Available theoretical tools in search for the critical point of the QCD phase diagram

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Outline

- what we know

- theory vs. experiment

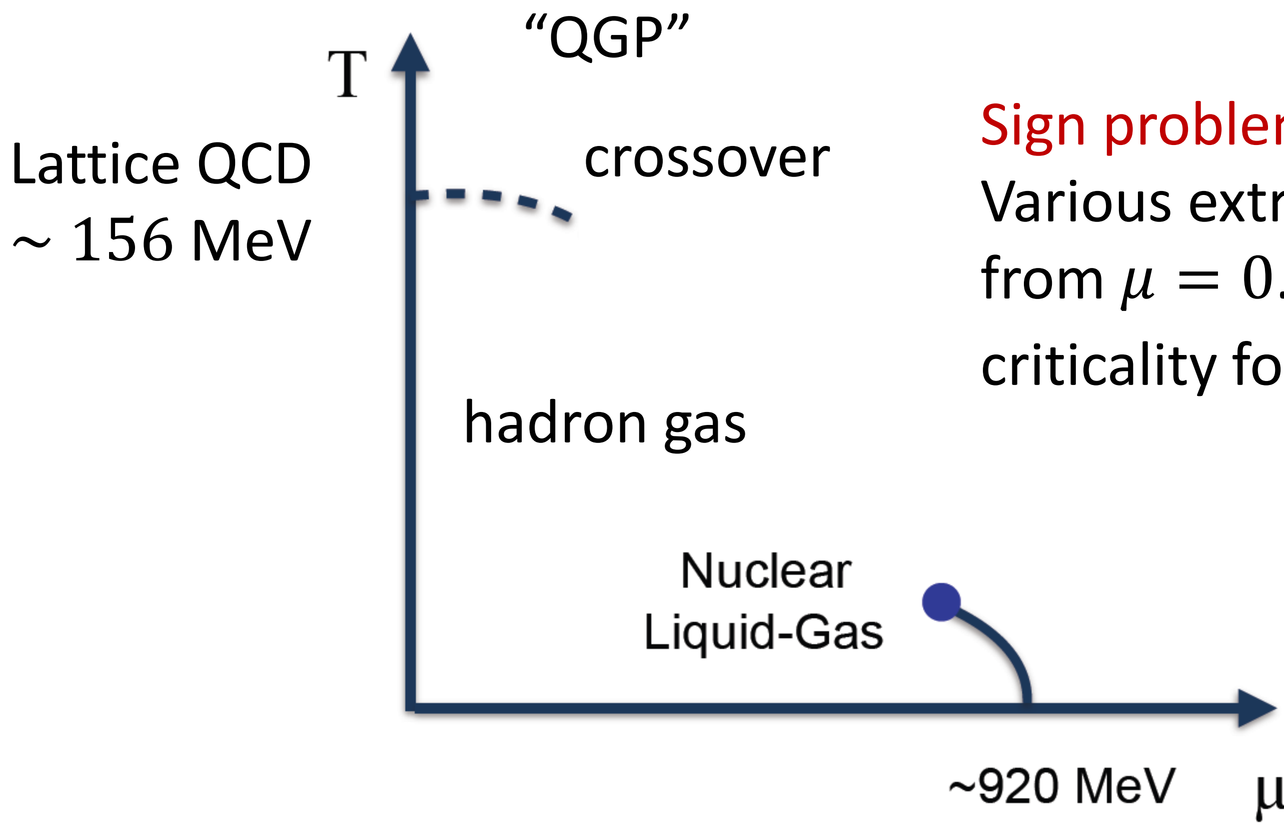
- cumulants, factorial cumulants

- STAR data

- critical point vs. theory

- summary

QCD phase diagram



Sign problem

Various extrapolations from $\mu = 0$. No sign of criticality for $\frac{\mu}{T} < 3$

Figured taken from V.Koch

Hopes

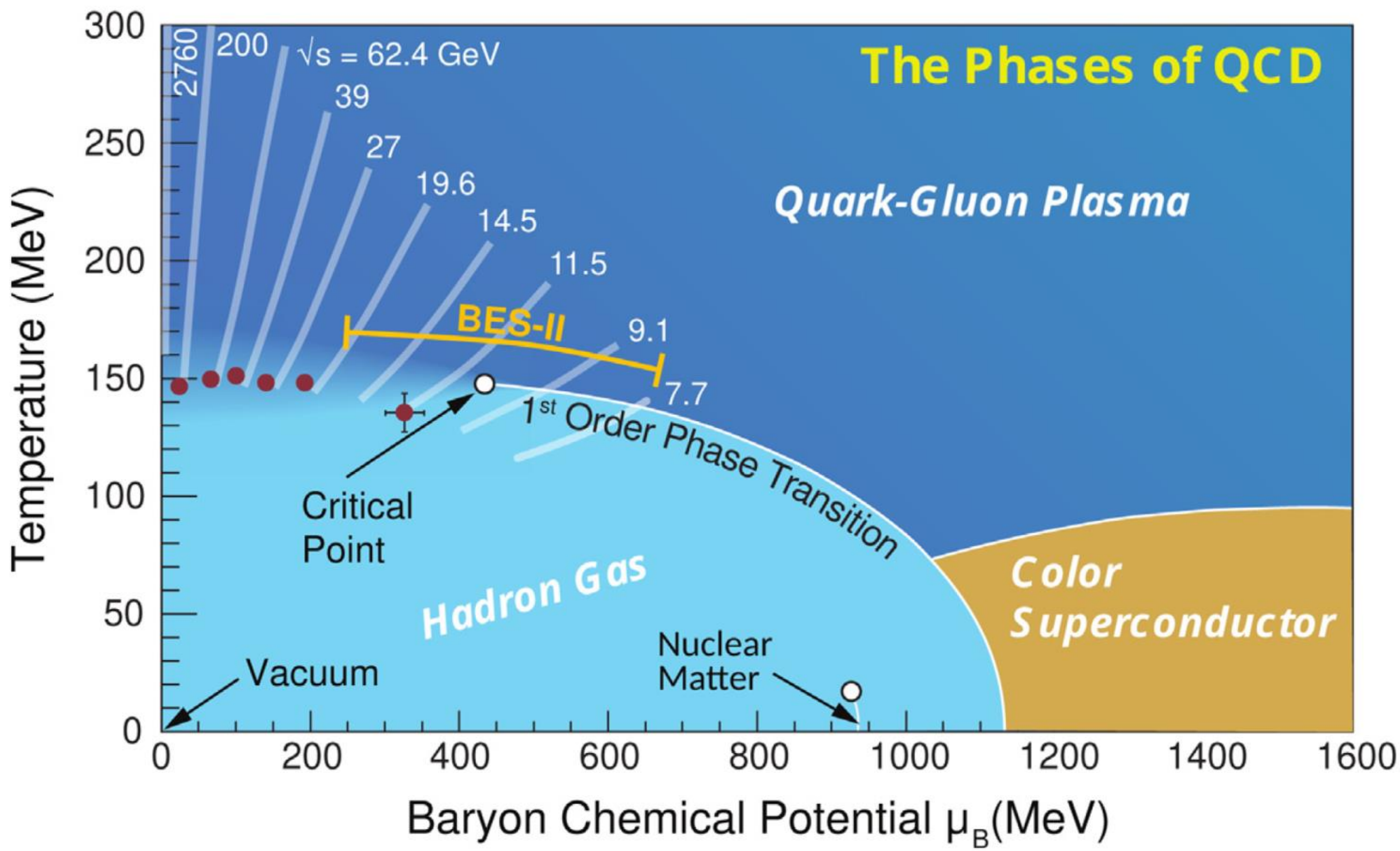
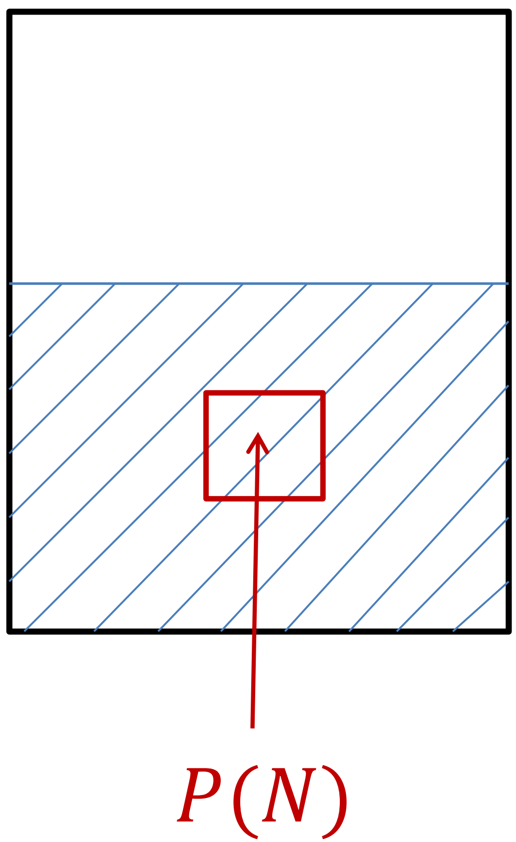
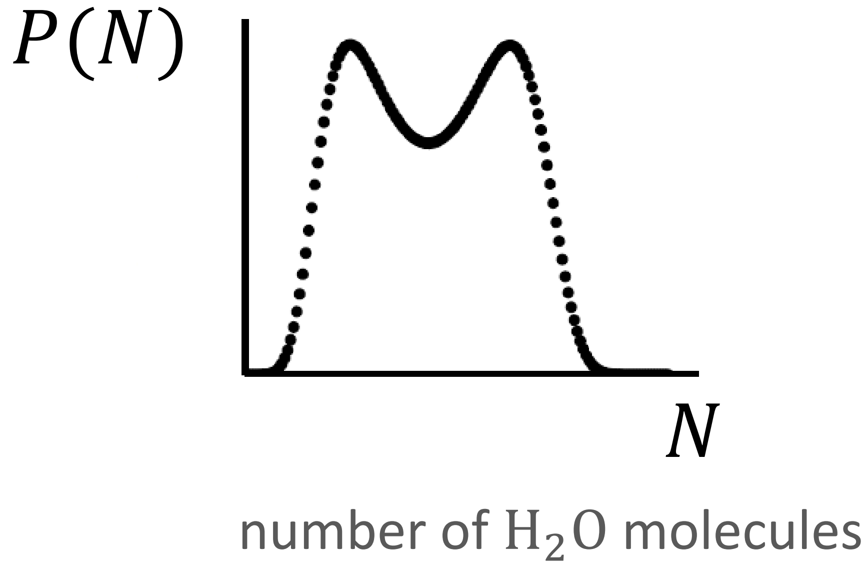


