SQM 2024

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Theoretical aspects of quarkonia production in heavy ion collisions

Jiaxing Zhao (**SUBATECH** & HFHF)

jzhao@subatech.in2p3.fr

06/06/2024

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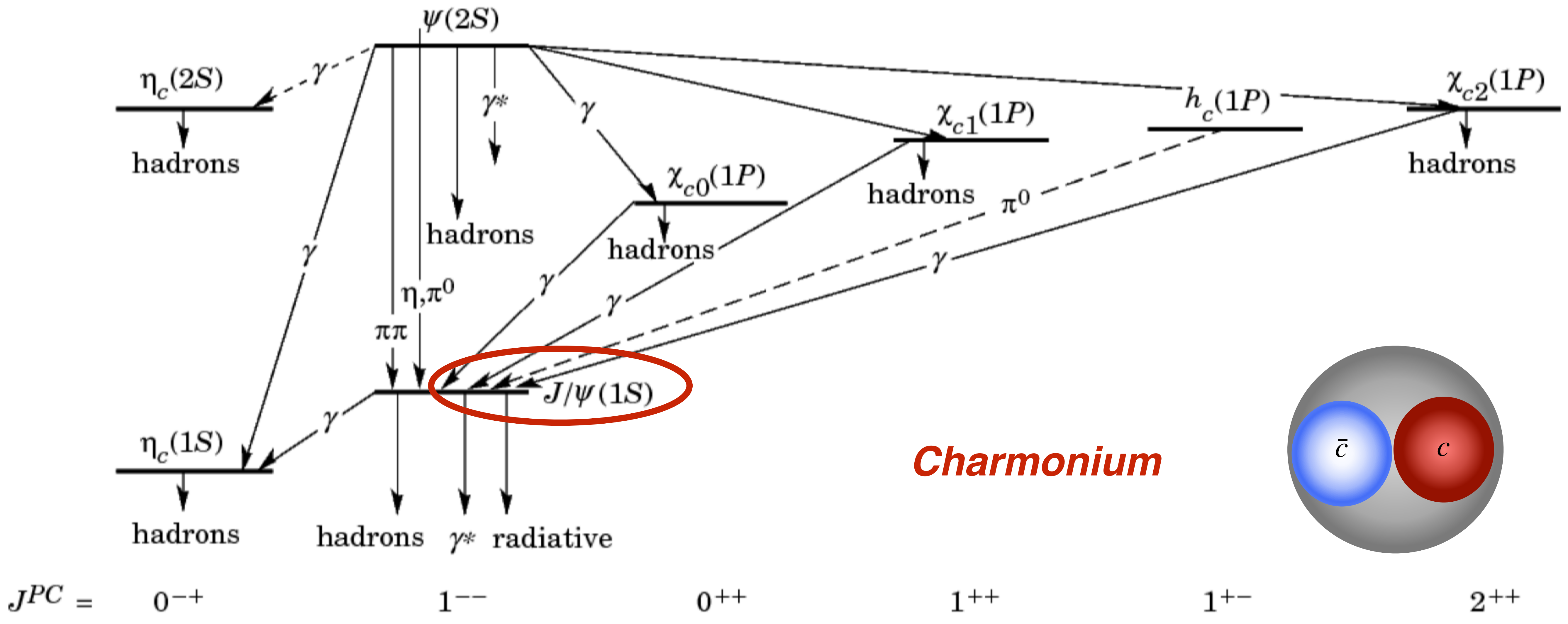
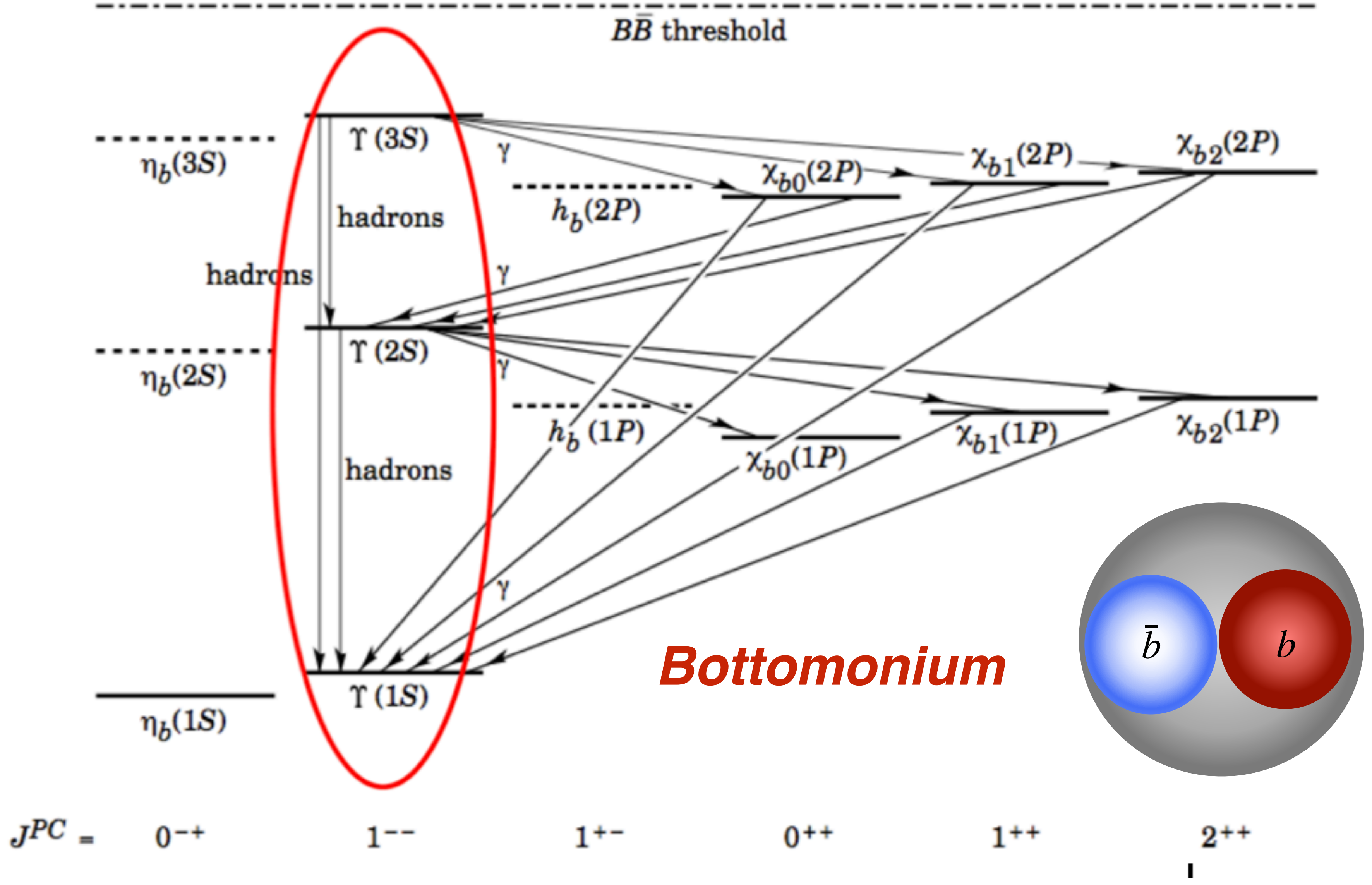
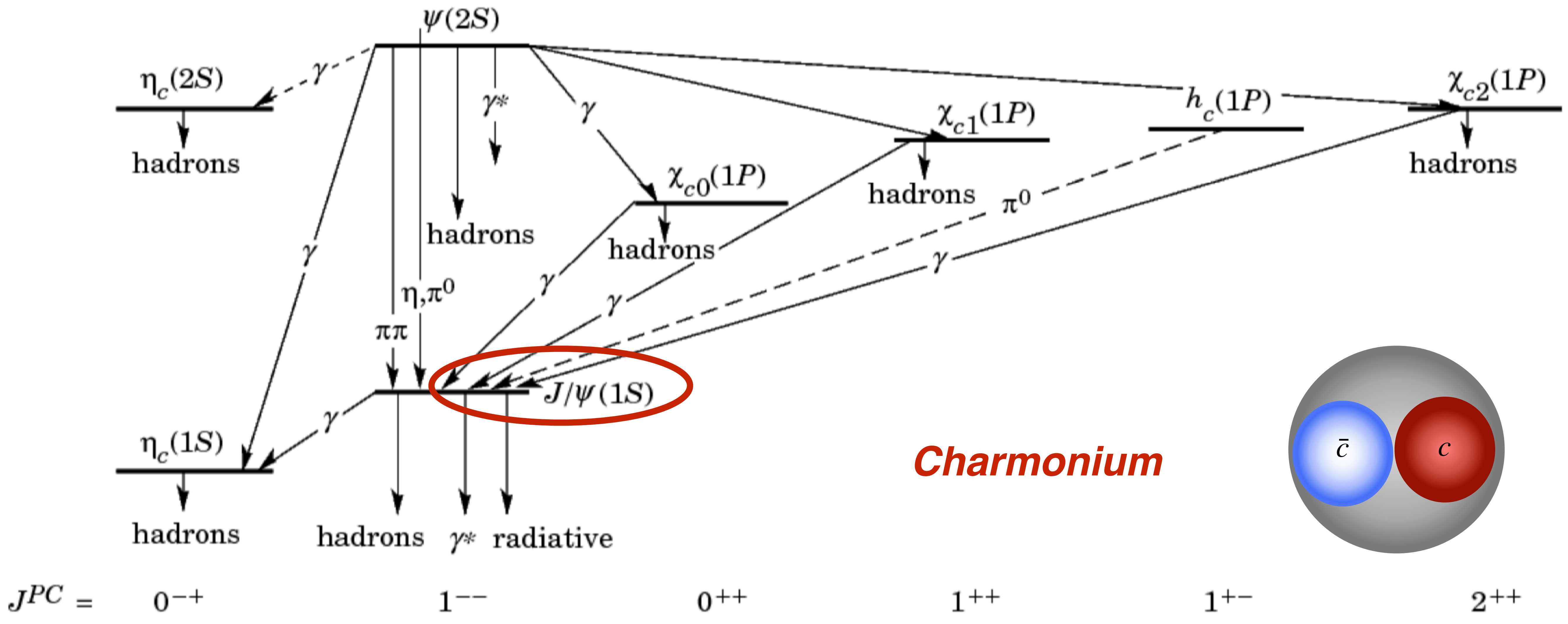


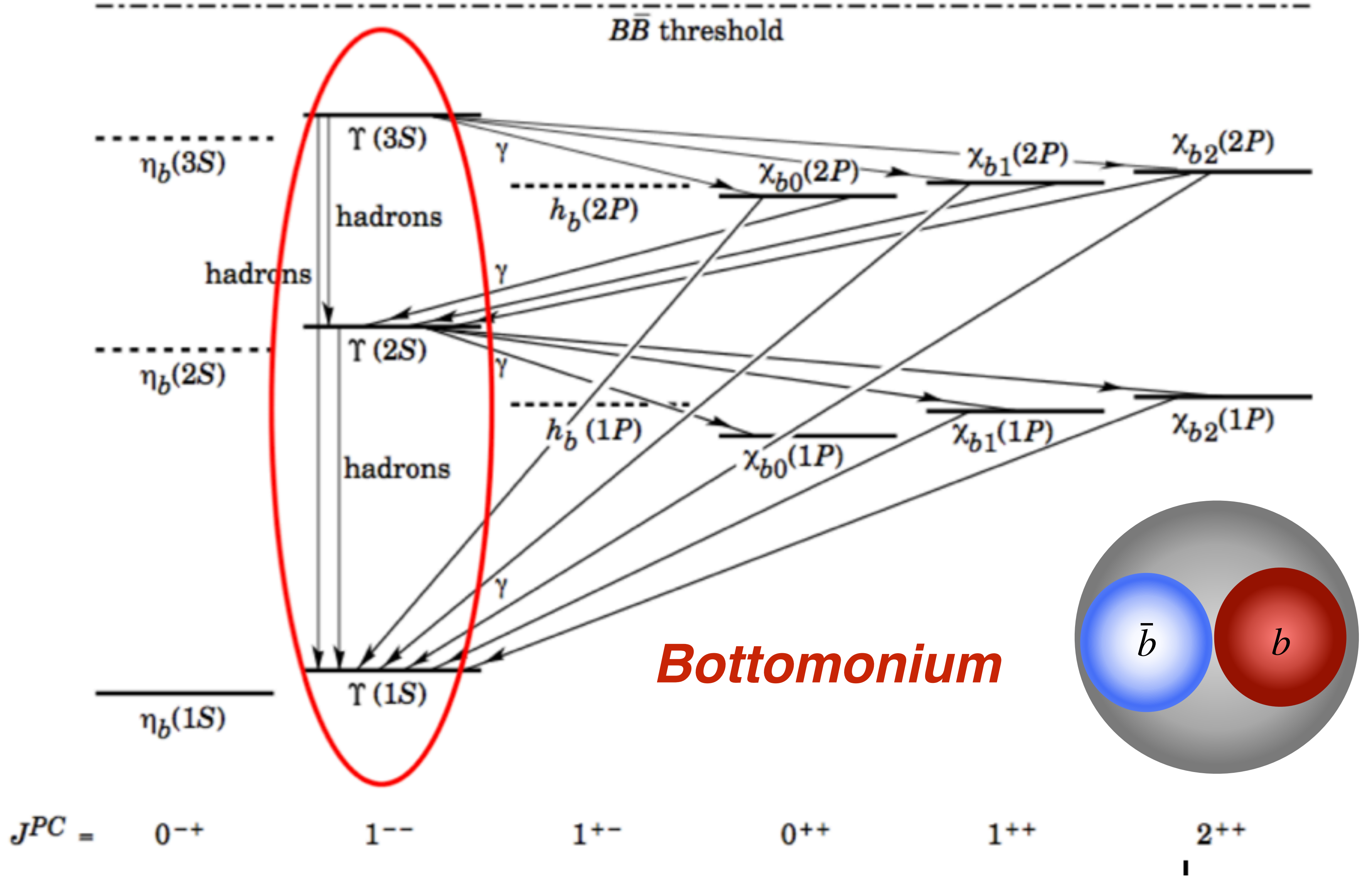


Why are they important ?

- ➡ Heavy quarks/quarkonoium are mostly produced in the early stage of heavy ion collisions
- ➡ Heavy quark -> scale separation -> Heavy Quark Effect Theory

Quarkonia suppression has been considered as a smoking gun of the QGP (Matsui, Satz at 1986, …)
From yield and distribution -> deduce in-medium properties and infer the fundamental interaction in QCD matter !





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Outline:

- ✤ **Vacuum and finite-temperature properties of quarkonium**
- ✤ **Quarkonium real-time evolution in hot QCD medium**

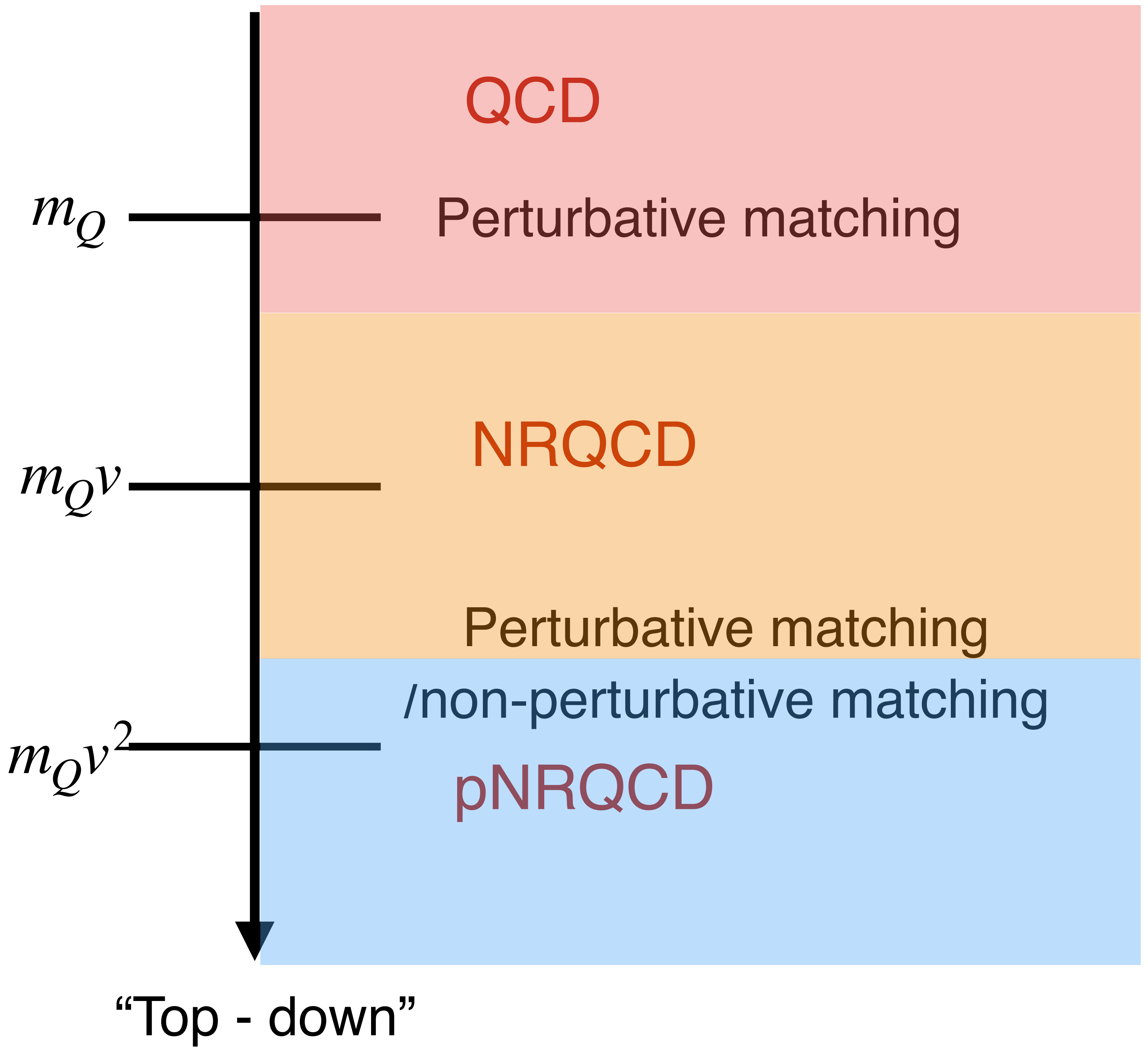
Vacuum and finite-temperature properties of quarkonium

Quarkonium static properties in a vacuum

From QCD to the potential model

$m_c \sim 1.5\ \mathrm{GeV}, m_b \sim 4.7\ \mathrm{GeV}$

Separation of scales: $m_Q \gg m_Q v \gg m_Q v^2$



$$\mathcal{L}_{pNRQCD} = \int d^3r \mathrm{Tr}\Big[S^\dagger(i\partial_0 - H_S)S + O^\dagger(i\partial_0 - H_O)O\Big] + V_A(r)\mathrm{Tr}[O^\dagger \mathbf{r}\cdot g\mathbf{E}S + S^\dagger \mathbf{r}\cdot g\mathbf{E}O] + \frac{V_B(r)}{2}\mathrm{Tr}[O^\dagger \mathbf{r}\cdot g\mathbf{E}O + O^\dagger O\mathbf{r}\cdot g\mathbf{E}] + \mathcal{L}'_g + \mathcal{L}'_l$$

Singlet field S; Octet field O.

$$H_S = \{c_1^s(r), \frac{\mathbf{p}^2}{2\mu}\} + c_2^s(r)\frac{\mathbf{P}^2}{2M} + V_S^{(0)} + \frac{V_S^{(1)}}{m_Q} + \frac{V_S^{(2)}}{m_Q^2},$$

$$H_O = \{c_1^o(r), \frac{\mathbf{p}^2}{2\mu}\} + c_2^o(r)\frac{\mathbf{P}^2}{2M} + V_O^{(0)} + \frac{V_O^{(1)}}{m_Q} + \frac{V_O^{(2)}}{m_Q^2}.$$

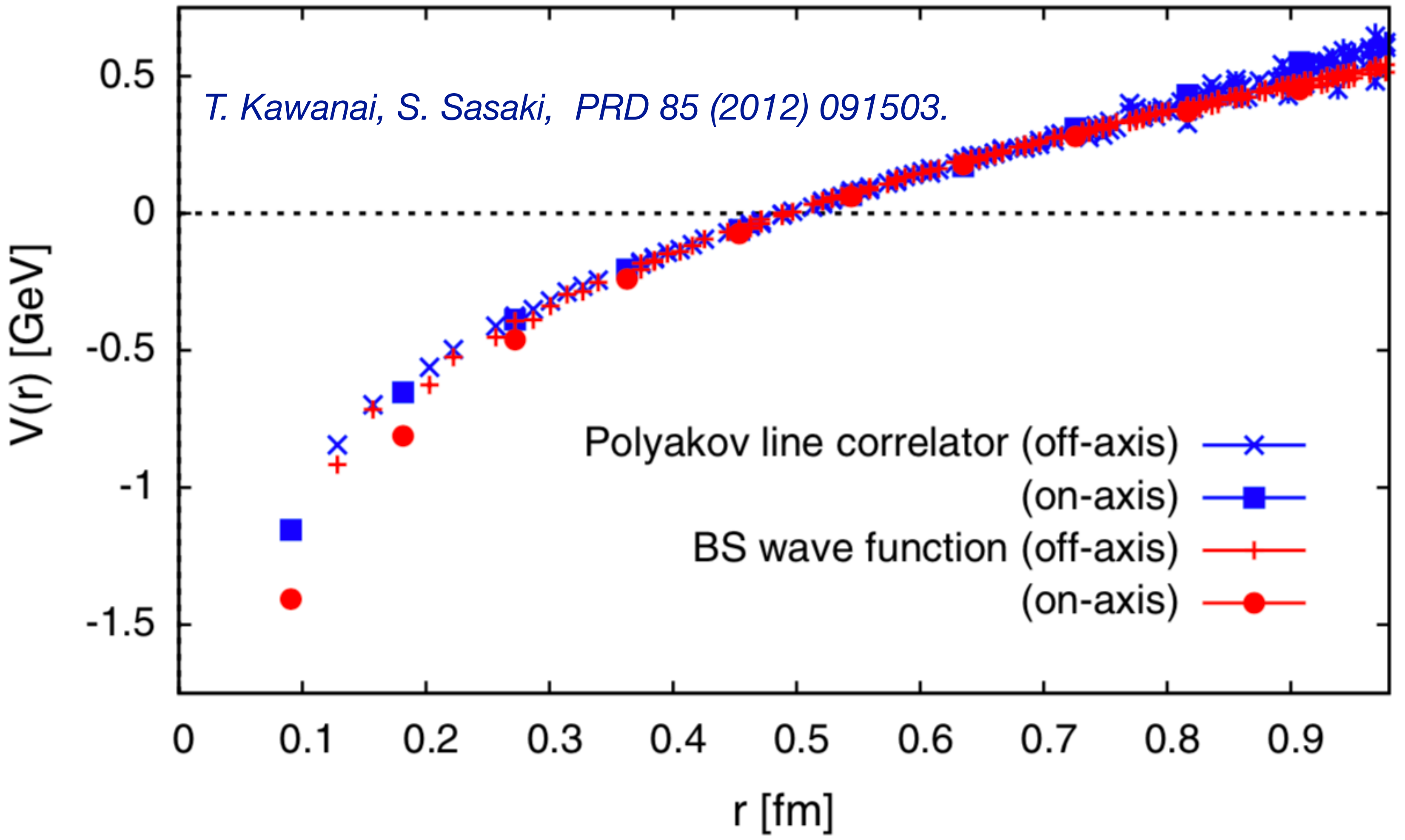
The potential model： two-body Schrödinger equation

