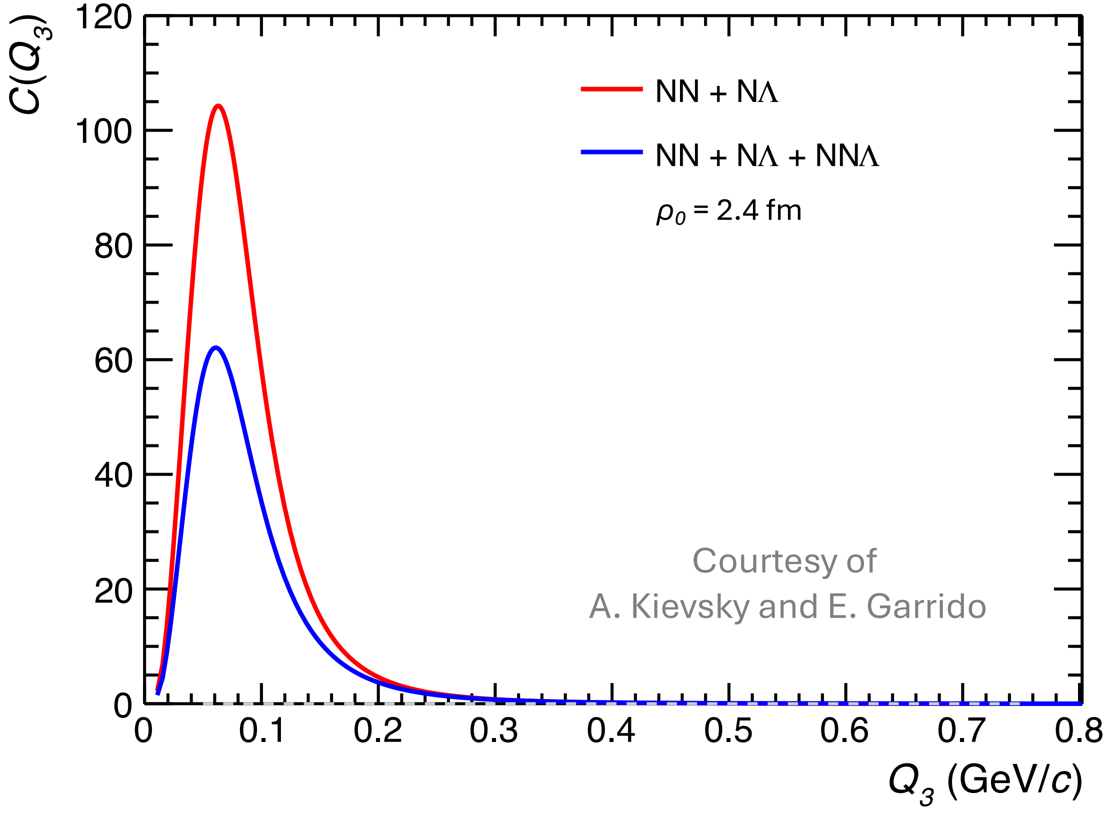
- **New data by ALICE** (Run 3 2022 data)
- By the end of Run 3: 150 times larger statistical triplets sample expected compared to Run 2 due to developed software triggers!

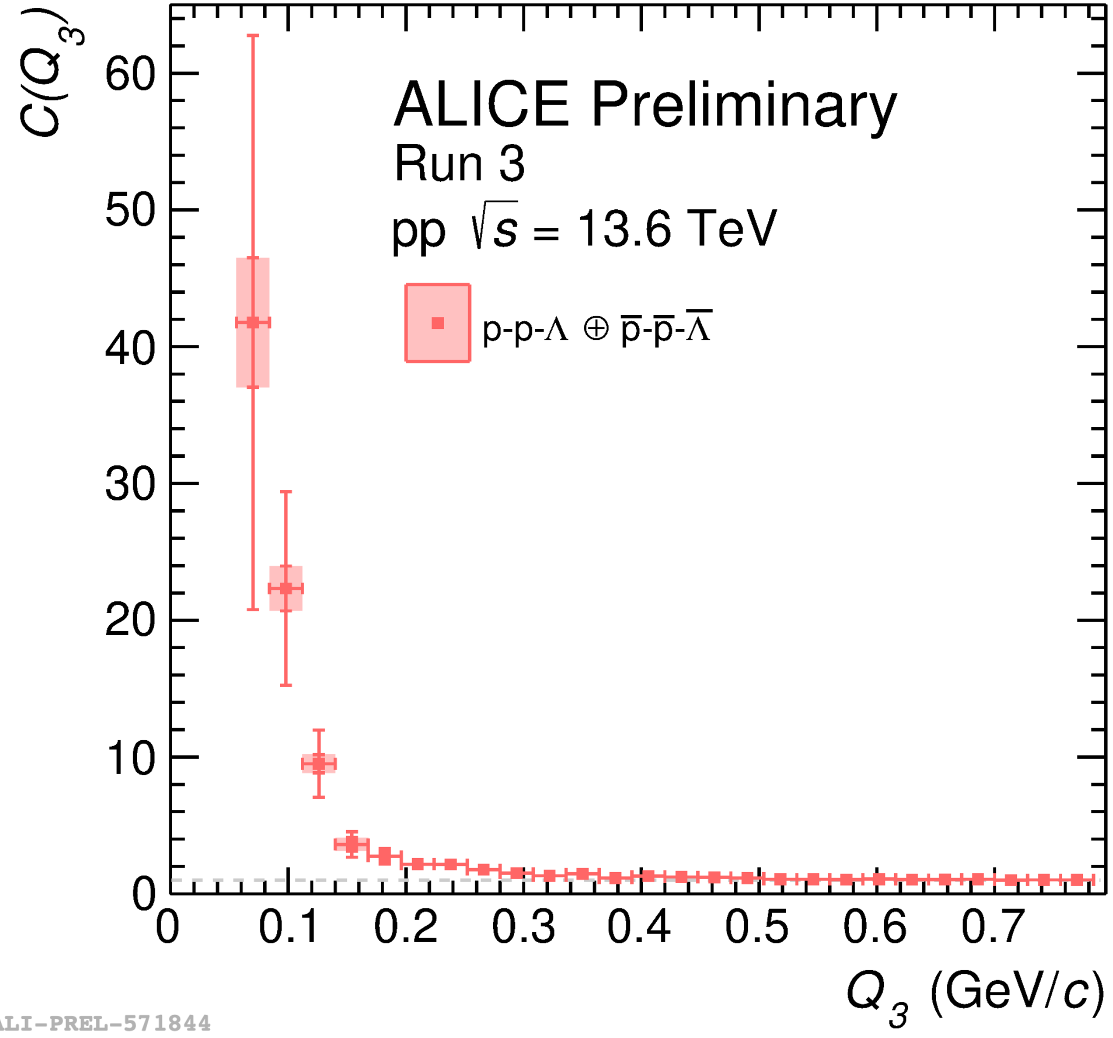


Talk by Laura Serksnyte/Anton Riedel 4 Jun, 17:30

p-p-Λ correlation function

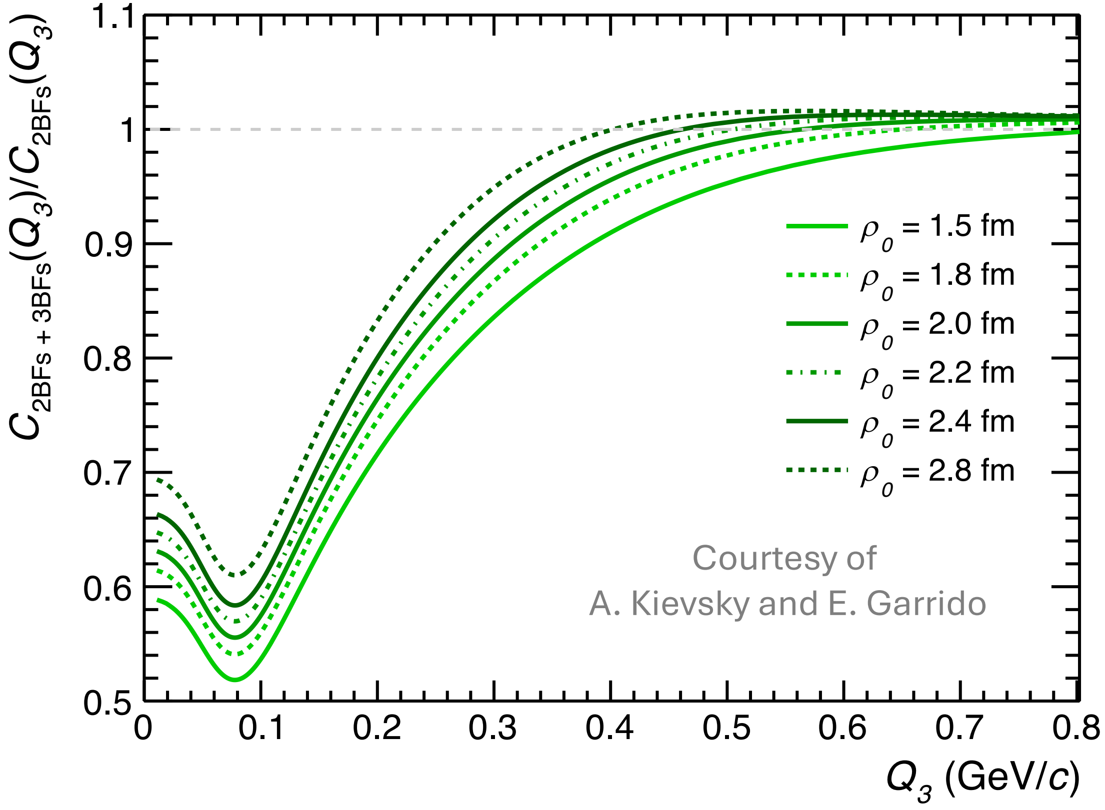
- First theoretical predictions:
  → NΛ interaction from NLO19
  → NNΛ interaction fixed to hypertrition BE