Figure from Phys. Rept. 853 (2020) (AB, S.Esumi, V.Koch, J.Liao, M.Stephanov, N.Xu)

Consider water vapour transition



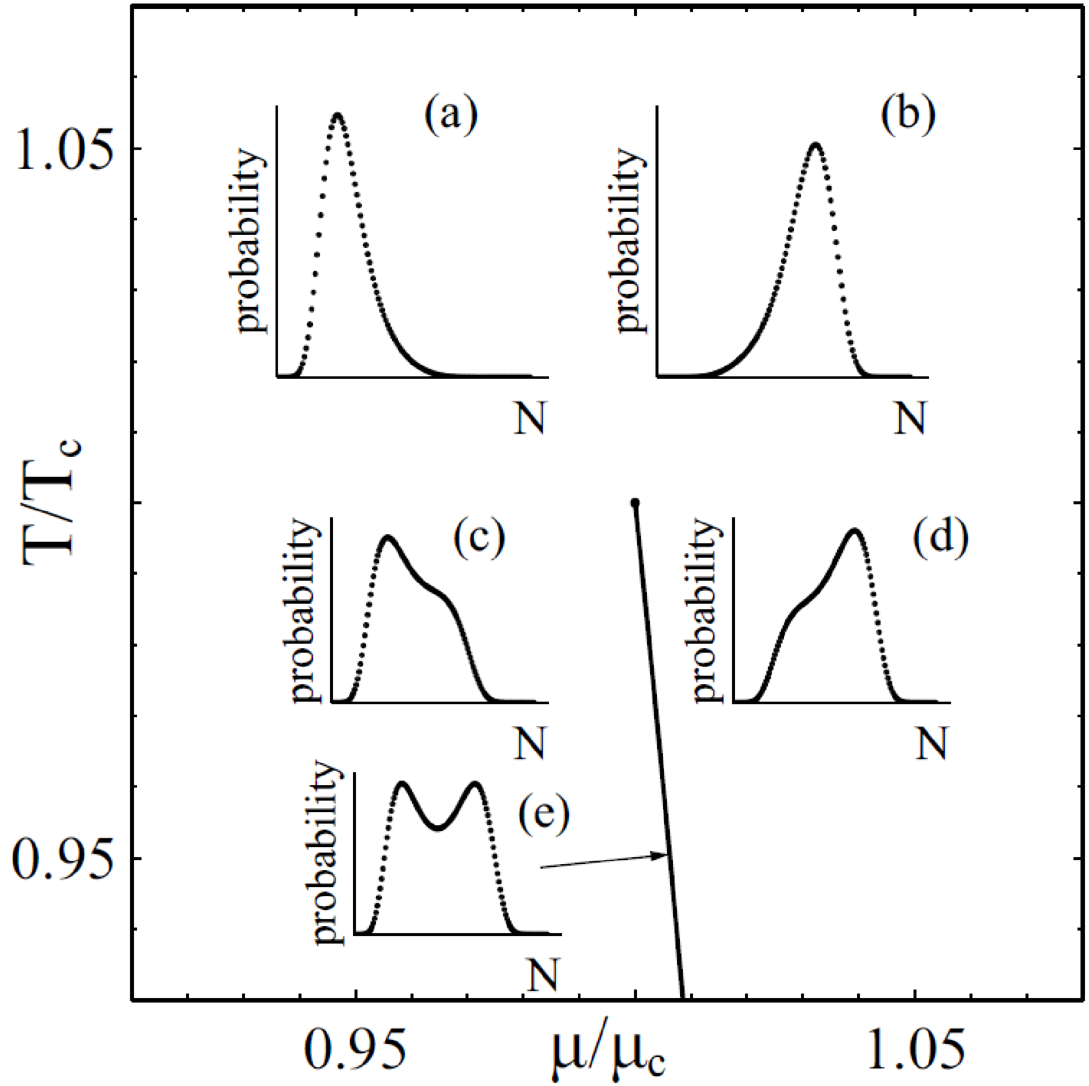
right at the phase transition



so we measure multiplicity distributions

In QCD we use, e.g., **net-baryon**, net-charge, net-strangeness

A finite volume van der Waals model



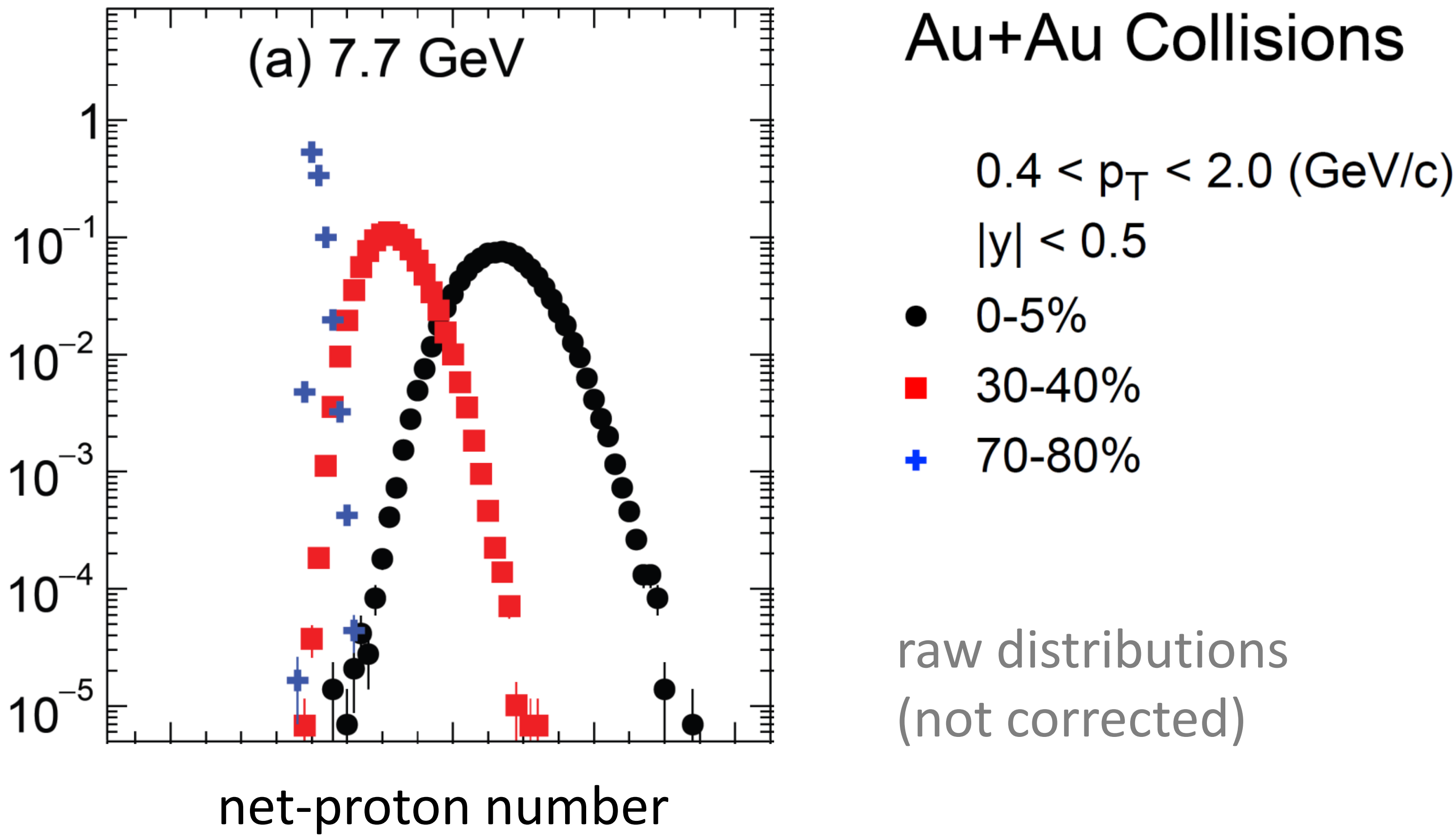
AB, V. Koch, D. Oliinychenko, J. Steinheimer, PRC 98 (2018) 054901

Theory vs. experiment

Theory	**Experiment**
Coordinate space	Momentum space
Fixed volume	Expanding and fluctuating volume
Long-lived	Extremely short-lived
Conserved charges	Non-conserved numbers

Lots of detector problems + various corrections (e.g. volume fluctuation)

So we measure multiplicity distributions...



raw distributions
(not corrected)

STAR Collaboration, PRC 104 (2021) 2, 024902

For baryons absolutely minimal goal is to see any deviations from Poisson (Skellam) distribution.

