

SQM 2024 (Strasbourg, France)

Student Lecture: Heavy quarks and Quarkonia (Theory and Phenomenology) for the non experts

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Disclaimer: This brief STUDENT lecture...

- ... is just meant to be an introduction to the topic for fresh students so that they can get an overall picture and better appreciate the talks and get an historical perspective
- => NOT intended to enter in any technical aspect
- => NOT intended to resolve ambiguities or express my personal opinion (just a bit)
- => NOT intended to cover all the the fascinating topics

The Menu

➤ Heavy Quarks in the QGP and Open Heavy Flavors

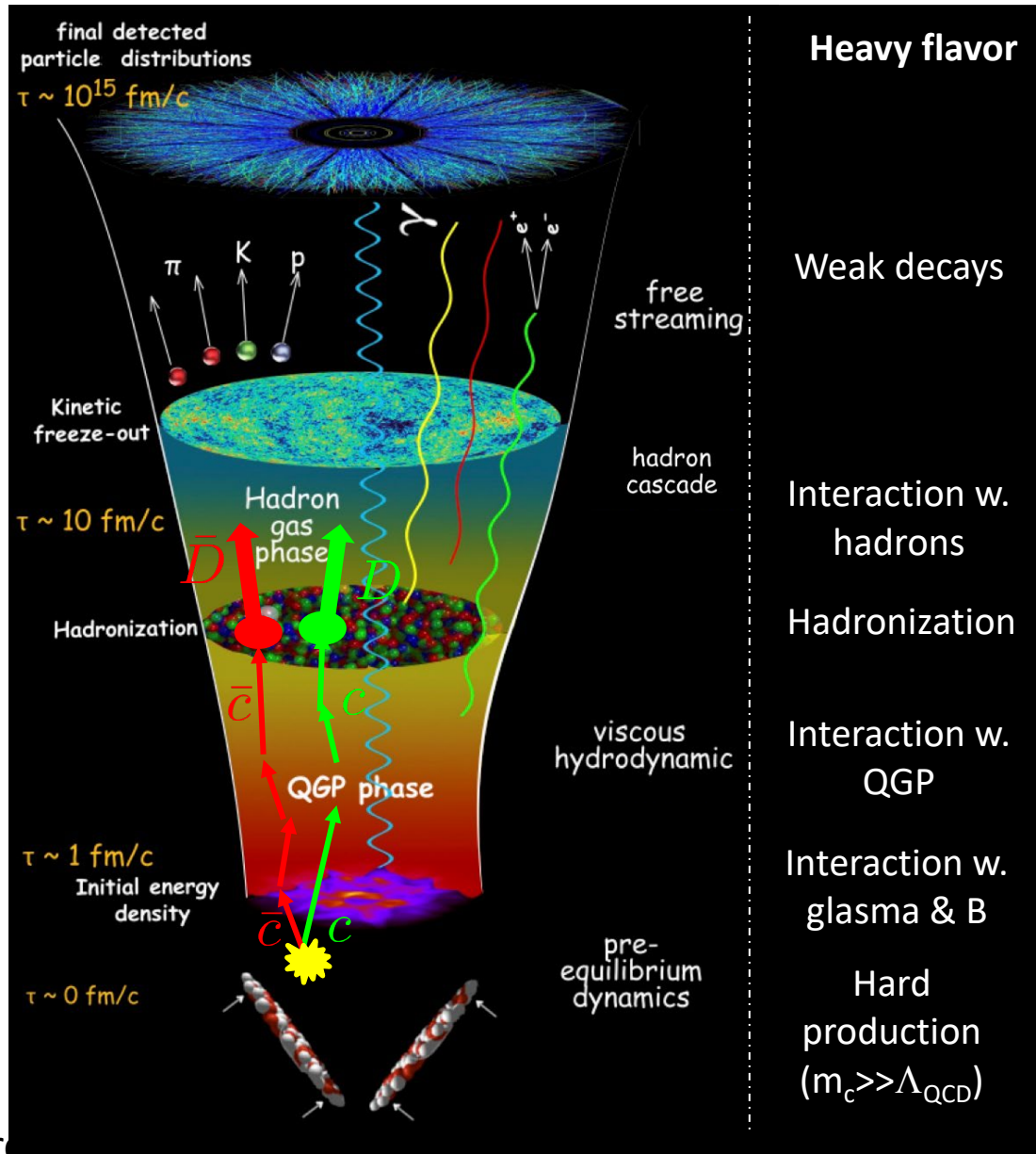
➤ Quarkonia (Φ) production in AA



Effect through $Q + \bar{Q} \rightarrow \Phi$

Standard model of URHIC

Established before 2010



- Produced early ($t \approx 1/m_Q$)
 - => No further $Q\bar{Q}$ generation in ensuing QGP
 - Initial production well controlled (advantage of $m_Q \gg \Lambda_{\text{QCD}}$)
 - But early phase might not be so innocent (magnetic field, CGC-glasma,...)

➤ Experience the full deconfined phase + hadronic phase

- probes the QGP on harder scales than the other hadronic observables *while not fully thermalized* ($t_{\text{relax}} \propto m_Q/T^2$)
- accumulates several effects => need to compare different systems to better differentiate them

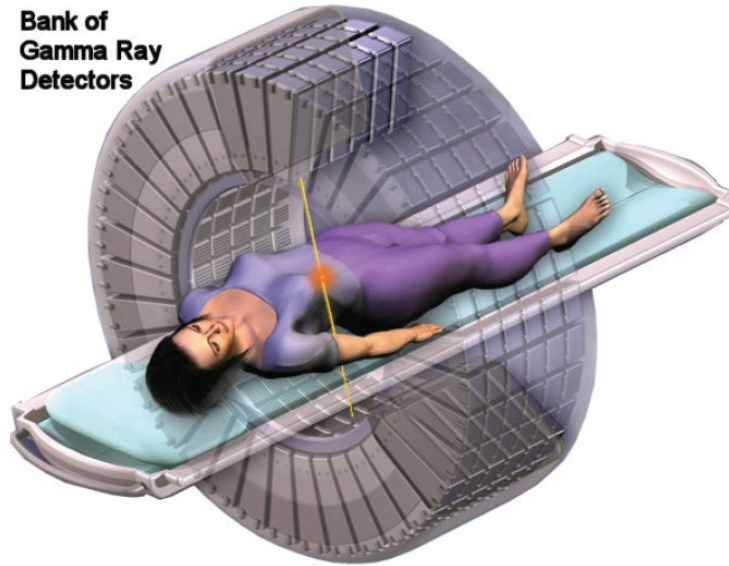
- Produced over a wide range of rapidities and p_T
 - increased richness in scrutinizing the interaction of HQ with medium...
 - but also sets more challenges (interactions for $p_T \ll m_Q$, $p_T \approx m_Q$, $p_T \gg m_Q$, appropriate transport theory ?).

- Nowadays turning into precision physics thanks to abundance of RHIC and LHC results !!!

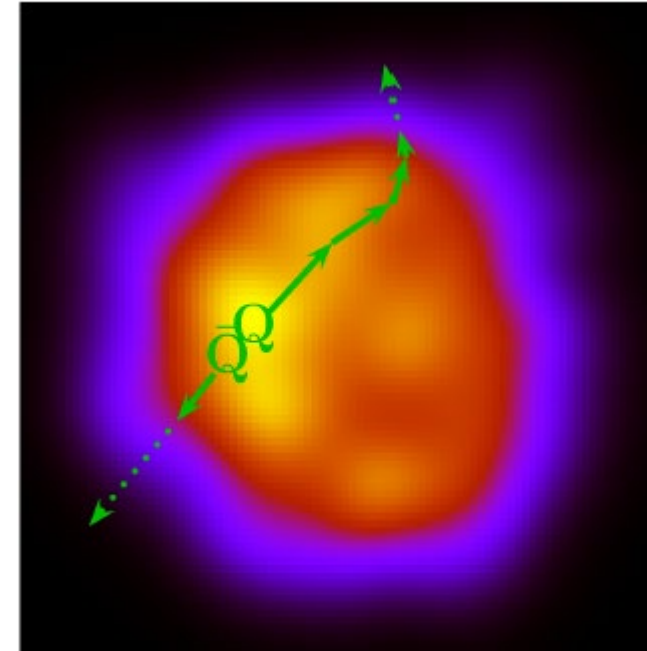
Since ≈ 2010

HQ lectur

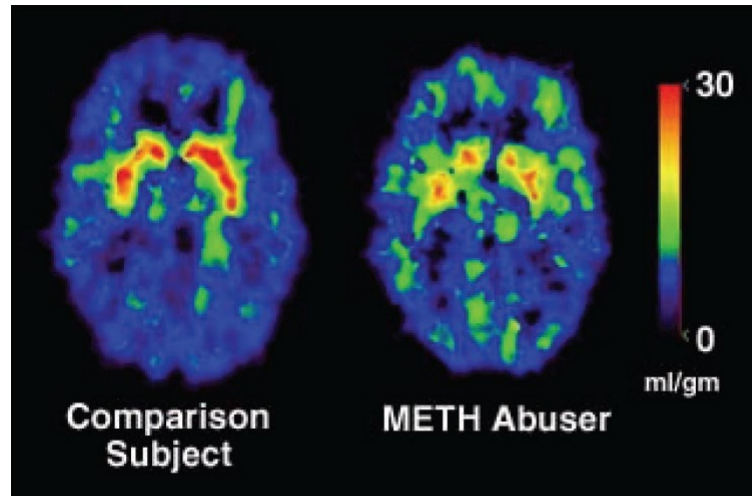
Extracting density profiles with HF Tomography ?



Schematic diagram of a PET scanner



QGP tomography with Q-Qbar pairs

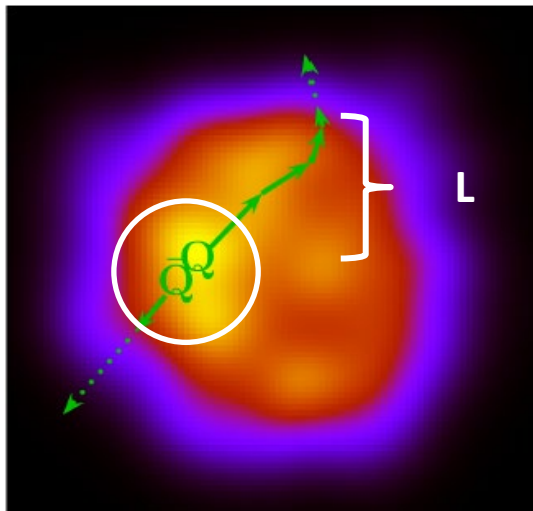


PET scan showing abnormal brain function of a METH user

Seems pretty attractive concept...

← Well formulated inverse problem.

Hard probes: Nuclear modification factor (R_{AA})



Naive assumption: $\frac{dp}{dl} = \rho \times f(p, \dots)$



$$\Delta p(p, \dots) = \int_0^L dl \frac{dp}{dl} = f(p, \dots) \times \int_0^L dl \rho$$

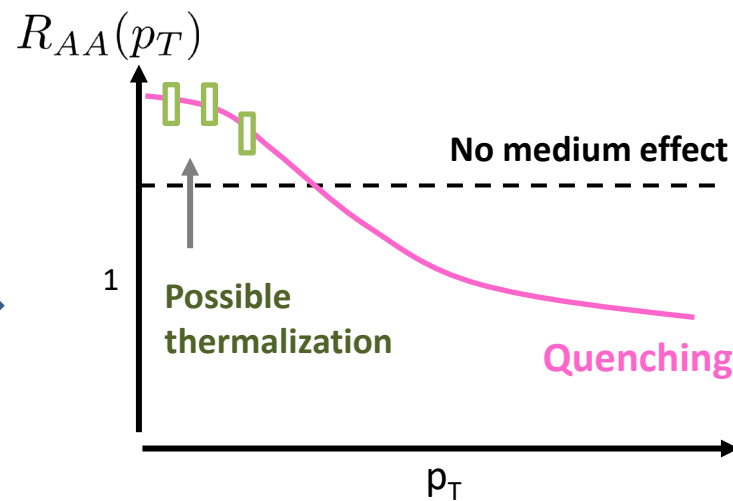
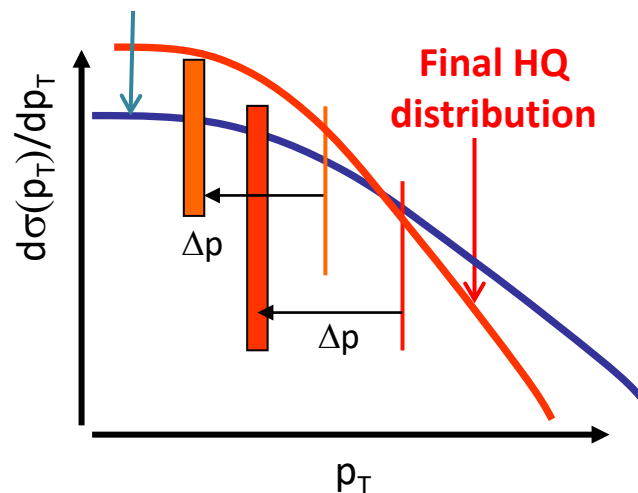
tomography

Experimental observable

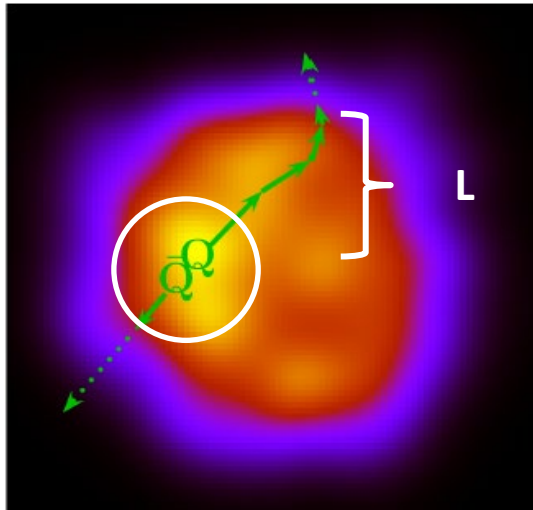
$$R_{AA}(p_T) = \frac{\frac{dN_{AA}}{dp_T}}{\langle N_{col} \rangle \times \frac{dN_{pp}}{dp_T}} \approx \frac{\frac{d\sigma_{pp}}{dp_T}(p_T + \Delta p(p_T))}{\frac{d\sigma_{pp}}{dp_T}(p_T)}$$

Model

Initial HQ distribution



Hard probes: Nuclear modification factor (R_{AA})



Naive assumption: $\frac{dp}{dl} = \rho \times f(p, \dots)$

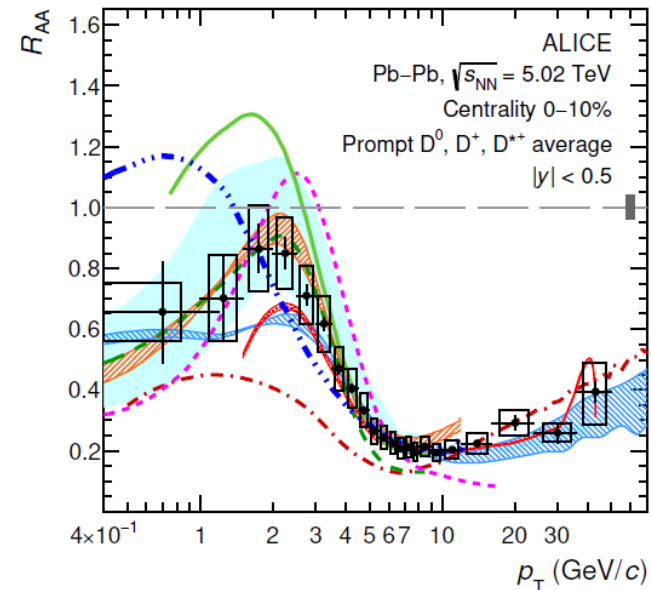
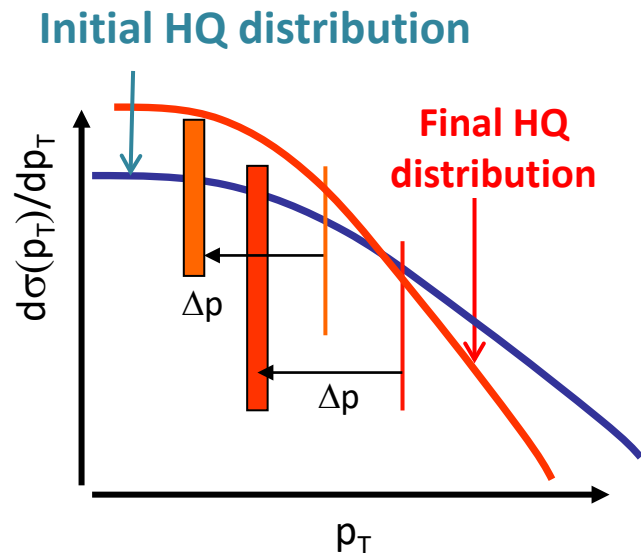
$$\Delta p(p, \dots) = \int_0^L dl \frac{dp}{dl} = f(p, \dots) \times \int_0^L dl \rho$$

tomography

Experimental observable

$$R_{AA}(p_T) = \frac{\frac{dN_{AA}}{dp_T}}{\langle N_{col} \rangle \times \frac{dN_{pp}}{dp_T}} \approx \frac{\frac{d\sigma_{pp}}{dp_T}(p_T + \Delta p(p_T))}{\frac{d\sigma_{pp}}{dp_T}(p_T)}$$

ALICE, JHEP 01 (2022) 174

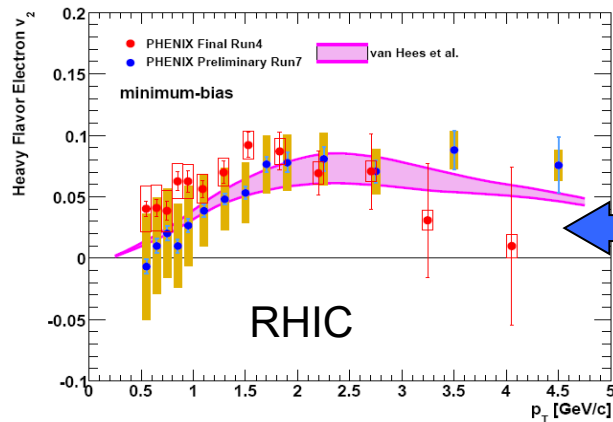


Model

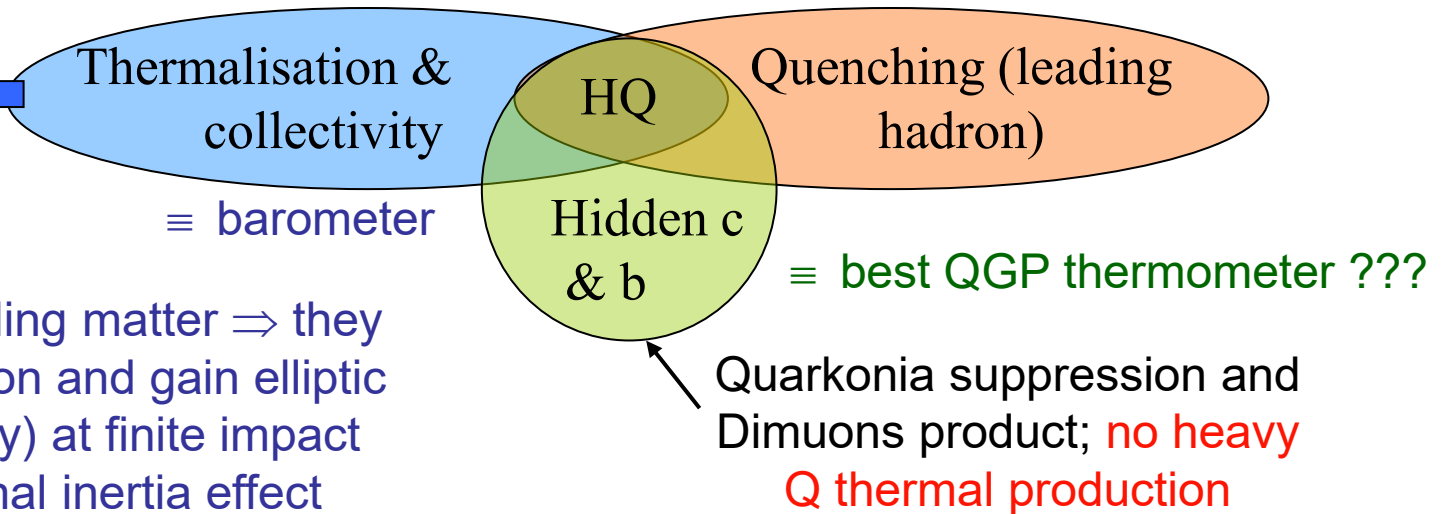
Why heavy flavors in A-A ?



➤ The Swiss knife of QGP hard probing !!!



The Trilogy:



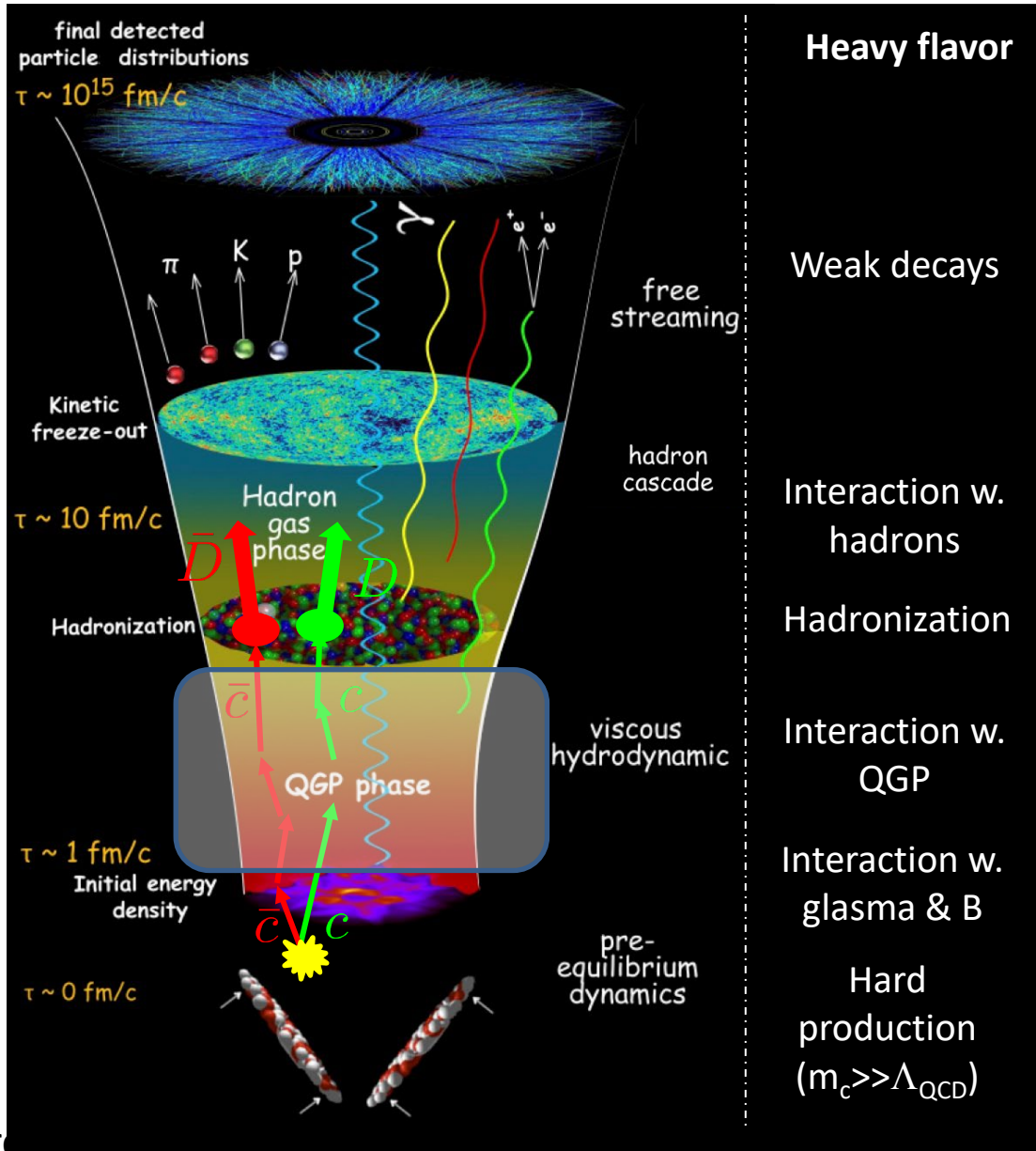
HQ are imbedded in expanding matter \Rightarrow they participate to collective motion and gain elliptic flow (v_2 : azimuthal asymmetry) at finite impact parameter... with additional inertia effect

➤ Mandatory to understand HQ evolution in QGP ($\frac{dp}{dl} = \rho \times f(p, \dots)$) & quarkonia production

Standard model of URHIC

Established before 2010

Since \approx 2010



Challenge:

Description of HQ E-loss / equilibration from fundamental theory.

In fact we are at the same time probing the system but also using the results to better understand our probe (and the coupling to QGP) at the same time !
=> useful to rely on other methods to constrain the bulk.

HQ Eloss: some models...

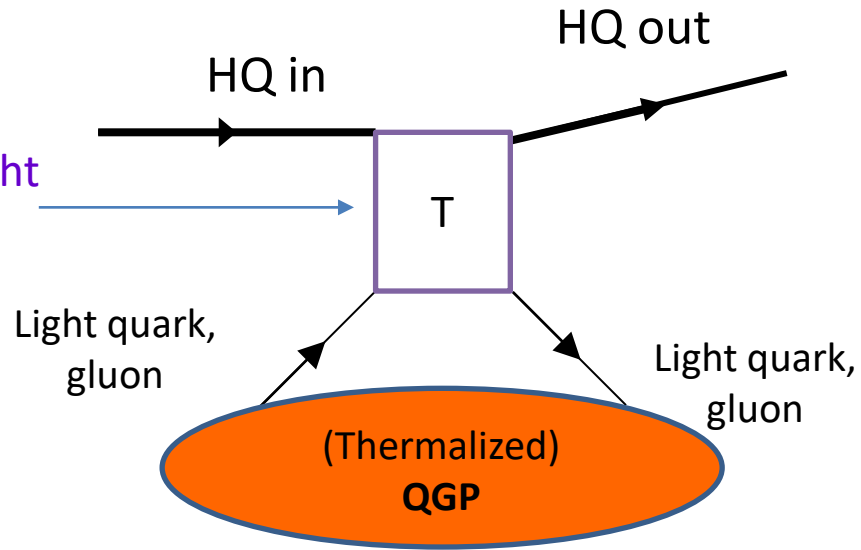
- Only a few !!! Sorry for this
- Not necessarily by historical appearance

HQ Eloss: Collisional (elastic) vs Radiative

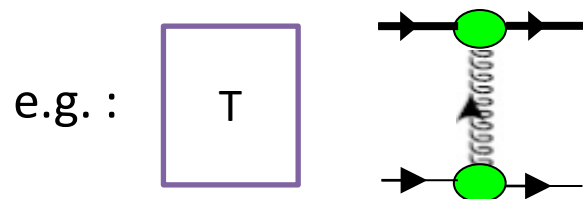
Strictly speaking: Both terms apply to pQCD processes (small and moderate coupling), or to pQCD-inspired processes for which quasi-particles still exists

Collisional (elastic): $2 \rightarrow 2$

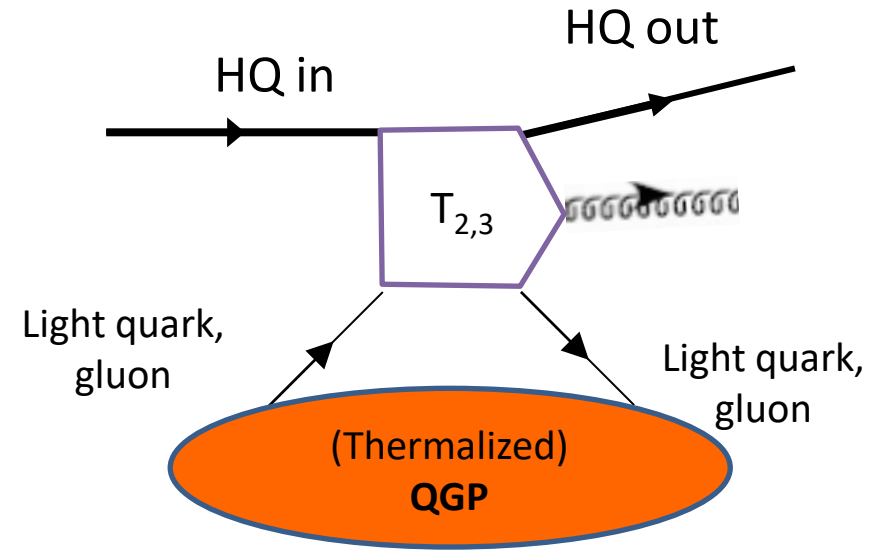
The heavy-light interaction



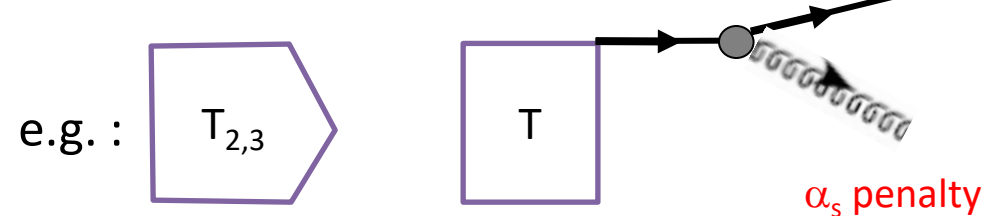
Energy flows from HQ \leftrightarrow medium



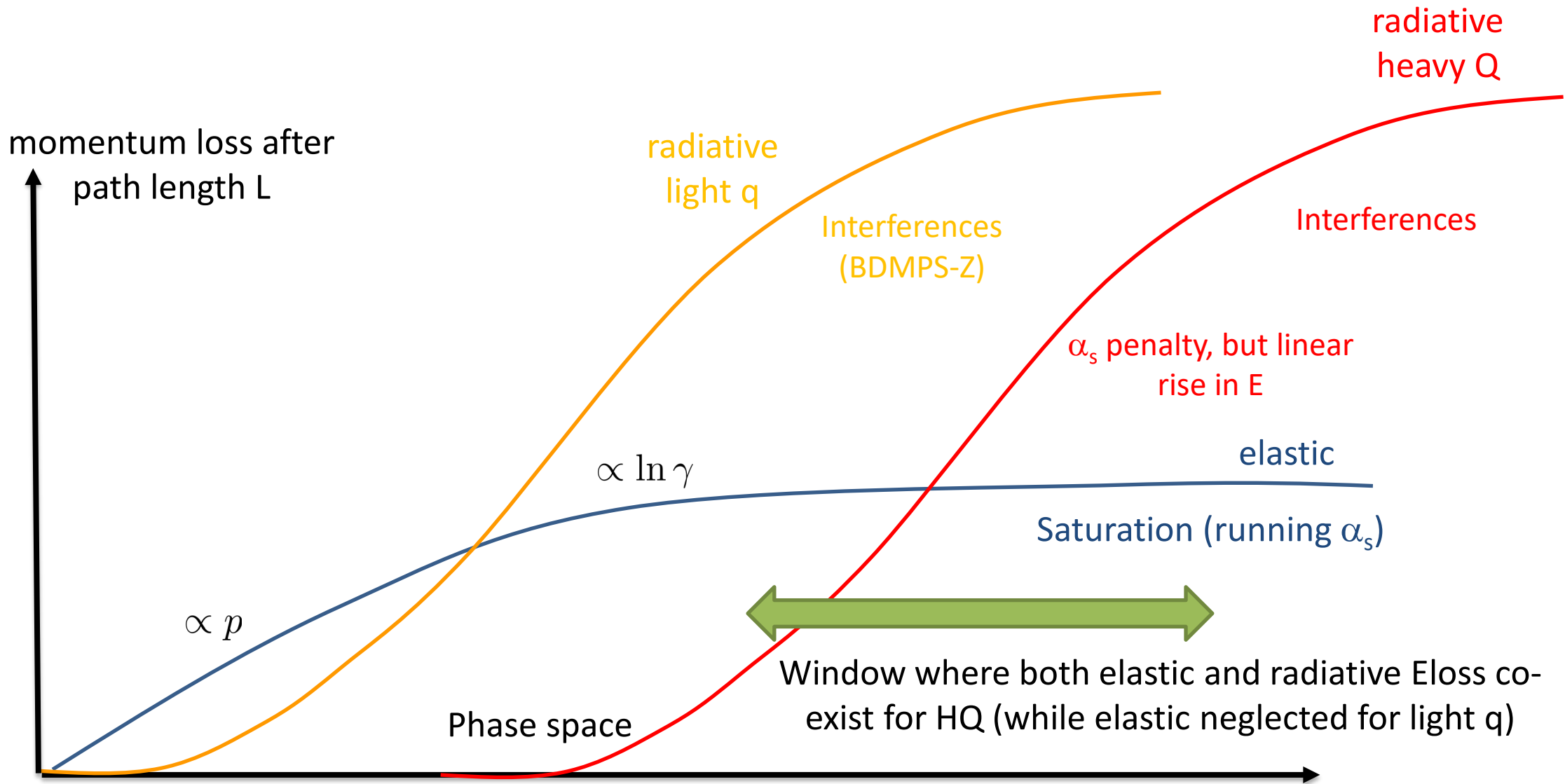
Radiative : $2 \rightarrow 3$



Most of energy flows from HE HQ \rightarrow radiated gluon



Collisional (elastic) vs Radiative



Looking at some concrete models...

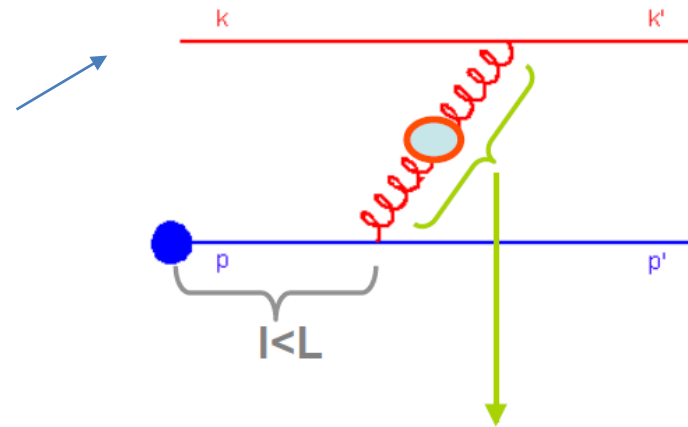
Parton rapidity

The dynamical energy loss formalism (Djordjevic)

M. D., Phys.Rev.C74:064907,2006

Collisional energy loss+

Dynamical QGP parton (as opposed to fixed scattering centers in other Eloss models)



L: path length in the QGP

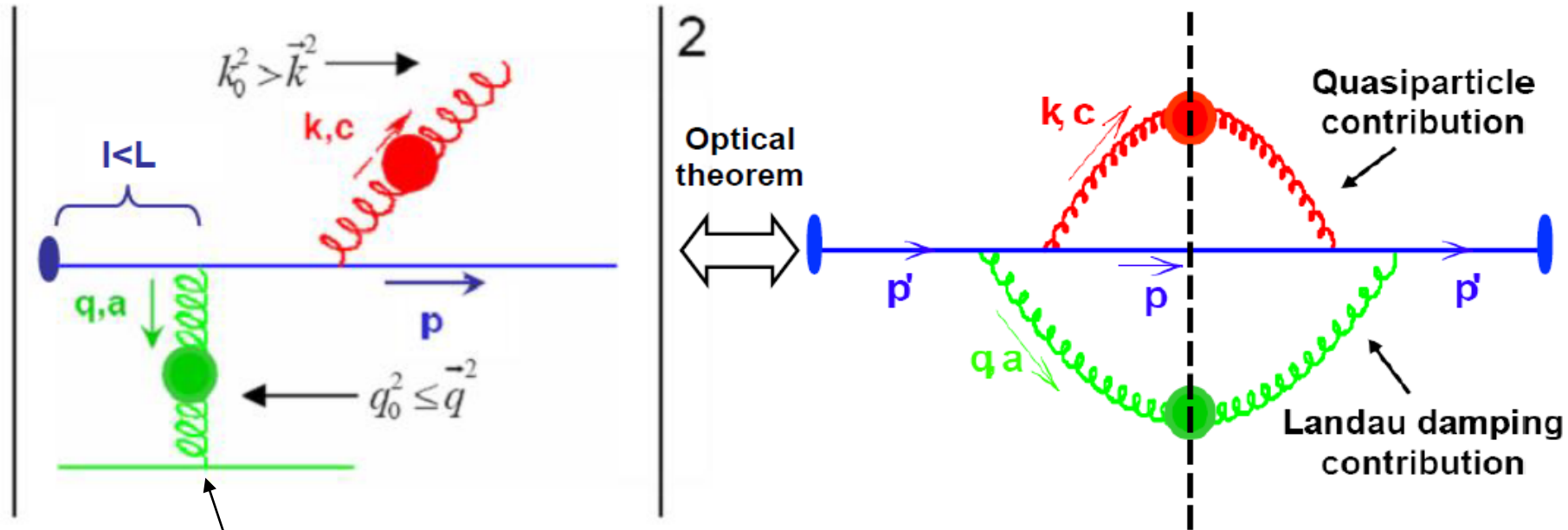
The effective gluon propagator:

$$D^{\mu\nu}(\omega, \vec{q}) = -P^{\mu\nu} \Delta_T(\omega, \vec{q}) - Q^{\mu\nu} \Delta_L(\omega, \vec{q})$$

Transverse and longitudinal gluon propagators
in Hard Thermal Loop approximation

The dynamical energy loss formalism (Djordjevic)

Radiative energy loss: radiation of one gluon induced by one collisional interaction with the medium.



first (lowest) order in number of scattering centers: first opacity expansion

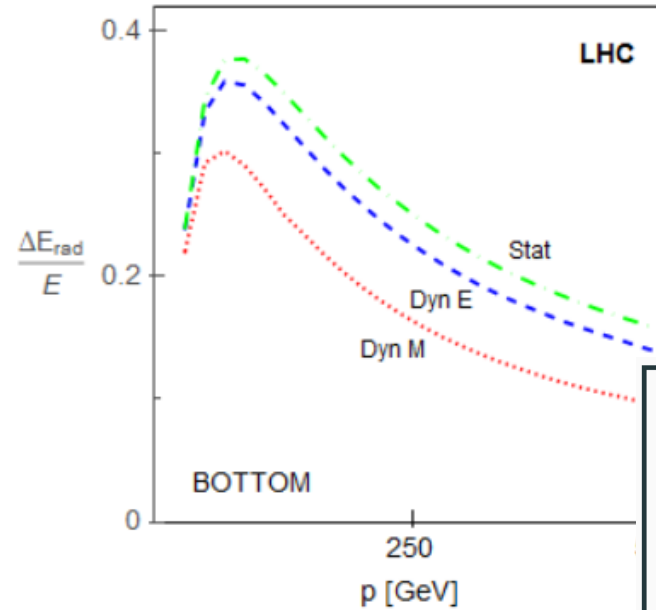
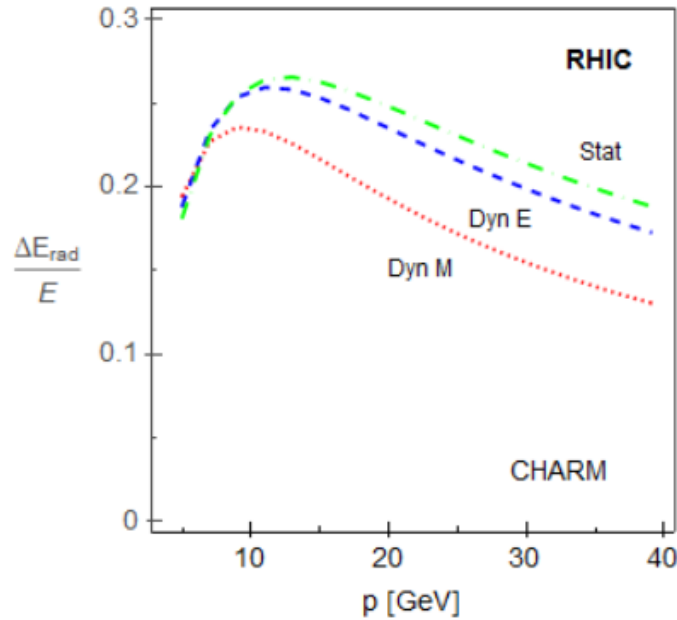
Effect of chromo-electric and chromo-magnetic contributions

$$\frac{\Delta E_{\text{dyn}}}{E} = \frac{C_R \alpha_s}{\pi} \frac{L}{\lambda_{\text{dyn}}} \int dx \frac{d^2 k}{\pi} \frac{d^2 q}{\pi} \frac{\mu^2}{q^2 (q^2 + \mu^2)} \left(1 - \frac{\sin \frac{(k+q)^2 + \chi}{xE^+} L}{\frac{(k+q)^2 + \chi}{xE^+} L} \right) \times 2 \frac{(k+q)}{(k+q)^2 + \chi} \left(\frac{(k+q)}{(k+q)^2 + \chi} - \frac{k}{k^2 + \chi} \right),$$

Interference of various contributions along the path length (strong reduction at short path length)

The dynamical energy loss formalism (Djordjevic)

Radiative energy loss

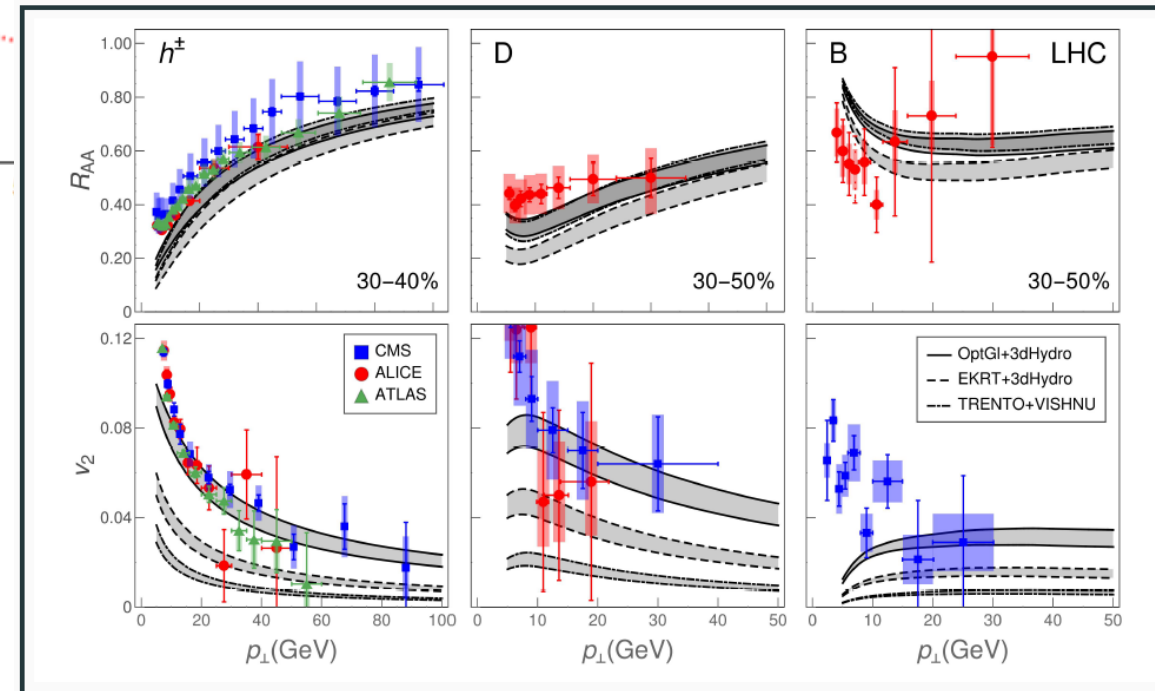


M.D., J.Phys.G39:045007, 2012

Dyn = Dyn M + Dyn E > Stat => **larger Energy Loss with the dynamical formalism**

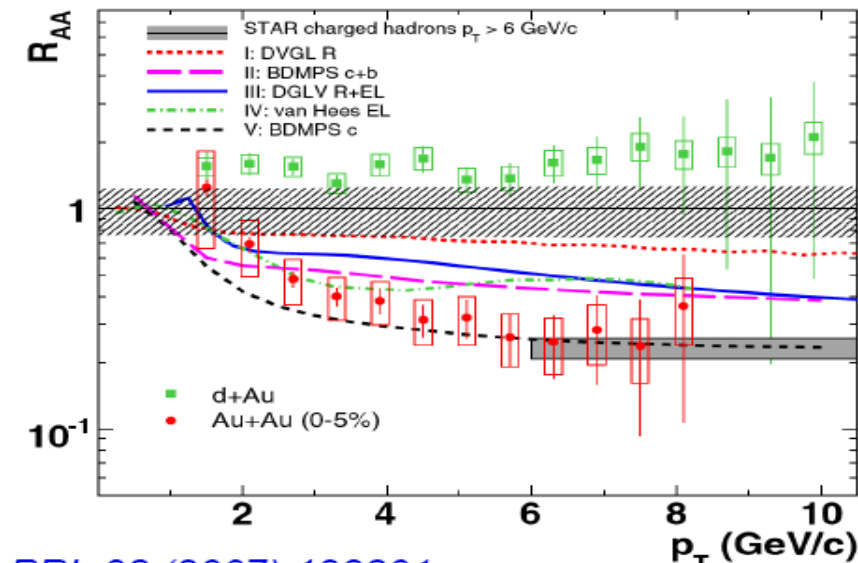
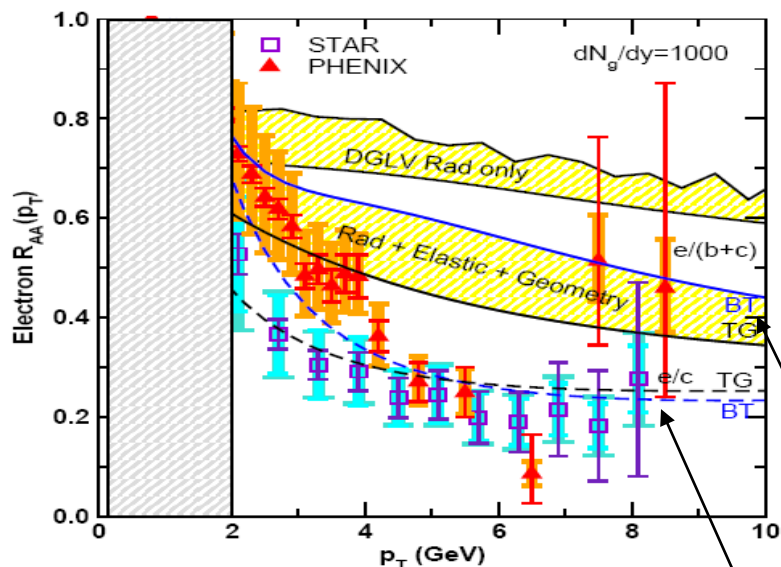
Evolved towards the DREENA framework

ebe-DREENA framework as a QGP tomography tool, Dusan Zigic (Explore QGP, Belgrade, 2023)



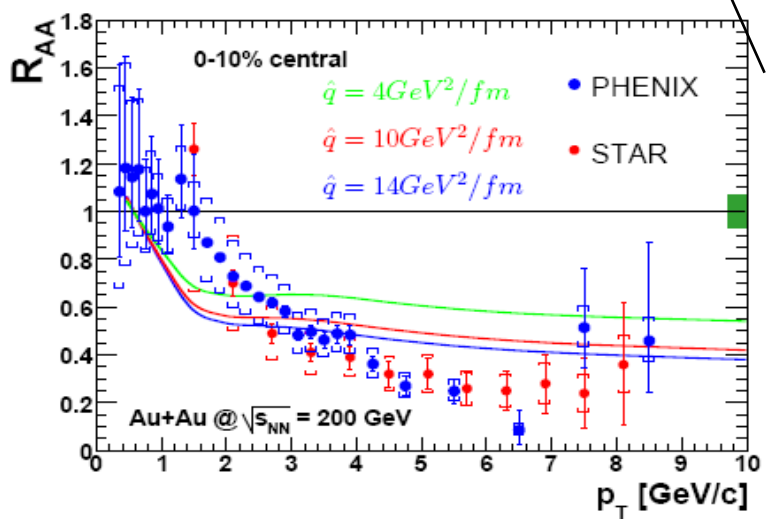
DREENA-A

Back to the future (2005)... more Eloss than expected from pQCD, even adding elastic part (often neglected up to then)