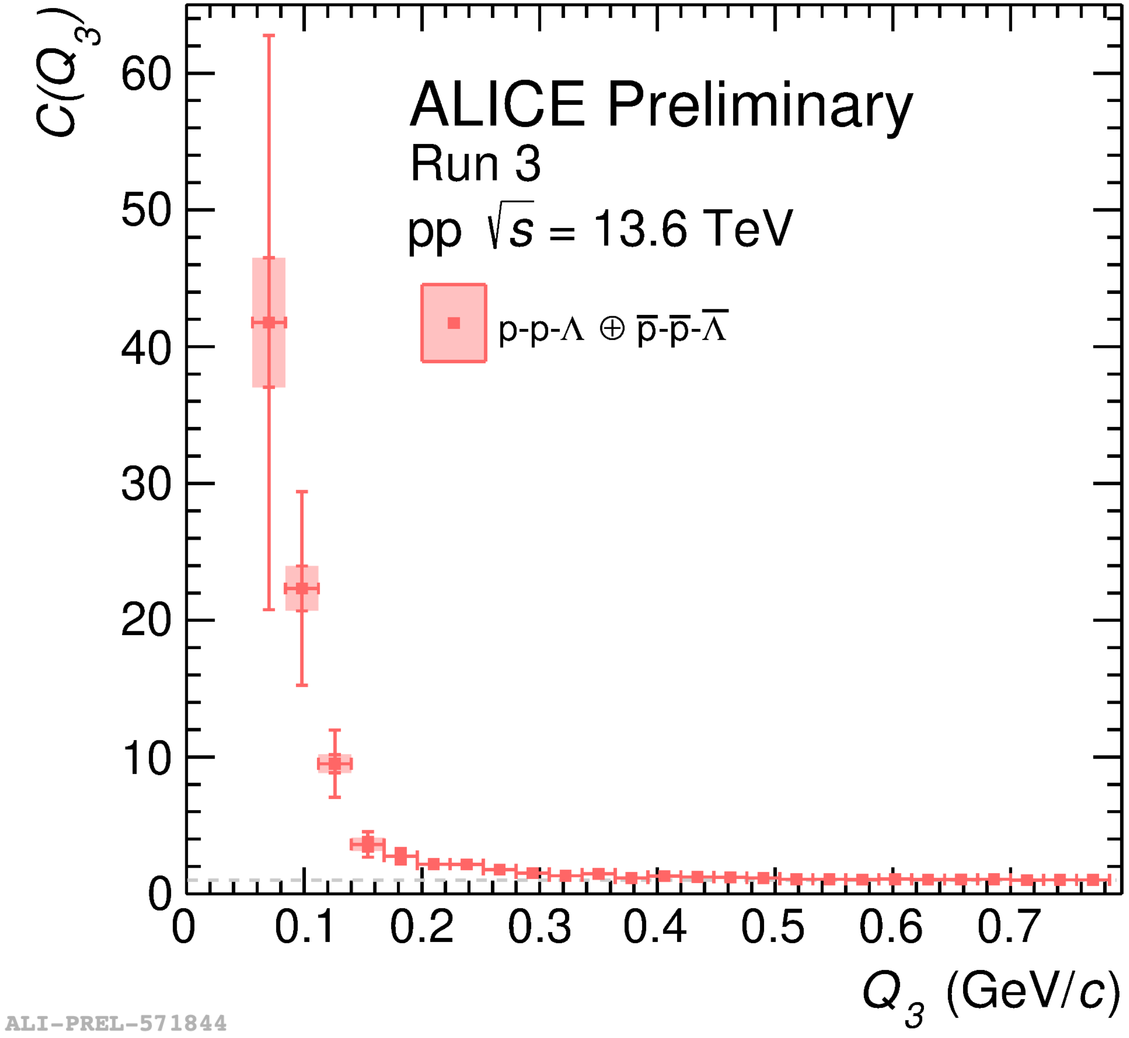




p-p-Λ correlation function

- First theoretical predictions:
  → effect up to 50% due to 3BFs
  (preliminary with *s*- and *p*-waves)





Femtoscopy to study bound states

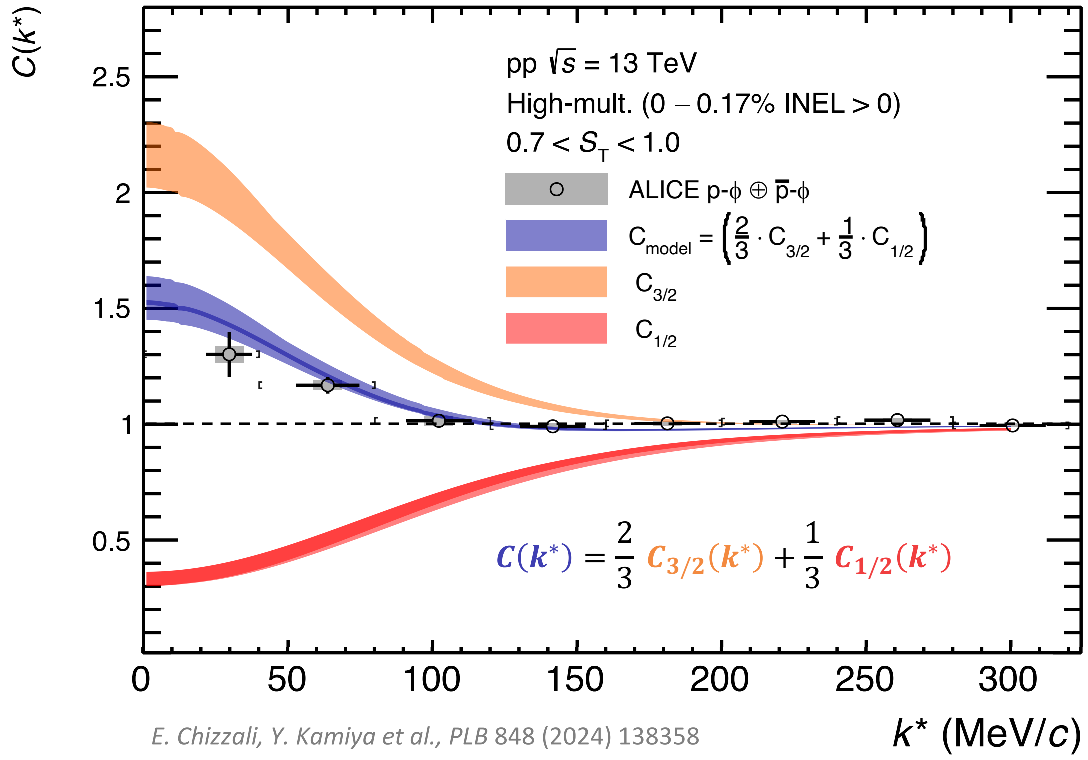
p-ϕ correlation function:

- Spin-3/2 interaction: elastic channel
  → Lattice QCD potentials (HAL QCD)
  Yan Lyu et al., Phys. Rev. D 106 (2022) 074507

- Spin-1/2 interaction: inelastic channels (Nϕ–ΛK, Nϕ–ΣK)
  → Complex potential fitted to the data

- Attractive potential with C(k*) < 1 provides **indication of a p-ϕ bound state**
  → Binding Energy = 14.7–56.6 MeV
  E. Chizzali, Y. Kamiya et al., PLB 848 (2024) 138358

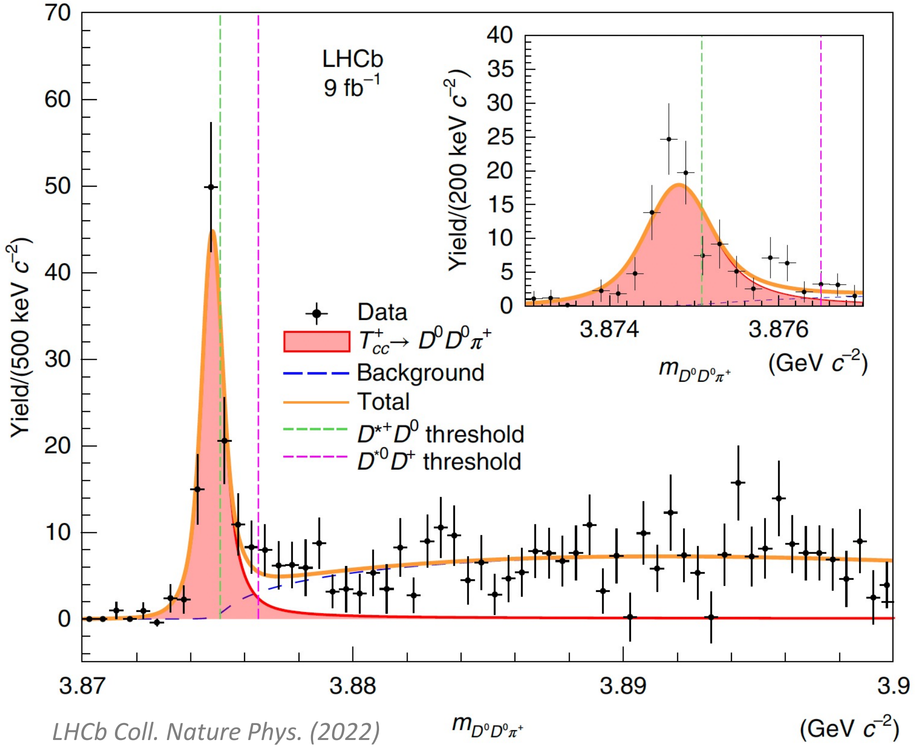
Correlation analysis as alternative to the standard invariant mass analyses to study bound states



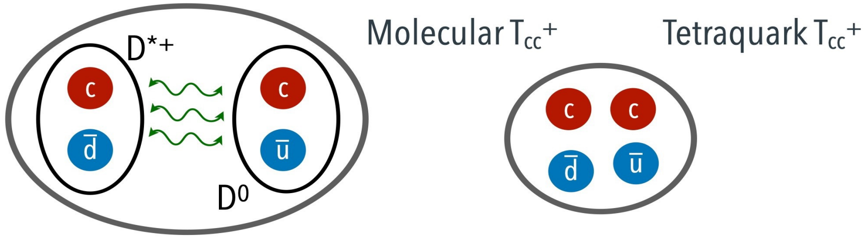
E. Chizzali, Y. Kamiya et al., PLB 848 (2024) 138358