It is difficult to see something in multiplicity distributions.
If there is any signal, it is likely small.

We usually characterize $P(N)$ by:

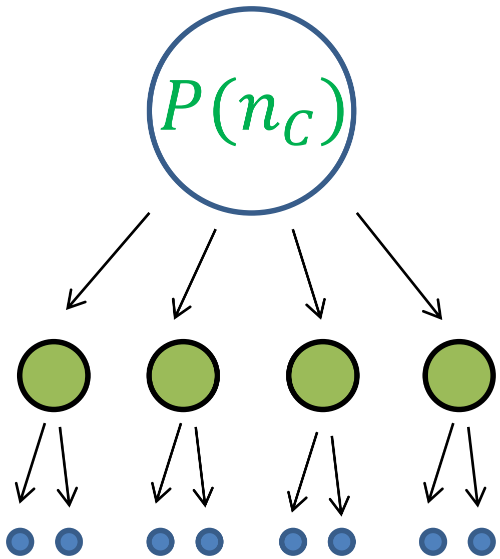
- cumulants κ_n

- factorial cumualnts, C_n (or $\hat{C}_n$)

- factorial moments F_n (mean number of pairs, triplets, etc.)

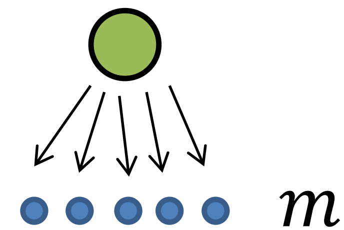
Warning: STAR uses opposite notation $\kappa_n \leftrightarrow C_n$

Factorial cumulants – example



Poisson

m particle cluster



$$C_2 \neq 0$$

$$C_k = 0, k > 2$$

$$C_{2,3,\ldots,m} \neq 0$$

$$C_k = 0, k > m$$

factorial cumulants

$$C_k = \frac{d^k}{dz^k} \ln\left(\sum_n P(n) z^n\right)\Big|_{z=1}$$

Two-particle correlation function

$$\rho_2(y_1, y_2) = \rho(y_1)\rho(y_2) + C_2(y_1, y_2)$$

Integrating both sides over some bin in rapidity

$$\langle n(n-1)\rangle = \langle n\rangle^2 + C_2$$

$$C_2 = \int C_2(y_1, y_2)\, dy_1 dy_2$$

Analogously with multiparticle correlations

Factorial cumulants are integrated multiparticle correlation functions

Factorial cumulants vs. cumulants

factorial cumulant $$C_i = \frac{d^i}{dz^i} \ln\left(\sum_n P(n) z^n\right) \Big|_{z=1}$$

cumulant $$\kappa_i = \frac{d^i}{dt^i} \ln\left(\sum_n P(n) e^{tn}\right) \Big|_{t=0}$$

Poisson

$C_i = 0, \; i > 1$

$\kappa_i = \langle n \rangle$

cumulants naturally appear in statistical physics

$$\ln(Z) = \ln\left(\sum_i e^{-\beta(E_i - \mu N_i)}\right)$$

Cumulants naturally appear in statistical physics

$$\ln(Z) = \ln\left(\sum_i e^{-\beta(E_i - \mu N_i)}\right)$$

$$\kappa_n = \frac{\partial^n \ln(Z)}{\partial(\mu_B/T)^n}$$

$$\chi_n = \frac{\kappa_n}{VT^3}$$ susceptibility

On the experimental side we need to measure various fluctuation observables and hope to see some nontrivial energy or/and system-size dependence

see, e.g.,
Stephanov, Rajagopal, Shuryak, PRL (1998)
Stephanov, PRL (2009)
Skokov, Friman, Redlich, PRC (2011)

There are many results:

ALICE, STAR, HADES	NA61/SHINE
Cumulants, factorial cumulamts	Intermittency Talk by A. Rybicki (Monday)
Proton v_1 (STAR)	Cumulants, scaled variance
HBT radii (STAR) R.A. Lacey, PRL 114 (2015) 142301	Strongly intensive variables

Cumulants (one species of particles)

$$\kappa_2 = \langle N \rangle + C_2$$

$$\kappa_3 = \langle N \rangle + 3C_2 + C_3$$

$$\kappa_4 = \langle N \rangle + 7C_2 + 6C_3 + C_4$$

Cumulants mix integrated correlation functions of different orders

Cumulants might be dominated by $\langle N \rangle$ and C_2

See, e.g.,
B. Ling, M. Stephanov, PRC 93 (2016) 034915
AB, V.Koch, N.Strodthoff , PRC 95 (2017) 054906

Short-range rapidity correlations

$$C_i \sim \langle N \rangle \sim \Delta y$$

$$\kappa_i \sim \langle N \rangle \sim \Delta y$$

Long-range rapidity correlations

$$C_i \sim \langle N \rangle^i \ \sim \ (\Delta y)^i$$

good observable with long-range rapidity corr. $\dfrac{C_i}{\langle N \rangle^i}$

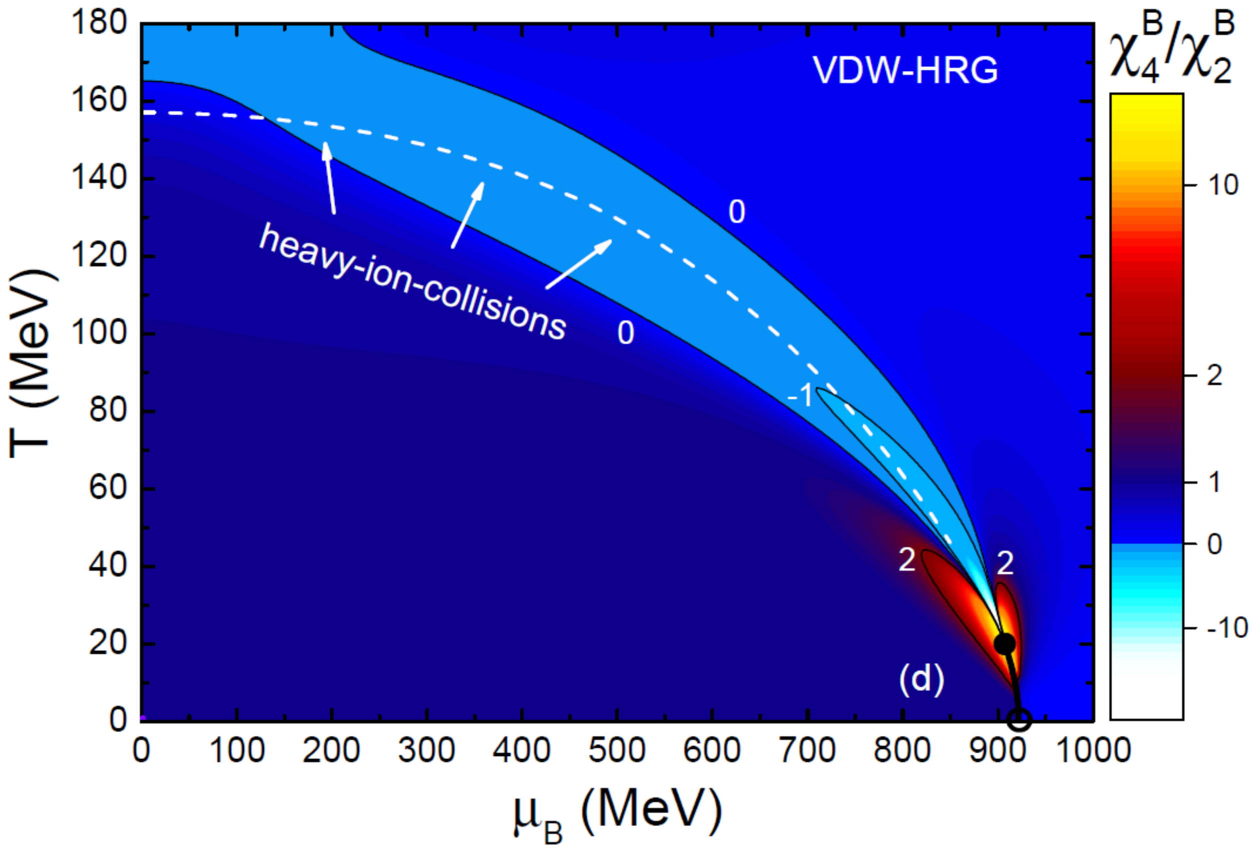
κ_i is complicated

$$\kappa_4 = \ \langle N \rangle + \left(\sim\langle N \rangle^2\right) + \left(\sim\langle N \rangle^3\right) + \left(\sim\langle N \rangle^4\right)$$

$$\kappa_2 = \ \langle N \rangle + \left(\sim\langle N \rangle^2\right)$$

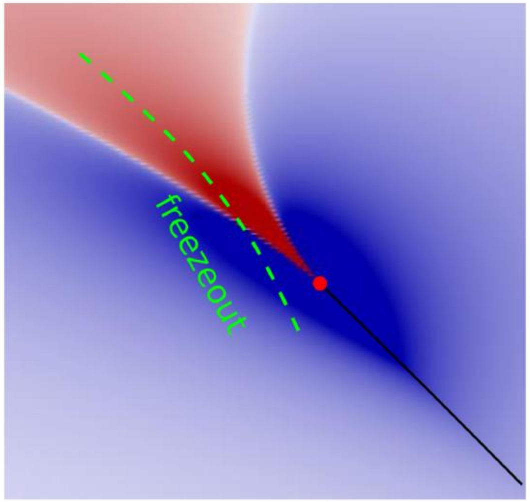
polynomial in Δy

HRG with attractive and repulsive Van der Waals interactions between (anti)baryons