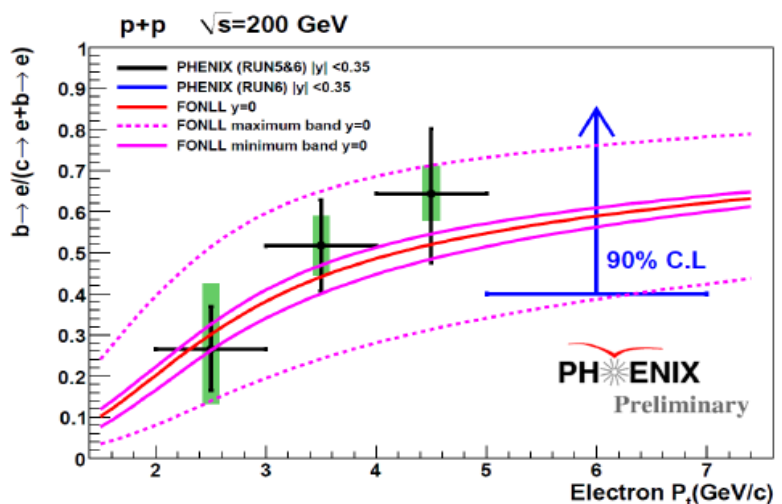


PRL 98 (2007) 192301

Elastic Eloss strikes back (Mustafa 05, Dutt Mazumber 05)



b quark is the puzzle and is definitively there:



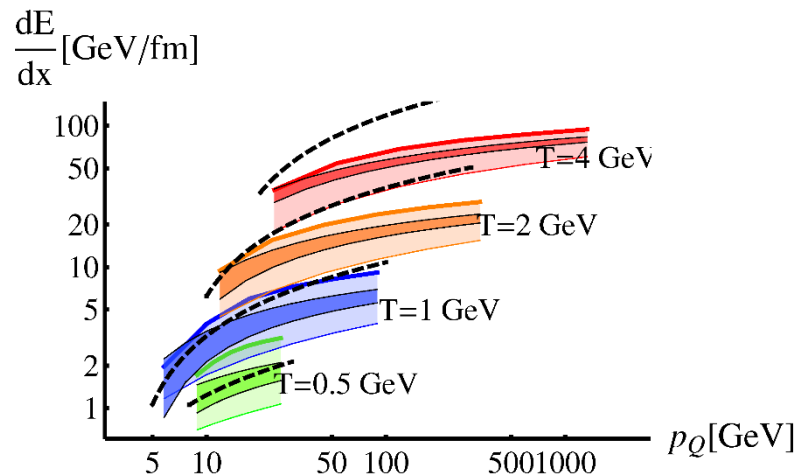
Meanwhile: $dN/dy(y \approx 0)$ scales like N_{bin}

pQCD inspired models (f.i. Nantes, 2008)

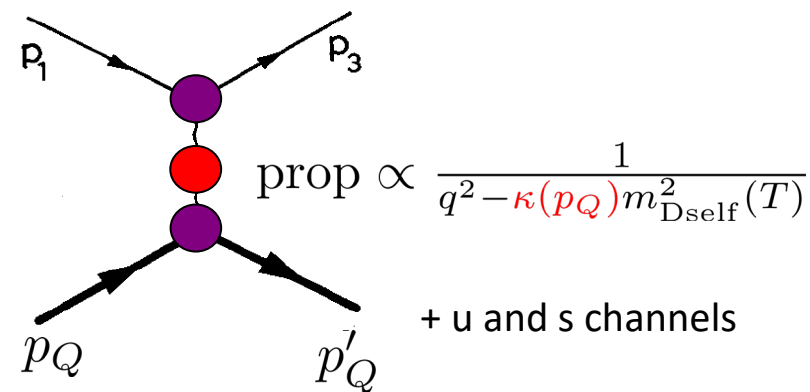
Collisional component

- One-gluon exchange model: reduced IR regulator $\kappa m_{\text{Dself}}^2$ in the hard propagator, fixed on HTL
Energy loss at large momentum (maximal insensitivity of dE/dx on q^*)
- Running coupling $\alpha_{\text{eff}}(t)$
- self consistent Debye mass

$$m_{\text{Dself}}^2(T) = (1+n_f/6) 4\pi\alpha_{\text{eff}}(m_{\text{Dself}}^2)T^2$$

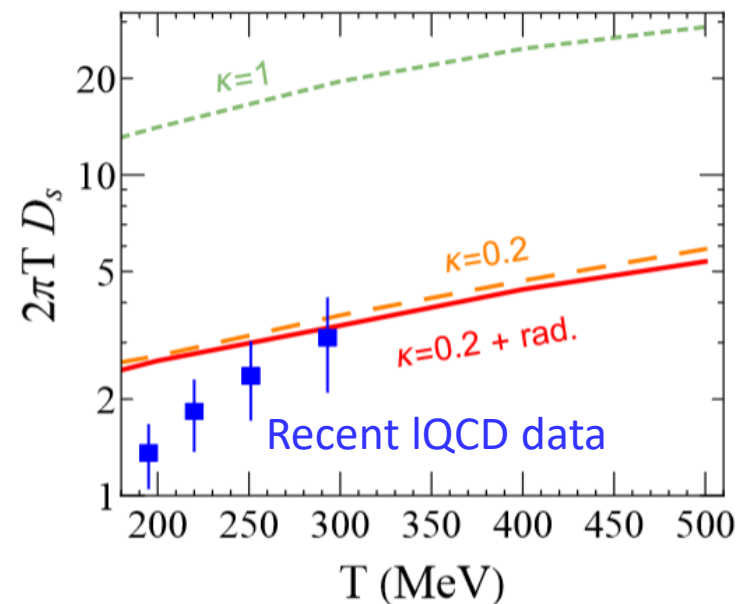


Comparison with Peigné-Peshier (2008) at finite momentum



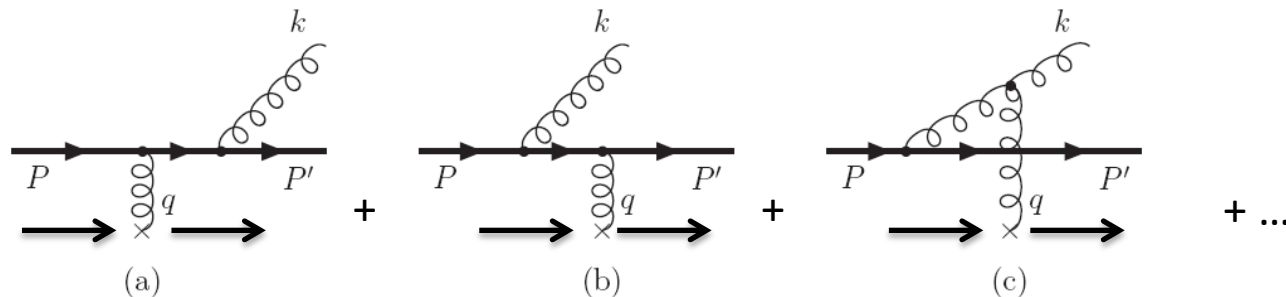
$$\text{prop} \propto \frac{1}{q^2 - \kappa(p_Q)m_{\text{Dself}}^2(T)}$$

$$D_s = \lim_{p_Q \rightarrow 0} T/(M_Q \eta_D) \quad \eta_D : \text{drag coefficient}$$



pQCD inspired models (f.i. Nantes, 2010)

Radiative component

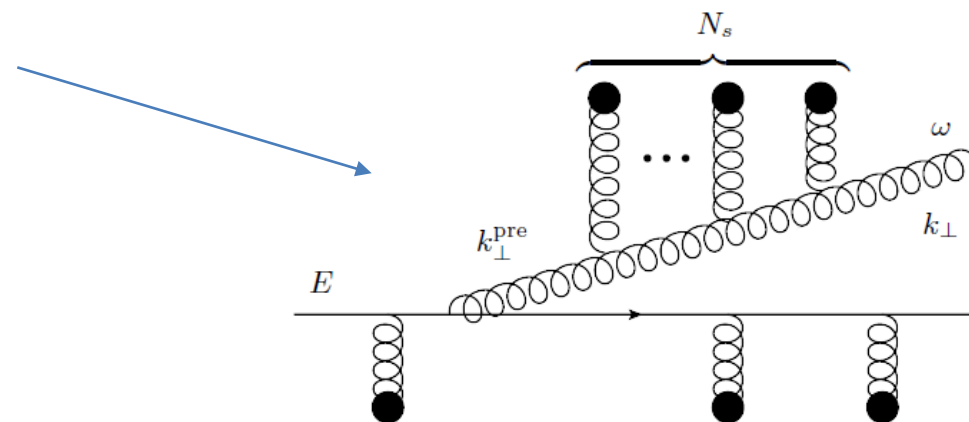


- Extension of Gunion-Bertsch approximation beyond mid-rapidity and to finite mass m_Q) distribution of induced gluon radiation per collision ($\Delta E_{\text{rad}} \propto E L$):

$$P_g(x, \mathbf{k}_\perp, \mathbf{q}_\perp, m_Q) = \frac{3\alpha_s}{\pi^2} \frac{1-x}{x} \left(\frac{\mathbf{k}_\perp}{\mathbf{k}_\perp^2 + x m_Q^2} - \frac{\mathbf{k}_\perp - \mathbf{q}_\perp}{(\mathbf{k}_\perp - \mathbf{q}_\perp)^2 + x m_Q^2} \right)^2$$

- LPM / BDMPS-Z effect for intermediate HQ-energy

Implemented in EPOSn-HQ through Boltzmann transport

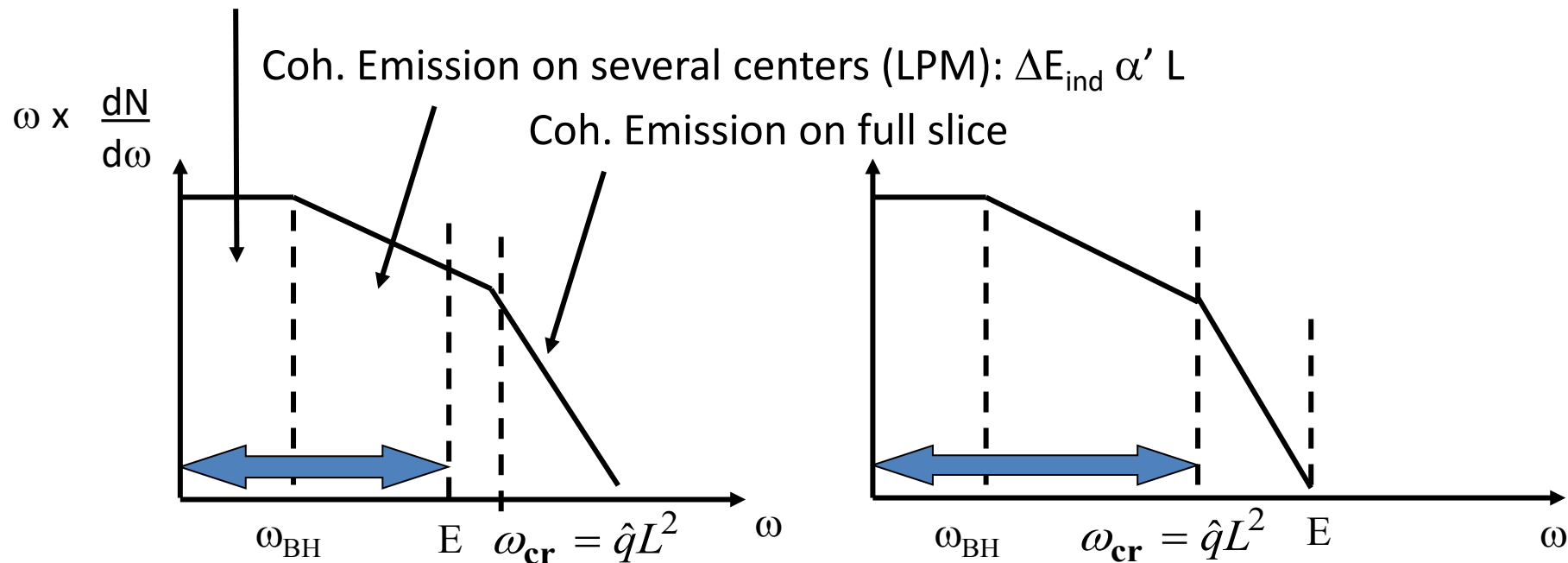


Some open heavy flavor history

01 (DK): contrarily to previous, HQ induced radiation is suppressed w.r.t. light quark: DEAD CONE EFFECT

gluon radiation from *massless* parton

Independent Emission on individual centers (BH): $\Delta E_{\text{ind}} \propto E \times L$



Intermediate E: $\Delta E_{\text{ind}} \propto \sqrt{E\hat{q}} \times L$

Large E: $\Delta E_{\text{ind}} \propto \hat{q} \times L^2$

Some open heavy flavor history

Soft gluon emission in hard process

$$\frac{d\theta}{\theta} \rightarrow \frac{d\theta}{\theta} \times \frac{1}{\left(1 + \left(\frac{M}{E\theta}\right)^2\right)^2}$$

gluon radiation from *massive* parton (initial Bremsstrahlung)

taken at $\theta^2 = \langle \theta^2 \rangle_{\text{BDMPS}} = \sqrt{\frac{\hat{q}}{\omega^3}}$ for $t_f \sim \sqrt{\frac{\omega}{\hat{q}}}$

Suppression factor $DC(\theta)$



Suppression factor $DC(\omega)$

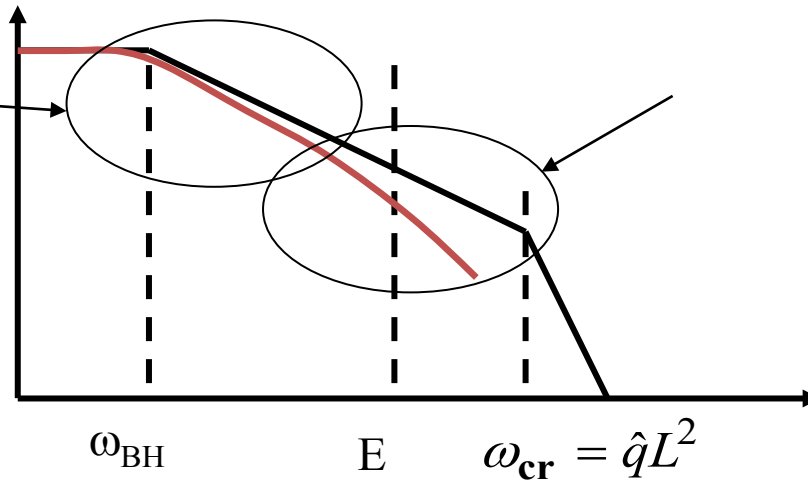


$$\frac{dN}{d\omega} \rightarrow \frac{dN}{d\omega} \times DC(\omega)$$

$$DC(\omega) = \frac{1}{\left(1 + \left(\frac{\omega}{\omega_{\text{DC}}}\right)^{3/2}\right)^2}$$

New scale $\omega_{\text{DC}} = \left(\frac{E^4 \hat{q}}{M^4}\right)^{1/3}$

Moderate for quenching



Important for ΔE

So-called “mass effect”



Quasi particle models (f.i DQPM)

- Nonperturbative effects near T_c are captured by $\alpha_s(T)$, leading to thermal masses/widths, determined from fits to IQCD EoS.

A. Peshier et al. PLB 337 (1994), PRD 70 (2004); M. Bluhm et al. EPJC 49 (2007); W. Cassing et al. NPA 795 (2007)

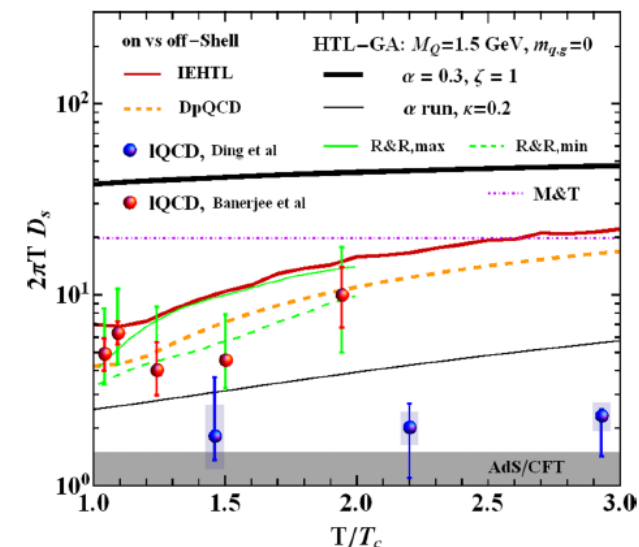
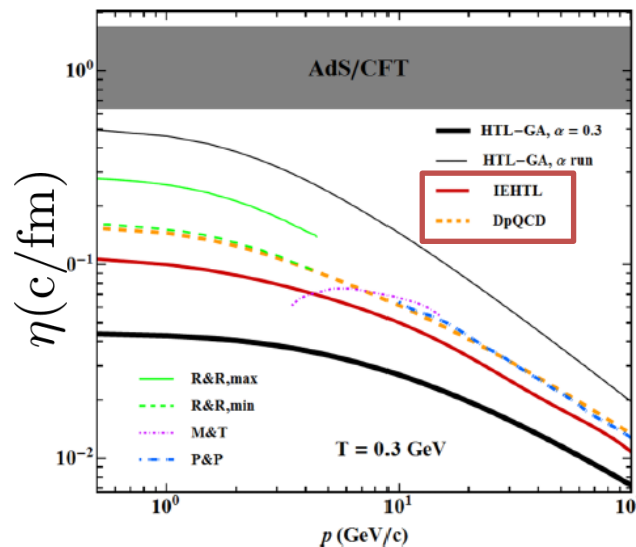
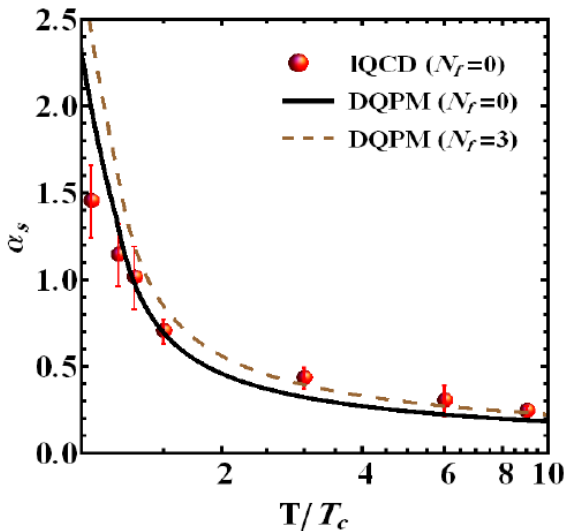
- Coupling between the effective DOF is then taken as $\alpha_s(T) \Rightarrow$ Relaxation rates larger than in pQCD for all T relevant for QGP, slightly smaller than the ones from TAMU

H. Berrehrah et al, PHYSICAL REVIEW C 90, 064906 (2014)

- Implemented for HF dynamics in e.g. PHSD (full off-shell, off-equilibrium transport).

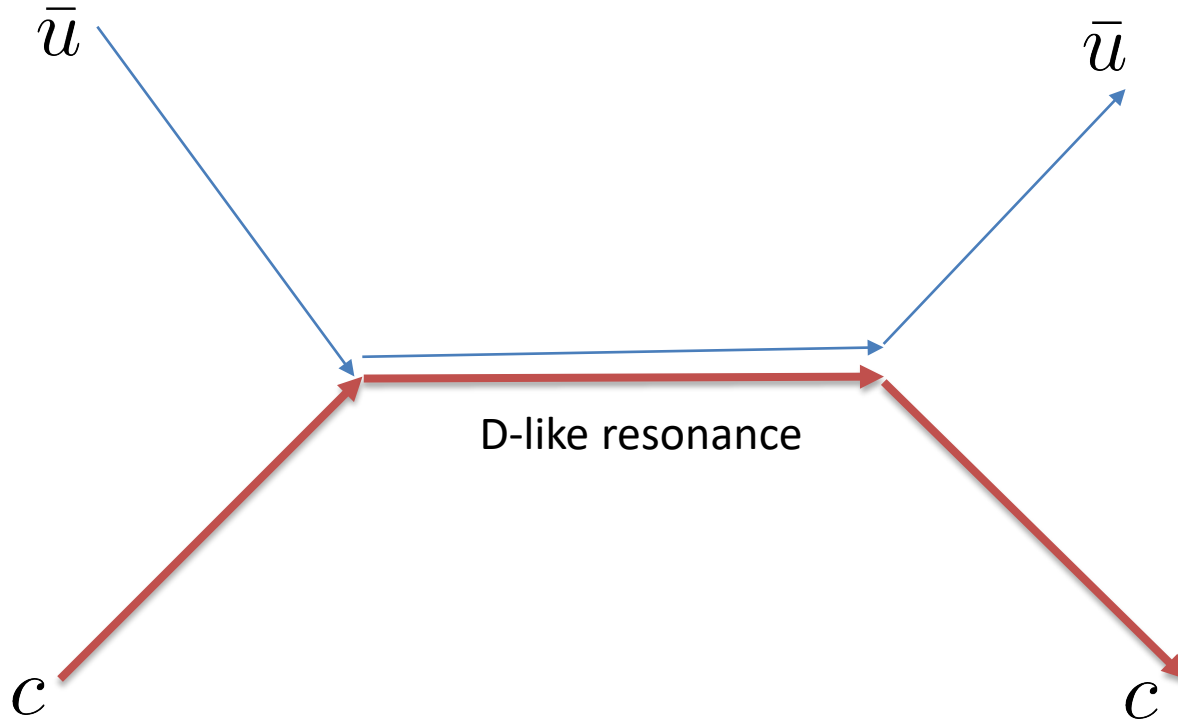
T. Song et al. PRC 92 (2015), PRC 93 (2016)

See also CATANIA



more thermalisation than expected from pQCD, some ways out

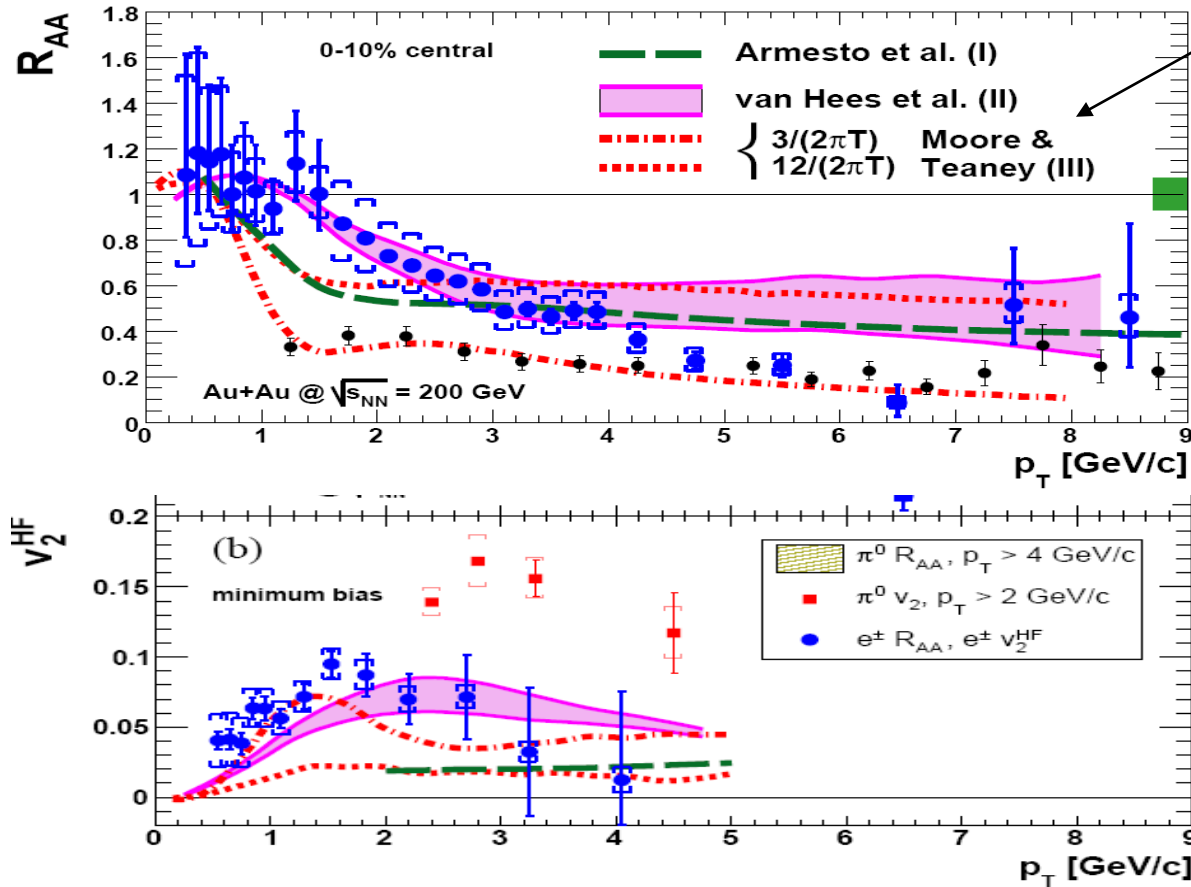
- Rapp and Van Hees (2004 ->): pQCD collisional + additional « strength » from quasi-bound D-like states, resorting to Langevin Dynamics



$$\Delta \vec{p} = -\vec{A} \Delta t + \underbrace{\vec{\xi}}_{\text{Random force (fluctuations)}} \quad \text{for each } \Delta t$$

more thermalisation than expected from pQCD, some ways out

RHIC, Au+Au @ 200 GeV/p (≈ 2005)

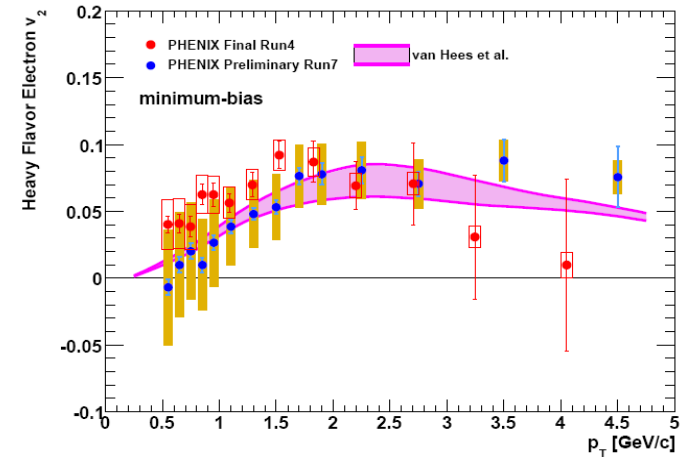


D: spatial diffus coeff ($\text{fm}^2/\text{fm}/c$)

pQCD:
$$D \approx \frac{6}{2\pi T} \left(\frac{0.5}{\alpha_s} \right)^2$$

$\approx 24/(2\pi T)$

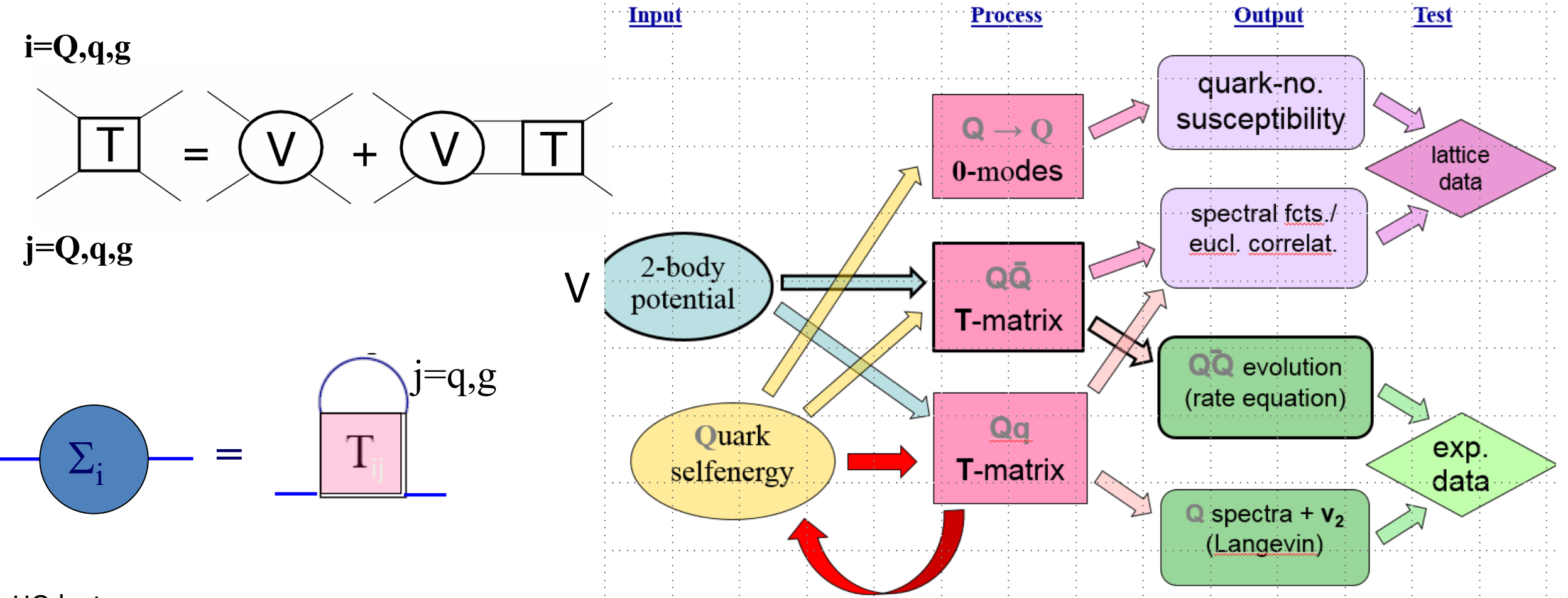
Charm flows !!!



3 representative approach of HQ modelling : pQCD, non perturbative model, data-driven

more thermalisation than expected from pQCD, some ways out

- Rapp and Van Hees (2004 ->): pQCD collisional + additional « strength » from quasi-bound D-like states; then (2008) systematically developed using the T-matrix resummation of a bona-fide 2 body potential including non-perturbative contributions



Potential models (TAMU)

- Thermodynamic T-matrix approach, $T = V + VGT$, given by a two-body driving kernel V , estimated from the IQCD internal/free energy for a static Q-Qbar pair; increase of coupling with QGP at small momentum

D. Cabrera, R. Rapp PRD 76 (2007); H. van Hees, M. Mannarelli, V. Greco, R. Rapp PRL 100 (2008)

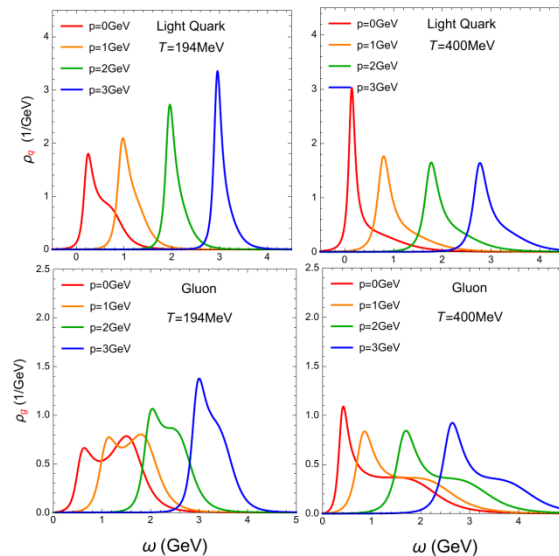
- Comprehensive sQGP approach for the EoS, light quark & gluon spectral functions, quarkonium correlators and HQ diffusion.

F. Riek, R. Rapp PRC 82 (2010); S. Liu, R. Rapp arxiv:1612.09138

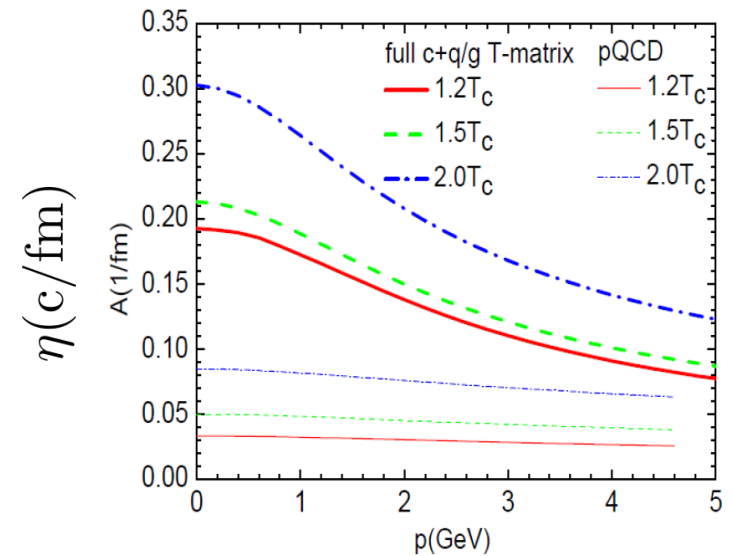
- Resonance correlations in the T-matrix naturally lead to recombination (resonance recombination model) near T_c from the same underlying interactions!

M. He, R. Fries, R. Rapp PRC 82 (2010), PRC 86 (2012)

- Implementation through Langevin dynamics in hydro evolution or in URQMD also corresponds to the disappearance of well defined quasi particles (for which Boltzmann breaks down while Langevin still holds)



No good q-particle at low p



Large coupling at small p_Q

The weak to strong axis for HQ

Bona fide pQCD
(WHDG, ASW, ...)

$\hat{q} \approx 1 \text{ GeV}^2/\text{fm}$

So-called “Failure of pQCD approach” aka “the non photonic single electron puzzle”

“Optimized” pQCD

Running α_s (Peshier, Gossiaux & Aichelin, Uphoff & Greiner)

Distorsion of heavy meson fragmentation functions due to the existence of bound mesons in QGP, R. Sharma, I. Vitev & B-W Zhang 0904.0032v1 [hep-ph]

Bound states diffusion or non-perturbative, lattice potential scattering models (see R. Rapp and H Van Hees 0903.1096 [hep-ph] for a review)

Lesson n°1 (pre LHC):

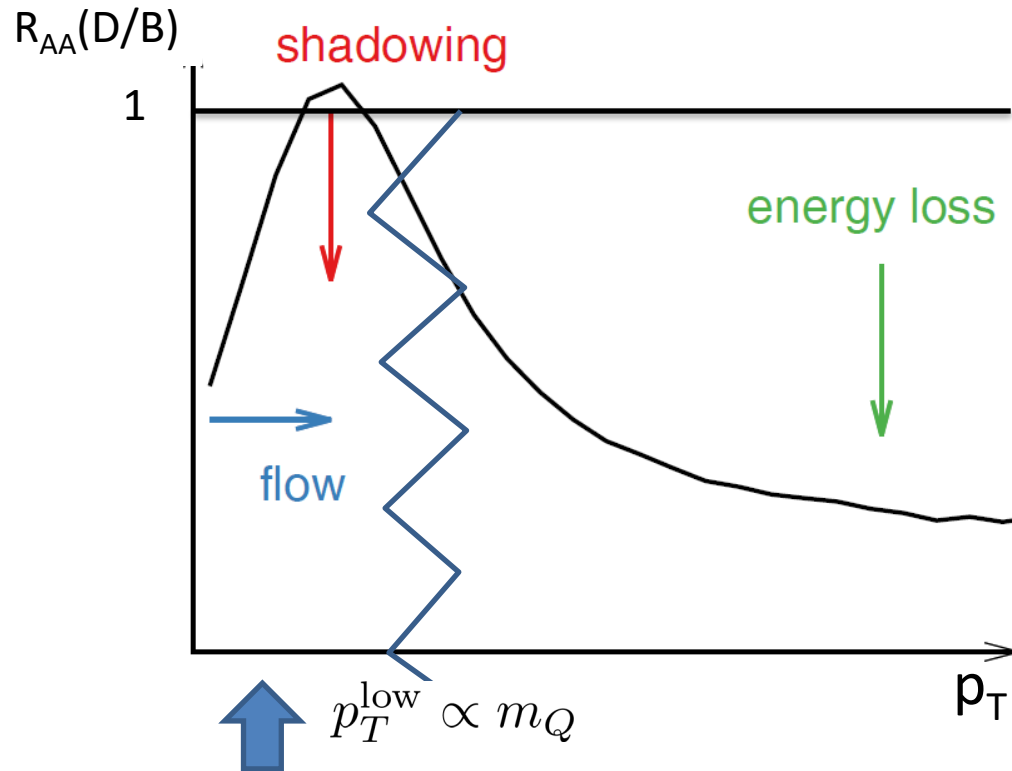
Several models containing either non perturbative features or tunable parameters are able to reproduce the HQ data, but many questions remain... and how to reconcile them all stays a challenge

QPM (Catania, PHSD)

AdS/CFT (Akamatsu et al, W. Horowitz)

Basic Consequences of HQ interaction with QGP for the R_{AA}

The pattern seen in the data



- Dominated by elastic interactions
- $m_Q \gg T \Rightarrow$ needs « many » collisions to equilibrate
- Physics close to « Langevin »

The acknowledged effects

Flow bump: due to

- *(radial) flow of the medium* and coupling at small p_T
- *recombination with light quarks*

shadowing: due to *initial state nuclear effects*

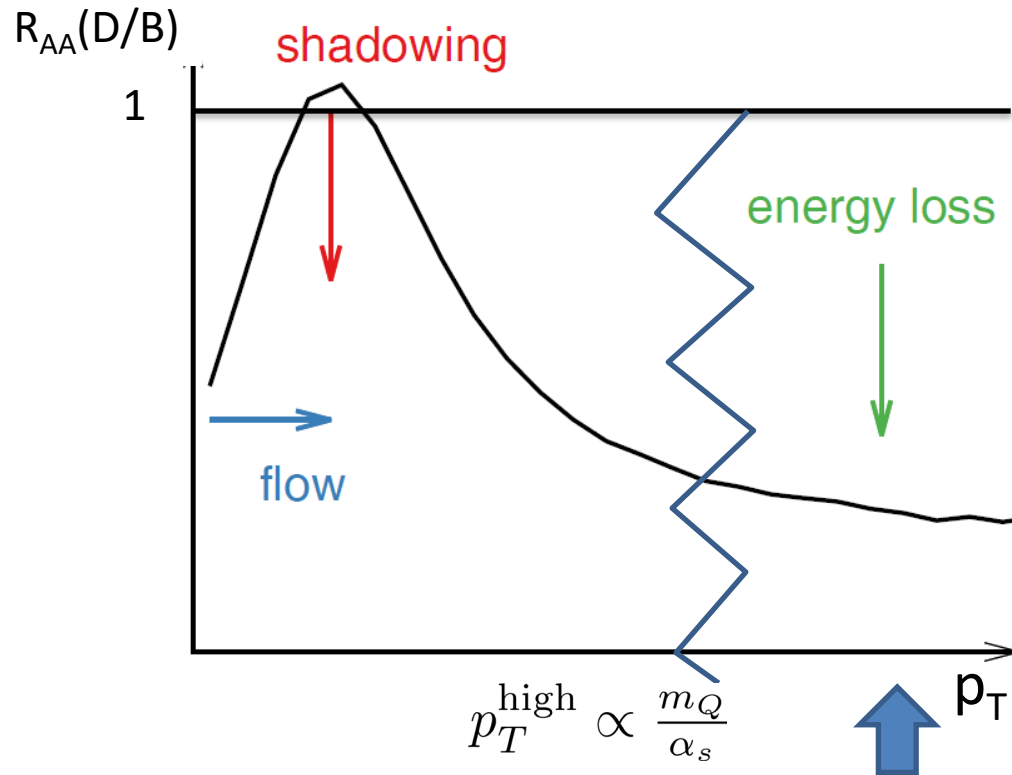
Quenching & energy loss: due to

- elastic and *inelastic* scatterings
- *opacity of the medium*

Italic: extrinsic to the HF coupling with QGP AKA « energy loss model »

Basic Consequences of HQ interaction with QGP for the R_{AA}

The pattern seen in the data



The acknowledged effects

Flow bump: due to

- (radial) flow of the medium and coupling at small p_T
- recombination with light quarks

shadowing: due to initial state nuclear effects

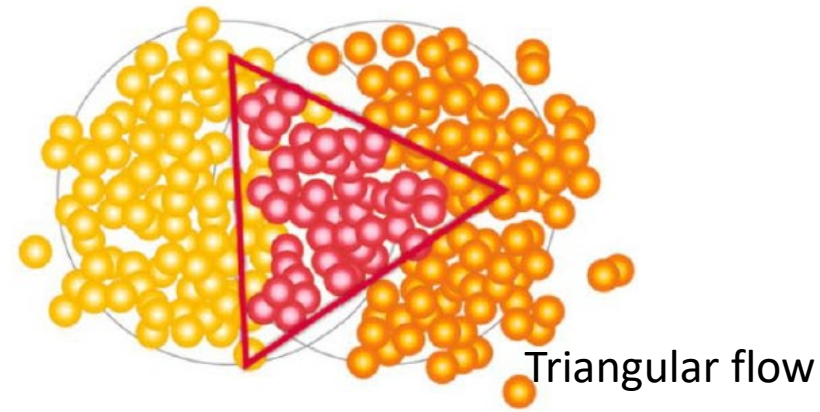
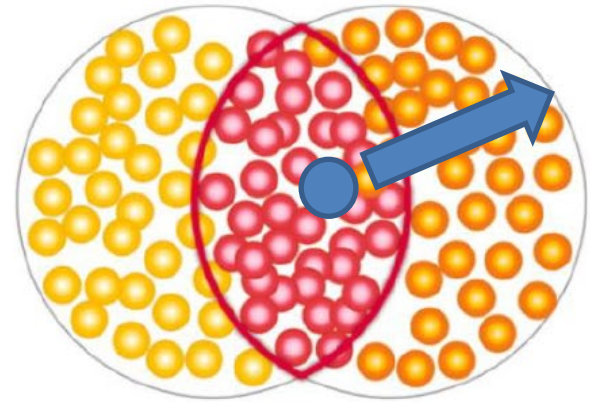
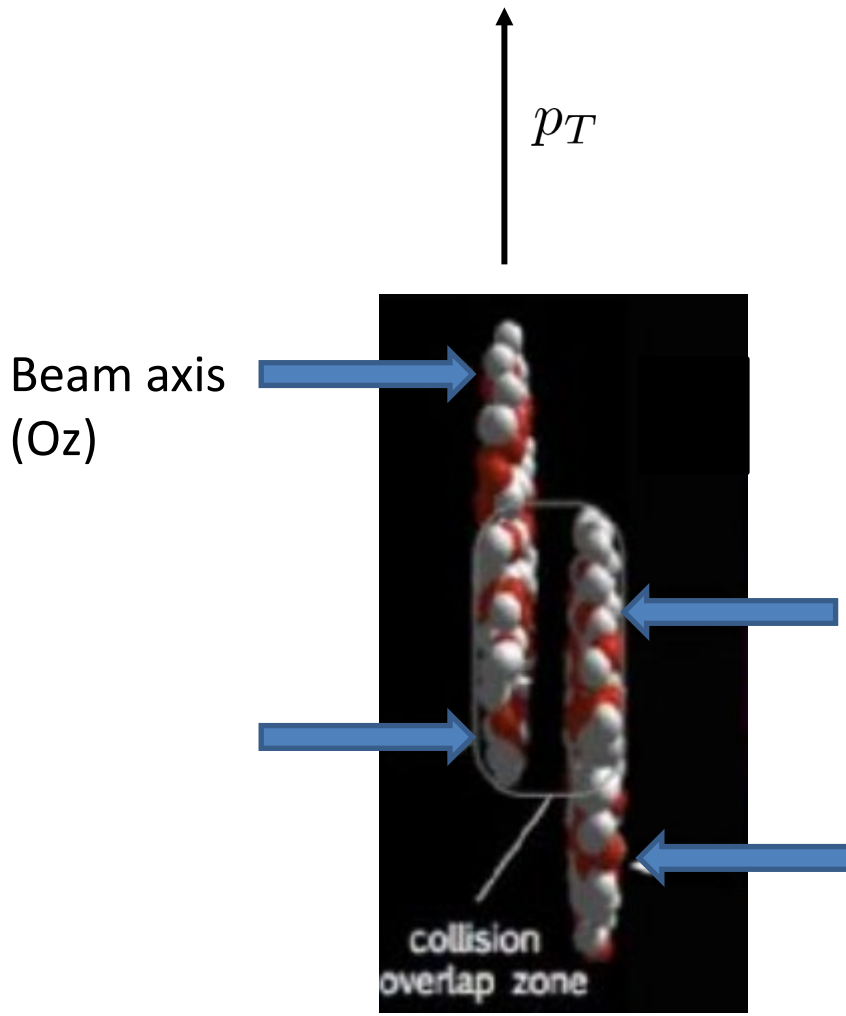
Quenching & energy loss: due to

- elastic and inelastic scatterings
- opacity of the medium

- **Dominated by radiative energy loss** (with important coherence effects: $\Delta E_{\text{rad}} \propto C_A \hat{q} L^2$)
- Eikonal regime (propagation along straight lines)
- 1 single transport coefficient dominates the whole physics: $\hat{q} \propto \kappa_T$
- HQ do not equilibrate with the medium
- **m_Q becomes a subscale of the physics** ($m_Q \ll p_T$)

Basic Consequences of HQ interaction with QGP for the R_{AA}

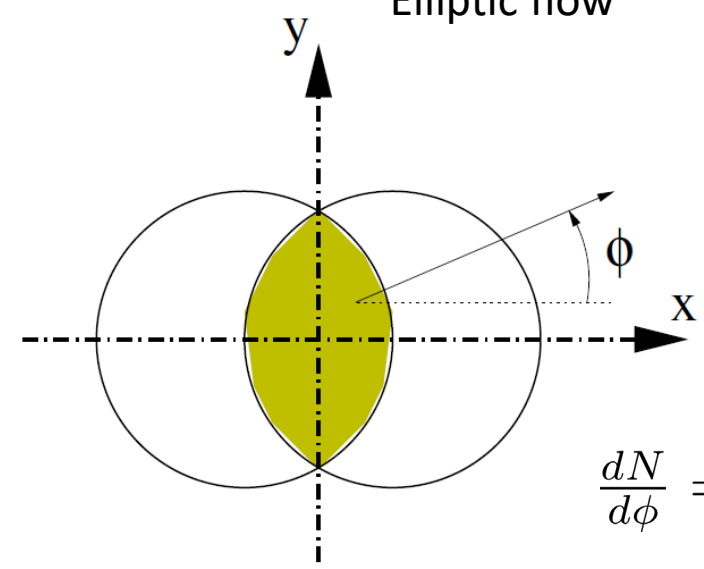
Initial stage of the collisions seen in the transverse plane: Non spherical initial spatial distribution due to eccentricity + fluctuations



... later on converted in anisotropies due to the fluid dynamics evolution.