Talk by K. Kuroki 5 June 08:50

D*D correlation and link to molecular states



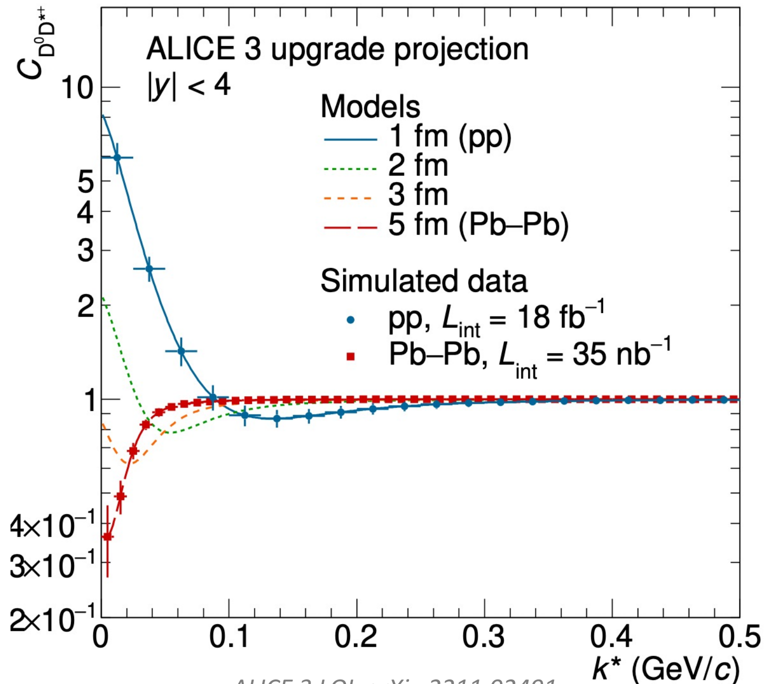
LHCb Coll. Nature Phys. (2022)



T^{+}_{CC} : molecular state ?

Inversion of the sign of the scattering length of the $D^{*+}D^0$ pair translates into an inversion of the correlation function from pp to Pb-Pb collisions at the LHC

Y. Kamiya et al. EPJA 58 (2022)

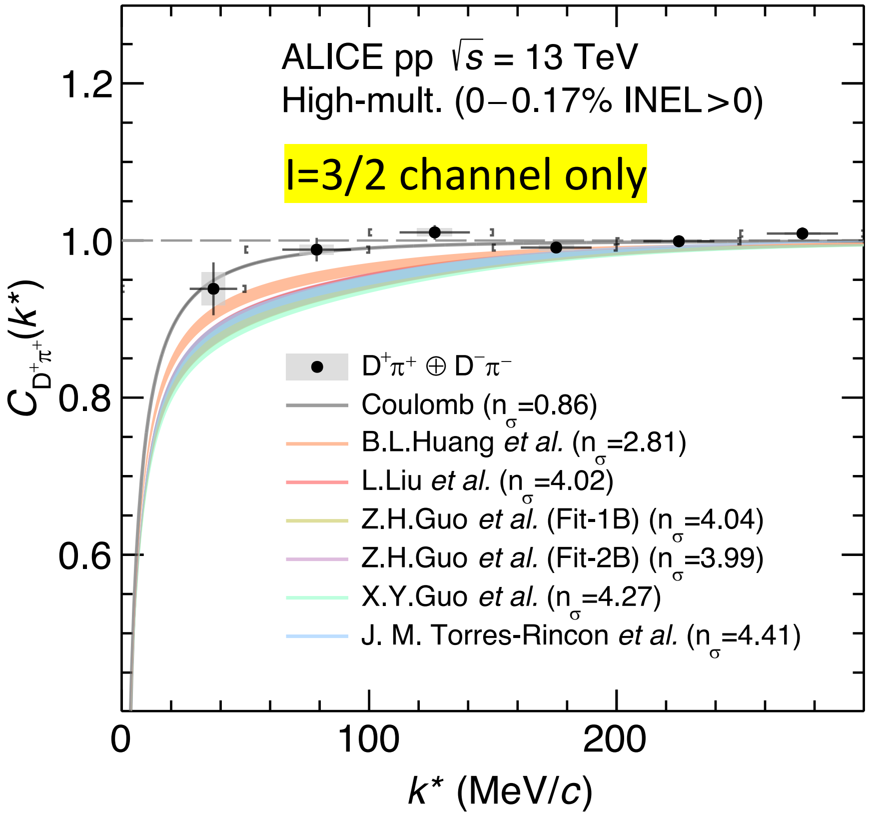


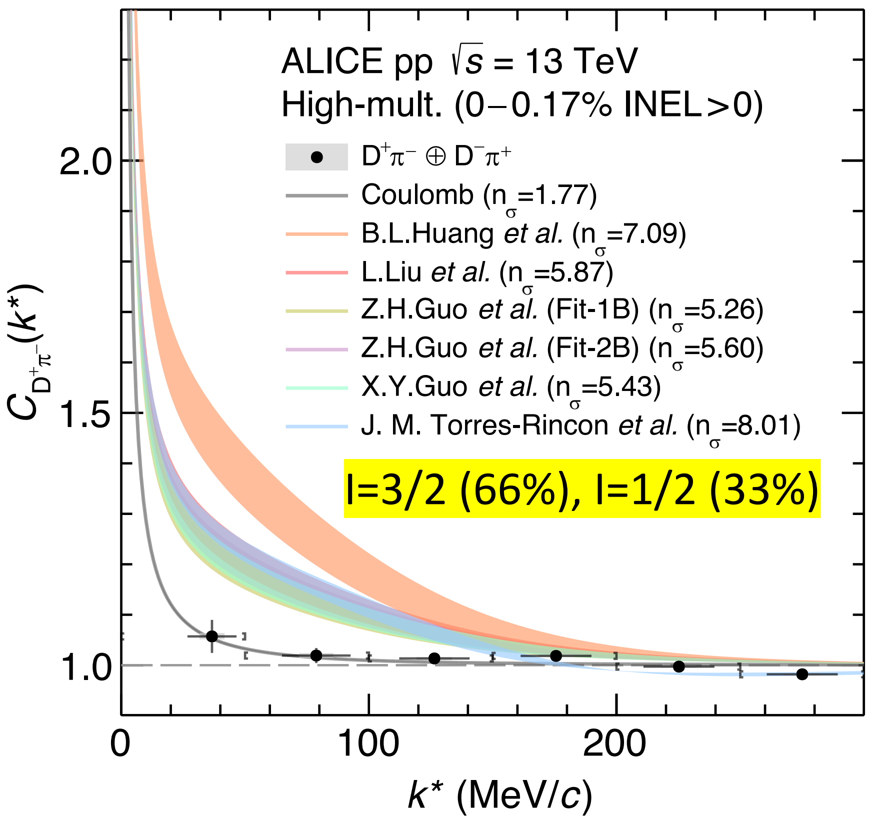
ALICE 3 LOI, arXiv:2211.02491

First measurement of Dπ correlation functions

- Coulomb-only interaction favoured
- Tension with theory models

L. Liu et al, Phys. Rev. D87 (2013) 014508
X.-Y. Guo et al, Phys. Rev. D 98 (2018) 014510
Z.-H. Guo et al Eur. Phys. J. C 79 (2019) 13
B.-L. Huang et al, Phys. Rev. D 105 (2022) 036016
J. M. Torres-Rincon et al, Phys. Rev. D 108 (2023) 096008





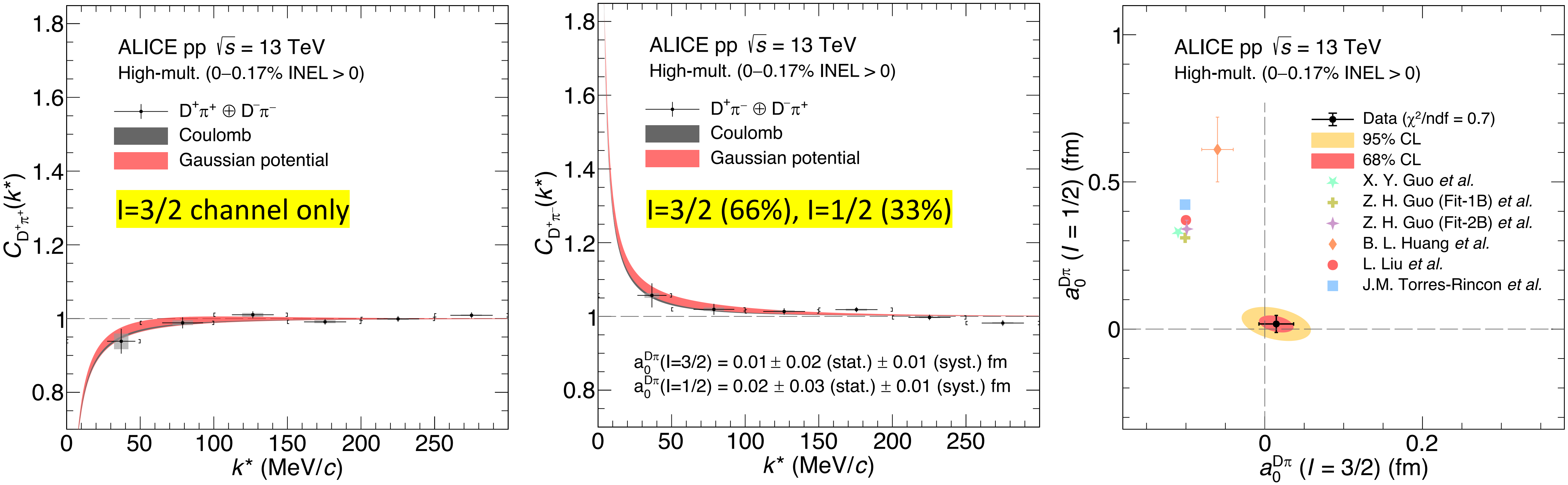
ALICE Coll., arXiv:2401.13541 (2023)

***Talk by** Juan Torres-Rincon 6 Jun, 18:00*

Dπ correlation function fit

- $D^+\pi^+$ and $D^+\pi^-$ share I=3/2 channel→ simultaneous fit
- Vanishing scattering parameters in both isospin channels
- Tension with theory especially in I=1/2 channel