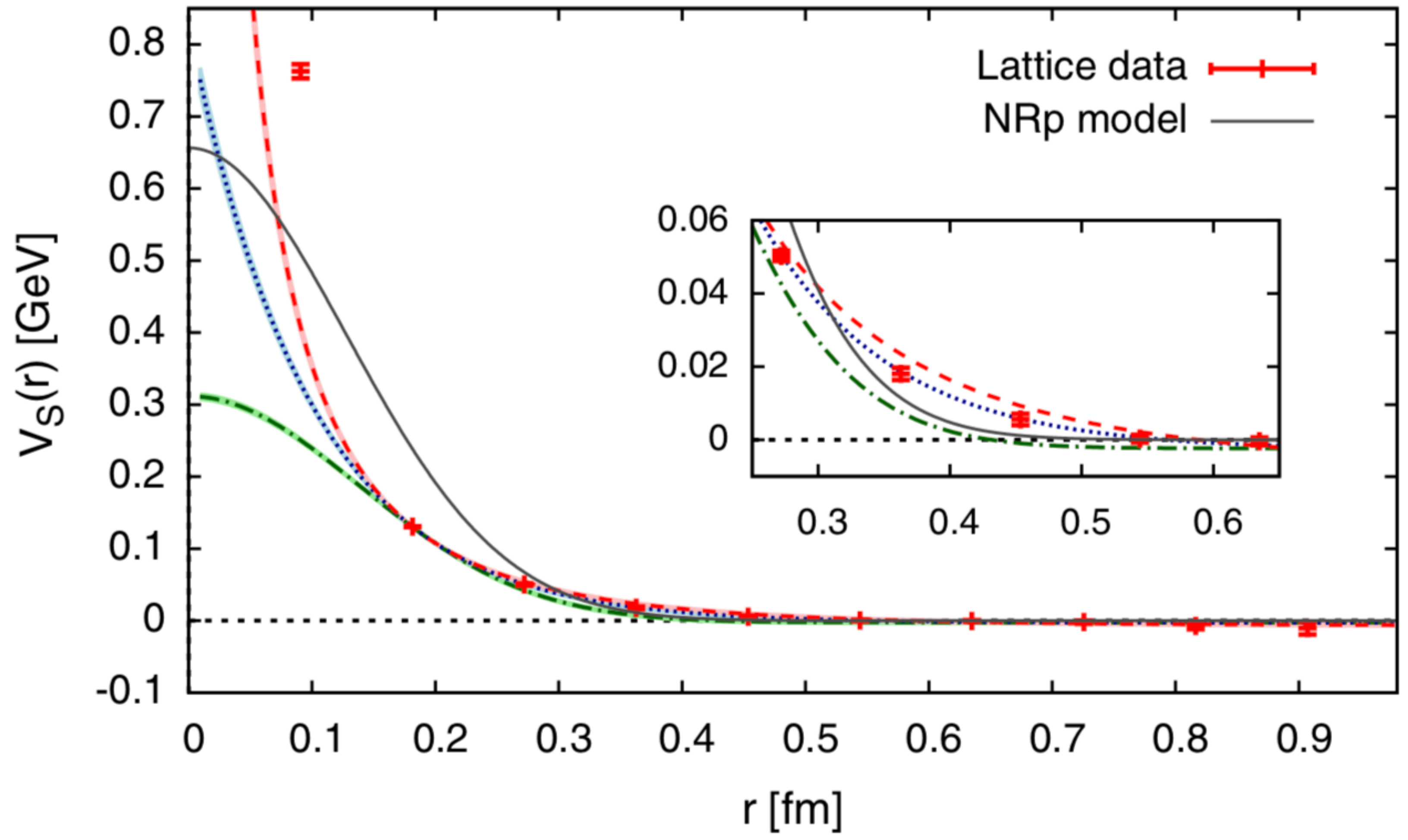
$$\left[\frac{\hat{p}_1^2}{2m_1} + \frac{\hat{p}_2^2}{2m_2} + V(\mathbf{r}_1, \mathbf{r}_2, \mathbf{s}_1, \mathbf{s}_2)\right]\psi = E\psi$$

See for e.g.:
W. Caswell, G. Lepage, Phys. Lett. B 167 (1986) 437.
N. Brambilla, A. Pineda, J. Soto, A. Vairo, Nucl. Phys. B 566 (2000) 275.

Quarkonium static properties in a vacuum

Cornell potential + Spin-spin interaction

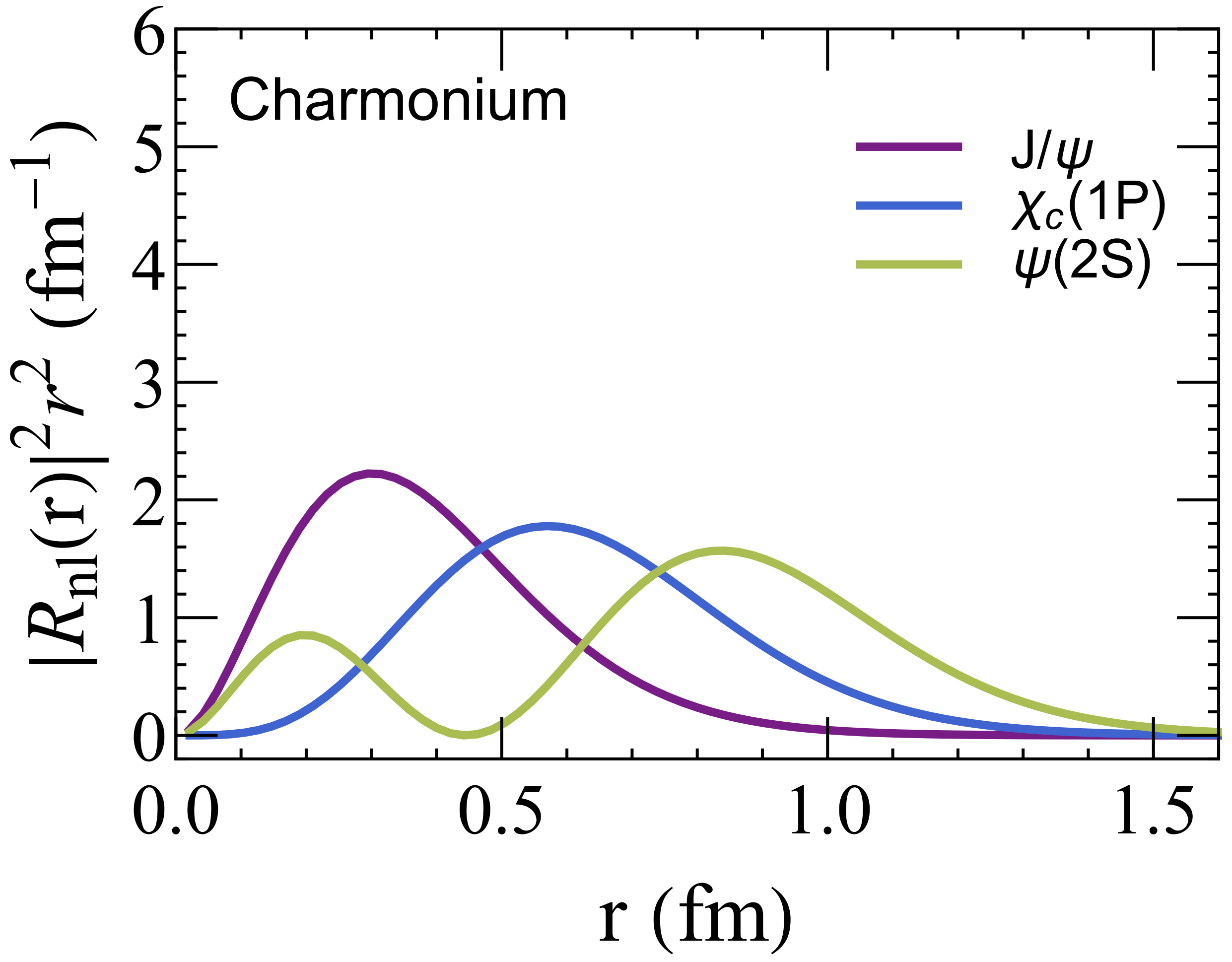


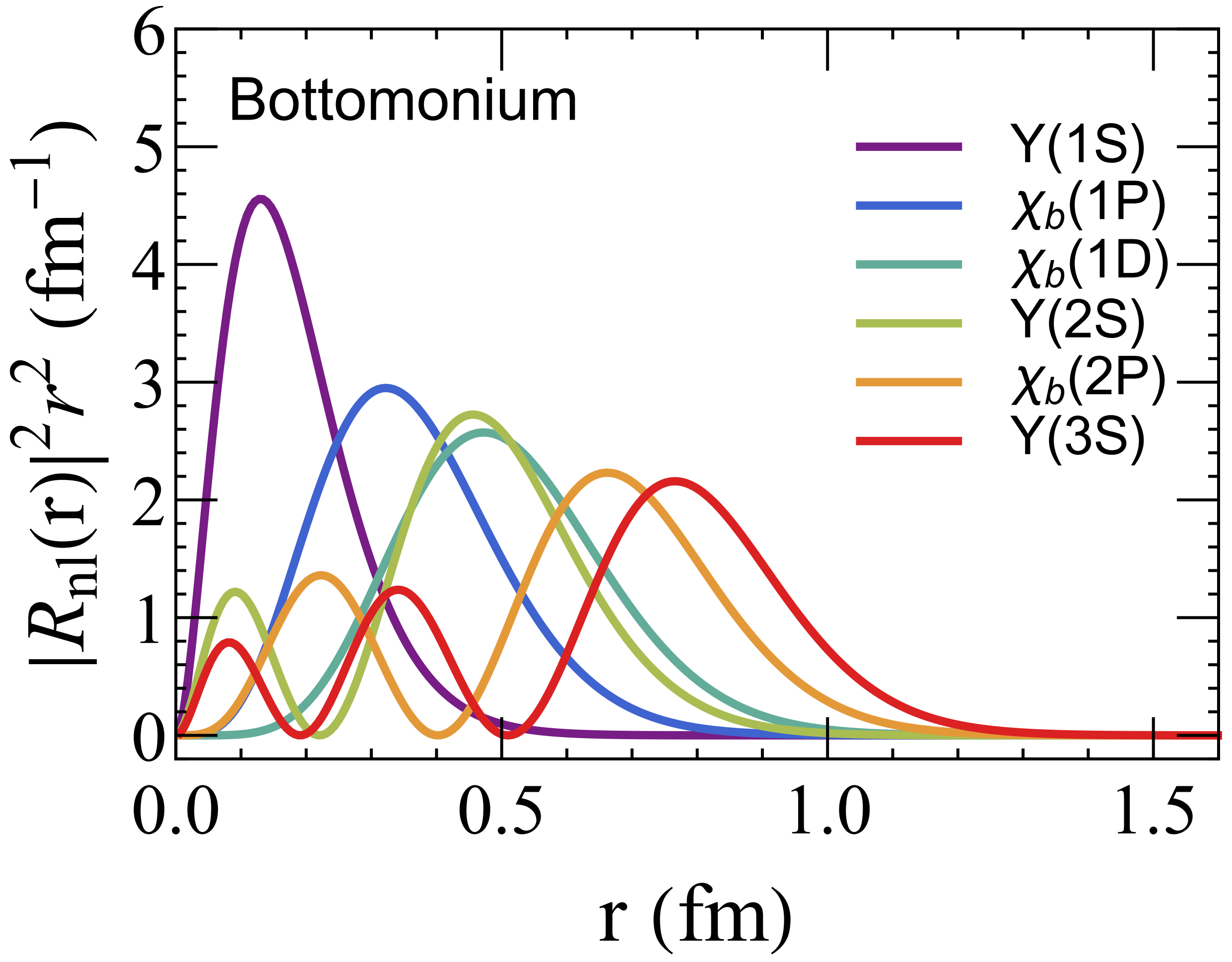


States	$\eta_c(1S)$	$J/\psi(1S)$	$h_c(1P)$	$\chi_c(1P)$	$\eta_c(2S)$	$\psi(2S)$	$h_c(2P)$	$\chi_c(2P)$
$M_{Exp.}$(GeV)	2.981	3.097	3.525	3.556	3.639	3.686	-	3.927
$M_{Th.}$(GeV)	2.967	3.102	3.480	3.500	3.654	3.720	3.990	4.000
$\langle r\rangle$(fm)	0.365	0.427	0.635	0.655	0.772	0.802	0.961	0.980
States	$\eta_b(1S)$	$\Upsilon(1S)$	$h_b(1P)$	$\chi_b(1P)$	$\eta_b(2S)$	$\Upsilon(2S)$	$\chi_b(2P)$	$\Upsilon(3S)$
$M_{Exp.}$(GeV)	9.398	9.460	9.898	9.912	9.999	10.023	10.269	10.355
$M_{Th.}$(GeV)	9.397	9.459	9.845	9.860	9.957	9.977	10.221	10.325
$\langle r\rangle$(fm)	0.200	0.214	0.377	0.387	0.465	0.474	0.603	0.680

JZ, *K. Zhou, S. Chen, P. Zhuang, PPNP.* 114 (2020) 103801.

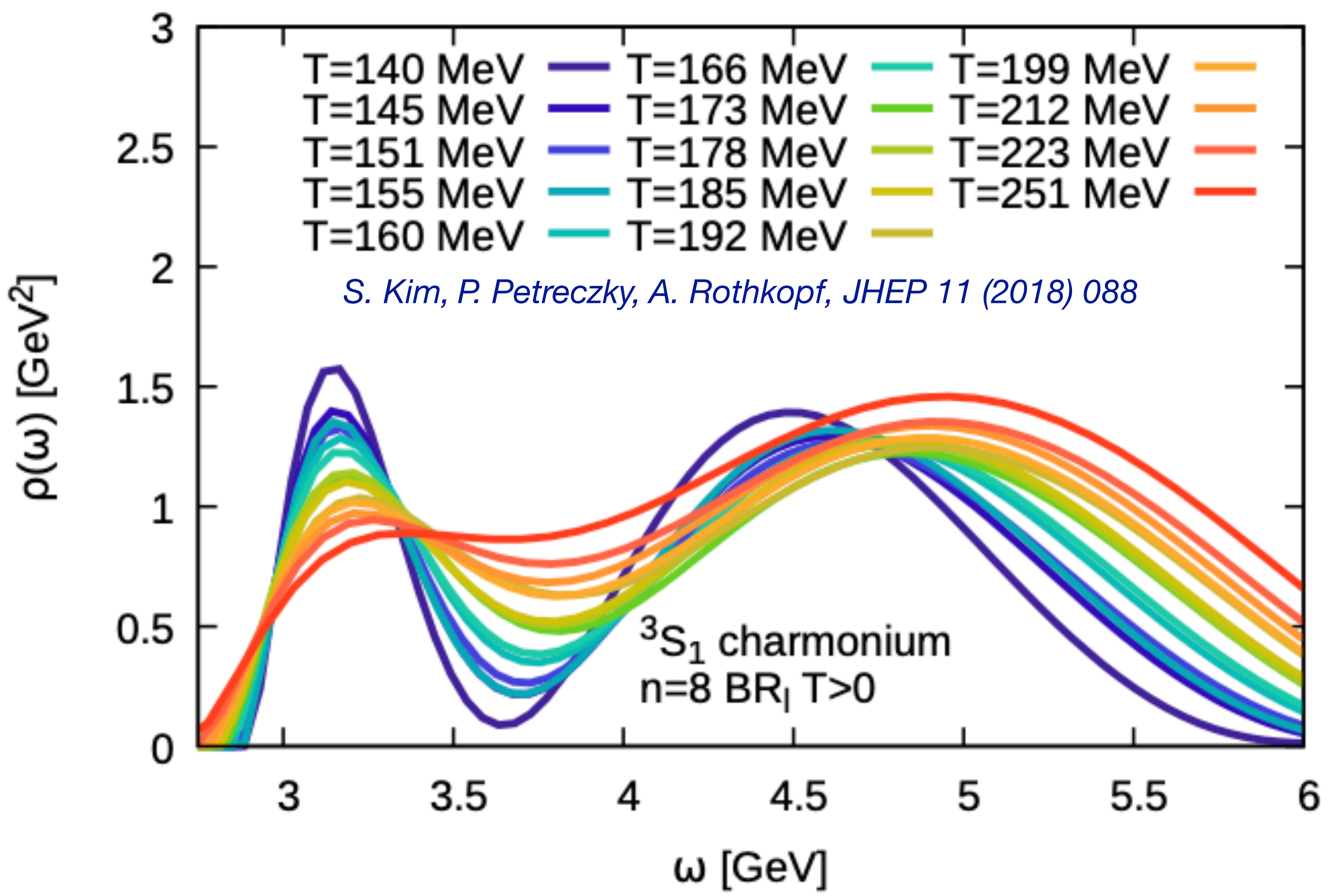
The mass spectra can be explained very well!





Quarkonium in the hot medium

All in-medium properties of quarkonium are encoded in their spectra function



peak position shifts and becomes broader as temperature increases.

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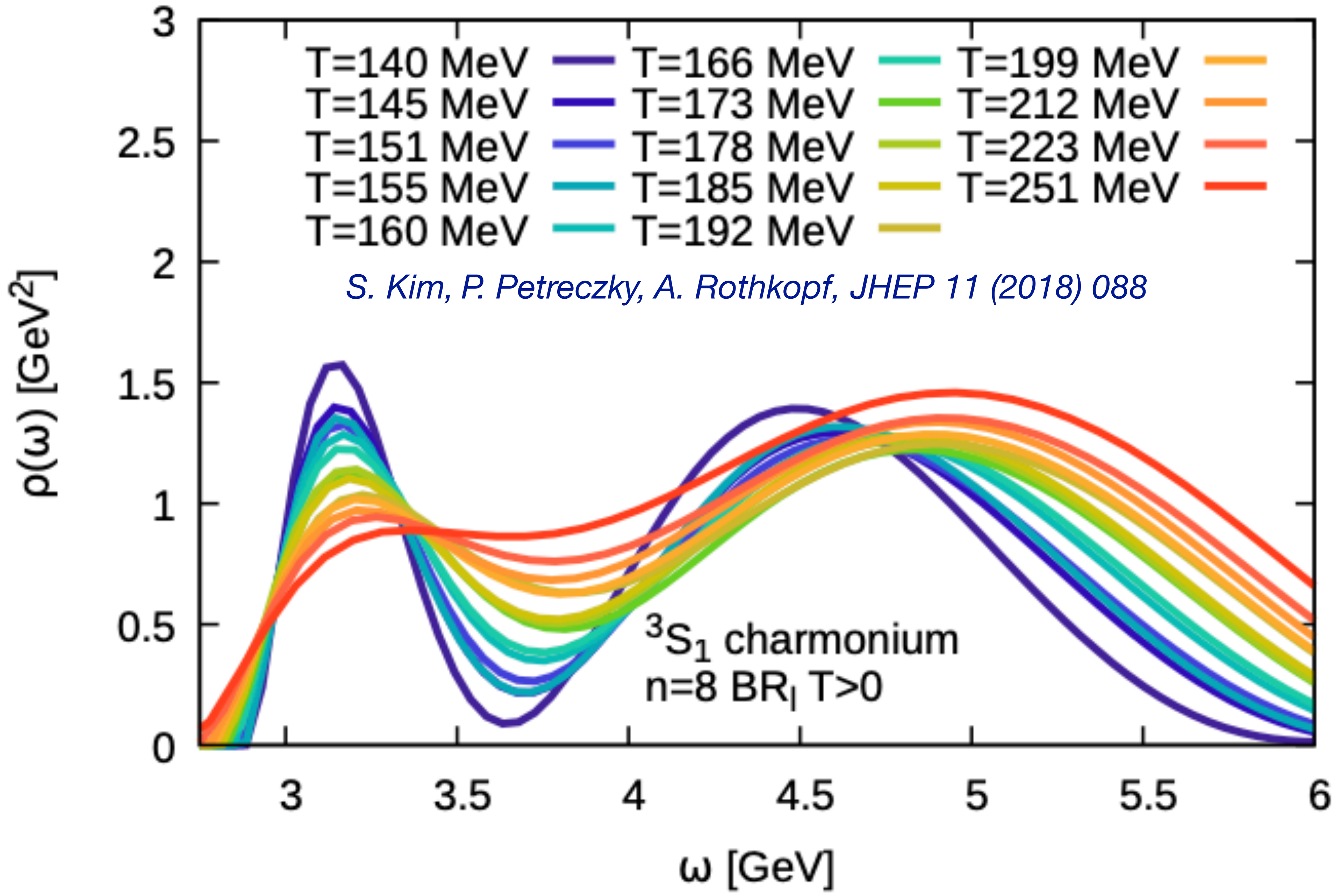
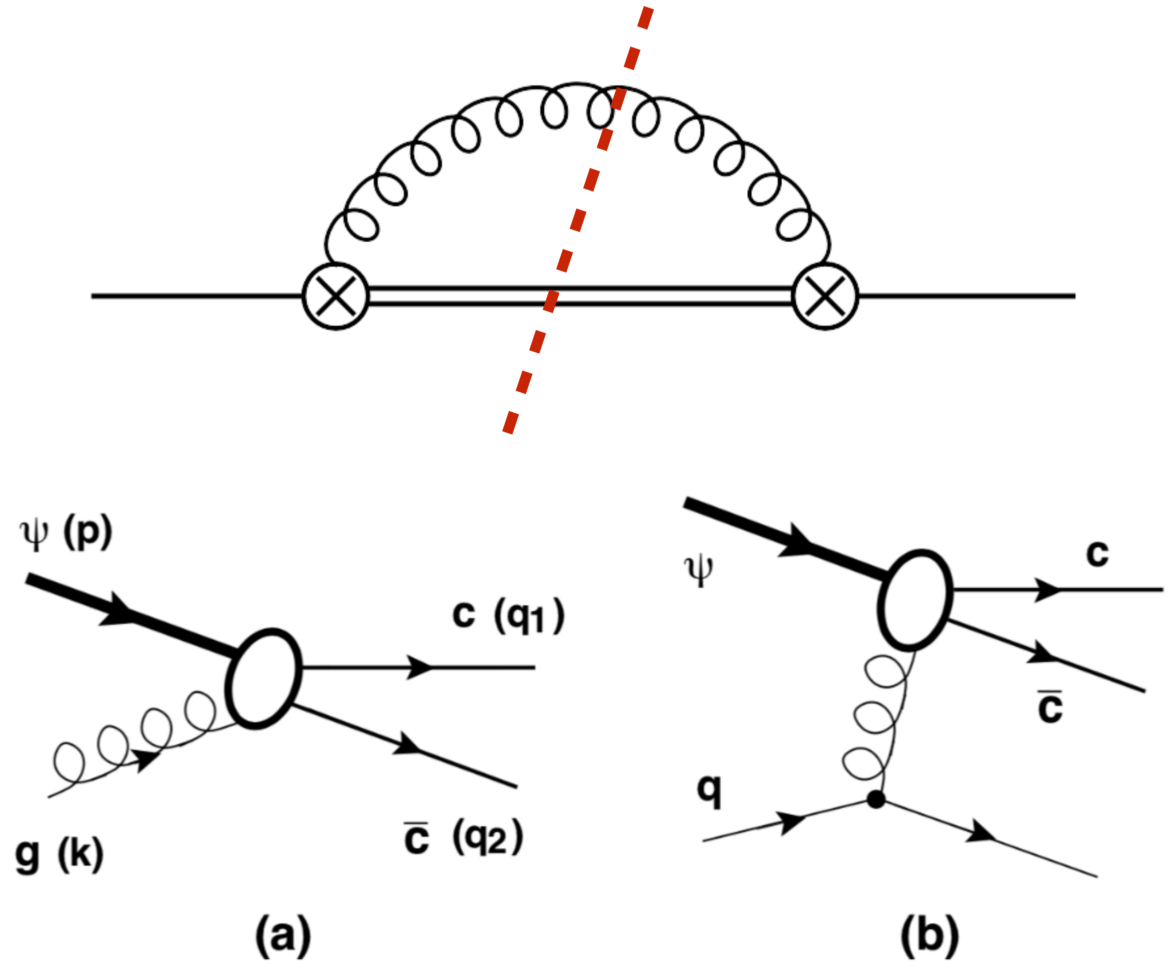


In the perturbative point of view:

Mass shift -> static color screening

Singlet-octet thermal break up -> gluon-dissociation

Landau damping-> inelastic scattering (quasifree limit)



peak position shifts and becomes broader as temperature increases.

