The LISA Global Fit

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Because of the signal overlaps, a global fit to all the signals has to be performed

PHYSICAL REVIEW D
covering particles, fields, gravitation, and cosmology

LISA data analysis using Markov chain Monte Carlo methods

Neil J. Cornish and Jeff Crowder
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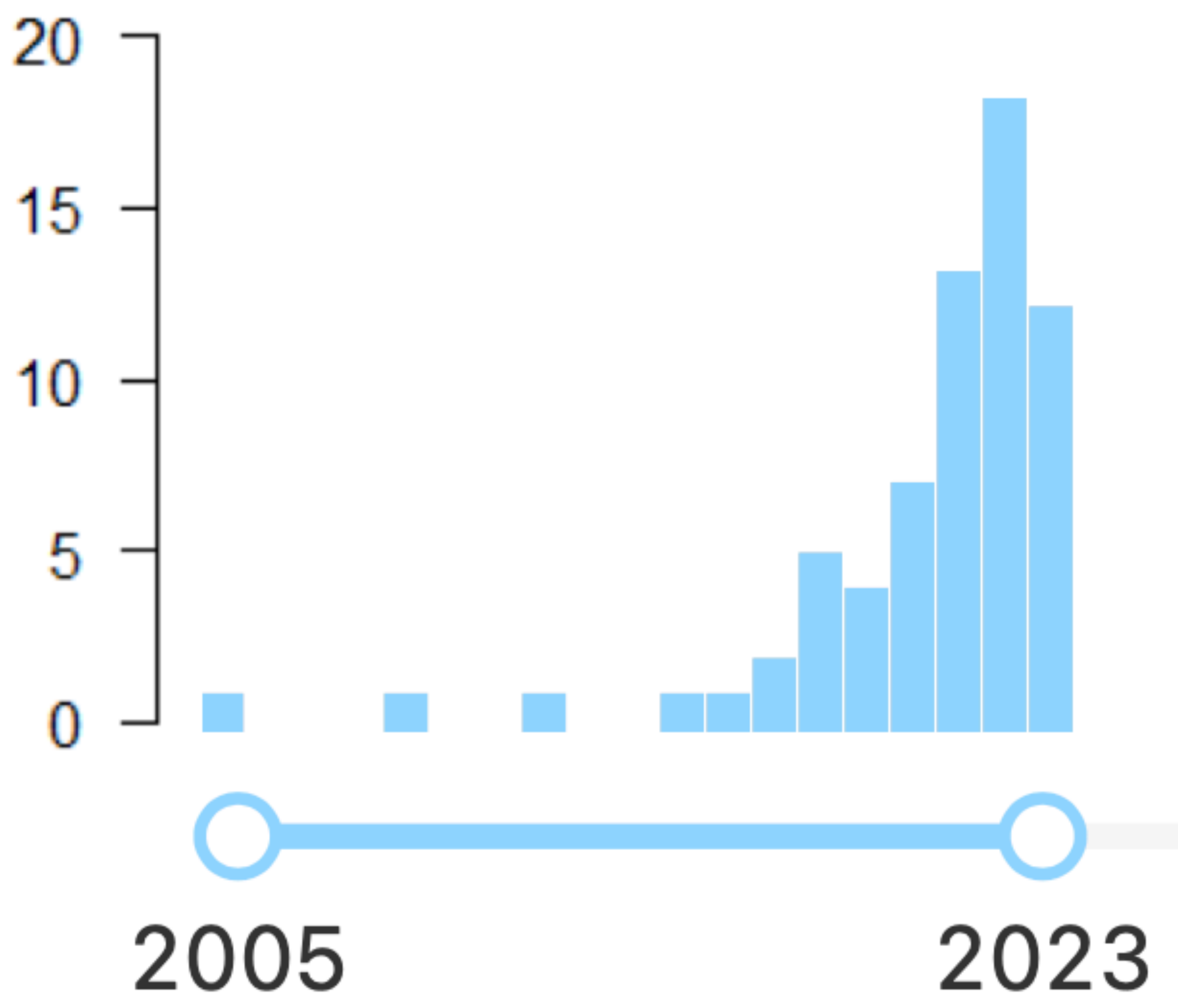
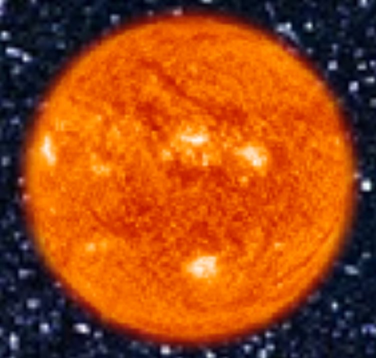
ABSTRACT

The Laser Interferometer Space Antenna (LISA) is expected to simultaneously detect many thousands of low-frequency gravitational wave signals. This presents a data analysis challenge that is very different to the one encountered in ground based gravitational wave astronomy. LISA data analysis requires the identification of individual signals from a data stream containing an unknown number of overlapping signals. Because of the signal overlaps, a global fit to all the signals has to be performed in order to avoid biasing the solution. However, performing such a global fit requires the exploration of an enormous parameter space with a dimension upwards of 50000. Markov Chain Monte Carlo (MCMC) methods offer a very promising solution to the LISA data analysis problem. MCMC algorithms