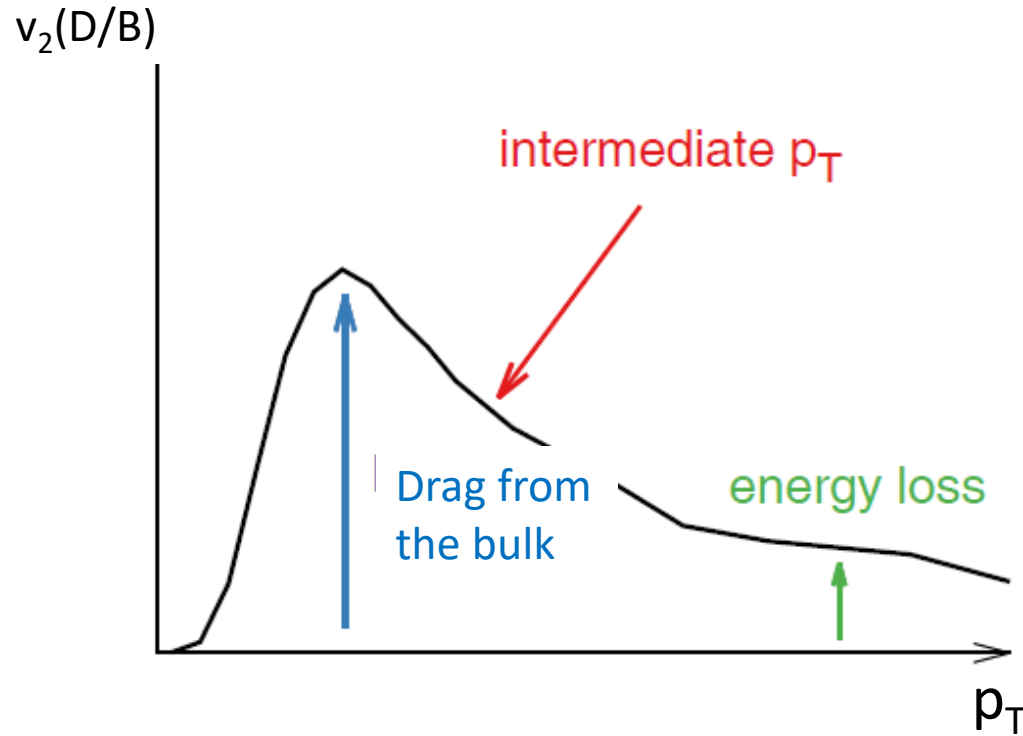
anisotropies in the final hadrons
azimuthal distributions (Fourier series)



$$\frac{dN}{d\phi} = \frac{N_0}{2\pi} (1 + 2v_2 \cos[2(\phi - \psi_{RP})] + \dots)$$

$$v_2 = \langle \cos[2(\phi - \psi_{RP})] \rangle$$

Basic Consequences of HQ interaction with QGP for the v_2



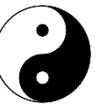
Small p_T : height of v_2 at low p_T sensitive to:

- Bulk anisotropy, mostly at the late times
- The drag force acting locally on HF

high p_T non-0 v_2 is due to anisotropic Eloss (same ingredients as for the R_{AA} + geometrical anisotropy of initial distribution of matter)

intermediate p_T : onset and offset of many competing effects.

!!! Alternative pointed out recently within transport model (AMPT & MPC) study: so-called « escape mechanism » characterized by a large v_2 component stemming from $N_{coll} \approx 1$



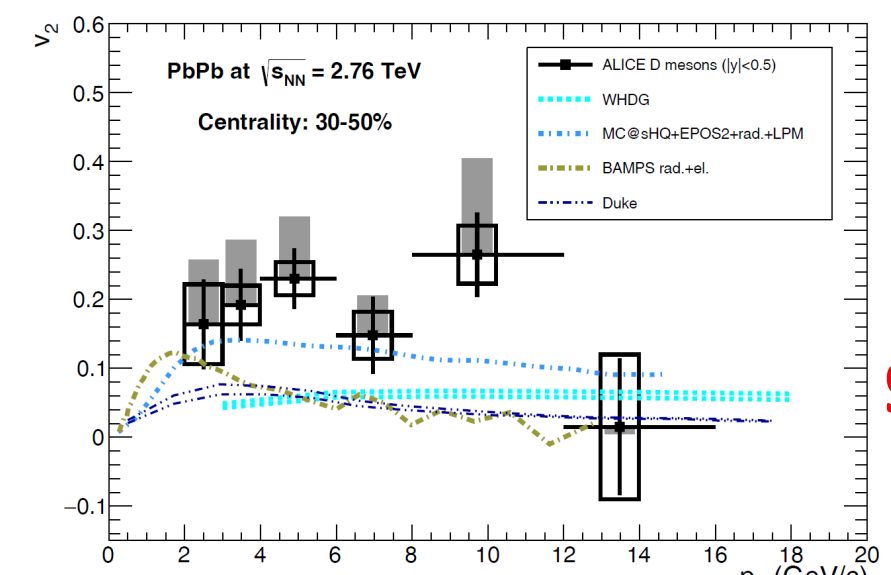
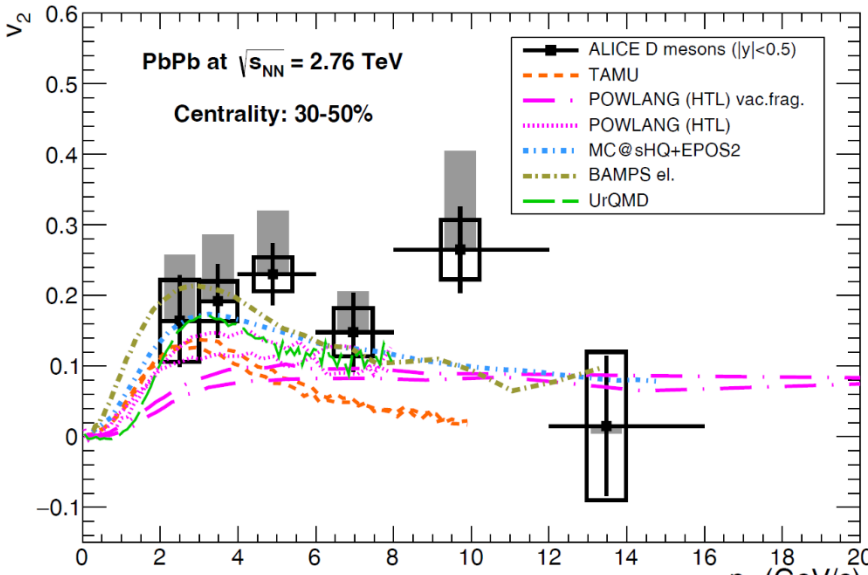
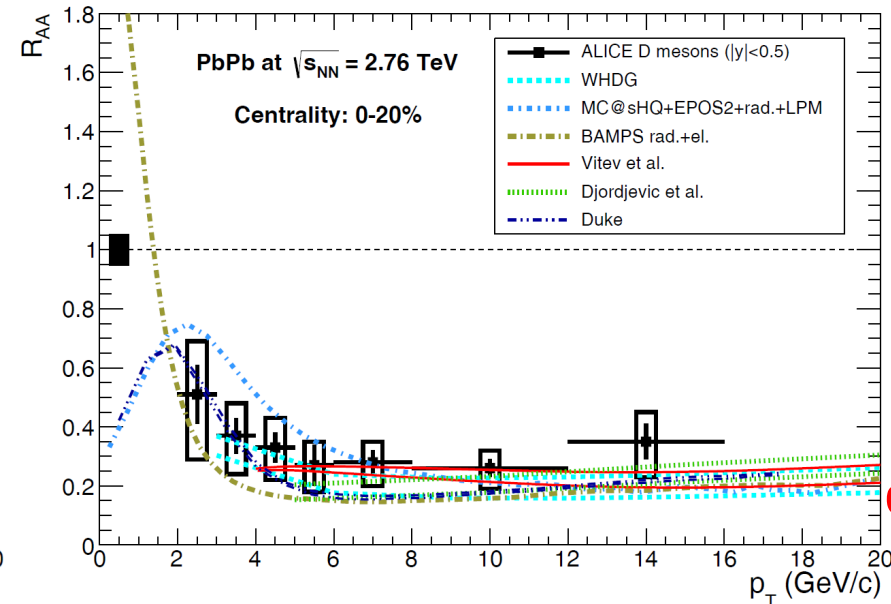
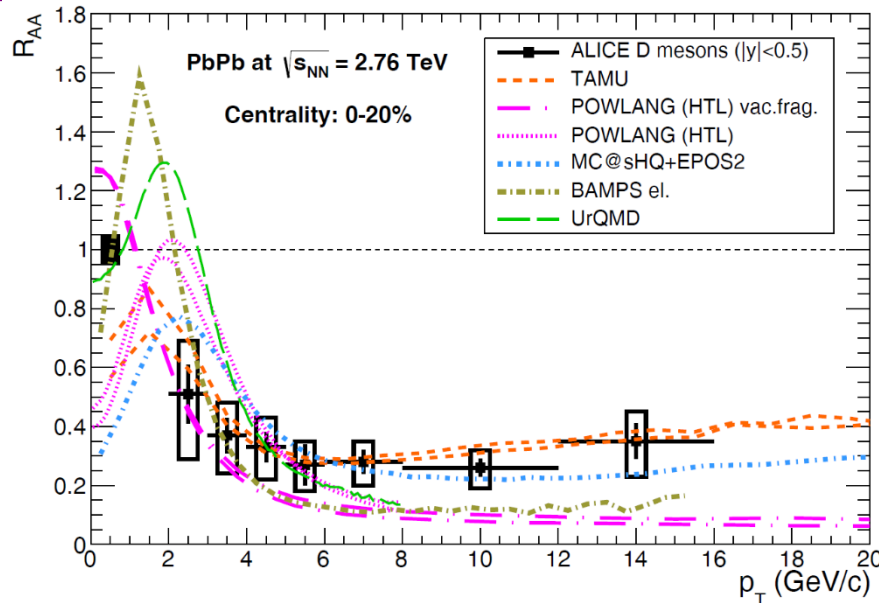
L. He et al, Physics Letters B753 (2016) 506

2 Important remarks:

- Any energy loss model, even the roughest one, will generate these typical structures in the R_{AA} and the v_2 . Getting a correct **quantitative** agreement is much more involved.
- Quantitative predictions also depends on some « extra ingredients » (hydro, initial conditions,...)

Models vs DATA at LHC (Saporo Gravis Report compilation)

Purely elastic scatterings

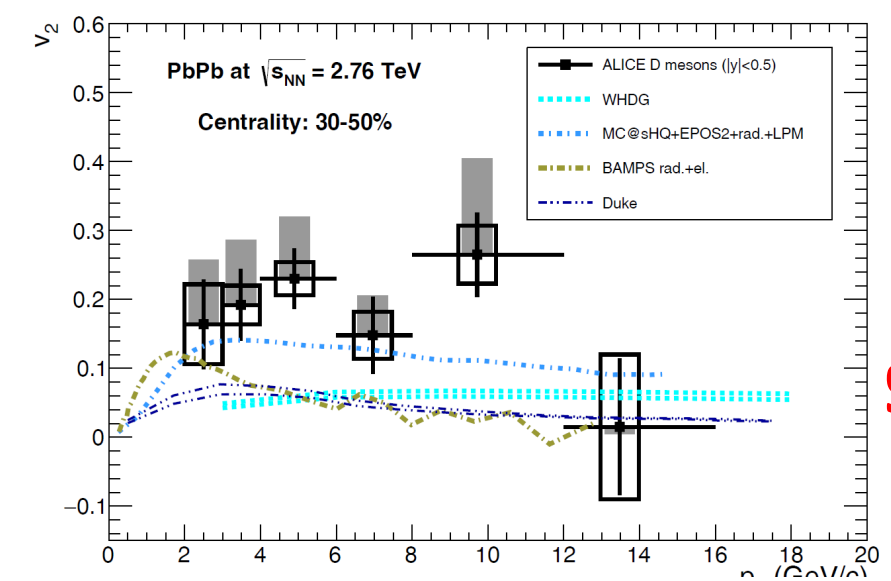
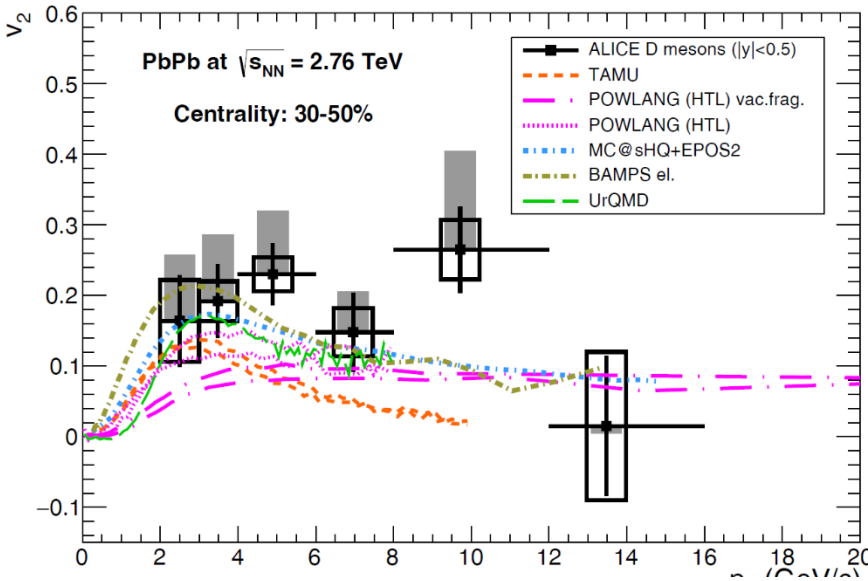
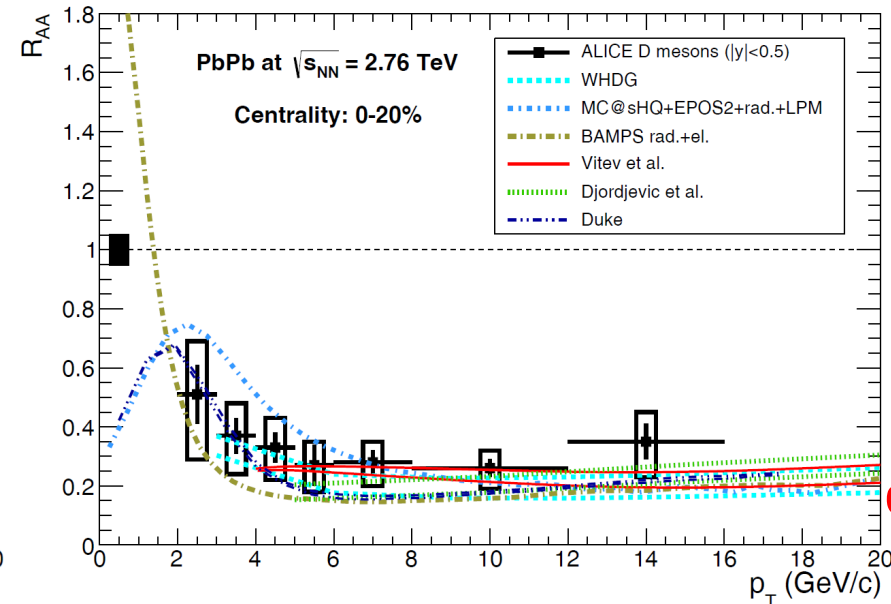
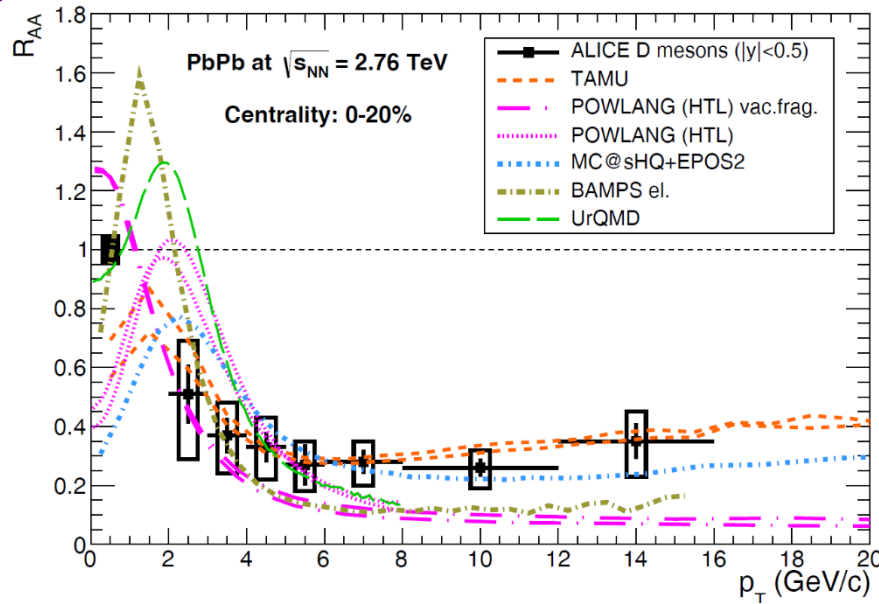


Elastic scatterings + radiative energy loss

Despite various prescriptions for Energy loss, a lot of models can cope with the data

Models vs DATA at LHC (Saporo Gravis Report compilation)

Purely elastic scatterings



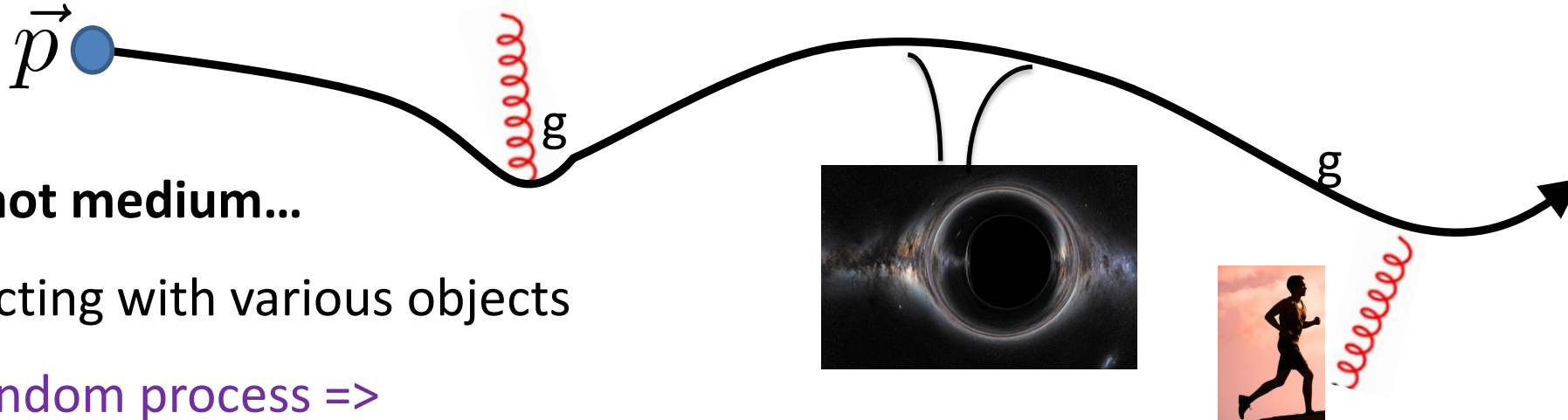
Elastic scatterings + radiative energy loss

Some historical advocated tension between R_{AA} and v_2

Going a bit deeper...

- How can we compare the energy loss models ?
- Are there any energy loss calculation stemming from fundamental principles...
- ...and that does not rely on any assumption/vision of the QGP in terms of (effective) degrees of freedom ?

Transport coefficients



HQ in hot medium...

... interacting with various objects

Quasi random process =>

$$-\frac{d}{dt} \langle \vec{p} \rangle = \vec{A}(\vec{p}, T) = \eta_D(\vec{p}, T) \times \vec{p} \quad \eta_D [\text{fm}^{-1}] : \text{Relaxation rate}$$

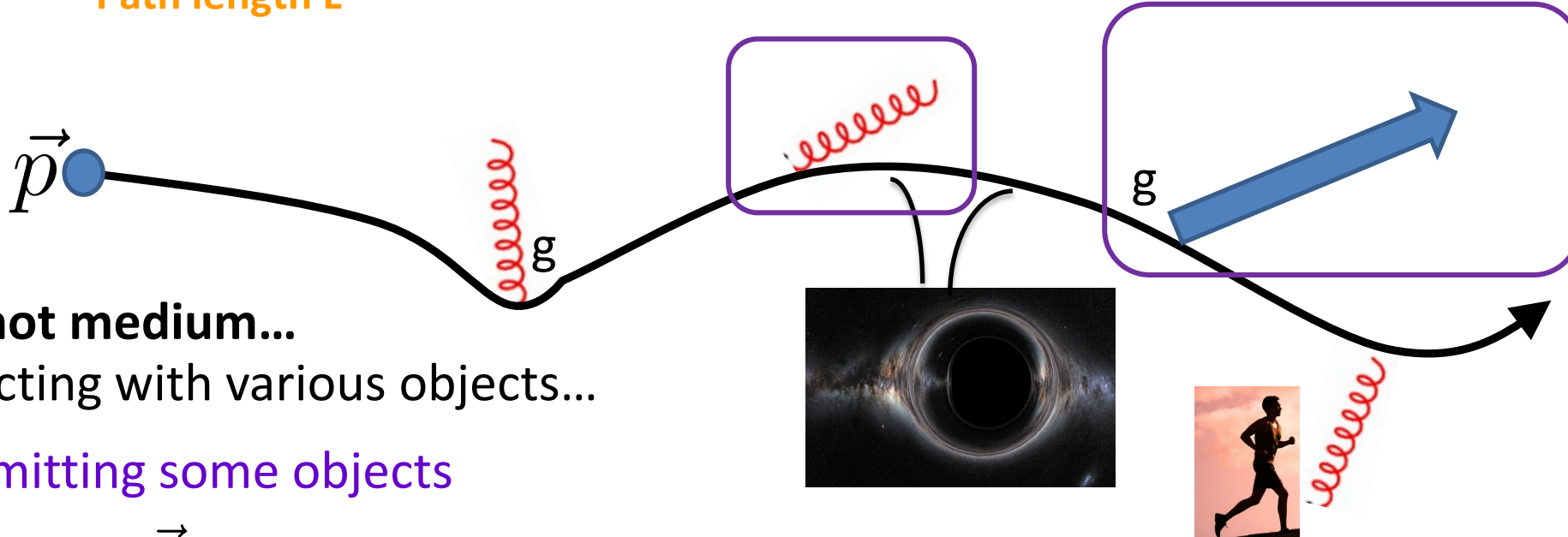
$$\frac{d}{dt} \langle \vec{p}_{T,i} \vec{p}_{T,j} \rangle = \kappa_T(\vec{p}, T) \delta_{i,j} \quad \kappa_T [\text{GeV}^2 \text{fm}^{-1}] : \text{Transverse diffusion coef. (p space); } \hat{q} = 2\kappa_T = 4B_0$$

$$\text{Similar in longitudinal direction} \quad \kappa_L [\text{GeV}^2 \text{fm}^{-1}] : \text{Longitudinal diffusion coef.}$$

In general, no relation between these coefficients except $\kappa_T = \kappa_L$ for $p=0$.

Transport coefficients and inelastic processes

Path length L



HQ in hot medium...

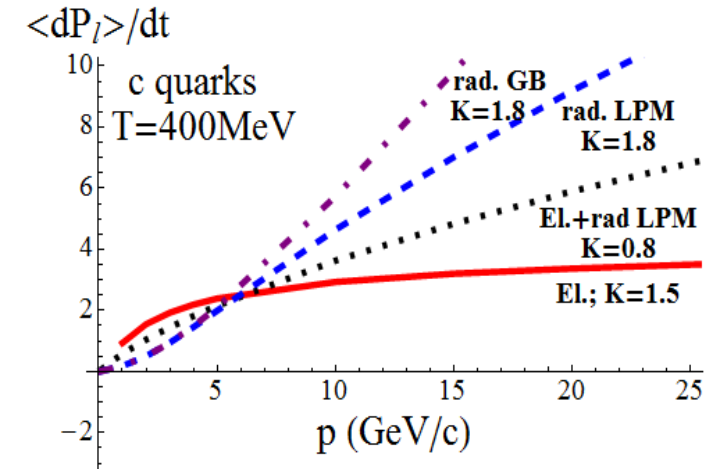
... interacting with various objects...

... and emitting some objects

$$\Delta \langle \vec{p} \rangle = \vec{A}(\vec{p}, T) \times L + \underbrace{(\Delta \vec{p})_{\text{rad}}}_{\text{radiated part}}$$

- contribution from « radiated » part
- In most of existing schemes: $(\Delta \vec{p})_{\text{rad}} = \mathcal{F}(\underbrace{\eta_D, \kappa_T, \kappa_L}_{\text{transport coeff.}}, p, L)$

Seeked transport coeff.



!!! In this case, the relaxation rate $\ll \eta_D$

Transport coefficients at low momentum $p \approx m_Q$

Langevin regime => Einstein relation: $\kappa(0) = 2TE_Q\eta_D(0)$ $\langle r^2(t) \rangle = 2dD_s t$

For historical reasons, physics displayed as a function of $2\pi T$ x the spatial diffusion coefficient

$$\underbrace{(2\pi T)D_s}_{\text{Gauge for the coupling strength}} = \frac{4\pi T^3}{\kappa} = \frac{2\pi T^2}{E_Q\eta_D} \quad \rightarrow \quad \tau_{\text{relax}} = \eta_D^{-1} = (2\pi T)D_s \times \frac{E_Q}{2\pi T^2}$$

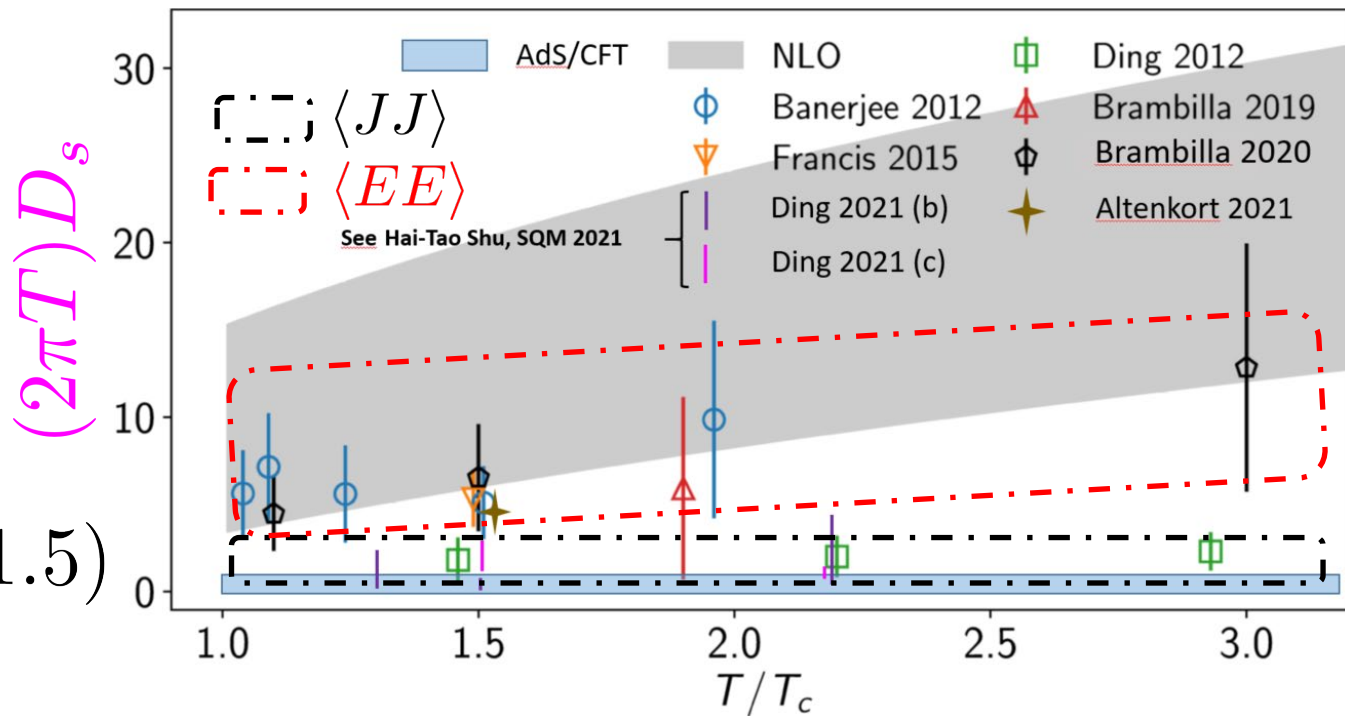
Gauge for the coupling strength

IQCD results

The sole direct rigorous calculation of the transport coeff to my knowledge...
but no dependence on the momentum

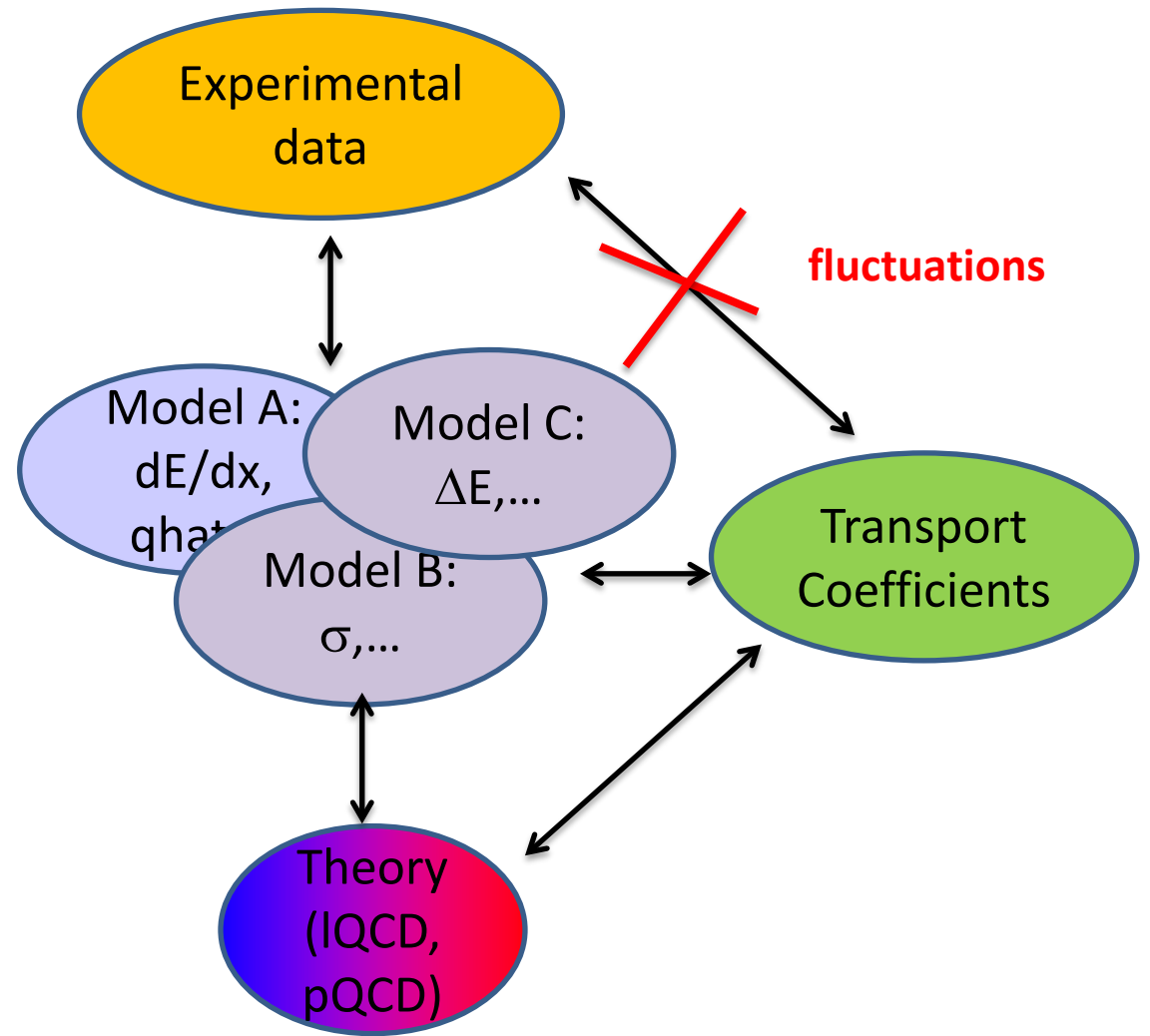
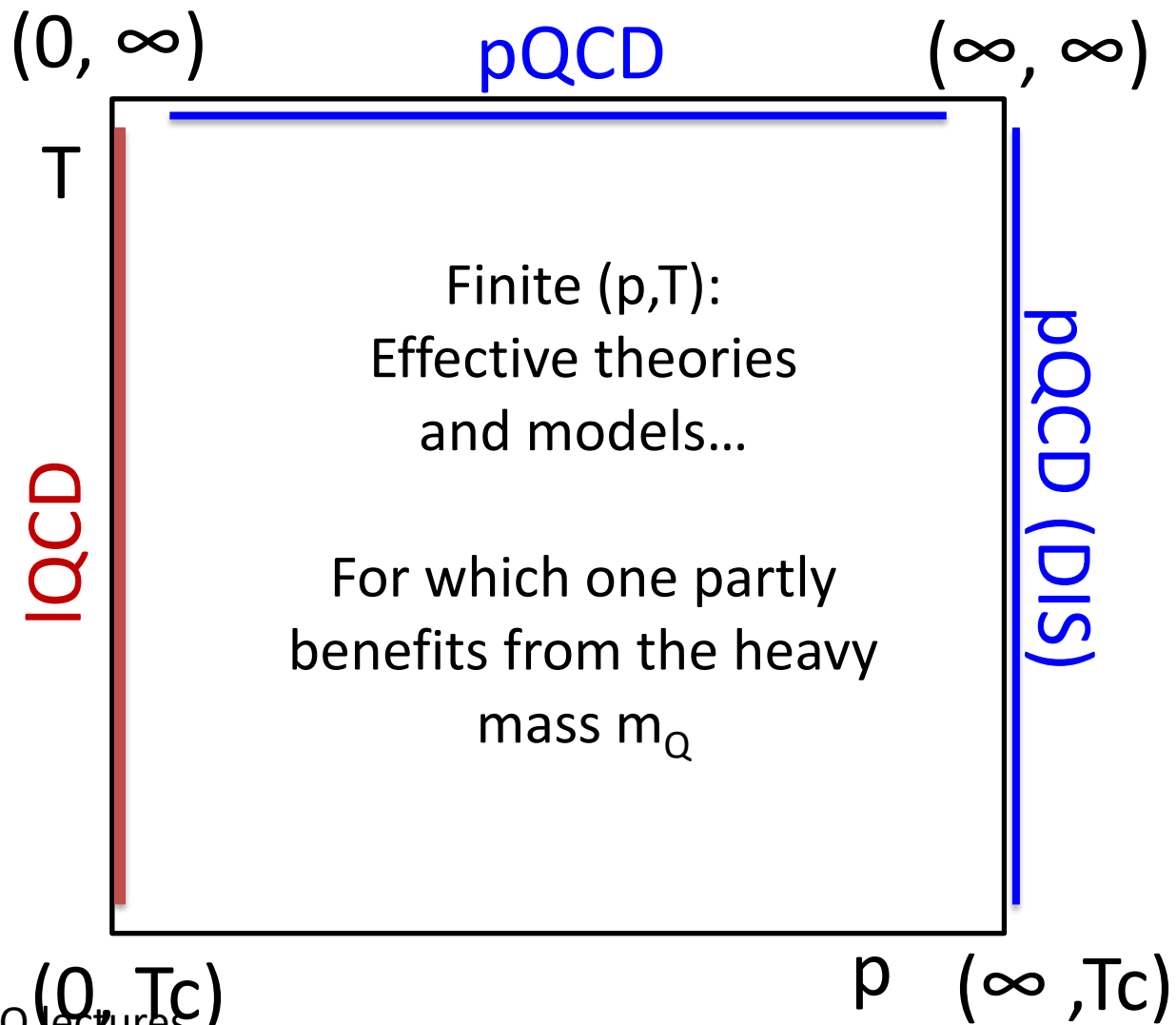
$$\tau_{\text{relax}}(T_c) \approx m_Q [\text{GeV}] \times (3 \pm 1.5)$$

For b: Indeed a hard probe !



Landscape of HF theory and modeling in URHIC

Transp. coeff. $\eta_D, \kappa_T, \kappa_L$



Transport coefficients: theory vs models

Langevin regime => Einstein relation: $\kappa = 2TE_Q\eta_D$

$$\langle r^2(t) \rangle = 2dD_s t$$

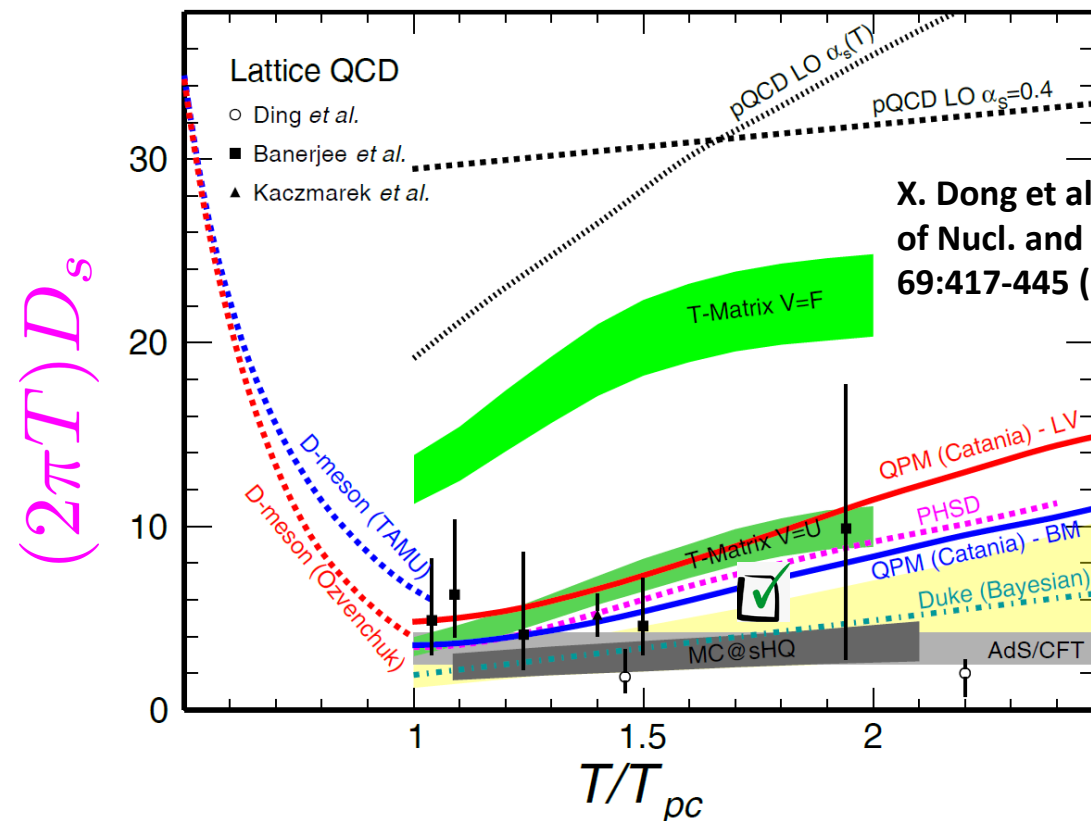
For historical reasons, physics displayed as a function of $2\pi T$ x the spatial diffusion coefficient

$$\underbrace{(2\pi T)D_s}_{\text{Gauge for the coupling strength}} = \frac{4\pi T^3}{\kappa} = \frac{2\pi T^2}{E_Q\eta_D} \quad \rightarrow \quad \tau_{\text{relax}} = \eta_D^{-1} = (2\pi T)D_s \times \frac{E_Q}{2\pi T^2}$$

Gauge for the coupling strength

Most of the models which reproduce exp. observables (✓) are compatible with IQCD constrains...

... But once again : this is just p=0 physics.



X. Dong et al. Annual Review of Nucl. and Part. Science 69:417-445 (2019)

Recent Collective actions beyond Sapore Gravis

- **Heavy Quark – Working Group** (convener: X-N Wang); in the spirit of the Jet Collaboration, the goal is, in a first stage, to :
 - **Collect and compare the transport coefficients from various models,**
 - Measure and understand their consequences by first studying a simpler brick problem
 - Estimate some systematics + uncertainties
- LBL-CCNU (XN Wang, S. Cao)
- Duke (S. Bass , S. Cao, M. Nahrgang, Y. Xu)
- Catania (V. Greco, S. Das, S. Plumari, F. Scardina)
- TAMU (R. Rapp, M. He)
- Frankfurt pHSD (E. Bratkovskaya, T. Song, H. Berrehrah)
- Nantes (J. Aichelin, PB Gossiaux, M. Nahrgang)



Phys.Rev.C 99 (2019) 5, 054907

Heavy-Quark Working Group



After 3 meetings, footprints of the physics start to emerge... but no firm conclusion yet



For step 1: Compare HQ spectra from different models in static medium with common initial condition

Models	note	basic	tune 1
LBL-CCNU ★	fix α_s	$\alpha_s = 0.3$	$\alpha_s = 0.26$
Duke ★	fix α_s	$\alpha_s = 0.3$	$\alpha_s = 0.23$
Catania QPM	$\alpha_s(T)$	$K = 1$	$K = 2$
Catania pQCD	$\alpha_s(T)$	$K = 1$	$K = 3.4$
TAMU	U -potential	no tuning	no tuning
Frankfurt PHSD	$\alpha_s(T)$	no tuning	no tuning
Nantes col. + rad. ★	$\alpha_s(q^2)$	$K = 1$	$K = 0.8$
Nantes col. only	$\alpha_s(q^2)$	$K = 1$	$K = 1.5$

★: Radiative included

Basic: original model

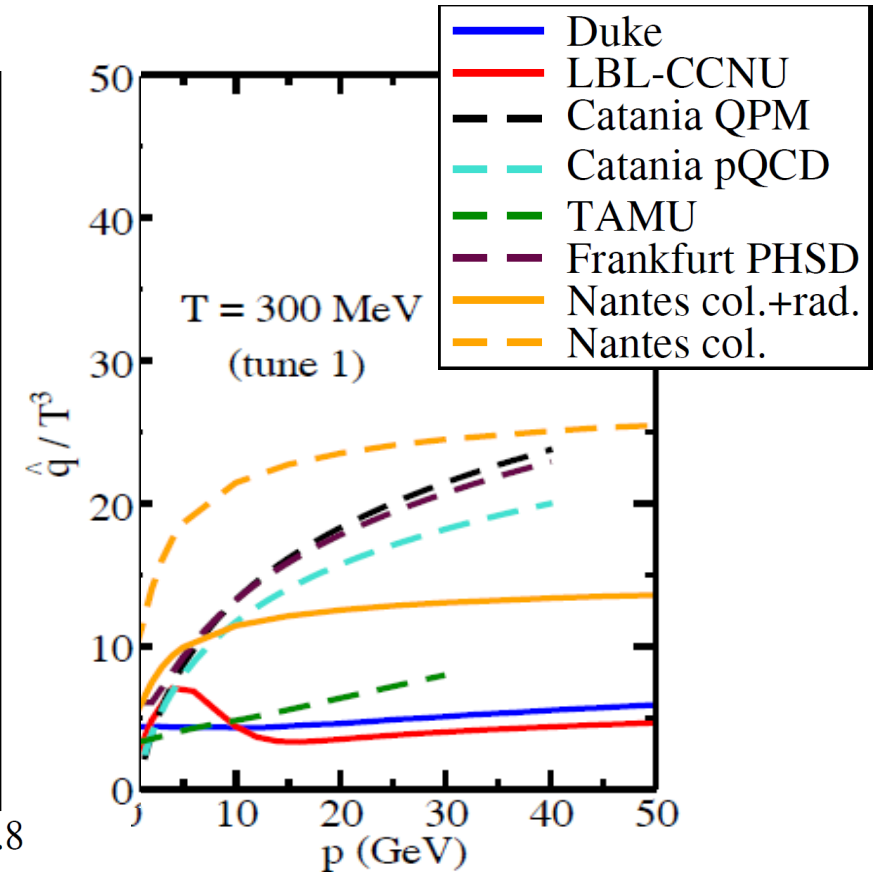
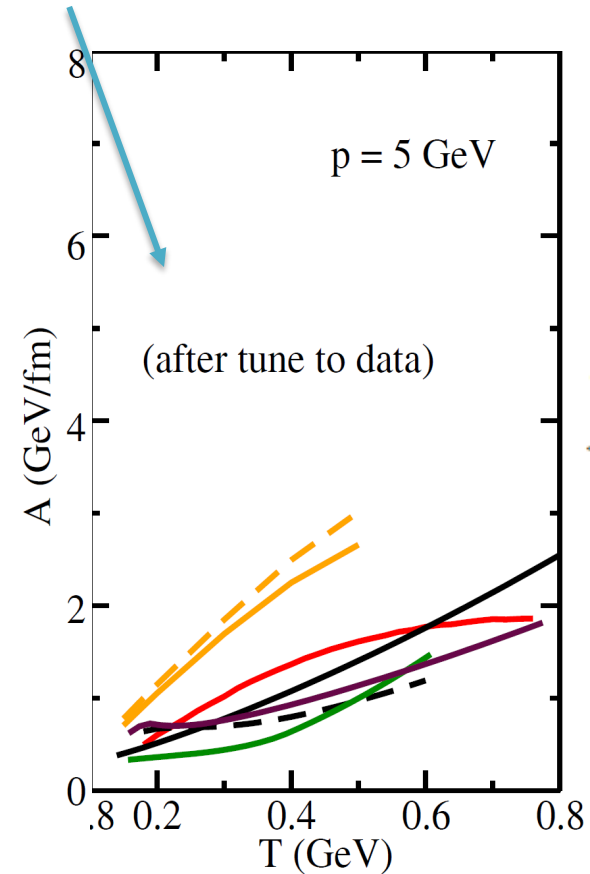
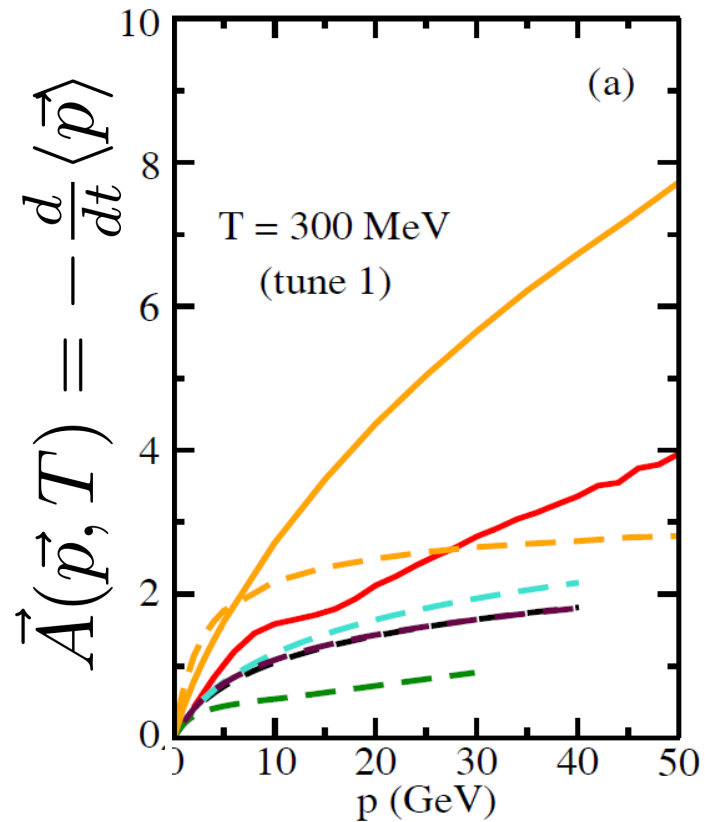
Tune 1: favorite tuning of each group in order to describe D meson data with their own ingredients (background, hadronization,...) ; K = rate multiplier

HQ Working Group

- Collect and compare the transport coefficients from various models:

What is used by various models to fit the data

c-quarks

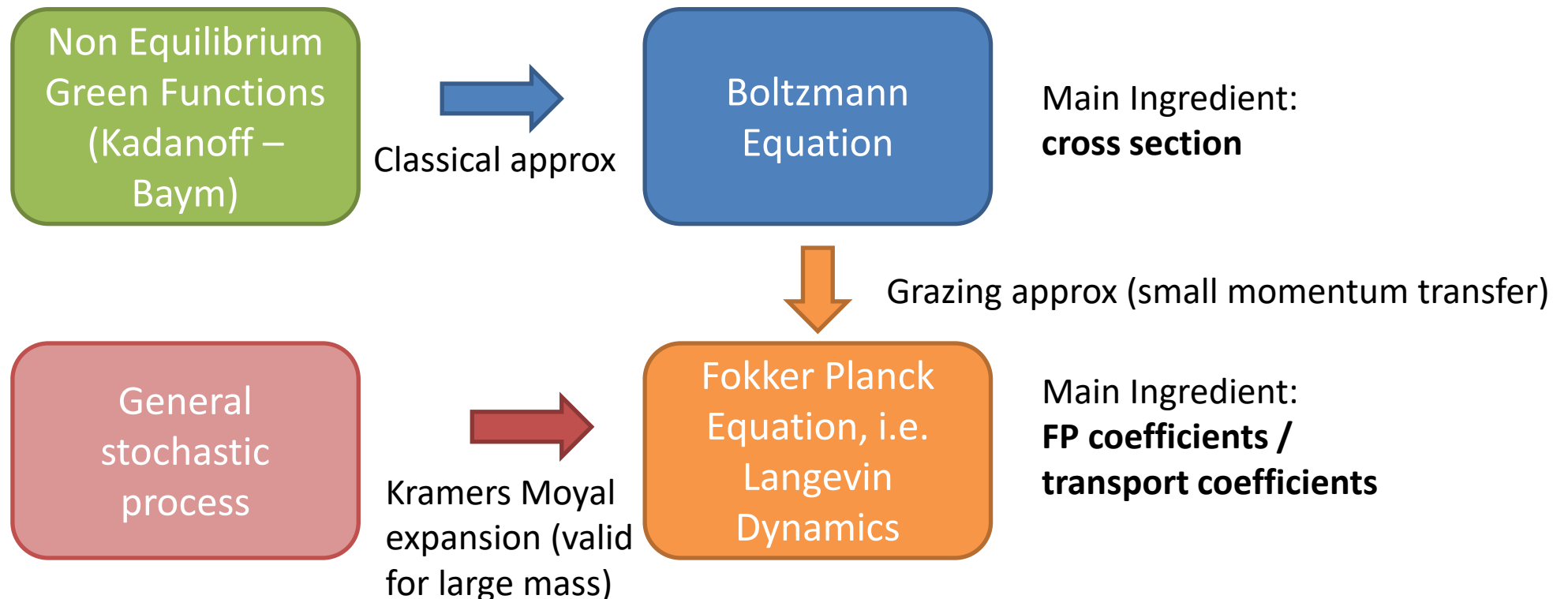


- Obviously not satisfying (from many perspectives) !
- Larger dispersion than the predictions for concrete observables... WHY ?
- Because of « extra ingredients », chosen differently in each model !!!