N. Brambilla, M. Escobedo, J. Ghiglieri, M. Laine, O. Philipsen, P. Romatschke, M. Tassler, P. Petreczky, et al, JHEP 03, 054 (2007). PRD 78, 014017 (2008). JHEP 09, 038 (2010). JHEP 1112 (2011) 116…

Heavy Quark Potential at finite temperature

In-medium properties can be absorbed in a temperature-dependent heavy quark potential.

If the heavy quarks interact with the medium for a very long time, the potential is equivalent to the free energy.

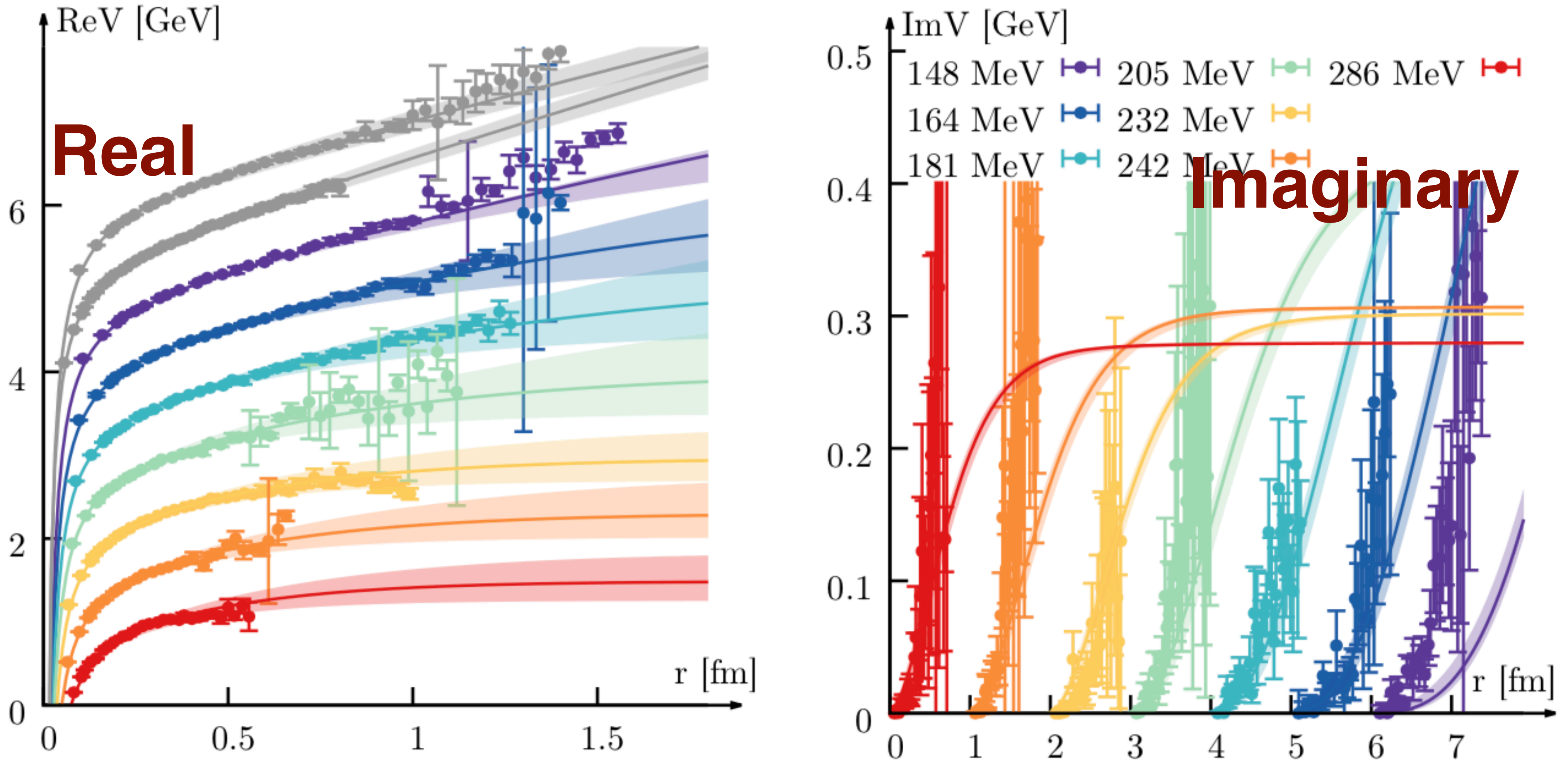
How the heavy potential is modified at scales comparable to the internal time scale of quarkonium?

✤ **In the weak-coupling regime (High temperature–> HTL,…)** *M. Laine, O. Philipsen, P. Romatschke, M. Tassler, JHEP 03 (2007) 054*

$$V(r,T) = -\frac{g^2 C_F}{4\pi}\left[m_D + \frac{\exp(-m_D r)}{r}\right] - \underbrace{\frac{i g^2 T C_F}{4\pi}\phi(m_D r)}_{\text{Imaginary}} \qquad \phi(x) = 2\int_0^\infty \frac{dz\, z}{(z^2+1)^2}\left[1 - \frac{\sin(zx)}{zx}\right]$$

Real *Imaginary*

✤ **In the strong-coupling regime (Lattice QCD,…)**



D. Lafferty, A. Rothkopf, Phys.Rev.D 101 (2020) 5, 056010 .

Obvious screening for the real part potential, the imaginary part larger than HTL results.

Heavy Quark Potential at finite temperature

Reconstructing spectral functions through Euclidean correlation functions is an ill-posed inverse problem. Big difference caused by the extraction strategy !

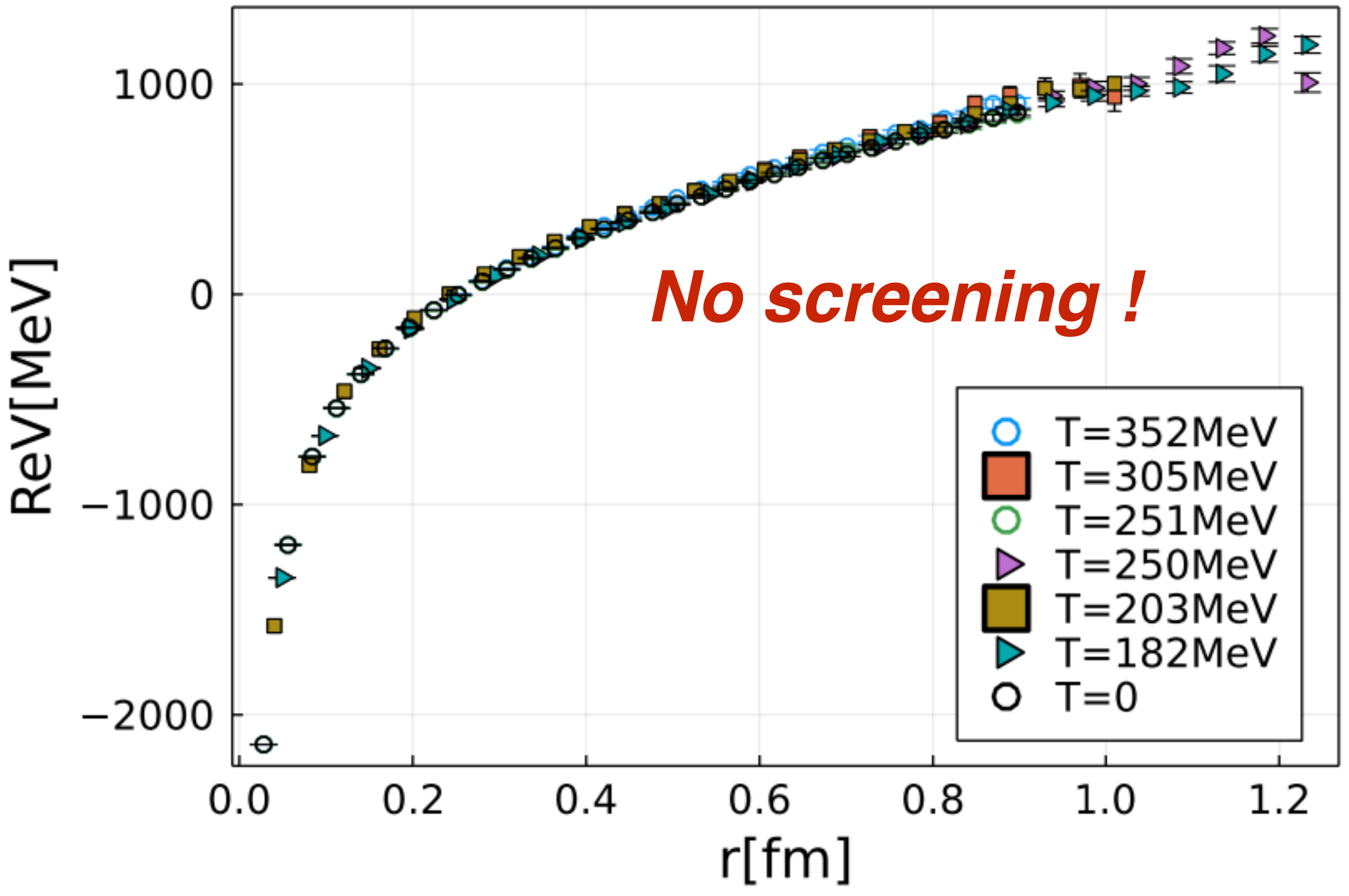
See for example: S. Shi, L. Wang and K.Zhou, Comput.Phys.Commun. 282 (2023) 108547.

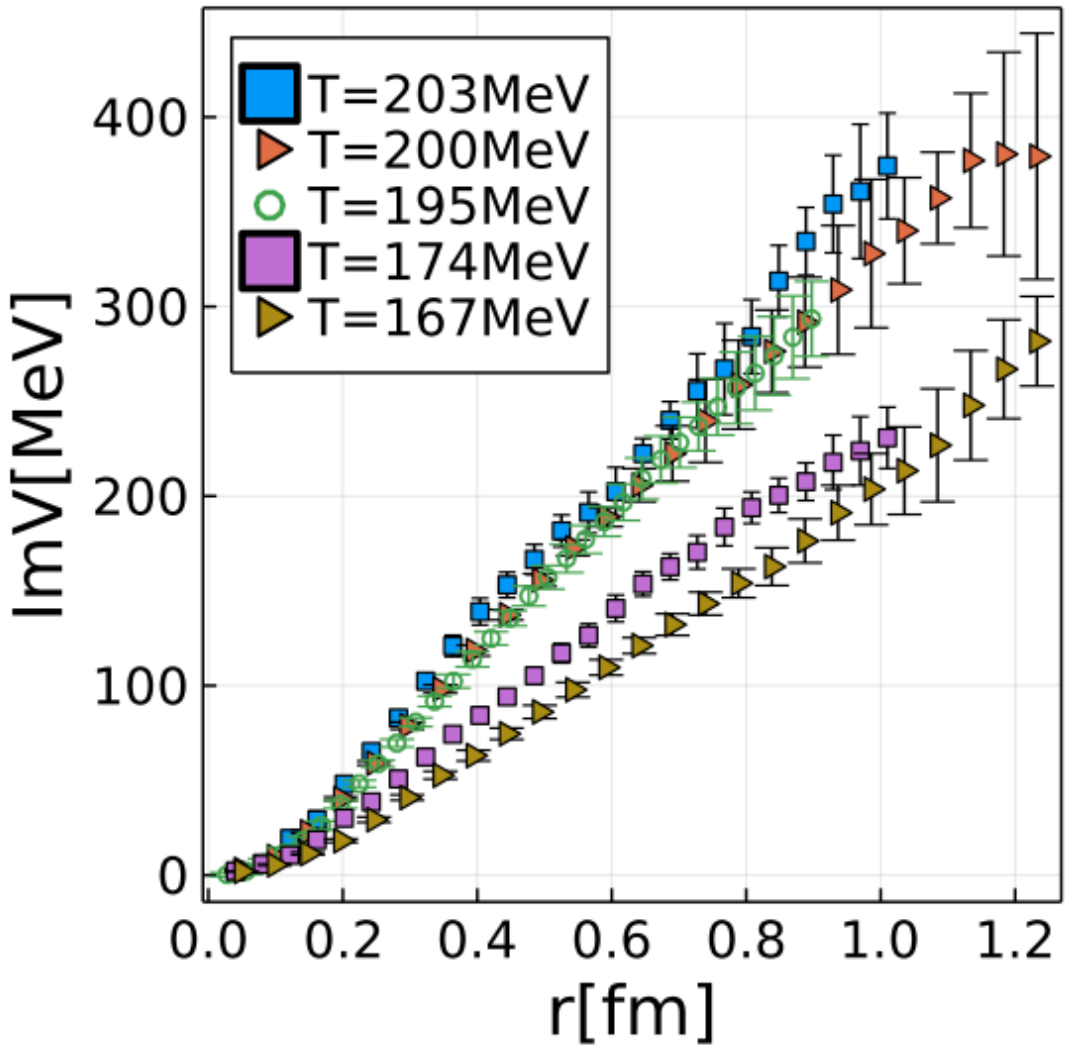
D. Bala et al, Phys.Rev.D 105, 054513 (2022).

Extract the spectral functions from correlators with four different methods:

1. Gaussian fit;
2. HTL inspired fit;
3. Pade fit;
4. Bayesian reconstruction (BR) method.

A physically appealing parametrization of spectrum -> Lorentzian form:





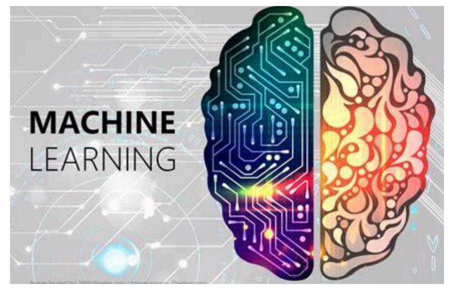
A.Bazavov, D. Hoying, O. Kaczmarek, R.N. Larsen, S. Mukherjee, P. Petreczky, A. Rothkopf, J.H. Weber , Phys.Rev.D 109, 074504 (2024)

Lattice QCD with dynamical fermions indicates no screening in static quark-antiquark potential !

Heavy Quark Potential at finite temperature

- **Extraction of the HQ Potential from bottomonium mass and width (Lattice NRQCD)**

Mass and Width



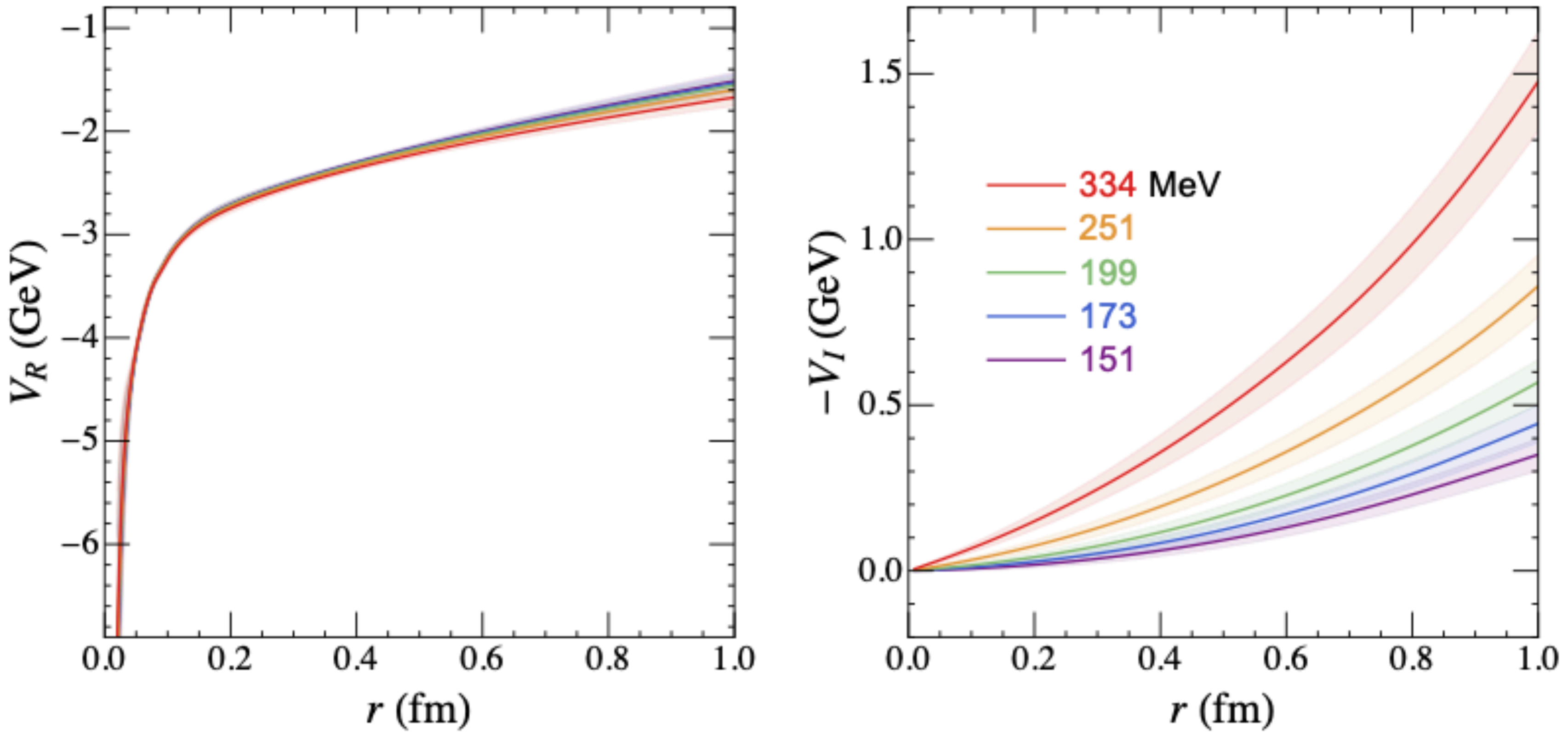
$M = 2m_b + Re[E]$
$\Gamma = -Im[E]$

Inverse problem ?
Deep Neural Networks

Heavy quark potential

$V(r,T) = V_R(r,T) + iV_I(r,T)$

$$-\frac{\nabla^2}{m_b}\psi_n + [V_R(T,r) + i\,V_I(T,r)]\,\psi_n = E_n\psi_n$$



*S. Shi, K. Zhou, **JZ**, S. Mukherjee, and P. Zhuang. PRD 105 (2022) 1, 1.*

Large imaginary part and very small screening effect !

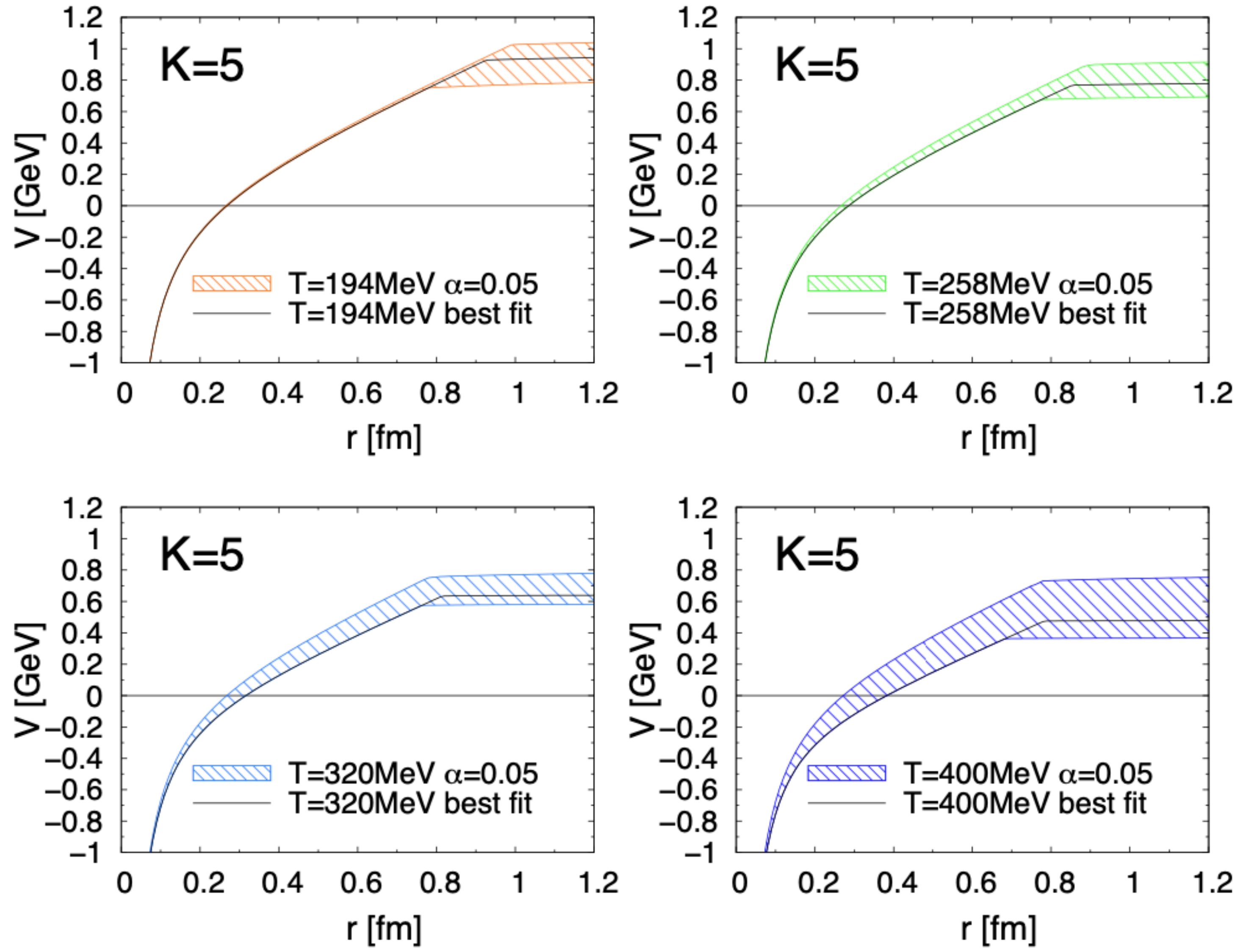
- **Extraction of the HQ Potential from Bottomonium Observables (R_{AA})**

$$V_{Q\bar{Q}}(r) = \begin{cases} -\frac{4}{3}\alpha_s\, e^{-m_D r}/r + \sigma r & , r < R_{SB} \\ -\frac{4}{3}\alpha_s\, e^{-m_D r}/r + \sigma R_{SB} & , r > R_{SB}\,. \end{cases}$$

$$m_D = aT_o\tilde{T},$$
$$m_S = m_S^{vac} + T_o\left[c\tilde{T} - (c-b)\left(\sqrt{\tilde{T}^2 + d^2} - d\right)\right],$$

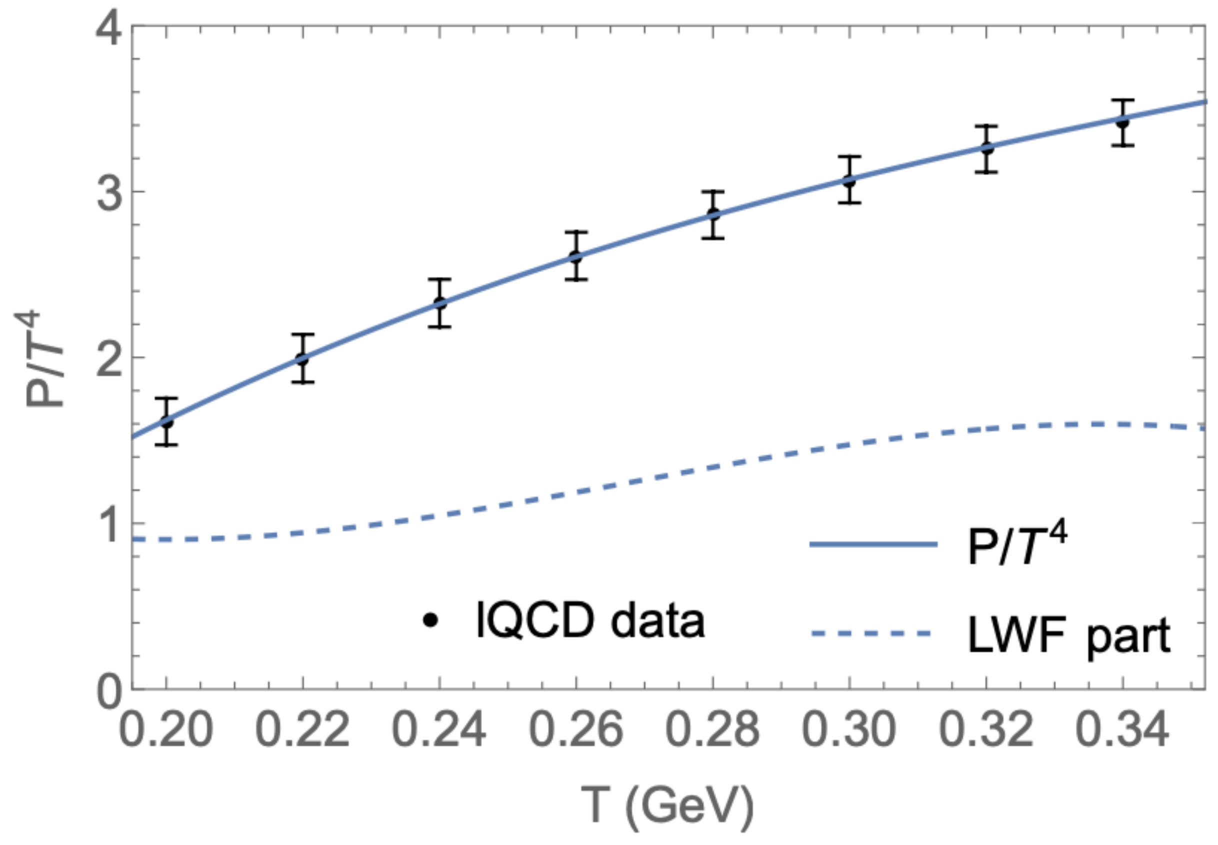
X. Du, S. Liu, R. Rapp. Phys.Lett.B 796 (2019) 20-25.