***Talk by** Juan Torres-Rincon 6 Jun, 18:00*



ALICE Coll., arXiv:2401.13541 (2023)

Conclusions and Outlook

- Exciting results from femtoscopy
  → Important experimental input to understand the many facets of QCD in strange and charm sector
  - Most precise p-Λ data at low momenta
  - First extraction of the p-Λ scattering parameters using femtoscopy and scattering data
  - First measurements of three-particle correlation functions
  - Indication of a p-ϕ bound state using correlation techniques
  - First measurements of D meson correlations

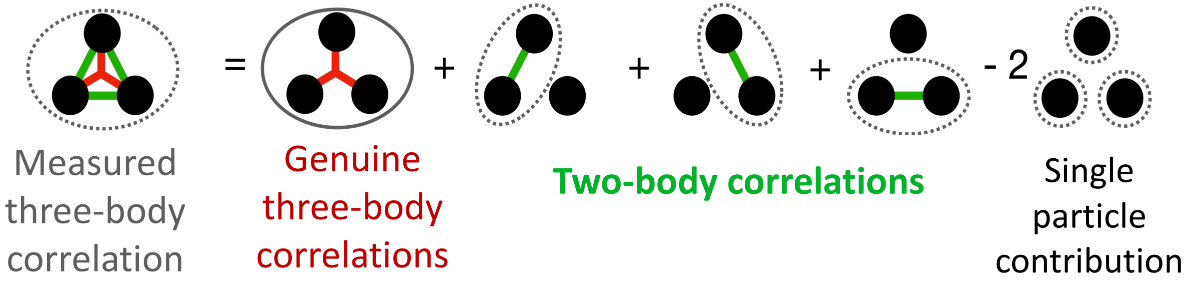
- On-going Run 3 and future Run 4
  - Access to precise data on three-particle interactions and interactions with charm mesons
  - Sensitivity to the effect of three-body forces in the correlation functions

Backup

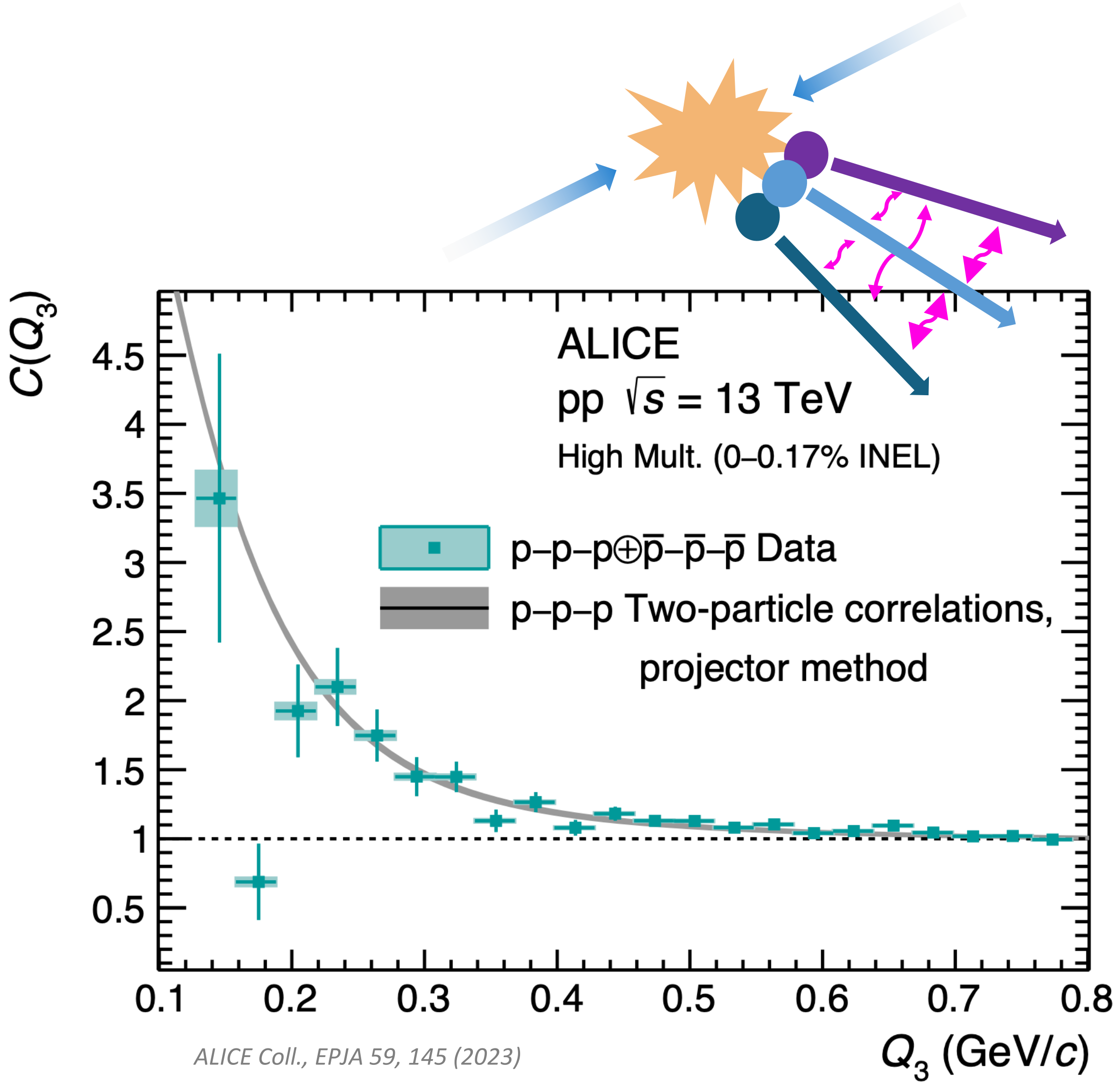
p-p-p correlation function

- Cumulant method provides first hint of effects beyond two-body correlations

R. Kubo, J. Phys. Soc. Jpn. 17, 1100-1120 (1962)
R. Del Grande et al. EPJC 82 (2022) 244



- A deviation of nσ = 6.7 from lower-order contributions
- Theoretical predictions necessary to understand the origin of the deviation further

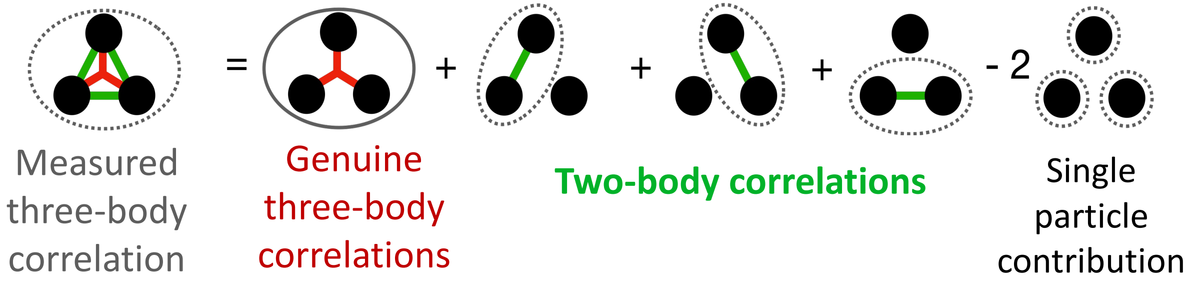


ALICE Coll., EPJA 59, 145 (2023)