Various approaches to transport

Bottom-up schemes (microscopic \rightarrow mesoscopic):

- Assume (effective) degrees of freedom and (effective) interactions
- Take insights and constraints from the fundamental QCD theory, but often inholds some free parameter
- Rely on more or less sophisticated realizations of the transport theory



Why Fokker – Planck (AKA Langevin forces) ?

Bona fide answer: because HQ are heavy => long relaxation times => accumulate many collisions before thermalization => the “details” are averaged (central limit theorem) .

μ -model

$$R(\mathbf{p}, t) = \frac{1}{2E_{\mathbf{p}}} \int \frac{d^3\mathbf{q}}{(2\pi)^3 2E_{\mathbf{q}}} \int \frac{d^3\mathbf{q}'}{(2\pi)^3 2E_{\mathbf{q}'}} \int \frac{d^3\mathbf{p}'}{(2\pi)^3 2E_{\mathbf{p}'}} \\ \times \frac{1}{\gamma_c} \sum \boxed{|\mathcal{M}|^2} (2\pi)^4 \delta^4(p + q - p' - q') \\ \times [f(\mathbf{p}')g(\mathbf{q}')\tilde{g}(\mathbf{q}) - f(\mathbf{p})g(\mathbf{q})\tilde{g}(\mathbf{q}')],$$



$$\left\{ \begin{array}{l} A_i = \langle\langle (p - p')_i \rangle\rangle \\ \kappa_{i,j} = \langle\langle (p - p')_i (p - p')_j \rangle\rangle \end{array} \right.$$

Recovers the averages from the μ -model

mesoscopic model (FP equation)

$$\frac{\partial f}{\partial t} = \vec{\nabla}_p \cdot \left[\vec{A}f + \frac{1}{2} \vec{\nabla}_p (\hat{\kappa} f) \right]$$



distribution f in phase space... which fulfills

$$\frac{d}{dt} \langle \vec{p} \rangle_f = - \langle \vec{A}(T) \rangle_f \\ \frac{d}{dt} \langle \vec{p}_i \vec{p}_j \rangle_f = \langle \kappa_{ij}(T) \rangle_f$$

... also because it is much easier to solve than sampling the rate !

MC simulation then writes:

$$\Delta \vec{p} = -\vec{A} \Delta t + \underbrace{\vec{\xi}}_{\text{Random force (fluctuations)}} \quad \text{for each } \Delta t$$

Langevin vs Boltzmann Dynamics for HQ at intermediate p_T



I am the most faithful to IQCD



I allow to grasp the main aspects of the physics with a limited set of transport coefficients



“Hard” tails in the distributions are not well taken into account by the first moments



Transport coefficients derived from differential microscopic rates do not systematically satisfy Einstein relations => need a correction “by hand”



I am the most faithful to pQCD for hard transfers



I rely on a quasi-particle picture that may not apply in view of the widths in the spectral distributions



I can describe the hard tails in the distributions



I naturally drive the HQ distributions towards the genuine equilibrium Boltzmann distribution

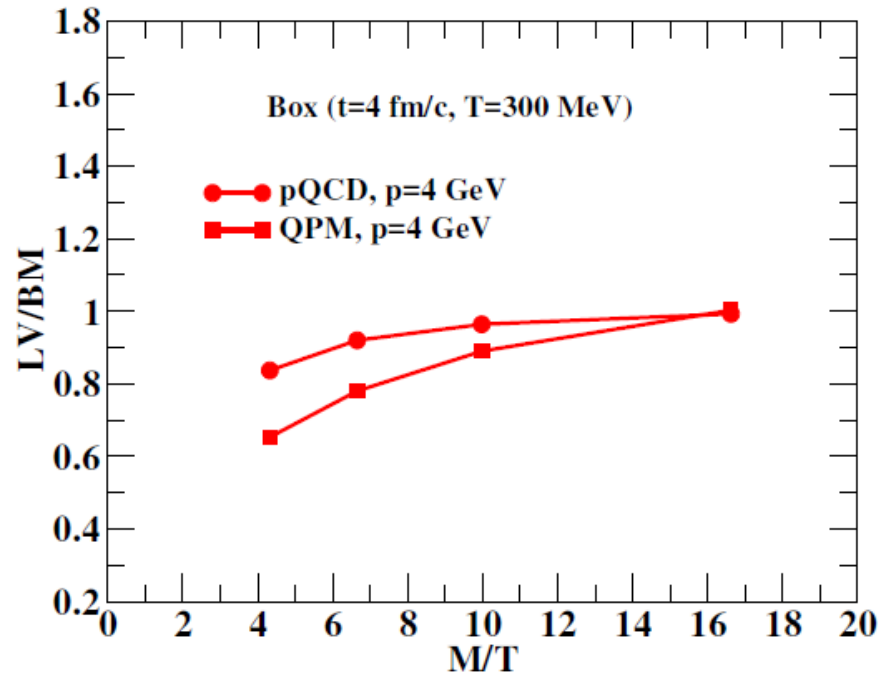
Fokker Planck

Boltzmann with μ model

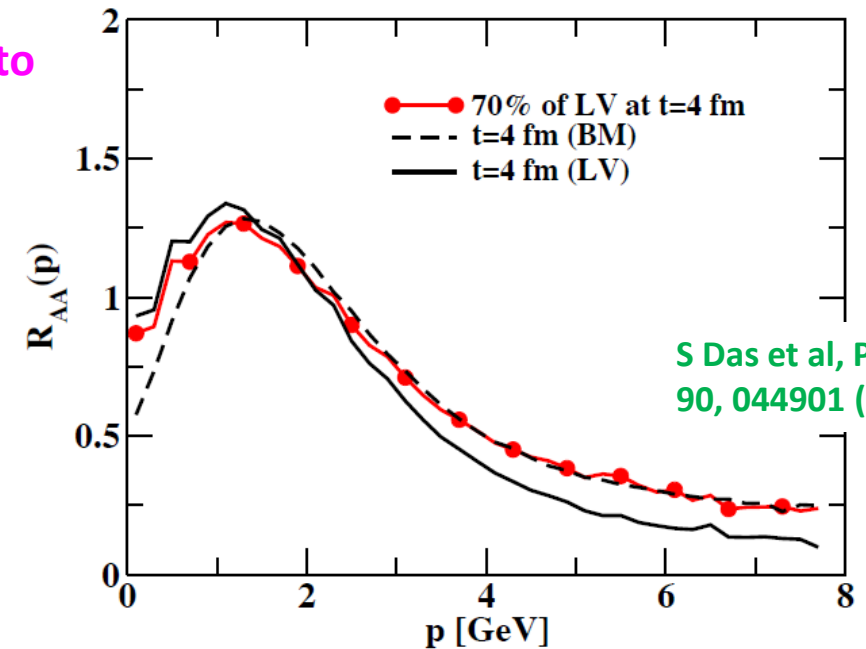
Boltzmann vs Langevin Dynamics

Langevin from Boltzmann view point:

- Lesson: For coarse grained observables like the R_{AA} and the v_2 , the agreement between the 2 transport schemes essentially depends on the isotropization strength of the cross section (i.e., the Debye mass of the gluon propagator)



Differences up to 40% found in QPM model

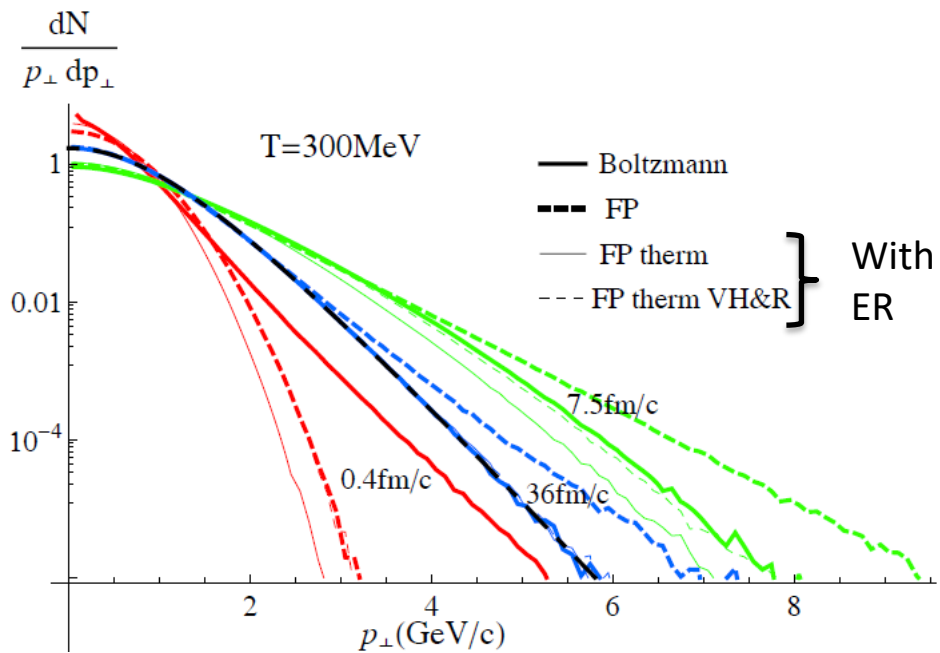
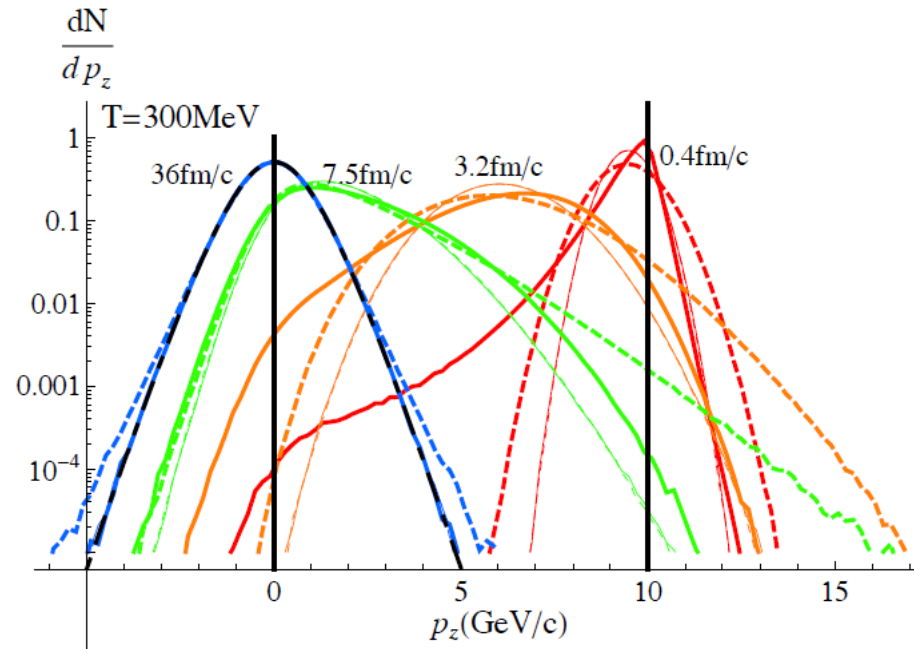


- For $m_D = g T \approx 2 T$ found f.i. in the Quasi Particle Model, extra coupling is found for the R_{AA} using LV, which can be suppressed by reducing the FP coefficients by $\approx 30\%$

Boltzmann vs Langevin Dynamics

Langevin from Boltzmann view point:

- For « exclusive process », momentum distributions differ significantly, even after imposing Einstein relation (ER):

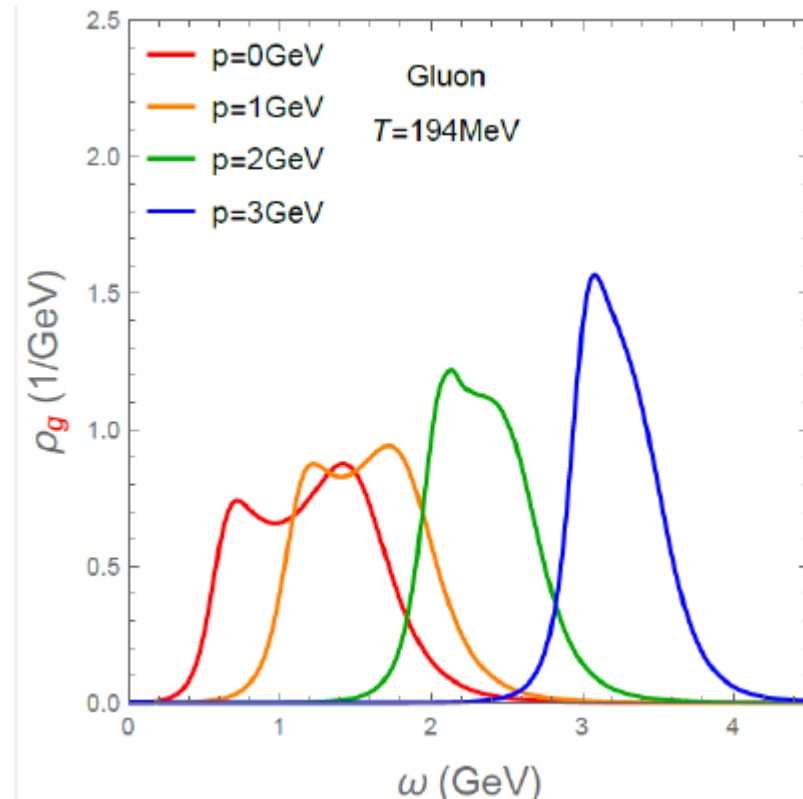


- These differences should be seen in observables like γ -HQ correlations

Boltzmann vs Langevin Dynamics

Boltzmann from Langevin view point:

- There are a lot of situations where Langevin dynamics applies, but not Boltzmann, thanks to the large mass of the particles.
- It is even a result proven for dynamical systems (conditions on the velocity applies as well)
- In a dense strongly coupled system, this is likely to be the case !



Gluon spectral functions (Liu & Rapp 2016)

Boltzmann vs Langevin Dynamics

Take home message for concrete 1-body observables:

R. Rapp et al. / Nuclear Physics A 979 (2018) 21–86

Extraction of heavy-flavor transport coefficients in QCD matter

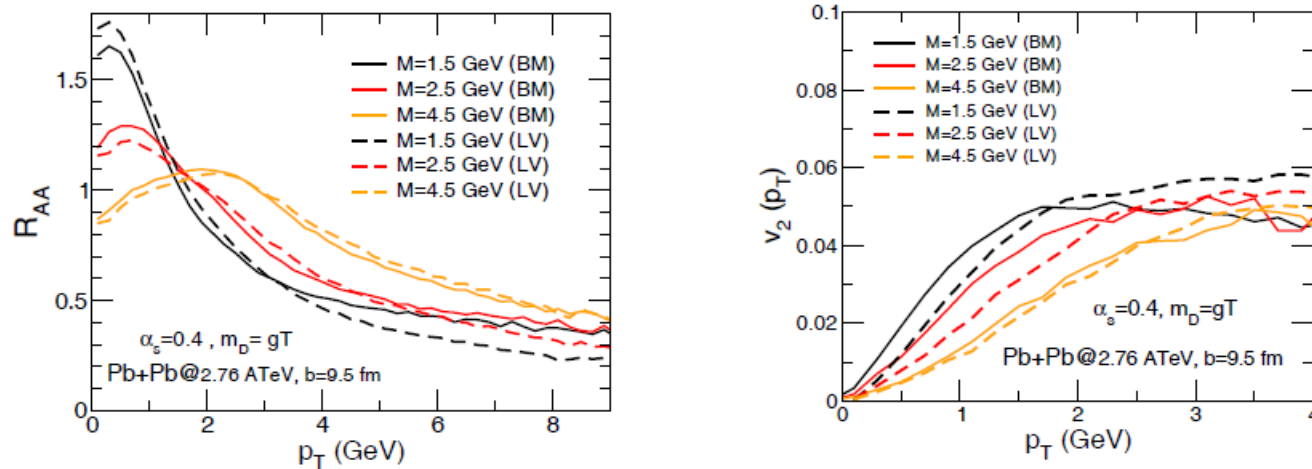


Figure 24: Nuclear modification factor (left panel) and elliptic flow (right panel) for heavy quarks in semi-central Pb+Pb($\sqrt{s_{NN}} = 2.76$ TeV) collisions (at $b = 9.5$ fm) for different values of the HQ mass, M_Q (indicated by the different line colors), in a Boltzmann (solid lines) and in a Langevin approach (dashed lines).

The adopted transport scheme mostly affects the c-quarks, Langevin leading to a reduced v_2

R. Rapp^{a,1}, P.B. Gossiaux^{b,*,1}, A. Andronic^{c,d,1}, R. Averbeck^{c,1},
 S. Masciocchi^{c,1}, A. Beraudo^e, E. Bratkovskaya^{c,f},
 P. Braun-Munzinger^{c,g}, S. Cao^h, A. Daineseⁱ, S.K. Das^{j,k},
 M. Djordjevic^l, V. Greco^{k,m}, M. Heⁿ, H. van Hees^f, G. Inghirami^{c,f,o,p},
 O. Kaczmarek^{q,r}, Y.-J. Lee^s, J. Liao^t, S.Y.F. Liu^a, G. Moore^u,
 M. Nahrgang^b, J. Pawlowski^v, P. Petreczky^w, S. Plumari^k, F. Prino^e,
 S. Shi^t, T. Song^x, J. Stachel^g, I. Vitev^y, X.-N. Wang^{r,z}

Recent Collective actions beyond Sapore Gravis

- **EMMI Rapid Reaction Task Force** (organizers: R. Rapp, PB Gossiaux, A. Andronic, R. Averbeck,, S. Maschiocchi):
 - Global strategy to extract the diffusion coefficient from the intercomparison between models and data
 - **Collect and analyse all ingredients from various models**
 - Identify constrains from IQCD
 - Initiate discussions to assess the limitations of some existing models.

R. Rapp^{*1}, P.B. Gossiaux^{*2}, A. Andronic^{*3,4}, R. Averbeck^{*3}, S. Masciocchi^{*3}, A. Beraudo⁵,
E. Bratkovskaya^{3,6}, P. Braun-Munzinger^{3,7}, S. Cao⁸, A. Dainese⁹, S.K. Das^{10,11},
M. Djordjevic¹², V. Greco^{11,13}, M. He¹⁴, H. van Hees⁶, G. Inghirami^{3,6,15,16}, O. Kaczmarek^{17,18}
Y.-J. Lee¹⁹, J. Liao²⁰, S.Y.F. Liu¹, G. Moore²¹, M. Nahrgang², J. Pawlowski²², P. Petreczky²³
S. Plumari¹¹, F. Prino⁵, S. Shi²⁰, T. Song²⁴, J. Stachel⁷, I. Vitev²⁵, and X.-N. Wang^{26,18}

Goal to attack the problem with a broad view right from the beginning...

R. Rapp et al, arXiv: 1803.03824
Nucl.Phys.A 979 (2018) 21-86

(20 monthes since first meeting)

But also :

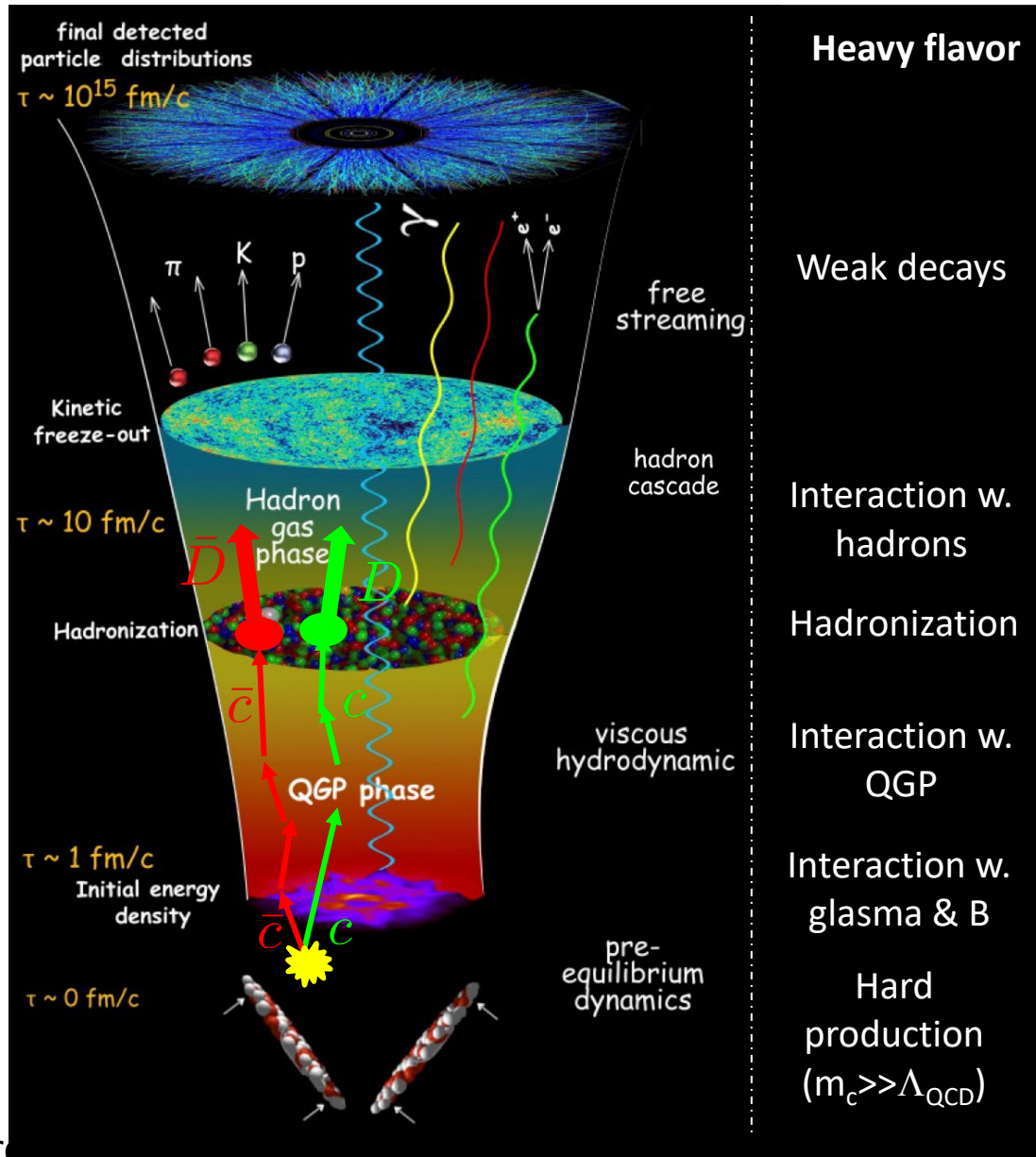
S. Cao et al, Phys.Rev.C 99 (2019), 054907 ;

T. Song et al, Phys. Rev. C 101 (2020) , 044903

Standard model of URHIC

Established before 2010

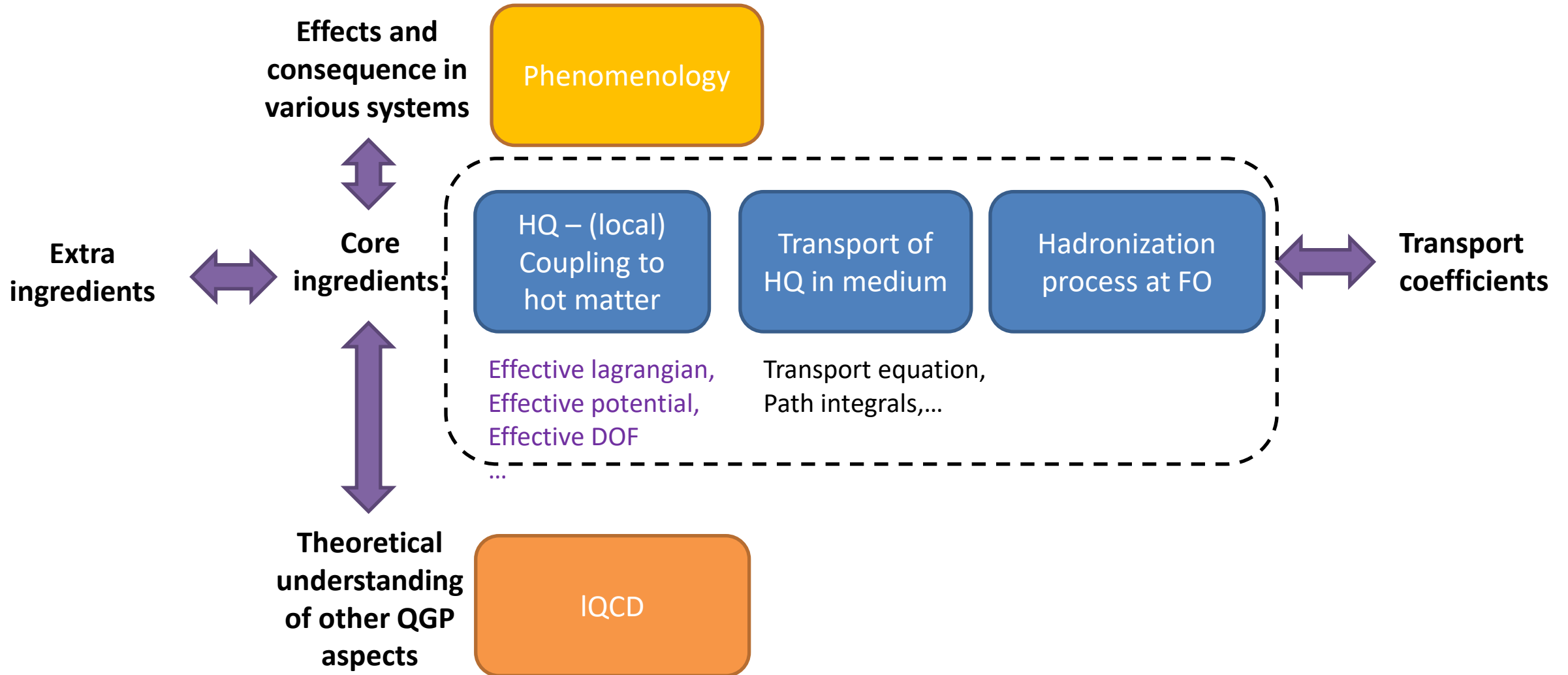
Since ≈ 2010



- Produced early ($t \approx 1/m_Q$)
 - => No further $Q\bar{Q}$ generation in ensuing QGP
 - Initial production well controlled (advantage of $m_Q \gg \Lambda_{\text{QCD}}$)
 - But early phase might not be so innocent (magnetic field, CGC-glasma,...)
- Experience the full deconfined phase + hadronic phase
 - probes the QGP on harder scales than the other hadronic observables *while not fully thermalized* ($t_{\text{relax}} \propto m_Q/T^2$)
 - accumulates several effects => need to compare different systems to better differentiate them
- Produced over a wide range of rapidities and p_T
 - increased richness in scrutinizing the interaction of HQ with medium...
 - but also sets more challenges (interactions for $p_T \ll m_Q$, $p_T \approx m_Q$, $p_T \gg m_Q$, appropriate transport theory ?).
- Nowadays turning into precision physics thanks to abundance of RHIC and LHC results !!!

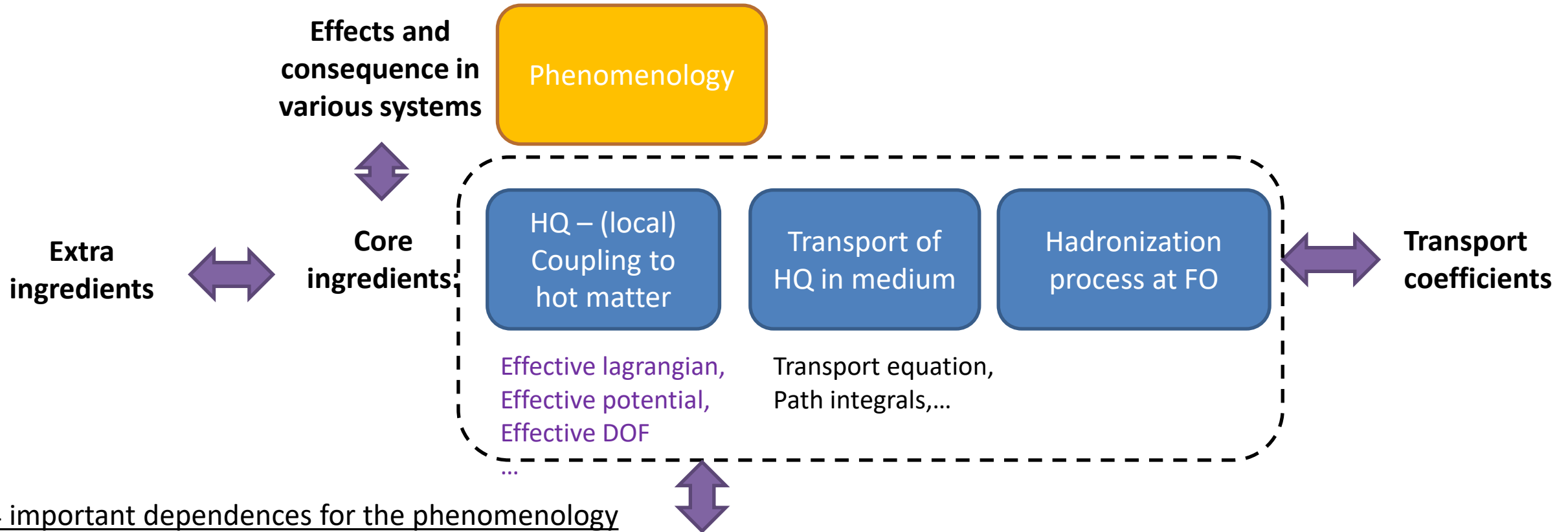
A bit of structure

- HQ propagation in QM & URHIC...



A bit of structure

- HQ propagation in QM & URHIC...



4 important dependences for the phenomenology

- Energy dependence : the saturation at large E explain the restoration $\rightarrow 1$ at large p_T
- Mass dependence \Rightarrow less thermalization for b quarks
- T dependence weights differently the initial stage and the late evolution (for which flows have developed)
- Path length dependence \Rightarrow makes it more transparent to the radiative in small systems ($\Delta E_{\text{rad}} \propto L^2$)

Models & Effective Theories

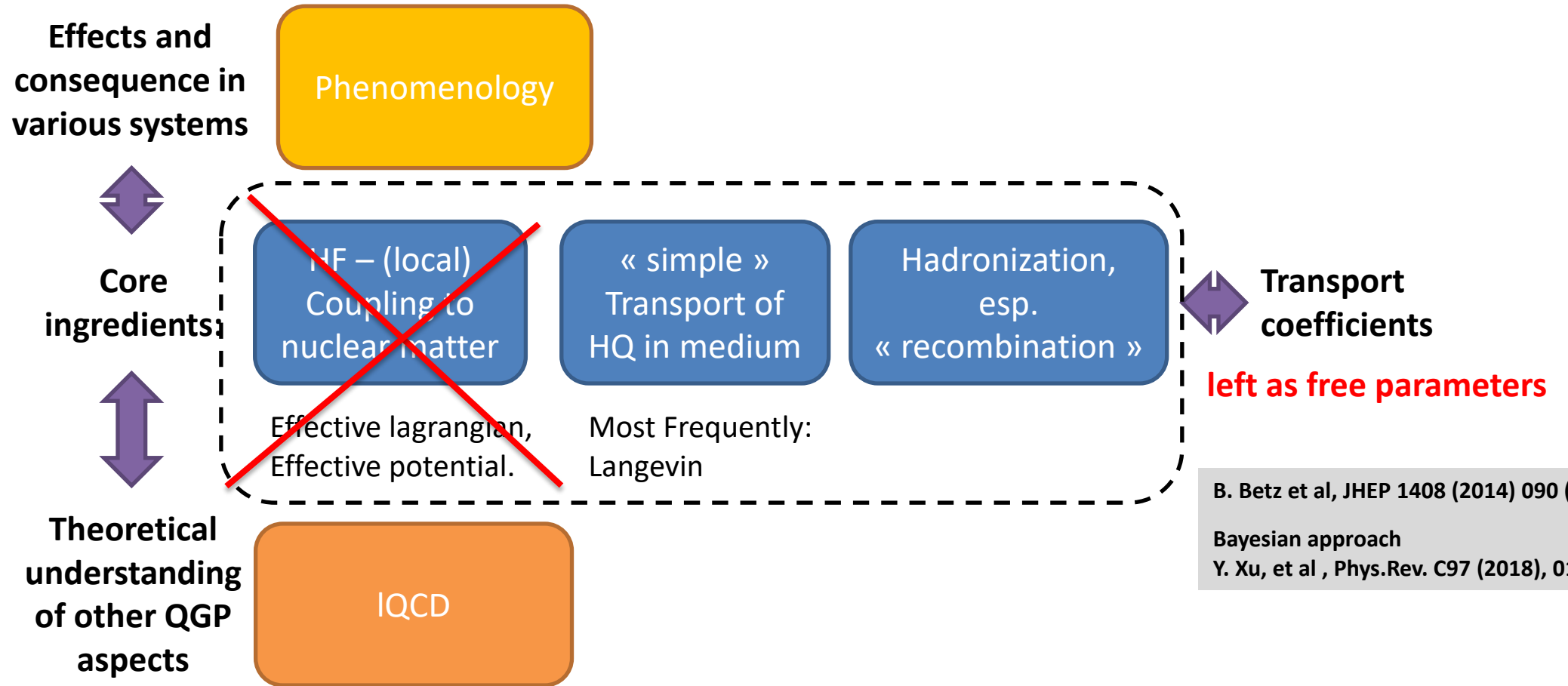
	elastic	Elastic + radiative	radiative	Other
Transport coefficient based (LV,...)	TAMU POWLANG HTL Catania LV	Duke, TAMU w rad.	ASW	ADS/CFT POWLANG IQCD <i>DABMOD</i> <i>S. Li et al, arXiv:1803.01508</i>
Cross section (or $ M ^2$) based (Boltzmann,...)	AMPT MC@sHQ el URQMD PHSD Catania BM	DREENA MC@sHQ el + rad BAMPS CUJET3 HYDJET++ Abir and Mustafa LBL-CCNU VNI/BMS LIDO	SCET _{G,M}	

Red: Transport models

Disclaimer : If your model does not appear here, please forgive me and contact me for completion
HQ lectures

A bit of structure

- No Model approach



Data driven extraction of transport coefficients

Y. Xu et al
arXiv:1710.00807v1

« Minimal model approach » : Bayesian analysis by the Duke group

$$\frac{d\vec{p}}{dt} = -\underbrace{\eta_D(p)\vec{p}} + \vec{\xi} + \vec{f}_g \leftarrow \frac{dN_g}{dxdk_{\perp}^2 dt} = \frac{2\alpha_s P(x)\hat{q}_g}{\pi k_{\perp}^4} \sin^2\left(\frac{t-t_i}{2\tau_f}\right) \left(\frac{k_{\perp}^2}{k_{\perp}^2 + x^2 M^2}\right)^4 \quad \text{Higher twist}$$

Usual Langevin $\langle \xi_i(t)\xi_j(t') \rangle = \kappa\delta_{ij}\delta(t-t')$ with $\kappa = \frac{2T^2}{D_s}$

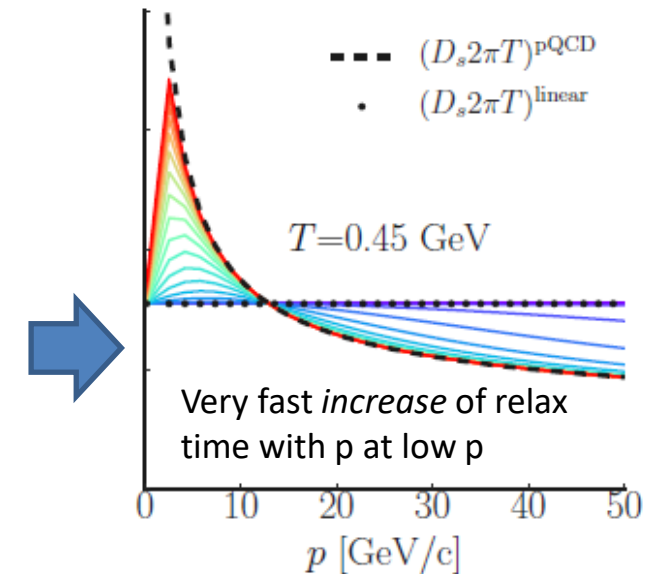
+ coal / frag hadronization and hadronic rescattering

$$D_s(T, p) = \frac{1}{1+(\gamma^2 p)^2} (D_s 2\pi T)^{\text{lin}}(T; \alpha, \beta) + \frac{(\gamma^2 p)^2}{1+(\gamma^2 p)^2} (D_s 2\pi T)^{\text{pQCD}}(T, p)$$

$$(D_s 2\pi T)^{\text{pQCD}} = 8\pi / (\hat{q}/T^3)$$

$$(D_s 2\pi T)^{\text{linear}} = \alpha \cdot (1 + \beta(T/T_c - 1))$$

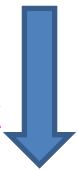
Encodes possible **Non Perturbative Effects** around T_c through parameters α (magnitude), β (slope) and γ (inverse momentum range of NP effects)



Duke “Bayesian approach”

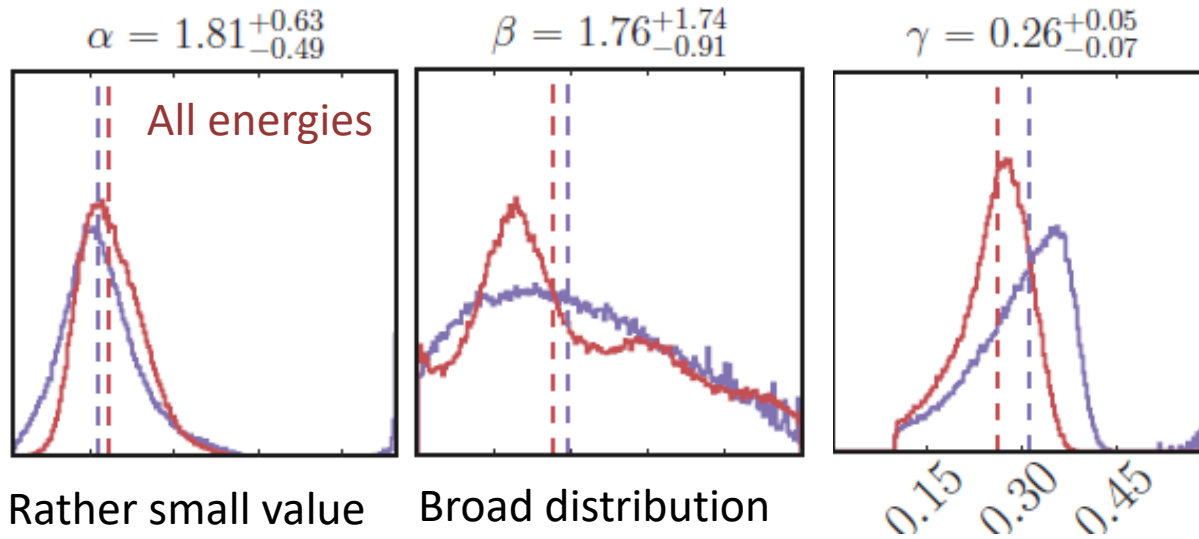
- Choice of 60 « prior » for which the physical observables are calculated
- Gaussian emulator to build a fast surrogate of physics
- Random walk throughout parameter space, with acceptance and rejection according to likelihood (with all uncertainties assumed to be uncorrelated).

Let the data speak



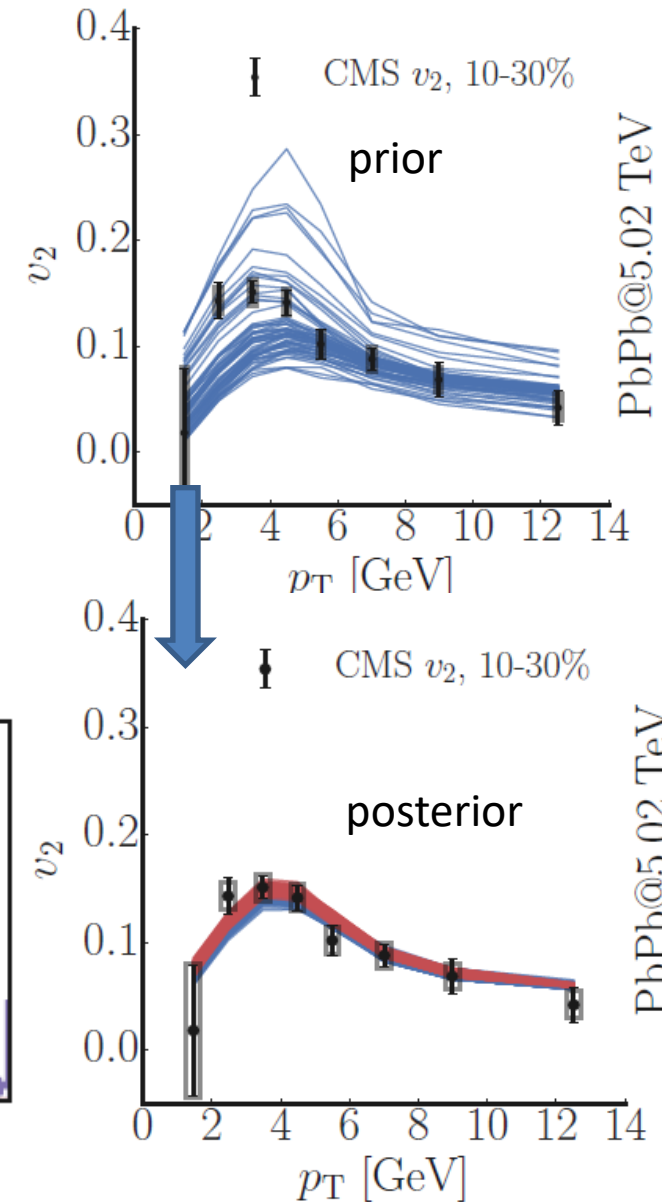
$$(D_s 2\pi T)^{\text{linear}} = \alpha \cdot (1 + \beta(T/T_c - 1))$$

RESULTS

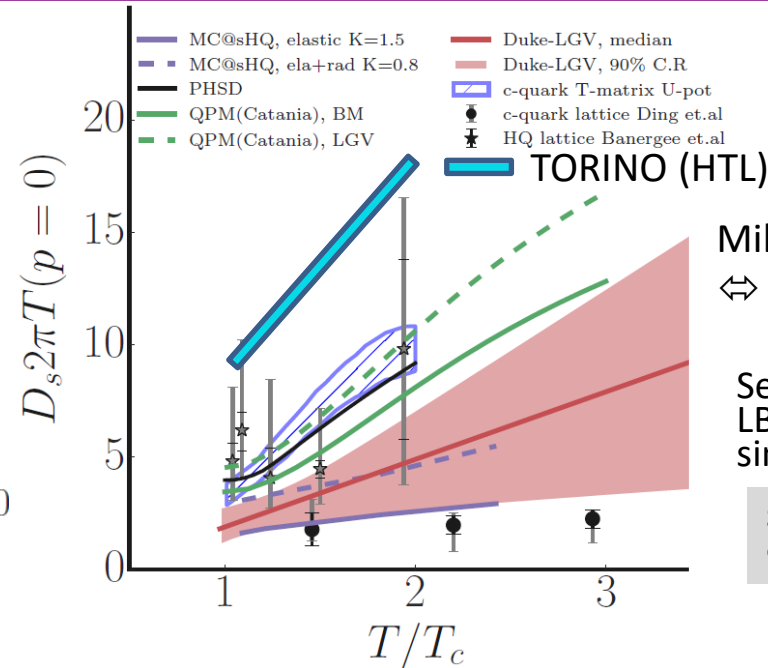
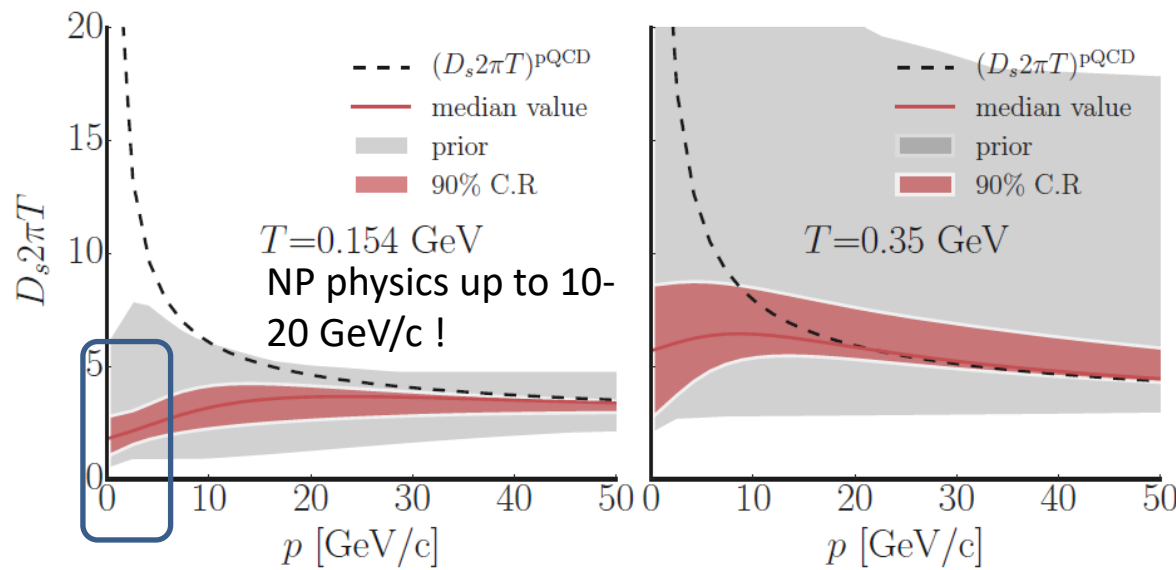


Rather small value
=> strong coupling !

Broad distribution



Duke "Bayesian approach" vs models



Mild lin. increase of $2\pi D_s T \dots$
 \Leftrightarrow physics beyond pQCD.