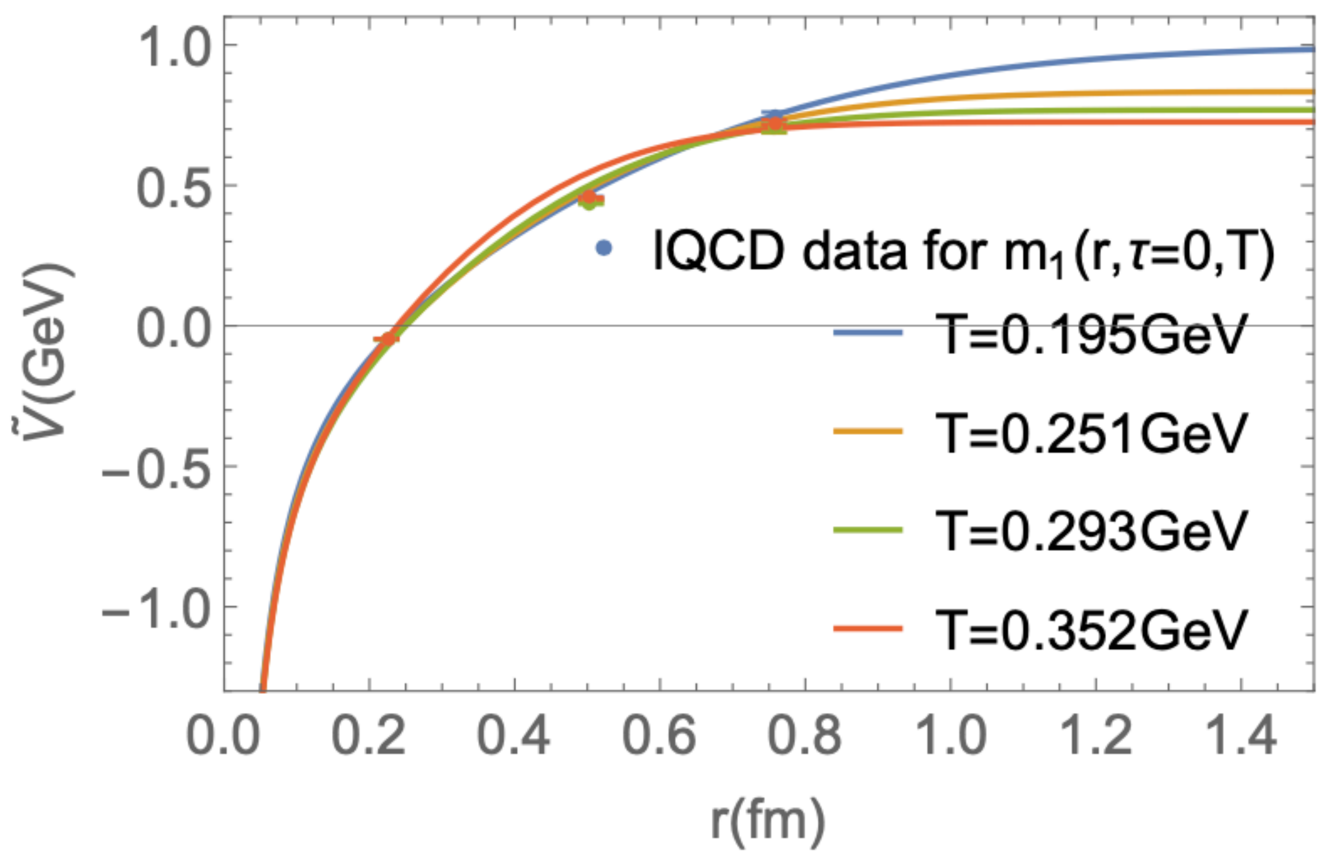
χ-square fit the exp. Data gives a rather strongly coupled (less color screening) potential !



Heavy Quark Potential at finite temperature

- **Extraction of the potential by fitting the Wilson line correlators and EOS in T-matrix approach**





Z. Tang, S. Mukherjee, P. Petreczky, R. Rapp. Eur.Phys.J.A 60 (2024) 4, 92.

A little screening at small and intermediate r!

- **HQ potential with HTL resummed perturbation method within the Gribov-Zwanziger approach**

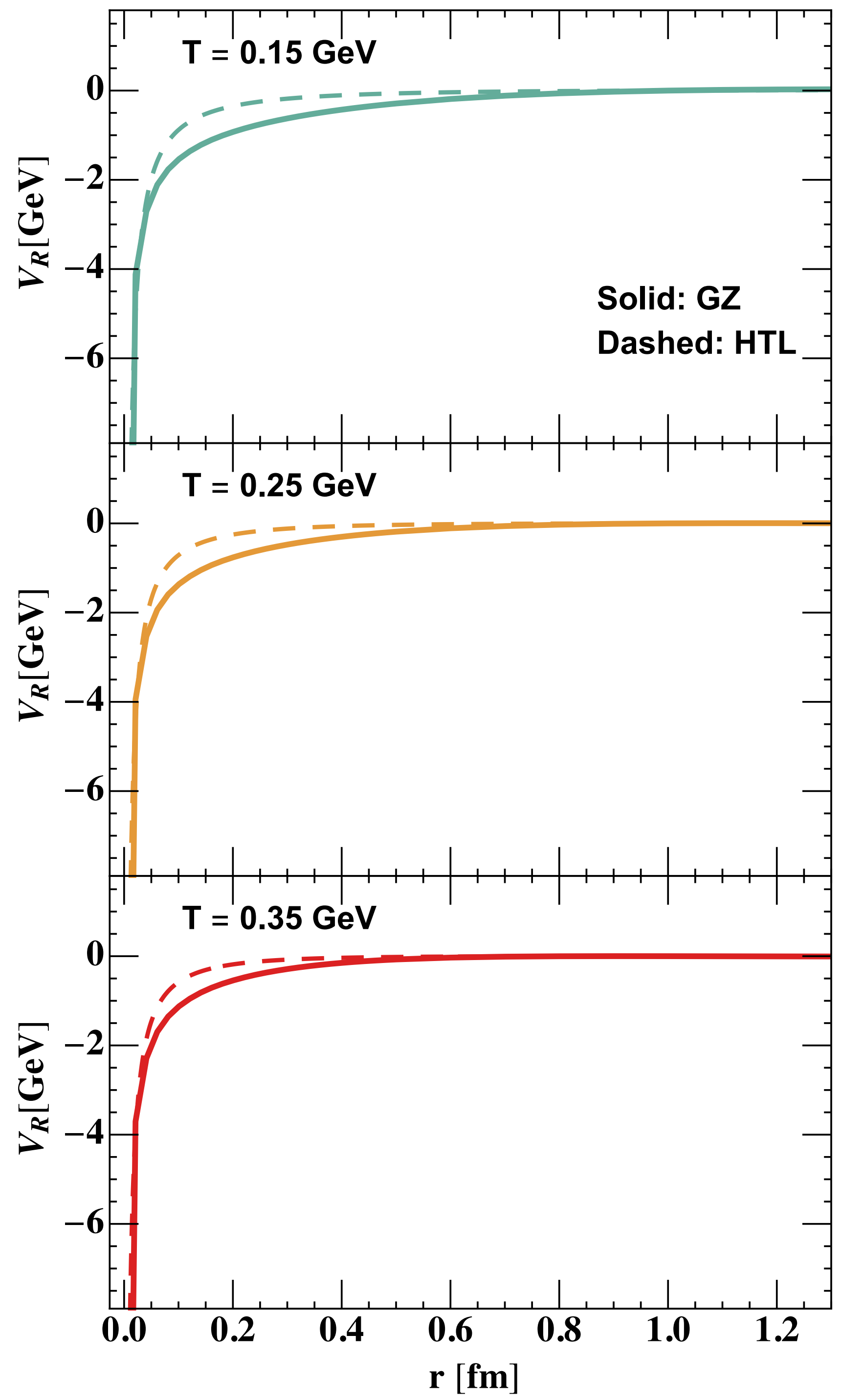
Gauge fixing in the quantization is not complete, there exist still gauge copies.
GZ approach for confinement of gluons, EOS of strongly coupled QCD medium.
(infrared improved gluon propagator-chromomagnetic scale- nonperturbative)

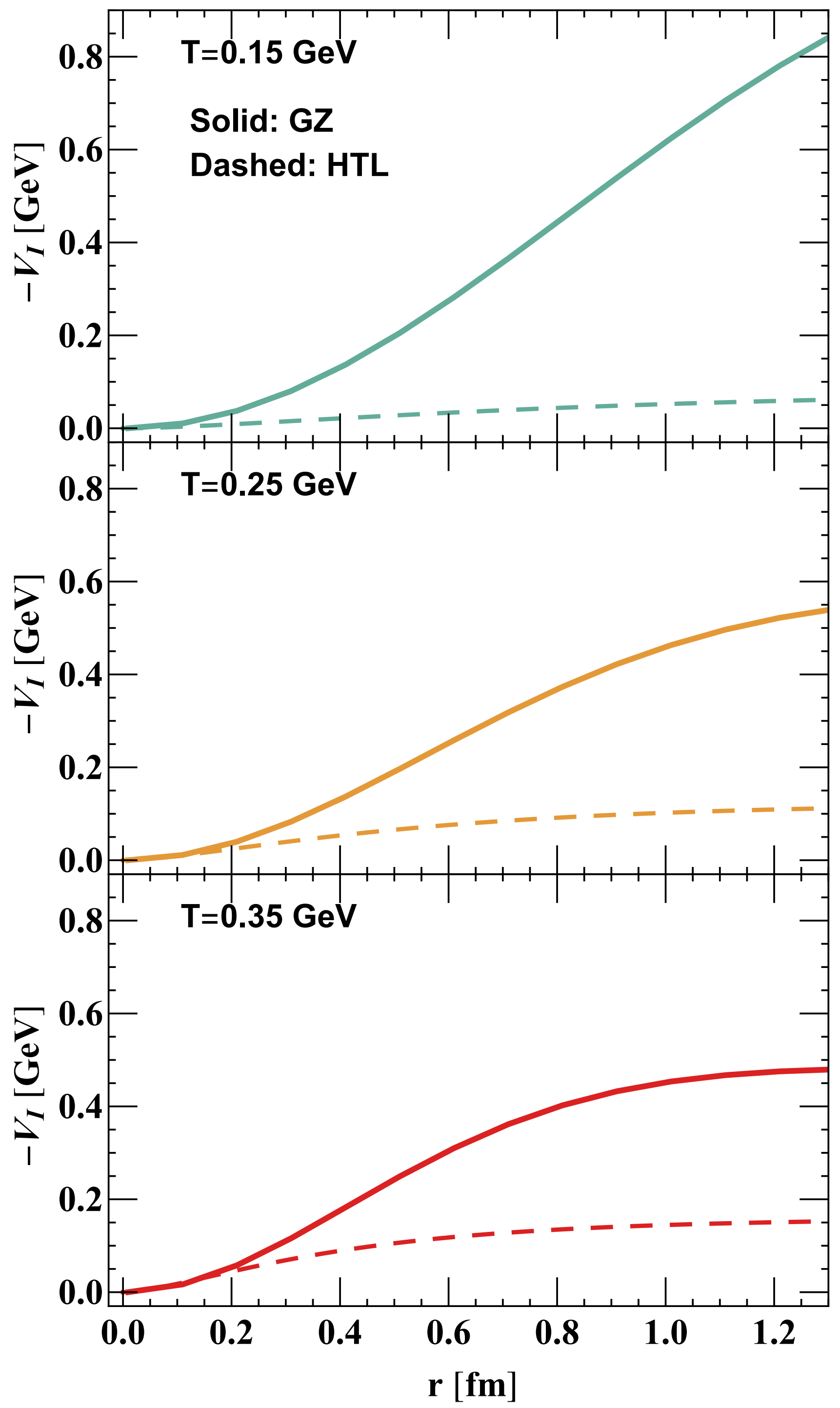
Gribov gluon propagator:

$$D^{ab}_{\mu\nu}(p) = \delta^{ab}\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\frac{p^2}{p^4 + m_G^4} \longrightarrow \text{Gribov mass}$$

Real part shows less color screening at small r and color confinement at large r; larger imaginary part than HTL!

*W. Wu, G. Huang, **JZ**, P. ZHuang. PRD 107 (2023) 11, 114033*
M.Debnath, R.Ghosh and N.Haque, Eur.Phys.J.C 84 (2024) 3, 313





Heavy Quark Potential at finite temperature

New heavy quark potential: No /a little color screening for the real part and a large imaginary part !

A different picture of quarkonium melting in the QGP -> dynamic dissociation plays an dominant role

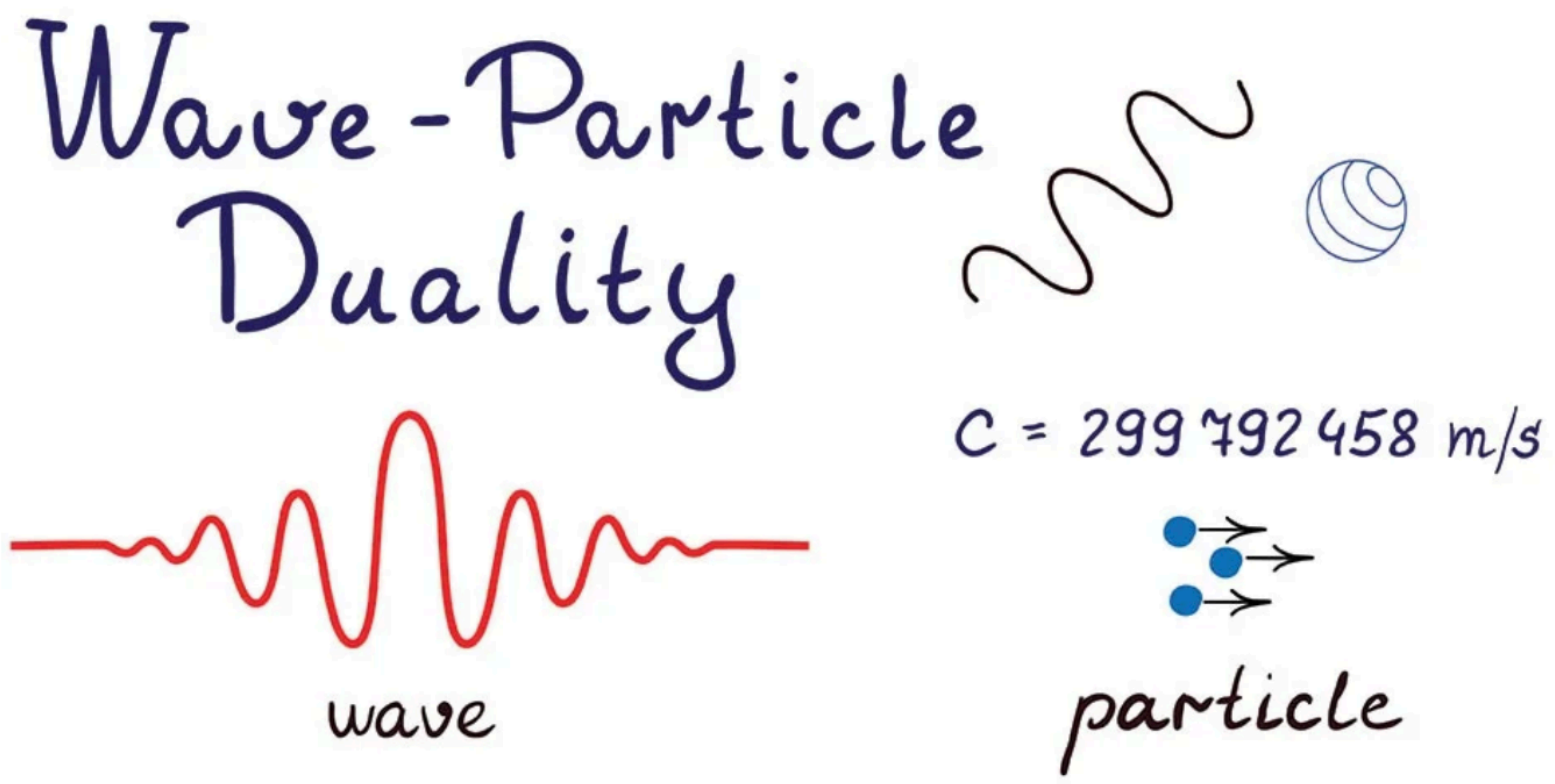
How can these two potentials
(small color screening+large imaginary part; large color screening+small imaginary part)
be distinguished in the experiment? Which observable?

More phenomenological studies with quarkonium real-time evolution in the QGP are needed !

Quarkonium real-time evolution in hot QCD medium

Quarkonium real-time evolution in heavy-ion collisions

Is quarkonium a wave or a particle in heavy ion collisions ?



Quarkonium real-time evolution in heavy-ion collisions

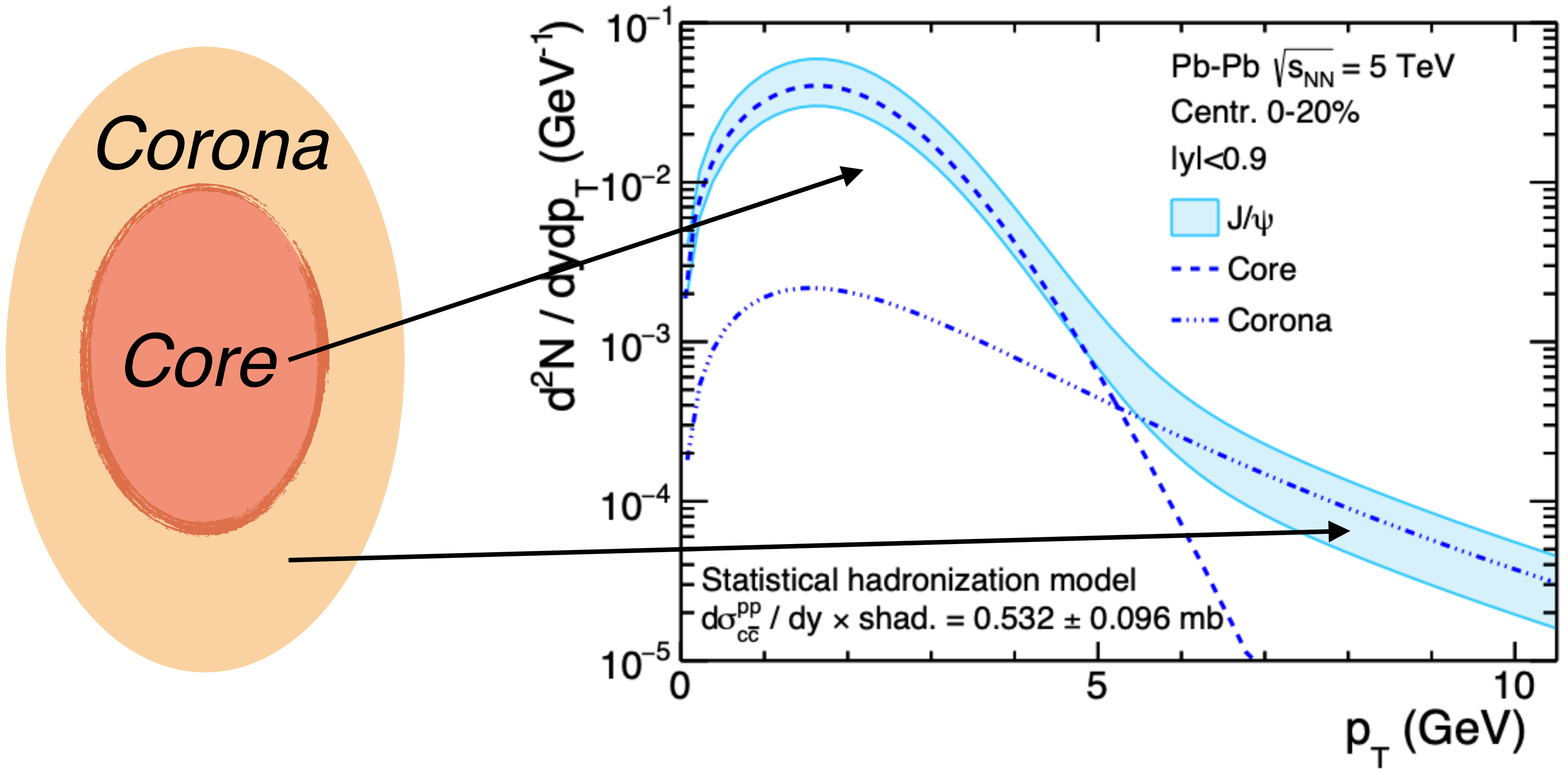
Assume the quarkonium is a **classical particle**!