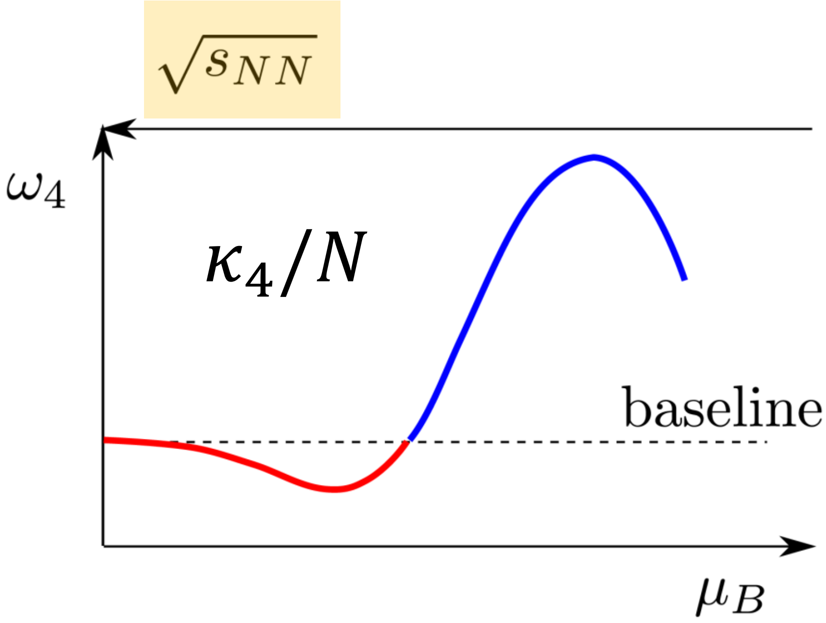


κ_4/κ_2

V. Vovchenko, M.I. Gorenstein, H. Stoecker, PRL 118 (2017) 182301



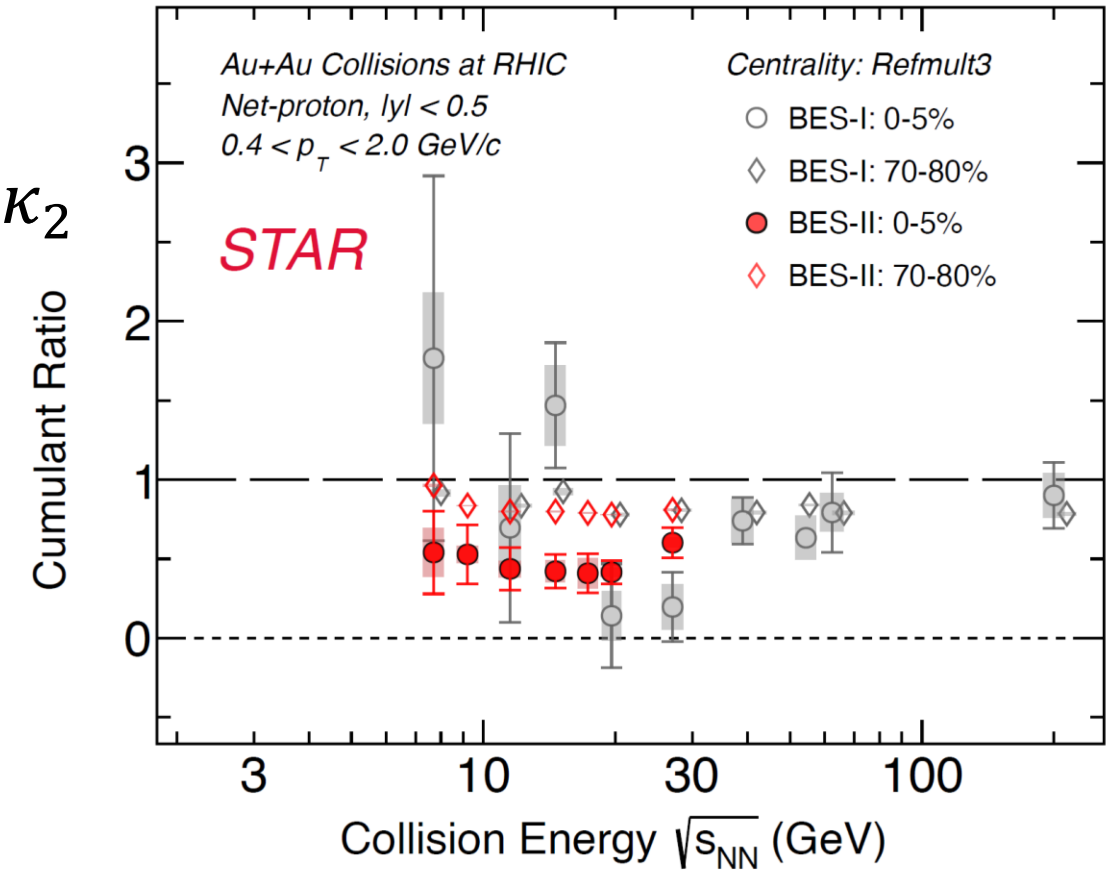
Density plot of the quartic cumulant obtained by mapping the Ising model into QCD.
Freezeout line is for demonstration only.



Normalized quartic cumulant of proton multiplicity

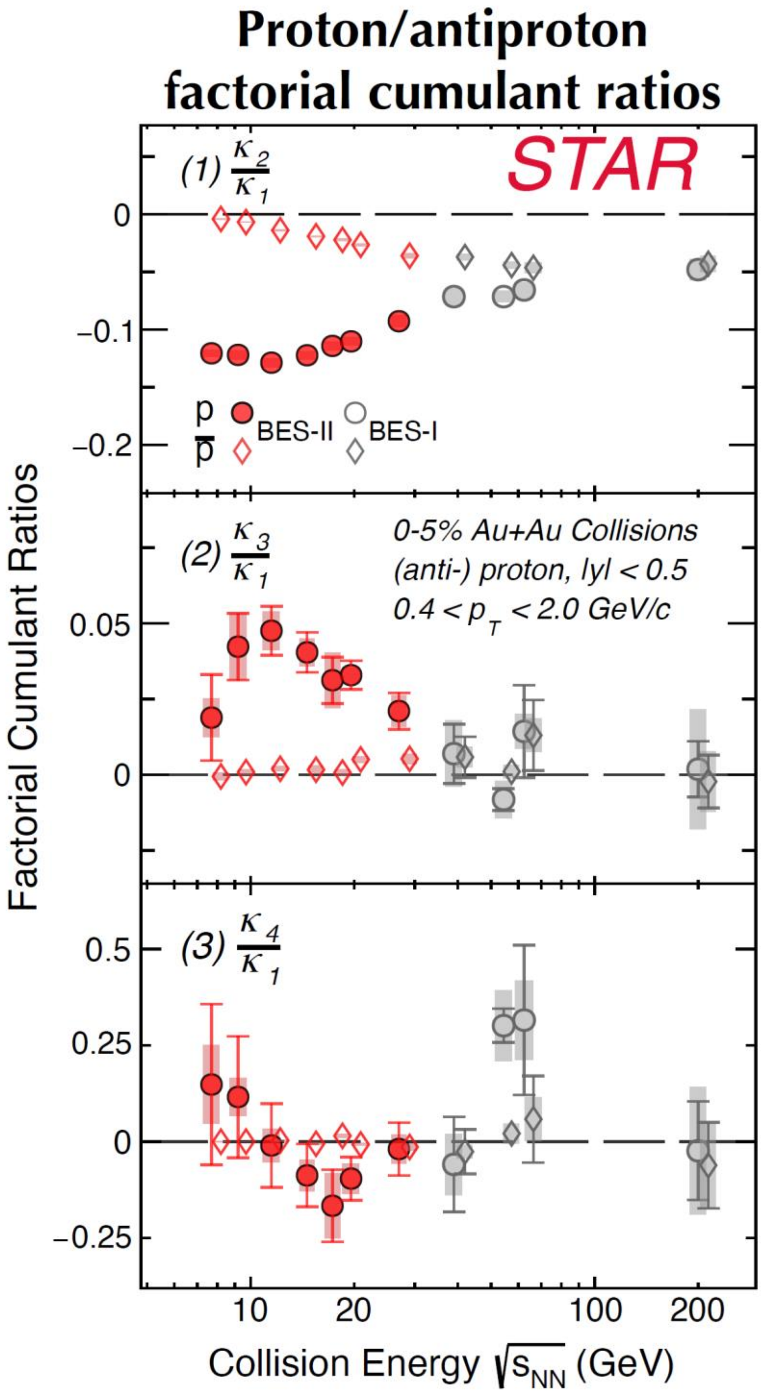
STAR Collaboration, BES-II

κ_4/κ_2



But we should look at the factorial cumulants …

Taken from Ashish Pandav, STAR Collaboration, CPOD 2024



Taken from Ashish Pandav, STAR Collaboration, CPOD 2024

Factorial cumulants look nontrivial (non-monotonic)

They are qualitatively consistent with the critical point physics.
Talk by M. Stephanov (Tuesday)

We need a good baseline model (with baron stopping etc.)