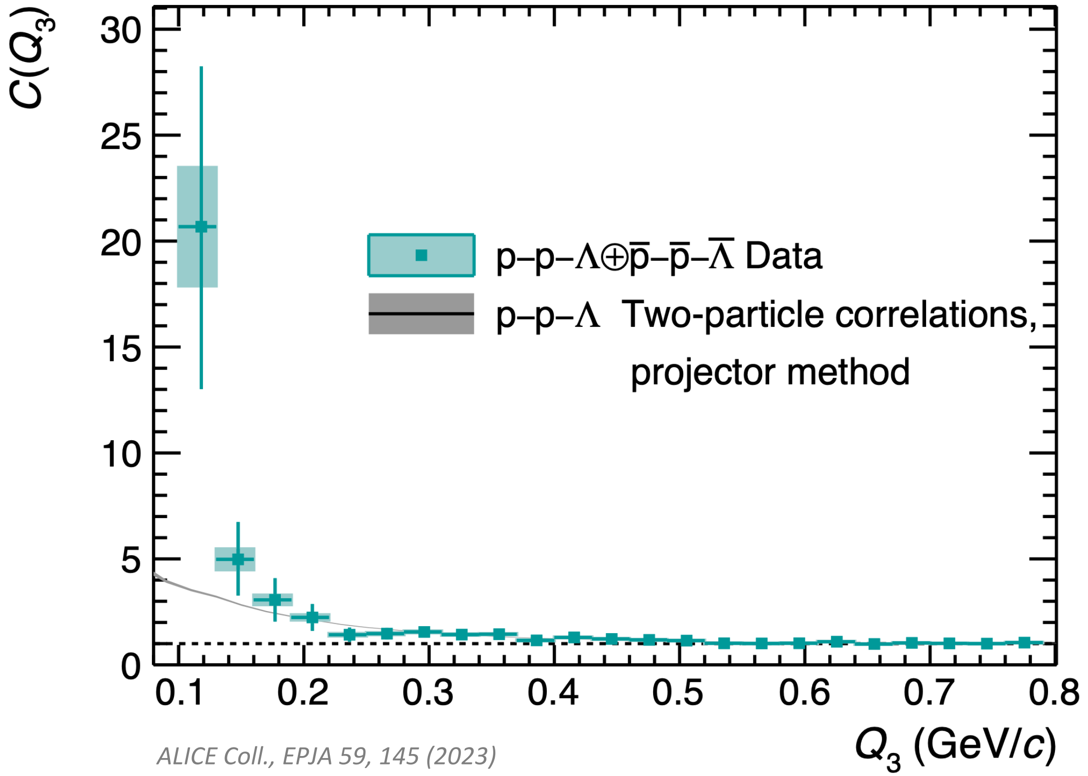
p-p-Λ correlation function

- Cumulant method provides first hint of effects beyond two-body correlations

R. Kubo, J. Phys. Soc. Jpn. 17, 1100-1120 (1962)
R. Del Grande et al. EPJC 82 (2022) 244

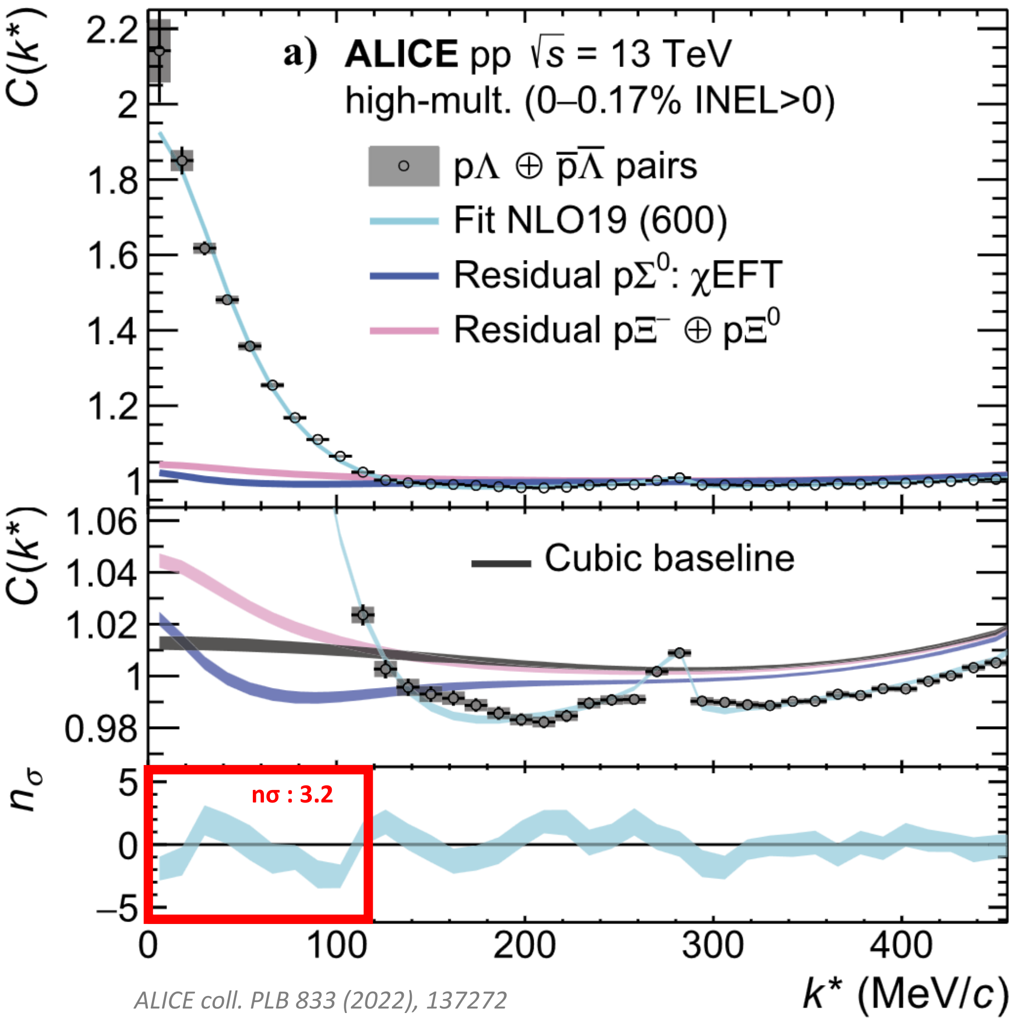


- Compatible with lower-order contributions (nσ = 0.8)



ALICE Coll., EPJA 59, 145 (2023)

The pΛ interaction in the femtoscopy era



ALICE coll. PLB 833 (2022), 137272

- New scenario for pΛ interaction
  → Weaker ΛN-ΣN coupling favoured, important for neutron stars

  D. Gerstung et al. Eur.Phys.J.A 56 (2020), 6, 175

- Most precise data on pΛ system at low momenta
  → Input for low energy effective models in the strange baryonic sector

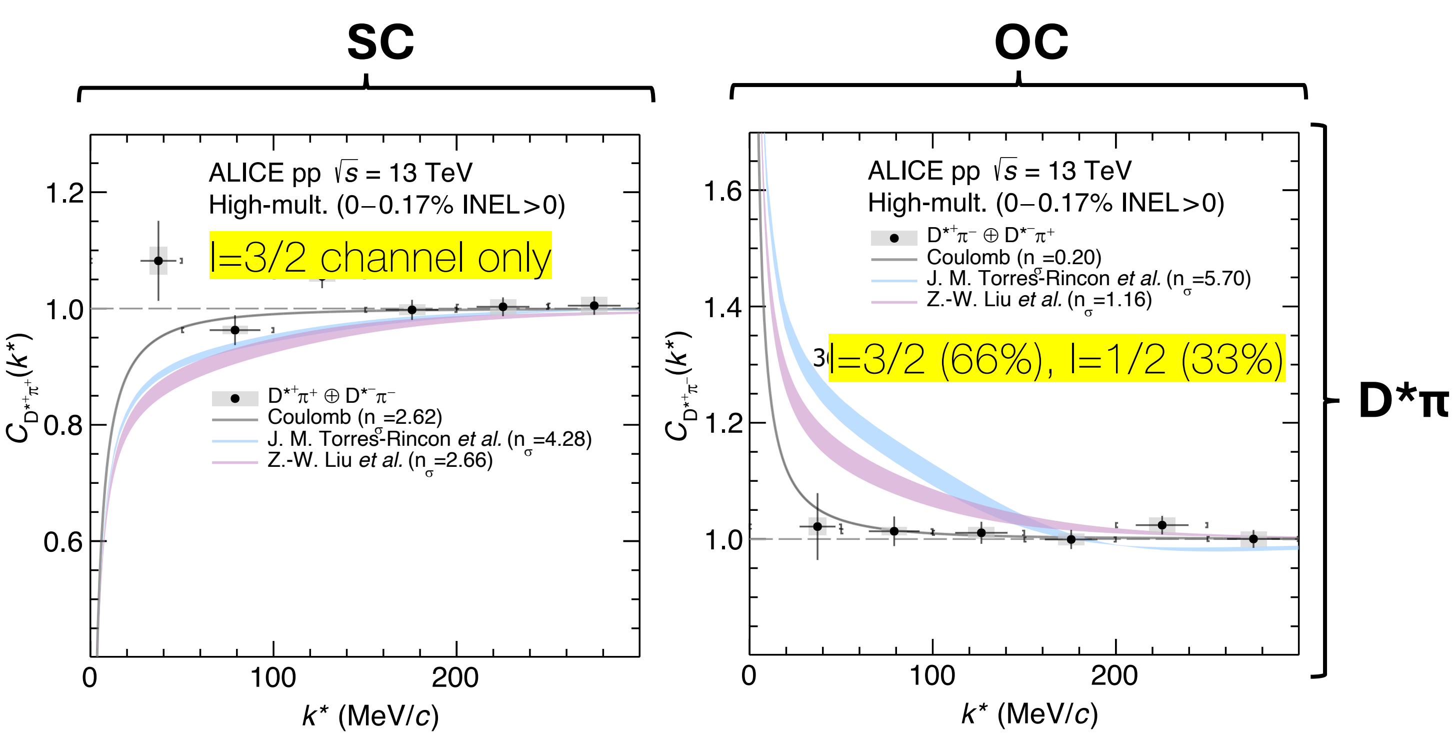
- More pieces needed for the hyperon puzzle in LHC Run 3 and Run-4
  → p$\Sigma^{+,-}$ and Λd interactions
  → Three-particle ppp and ppΛ interactions

  ALICE coll. arXiv: 2206.03344 (2023)

D*π interaction

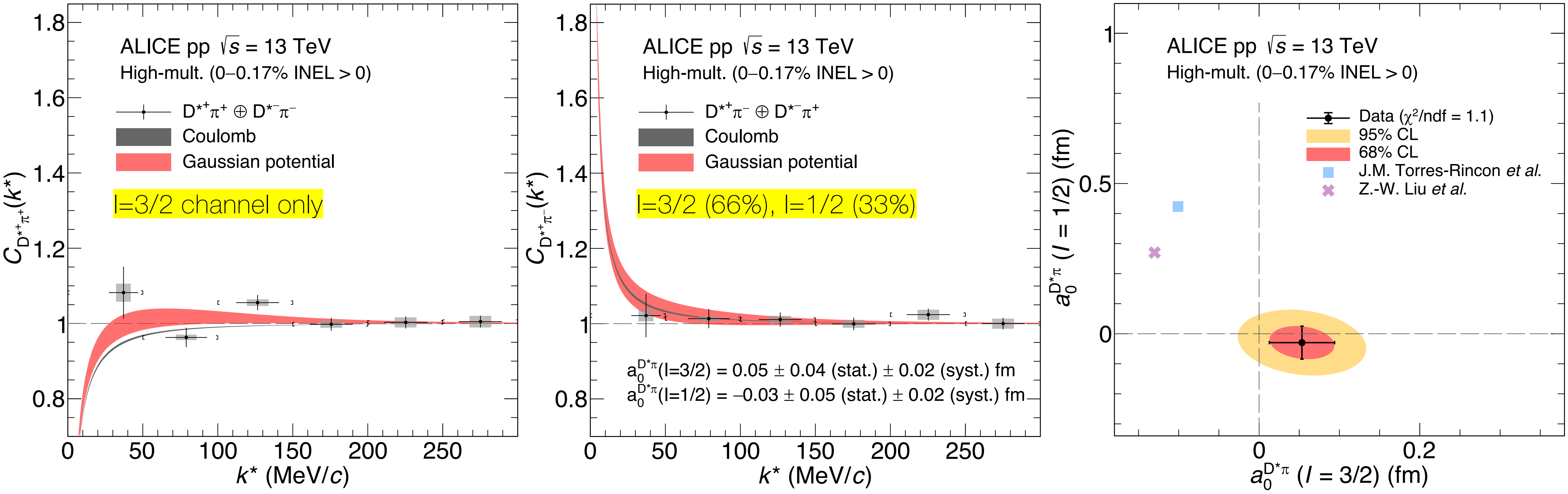
- Similar results as for D-π
  → heavy-quark spin symmetry
- D*π
  - Coulomb-only interaction favoured
  - Tension with theory model