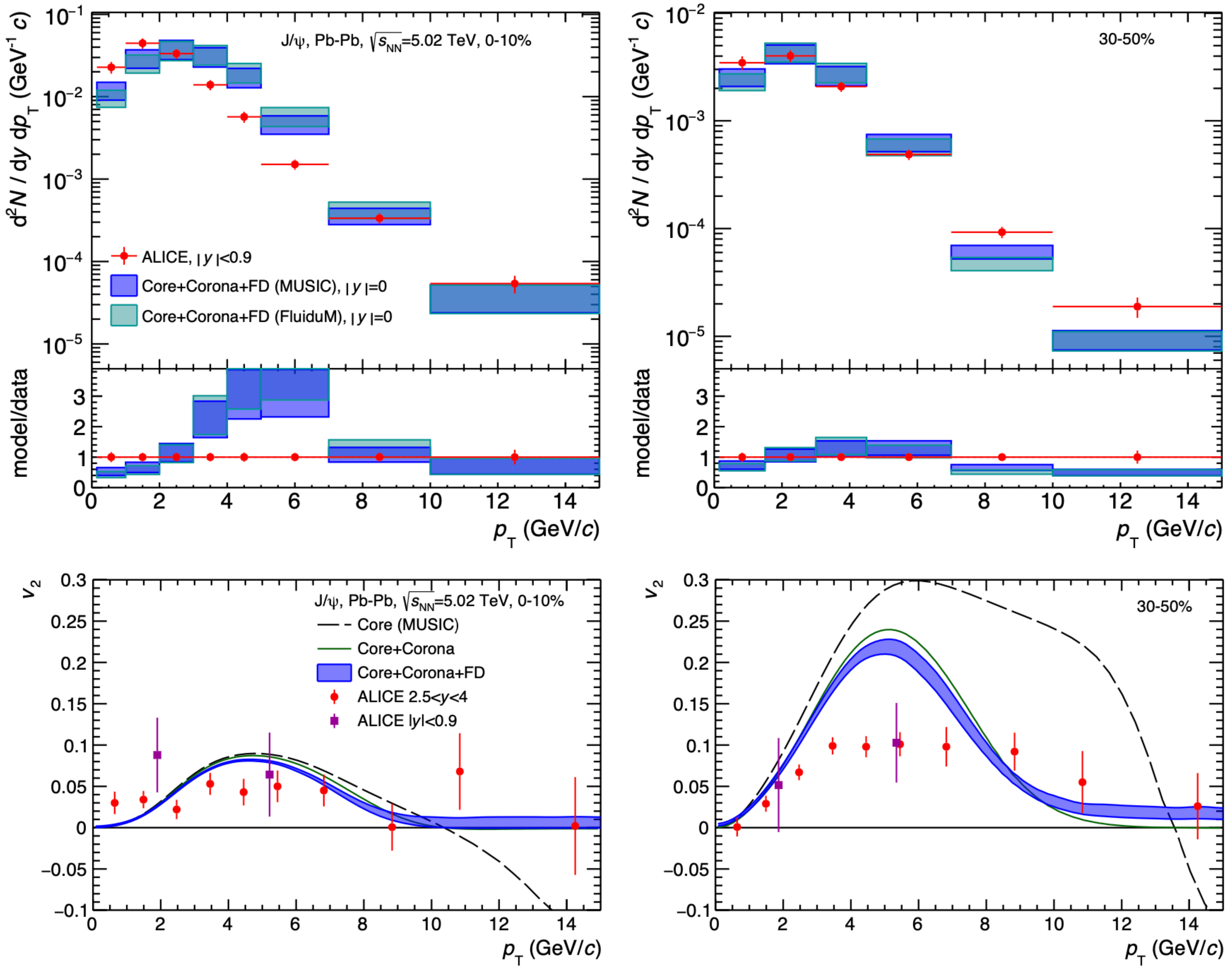
Charmonium: fully dissociated in the QGP and exclusively generated at the QCD phase boundary.

$$N_{c\bar{c}}^{dir} = \frac{1}{2} g_c N_{oc}^{th} \frac{I_1(g_c N_{oc}^{th})}{I_0(g_c N_{oc}^{th})} + \boxed{g_c^2 N_{c\bar{c}}^{th}.}$$

g_c : fugacity parameter

Recent: transverse momentum spectra are studied by a core-corona approach; core: Hydro+blast-wave/Cooper-Frye; corona: fit the spectra in pp





A. Andronic, P. Braun-Munzinger, H. Brunssen, J. Crkovska, J.Stachel, V. Vislavicius, and M. Volkl, arXiv:2308.14821; Phys.Lett.B 797 (2019) 134836; JHEP 07 (2021) 035

p_T distribution, anisotropic flow coefficients v_2 and v_3 for charmonia are investigated.

Quarkonium real-time evolution in heavy-ion collisions

Assume the quarkonium is a **classical particle**!

Charmonium are not fully dissociated. Dissociation and regeneration happen gradually in QGP.

➡Transport description (Boltzmann equation)

$$p^\mu \partial_\mu f_\psi = -\alpha E f_\psi + \beta E$$

$$\alpha = \frac{1}{2E_T}\int \frac{d^3\mathbf{p}_g}{(2\pi)^3 2E_g} W_{g\psi}^{c\bar{c}}(s) f_g(p_g, x)$$ Gluon-dissociation

$$\beta = \frac{1}{2E_T}\int \frac{d^3\mathbf{p}_g}{(2\pi)^3 2E_g}\frac{d^3\mathbf{p}_c}{(2\pi)^3 2E_c}\frac{d^3\mathbf{p}_{\bar{c}}}{(2\pi)^3 2E_{\bar{c}}} W_{c\bar{c}}^{g\psi}(s) f_c(p_c, x) f_{\bar{c}}(p_{\bar{c}}, x)(2\pi)^4 \delta^{(4)}(p + p_g - p_c - p_{\bar{c}})$$ Regeneration

Dissociation and regeneration are related to each other via the detailed balance.

➡Transport description (Rate equation)

$$\frac{dN_\psi(\tau)}{d\tau} = -\Gamma_\psi[N_\psi(\tau) - N_\psi^{\mathrm{eq}}(\tau)]$$

Dissociation rate

Include both gluon-dissociation and NLO (quasifree) process

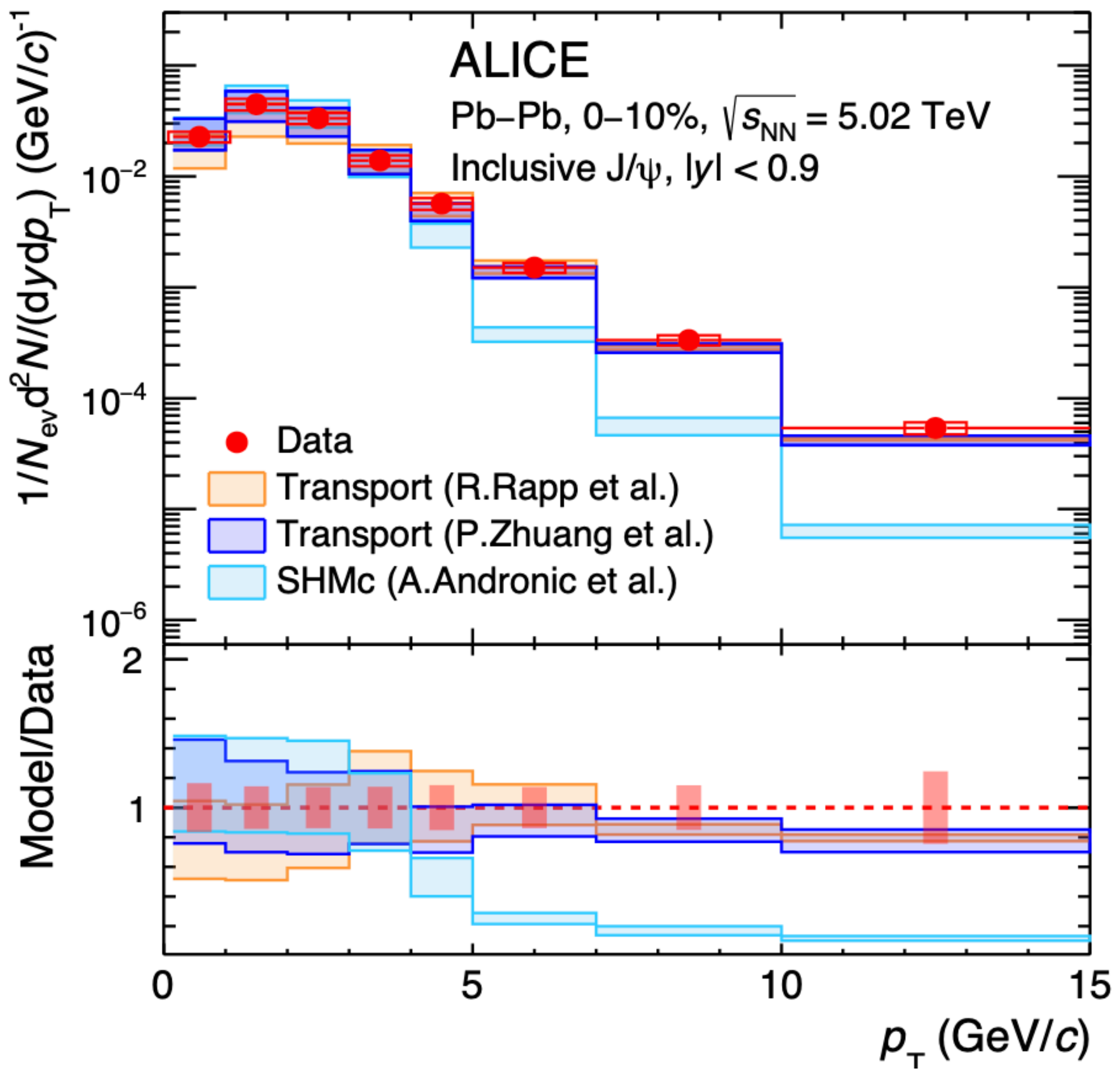
equilibrium limit of each state (Satisfied obviously.)

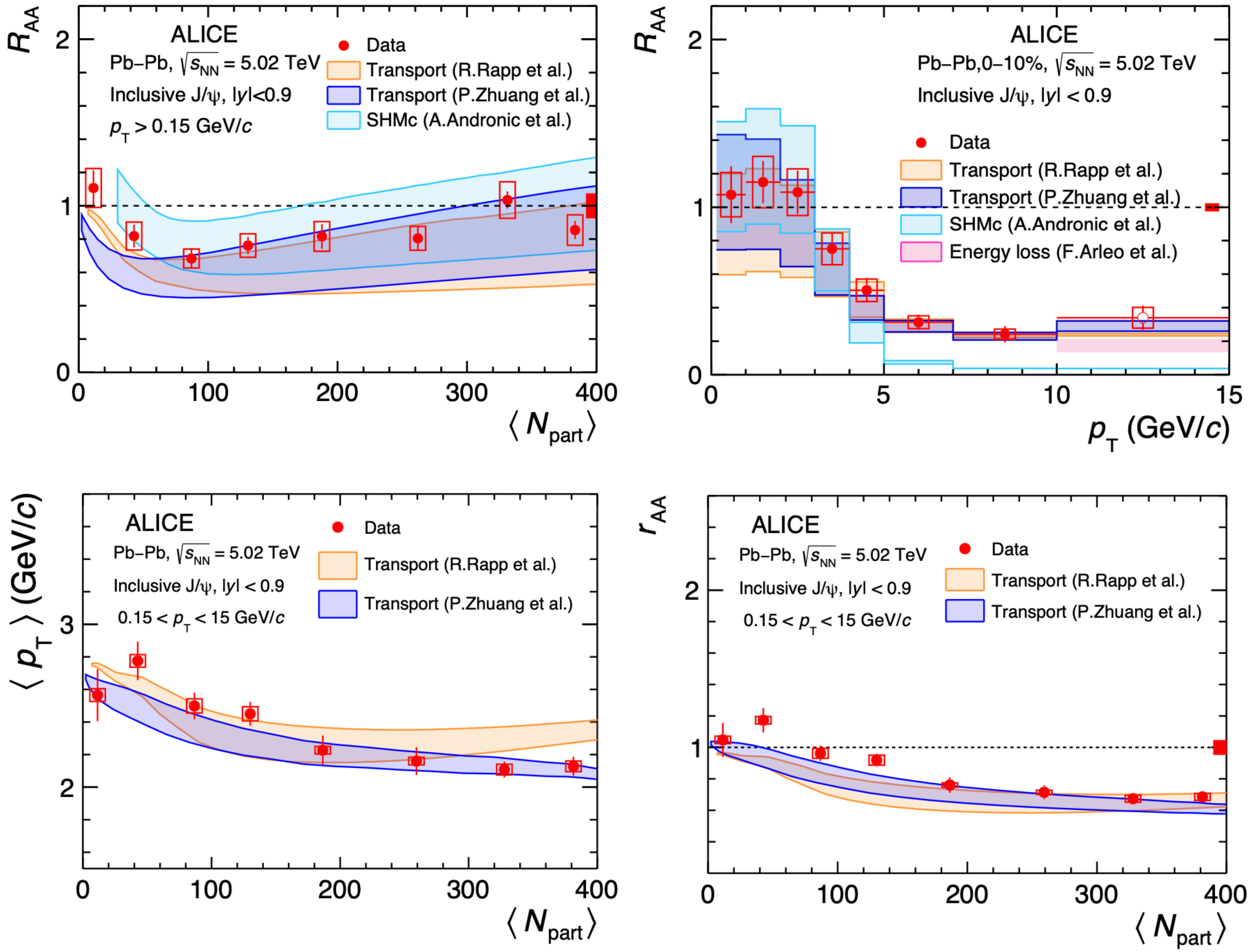
$$N_\psi^{\mathrm{eq}} = g_c^2 N^{\mathrm{eq}}$$

Quarkonium real-time evolution in heavy-ion collisions

Assume the quarkonium is a **classical particle**!

Models (such as TAMU, Tsinghua, Comover) can explain the experimental observables, like charmonium, spectra, R_{AA}, v_2, r_{AA}, $\langle p_T \rangle$ …**compared to the recent ALICE results.**





ALICE Collaboration, Phys.Lett.B 849 (2024) 138451

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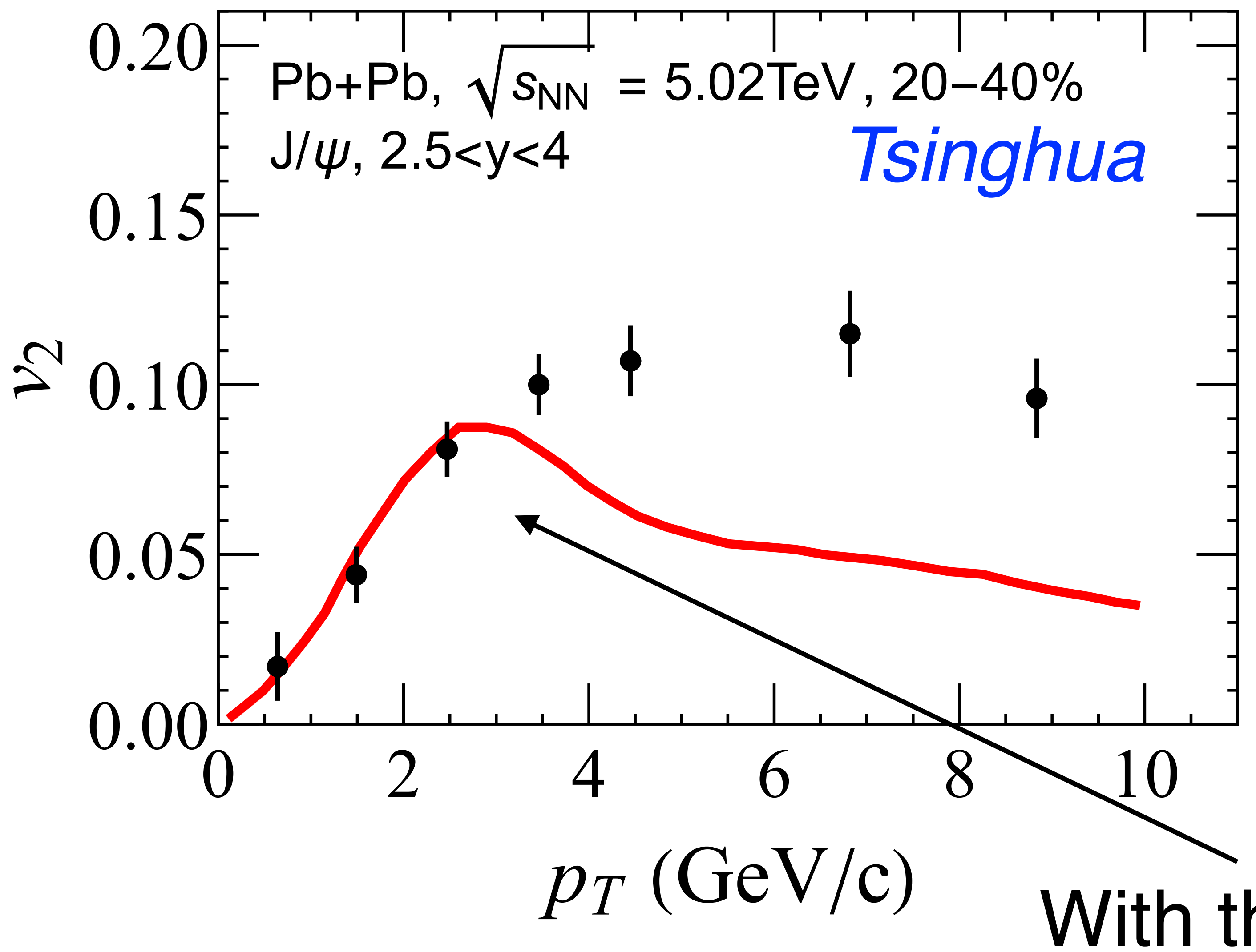
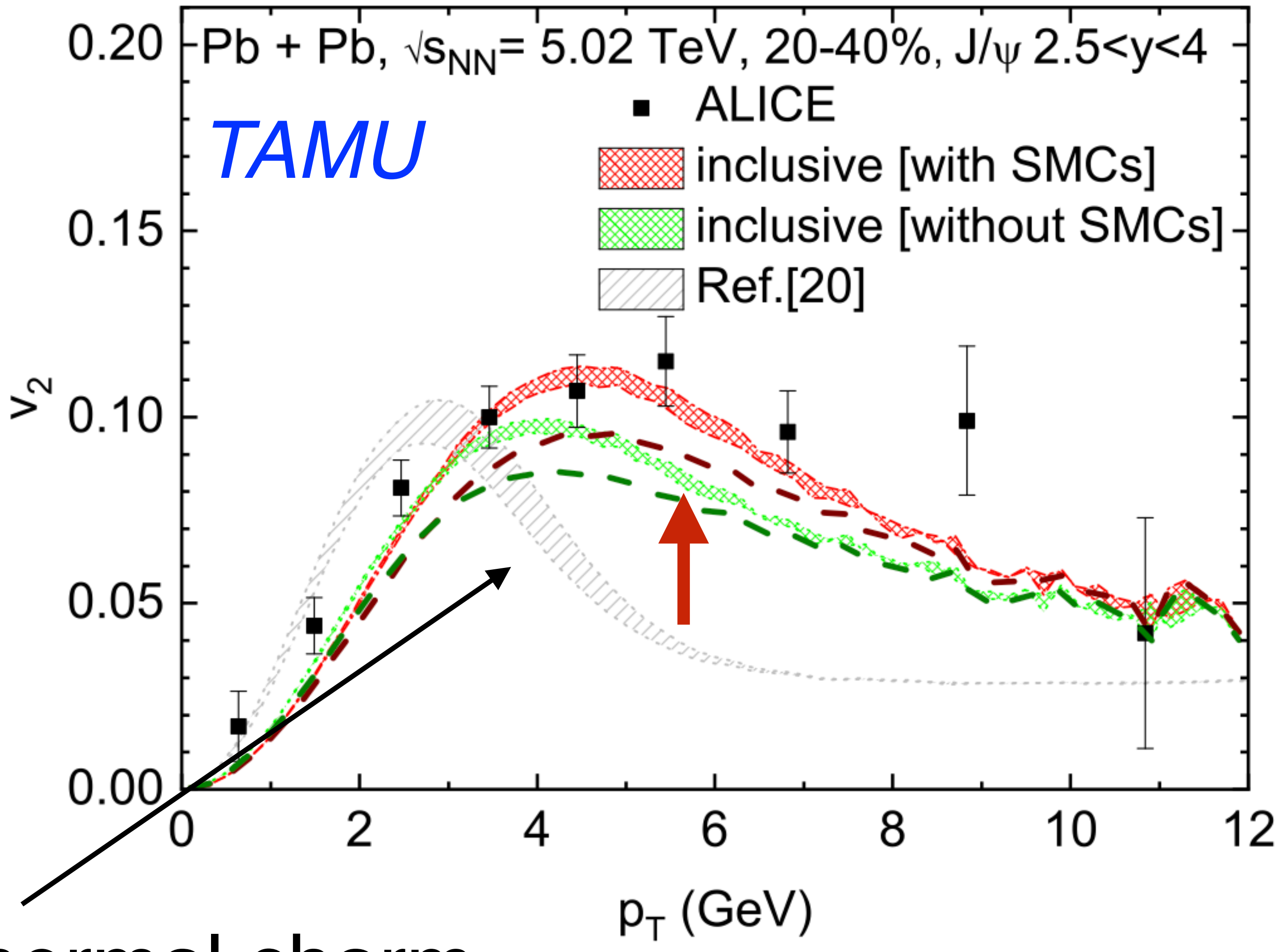
1. J/ψ v_2 with non-thermal charm distribution.





With thermal charm

Victor Valencia, poster section.
($v_2\{2\}$ and $v_2\{4\}$ of J/ψ)

space-momentum correlation (SMC) increases a bit v_2

2. Directed flow v_1 and Triangular flow v_3 of J/ψ.
3. Quarkonium polarization.
4. Probe the initial nuclear deformation.
5. Quarkonium with EM fields.
6. B_c, X(3872)

…

*B. Chen, M. Hu, H. Zhang, and **JZ**, PLB802 (2020) 135271; **JZ**, B. Chen, and P. Zhuang, PRC 105 (2022) 3, 034902*

D.Yang and X.Yao, arXiv:2405.20280; Y. Zhao, X. Sheng, S. Li and D.Hou, arXiv:2403.07468; JZ and B. Chen, arXiv:2312.01799

***JZ** and S. Shi, Eur.Phys.J.C 83 (2023) 6, 511.*

*See review: S. Iwasaki, M. Oka and K. Suzuki, Eur.Phys.J.A 57 (2021) 7, 222; **JZ**, K. Zhou, S. Chen, P. Zhuang, PPNP. 114 (2020) 103801.*

Shile Chen, Wed. 11:20

*B. Wu, Z. Tang, M. He, R. Rapp, Phys. Rev. C **109**(1), 014906 (2024). B. Wu, X. Du, M. Sibila, R. Rapp, Eur. Phys. J. A **57**(4), 122 (2021). **JZ** and P. Zhuang, arXiv: 2209.13475.*
A.Esposito, E. Ferreiro, A.Pilloni, A. Polosa and C. Salgado, Eur.Phys.J.C 81 (2021) 7, 669. Y. Guo, X. Guo, J. Liao, E. Wang and H. Xing, arXiv:2302.03828.

Miguel Angel Escobedo, Tue. 11:40

Quarkonium real-time evolution in heavy-ion collisions

Are quantum effects important? What are they?