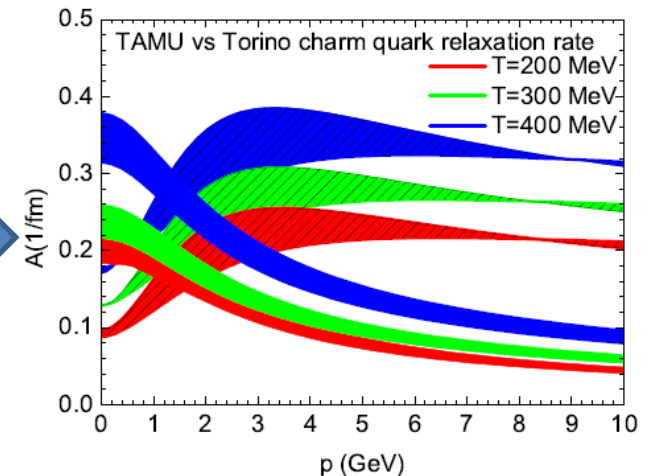
See as well analysis in the LBL-CCNU model with similar conclusions

S. Cao et al, Phys. Rev. C 94, 014909 (2016)

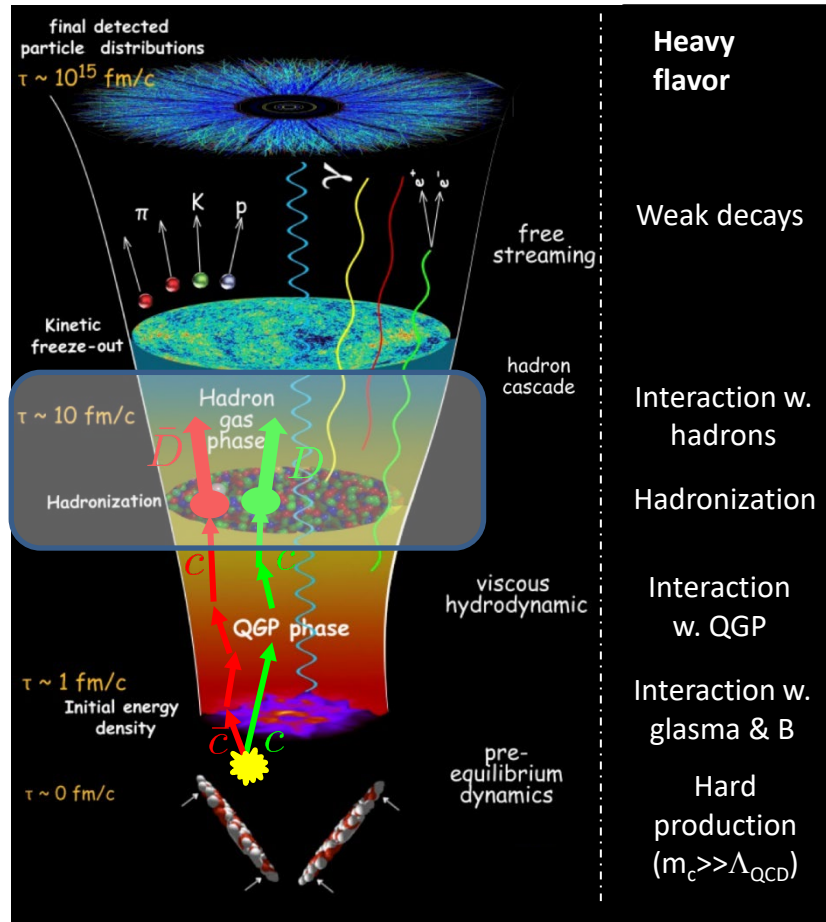
All together (IQCD, Bayesian analysis and most recent models) make a strong case for NP physics « around T_c » and at «low» p_T ... needs to be precised in the future

- Does not mean that all models inhold the same physics...
- $D_s(p=0)$ does not represent the full physics (different momentum dependences)
- D_s (finite p) in Duke's el + rad approach should not be compared to the same quantity in purely elastic models (additional contribution to energy loss due to the rad. part)

Prino and Rapp, J.Phys. G43 (2016), 093002



Heavy quarks as ideal hard probes:

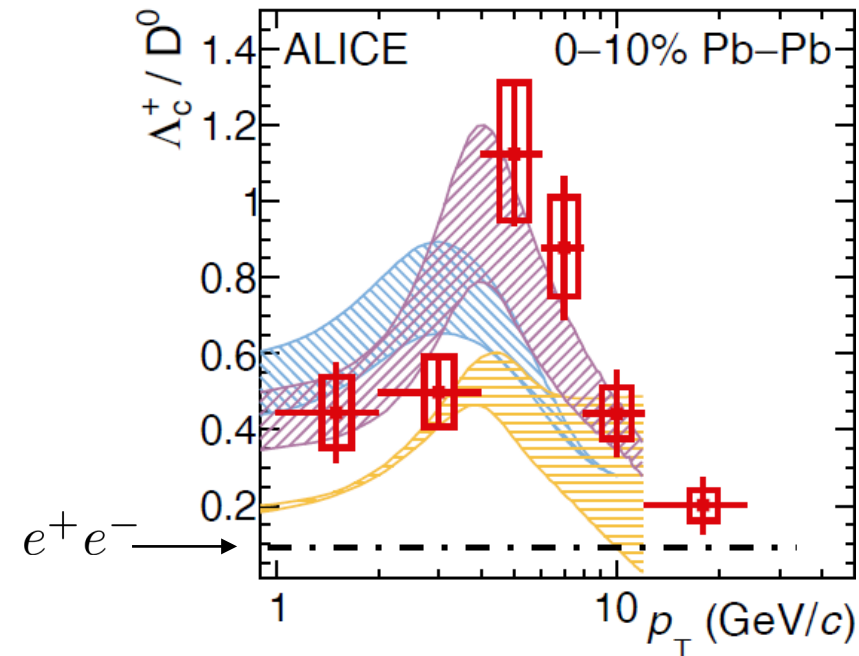


The recombination of heavy quark with some *existing* light quark(s) from the QGP is an essential mechanism at “low” $p_T < 5-10 \text{ GeV}/c$...

Mandatory to understand the Λ_c/D^0 ratio

- Evolution in the QGP DOES NOT modify the yield of initial Q and Qbar: Negligible annihilation rate !
- It only impacts their distribution in momentum
 - Hence, for usual observables like RAA and v_2 ... only the initial “1 body” distribution matters

Mostly like in elementary pp collisions...
- ... However hadronization is affected by the QGP
 - Other mechanism wrt usual fragmentation of HQ in elementary collisions : **coalescence / recombination**



HQ-Hadronization (would deserve a full talk)

Acknowledged:

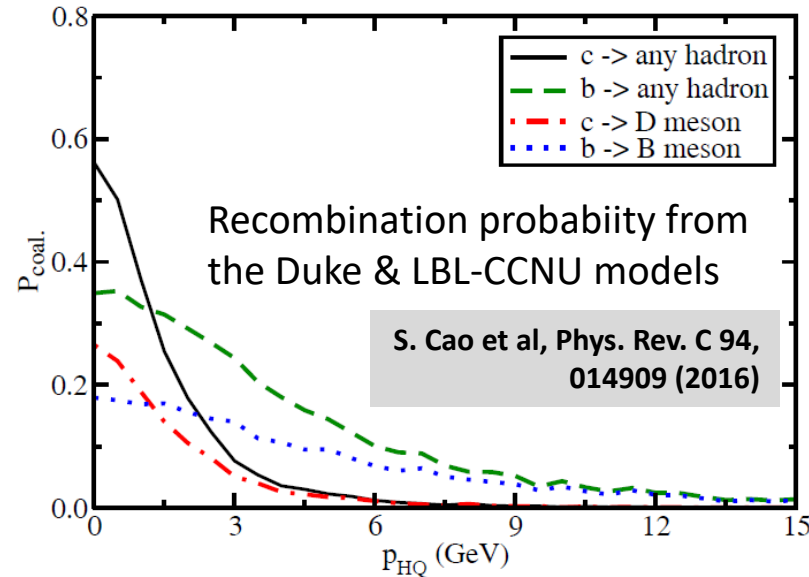
- towards the end of QGP, hadronization of (of equilibrium) HQ can proceed through a **dual mechanism**:

Low p_T :

- The quark partner(s) are already present in the hot cooling medium
- New specific recombination mechanism; no obvious calibration**
- The footprint of reconfinement (?!)
- Crucial to explain the flow bump in $R_{AA}(D)$ and sizable $v_2(D)$ => **large impact.**

Uncertain (and not disputed enough):

- Genuine physical recombination process !



High p_T :

- The quark partner(s) needed to create the HF-hadron have to be generated from the vacuum
- « usual » fragmentation calibrated on p+p and e^+e^- data (Petersen,...)

HQ - Recombination

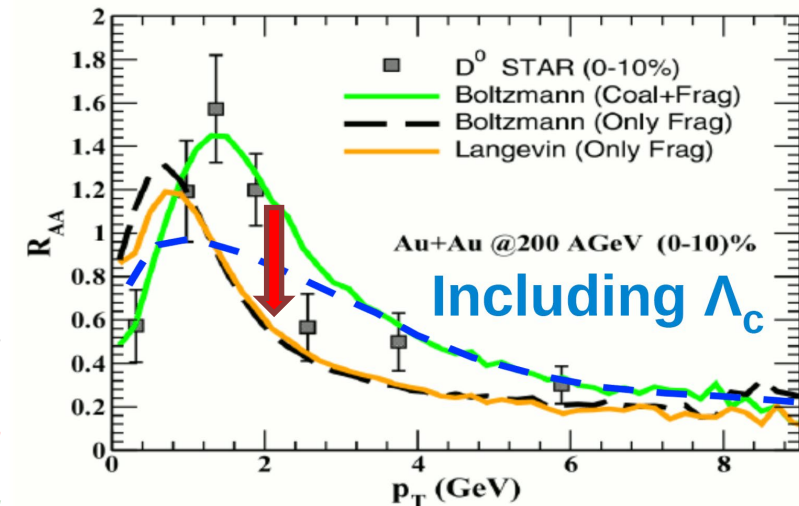
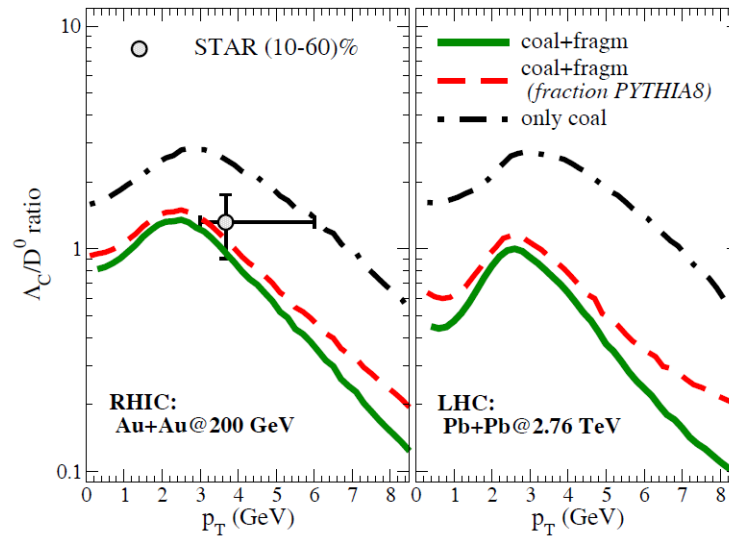
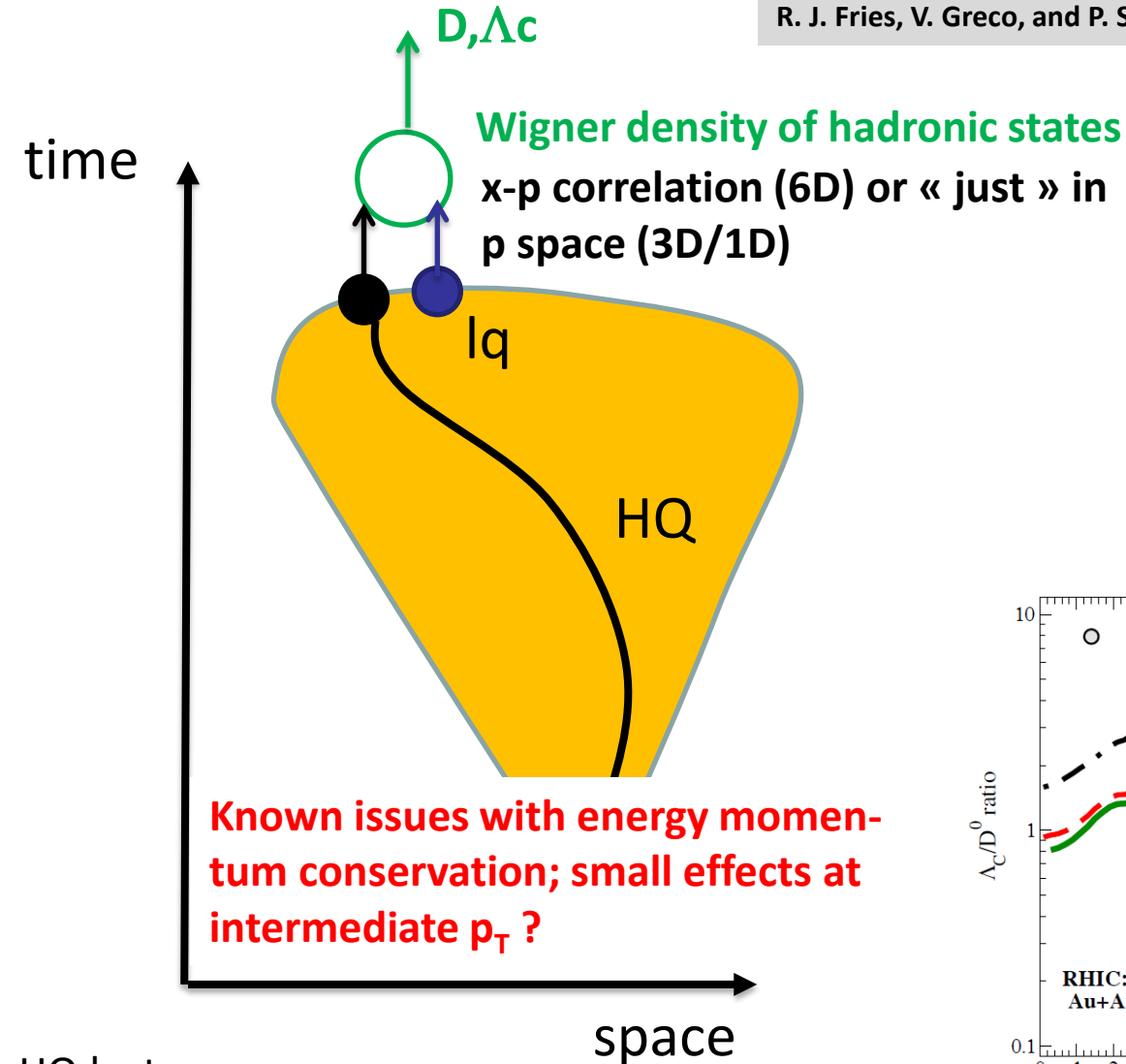
Instantaneous coalescence:

Greco, Ko & Levai. Phys. Rev. C 68 (2003) 034904
 V. Greco, C. Ko, and R. Rapp, Phys. Lett. B 595 (2004) 202
 Y. Oh et al, Phys. Rev. C 79 (2009) 044905
 R. J. Fries, V. Greco, and P. Sorensen, Ann. Rev. Nucl. Part. Sci. 58 (2008) 177

S Plumari et al. arXiv:1712.00730

Latest Catania's coalescence model:

- Full 6D coalescence
- New normalization to impose $P_{\text{coal}} \rightarrow 1$ for $p_T \rightarrow 0$
- Resonance decay
- Mini jet contribution
- Inclusion of Λ_c baryonic states
 - \Rightarrow reduction of $R_{AA}(D)$ at small p_T
 - \Rightarrow increase of Λ_c/D^0 wrt pp and pPb.



HQ - Recombination

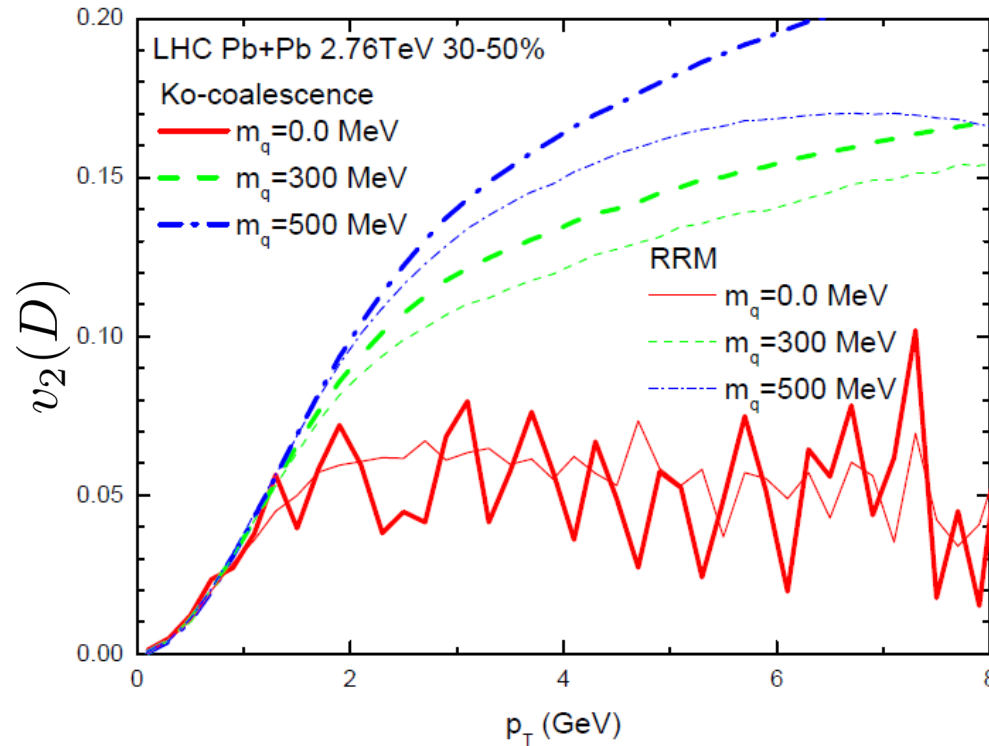
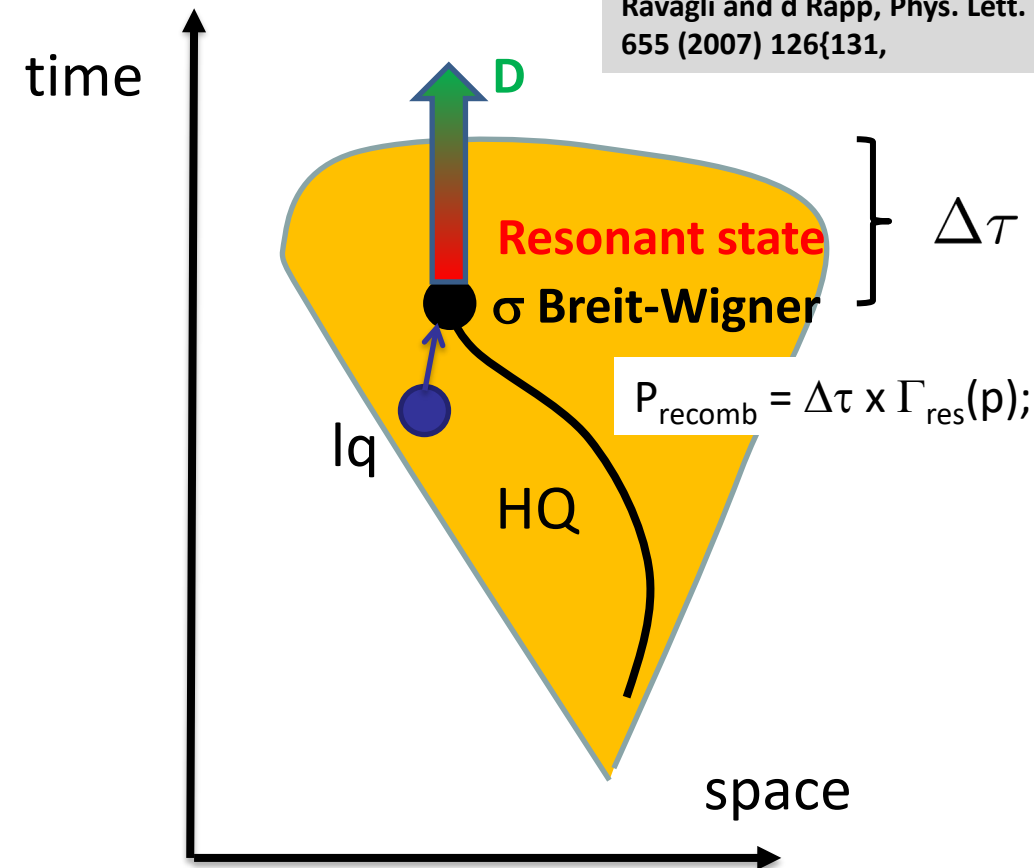
Resonance Recombination Model:

$$\left(\frac{\partial}{\partial t} + \vec{v} \cdot \vec{\nabla}\right) F_M(t, \vec{x}, \vec{p}) = -\frac{\Gamma}{\gamma_p} F_M(t, \vec{x}, \vec{p}) + \beta(\vec{x}, \vec{p})$$

Ravagli and d Rapp, Phys. Lett. B 655 (2007) 126{131,

- Dynamical 2->1 process, implemented in the asymptotic limit of the kinetic equation
- A possible way to solve energy-momentum conservation
- Process governed by the interaction of HQ with QGP around $T_c \Rightarrow$ natural link with the energy loss model.

In EMMI RRTF, comparison between Instant. Coal. & RRM



starting from the same bulk and from the same c spectrum

Significant differences found both for D meson p_T spectrum and v_2 .

R. Rapp et al, arXiv: 1803.03824

EMMI RRTF : Consequences from various Hadronization Mechanisms

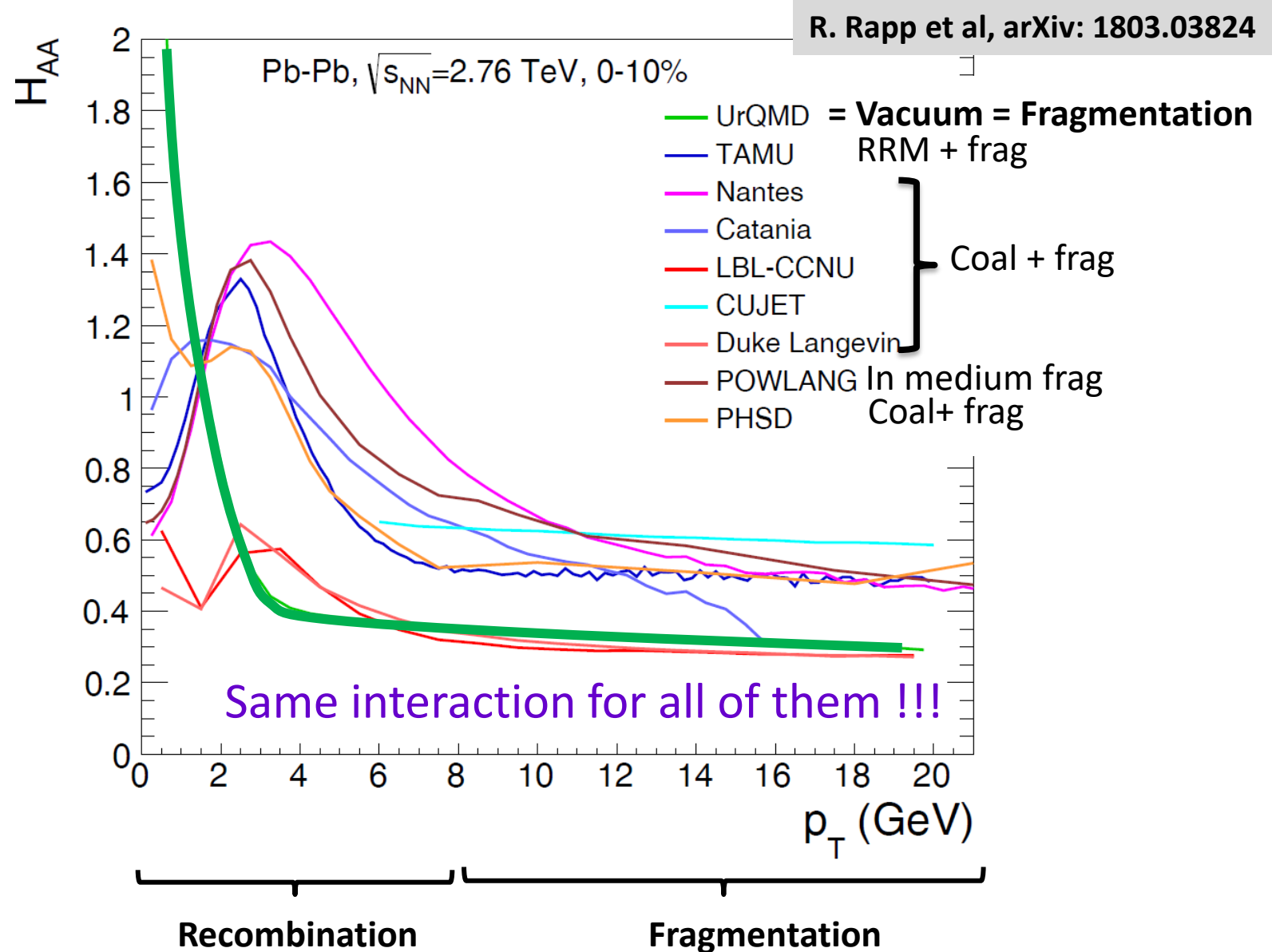
We define and display the H_{AA} quantity

$$H_{AA} = \frac{\frac{dN_D}{dp_T}}{\frac{dN_{c \text{ final}}}{dp_T}}$$

...which exhibits at best the specific effects of hadronization :

Significant uncertainties !

=> Yes, one can for sure put more constrains with D_s and Λ_c , but probably one has also to converge on more robust schemes for « basic » D mesons

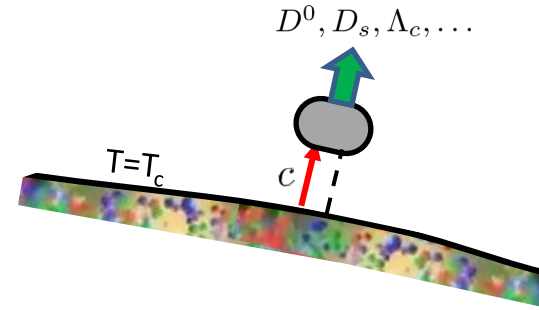
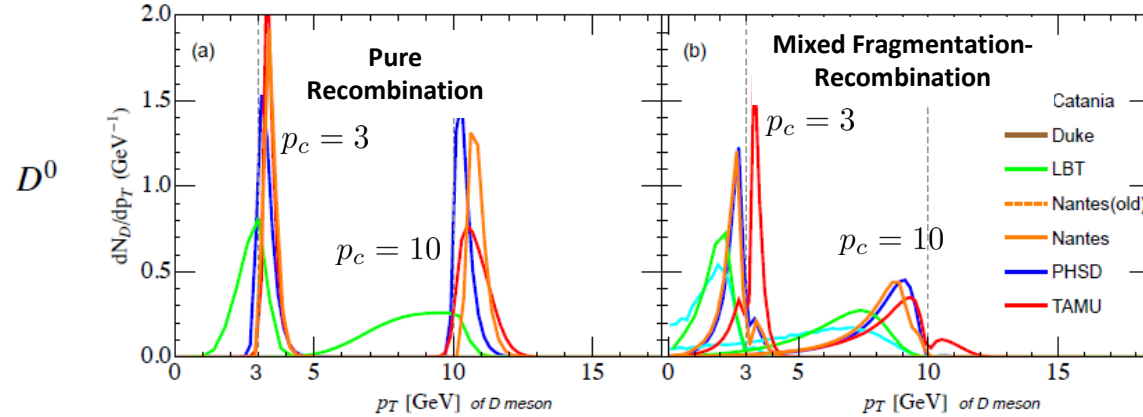


Hadronization of heavy quarks

- Recent effort of theorists to compare their hadronization schemes at the end of the QGP

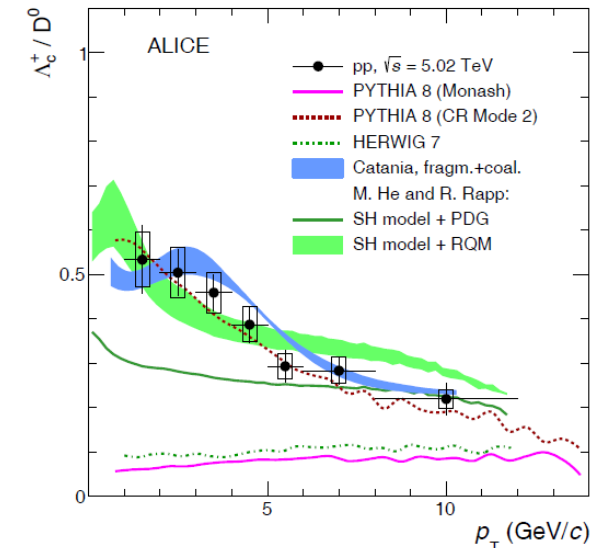
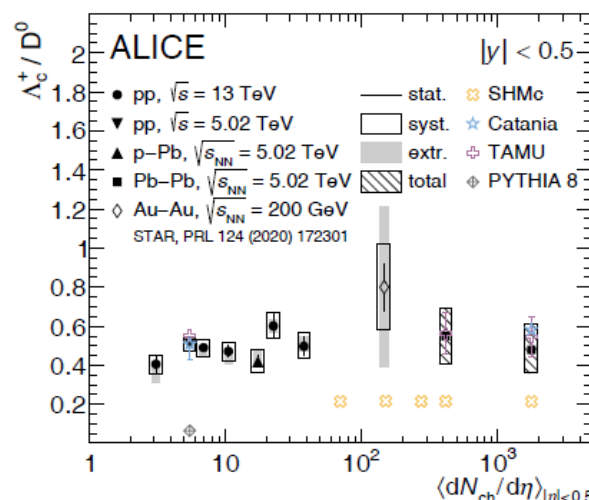
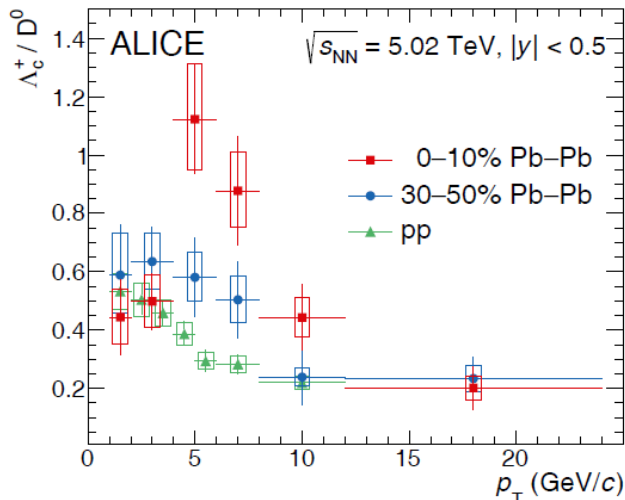
dN_D/dp_T of the direct D^0 meson produced by a c -quark with $p_T = 3$ GeV and 10 GeV

Jiaxing Zhao et al., 2311.10621



- Diversity => things to learn ! ... Hadrochemistry of Heavy Flavor will be a major subject of investigation for ALICE 3.

- But also in small systems like pp (many signs of **collectivity in small systems => QGP ?**)



Alternate observables

Key message:

Usual “1 body” observables like R_{AA} and v_2 do not allow a full discrimination between the models

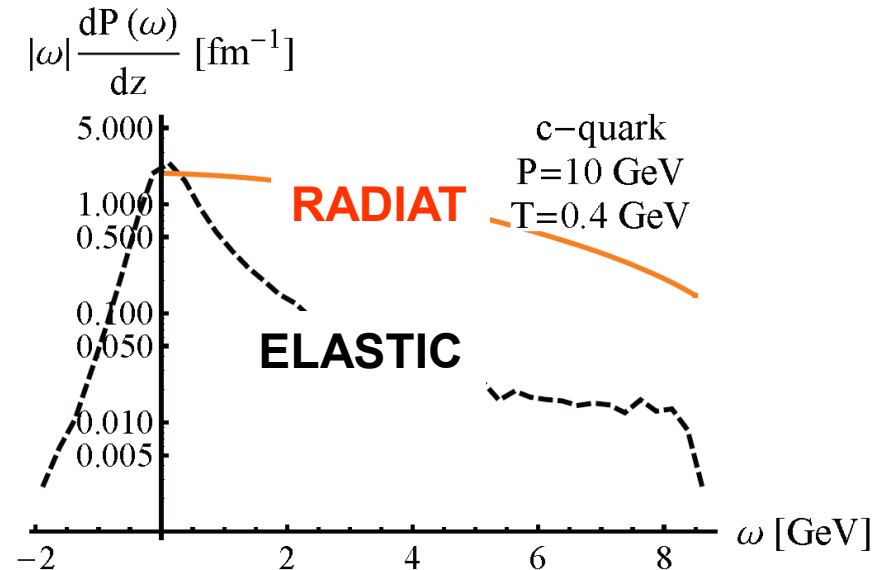
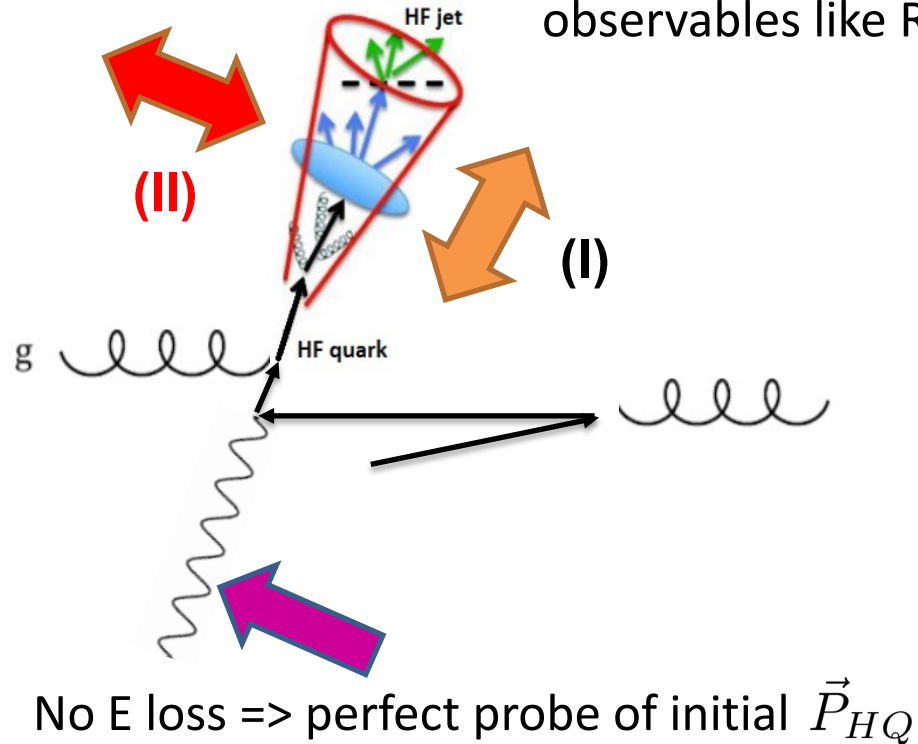
Many models can reproduce these observables, sometimes at the price of some tuning or extra ingredient

One needs to turn to **more exclusive observables that will to better allow discriminating the various aspects of the HQ “energy loss”**.

γ - b/c jet: Best HF Correlation ever ?

➤ γ - D/B/c jet /b jet:

In QGP: **Longitudinal and transverse (\hat{q}) fluctuations** of the HQ, which crucially depend on the Eloss mechanism and cannot be measured in usual observables like R_{AA} or v_2

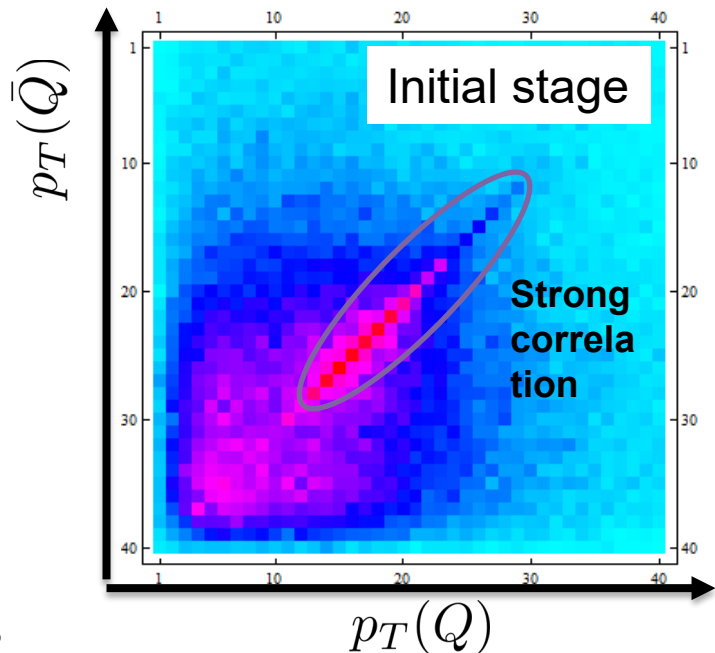
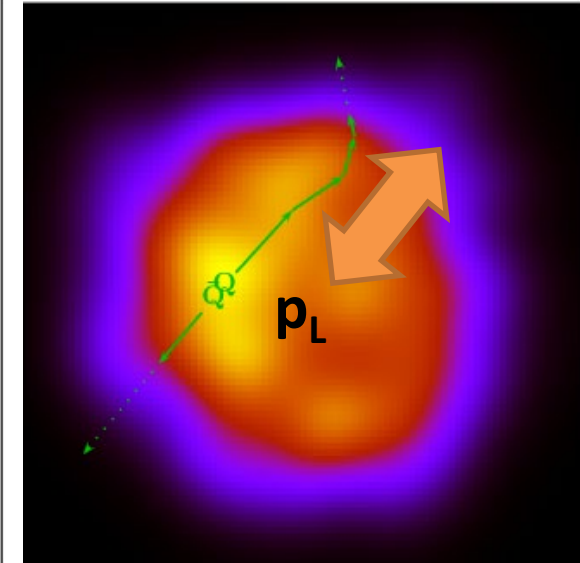
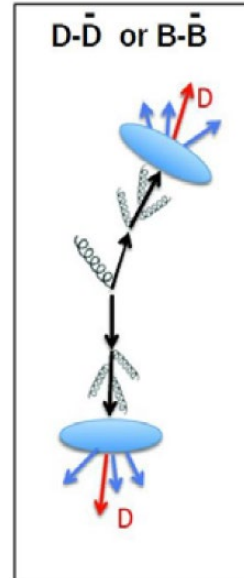


differential probability to loose energy ω per unit time

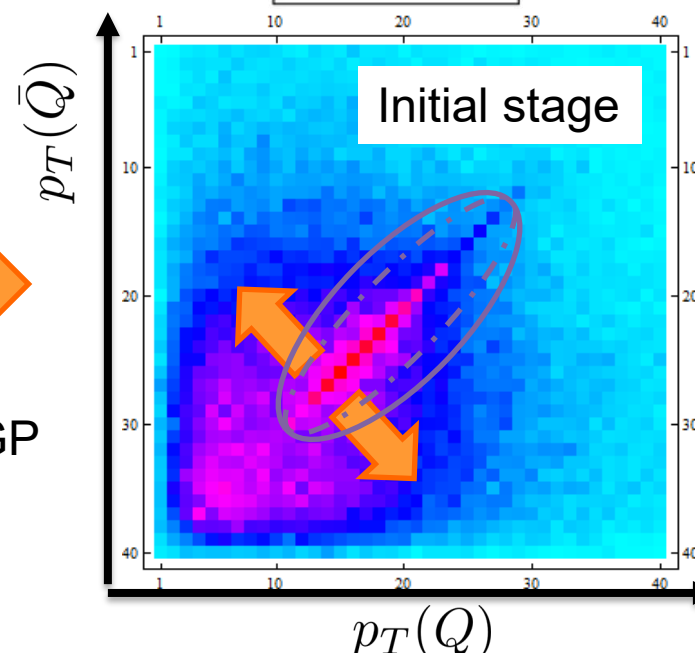
➤ Of course: NLO effect in the production mechanisms makes it not so trivial (not to speak about exp. Issues... RUN3 ? RUN4 ?)

Next best thing: HF-HF correlations

- Back to back D/Dbar or B/Bbar: As compared to γ -D/B: “triggering” itself is affected but symmetry between both particles could limitate the various effects:
- Large number of c-cbar from various NN collisions => large uncorrelated background
- Competing effects due to energy loss: ...



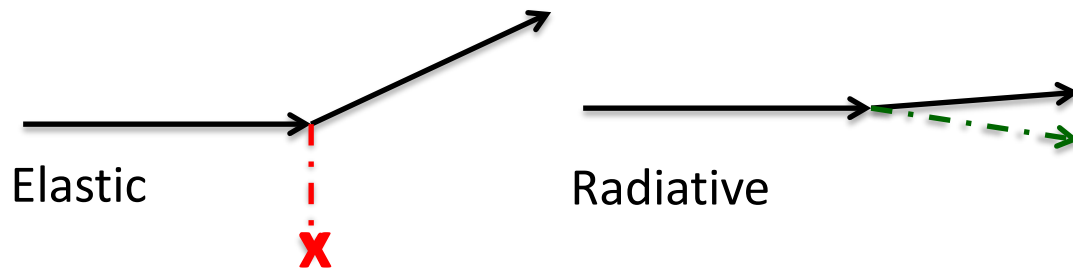
Evolution
in hot QGP
medium



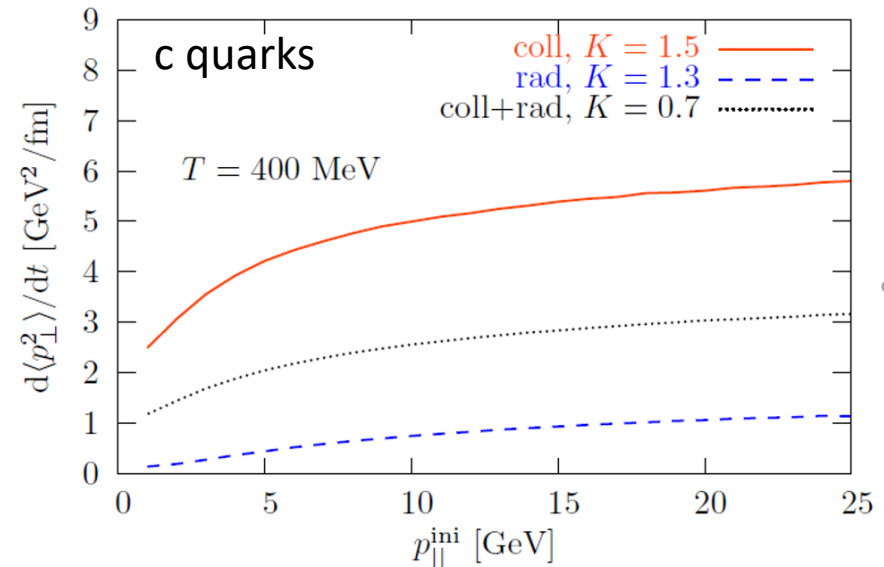
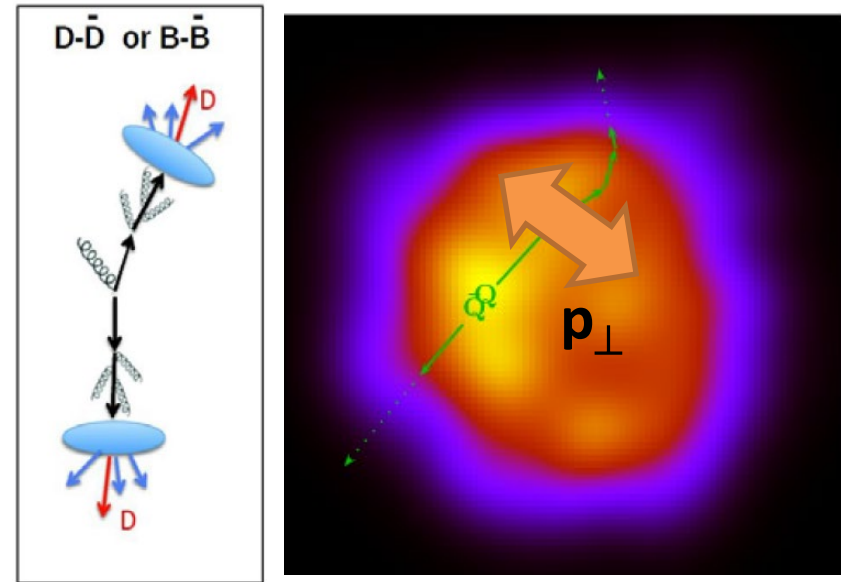
- decorrelation due to various path lengths + fluctuations: **reduction**

Next best thing: HF-HF correlations

- Back to back D/Dbar or B/Bbar: As compared to γ -D/B: “trigger” itself is affected but symmetry between both particles limitates the various effects.
- Elastic Eloss vs radiative Eloss: **The purely collisional scatterings lead to a larger average $\langle p_{\perp}^2 \rangle$ than the radiative “corrections”** (need for large scattering to be efficient)... although both types can give correct agreement with the data at intermediate p_T .



- Expected consequences for azimuthal correlations (probe of B_T : good: **complimentary** to usual RAA and v_2)

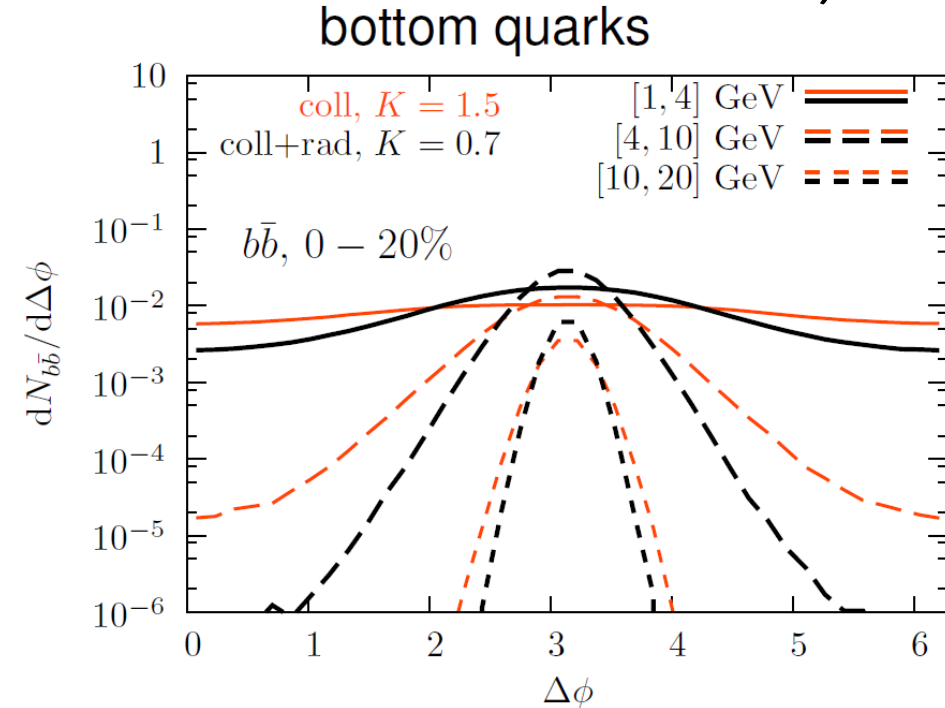
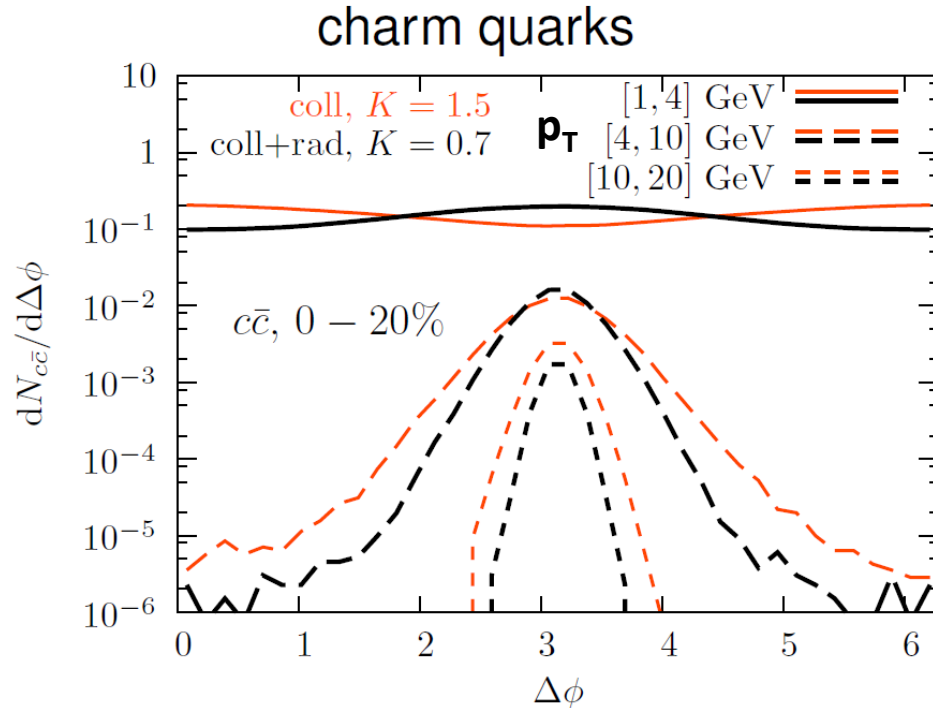


Tuned to reproduce the R_{AA}

Next best thing: azimuthal correlations

- Assumption of back 2 back emission of initial QQbar (naïve LO...)