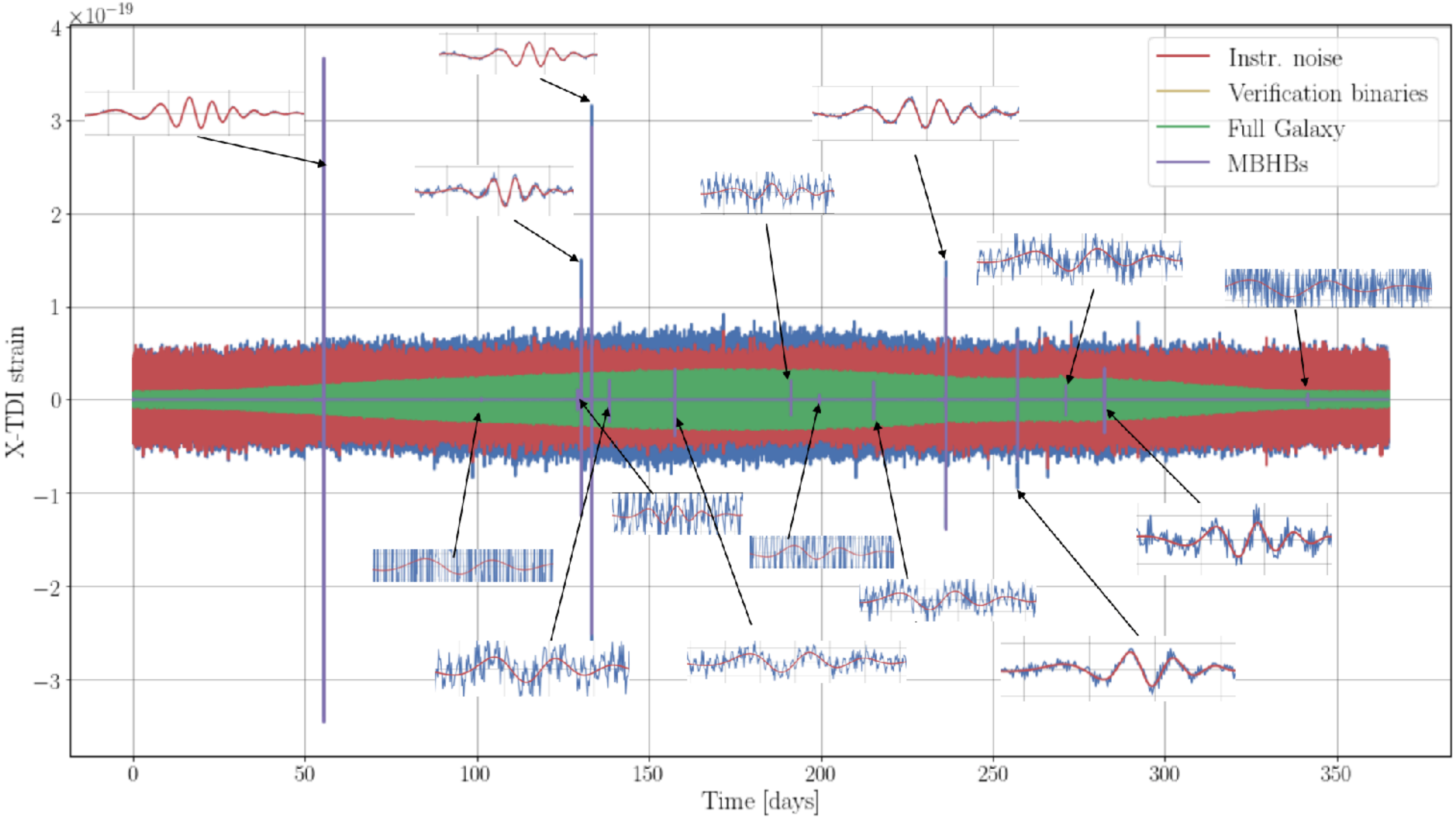


LISA is not LIGO in Space



- Millions of overlapping signals
- Unknown number of detectable sources
- Non-stationary and non-Gaussian noise
- Data gaps and disturbances
- Time varying instrument response
- Complex signals, multiple harmonics