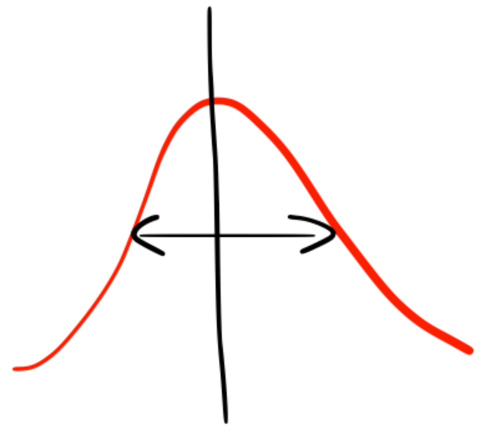
- Quantum coherence and decoherence

  Superposition state of various eigen states,…
  Usually absorbed into a phenomenological parameter "formation time" in the transport approach.

$$\Psi = \sum_i c_i \psi_{nl}$$

- Define and evolve a particle with a large width in the hot medium

$$\Gamma \gtrsim E$$



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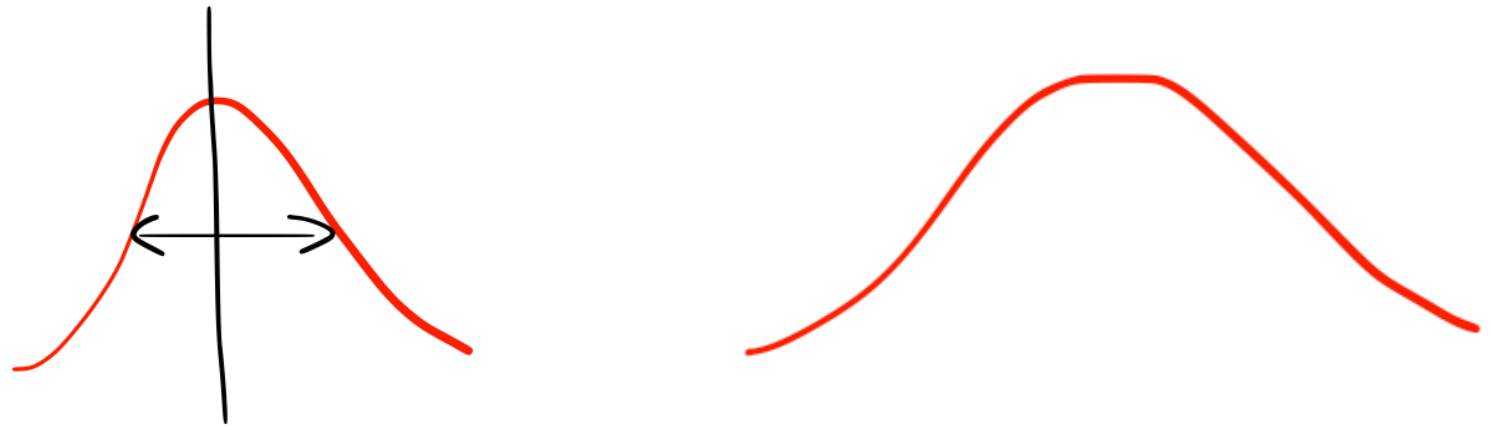


Assume the quarkonium is a **quantum wavefunction**!

Models (such as time-dependent Schrödinger equation + complex potential) have been used to describe the bottomonium evolution and production in heavy ion collisions.

no regeneration from uncrorrelated $b\bar{b}$ ($\leq$ 1 pair/event)

A. Islam and M. Strickland, JHEP 21, 235 (2020); Phys.Lett.B 811 (2020) 135949;
*L. Wen and B. Chen, Phys. Lett. B 839, 137774 (2023); G. Chen, B. Chen and **JZ**, arXiv:2402.11316;…*

Quarkonium real-time evolution in heavy-ion collisions

“pure” state (**wavefunction**) -> “mixed” state (**density operator**)

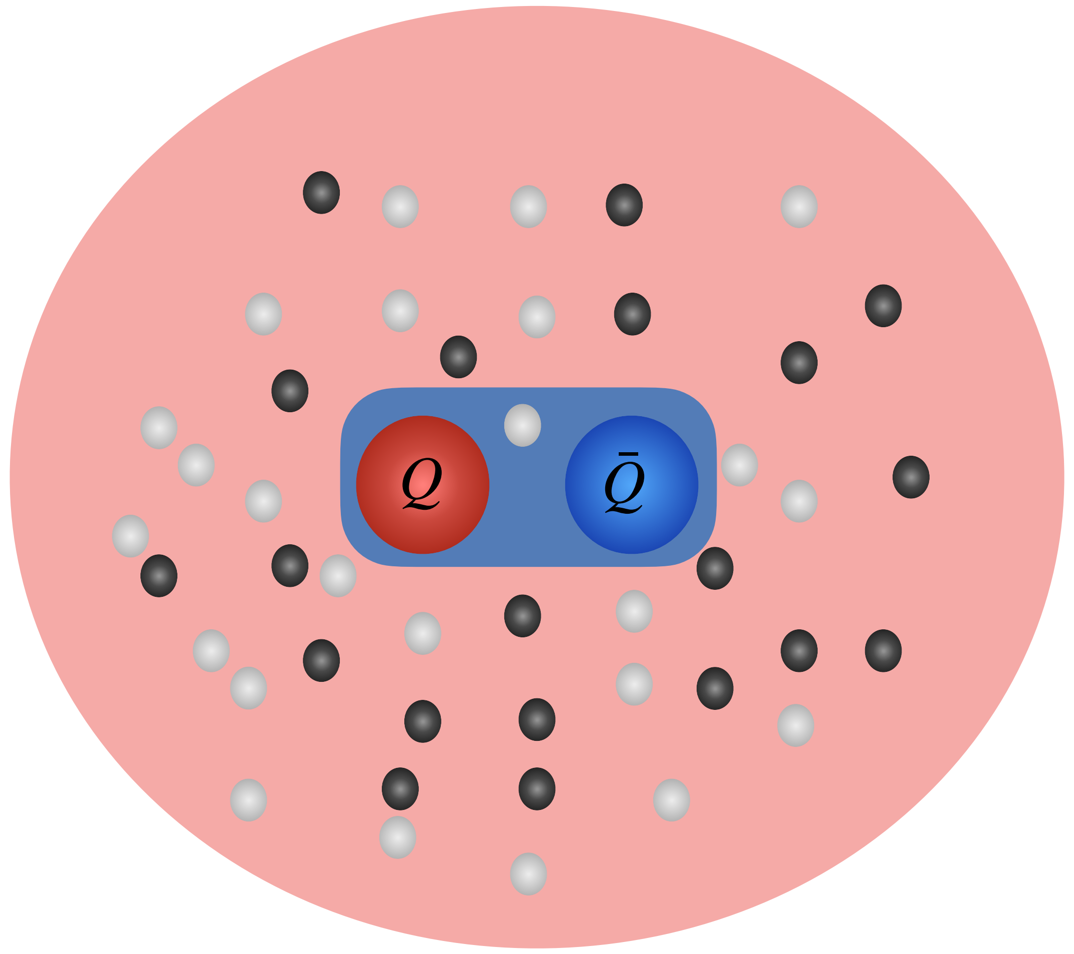
Open quantum system (OQS)

$$\hat{\rho}_{tot} = \sum_i p_i |\psi_i\rangle\langle\psi_i|$$

von Neumann equation: $$\frac{d\hat{\rho}_{tot}}{dt} = -i[\hat{H}_{tot}, \hat{\rho}_{tot}]$$

$$\hat{H}_{tot} = \hat{H}_s \otimes I_e + I_s \otimes \hat{H}_e + \hat{H}_{int},$$

Subsystem **Environment** **Interaction**



Trace over the environment degrees of freedom :

Quantum master equation

$$i\hbar\dot{\hat{\rho}}_s(t) = \mathrm{Tr}_e[\hat{H}_{tot}, \hat{\rho}_{tot}] = [\hat{H}_s, \hat{\rho}_s] + \mathrm{Tr}_e[I_s \otimes \hat{H}_e + \hat{H}_{int}, \hat{\rho}_{tot}]$$

➡ Separation of time-scales:

Environment relaxation time scale $\tau_e \sim \frac{1}{\pi T}.$

Intrinsic time scale of subsystem $\tau_s \sim \frac{1}{E_{bind}}.$

Subsystem relaxation time scale $\tau_r \sim \frac{1}{\eta} \approx \frac{M}{T^2}.$

Quarkonium real-time evolution in heavy-ion collisions

$\tau_e \sim 1/(\pi T)$, $\tau_s \sim 1/E_{bind}$, $\tau_r \sim M/T^2$

Non-Markovian

Markovian approximation: $\tau_e \ll \tau_r$ **, memory lose; good for HIC**

Quantum master equation –> Lindblad equation

T

$\sim m_Q$

$\tau_e \ll \tau_s$

$(E_{bind} \ll m_D)$

$\tau_s \ll \tau_r$

$(E_{bind} \gg m_D)$

Quantum Brownian Motion

A broader peak (e.g. excited states)

Short quantum decoherence time

Classical limit:

Fokker-Planck equation

Munich-Kent

Nantes

Osaka

…

Quantum optical Limit

Well defined quarkonium state

Long quantum decoherence time

Classical limit:

Boltzmann equation/ Rate equation

TAMU

Tsinghua

Comover

Duke-MIT

…

Recent review papers: *R. Sharma, Eur.Phys.J.ST 230 (2021) 3, 697-718* *Y. Akamatsu, Prog.Part.Nucl.Phys. 123 (2022) 103932.* *X. Yao, Int.J.Mod.Phys.A 36 (2021) 20, 2130010.*

Duke-MIT Approach

- ✦ **pNRQCD+OQS works in quantum optical limit** $M \gg Mv \gg Mv^2 \gtrsim T \gtrsim m_D$
- ✦ **A semi-classical (gradient) expansion and w/o quantum effect anymore**
- ✦ **Used for bottomonium.** X. Yao, T. Mehen,W. Ke, Y. Xu, S.Bass, B. Muller. Phys.Rev.D 99 (2019) 9, 096028; JHEP 01 (2021) 046.

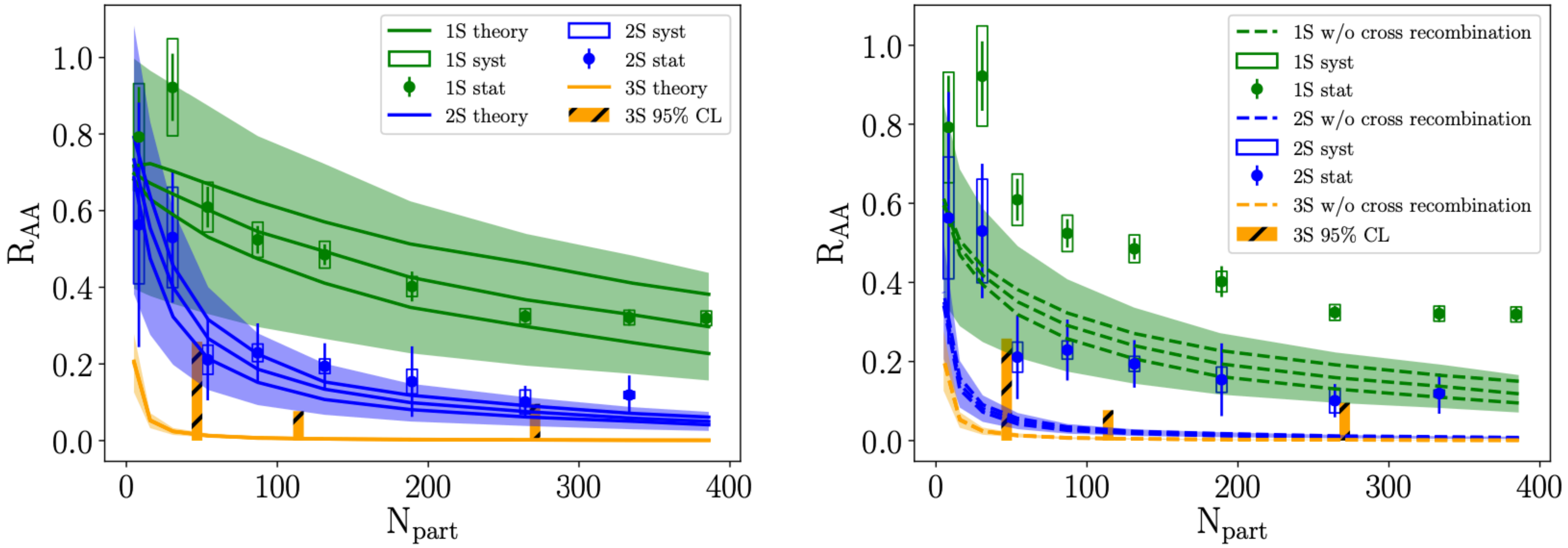
$$\rho_S(t) = \rho_S(0) + \sum_{a,b,c,d} \gamma_{ab,cd}(t)\Big(L_{ab}\rho_S(0)L^\dagger_{cd} - \frac{1}{2}\{L^\dagger_{cd}L_{ab}, \rho_S(0)\}\Big) - i\sum_{a,b}\sigma_{ab}(t)[L_{ab}, \rho_S(0)] + \mathcal{O}(H_I^3)\,.$$

Lindblad equation

$$f_{nl}(\boldsymbol{x}, \boldsymbol{k}, t) \equiv \int \frac{\mathrm{d}^3k'}{(2\pi)^3} e^{i\boldsymbol{k}'\cdot\boldsymbol{x}} \langle \boldsymbol{k} + \frac{\boldsymbol{k}'}{2}, nl, 1|\rho_S(t)|\boldsymbol{k} - \frac{\boldsymbol{k}'}{2}, nl, 1\rangle$$

$$\frac{\partial}{\partial t} f_{nl}(\boldsymbol{x}, \boldsymbol{k}, t) + \boldsymbol{v}\cdot\nabla_{\boldsymbol{x}} f_{nl}(\boldsymbol{x}, \boldsymbol{k}, t) = \mathcal{C}^{(+)}_{nl}(\boldsymbol{x}, \boldsymbol{k}, t) - \mathcal{C}^{(-)}_{nl}(\boldsymbol{x}, \boldsymbol{k}, t)$$

Similar to the TAMU and Tsinghua model



Importance of recombination from correlated $b\bar{b}$!

Gives a connection between the OQS and Boltzmann equation in the quantum optical limit!

Munich-Kent Approach

- **pNRQCD+OQS works in Quantum Brownian motion Regime** $M \gtrsim 1/a_0 \gg \pi T \sim m_D \gg E_{bind}$
- **Expansion of E_{bind}/T from LO to NLO; the quantum jumps are now implemented.**
- **Used for bottomonium.**

N.Brambilla, M.Escobedo, M.Strickland, A.Vairo, J.Weber, Phys.Rev.D 104 (2021) 9, 094049; JHEP 05 (2021) 136;JHEP 08 (2022) 303; Phys.Rev.D 108 (2023) 1, L011502.

$$\frac{d\rho(t)}{dt} = -i[H,\rho(t)] + \sum_n \left(C_n \rho(t) C_n^\dagger - \frac{1}{2}\left\{C_n^\dagger C_n, \rho(t)\right\}\right)$$

$$\rho(t) = \begin{pmatrix} \rho_s(t) & 0 \\ 0 & \rho_o(t) \end{pmatrix},$$

$$H = \begin{pmatrix} h_s & 0 \\ 0 & h_o \end{pmatrix} + \frac{r^2}{2}\gamma \begin{pmatrix} 1 & 0 \\ 0 & \frac{N_c^2-2}{2(N_c^2-1)} \end{pmatrix},$$

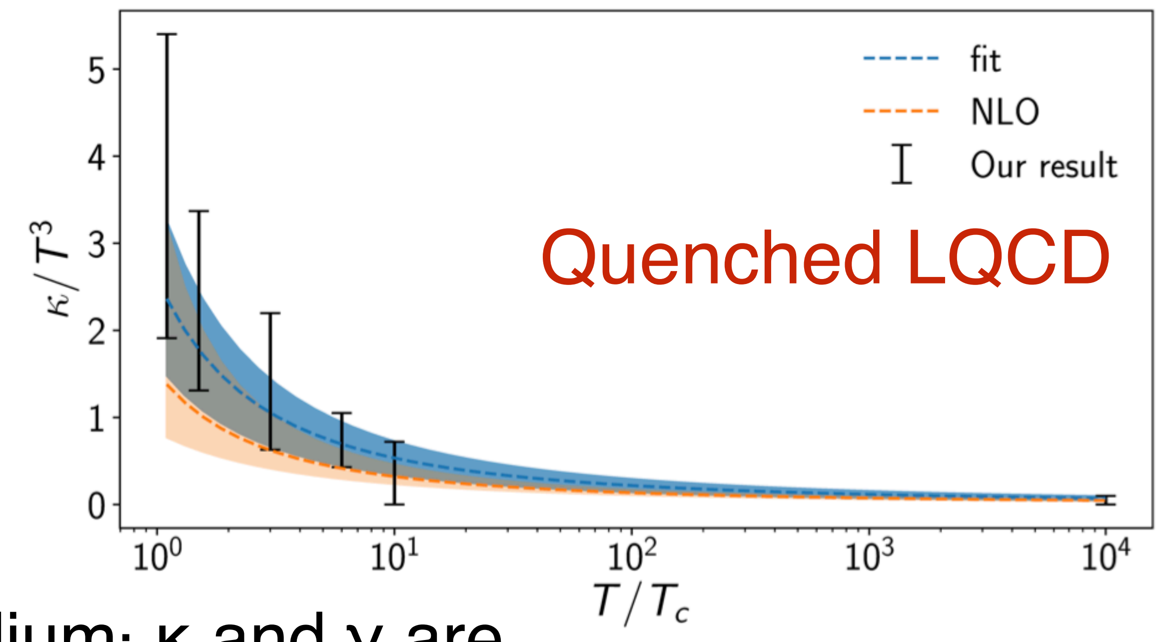
$$C_i^0 = \sqrt{\frac{\kappa}{N_c^2-1}}\, r^i \begin{pmatrix} 0 & 1 \\ \sqrt{N_c^2-1} & 0 \end{pmatrix},$$

$$C_i^1 = \sqrt{\frac{(N_c^2-4)\kappa}{2(N_c^2-1)}}\, r^i \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}.$$

Quantum jump

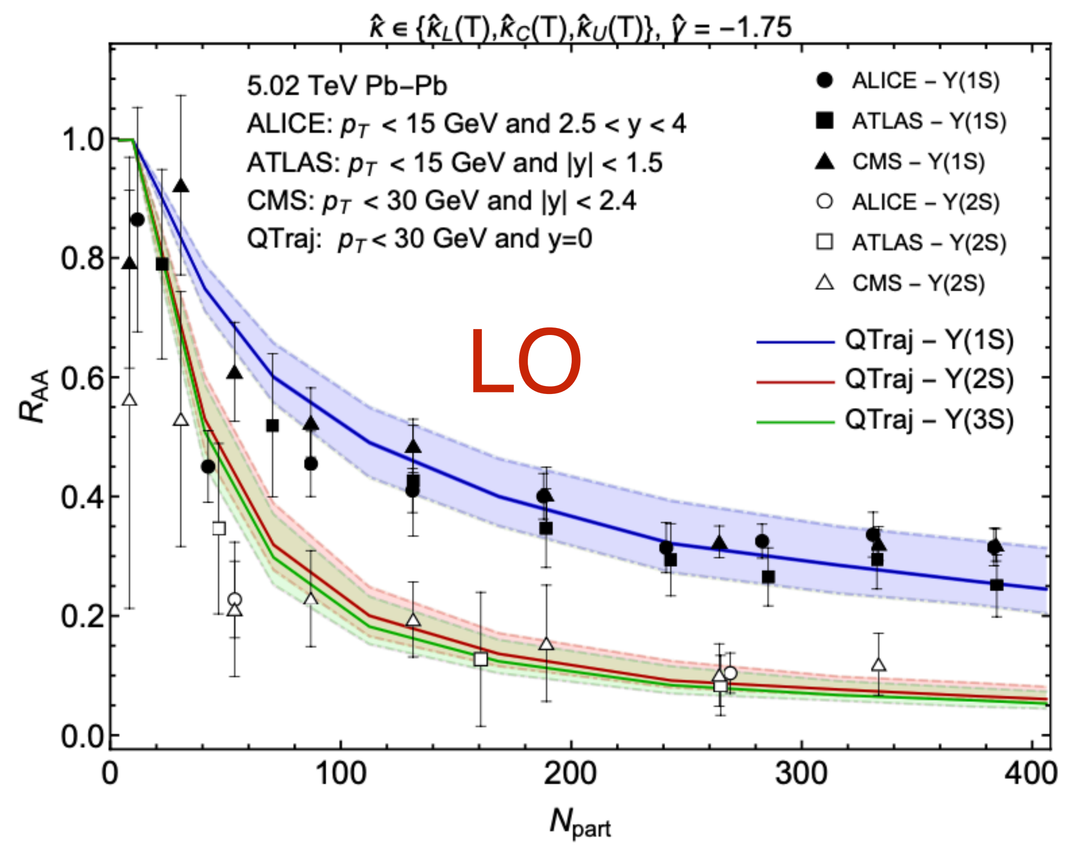
$$\kappa = \frac{g^2}{18}\int_0^\infty dt \left\langle \left\{ \tilde{E}^{a,i}(t,\mathbf{0}), \tilde{E}^{a,i}(0,\mathbf{0}) \right\} \right\rangle,$$

$$\gamma = -i\frac{g^2}{18}\int_0^\infty dt \left\langle \left[\tilde{E}^{a,i}(t,\mathbf{0}), \tilde{E}^{a,i}(0,\mathbf{0}) \right] \right\rangle$$

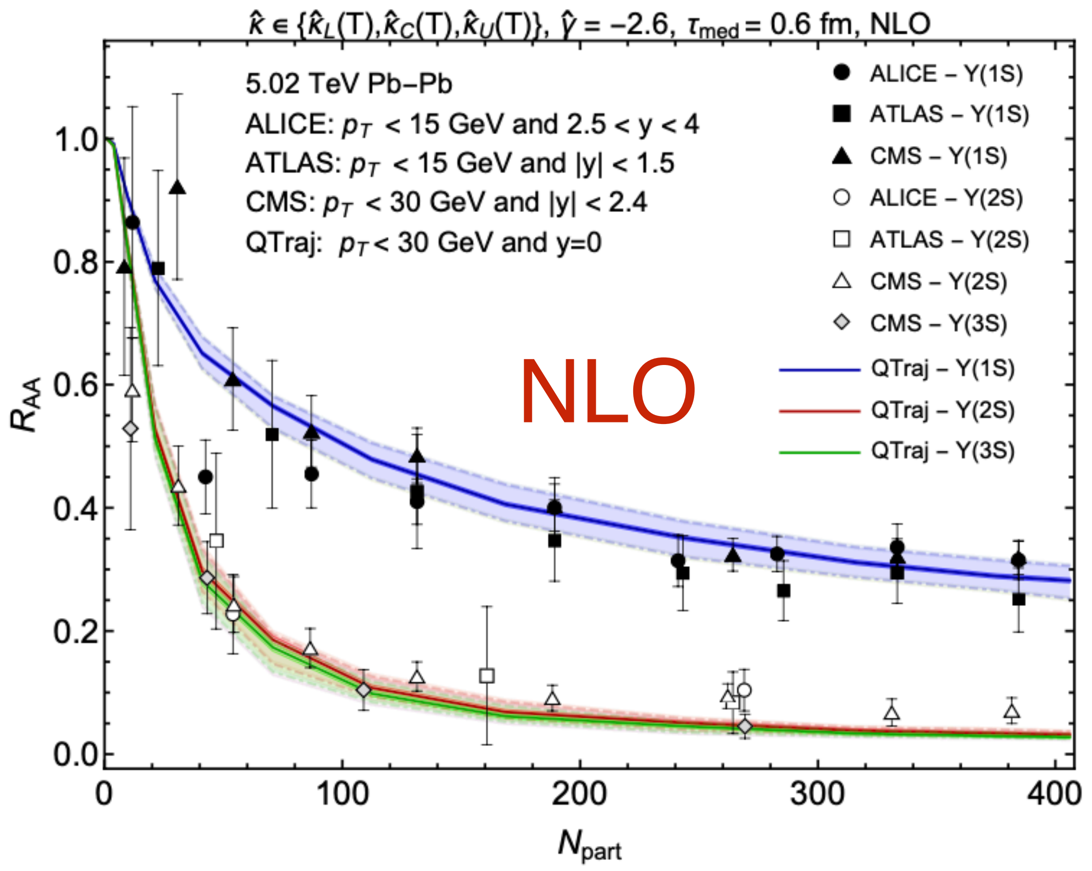


Strong coupling between heavy quark and medium; κ and γ are related to the thermal width Γ and mass shift δM of the bottomonium.