We need a good dynamical model with criticality

Antiprotons? $\frac{C_i}{\langle N \rangle^i}$

Finite-size scaling of proton cumulants

A. Sorensen, P. Sorensen, 2405.10278 [nucl-th]

$$\chi(L,t) = L^{\gamma/\nu}\Phi(tL^{1/\nu})$$

χ – susceptibility

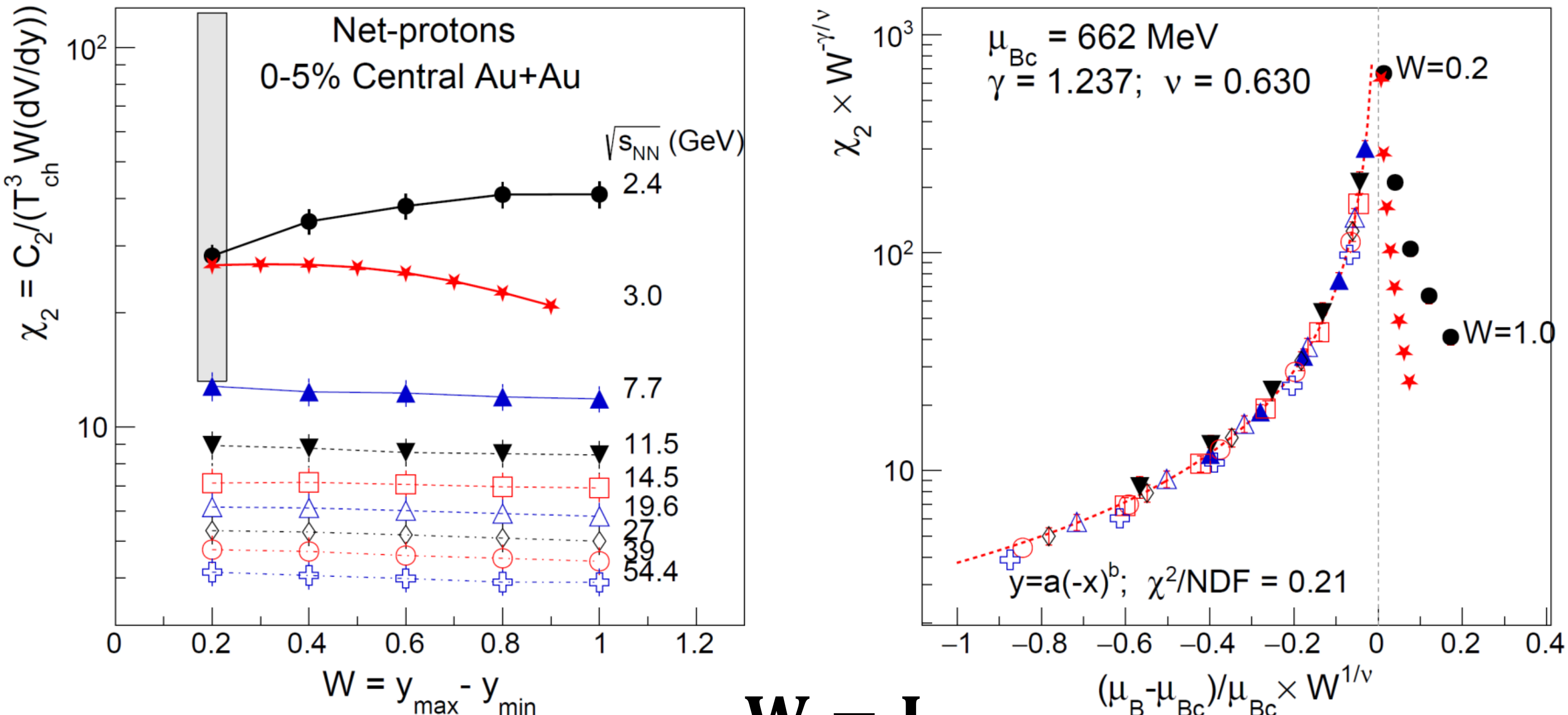
L – characteristic size of the system

t – distance from CP, for example $(\mu_B - \mu_{B,\mathrm{c}})/\mu_{B,\mathrm{c}}$

Φ – unknown universal function

γ and ν are the critical exponents. QCD is expected to be in the 3-D Ising universality class, where $\gamma = 1.237$ and $\nu = 0.630$.

A. Sorensen, P. Sorensen, 2405.10278 [nucl-th]



$$\mathbf{W = L}$$

CP at $\mathbf{565 < \mu_B < 685}$ MeV ?

What about new STAR data?

It is important to do similar analysis with, e.g., UrQMD (no CP) and show that it fails!

Multi-point Pade approximation

AIC - Akaike information criterion

crossover temperature

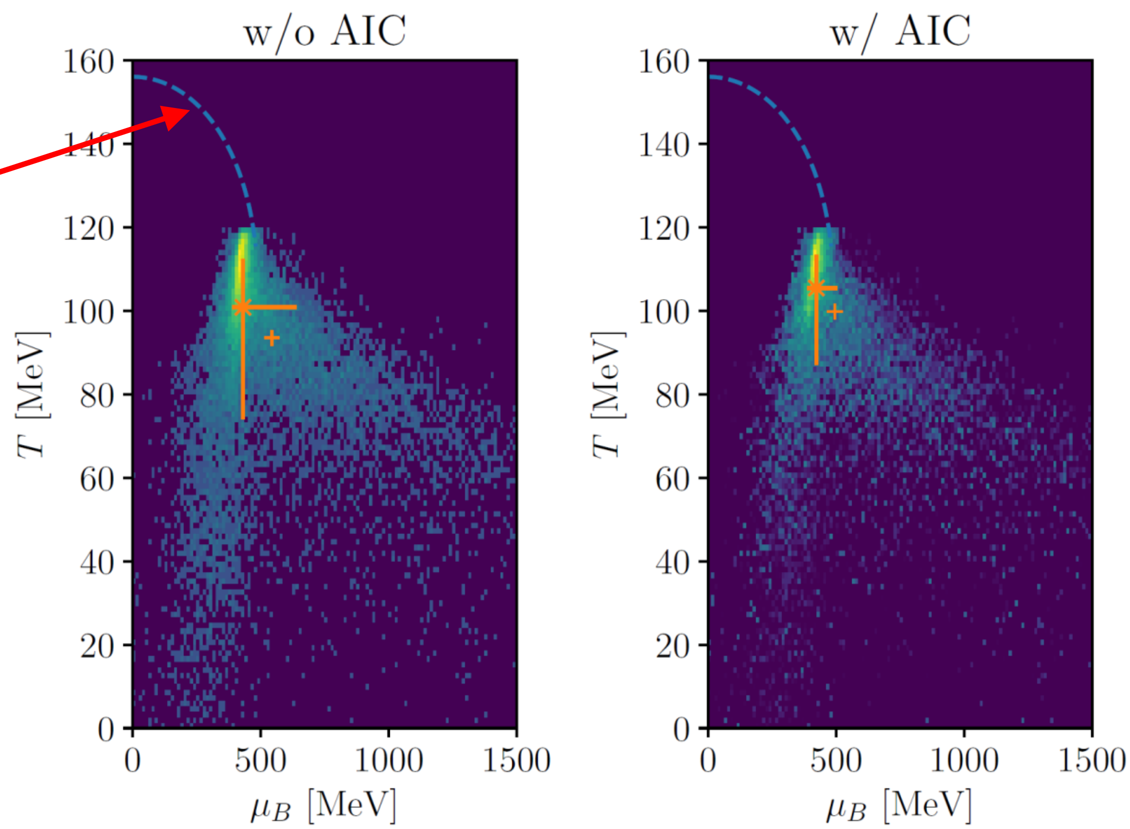
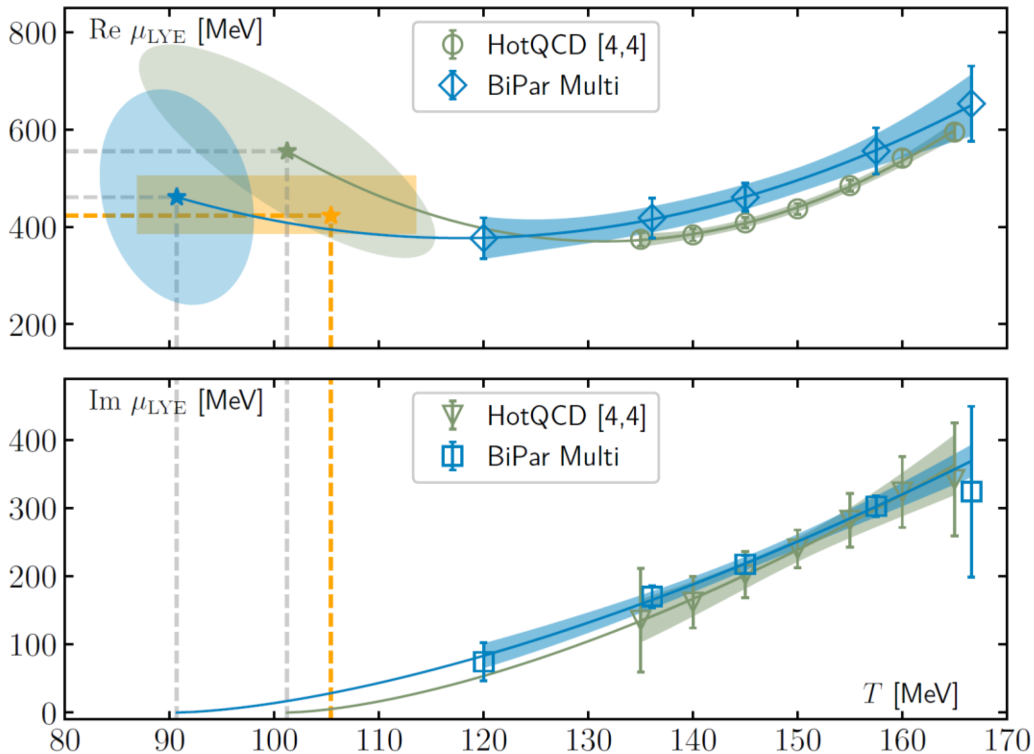