LISA Data Challenge: Sangria Edition



The Global Solution

Likelihood function for Gaussian noise

$$p(\mathbf{d}|\vec{\lambda}) = \frac{1}{\sqrt{(2\pi)^M \det\mathbf{C}}}\, e^{-\frac{1}{2}(\mathbf{d}-\mathbf{h})\cdot\mathbf{C}^{-1}\cdot(\mathbf{d}-\mathbf{h})}$$

$$\mathbf{h} = \sum_{i=1}^{N} \mathbf{h}_i = \text{GW signal model}$$

N unknown, mix of signal types

$$\mathbf{C} = \text{noise correlation matrix}$$

Jointly inferred with signal model . Up to M^3 cost to invert

$$\vec{\lambda} = \text{model parameters}$$

Signal and noise $\mathcal{O}(10^6)$ parameters

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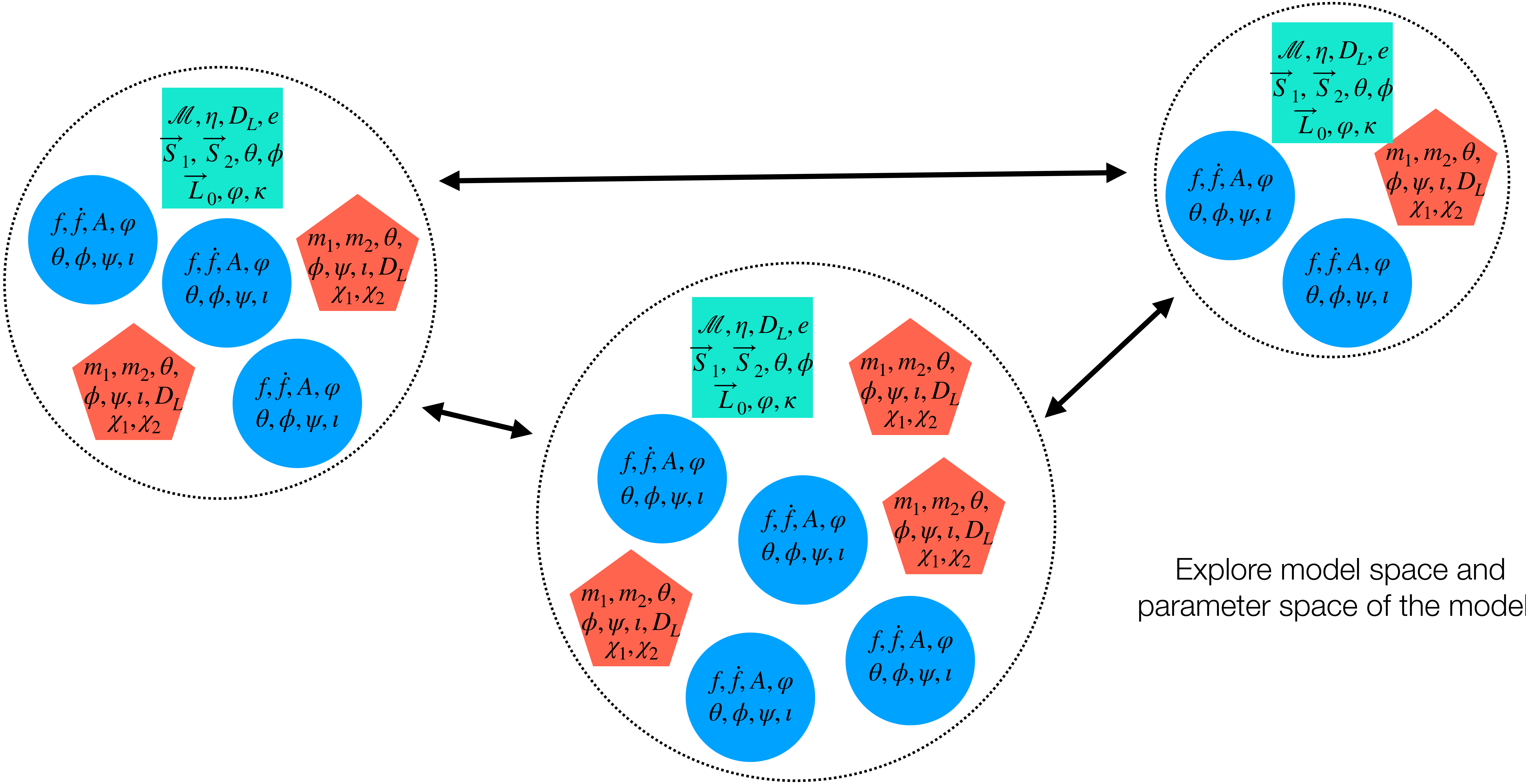
$$\log L = (d\,|\,h) - \frac{1}{2}(h\,|\,h) = \sum_i^N \log L_i - \frac{1}{2}\sum_{i\neq j}(h_i\,|\,h_j)$$