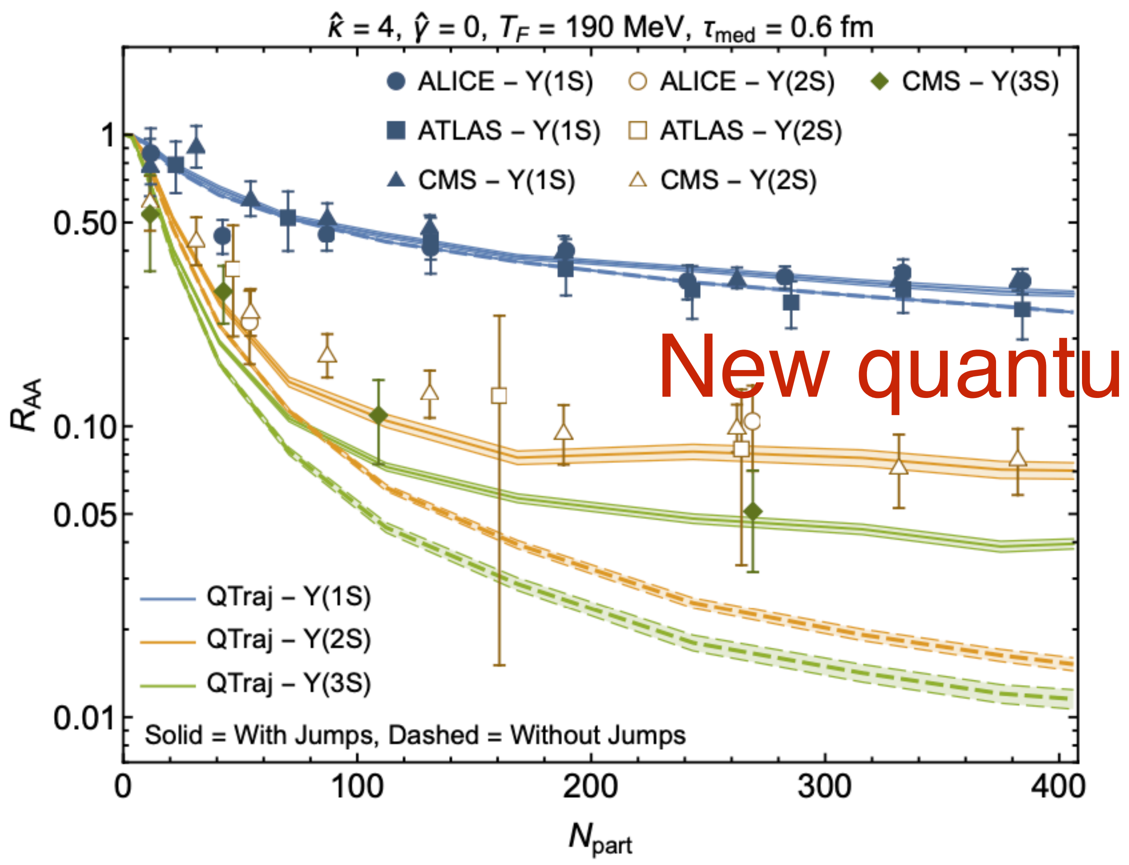
T_f = 250MeV



T_f = 190MeV



T_f = 190MeV, $\hat{\kappa} = 4, \hat{\gamma} = 0$ -> w/o screening



The new results with quantum jumps and w/o color screening agree well with the R_{AA} and double ratios!

Nantes Approach

- NRQCD+OQS works in Quantum Brownian motion Regime $M \gg T \sim m_D \gtrsim E_{bind}$
- Expansion of τ_e/τ_s.
- Used for bottomonium and charmonium in 1D.

Stéphane Delorme, Tue. 2:20

*J. Blaizot, M. Escobedo, JHEP **06**, 034 (2018).*
*S. Delorme, T. Gousset, R. Katz, P.B. Gossiaux, Acta Phys. Pol. B Proc. Suppl. **16**, 1–112 (2023) ;*
*Eur. Phys. J. A **58**(10), 198 (2022); arXiv:2402.04488.*

$$\frac{\mathrm{d}\mathcal{D}_Q}{\mathrm{d}t} = \mathcal{L}\,\mathcal{D}_Q, \qquad \mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3$$

Real part potential

$$\mathcal{L}_0\mathcal{D} = -i\,[H_Q, \mathcal{D}]$$

$$\mathcal{L}_1\mathcal{D} = -\frac{i}{2}\int_{xx'} V(x-x')\,[n_x^a n_{x'}^a, \mathcal{D}]$$

$$\mathcal{L}_2\mathcal{D} = \frac{1}{2}\int_{xx'} W(x-x')\,(\{n_x^a n_{x'}^a, \mathcal{D}\} - 2n_x^a \mathcal{D} n_{x'}^a)$$ fluctuations

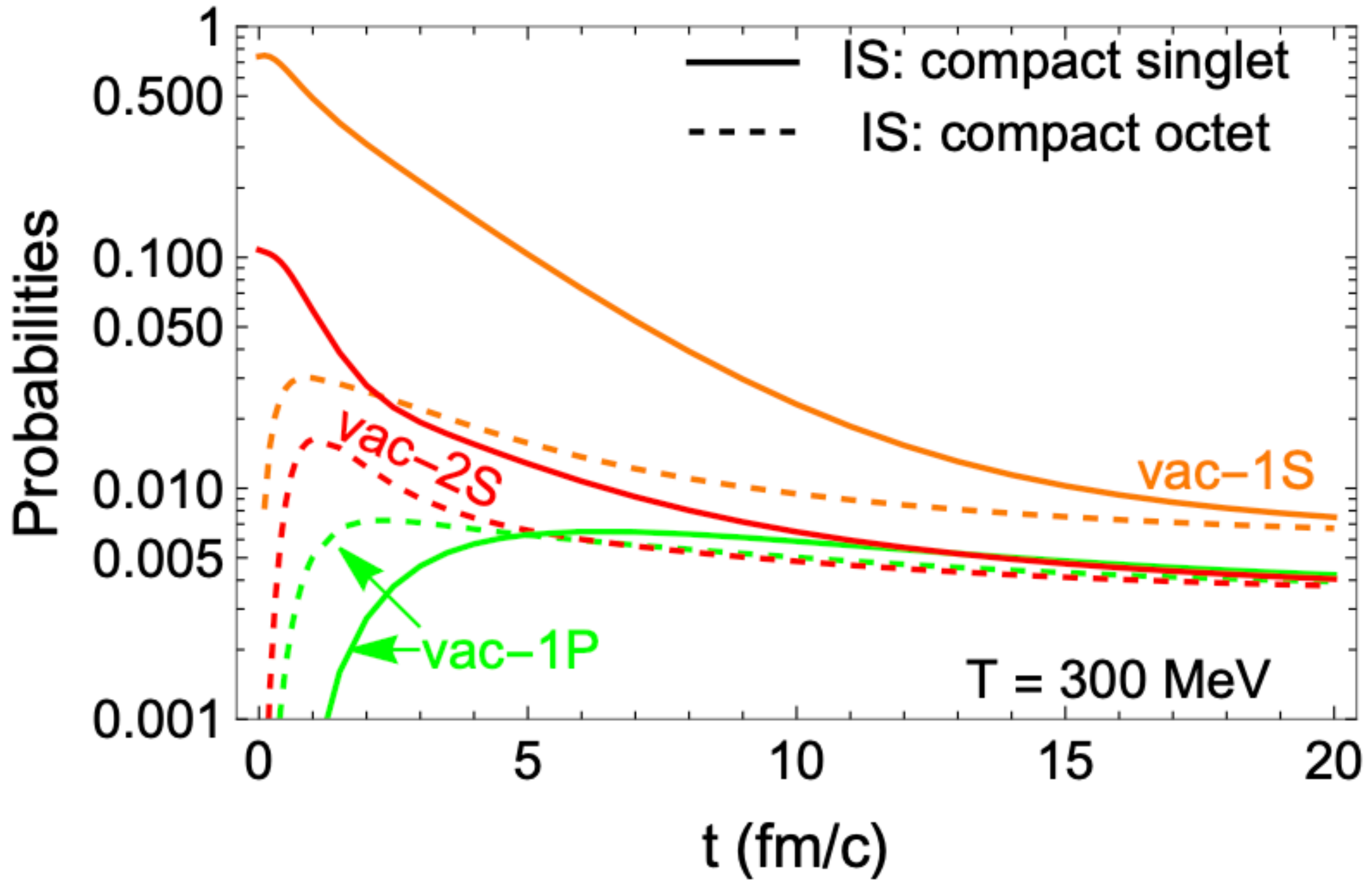
$$\mathcal{L}_3\mathcal{D} = -\frac{i}{4T}\int_{xx'} W(x-x')\left(\dot{n}_x^a \mathcal{D} n_{x'}^a - n_x^a \mathcal{D}\dot{n}_{x'}^a + \frac{1}{2}\{\mathcal{D}, [\dot{n}_x^a, n_{x'}^a]\}\right)$$ dissipation

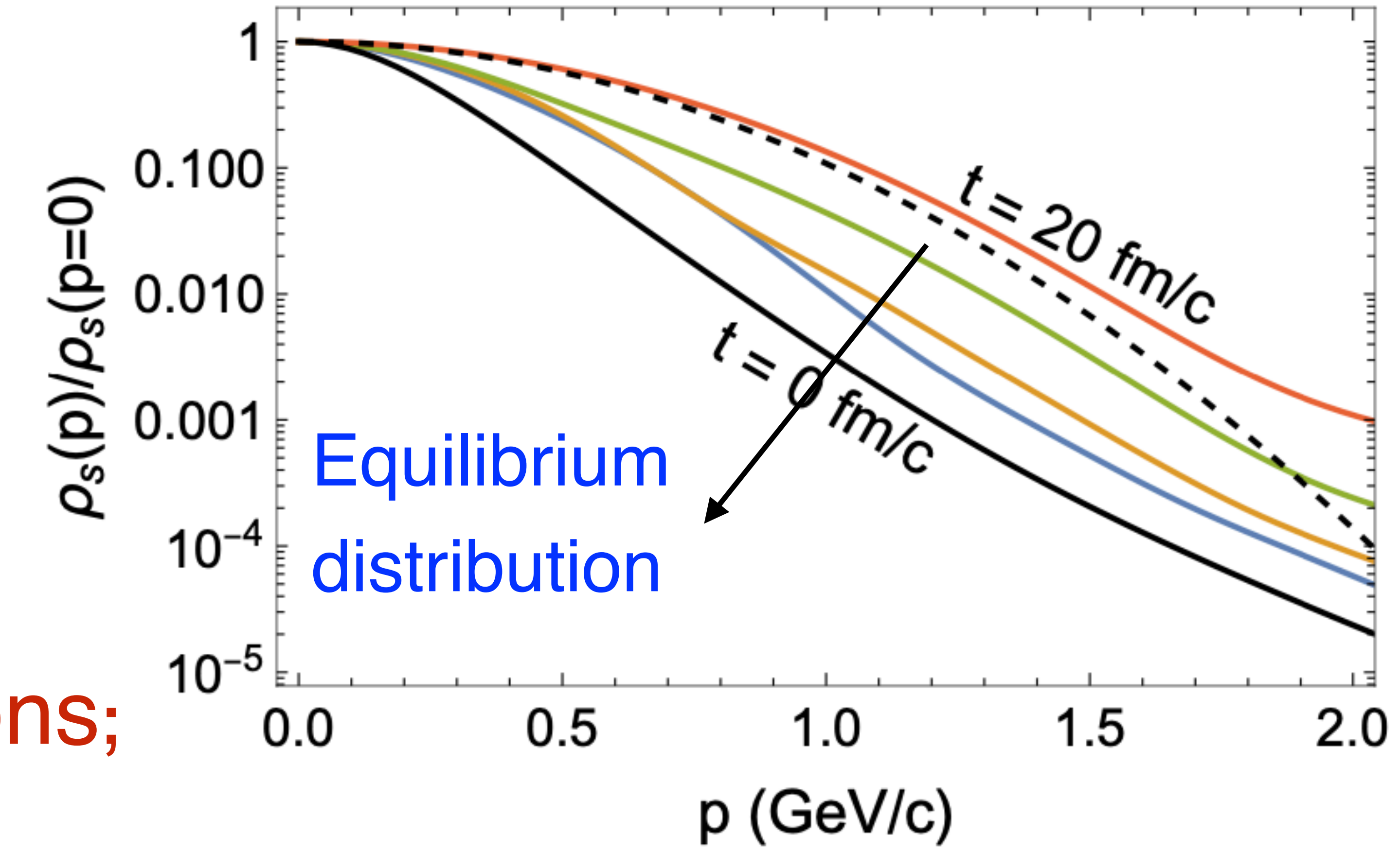
$$\mathcal{L}_4\mathcal{D} = \frac{1}{32T^2}\int_{xx'} W(x-x')\,(\{\dot{n}_x^a \dot{n}_{x'}^a, \mathcal{D}\} - 2\dot{n}_x^a \mathcal{D}\dot{n}_{x'}^a)\,.$$

preservation of positivity

Imaginary part potential

$$\frac{d}{dt}\begin{pmatrix}\mathcal{D}_s \\ \mathcal{D}_o\end{pmatrix} = \mathcal{L}\begin{pmatrix}\mathcal{D}_s(\mathbf{s},\mathbf{s}',t) \\ \mathcal{D}_o(\mathbf{s},\mathbf{s}',t)\end{pmatrix}, \qquad \mathcal{L} = \begin{pmatrix}\mathcal{L}_{ss} & \mathcal{L}_{so} \\ \mathcal{L}_{os} & \mathcal{L}_{oo}\end{pmatrix}$$





Beyond the dipole approximation;
The equations are solved with different initial states and medium configurations;
Equilibrium is checked.

Osaka Approach

- NRQCD+OQS works in Quantum Brownian motion Regime $M \gg T \sim m_D \gtrsim E_{bind}$
- Weak coupling (strict) and go beyond the weak coupling (approximation)
- Used for bottomonium.

T. Miura, Y. Akamatsu, M. Asakawa, et al, PRD 87 (2013) 045016; PRD 91 (2015) 5, 056002.; PRD97 (2018), 014003.; Phys.Rev.D 106 (2022) 7, 074001.

$$\frac{d}{dt}\rho_r(t) = -i[H_{\rm eff}^{(r)}, \rho_r] + \sum_{\vec{k}a}\left(2L_{\vec{k}a}^{(r)}\rho_r L_{\vec{k}a}^{(r)\dagger} - L_{\vec{k}a}^{(r)\dagger}L_{\vec{k}a}^{(r)}\rho_r - \rho_r L_{\vec{k}a}^{(r)\dagger}L_{\vec{k}a}^{(r)}\right),$$

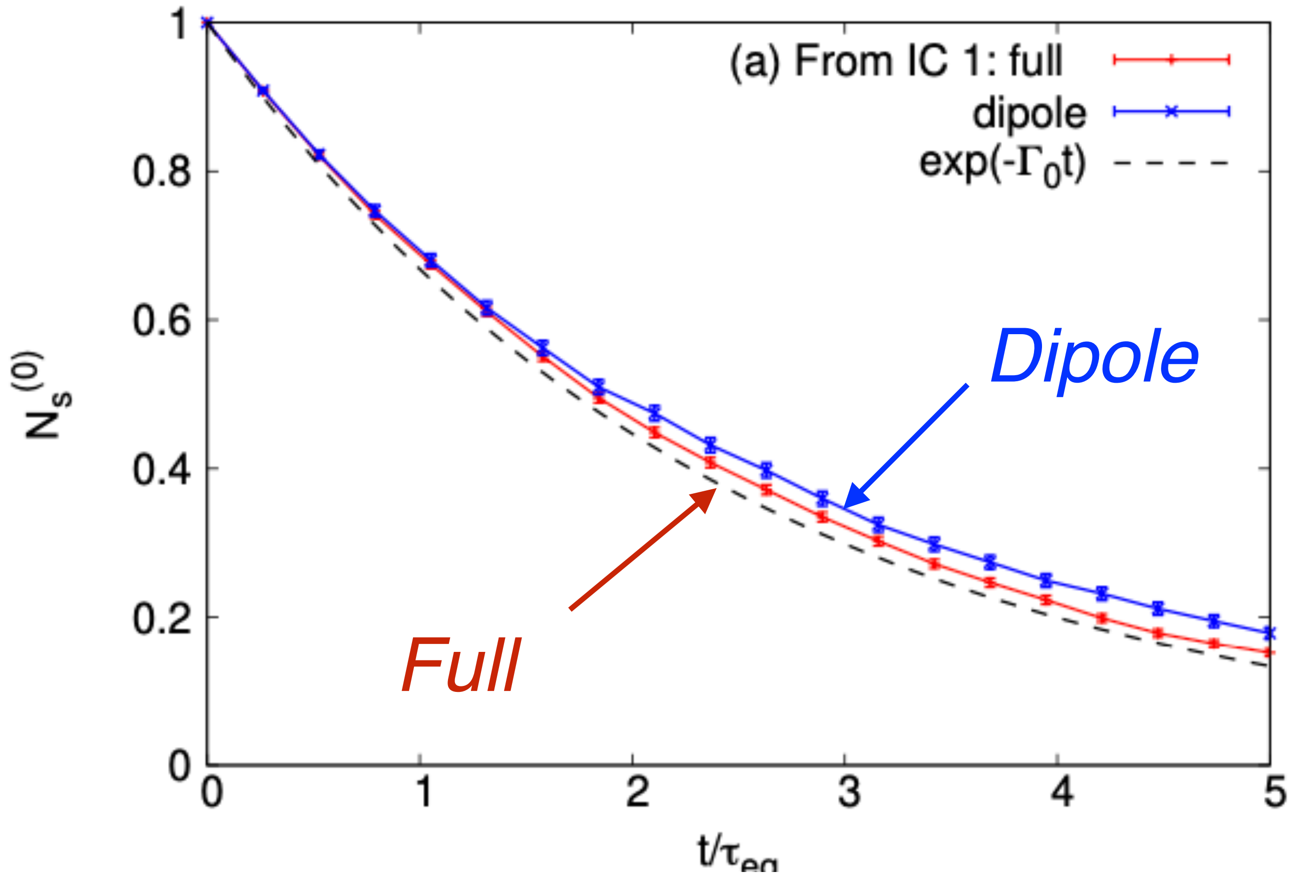
$$H_{\rm eff}^{(r)} = \frac{\vec{p}^2}{M} + V(\vec{r})(t^a \otimes t^{a*}) - \frac{1}{4MT}\left\{\vec{p}, \vec{\nabla}D(\vec{r})\right\}(t^a \otimes t^{a*}),$$

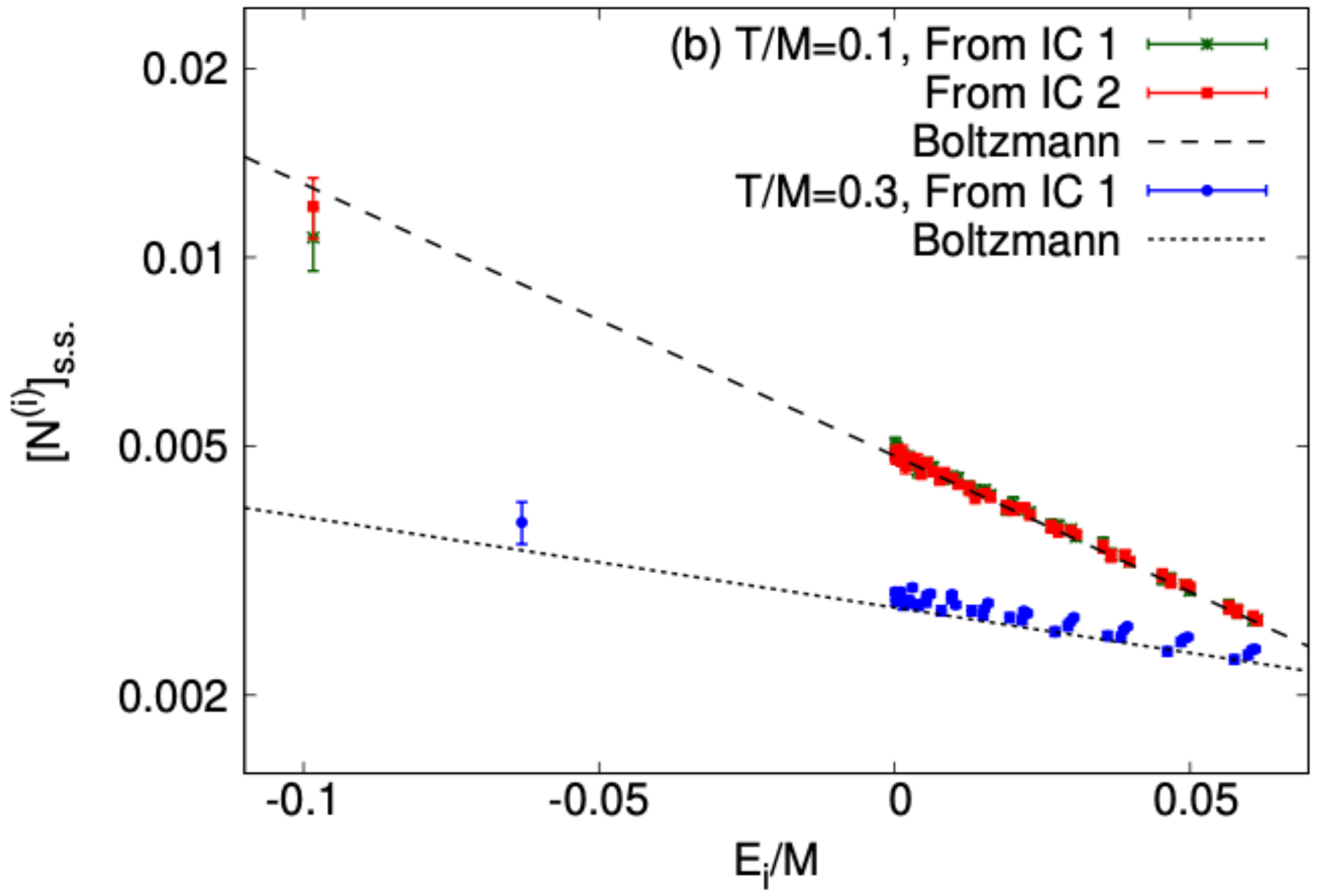
$$L_{\vec{k}a}^{(r)} = \sqrt{\frac{\tilde{D}(\vec{k})}{2L^3}}\left[1 - \frac{\vec{k}}{4MT}\cdot\left(\frac{1}{2}\vec{P}_{\rm CM} + \vec{p}\right)\right] e^{\frac{i\vec{k}\cdot\vec{r}}{2}}(t^a \otimes 1)$$
$$- \sqrt{\frac{\tilde{D}(\vec{k})}{2L^3}}\left[1 - \frac{\vec{k}}{4MT}\cdot\left(\frac{1}{2}\vec{P}_{\rm CM} - \vec{p}\right)\right] e^{-\frac{i\vec{k}\cdot\vec{r}}{2}}(1 \otimes t^{a*}).$$