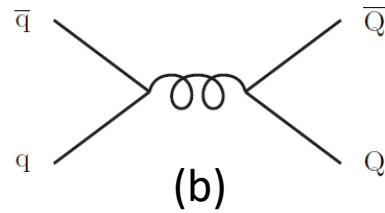
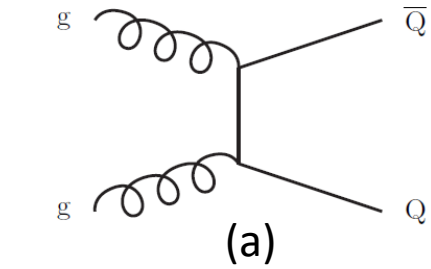
Nahrgang et al.
Phys. Rev. C 90,
024907 (2014)



- Indeed, rather large differences found for both b and c, and all kind of p_T cuts (... but good to see there is an effect though,...)
- For the smallest p_T bin and elastic energy loss, we even find an inversion of the correlation (“hot partonic wind” push; v_0 bulk => v_1 correl; underlying event)

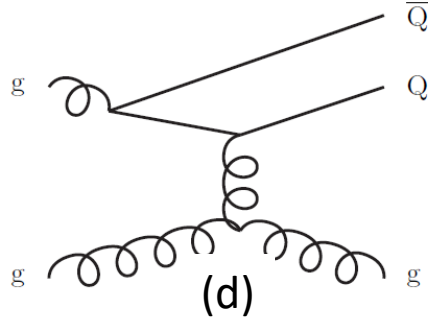
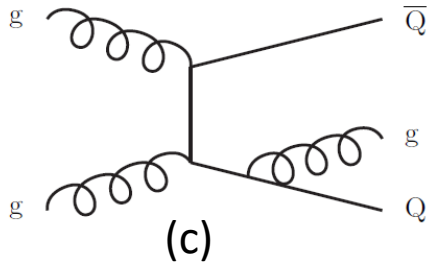
Next best thing: azimuthal correlations

➤ ...but higher orders can have a significant impact:



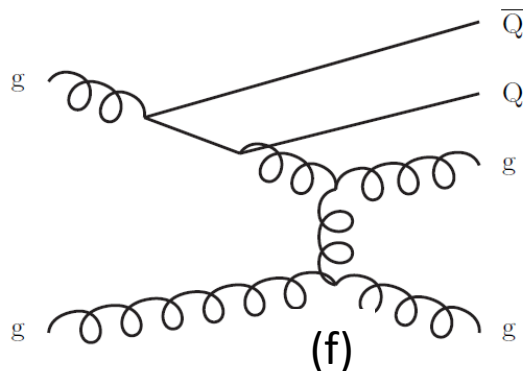
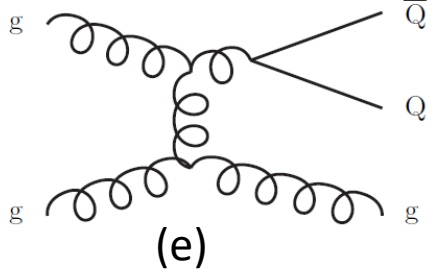
➤ LO; (a): back to back peak

➤ NLO;



(c): “blurring” of B2B peak

(d): “flavor excitation”: no strong azimuthal correlation expected

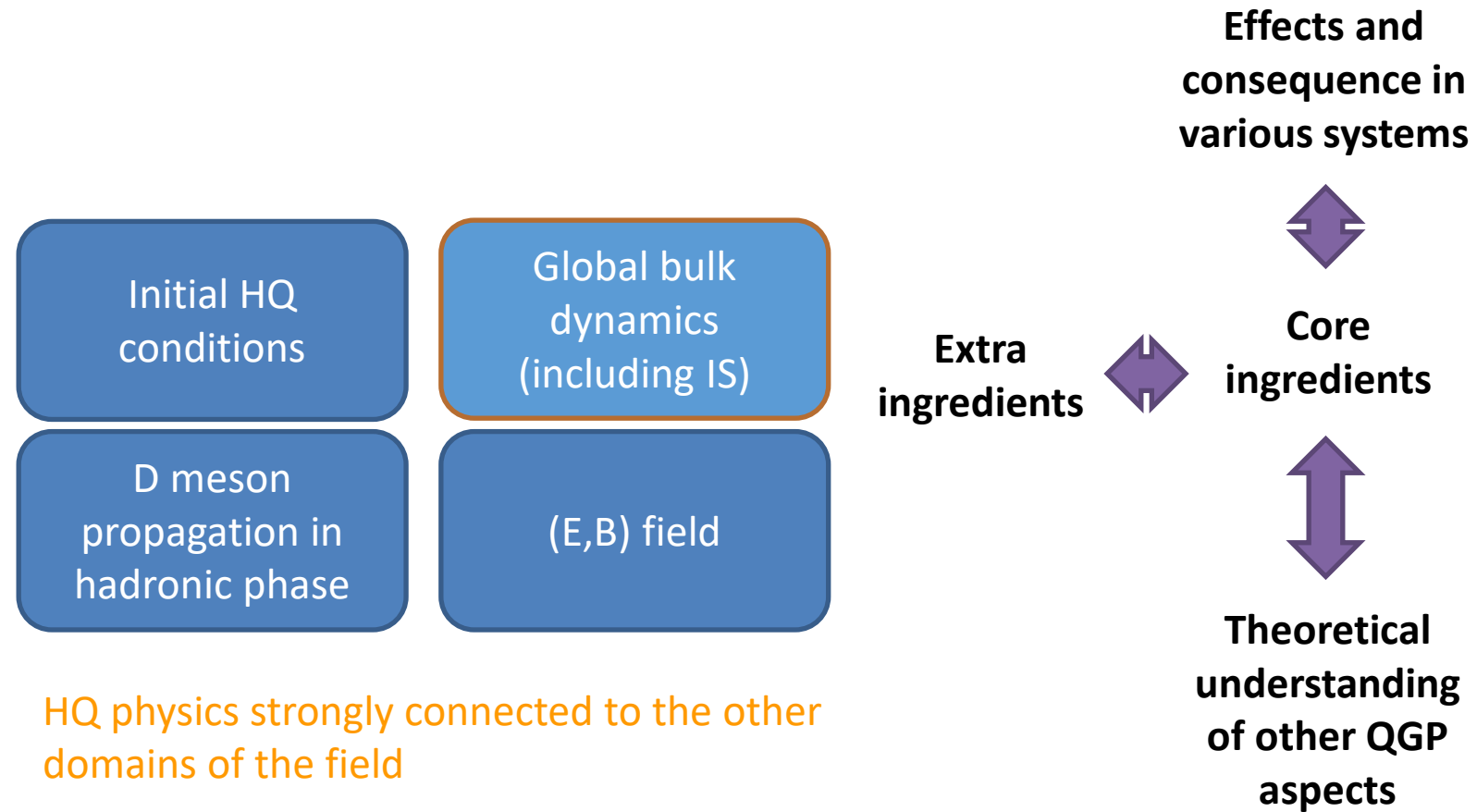


(e): gluon splitting: strong peak around $\Delta\phi=0$

(f): higher order FE; both Q and Qbar in the “remnant” region

A bit of structure

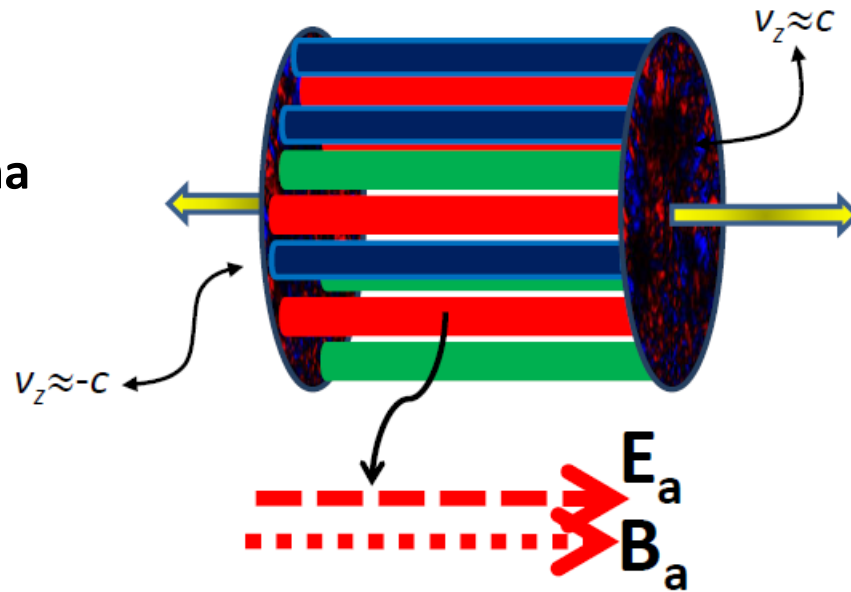
- HQ propagation in QM & URHIC...



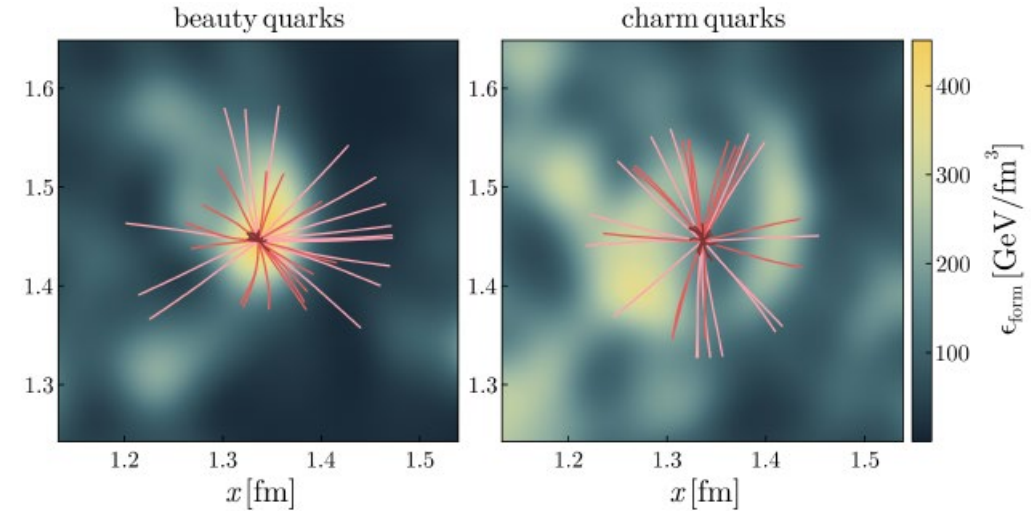
Early stage evolution of HQ in the Glasma phase

- Diffusion of heavy quarks in the early stages of high energy nuclear collisions

Early stage : **Glasma**



D. Avramescu et al, Phys. Rev. D 107 (2023), 114021



trajectories of heavy quarks propagating in a single Glasma flux tube

- Diffusion of HQs in the early stage of high energy collisions is affected by the strong fields: coherence memory effects are substantial. Most recent work:

Heavy-Quark Momentum Broadening in a Non-Abelian Plasma away from Thermal Equilibrium

May 2024 · [Physical Review Letters](#) 132(22)

DOI: [10.1103/PhysRevLett.132.222301](https://doi.org/10.1103/PhysRevLett.132.222301)

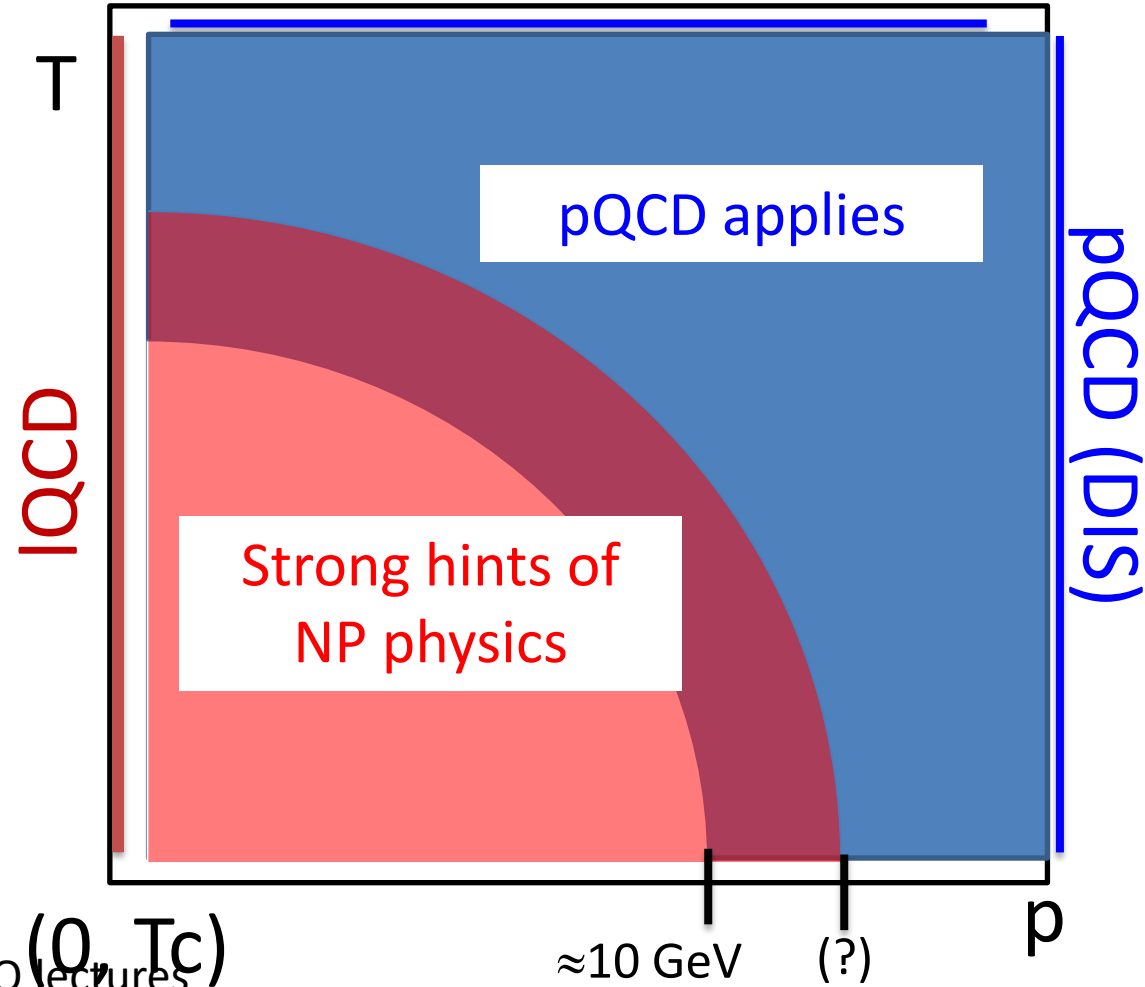
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Harshit Pandey · Sören Schlichting · Sayantan Sharma

Conclusions for Open Heavy Flavors

HQ in QGP

pQCD



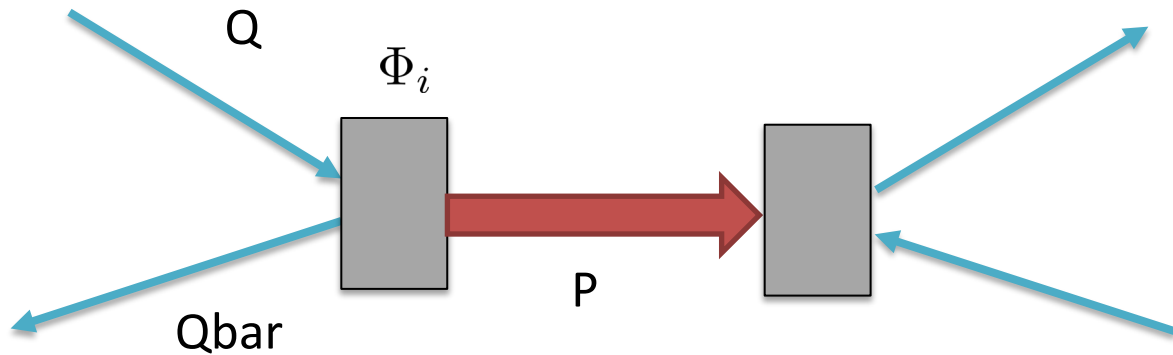
- Existing models offer the possibility to describe most of the OHF experimental AA data while being compatible with existing theory constrains...
- ... however with unequal precision and no consensus on the physical NP content
- Improvements and quantitative understanding is on their way, but it will still take some time and a lot of efforts => need for resources, bright people and collective work.
- Open Heavy Flavors are maybe not an ideal probe of QGP yet, but they are quite fascinating and offer bright future for the field, with multiple interconnections.

What is a quarkonia ?

What is a quarkonium ? Bound state of Q and Qbar quarks

Bound states in QFT ? $\hat{H}|\Psi\rangle = E|\Psi\rangle$?

More convenient : analyse the Q-Qbar scattering amplitude S_2 for a given total 4-momentum P



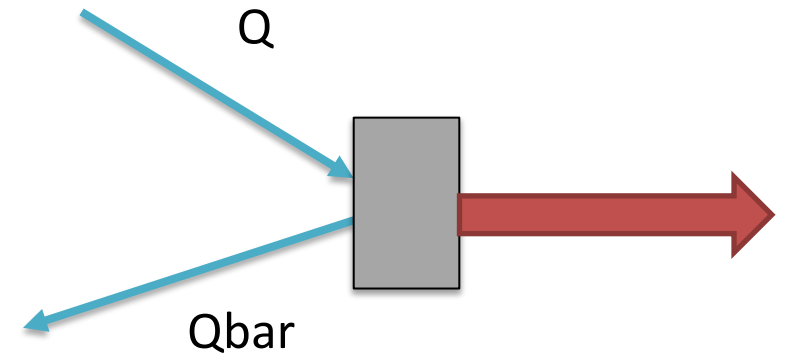
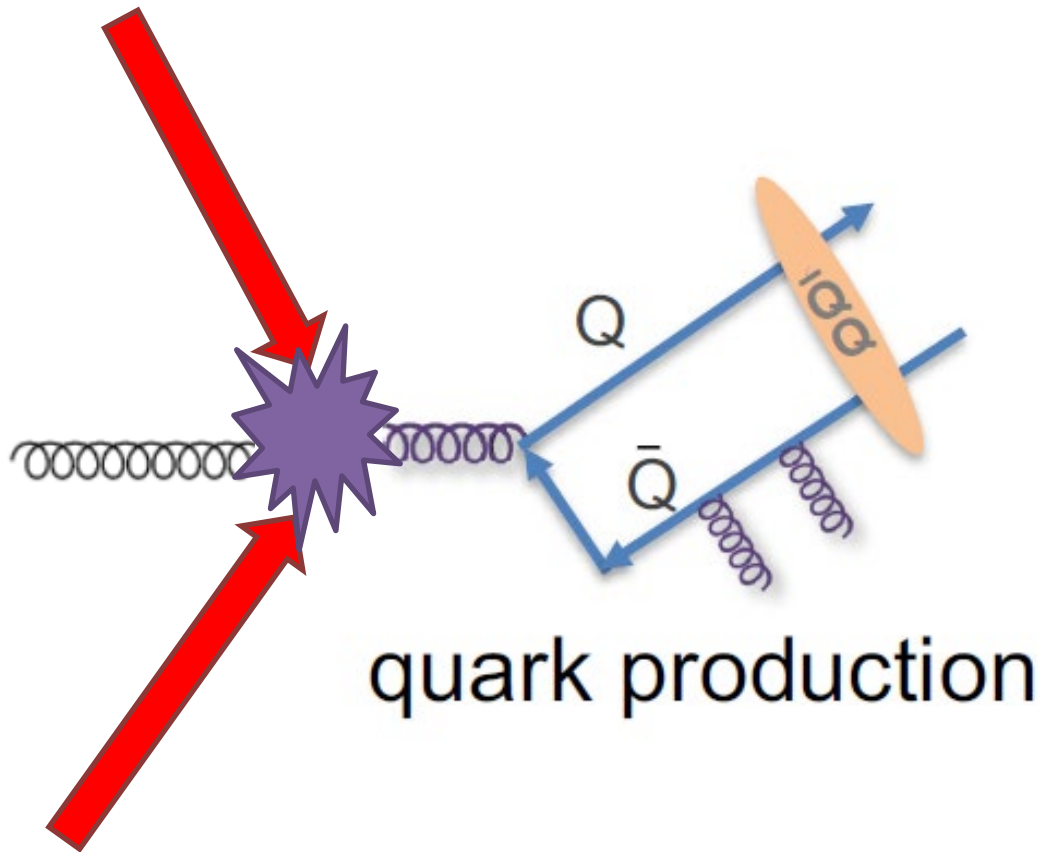
And decompose it as

$$S_2 = \sum_i |\Phi_i\rangle \frac{1}{P^2 - M_i^2} \langle \Phi_i| + \underbrace{\dots}_{\text{continuum}}$$

Then : $\left\{ \begin{array}{l} M_i : \text{bound state mass} \\ \Phi_i \text{ (Bethe - Salpeter) amplitude for Q-Qbar} \rightarrow \text{bound state transition} \end{array} \right.$

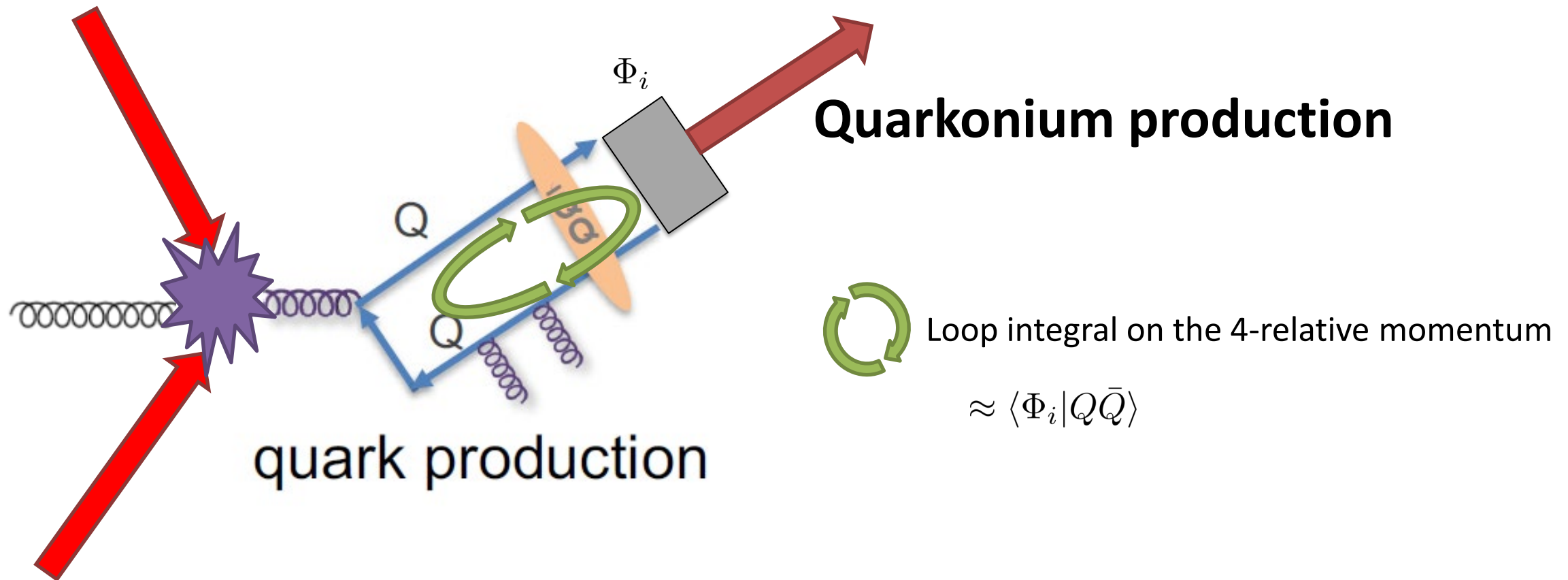
What is a quarkonia ?

This amplitude can thus be used to evaluate S – matrix and cross section for bound state formation in collisions



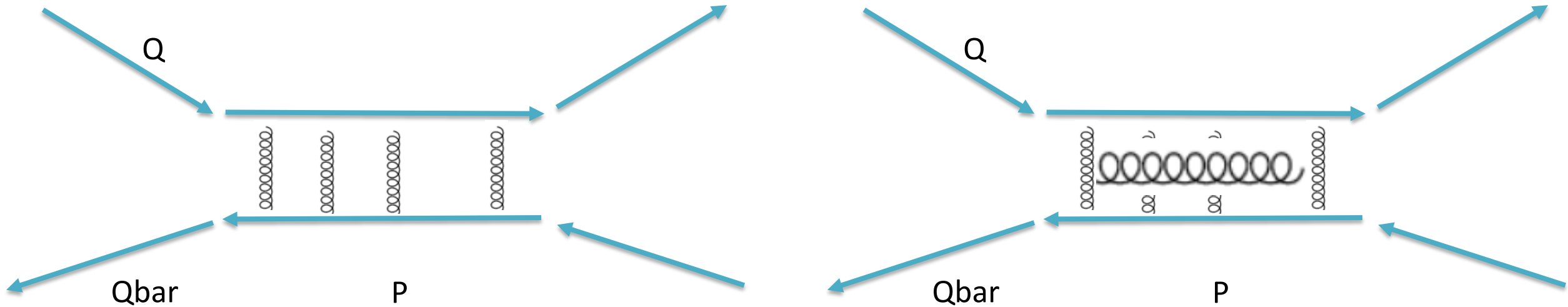
What is a quarkonia ?

This amplitude can thus be used to evaluate S – matrix and cross section for bound state formation in collisions



What is a quarkonia ?

Evaluating the scattering amplitude from first principles:



« Easy »; potential like

« complicated »

One has to pay the price for QFT: the number of quanta at given « time » is not fixed:

$$|\Psi\rangle = |Q\bar{Q}\rangle + |Q\bar{Q}g\rangle + |Q\bar{Q}q\bar{q}\rangle \dots$$



Possible strategies :

- ressumation
- effective theories
- models

What is a quarkonia ?

$|\Psi\rangle = |Q\bar{Q}\rangle + |Q\bar{Q}g\rangle + |Q\bar{Q}q\bar{q}\rangle \dots \rightarrow$ Possible strategies :

- ressumation
- effective theories
- models

 NRQCD

Benefit from $m_Q \gg \Lambda_{\text{QCD}}$: small velocities

$$m_Q \gg \Lambda_{\text{QCD}} \quad \& \quad m_Q \gg m_Q v \sim |\mathbf{p}_{\text{rel}}| \gg m_Q v^2 \sim E_{\text{bind}}$$

1. Integrate out the momentum scales above $m_Q \Rightarrow$ Non relativistic description \Rightarrow no more $g \rightarrow Q+Q\bar{q}$ vertex in diagrams (but contact terms) \Rightarrow NRQCD

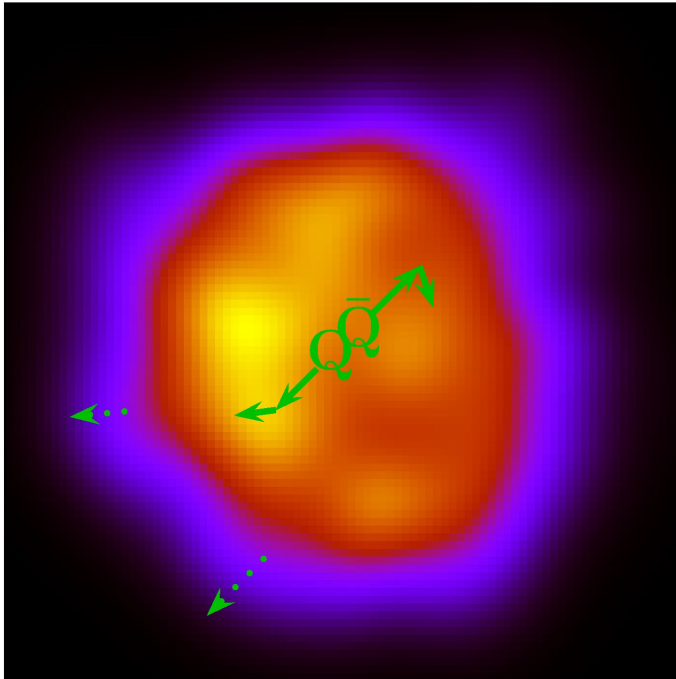
2. Integrate scales $\approx m_Q v \Rightarrow$ eliminates on shell gluon propagation \Rightarrow potential like

pNRQCD $L_{\text{pNRQCD}} = L_{\text{NRQCD}}^{\text{US}} + L_{\text{pot}}$

Projection on the Q-Qbar sector \Rightarrow singlet and octet states

$$L_{\text{pot}} = - \int d^3\mathbf{x}_1 d^3\mathbf{x}_2 \psi^\dagger(t, \mathbf{x}_1) \chi(t, \mathbf{x}_2) V(\mathbf{r}, \mathbf{p}_1, \mathbf{p}_2, \mathbf{S}_1, \mathbf{S}_2) \chi^\dagger(t, \mathbf{x}_2) \psi(t, \mathbf{x}_1)$$

What is a quarkonia... in a hot QGP medium ?



Answer may vary depending on how hot is the QGP, and how long you observe



Not to high T, not too long : Same as in vacuum + some external perturbation



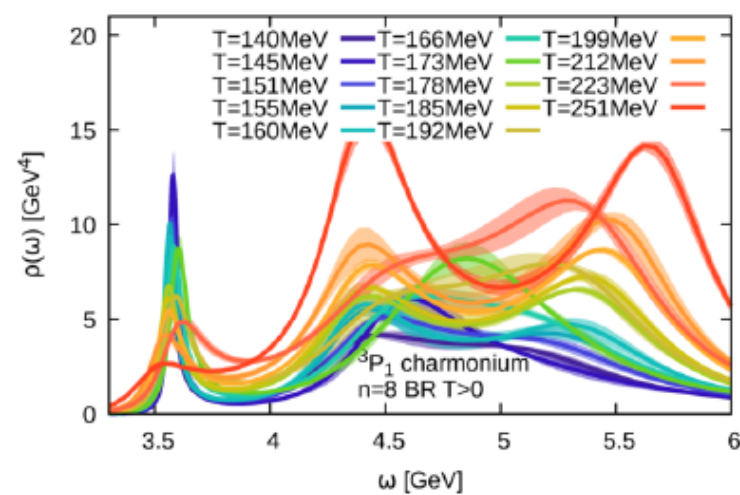
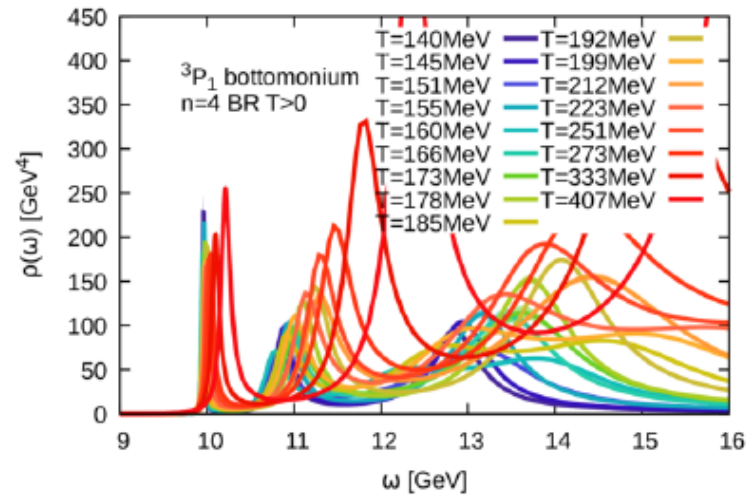
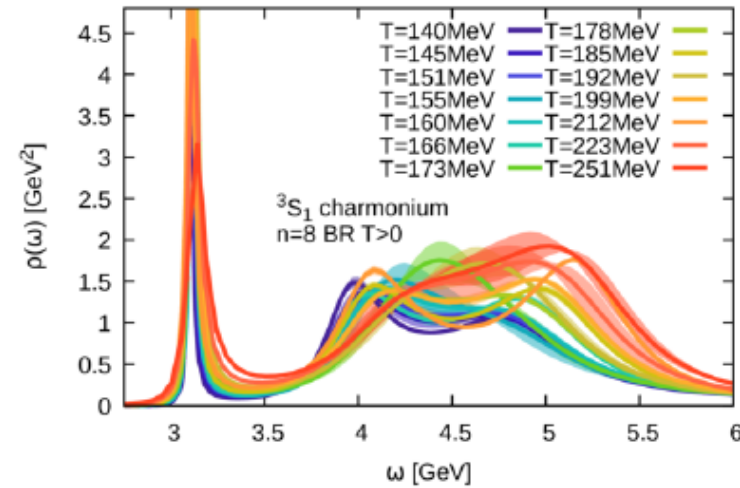
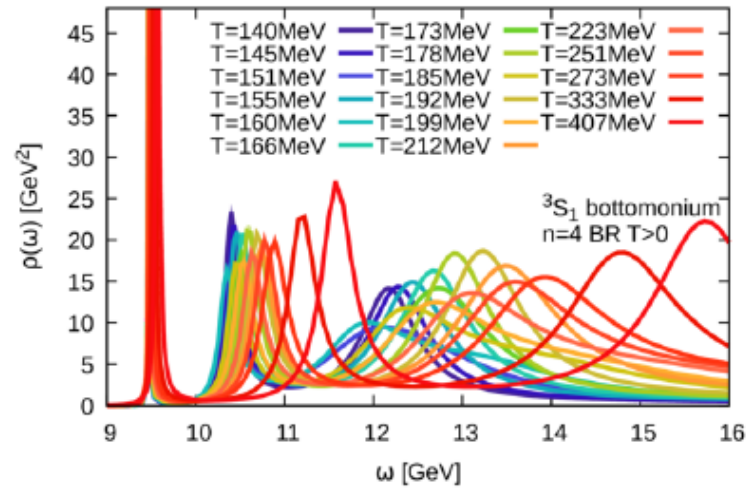
If not : probably better to speak a $Q\bar{Q}$ pair



When is it legitimate to speak of a bound state ?... And deal with it as such in the transport theory. Answer may vary depending on the fundamental ingredients

IQCD perspective : spectral function

Kim et al, JHEP11(2018)088



Many such kind of results in the literature

Rich structure : broadening and mass shift. What are the underlying “ingredients” ?

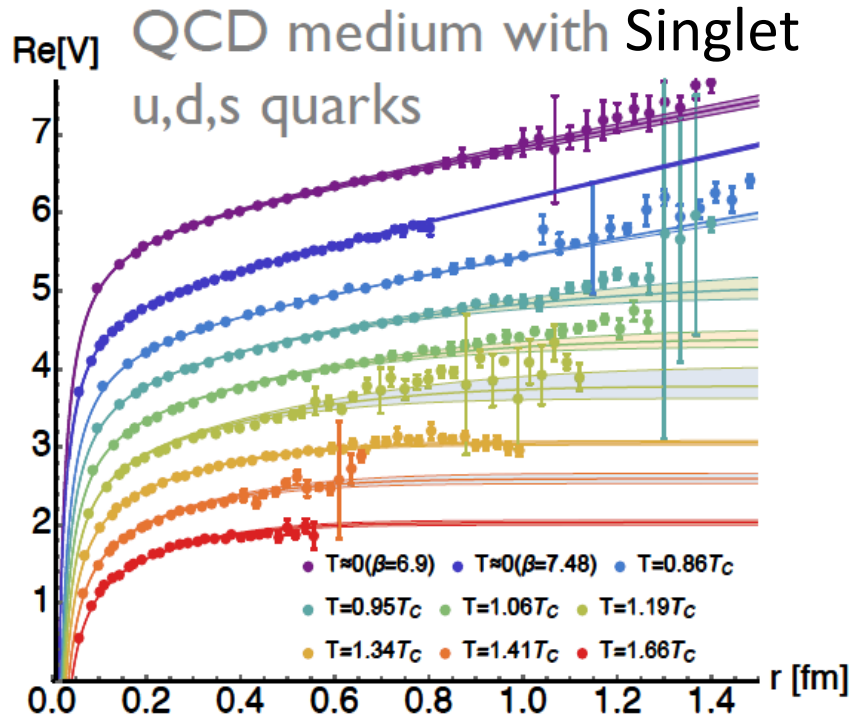
The 3 pillars of quarkonia production in AA



↳ Implicitly in the pNRQD EFT.

Screening of the real potential

Protential (recent IQCD calculations)



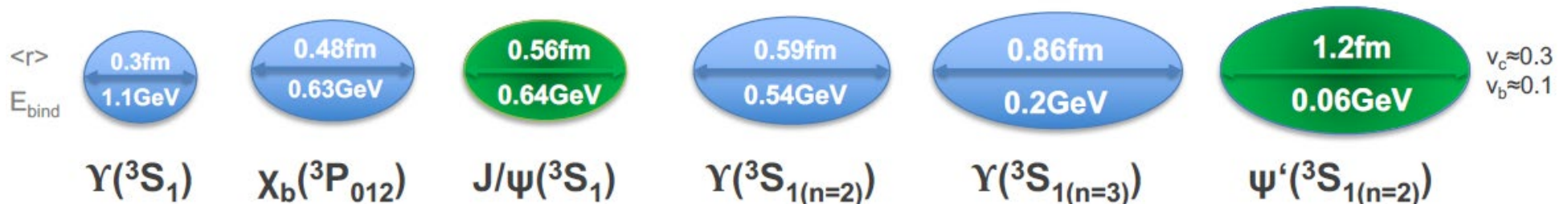
At $T=0$, well described by the Cornell shape:

$$V(r) = -\frac{\alpha}{r} + Kr$$

Quarkonia scales

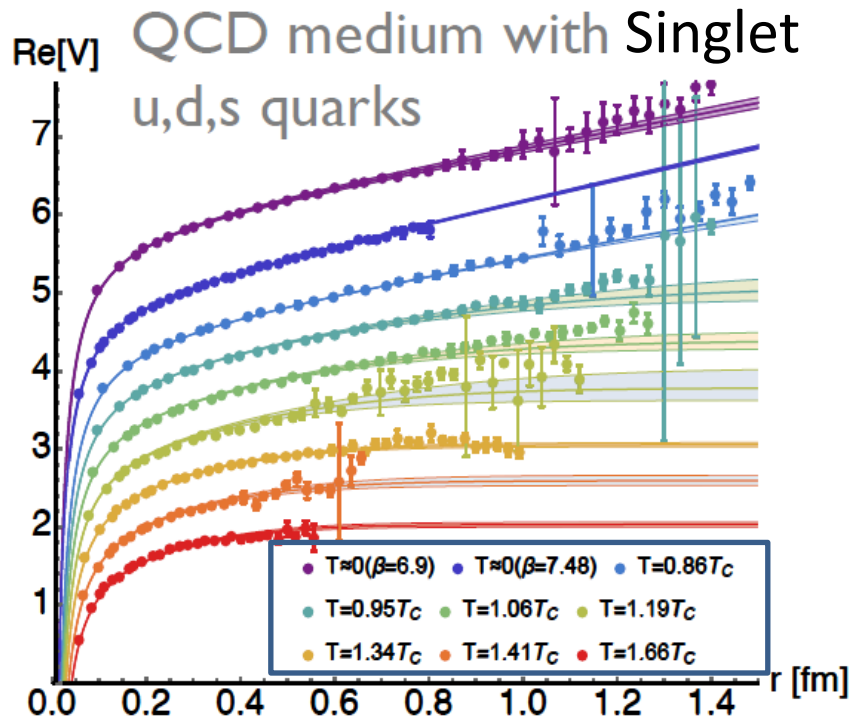
- m_Q
- **In vacuum:** Binding energy / separation energy btwn levels: $\Delta E \propto m_Q g^4$ (Coulomb part) $\Rightarrow v \propto g^2$
- Radius : $(m_Q g^2)^{-1}$
- For a linear potential $\hbar\omega_0 = \left(\frac{\hbar^2 K_l^2}{m_b/2}\right)^{\frac{1}{3}} \approx 0.504 \text{ GeV}$

$$\hookrightarrow v \propto \left(\frac{K_l}{m_b^2}\right)^{\frac{1}{3}}$$



Screening of the real potential

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At $T=0$, well described by the Cornell shape:

$$V(r) = -\frac{\alpha}{r} + Kr$$

Quarkonia scales

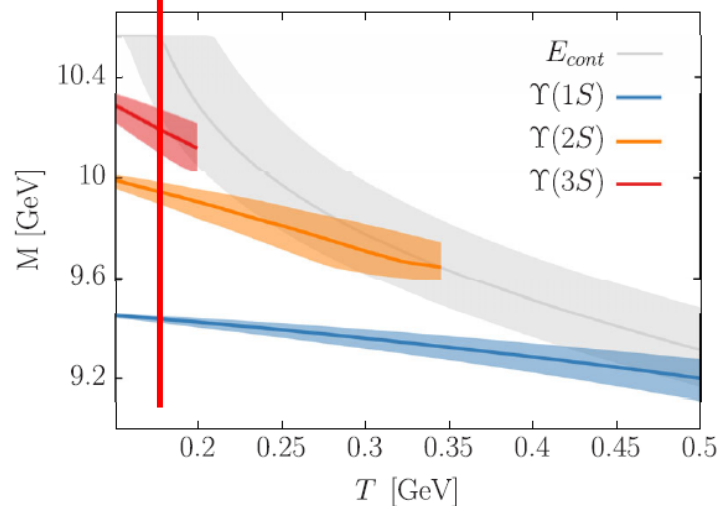
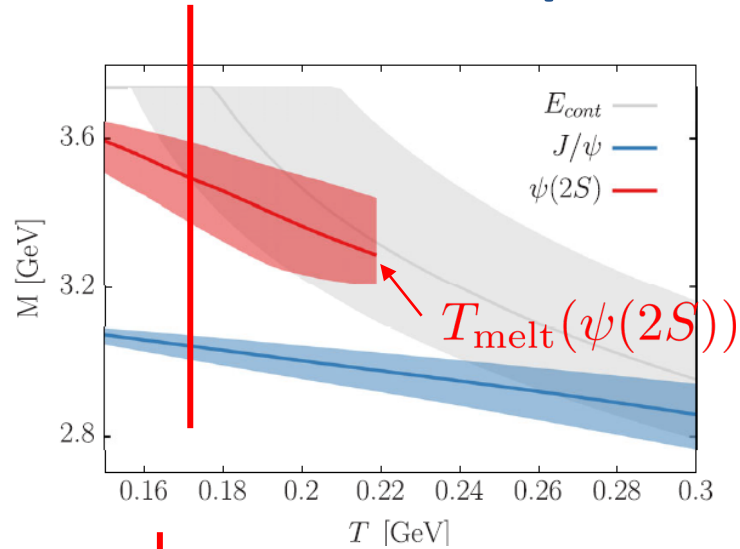
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$$\hookrightarrow v \propto \left(\frac{K_l}{m_b^2}\right)^{\frac{1}{3}}$$

Compact and tightly bound states (at least for the lowest ones) \Rightarrow could survive QGP at low/mid T as well as to interactions with hadronic matter.

Screening of the real potential

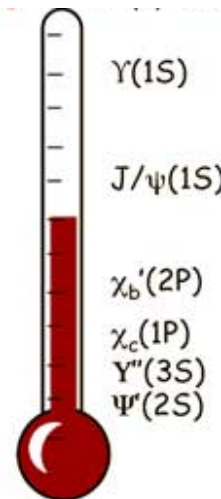
Recent In-medium spectrum (Lafferty and Rothkopf 2020)



« all or nothing scenario »:

- If $T_{\text{early QGP}} > T_{\text{melt}} \Rightarrow$
the state is not produced
- If $T_{\text{early QGP}} < T_{\text{melt}} \Rightarrow$
the state is produced like in pp

\Rightarrow *SEQUENTIAL SUPPRESSION; Quarkonia as early QGP thermometer*

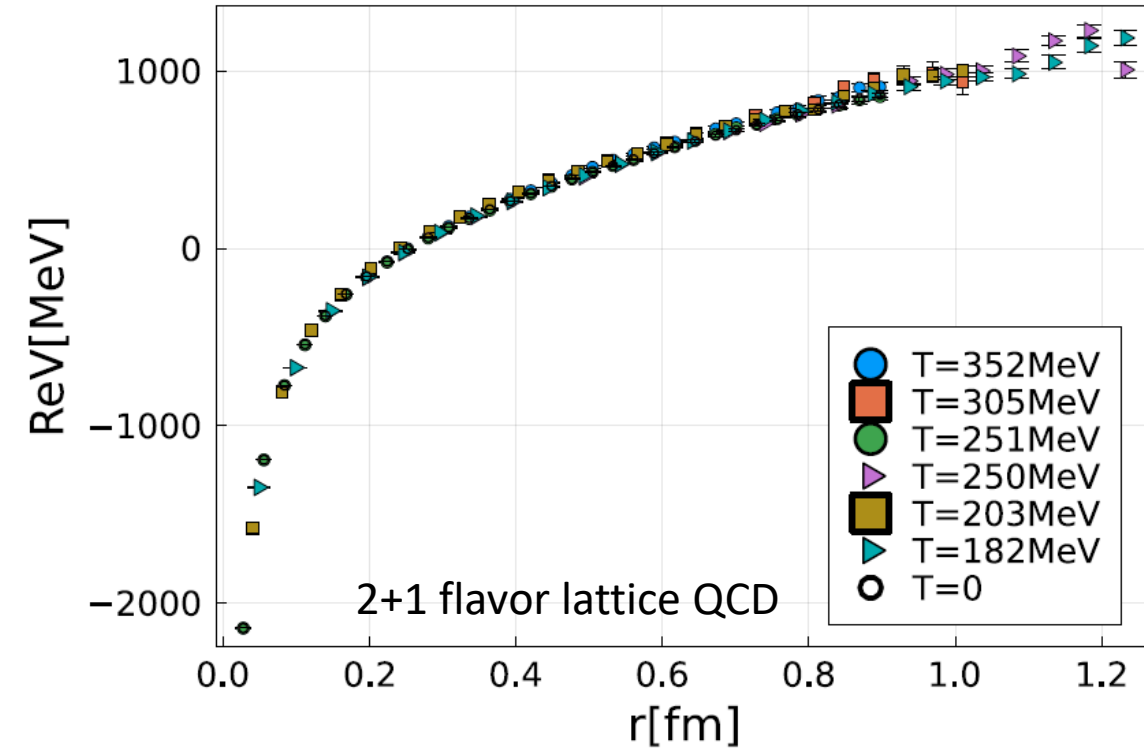


Most prominently : probing new state of matter in AA collision: Original idea by Matsui and Satz (86)...

... and advertized as a motivation in hundreds of talks (and papers) since then

Screening of the real potential

Recent news : the real potential is not screened at temperatures reached in AA collisions !!!