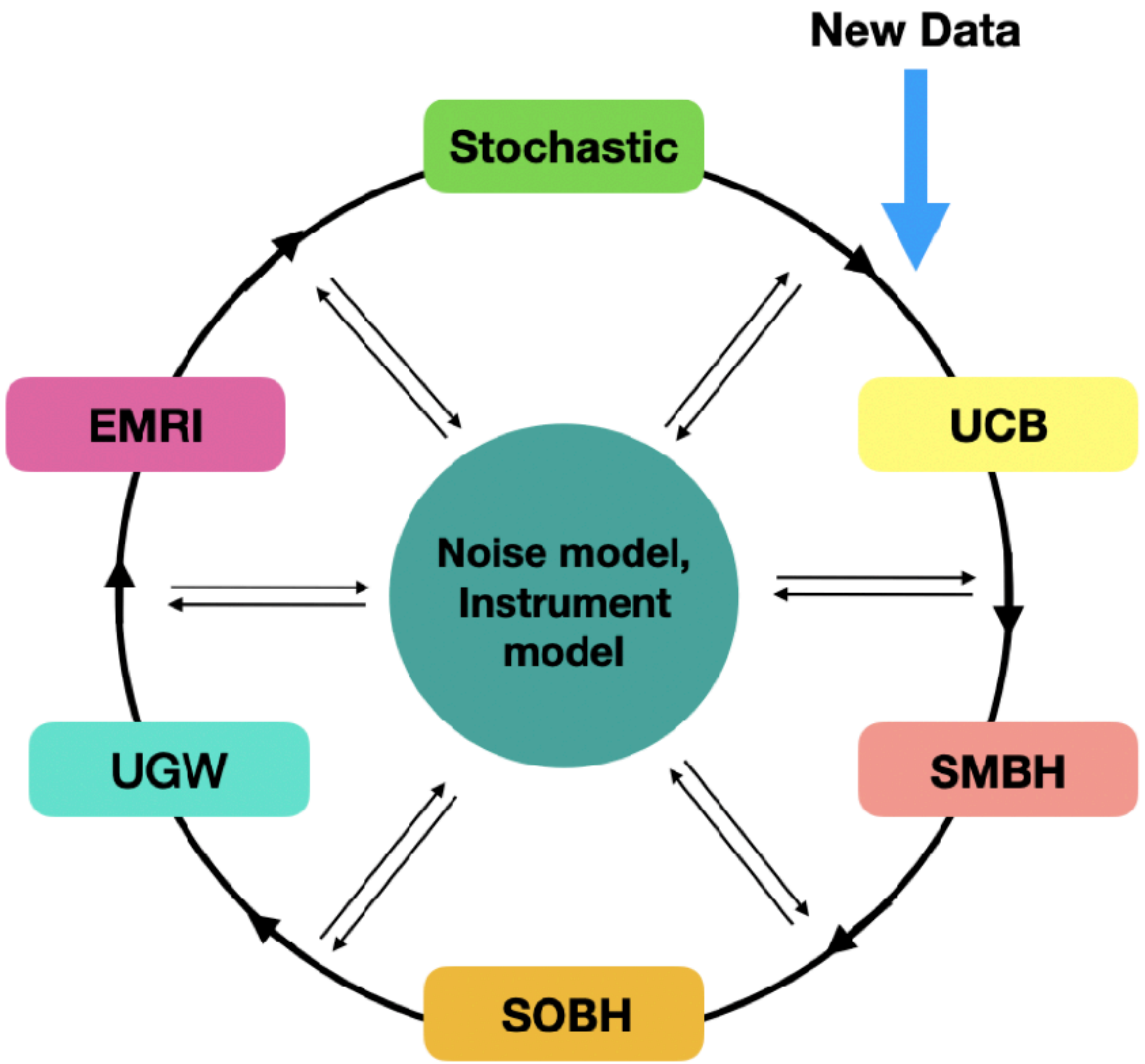
Why we need a global solution

Trans-dimensional Inference



Explore model space and parameter space of the models

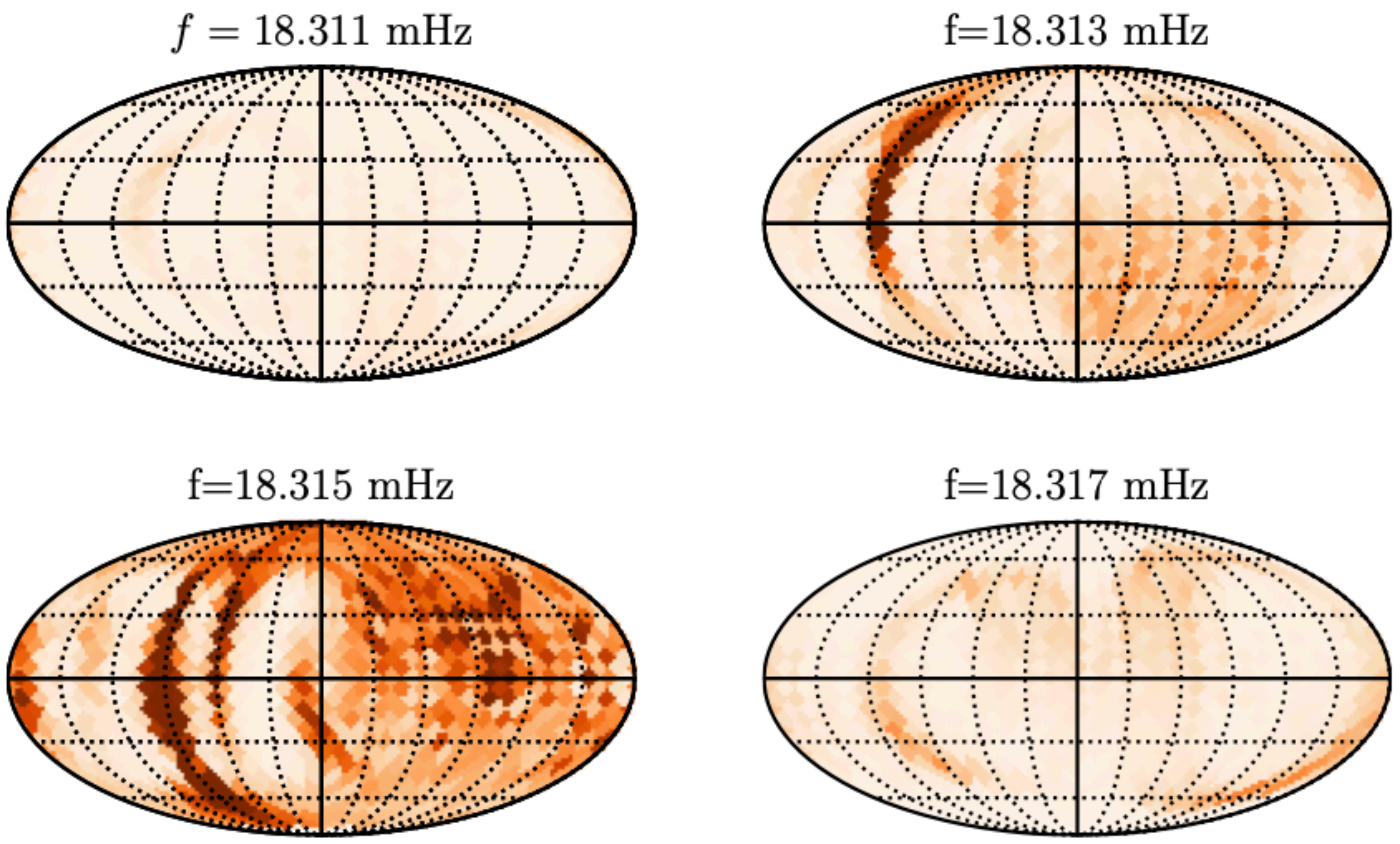
Global Fit via Blocked Sampling



- Transdimensional Markov Chain Monte Carlo (RJMCMC)