FIG. 1. Probability distribution of the QCD critical point from extrapolating Lee-Yang singularities to the real domain using universal scaling. For a detailed description see the text.

D. A. Clarke, P. Dimopoulos, F. Di Renzo, J. Goswami, C. Schmidt, S. Singh, K. Zambello, 2405.10196 [hep-lat]

Using the multi-point Pade approach, the Lee-Yang edge singularities are located in the QCD pressure in the complex baryon chemical potential.

$$(T^{\mathrm{CEP}}, \mu_B^{\mathrm{CEP}}) = (105^{+8}_{-18}, 422^{+80}_{-35})\ \mathrm{MeV}$$



"An important limitation of the estimate presented here is that they are not fully extrapolated to the continuum limit"

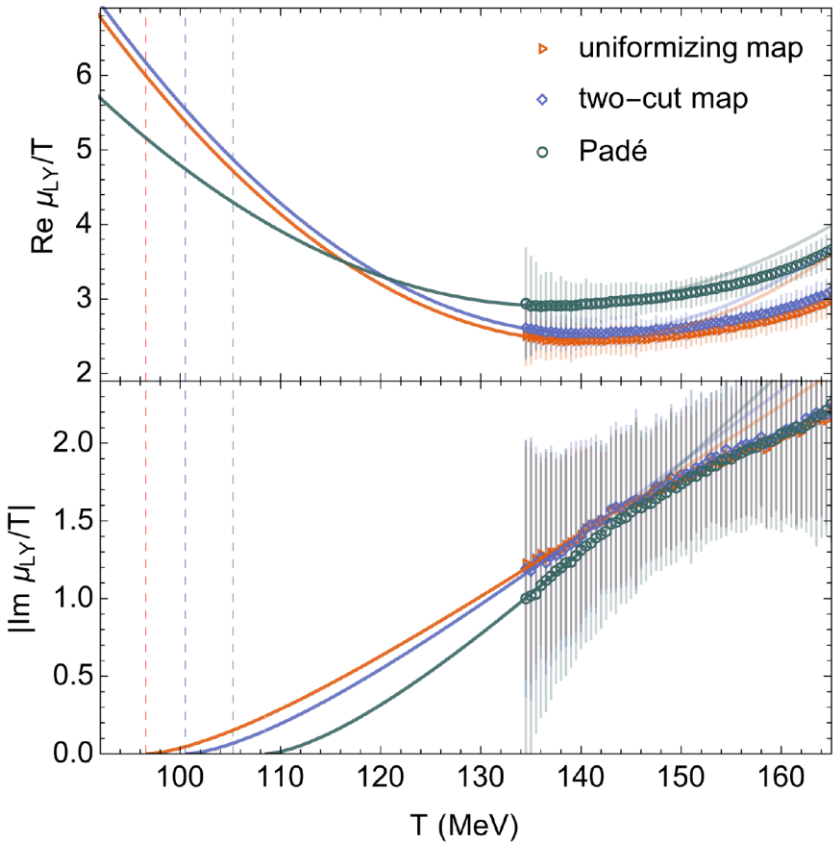
Can we conclude that there is the QCD critical point?

D. A. Clarke, P. Dimopoulos, F. Di Renzo, J. Goswami, C. Schmidt, S. Singh, K. Zambello, 2405.10196 [hep-lat]

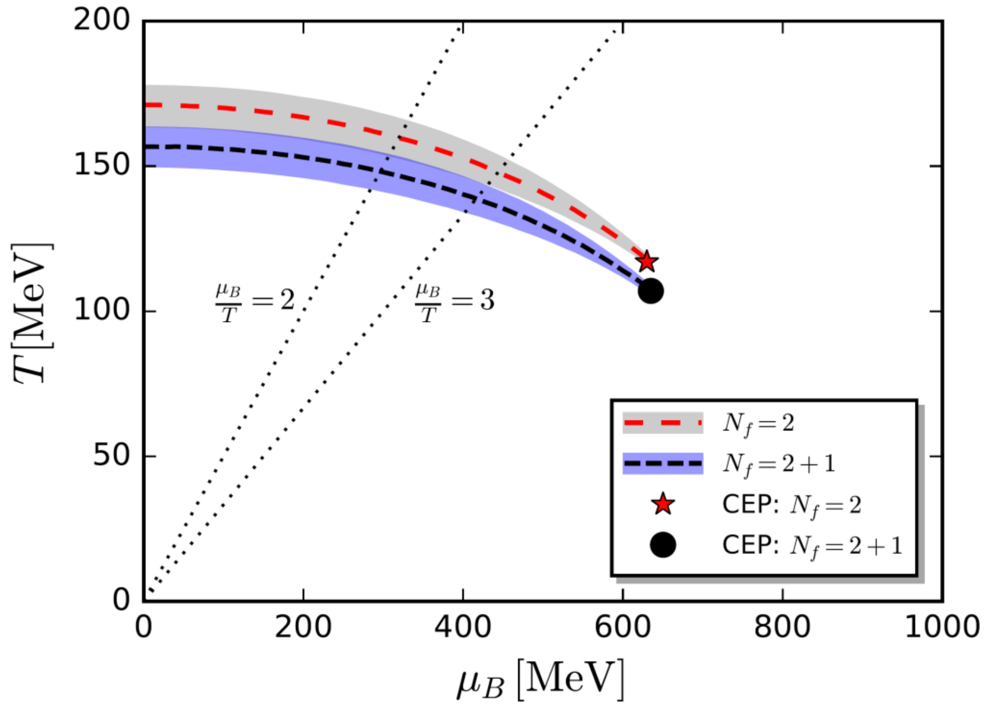
Similar results were obtained earlier

Gokce Basar, 2312.06952

We analyze the trajectory of the Lee-Yang edge singularities of the QCD equation of state in the complex baryon chemical potential (μ_B) plane… By extrapolating from this information, we estimate for the location of the QCD critical point, $\boldsymbol{T_c \approx 100}$ ***MeV,*** $\boldsymbol{\mu_c \approx 580}$ ***MeV***.