J. M. Torres-Rincon et al, Phys. Rev. D 108 (2023) 096008
Z.-W. Liu et al, Phys. Rev. D 84 (2011) 034002



D*π correlation function fit

arXiv:2401.13541

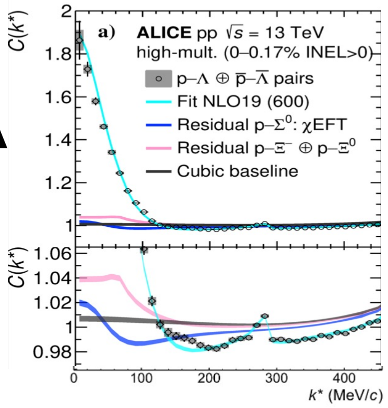


- Vanishing scattering parameters within uncertainties
- Scattering parameters compatible with Dπ results → Heavy-quark spin symmetry

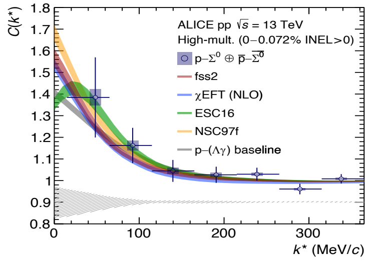
An example of EoS for neutron stars

Correlation
2-body interaction

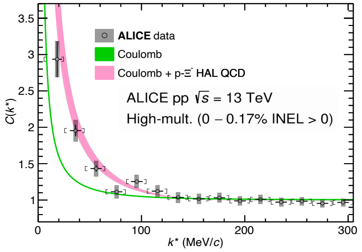
pΛ



pΣ

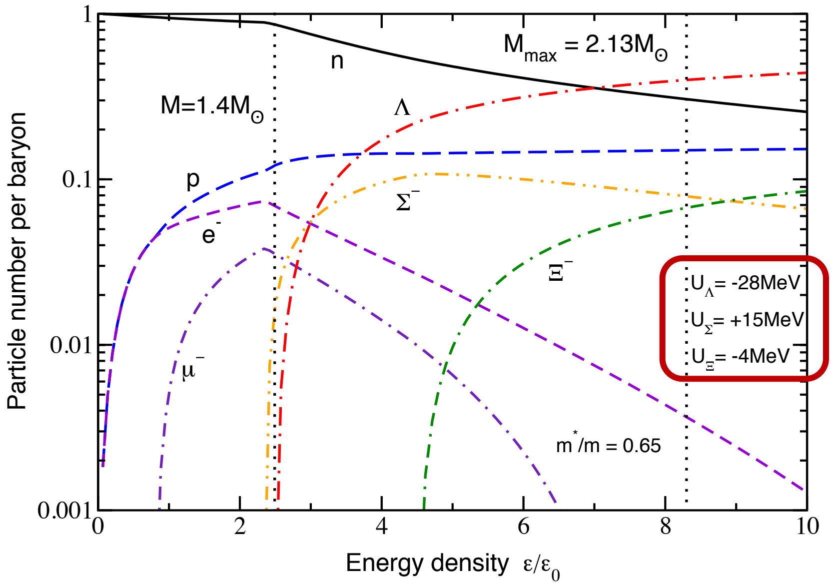


pΞ



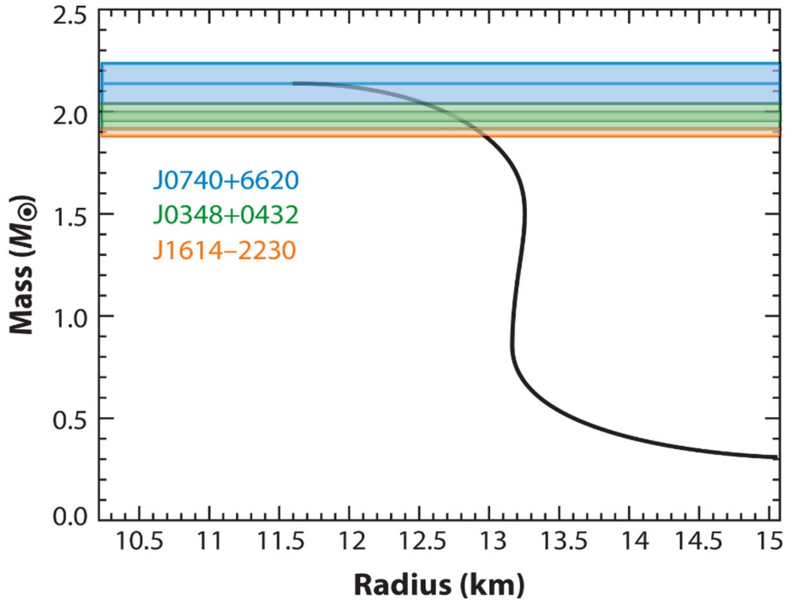
Single-partical potentials
EoS

Courtesy J. Schaffner-Bielich 2020



Mass vs Radius relation
for hyperon stars

Courtesy J. Schaffner-Bielich and B. Dönigus 2020



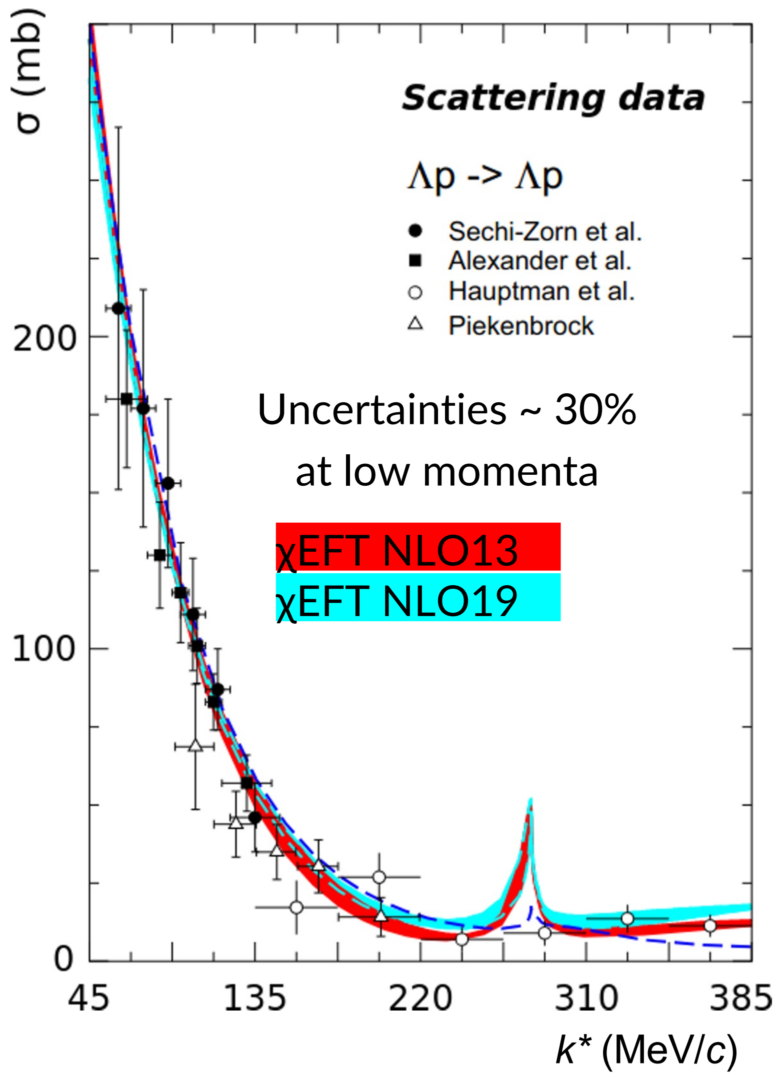
L. Fabbietti et al. Ann.Rev.Nucl.Part.Sci. 71 (2021)

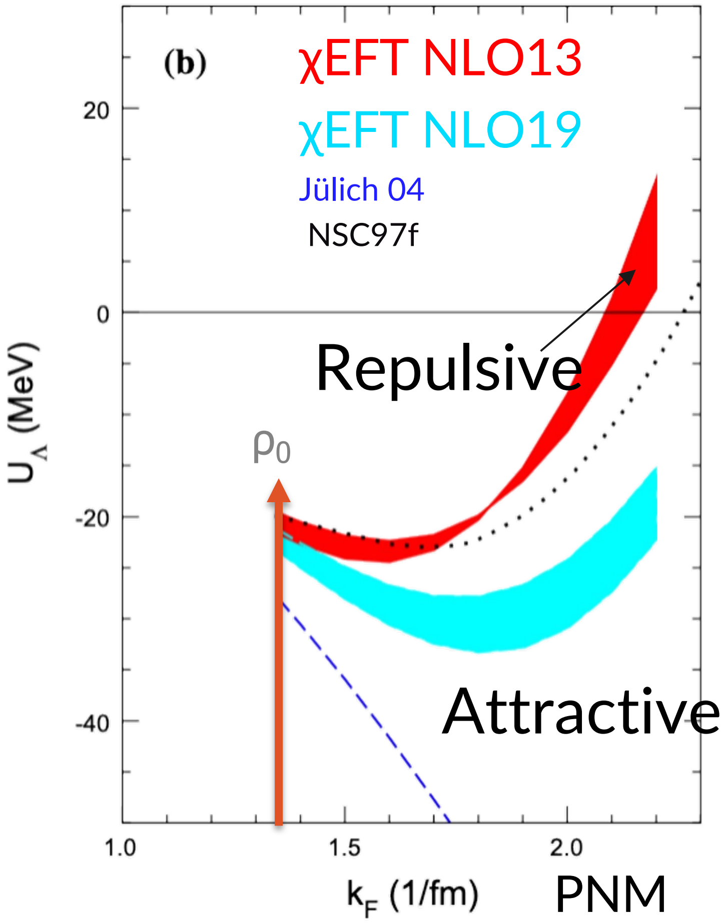
This is only an example. Experimental uncertainties need to be propagated and some interactions are missing ...