Bazazov et al 2023 (Hot QCD collaboration)

How to define properly a “potential” on the lattice ?

Historically : thermodynamical potential like the free energy (in presence of a static dipole) or the total internal energy.

Modern approach : evaluate the Wilson loop and connect it to the r-dependent spectral density

$$W(\tau, r, T) = \int_{-\infty}^{+\infty} d\omega e^{-\omega\tau} \rho_r(\omega, T)$$

A “peak” contribution in the spectral density modelled as

$$\rho_r^{\text{peak}}(\omega, T) = \frac{1}{\pi} \text{Im} \frac{A_r(T)}{\omega - \text{Re}V(r, T) - i\Gamma(\omega, r, T)}$$

=> Lattice data then unfolded with this Ansatz.

Does not seem quite intuitive, may not be the end of the story

Screening of the real potential

Recent news : the

... screened at temperatures reached in AA collisions !!!

How to define properly a "potential" on the lattice ?

... thermodynamical potential like the free energy (in ...) or the total internal energy.

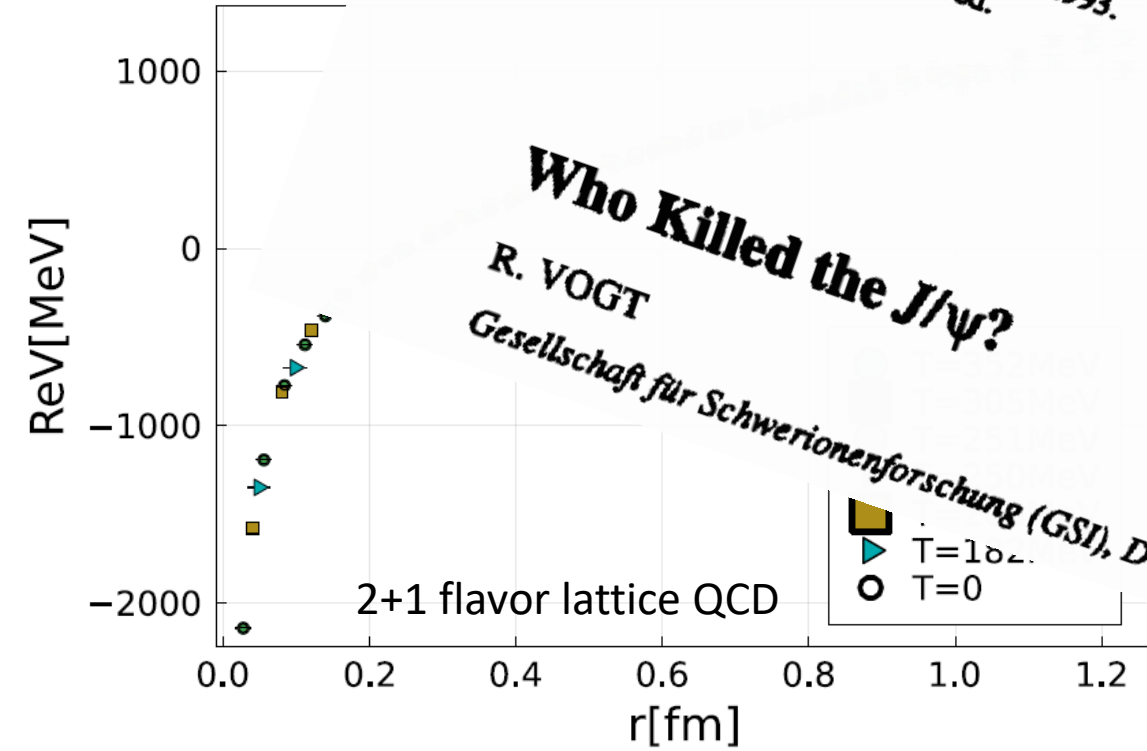
... loop and connect it to

0146-6410/93 \$24.00
© 1993 Pergamon Press Ltd

... lled as

$$\rho_T^{\text{peak}}(\omega, T) = \pi \overline{\dots; T}$$

=> Lattice data then deconvoluted with ... satz.

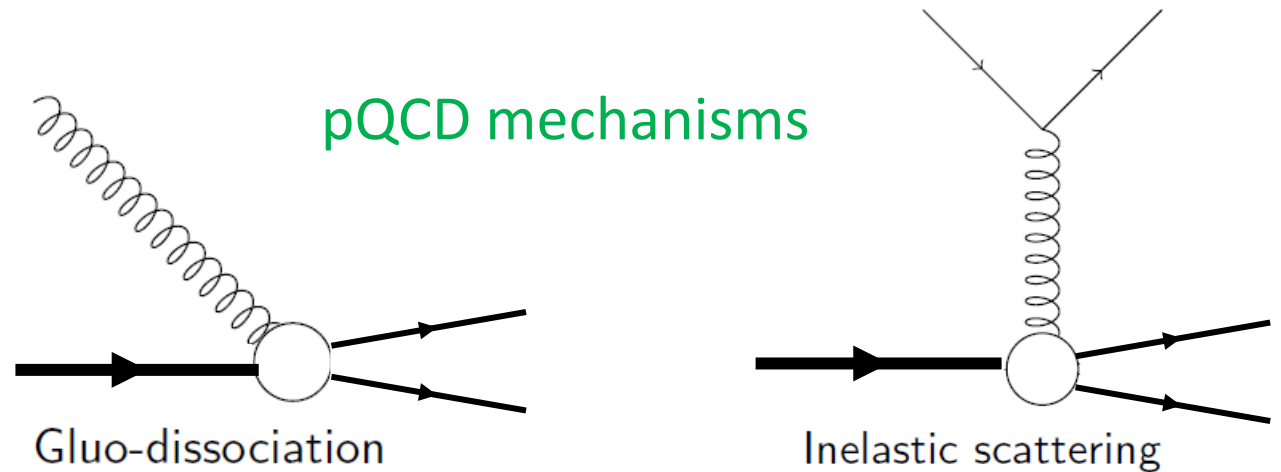
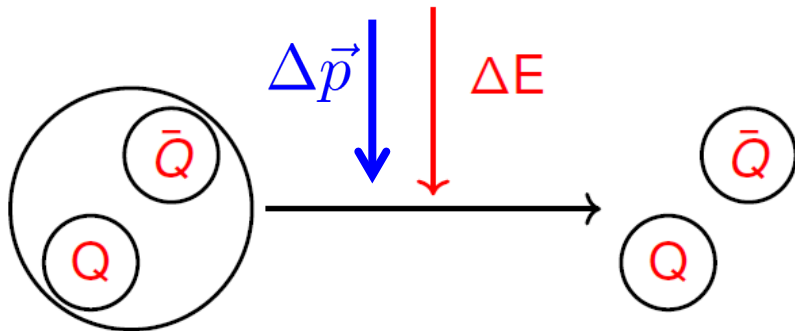


Bazazov et al 2023 (Hot QCD collaboration)

Does not seem quite intuitive, may not be the end of the story

Collisions with the QGP

- Besides arguments based on the Debye mass / screening, it was pointed out already in the 90's that interactions with partons in the QGP could lead to dissociation of bound states (whose spectral function thus acquire some width Γ corresponding to the dissociation rate)
- Energy-momentum exchange with the QGP (gluo-dissociation, q – quarkonia quasi elastic scattering)

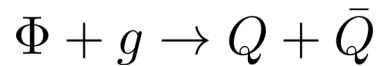
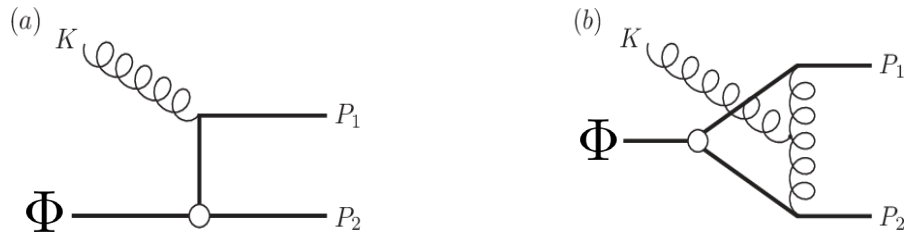


- => pair dissociation => **Suppression**
- ↔ loss of probability of the quarkonia ... Often described by some imaginary potential W in modern approaches

A central quantity: the decay rate Γ

Many approaches

pQCD view (Bhanot & Peskin), later on consolidated by NRQCD (Brambilla & Vairo)



Dissociation cross section σ



$$\Gamma_{\Phi}(T) = \langle \sigma n_g \rangle_T$$

Other mechanisms : $x + \Phi \rightarrow x + Q + \bar{Q}$

QFT/Lattice QCD

Time correlator

$$\mathcal{C}_{>}(t, \vec{r}) \approx \langle \psi(t, \frac{\vec{r}}{2}) \bar{\psi}(t, -\frac{\vec{r}}{2}) \psi(0, 0) \bar{\psi}(0, 0) \rangle$$

Satisfies Schroedinger equation with complex potential $V+iW$. Breakthrough by Laine et al. (2006)



$$\Gamma_{\Phi}(T) = -2 \langle \Phi | W | \Phi \rangle$$

Concept better suited as it genuinely encodes the “in medium” propagation

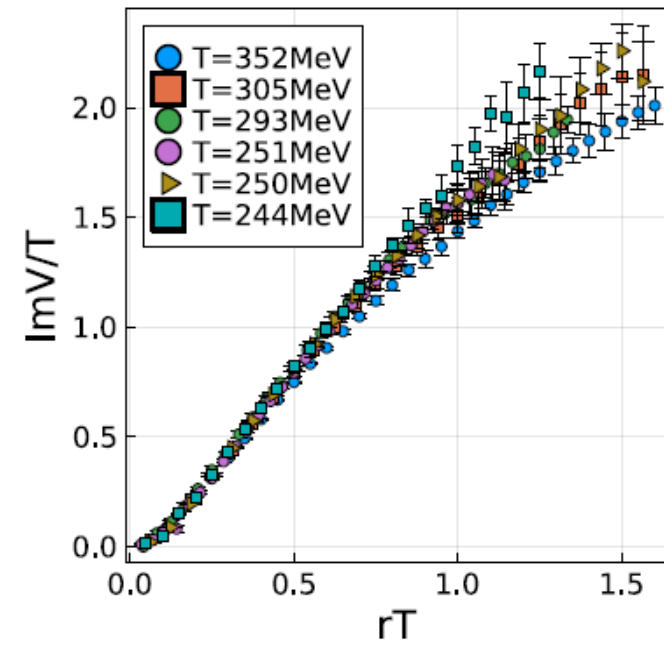
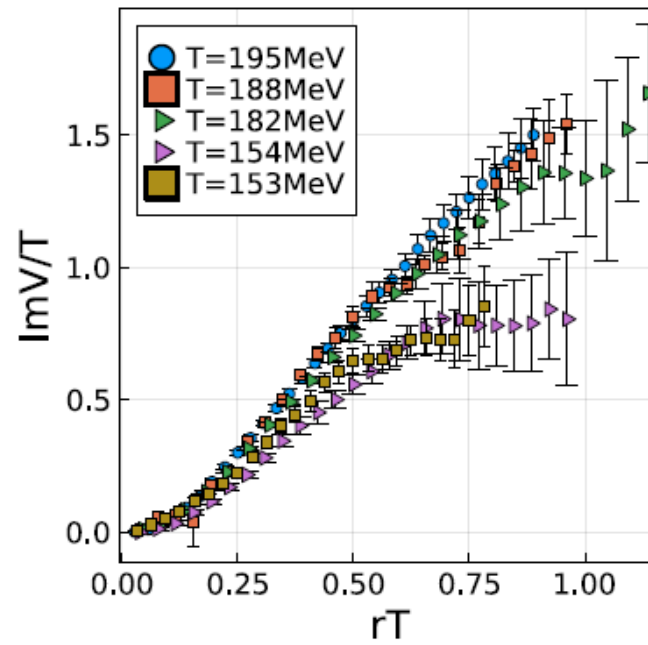
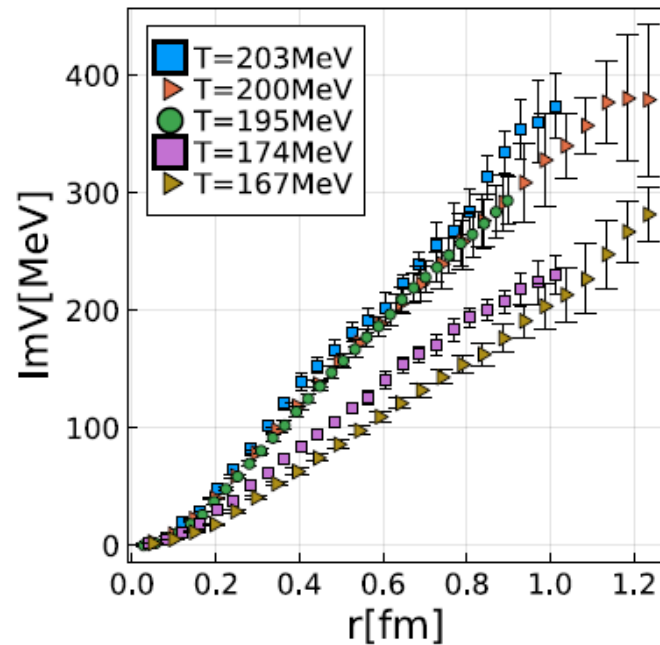
$$\Rightarrow \text{Simple decay law : Prob survival} = \exp\left(-\int_{t_0}^{t_{\text{fin}}} \Gamma(T(t)) dt\right)$$

A central quantity: the decay rate Γ

Recent IQCD calculations of $W(r) = \text{Im}(V(r))$ (at $\omega=0$)

$$\rho_r^{\text{peak}}(\omega, T) = \frac{1}{\pi} \text{Im} \frac{A_r(T)}{\omega - \text{Re}V(r, T) - i\Gamma(\omega, r, T)}$$

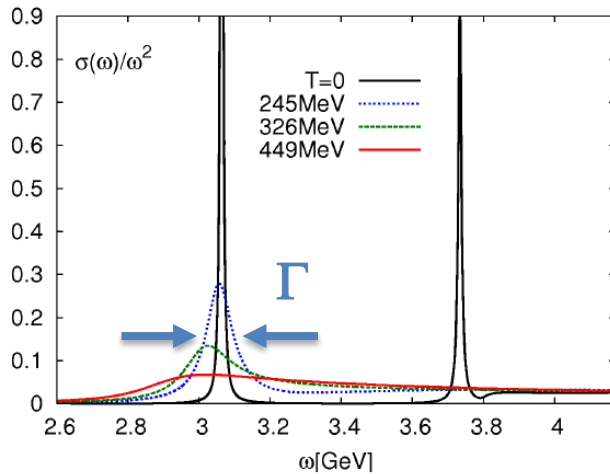
Bazazov et al 2023 (Hot QCD collaboration)



- Nice r T scaling
- Dipole structure at small r , no saturation seen at “large” r

Quarkonia at finite T

- Pheno: Yet, these pictures might still be compatible with the notion of sequential « suppression »...
- However, this notion has to be made more precise : (LQCD) spectral function IQCD



$$\rho(\omega, p, T) = \frac{1}{2\pi} \text{Im} \int_{-\infty}^{\infty} dt e^{i\omega t} \int d^3x e^{ipx} \langle [J(x, t), J(0, 0)] \rangle_T$$

At $T=245$ MeV, ψ' has disappeared but J/ψ still surviving for $\approx 1/\Gamma \approx$ a couple of fm/c ... which needs to be compared with the local QGP cooling time τ_{cool} : $\Gamma \times \tau_{\text{cool}} > 1 \Leftrightarrow$ suppressed

- N.B.: The opposite phenomenon might also be relevant: some state above the « melting » temperature can survive (for a short while $< 1/\Gamma$) before getting lost definitively.
- Key question : do the quarkonia states (chemically) equilibrate with the QGP ?

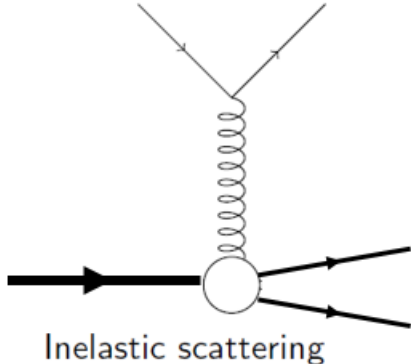
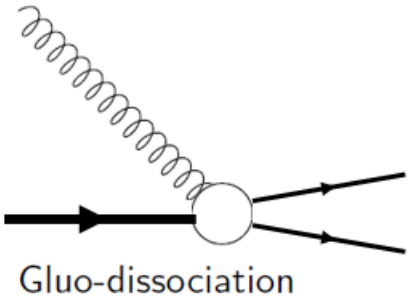
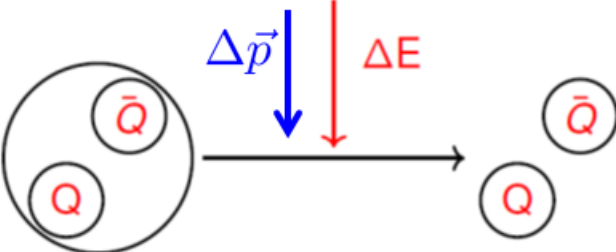


Will it melt
(even party) ?

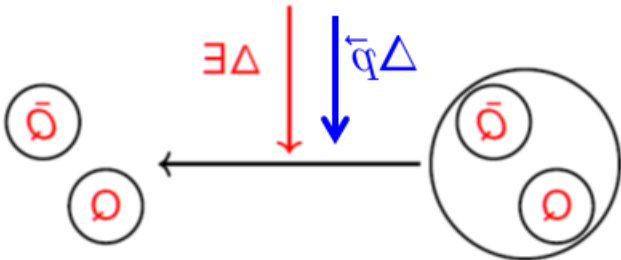
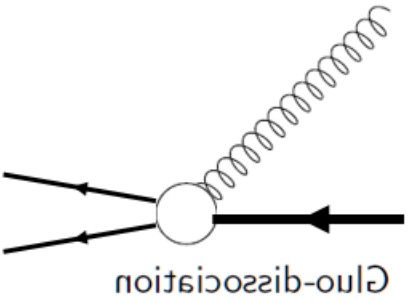
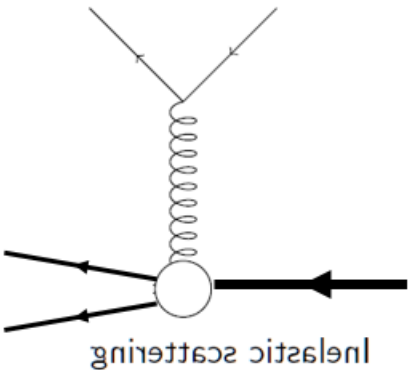
Modern era

If life was not complicated enough...: Regeneration

Detailed balance :

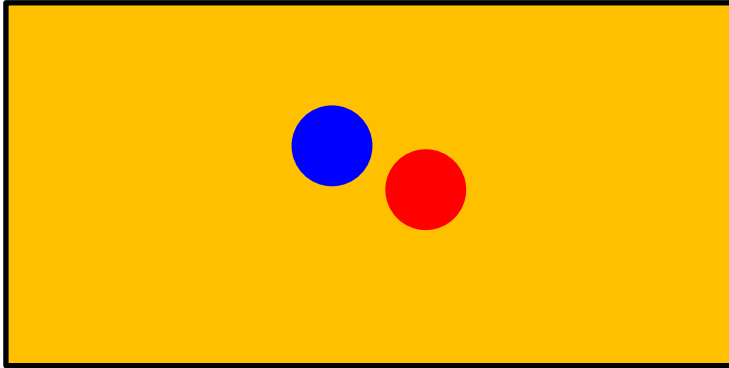


Reverse mechanisms

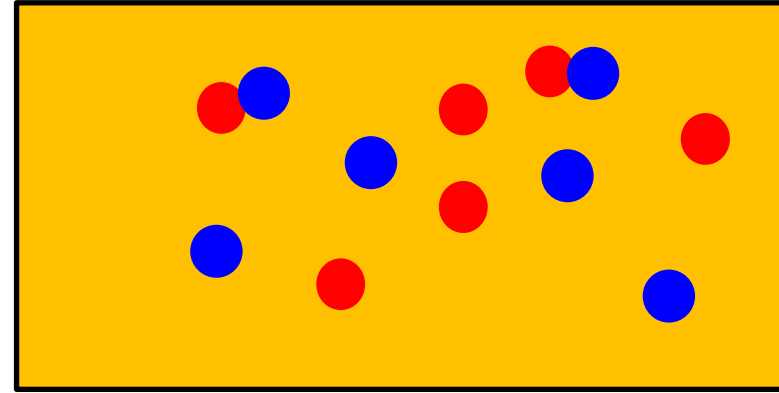


Regeneration: Dilute vs Dense

Bottomia

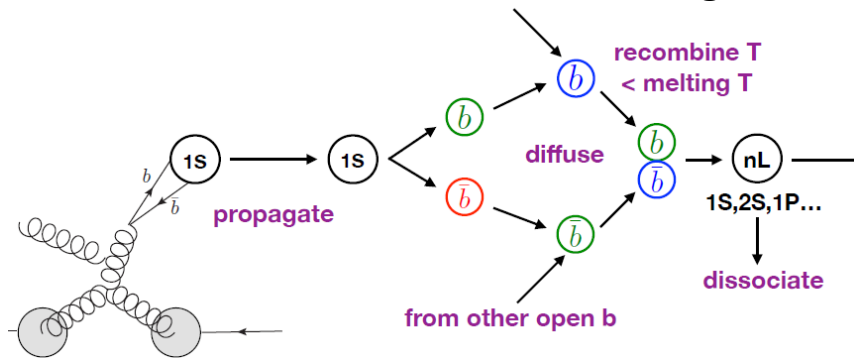


Charmonia



No exogenous recombination : only the b - \bar{b} pairs which are initially close together will emerge as bottomia states

In some SC formalisms : intermediate regeneration

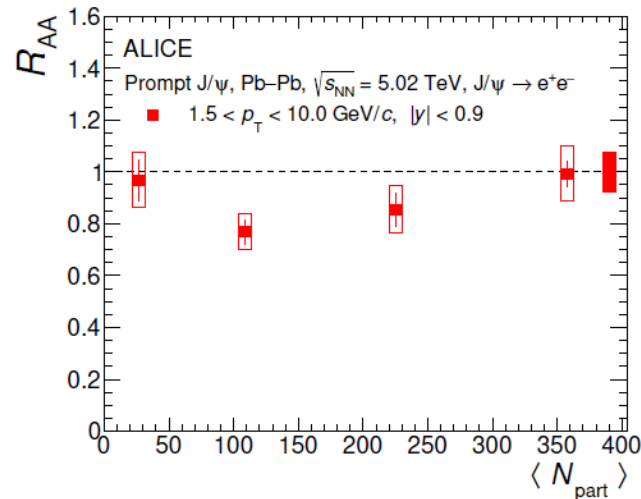
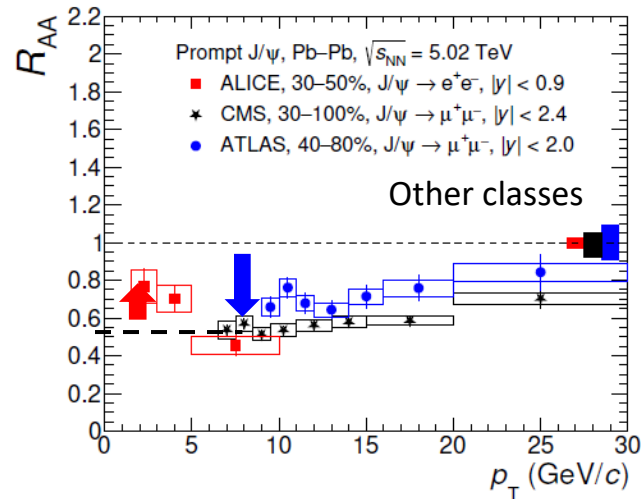
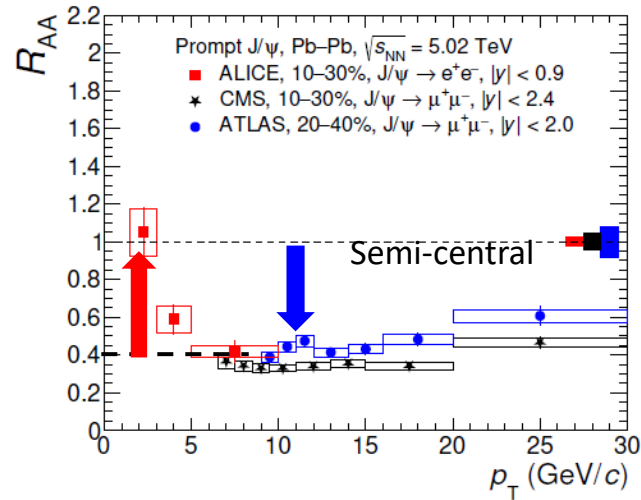
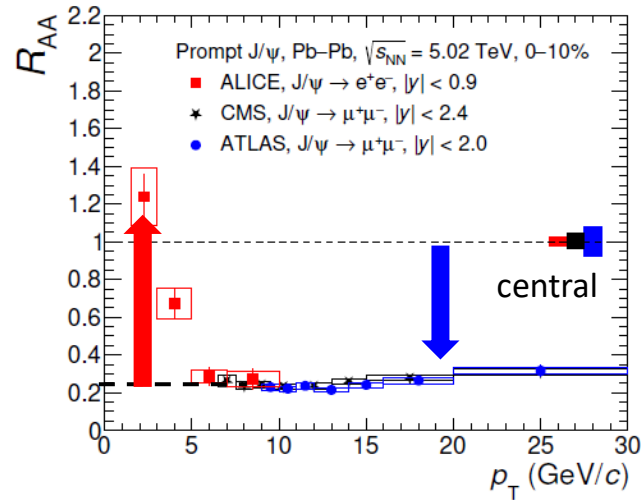


Yao, Mehen, Müller

Exogenous recombination : c & \bar{c} initially far from each other may recombine and emerge as charmonia states

- No full quantum treatment => semi-classical approximation
- Key questions : when does the recombination (dominantly) happen ? Crucial role of the binding force.
- Are the c and \bar{c} distributions equilibrated at this time ?
- One extreme viewpoint : regeneration happens at the end of the QGP (Statistical Hadronization Model: ask if you want)

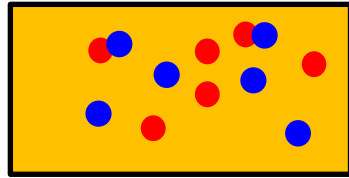
What experiment tells us



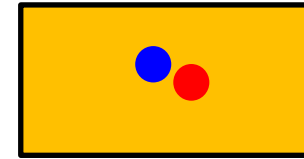
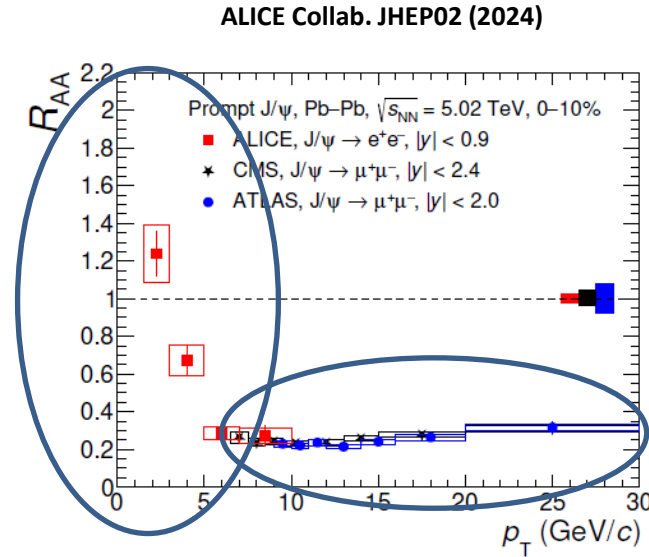
ALICE Collab. JHEP02 (2024) 066

- Quarkonia production in AA strongly affected by the presence of the QGP => good probe of the QGP properties on small scales ($1/M_Q$)
- Increasing suppression with centrality at intermediate and high p_T
- Increasing yield with centrality at low p_T
- Increasing experimental precision => need for the models to gain in accuracy

What experiment tells us

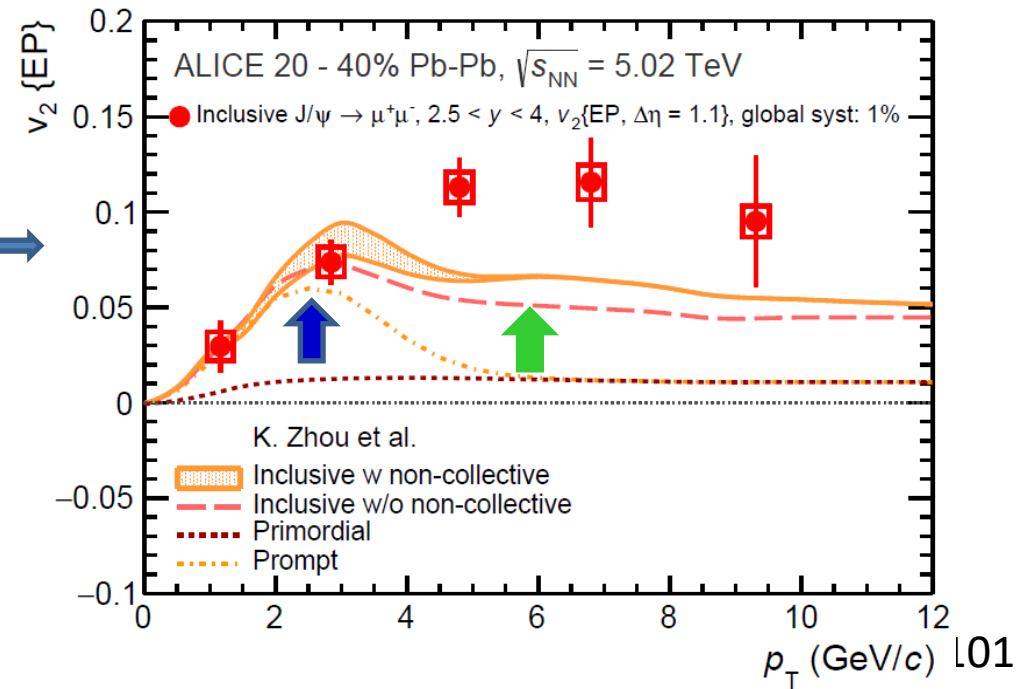


Dense (in phase space)
=> recombination

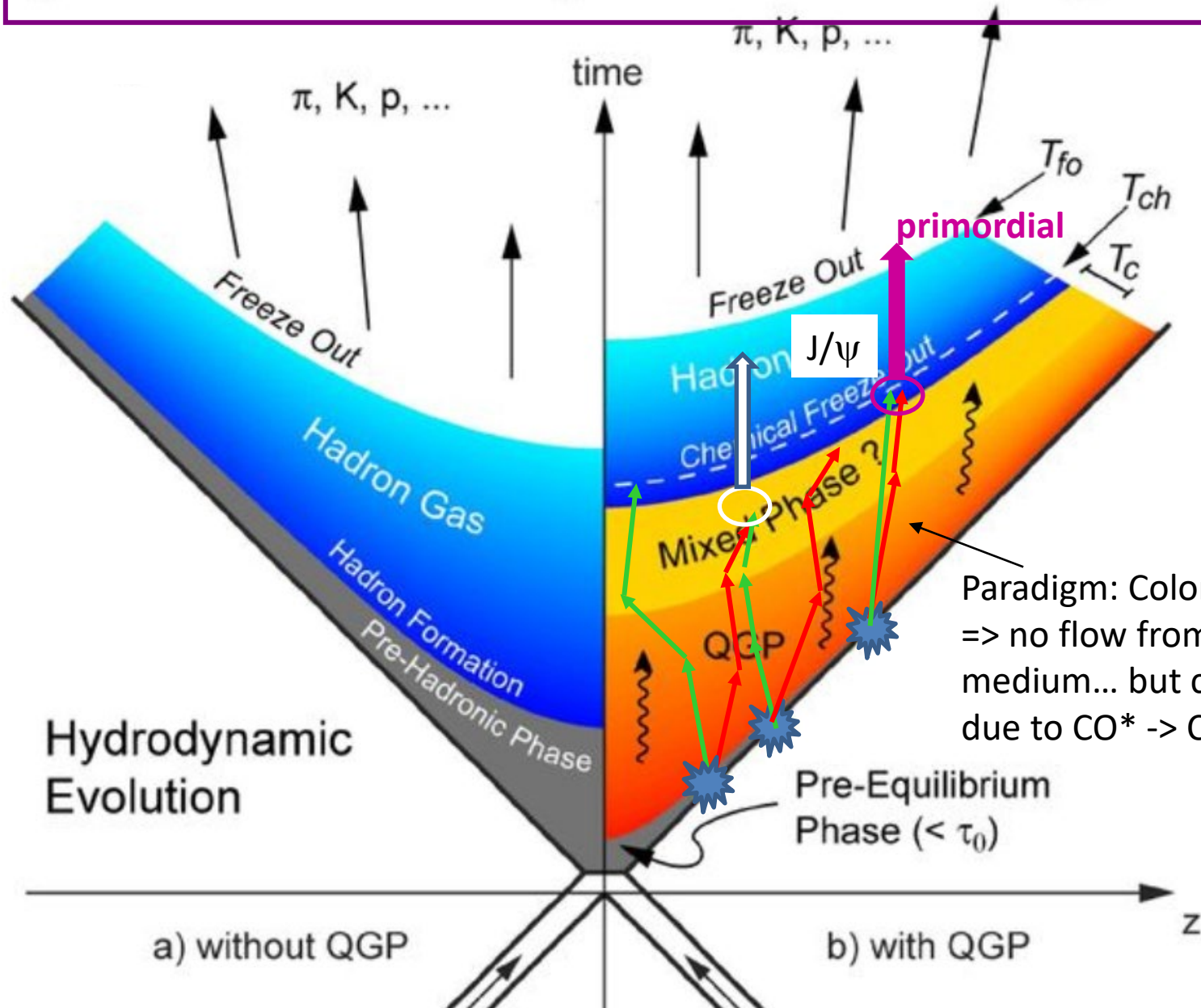


Dilute (in phase space)

Alternate possible explanation : p_T -dependent absorption cross section :
not excluded, but not favored by the finite v_2 observed for J/ψ by ALICE →



Quarkonia in transport models

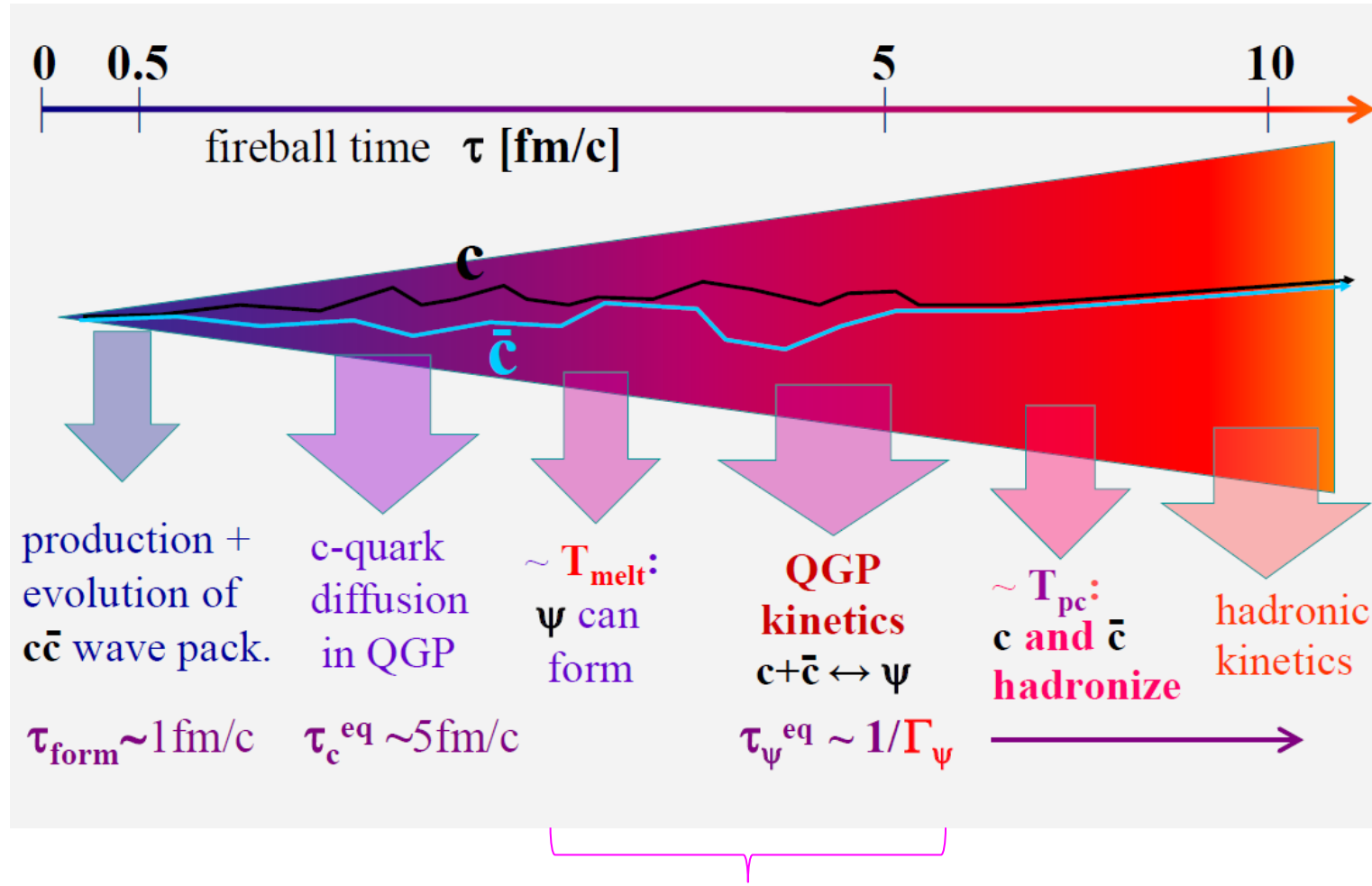


The working horse of most concrete predictions for quarkonia production in AA, especially for charmonia

Paradigm: Color singlet
=> no flow from the medium... but could be due to $CO^* \rightarrow CS$

Quarkonia in transport models

Rapp and Du *Nucl.Phys.A* 967 (2017) 216-224



“in medium” quarkonia (bound states of possibly screened potential)

Reaction rate approach by TAMU

Rate equation for quarkonia (main dof) :

(the gain and loss are not evaluated locally in phase space)

$$\frac{dN(t)}{dt} = -\Gamma(T(t)) (N(t) - N^{\text{eq}}(T(t)))$$

Loss

Gain

Automatically takes care of the recombination at time t **assuming HQ are in thermal equilibrium**

Can again be split into 2 components :

$$\frac{dN^{\text{prim}}(t)}{dt} = -\Gamma(T(t)) N^{\text{prim}}(t) \quad \text{with} \quad N^{\text{prim}}(t_0) = N_0$$

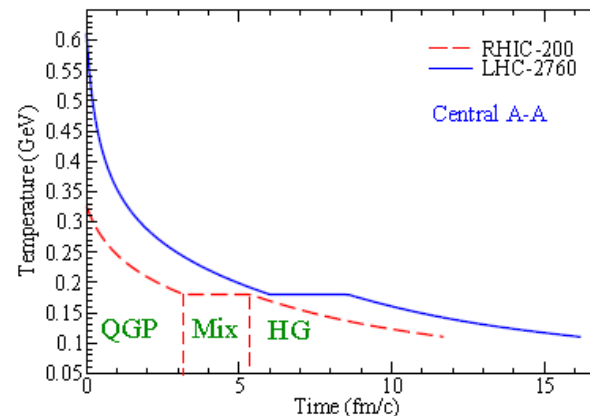
$$\frac{dN^{\text{regen}}(t)}{dt} = -\Gamma(T(t)) [N^{\text{regen}}(t) - N^{\text{eq}}(T)] \quad \text{with} \quad N^{\text{regen}}(t_0) = 0$$

- Naturally interpolates between simple suppression and regeneration
- Dissociation Rate from usual cross sections with quasi particle masses compatible with IQCD EOS

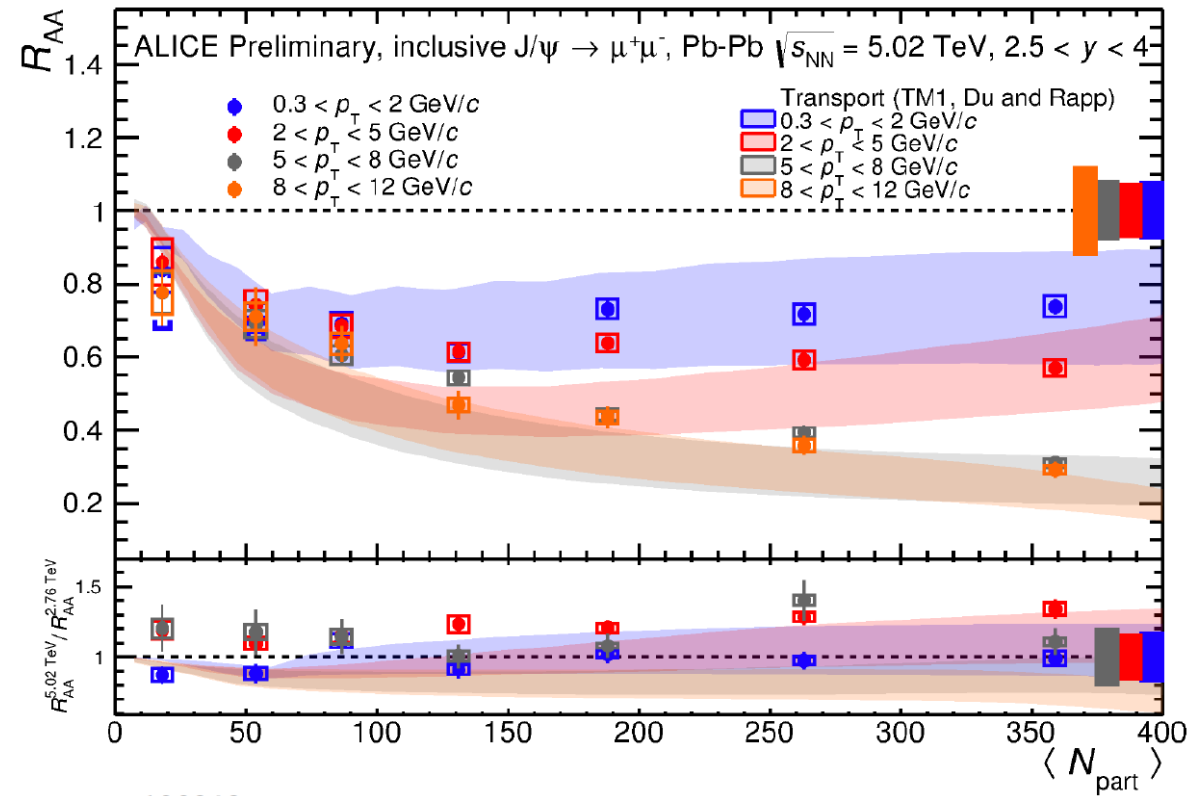
$$\Gamma_{\Psi}(T) = \int \frac{d^3p}{(2\pi)^3} v_{i\Psi} \sigma_{i\Psi \rightarrow X} f_i(m_i, T)$$

Parton Density

- Mostly Implemented in fireball model
- Including contribution from hadron gas



Reaction rate approach by TAMU



ALI-PREL-120949

- In transport theory, primordial component is mandatory to reproduce the absolute production as a function of centrality & p_T class



Not simple statistical hadronization at the end of the QGP

A consistent picture emerging in the charmonia sector

Good overall consistency of the following facts:

- Increase of J/ψ production from RHIC \rightarrow LHC
- Mostly at low p_T where regeneration is expected
- **Finite $v_2(J/\psi)$ observed for the first time with 5σ confidence**
- Washing out of the spectral function (only J/ψ survive for $T < 0.25$ GeV)
- Statistical ratio achieved for $\psi' / J/\psi$ at LHC for central and semi-central

Not paying too much attention at CNM effects:

With the interpretation that a large fraction of direct quarkonia are produced through recombination (also see in transport models)

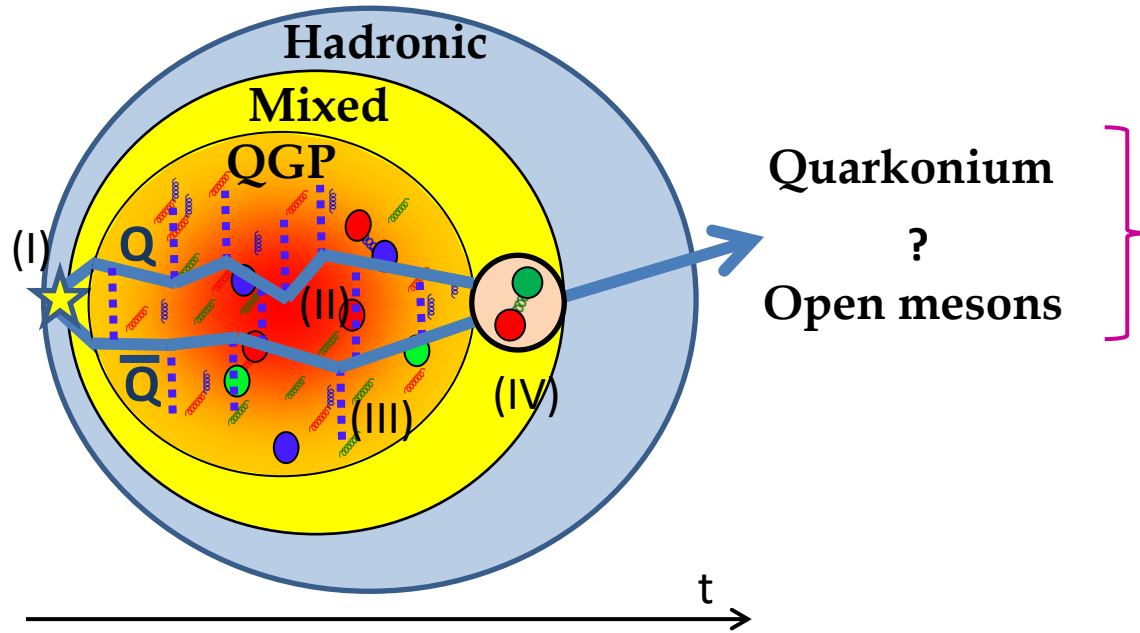
N.B.: if state not dissociated / tightly bound, then little recomb. as well (small Γ) \Rightarrow cannot benefit from the $v_2(c)$, except it some significant elastic scattering (no sign for this)

$$\frac{dN_\Phi}{dt} = -\Gamma(T(t)) (N_\Phi - N_{\Phi,eq}(T(t)))$$

Remaining challenges:

- v_2 at finite (5-10 GeV) p_T (A lot of effects can destroy the Onium, but how to give it v_2 ?)
- role of the magnetic field (not discussed)

The full scheme for microscopic approaches



Strictly speaking, only resolved at the end of the evolution



Beware of quantum coherences during the whole evolution !



Especially at early time...

In practice, what counts is the so-called decoherence time, not the "Heisenberg time"

Complicated QFT problem (also due to the evolving nature of the QGP that mixes several scales)... only started to be addressed at face value recently

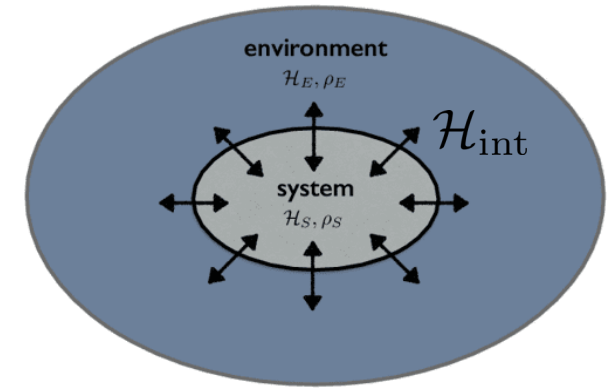
- 1) Initial state
- 2) (Screened) interaction between both HQ
- 3) Interactions with surrounding QGP partons
- 4) Projection on the final quarkonia

How to proceed ?