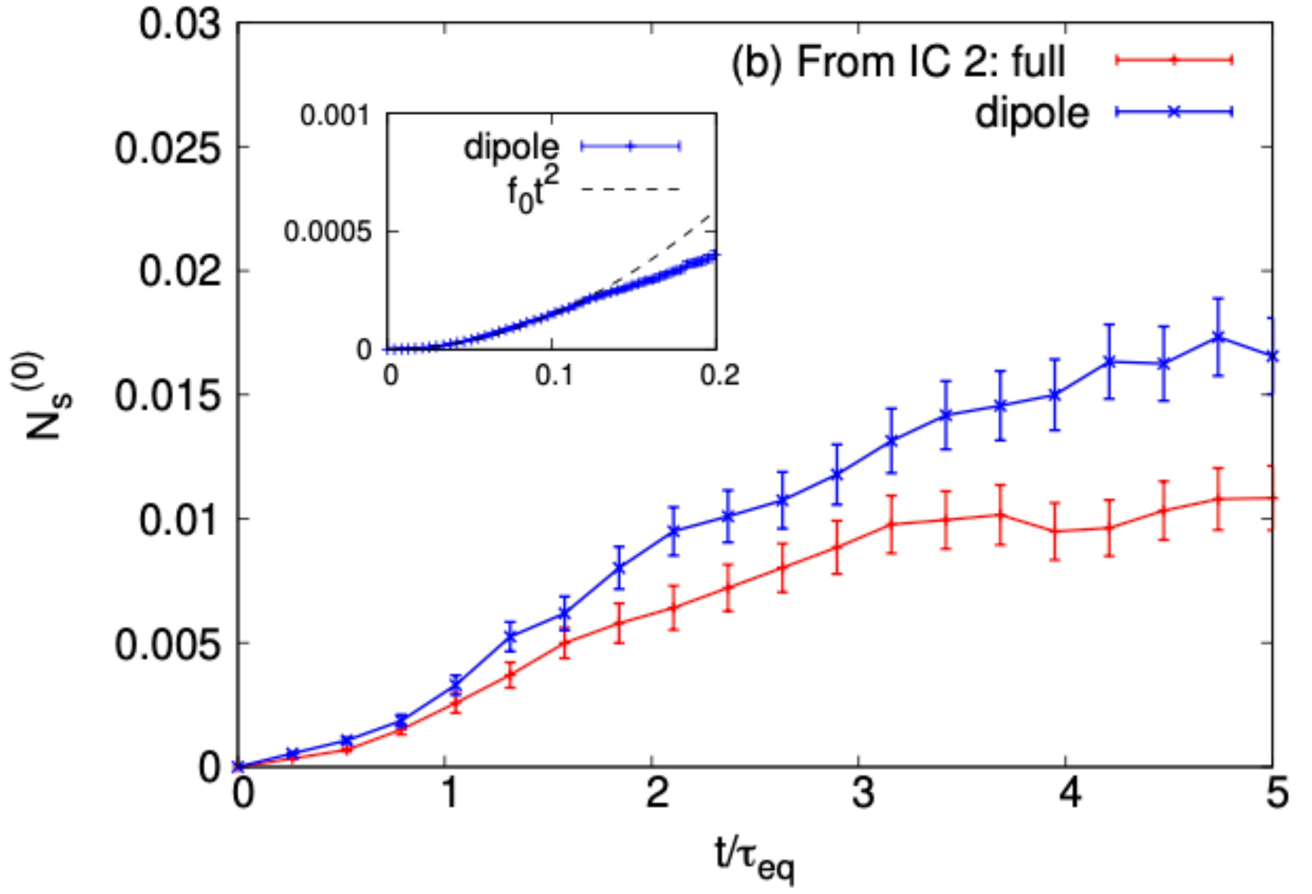
Beyond the weak coupling and assume the real and imaginary potential:

$$V(x) = -\frac{\alpha}{\sqrt{x^2 + x_{\rm c}^2}}{\rm e}^{-m_{\rm D}|x|},$$

$$D(x) = \gamma \exp(-x^2/\ell_{\rm corr}^2).$$







Beyond the dipole approximation;
The dipole approximation is an efficient alternative method, but it depends on the initial condition!
Equilibrium is satisfied.

Quarkonium real-time evolution in heavy-ion collisions

$\tau_e \sim 1/(\pi T),\ \tau_s \sim 1/E_{bind},\ \tau_r \sim M/T^2$

Non-Markovian

Markovian approximation: $\tau_e \ll \tau_r$ **, memory lose**

Quantum master equation –> Lindblad equation

$\sim m_Q$

$\tau_e \ll \tau_s$

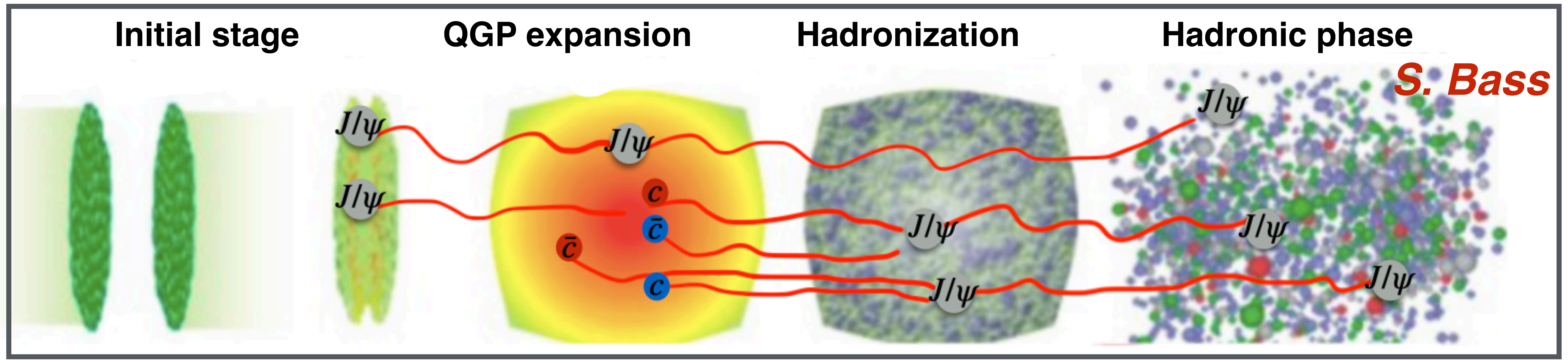
$(E_{bind} \ll m_D)$

$\tau_s \ll \tau_r$

$(E_{bind} \gg m_D)$

Quantum Brownian Motion

Quantum optical Limit



Further needs:

1. Connect the Quantum Brownian Motion at high temperture to the Quantum optical regime at low temperature.
2. One pair –> many pairs and regeneration from uncorrelated heavy quarks.

PHSD-Nantes Approach

- Start from the OQS and works from the QBM to QOL
- N pairs of $Q\bar{Q}$. Used for bottomonium and charmonium.
- N-body Wigner density is approximated as a classical phase space distribution

Taesoo Song, Tue. 2:00

D. Villar, **JZ**, J. Aichelin, and P. Gossiaux, Phys.Rev.C 107 (2023) 5, 054913.
T. Song, J. Aichelin, and E. Bratkovskaya, Phys.Rev.C 107 (2023) 5, 054906.
T. Song, J. Aichelin, **JZ**, P. Gossiaux and E. Bratkovskaya,PRC108 (2023) 5, 054908.

Probability that at time t the state Φ is produced:

$$P^{\Phi}(t) = \mathrm{Tr}[\rho^{\Phi}\hat{\rho}_{tot}(t)] \qquad \rho^{\Phi} = |\Phi><\Phi|$$

So: $$P^{\Phi}(t) = P^{\Phi}(0) + \int_0^t \Gamma^{\Phi}(t)dt$$

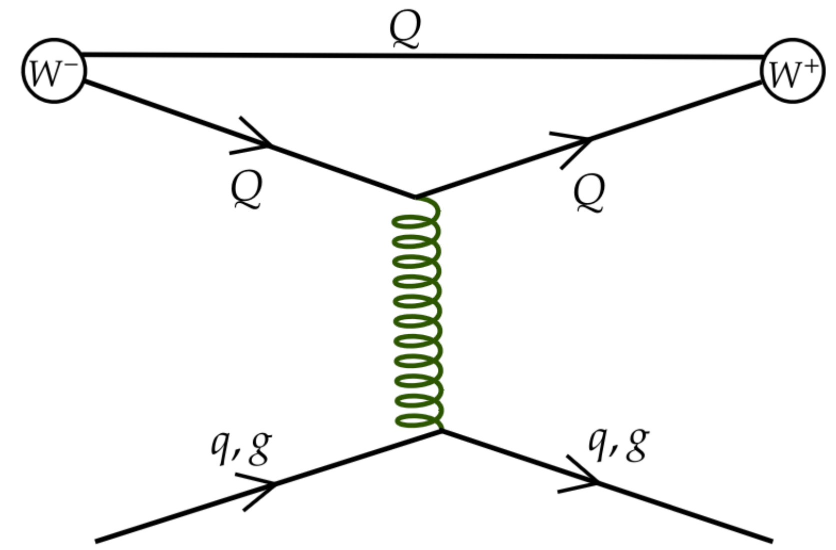
$$\Gamma^{\Phi} = \frac{dP^{\Phi}}{dt} = \frac{d}{dt}\mathrm{Tr}[\rho^{\Phi}\rho_{tot}] \approx \frac{d}{dt}\prod \frac{d^3r_id^3p_i}{(2\pi)^{3N}} W^{\Phi}(r,p)W_N(r_1,p_1,\ldots,r_N,p_N)$$

$$W_N \approx W_N^{C(\mathrm{classical})} = \prod_{i=1}^{N} \delta(\mathbf{r}_i - \mathbf{r}_i^*(t))\delta(\mathbf{p}_i - \mathbf{p}_i^*(t))$$

Collisional rate

$$\Gamma_{\mathrm{coll}}(t) = \sum_k^2 \sum_{i\geq 3}^N \sum_n \delta(t - t_{ki}(n)) \int \prod_{j=1}^N d^3\mathbf{r}_j d^3\mathbf{p}_j W^{\Phi}(\mathbf{r}_1,\mathbf{r}_2,\mathbf{p}_1,\mathbf{p}_2)\left[W_N^C(\{\mathbf{r}_j\},\{\mathbf{p}_j\},t+\epsilon) - W_N^C(\{\mathbf{r}_j\},\{\mathbf{p}_j\},t-\epsilon)\right]$$

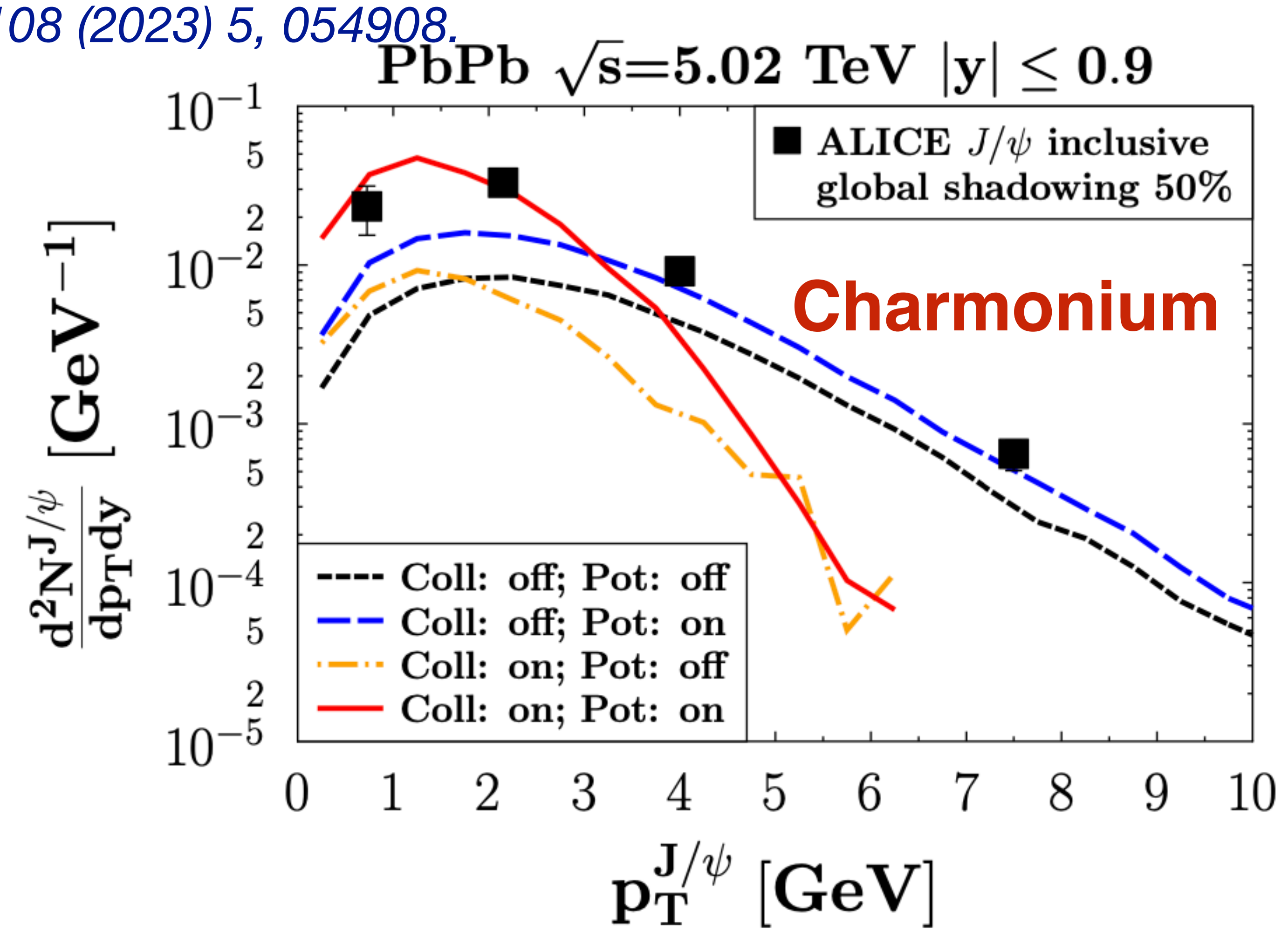
W^+ $\qquad W^-$

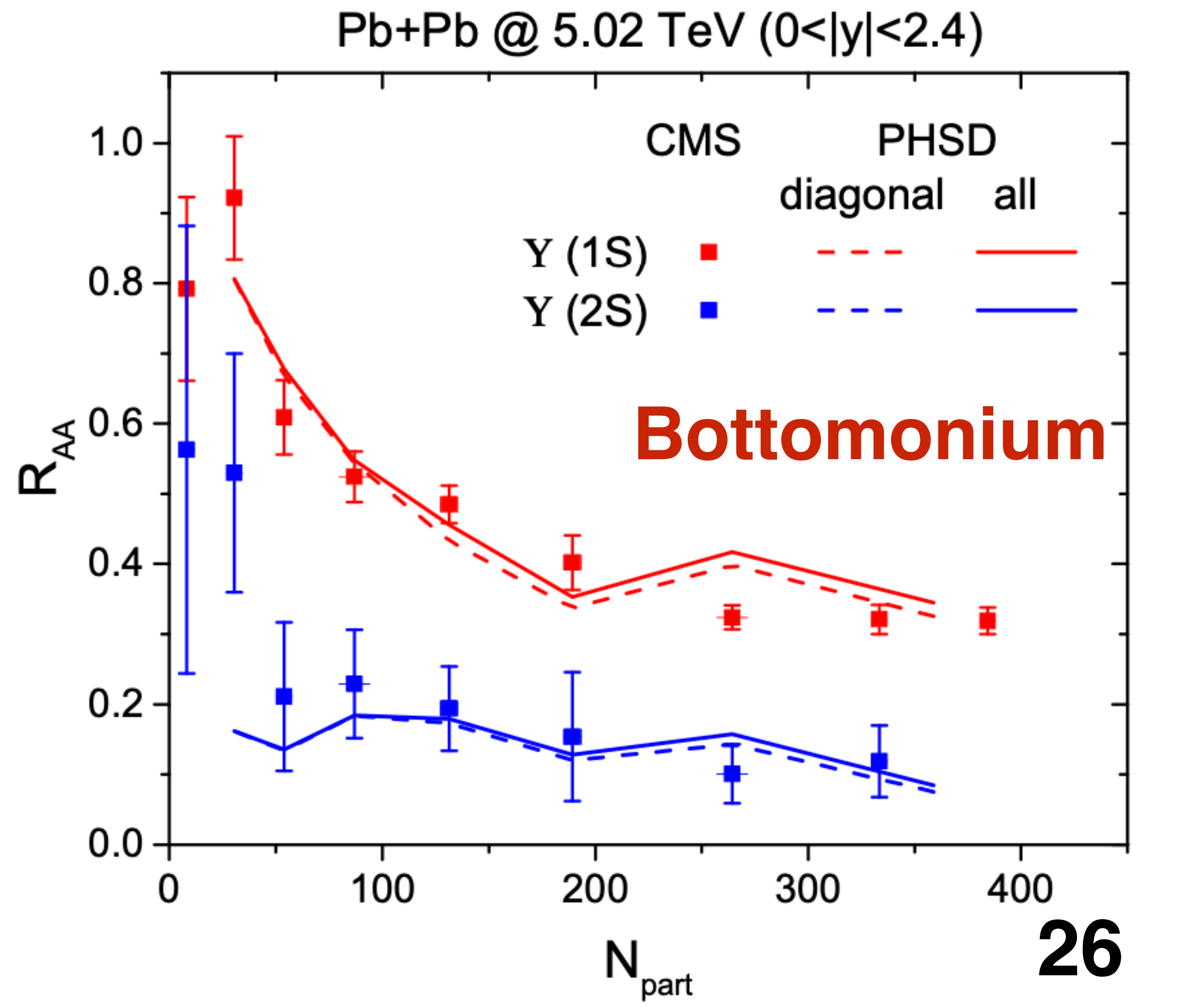


Wigner density of quarkonium states is temperature or time dependent->another term:

Local rate

$$\Gamma_{\mathrm{local}}(t) = \int \prod_{j=1}^N d^3\mathbf{r}_j d^3\mathbf{p}_j \dot{W}^{\Phi}(\mathbf{r}_1,\mathbf{r}_2,\mathbf{p}_1,\mathbf{p}_2,T(t))W_N^C(\{\mathbf{r}_j\},\{\mathbf{p}_j\},t)$$

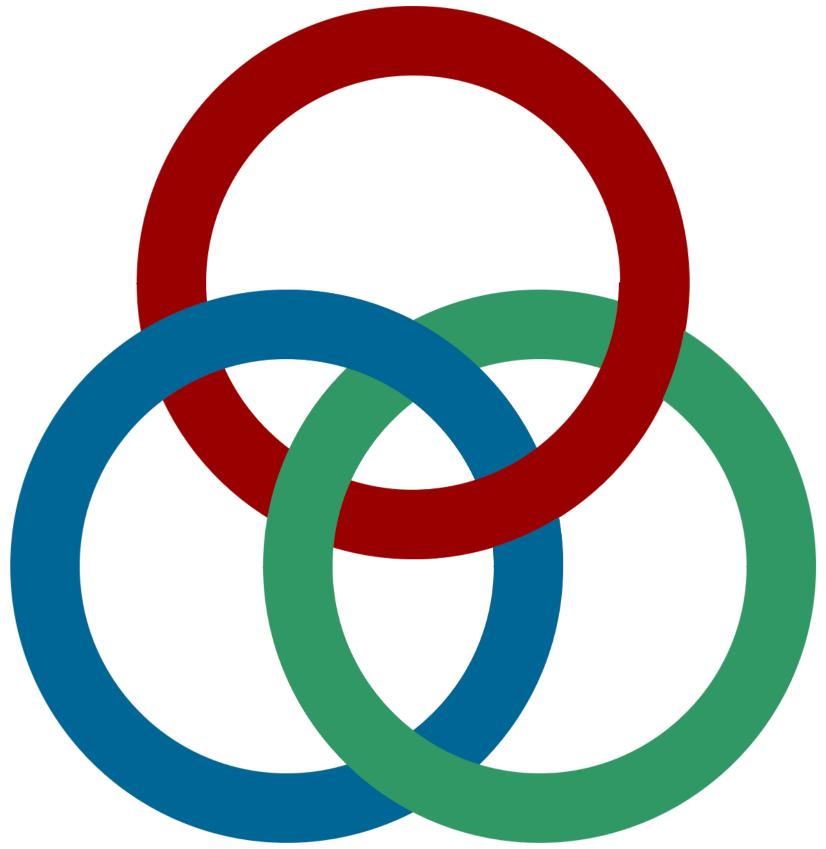




Bulid a unified framework

To learn more: To combine the light with heavy, open heavy flavor with quarkonium.

Light hadrons



Thermal medium properties:
EOS, lifetime,temperature,
velocity, shear viscosity…

- Catania model
- Duke model
- EPOS4 model
- LBT model
- PHSD model
- TAMU model
- Torino model
- …

Open heavy flavor

Quarkonium

- Comover model
- TAMU model
- Tsinghua model
- OQS based approaches:
- Munich-Kent
- Nantes
- Osaka
- PHSD-Nantes
- …

Heavy quark energy loss
Correlations
Quark number conservation
…

Vincenzo Greco, Thu. 9:30

➡ **EPOS4 is now ready for light and open heavy flavors, and the quarkonium part is coming soon.**

K. Werner, PRC 109 (2024) 1, 014910
***JZ,** J.Aichelin, P.B. Gossiaux, V. Ozvenchuk, K.Werner, arXiv:2401.17096*
***JZ,** J.Aichelin, P.B. Gossiaux, K.Werner, Phys.Rev.D 109 (2024) 5, 054011*

Pol-Bernard Gossiaux, Tue. 9:30

Summary

- The vacuum properties are well described by the potential model. The in-medium properties can mostly be absorbed in the finite-temperature potential, which has both real and imaginary parts. Recent studies show: The HQ potential has no/a small color screening effect and a large imaginary part.

- With the assumption of a classical particle of quarkonium, the transport model as well as the statistical model (with corona effect) can describe quite well the experimental data, which help us to understand the HQ in-medium potential, HQ energy loss, QGP properties,… Also extended to B_c and $X(3872)$.

- Aiming to include the quantum effects and to build a genuine first principles based real time evolution framework, OQS is used and developed in different ways based on heavy quark effective theories. Much progress has been made in the Quantum Brownian Motion regime, where a bound state is difficult to define.

I'm sorry if your work progress was not included. Feel free to tell me later.

Recent comparison paper

Eur. Phys. J. A (2024) 60:88
https://doi.org/10.1140/epja/s10050-024-01306-6

THE EUROPEAN PHYSICAL JOURNAL A

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Review

Comparative study of quarkonium transport in hot QCD matter

A. Andronic[1,a], P. B. Gossiaux[2,b], P. Petreczky[3,c], R. Rapp[4,d], M. Strickland[5,e], J. P. Blaizot[6], N. Brambilla[7], P. Braun-Munzinger[8,9], B. Chen[10], S. Delorme[11], X. Du[12], M. A. Escobedo[13,12], E. G. Ferreiro[12], A. Jaiswal[14], A. Rothkopf[15], T. Song[8], J. Stachel[9], P. Vander Griend[16], R. Vogt[17], B. Wu[4], J. Zhao[2], X. Yao[18]

Thanks for your attention