- What about the three-body strong interaction?

|S| = 1: p–Λ interaction

- Low statistics and not available at low momenta
- ΛN-ΣN coupled system → two-body coupling to ΣN is not (yet) measured
- ΣN coupling strength relevant for EoS
  - Strongly affects the behaviour of Λ at finite density
  - Implications for ΛNN interactions
- NLO19 predicts weak coupling NΛ-NΣ
  - Attractive Λ interaction in neutron matter



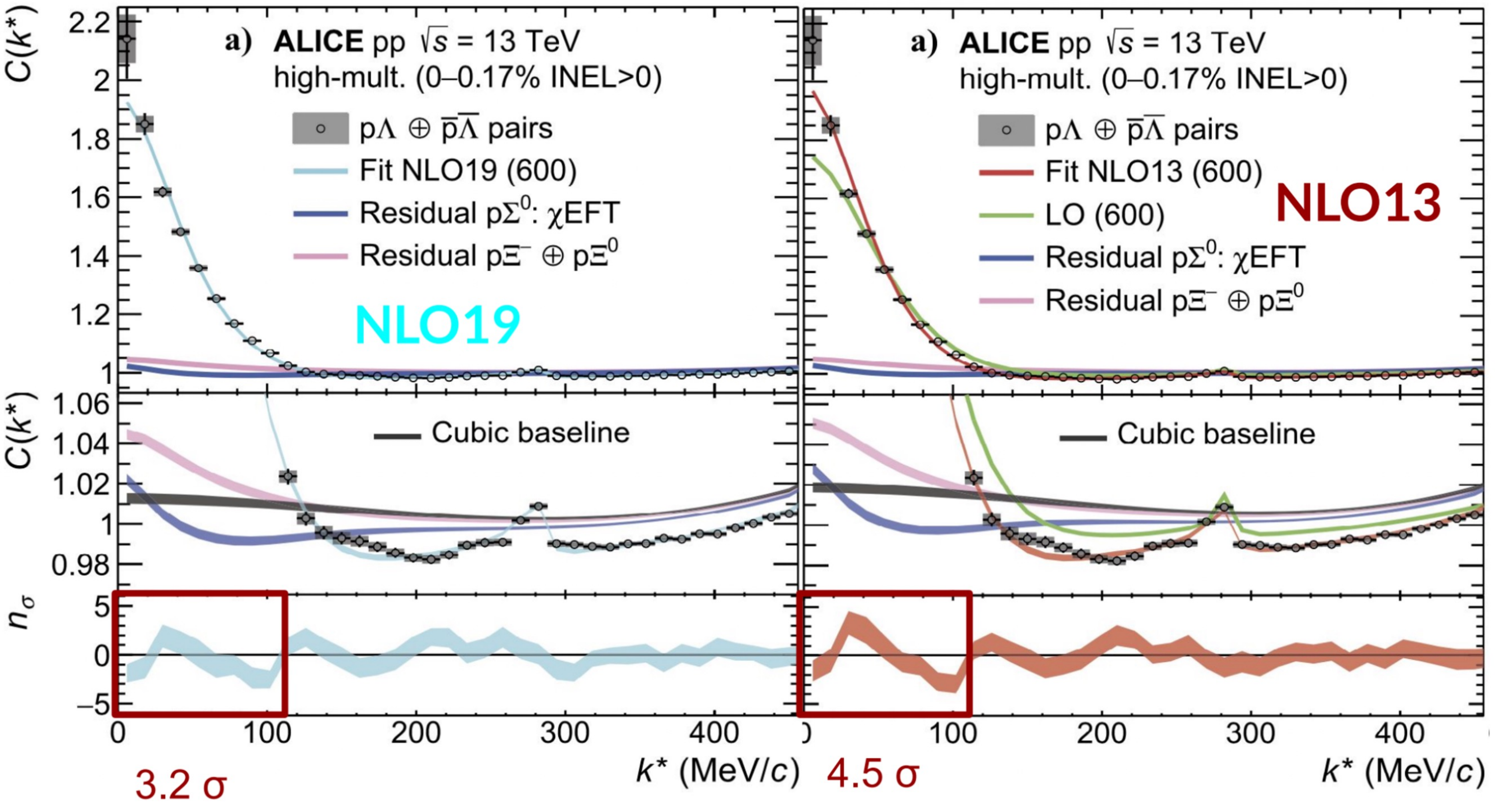


J.Haidenbauer, N.Kaiser et al. NPA 915 24 (2013)
J.Haidenbauer, U. Meiβner EPJA 56 (2020)

|S| = 1: p–Λ interaction

Comparison with χEFT potentials

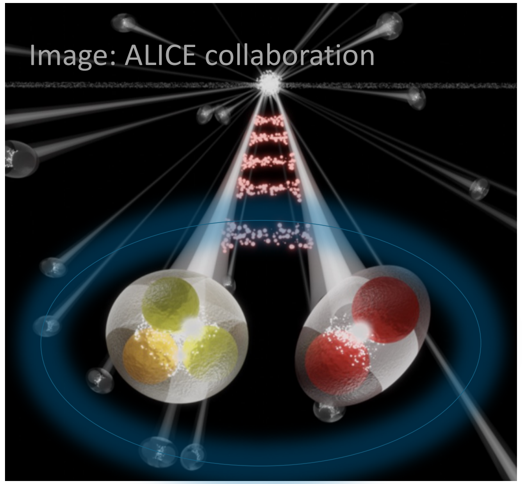
- Sensitivity to different ΣN coupling strength
- NLO19 favoured (n_σ = 3.2) → attractive interaction of Λ at large densities



ALICE Coll. PLB 833 137272 (2022)

p-ϕ bound state

- Predicted by various theoretical calculations
- No experimental evidence
  - Standart method of invariant mass measurment not yet available
- Accessible by studying **interaction** among constituents



	System	E_B [MeV]
QCD Van der Waal using Yukawa type Potential[1]	ϕN	1.8
Chiral quark model[2]	ϕN	3.0
Monte Carlo study of ϕ photoproduction from nuclear targets[3]	ϕN	2.5
Quark delocalization color screening model[4]	ϕN	0.3-8.8
Unitary coupled-channel approximation anchored to ALICE pϕ scattering data[5]	ϕN	9.0
Phenomenological potential+variational method[6]	ϕN	9.3/9.23
	ϕNN	10.0/17.5
Phenomenological potential+variational method[7]	ϕN	9.5
	ϕNN	39.8
	ϕϕNN	124.6

[1]H. Gao, T.-S. H. Lee, and V. Marinov, Phys. Rev. C 63 (2001) 022201(R)
[2]F. Huang, Z.Y. Zhang, and Y.W. Yu, Phys. Rev. C 73 (2006) 025207
[3]H. Gao et al., Phys. Rev. C 95 (2017) 055202
[4]S. Liska, H. Gao, W. Chen, and X. Qian, Phys. Rev. C 75 (2007) 058201
[5]B.-X. Sun, Y.-Y. Fan, and Q.-Q. Cao, arXiv , 2206.02961 (2022)
[6]V. B. Belyaev, W. Sandhas, and I. I. Shlyk, Few-Body Syst. 44 (2008) 347
[7]S. A. Sofianos, G. J. Rampho, M. Braun, and R. M. Adam, J. Phys. G. 37 (2010) 085109

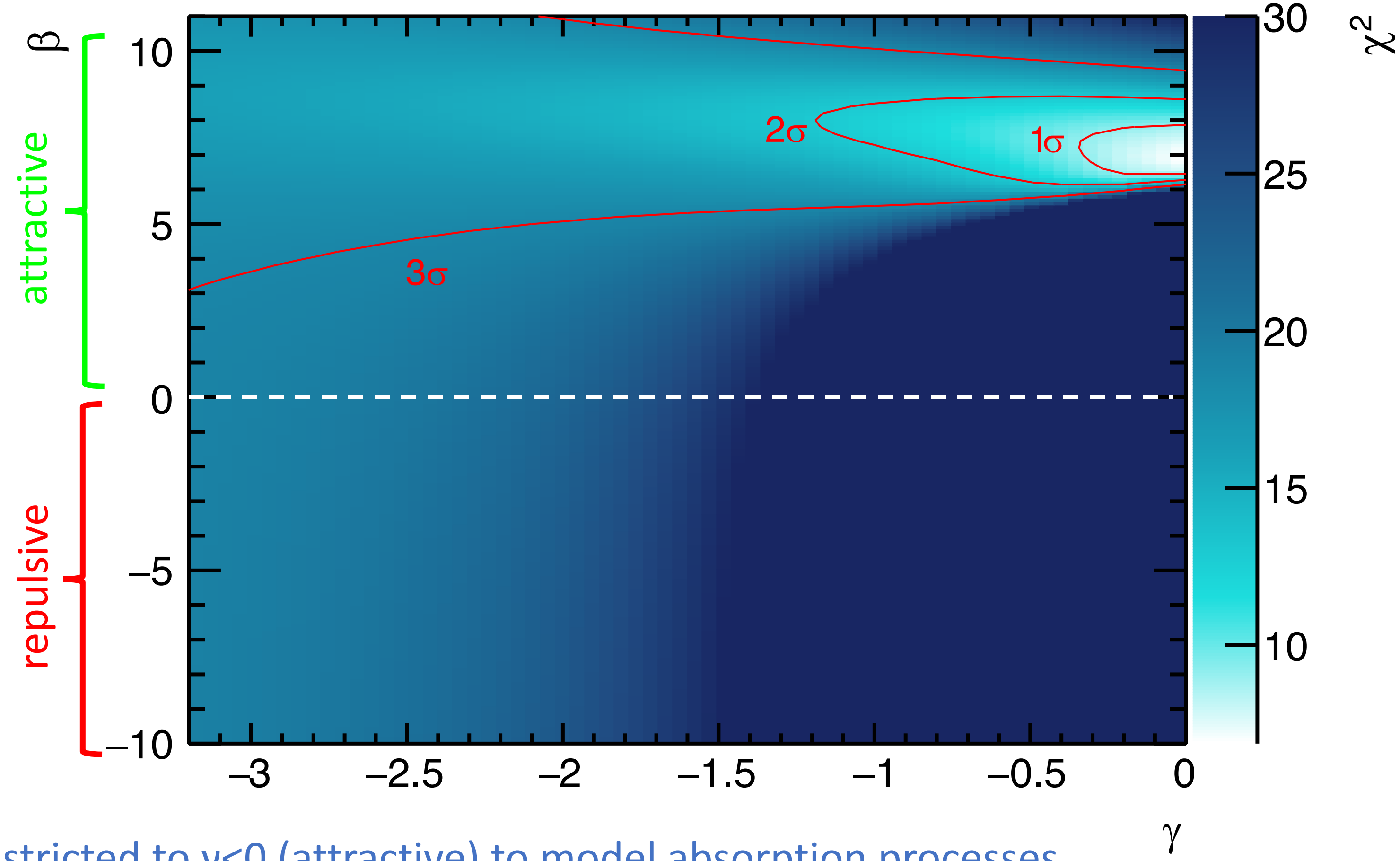
|S|=0 sector: p−φ spin dependent interactions

$$V_{\frac{1}{2}}(r) = V_{LATTIC,MOD}(r) + i \cdot \sqrt{f(r;b_3)} \cdot \frac{\gamma}{r} e^{-m_K \cdot r}$$

$$\beta \cdot V_{short}(r) + V_{2\pi}(r)$$

- Best fit to data obtained for attractive potential
  - $\beta = 7.0^{+0.8}_{-0.2}(stat.)^{+0.2}_{-0.2}(syst.)$
  - $\gamma = 0.0^{+0.0}_{-0.2}(stat.)^{+0.0}_{-0.2}(syst.)$
- Repulsive potential (β<0) excluded by over 3σ
- Within uncertainties room for inelastic contributions expected by theory

arXiv:2212.12690 [nucl-ex]