First incomplete QM treatments dating back to Blaizot & Ollitrault, Thews, Cugnon and Gossiaux; early 90's

Open Quantum Systems & Quantum Master Equations

Quite generally, system (Q-Qbar pair) builds correlation with the environment thanks to the Hamiltonian $\hat{H} = \hat{H}_{Q\bar{Q}}^{(0)} + \hat{H}_E + \hat{H}_{\text{int}}$ with $\hat{H}_E = \hat{H}_{QGP}$



Von Neumann equation for the total

density operator ρ

System + environment (QGP)
 $\rho(t=0) = \rho_{Q\bar{Q}} \otimes \rho_{QGP}$

$$\frac{d}{dt}\rho = -i[H, \rho]$$

Evolution of the total system
 $\rho(t) = U(t, 0) [\rho_{Q\bar{Q}} \otimes \rho_{QGP}] U(t, 0)^\dagger$

Trace out QGP degrees of freedom =>
 Reduced density operator $\rho_{Q\bar{Q}}$

System (Q \bar{Q} pair)
 $\rho_{Q\bar{Q}}(t=0)$

Evolution of the system
 $\rho_{Q\bar{Q}}(t) = \text{Tr}_{QGP} [U(t, 0)\rho(t=0)U(t, 0)^\dagger]$

Can be formulated differentially ./ time :

$$\frac{d\rho_{Q\bar{Q}}}{dt} = \mathcal{L}[\rho_{Q\bar{Q}}]$$

Definition of $\mathcal{L}[\cdot]$



Open Quantum Systems & Quantum Master Equations

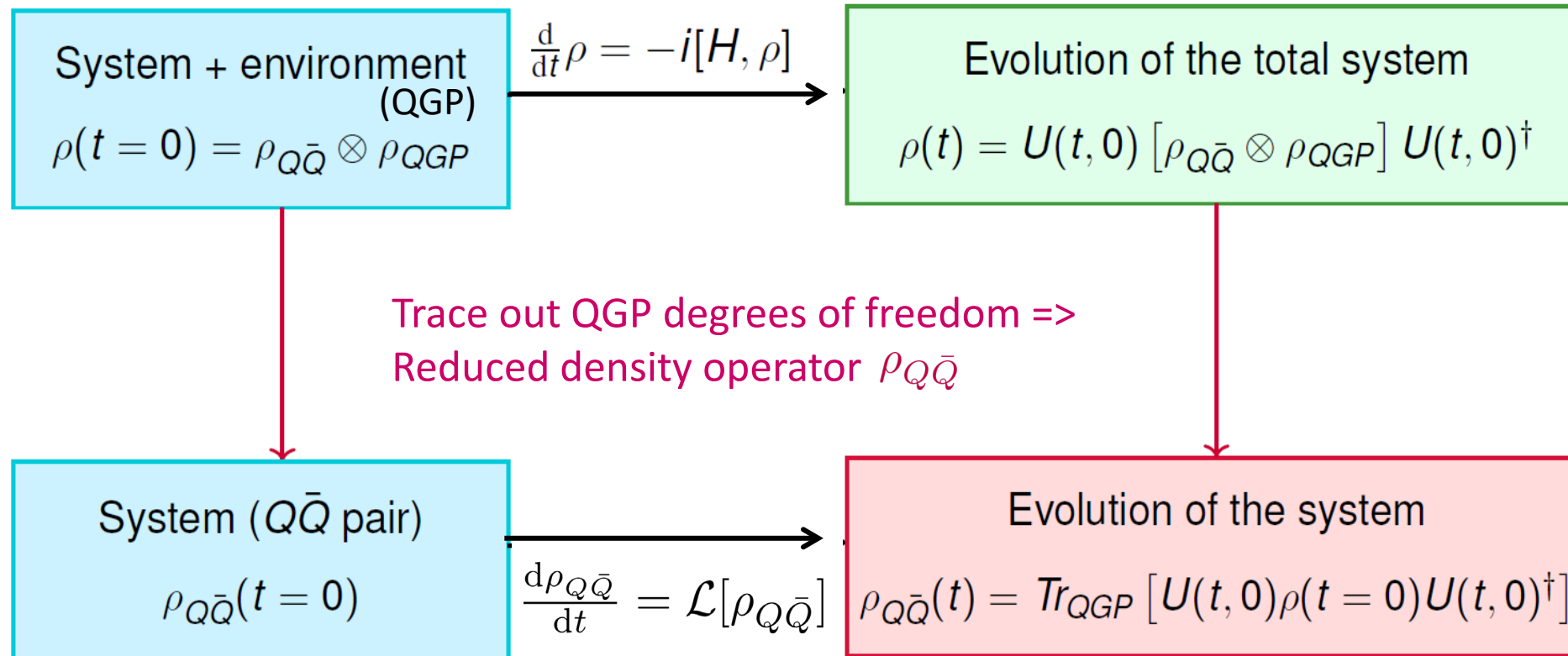
$$\hat{\rho}_{Q\bar{Q}} = \sum_{\alpha,\beta} d_{\alpha,\beta} |\alpha\rangle\langle\beta|$$

Quite generally, system (Q-Qbar pair) builds correlation with the environment thanks to the Hamiltonian $\hat{H} = \hat{H}_{Q\bar{Q}}^{(0)} + \hat{H}_E + \hat{H}_{\text{int}}$ with $\hat{H}_E = \hat{H}_{QGP}$

QME deal with the (coupled) evolution of probabilities ($d_{\alpha,\alpha}$) and coherences ($d_{\alpha,\beta \neq \alpha}$)

Von Neumann equation for the total

density operator ρ



However, $\mathcal{L}[\cdot]$ is generically a non local super-operator in time (linear map)

A special QME: The Lindblad Equation

There are many different QME... a special one :

$$\frac{d}{dt}\rho_{Q\bar{Q}}(t) = -i[H_{Q\bar{Q}}, \rho_{Q\bar{Q}}(t)] + \sum_i \gamma_i \left[L_i \rho_{Q\bar{Q}}(t) L_i^\dagger - \frac{1}{2} \{ L_i L_i^\dagger, \rho_{Q\bar{Q}}(t) \} \right]$$

γ_i Characterize the coupling of the system (Q-Qbar) with the environment

$H_{Q\bar{Q}} : \{Q, \bar{Q}\}$ kinetics + Vacuum potential V + Lamb shift / screening (every unitary term that is generated by tracing out the environment)

$\underbrace{\hspace{10em}}_{\hat{H}_{Q\bar{Q}}^{(0)}}$

L_i : Collapse (or Lindblad) operators, depend on the properties of the medium

3 important conservation properties :

$$\rho_{Q\bar{Q}}^\dagger = \rho_{Q\bar{Q}}$$

(Hermiticity)

$$\text{Tr}[\rho_{Q\bar{Q}}] = 1$$

(Norm)

$$\langle \varphi | \rho_{Q\bar{Q}} | \varphi \rangle > 0, \forall |\varphi\rangle$$

(Positivity)

... but in general, non unitary !!! (relaxation)

Nice feature : Can be brought to the form of a stochastic Schroedinger equation (quantum jump method : QTRAJ)

A special QME: The Lindblad Equation

Non unitary / dissipative evolution \equiv decoherence

$$\frac{d}{dt}\rho_{Q\bar{Q}}(t) = -i[H_{Q\bar{Q}}, \rho_{Q\bar{Q}}(t)] + \sum_i \gamma_i \left[L_i \rho_{Q\bar{Q}}(t) L_i^\dagger - \frac{1}{2} \{ L_i L_i^\dagger, \rho_{Q\bar{Q}}(t) \} \right]$$

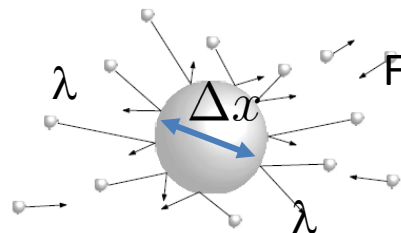
Genuine transitions :

- ✓ Singlet \leftrightarrow octet
- ✓ Octet \leftrightarrow octet

Can be reshuffled into non Hermitic effective hamiltonian

$$\hat{H}_{Q\bar{Q},\text{eff}} = \hat{H}_{Q\bar{Q}} - i \sum_j \gamma_j \frac{L_j L_j^\dagger}{2} \equiv \text{Dissociation width}$$

For **infinitely massive single Q** and environment wave length $\lambda \gg$ wave packet size Δx :



Fluctuations from env. \longleftrightarrow

$$\frac{\partial \rho_Q(x_Q, x'_Q)}{\partial t} = -F(x_Q - x'_Q) \rho_Q(x_Q, x'_Q)$$

Decoherence factor: $F \approx \kappa (x_Q - x'_Q)^2$

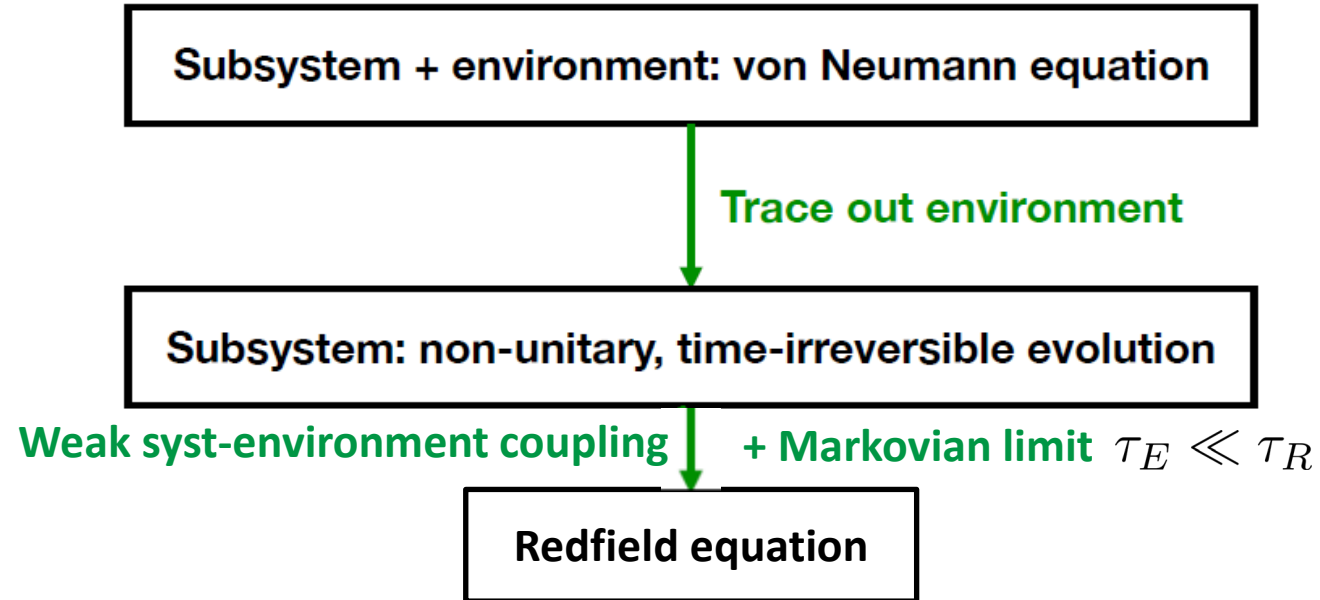
In Q world: smaller objects live longer !

At 1st order in $1/m_Q$: recoil corrections \longleftrightarrow friction / dissipation

HQ momentum diffusion coefficient (adjoint)

Pictorial summary

τ_E : environment autocorrelation time τ_S : system intrinsic time scale τ_R : system relaxation time



$$\frac{\partial}{\partial t} \rho_I(t) = -\frac{1}{\hbar^2} \sum_{m,n} \int_0^\infty d\tau \left(C_{mn}(\tau) \left[S_{m,I}(t), S_{n,I}(t-\tau) \rho_I(t) \right] - C_{mn}^*(\tau) \left[S_{m,I}(t), \rho_I(t) S_{n,I}(t-\tau) \right] \right)$$

Similar structure to the Linblad equation but with time delay effects

Pictorial summary

τ_E : environment autocorrelation time τ_S : system intrinsic time scale τ_R : system relaxation time

Subsystem + environment: von Neumann equation

Trace out environment

Subsystem: non-unitary, time-irreversible evolution

Weak syst-environment coupling + Markovian limit $\tau_E \ll \tau_R$

Redfield equation

Smallest time scales wins it all !

$\tau_S \ll \tau_R$
Quantum Optical Regime

$\tau_E \ll \tau_S$
Quantum Brownian Motion

Lindblad equation

Lindblad equation

Eigenstates of the HQ Hamiltonian

Not the same basis !

Phase space densities

Wigner transform + gradient expansion

Boltzmann equation

Fokker-Planck equation

Rate equations: \Leftrightarrow
transport models

Semi-classical approx : density matrix \approx diagonal

Good method for many $c\bar{c}$ pairs

QCD time scales

τ_E : environment autocorrelation time

$$\tau_E \approx \frac{1}{m_D} \approx \frac{1}{CT} \approx \frac{1}{T} \quad (\text{C taken as close to unity})$$

τ_S : system intrinsic time scale

$$\tau_S \approx \underbrace{\frac{1}{\Delta E}} \approx \frac{1}{m_Q v^2} \quad \text{with } v \approx \alpha_S \quad \dots \text{ at the beginning of the evolution}$$

Difference btwn energy levels

τ_R : system relaxation time

$$\Gamma = \tau_R^{-1} \sim 2\langle \psi | W | \psi \rangle \approx \alpha_S T \times \Phi(m_D r) \approx \alpha_S T \times \Phi\left(\frac{CT}{m_Q \alpha_S}\right)$$

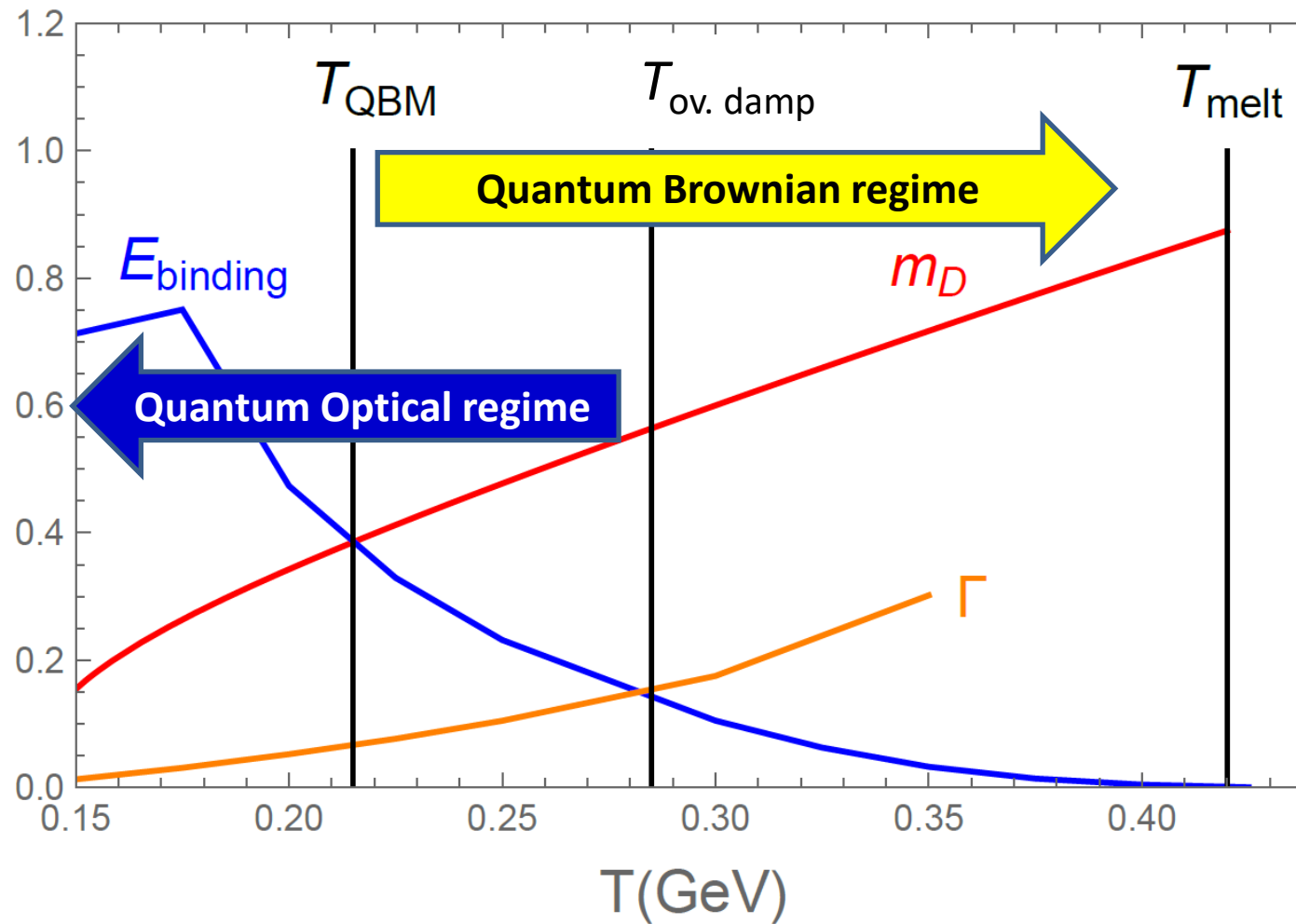
At “small” T ($T \lesssim \frac{m_Q \alpha_S}{C}$): dipole approximation : $\Gamma = \tau_R^{-1} \approx \frac{C^2 T^3}{\alpha_S m_Q^2}$



$$\boxed{\frac{\tau_R}{\tau_E} = \frac{\alpha_S m_Q^2}{CT^2} \gg 1} \quad \text{And} \quad \frac{\tau_R}{\tau_S} = \frac{\alpha_S^3 m_Q^3}{C^2 T^3} \gg 1 \quad \text{for } T \lesssim m_Q \frac{\alpha_S}{C^{2/3}}$$

Fine with the Markovian assumption

Two types of dynamical modelling



$c\bar{c}$ pair

Numbers extracted from a specific potential model : Katz et al, Phys. Rev. D 101, 056010 (2020)

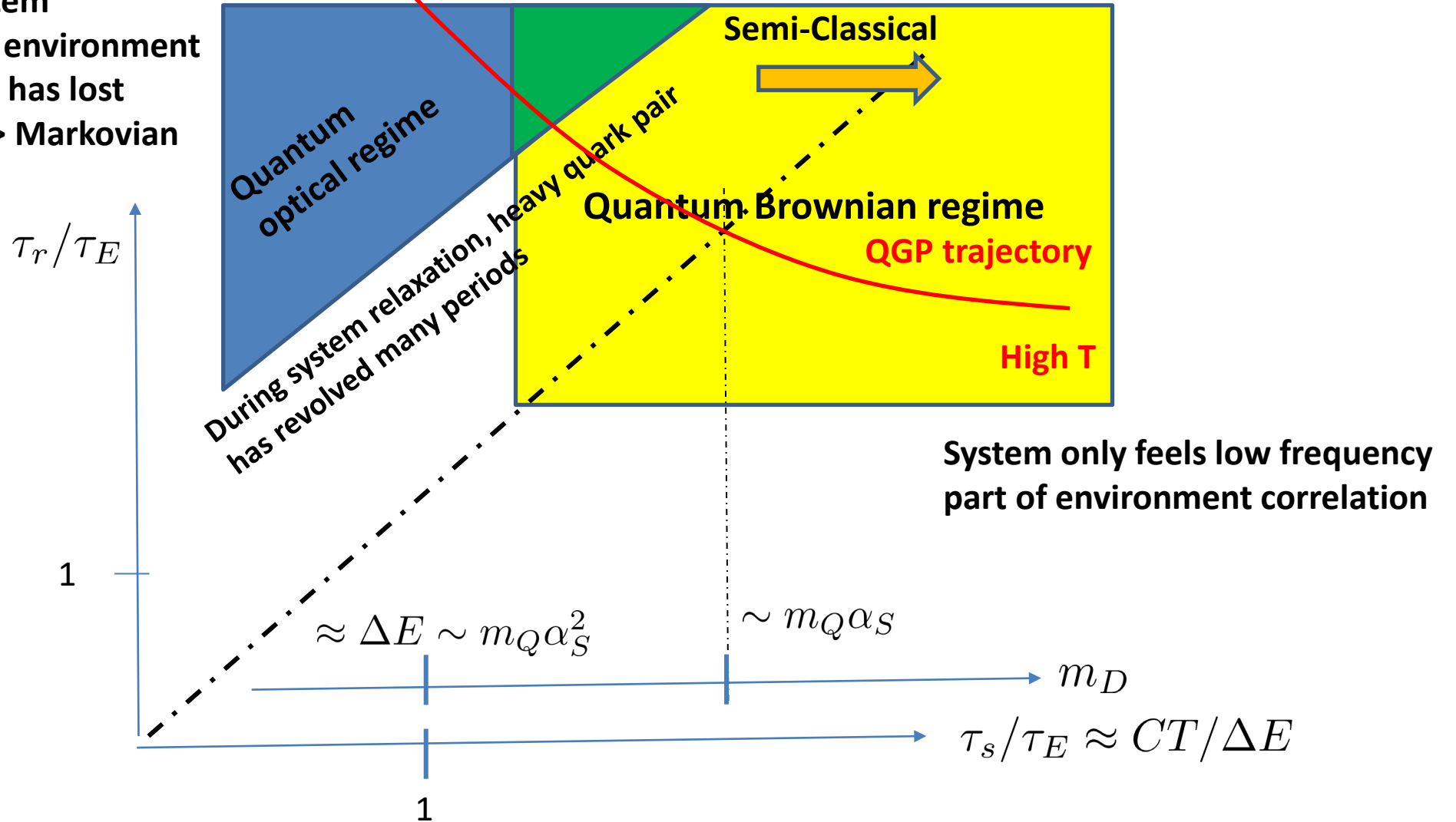
QCD time scales

$$\tau_E \approx \frac{1}{m_D} = \frac{1}{CT}$$

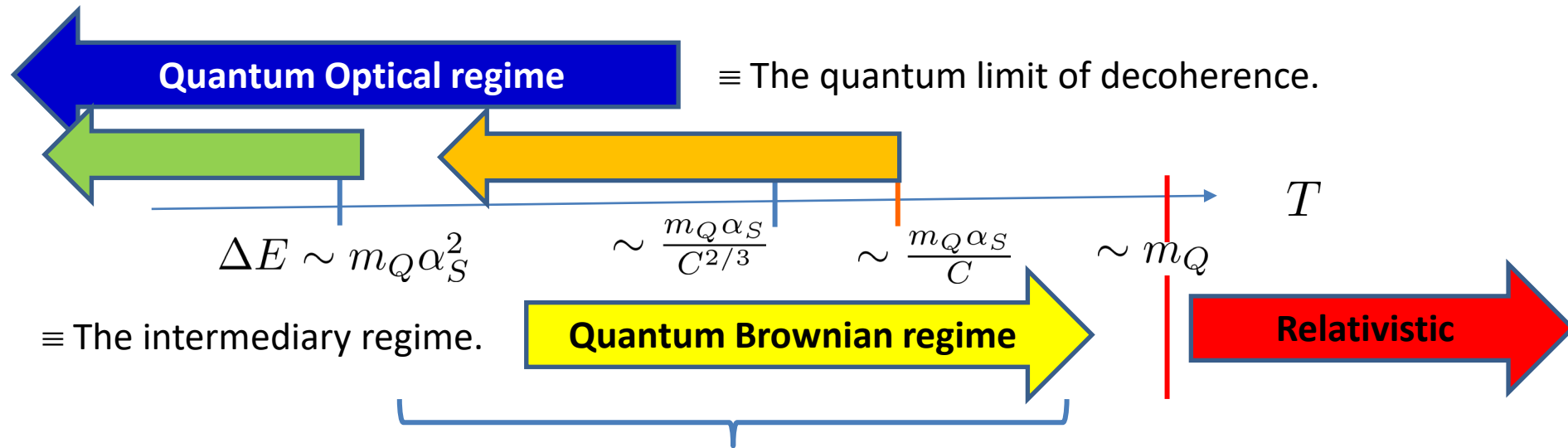
$$\tau_S^{\text{early}} \approx \frac{1}{m_Q \alpha_S^2}$$

$$\tau_R^{\text{early}} \approx \frac{\alpha_s m_Q^2}{C^2 T^3} \quad \text{for } T \lesssim \frac{m_Q \alpha_S}{C}$$

During system relaxation, environment correlation has lost memory => Markovian process

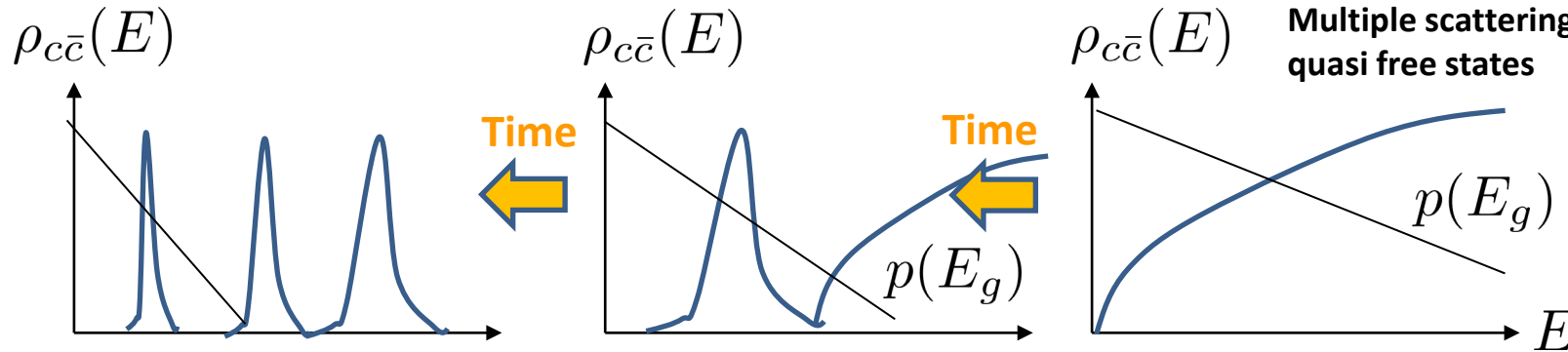


QCD Temperature scales



For these « large » temperatures, the Q-Qbar gain enough energy to overwhelm the real binding potential
 \Rightarrow larger distance \Rightarrow larger decoherence

dissociation of well identified levels by scarce "high-energy" modes (dilute medium \Rightarrow cross section ok)



In // : continuous evolution of the $Q\bar{Q}$ spectral function

T

Two types of dynamical modelling

$$m_D \ll E_{\text{bind}}$$

Quantum Optical Regime

$$m_D \sim E_{\text{bind}}$$

$$m_D \gg E_{\text{bind}}$$

Quantum Brownian Motion

- **Well identified resonances**
- Time long enough wrt quantum decoherence time (once we reach this regime)

Good description with transport models (TAMU, Tsinghua, Duke)

Central quantities :
2->2 and 2->3 Cross sections,
decay rates

Equilibrium : $\exp(-E_n/T)$ (theorem)

SC Approx: rate equations

?

- Correlations growing with cooling QGP
- **Best described in position-momentum space**
- Time short wrt quantum decoherence time ?

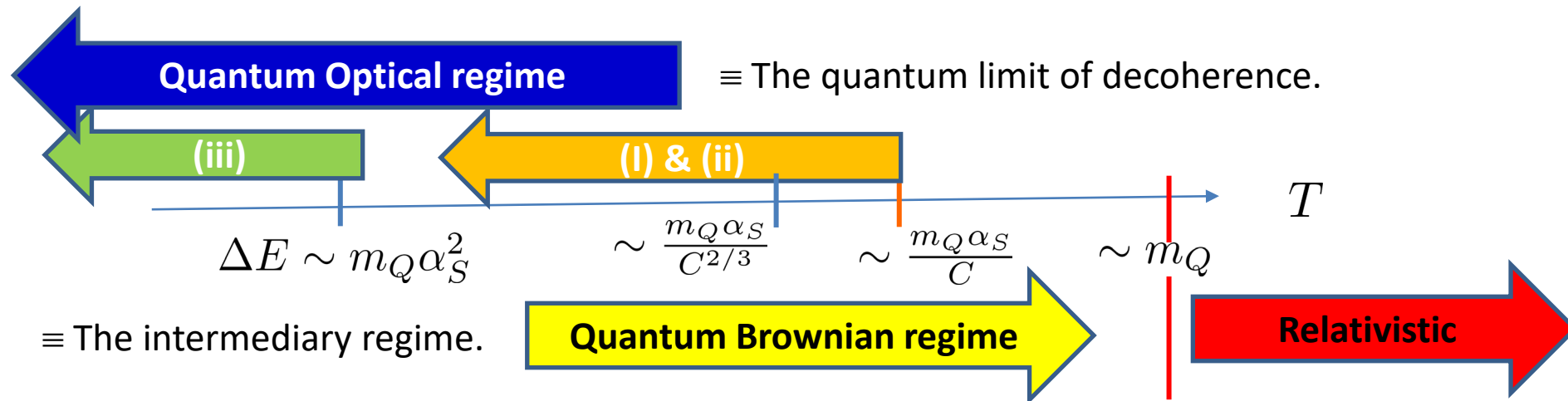
Quantum Master Equations for **microscopic dof (QS and Qbars)**

Equilibrium / asympt* : some limiting cases

SC Approx: Fokker-Planck equations
in position-momentum space

* Since one is facing both dissociation and recombination, obtaining a correct equilibrium limit of these models is an important prerequisite !!!

QCD Temperature scales



Refined subregimes when playing with the scales of NRQCD / pNRQCD (series of recent papers by N. Brambilla, M.A. Escobedo, A. Vairo, M Strickland et al, Yao, Müller and Mehen,...)

NRQCD: $Mv, \Lambda_{\text{QCD}}, T \ll \mu_{\text{NR}} \ll M$: most general scheme for markovian OQS !

- pNRQCD:
- (i) $1/r \gg T \sim \dot{m}_D \gg E$: « strongly coupled » QME same as small dipole limit of NRQCD (applies for small time evolution) :
 - (ii) $1/r \gg T \gg E \gg m_D$: « weakly coupled » : $g T \ll T$: essential contribution is gluo – dissociation from hard mode T : does not apply in QCD
 - (iii) $1/r \gg T \sim E \gg m_D$: Quantum optical regime
- (Singlet and octet quarkonium fields)

Recent OQS implementations (single $Q\bar{Q}$ pair)

regime	SU3 ?	Dissipation ?	3D / 1D	Num method	year	remark	ref
NRQCD \leftrightarrow QBM	No	No	1D	Stoch potential	2018		Kajimoto et al. , Phys. Rev. D 97, 014003 (2018), 1705.03365
	Yes	No	3D	Stoch potential	2020	Small dipole	R. Sharma et al Phys. Rev. D 101, 074004 (2020), 1912.07036
	Yes	No	3D	Stoch potential	2021		Y. Akamatsu, M. Asakawa, S. Kajimoto (2021), 2108.06921
	No	Yes	1D	Quantum state diffusion	2020		T. Miura, Y. Akamatsu et al, Phys. Rev. D 101, 034011 (2020), 1908.06293
	Yes ✓	Yes ✓	1D	Quantum state diffusion	2021		Akamatsu & Miura, EPJ Web Conf. 258 (2022) 01006, 2111.15402
	No	Yes	1D	Direct resolution	2021		O. Ålund, Y. Akamatsu et al, Comput. Phys. 425, 109917 (2021), 2004.04406
	Yes ✓	Yes ✓	1D	Direct resolution	2022		S Delorme et al, https://inspirehep.net/literature/2026925
pNRQCD (i)	Yes	No	1D+	Direct resolution	2017	S and P waves	N. Brambilla et al, Phys. Rev. D 96, 034021 (2017), 1612.07248
(i) Et (ii)	Yes	No	1D+	Direct resolution	2017	S and P waves	N. Brambilla et al, Phys. Rev. D 97, 074009 (2018), 1711.04515
(i)	Yes	No	Yes	Quantum jump	2021	See SQM 2021	N. Brambilla et al. , JHEP 05, 136 (2021), 2012.01240 & Phys.Rev.D 104 (2021) 9, 094049, 2107.06222
(i)	Yes ✓	Yes ✓	Yes ✓	Quantum jump	2022		N. Brambilla et al. 2205.10289
(iii)	Yes ✓	Yes ✓	Yes ✓	Boltzmann (?)	2019		Yao & Mehen, Phys.Rev.D 99 (2019) 9, 096028, 1811.07027
NRQCD & « pNRQCD »	Yes	Yes	1D	Quantum state diffusion	2022		Miura et al. http://arxiv.org/abs/2205.15551v1
Other	No	Yes	1D	Stochastic Langevin Eq.	2016	Quadratic W	Katz and Gossiaux

(Year > 2015)

Not exhaustive

See as well table in 2111.15402v1

...

Quantum Brownian Motion : The Blaizot-Escobedo QME

See S. Delorme's talk on Tuesday afternoon

Series expansion in τ_E/τ_S

Compact form: $\frac{d\mathcal{D}_Q}{dt} = \mathcal{L} \mathcal{D}_Q$ with $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 + \dots$

$$\left. \begin{aligned} \mathcal{L}_0 \mathcal{D}_Q &\equiv -i[H_Q, \mathcal{D}_Q], \\ \mathcal{L}_1 \mathcal{D}_Q &\equiv -\frac{i}{2} \int_{\mathbf{x}\mathbf{x}'} V(\mathbf{x} - \mathbf{x}') [n_{\mathbf{x}}^a n_{\mathbf{x}'}^a, \mathcal{D}_Q], \\ \mathcal{L}_2 \mathcal{D}_Q &\equiv \frac{1}{2} \int_{\mathbf{x}\mathbf{x}'} W(\mathbf{x} - \mathbf{x}') (\{n_{\mathbf{x}}^a n_{\mathbf{x}'}^a, \mathcal{D}_Q\} - 2n_{\mathbf{x}}^a \mathcal{D}_Q n_{\mathbf{x}'}^a), \\ \mathcal{L}_3 \mathcal{D}_Q &\equiv \frac{i}{4T} \int_{\mathbf{x}\mathbf{x}'} W(\mathbf{x} - \mathbf{x}') ([n_{\mathbf{x}}^a, \dot{n}_{\mathbf{x}'}^a \mathcal{D}_Q] + [n_{\mathbf{x}}^a, \mathcal{D}_Q \dot{n}_{\mathbf{x}'}^a]) \end{aligned} \right\} \begin{array}{l} \text{Mean field hamiltonian} \\ \text{Fluctuations, Linblad form} \\ \text{Dissipation} \end{array}$$

External "ingredients" : complex potential V + IW

N.B. : Friction is NOT of the Linbladian form => the evolution breaks positivity.

Positivity and Linblad form can be restored at the price **of extra subleading terms** :

$$\left\{ \left(n_{\mathbf{x}}^a - \frac{i}{4T} \dot{n}_{\mathbf{x}}^a \right) \left(n_{\mathbf{x}'}^a + \frac{i}{4T} \dot{n}_{\mathbf{x}'}^a \right), \mathcal{D}_{Q\bar{Q}} \right\} - 2 \left(n_{\mathbf{x}}^a + \frac{i}{4T} \dot{n}_{\mathbf{x}}^a \right) \mathcal{D}_{Q\bar{Q}} \left(n_{\mathbf{x}'}^a - \frac{i}{4T} \dot{n}_{\mathbf{x}'}^a \right)$$

\mathcal{L}_4

A consistent picture emerging in the bottomia sector

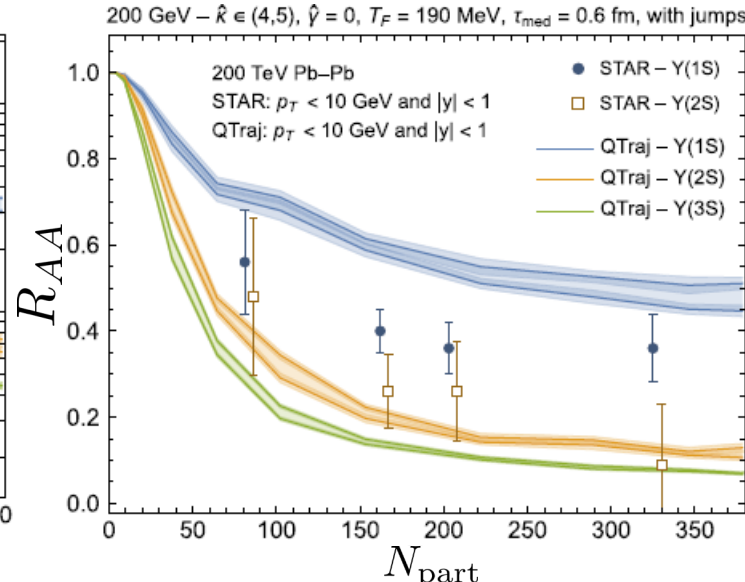
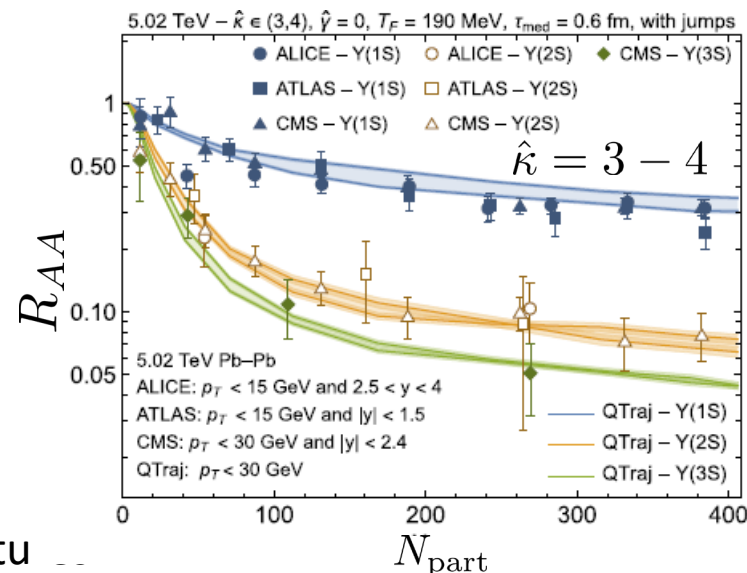
Beauty sector: good overall consistency of the following facts:

- Similar production of Y(1S) from RHIC -> LHC
- Higher states strongly suppressed
- Washing out of the spectral function (but the Y(1S) which survive up to $T = 0.45$ GeV)

Not paying too much attention at CNM effects:

With the interpretation that higher states (which contribute to the prompt Y(1S)) are suppressed both at RHIC and LHC in the QGP, while **the ground state Y(1S) survive and is thus a genuine hard QGP probe**; higher states could be produced (partly) through recombination

N.B.: No precise $v_2(Y)$ measured up to now. One would expect very small $v_2(Y(1S))$ and slightly larger $v_2(Y(2S))$... but will be hard to measure.



M. Strickland & S. Thapa, Phys. Rev. D 108, 014031 (2023)

Good agreement with suppression at LHC but not at RHIC

Other implementations : Osaka, Saclay, Nantes, Duke,...

The present challenges for Quarkonium modelling in URHIC

Meet the higher and higher precision
of experimental data (already beyond
the present model uncertainties)

Unravel the Q-Qbar interactions under
the influence of the surrounding QGP
and with the QGP



Develop a scheme able to deal with the evolution
of one (or many) $Q\bar{Q}$ pair(s) in a QGP, fulfilling all
fundamental principles (**quantum features**, gauge
invariance, equilibration,...)

Need for IQCD constraints / inputs

**Ultimately, go beyond the “one team
– one model” paradigm**

Collective work on quarkonia in AA

Eur. Phys. J. A (2024) 60:88
<https://doi.org/10.1140/epja/s10050-024-01306-6>

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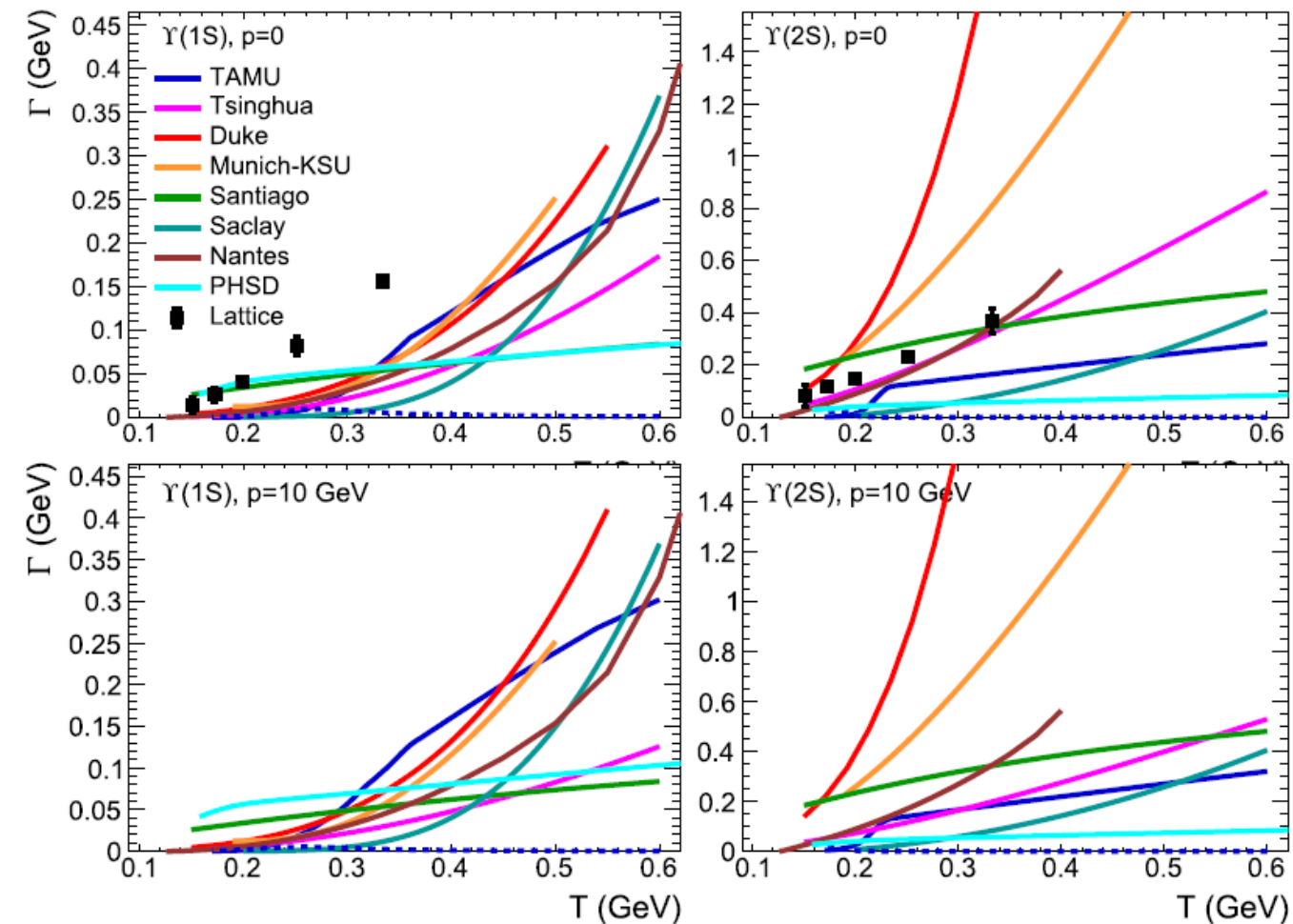


Review

Comparative study of quarkonium transport in hot QCD matter

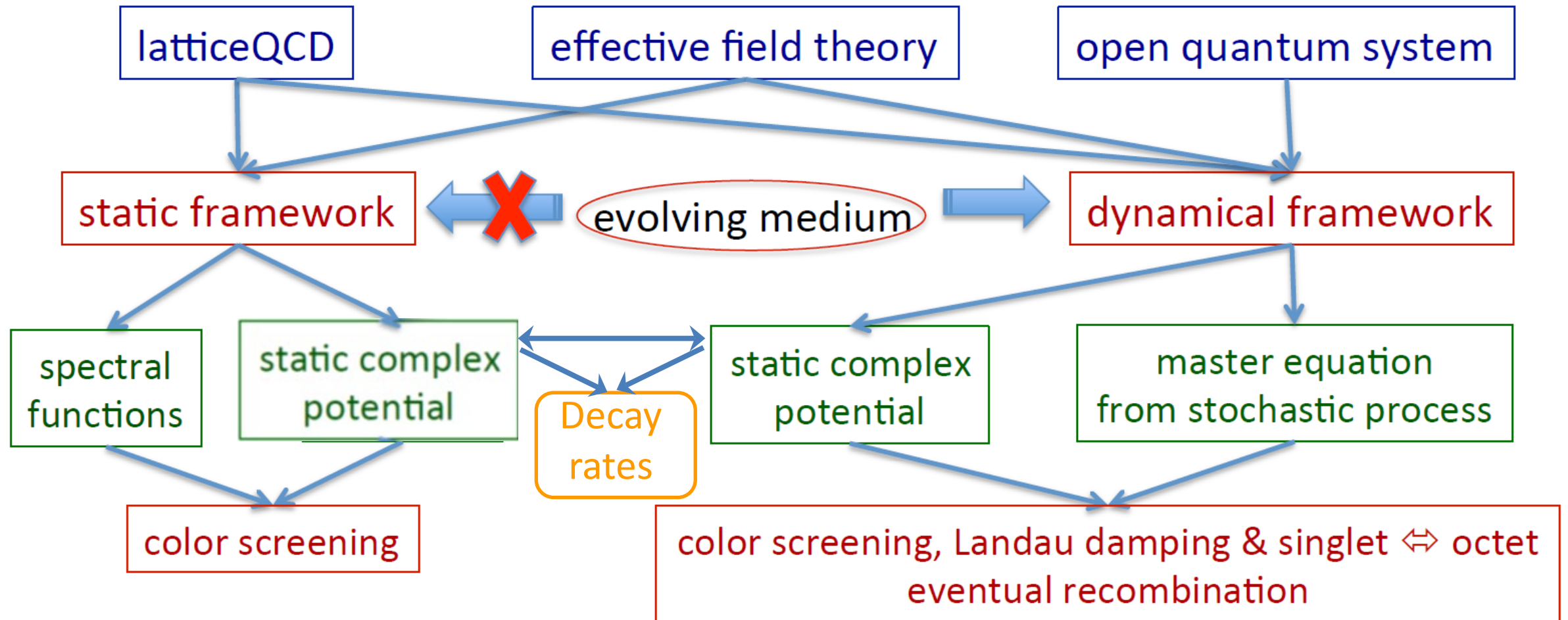
A. Andronic^{1,a}, P. B. Gossiaux^{2,b}, P. Petreczky^{3,c}, R. Rapp^{4,d}, M. Strickland^{5,e}, J. P. Blaizot⁶, N. Brambilla⁷, P. Braun-Munzinger^{8,9}, B. Chen¹⁰, S. Delorme¹¹, X. Du¹², M. A. Escobedo^{13,12}, E. G. Ferreira¹², A. Jaiswal¹⁴, A. Rothkopf¹⁵, T. Song⁸, J. Stachel⁹, P. Vander Griend¹⁶, R. Vogt¹⁷, B. Wu⁴, J. Zhao², X. Yao¹⁸

- A lot of diversity in the models... including those which are compatible with the experimental data
 - Underlying binding force between Q & Qbar
 - Binding energy
 - Whether, on the top of dissociation, some « melting » is allowed
 -
- Larger diversity for finite momentum.
- Some tension with the lattice calculations (R. Larsen et al., Phys. Lett. B **800**, 135119 (2020), [arXiv:1910.07374](https://arxiv.org/abs/1910.07374) [hep-lat])



Global picture (slightly adapted from E. Ferreiro; QM 2018)

Caveat I: we need firm theoretical understanding of quarkonium production in pp collisions



Global picture for quarkonia: the 2 faces of “theory”



Simple

(searching for new effects)

Sophisticated

(the need for precision physics)



Shadowing + 2-components
model (comovers +
statistical generation at FO)



Open quantum system with
 $H_{\text{interaction}}$ tuned on IQCD
constrains

Where should we position the « cursor » in order to claim hard probing QGP with quarkonia ?

I invite you to make you own judgment, with the help of this great conference !