- Blocked Metropolis Hastings—update each component of the signal/noise model in circular sweeps
- Only pass residuals - decouples the analysis types
- Update the fit every ~week as new data arrives

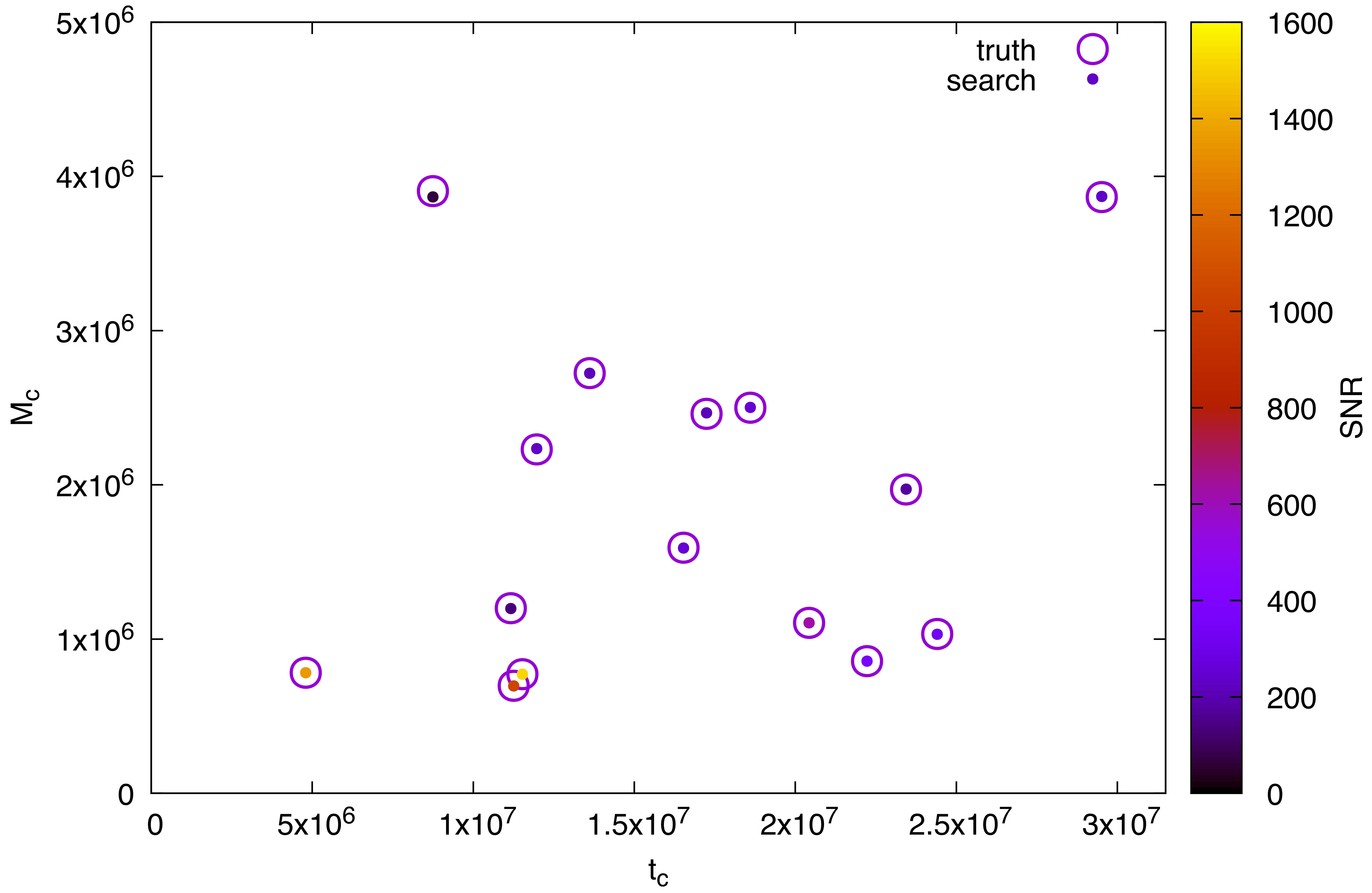
Low latency single-source search results used as proposals in global fit