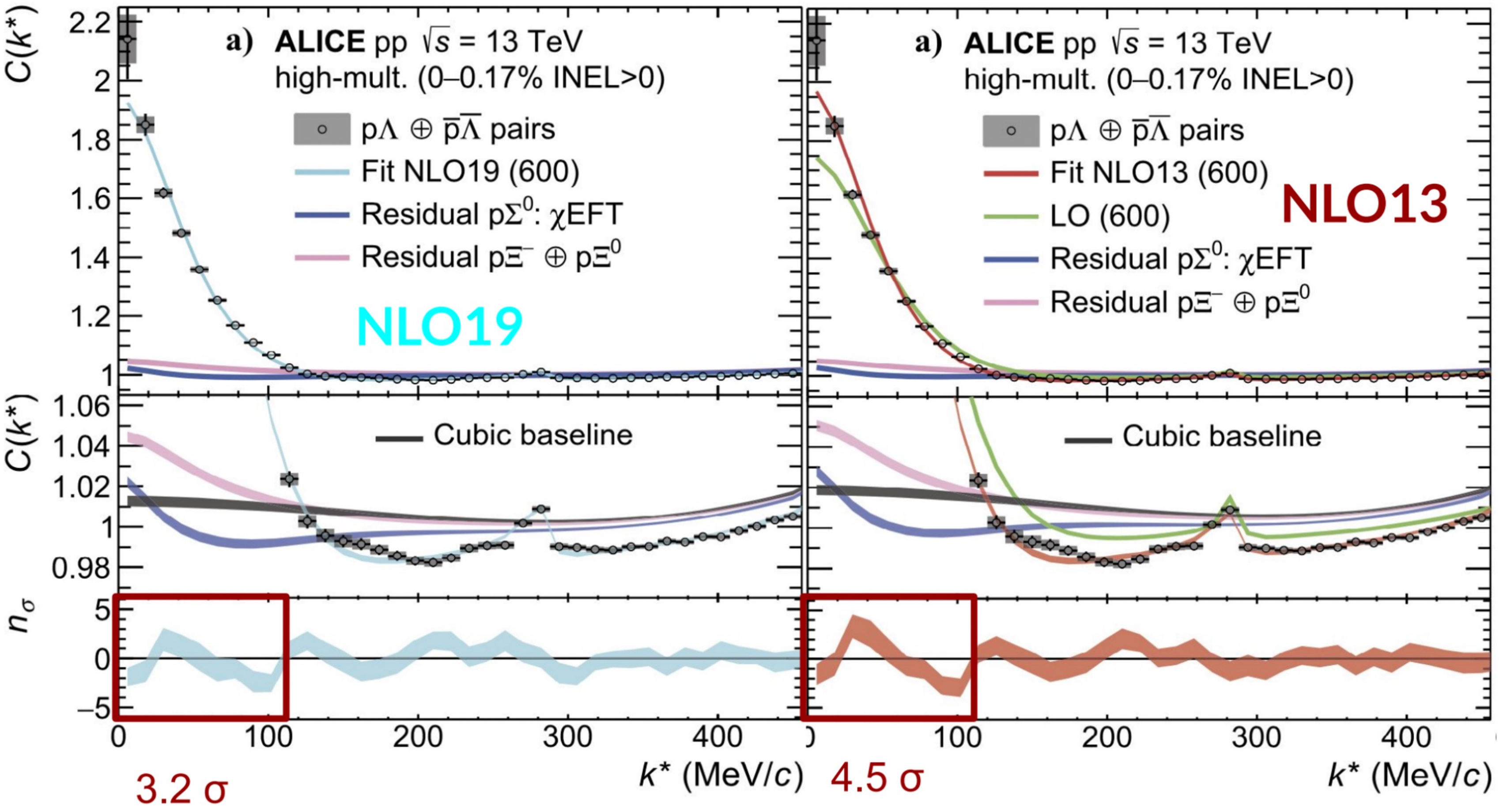


Imaginary Pot restricted to γ<0 (attractive) to model absorption processes

|S| = 1: p–Λ interaction

Comparison with χEFT potentials

- Sensitivity to different ΣN coupling strength
- NLO19 favoured (n_σ = 3.2) → attractive interaction of Λ at large densities

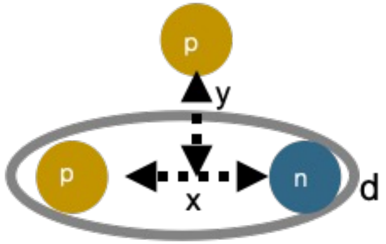


ALICE Coll. PLB 833 137272 (2022)

p–d correlation function: d as composite object

The three body wave function with proper treatment of 2N and 3N interaction at very short distances goes to a p–d state.

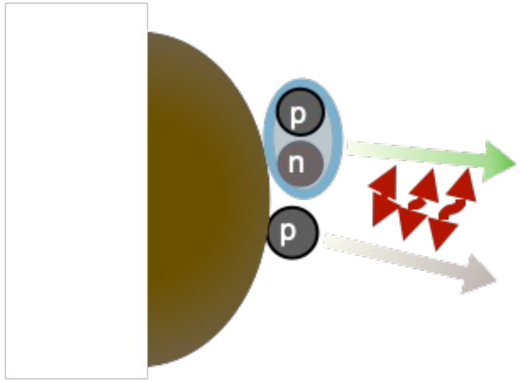
- **Three–body wavefunction for p–d:** $\Psi_{m_2,m_1}(x,y)$ describing three-body dynamics, anchored to p–d scattering observables.
  - x = distance of p-n system within the deuteron
  - y = p-d distance
  - m_2 and m_1 deuteron and proton spin



- $\Psi_{m_2,m_1}(x,y)$ three-nucleon wave function asymptotically behaves as p–d state:

$$\Psi_{m_2,m_1}(\boldsymbol{x},\boldsymbol{y}) = \Psi^{(\text{free})}_{m_2,m_1} + \sum_{LSJ}^{J\le\bar{J}} \sqrt{4\pi}\, i^L \sqrt{2L+1}\, e^{i\sigma_L} (1 m_2 \tfrac{1}{2} m_1 | S J_z)(L 0 S J_z | J J_z)\, \widetilde{\Psi}_{LSJJ_z} .$$

Asymptotic form — Strong three-body interaction



➡ $\widetilde{\Psi}_{LSJJ_Z}$ describe the configurations where the three particles are close to each other

➡ $\Psi^{(\text{free})}_{m_1,m_2}$ an asymptotic form of p-d wave function

Kievsky et al, Phys. Rev. C 64 (2001) 024002
Kievsky et al, Phys. Rev. C 69 (2004) 014002
Deltuva et al, Phys. Rev. C71 (2005) 064003

p–d correlation function

- **Starting with the PPN state that goes into pd state**:
  - Nucleons with the Gaussian sources distributions

Single-particle Gaussian emission source

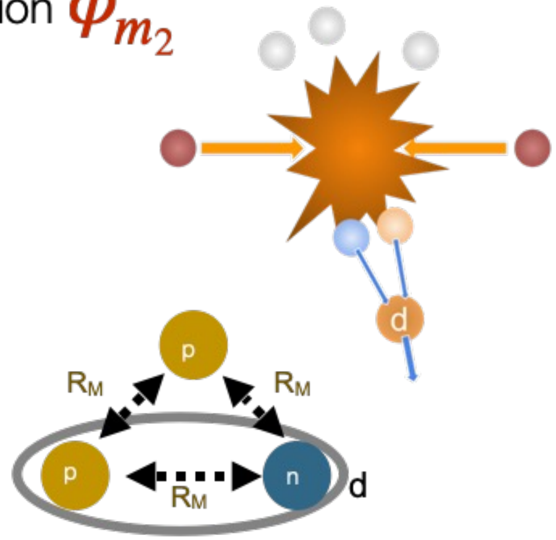
$$A_d C_{pd}(k) = \frac{1}{6}\sum_{m_2,m_1}\int d^3r_1 d^3r_2 d^3r_3\, S_1(r_1)S_1(r_2)S_1(r_3)|\Psi_{m_2,m_1}|^2\,,$$

  - Where A_d is the deuteron formation probability using deuteron wavefunction ϕ_{m_2}

$$A_d = \frac{1}{3}\sum_{m_2}\int d^3r_1 d^3r_2\, S_1(r_1)S_1(r_2)|\phi_{m_2}|^2\,,$$

  - Final definition of the correlation with p–p source size R_M:

$$A_d C_{pd}(k) = \frac{1}{6}\sum_{m_2,m_1}\int \rho^5 d\rho d\Omega\, \frac{e^{-\rho^2/4R_M^2}}{(4\pi R_M^2)^3}|\Psi_{m_2,m_1}|^2\,.$$



Mrówczyński et al *Eur. Phys. J. Special Topics 229, 3559 (2020)*