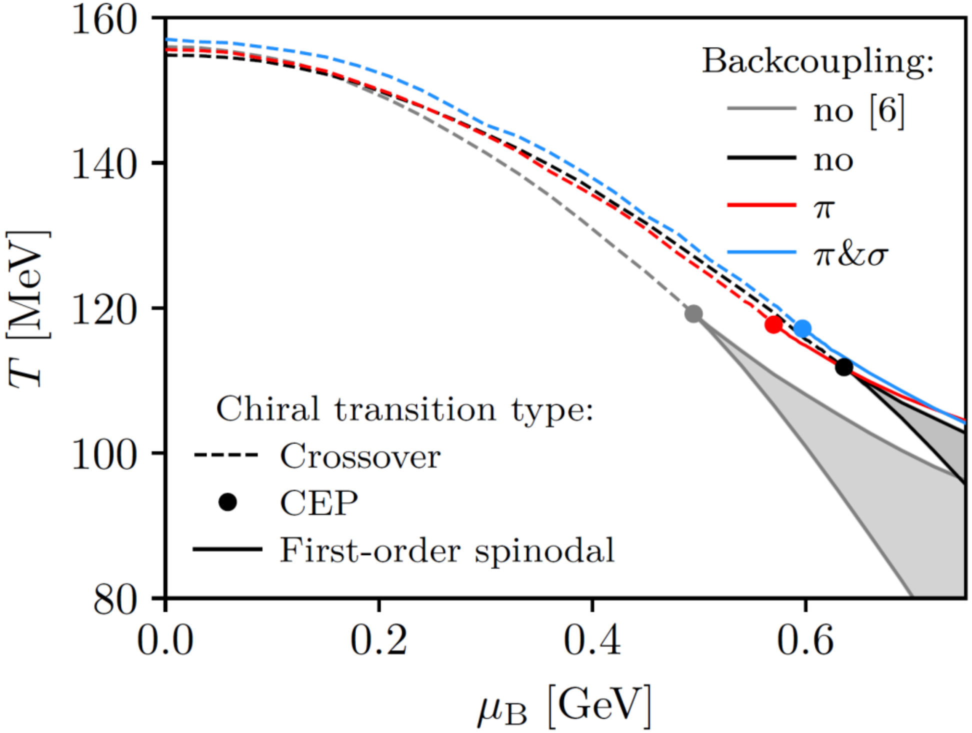
Functional renormalization group approach



W. Fu, J.M. Pawlowski, F. Rennecke, Phys. Rev. D 101 (2020) 5, 054032

We discuss the phase structure of QCD for $N_f = 2$ and $N_f = 2+1$ dynamical quark flavours at finite temperature and baryon chemical potential. It emerges dynamically from the underlying fundamental interactions between quarks and gluons in our work. To this end, starting from the perturbative high-energy regime, we systematically integrate-out quantum fluctuations towards low energies by using the functional renormalisation group. By dynamically hadronising the dominant interaction channels responsible for the formation of light mesons and quark condensates, we are able to extract the phase diagram for $\mu_B/T \lesssim 6$. We find a critical endpoint at $(T_{\rm CEP}, {\mu_B}_{\rm CEP}) = (107, 635)$ MeV.

Dyson-Schwinger equation



P.J. Gunkel, C.S. Fischer,
Phys. Rev. D 104 (2021) 5, 054022

$T_c \sim 117$ MeV

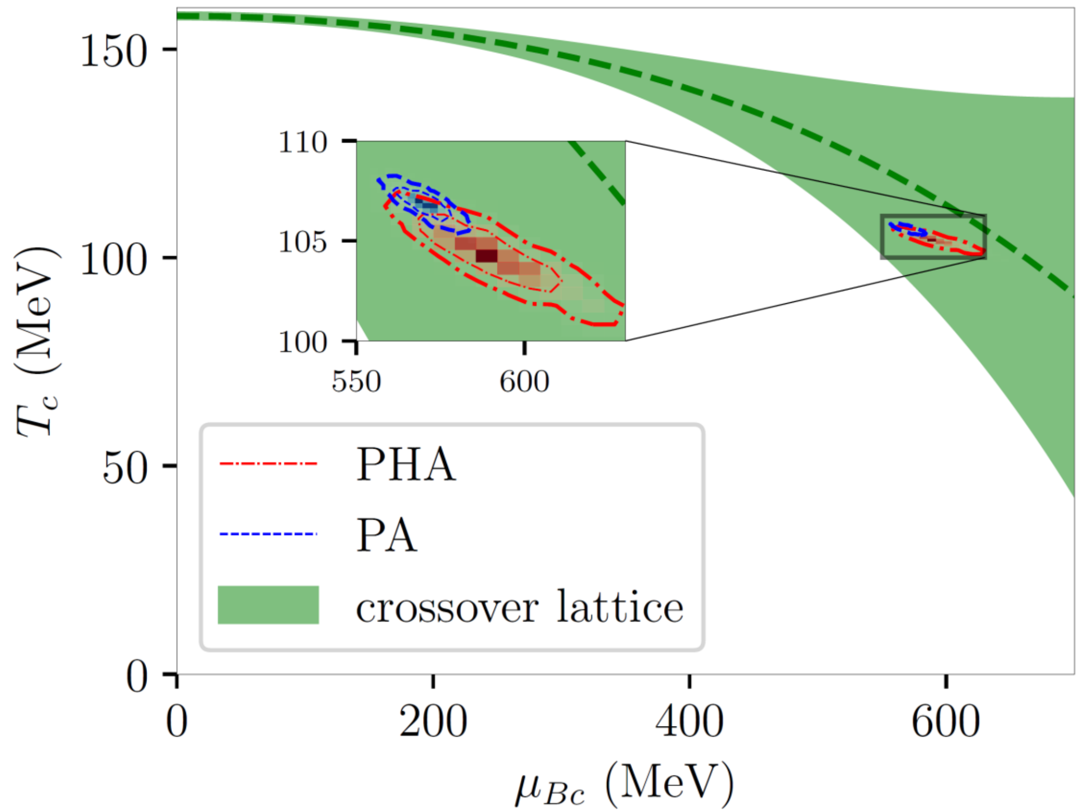
$\mu_{Bc} \sim 600$ MeV

Our results indicate that the location of the critical endpoint in the phase diagram is mainly determined by the microscopic degrees of freedom of QCD (in contrast to its critical properties).

…with regard to the critical behavior very close to the CEP, it is expected that macroscopic degrees of freedom (in particular the sigma meson) take over …

Holographic approach.

The gauge/gravity duality map of QCD onto a theory of dual black holes Talk by C. Ratti (Tuesday)



$T_c \sim 105$ MeV

$\mu_{Bc} \sim 580$ MeV

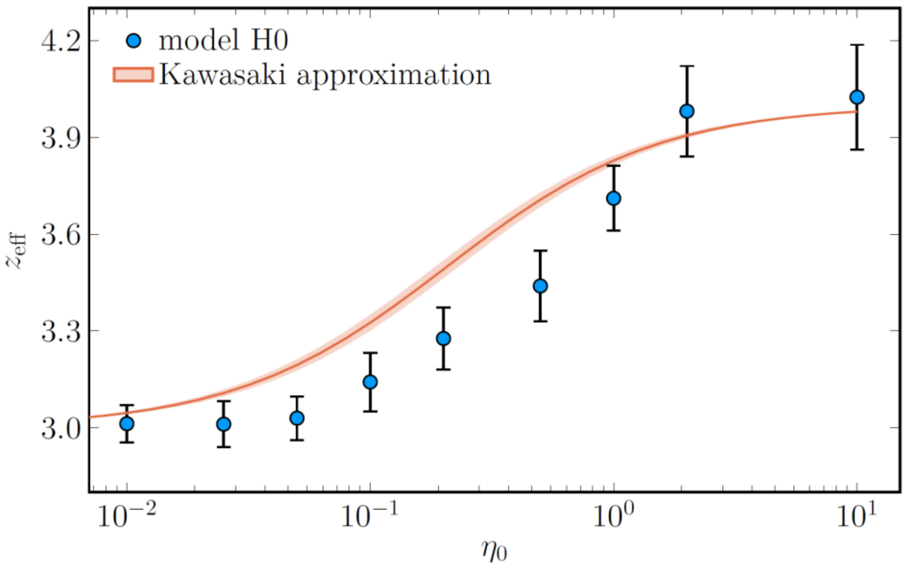
Model parameters fitted to various LQCD results at $\mu_B = 0$ ($\frac{s}{T^4}, \chi_2$)

M.Hippert, J.Grefa, T.A.Manning, J.Noronha, J.Noronha-Hostler, I.P. Vazquez, C.Ratti, R.Rougemont, M.Trujillo, 2309.00579

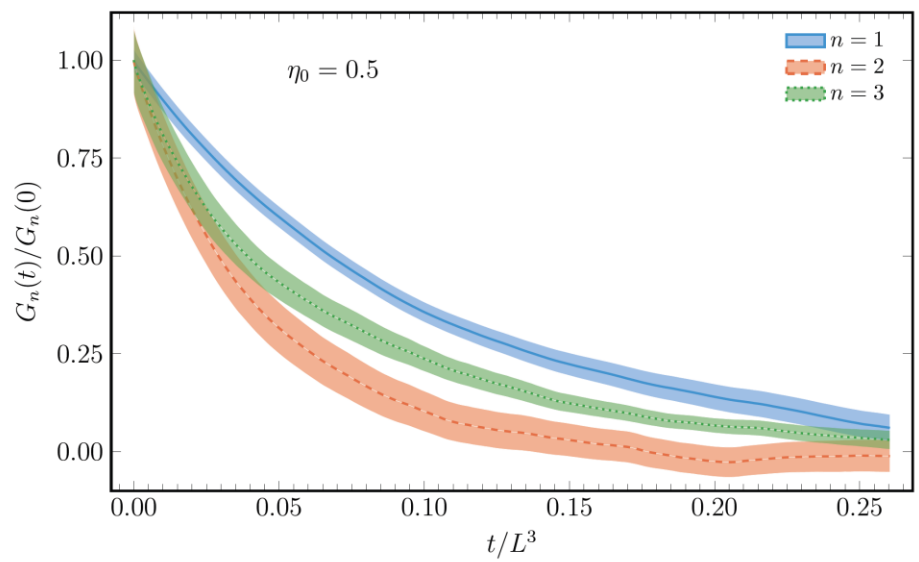
Progress towards dynamical models

C. Chattopadhyay,
J. Ott,
T. Schaefer,
V. Skokov,
2403.10608

Stochastic fluid dynamics near a critical point



Correlation function of the order parameter



The first non-perturbative determination of the dynamical critical exponent near the critical point of model H (same dynamical model as QCD).
The paper defines a strategy for how to simulate critical fluctuations in HIC.