F-statistic maps for GBs



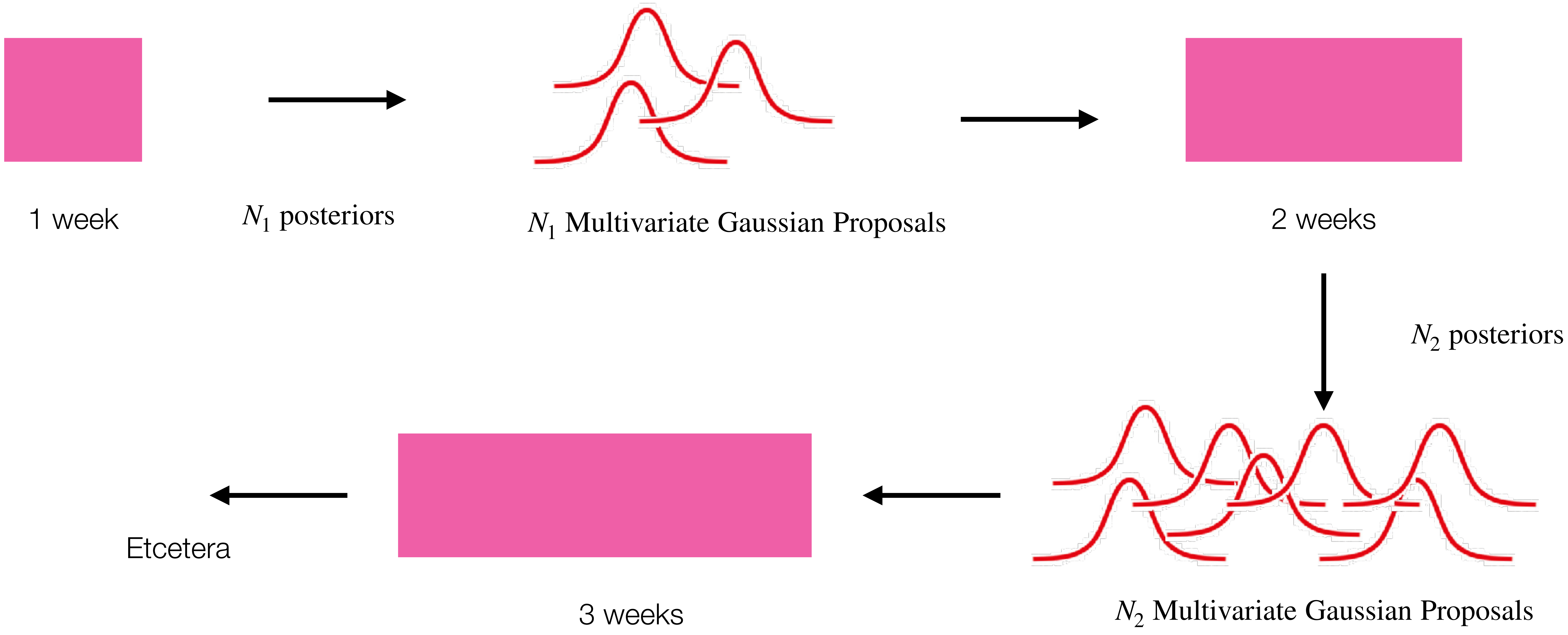
[Littenber, Cornish, Lackeos & Robson, arXiv:2004.08464]

Low latency BH search



[Cornish, arXiv: 2110.06238]

Building up the solution - “time annealing”