Progress towards dynamical models

HYDRO+ (hydrodynamics with critical modes)

M. Stephanov, Y. Yi, PRD 98, 036006 (2018)

K. Rajagopal, G. Ridgway, R. Weller, Y. Yin, PRD 102, 094025 (2020)

Recent review: M. Stephanov, Acta Phys. Polon. B 55 (2024) 5

We consider … a generic extension of hydrodynamics by a parametrically slow mode or modes ("Hydro+") and a description of fluctuations out of equilibrium.

Equation of state with critical point

M.Kahangirwe, S.A. Bass, E.Bratkovskaya, J.Jahan, P.Moreau, P.Parotto, D.Price, C.Ratti, O.Soloveva, M.Stephanov, PRD 109 (2024) 9, 094046

We present a novel construction of the QCD equation of state (EoS) at finite baryon density. Our work combines a recently proposed resummation scheme for lattice QCD results with the universal critical behavior at the QCD critical point. This allows us to obtain a family of equations of state in the range $0 \leq \mu_B \leq 700$ MeV and $25\ \text{MeV} \leq T \leq 800$ MeV, which match lattice QCD results near $\mu_B = 0$ while featuring a critical point in the 3D Ising model universality class.

Conclusions

Non-monotonic factorial cumulants from STAR

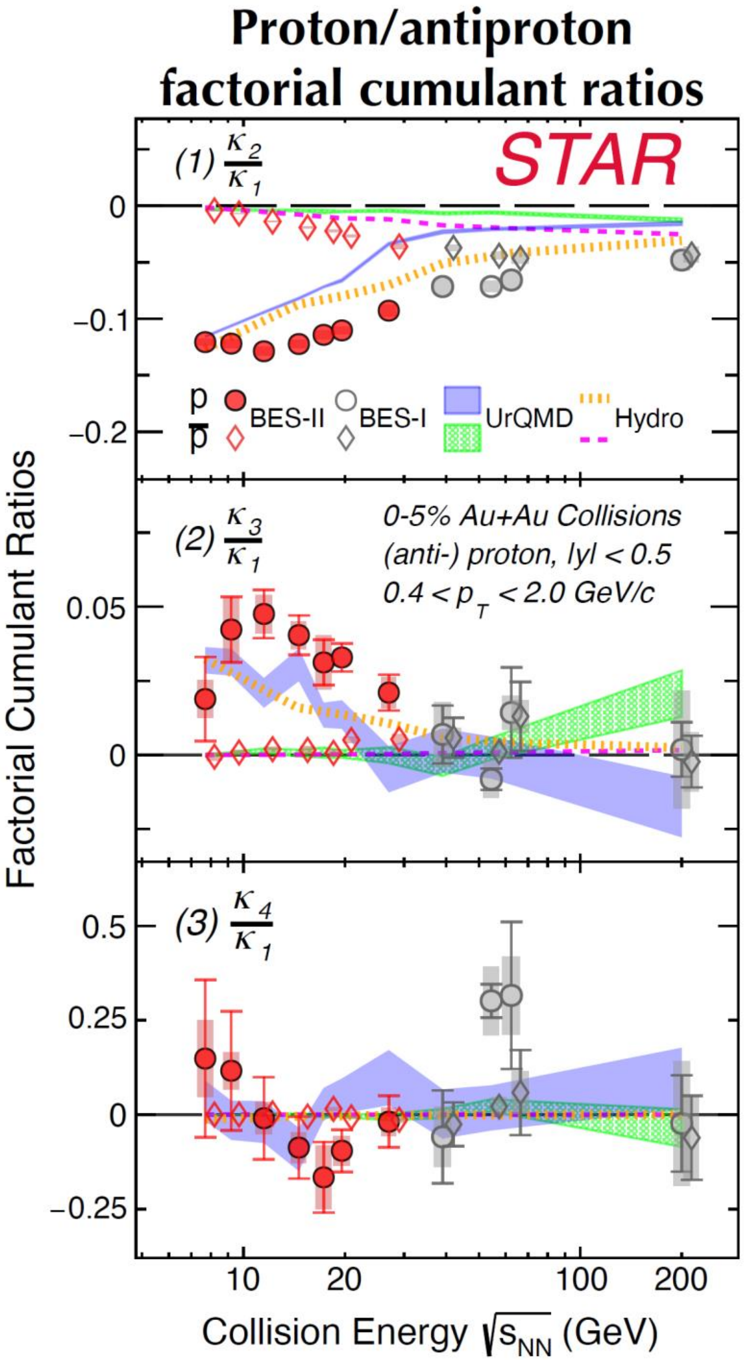
We need a good baseline model

We need a good dynamical model with criticality

Various theoretical and phenomenological approaches predict similar location of the QCD critical point

Progress towards simulating critical fluctuations in heavy-ion collisions

Backup

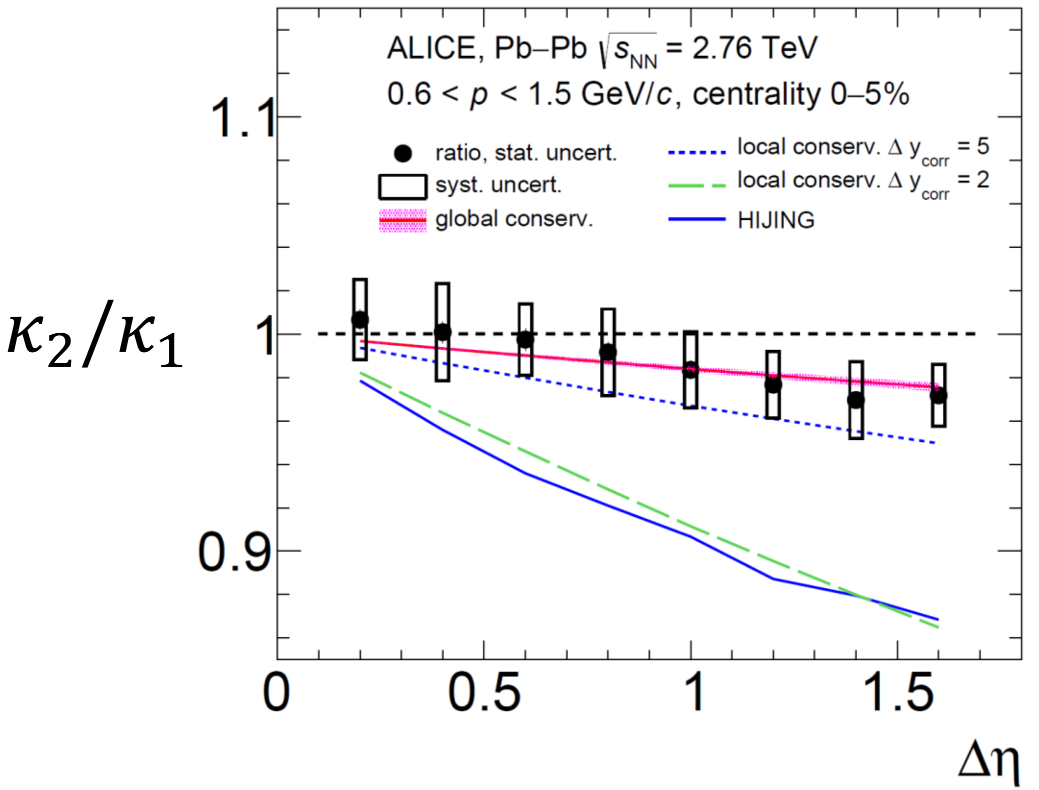


STAR data vs. models

Taken from Ashish Pandav, STAR Collaboration, CPOD 2024

ALICE Collaboration

PLB 807 (2020) 135564



Global (not local) baryon conservation. Something to understand. It would be good to measure proton, antiproton and mixed proton-antiproton factorial cumulants M.Barej, AB, PRC 102 (2020) 6, 064908

See O.Savchuk, V.Vovchenko, V.Koch, J.Steinheimer, H.Stoecker, PLB 827 (2022) 136983
Local conservation and $B\bar{B}$ annihilation.

Genuine three-particle correlation

$$\rho_3(y_1, y_2, y_3) = \rho(y_1)\rho(y_2)\rho(y_3) + \rho(y_1)\boldsymbol{C_2}(y_2, y_3) + \cdots$$

three possibilities

$$+ \boldsymbol{C_3}(y_1, y_2, y_3)$$

Integrating both sides

$$\langle n(n-1)(n-2)\rangle = \langle n\rangle^3 + 3\langle n\rangle \boldsymbol{C_2} + \boldsymbol{C_3}$$

factorial cumulant (integrated correlation function)

$$\boldsymbol{C_3} = \int \boldsymbol{C_3}(y_1, y_2, y_3)\, dy_1\, dy_2\, dy_3$$

and analogously for higher-order correlation functions