We used 1 month -> 3 months -> 6 months -> 12 months

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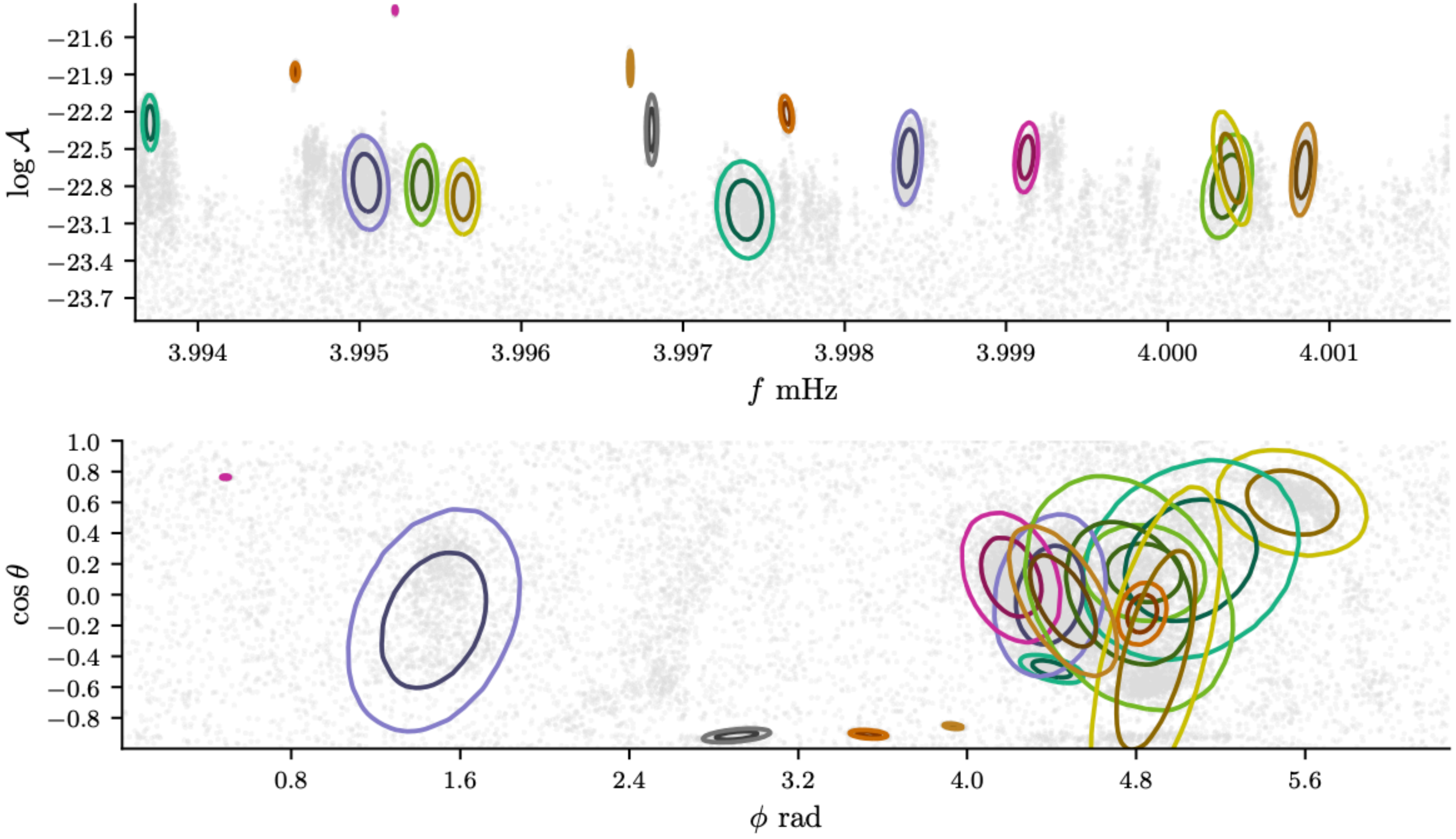

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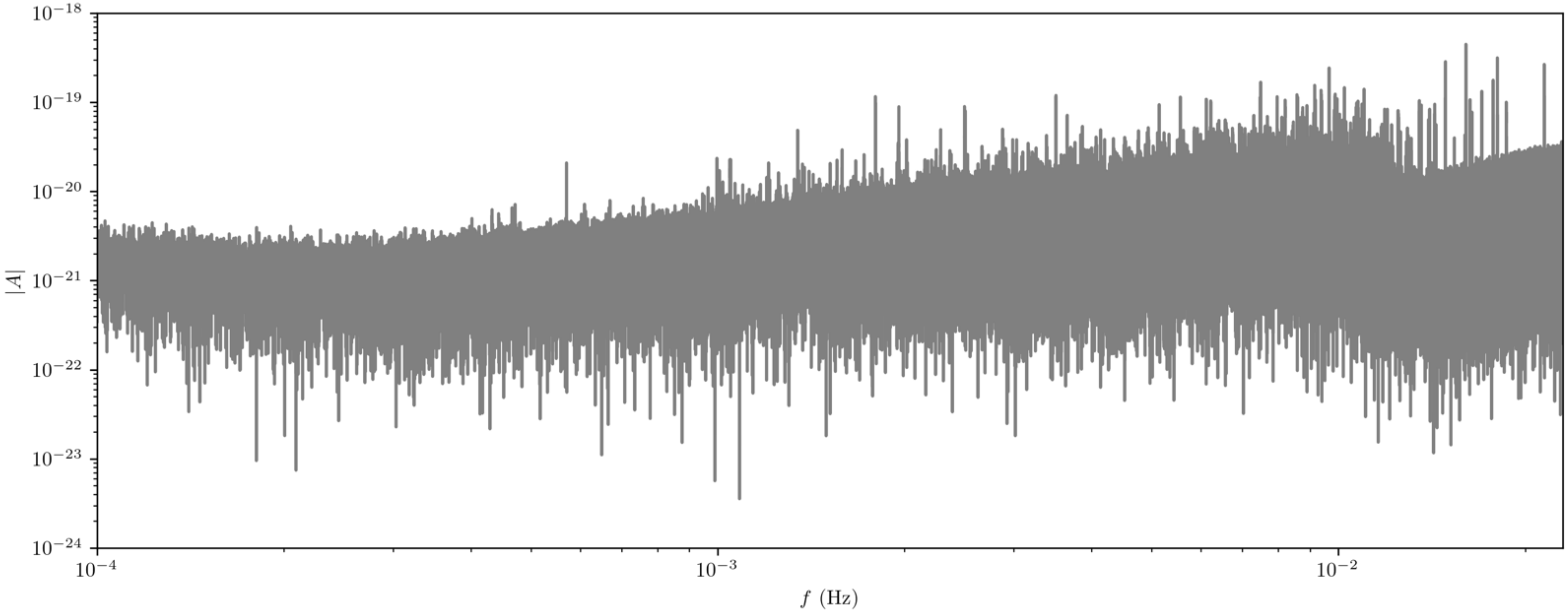
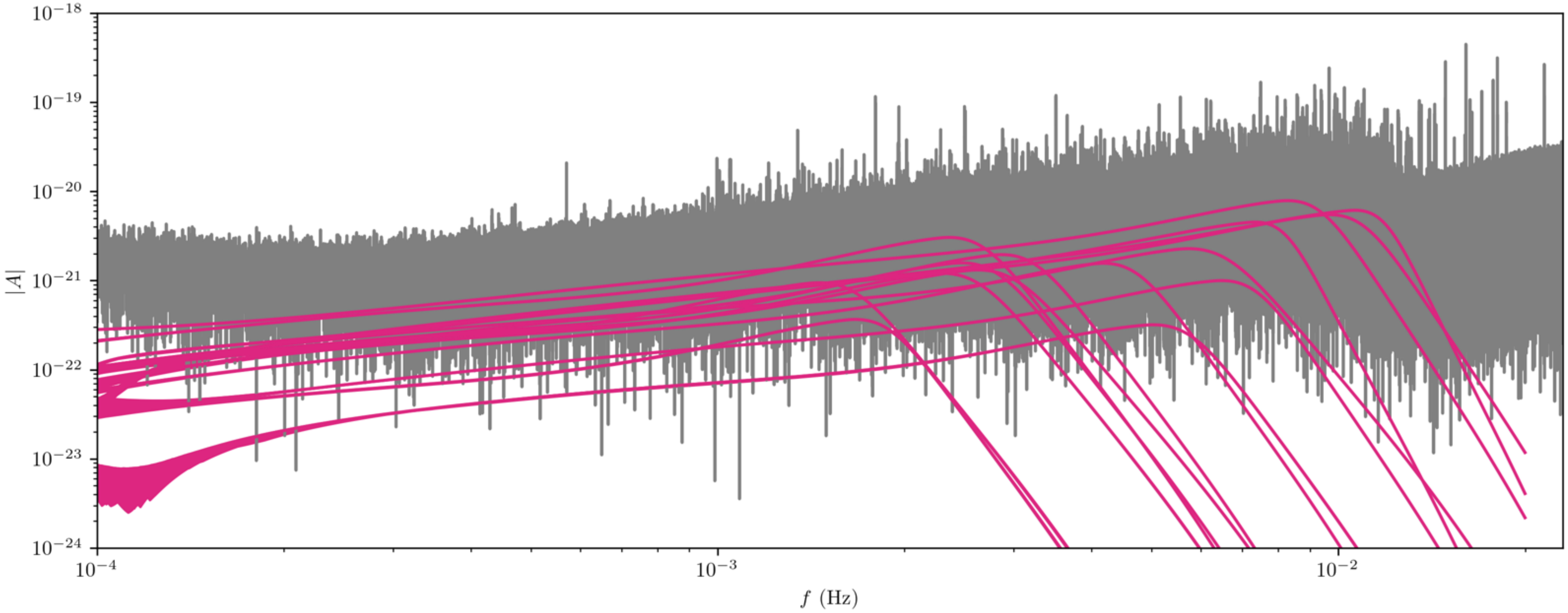
12 months of Sangria data - A TDI channel



[Litttenberg & Cornish, arXiv: 2301.03673]

12 months of Sangria data - A TDI channel



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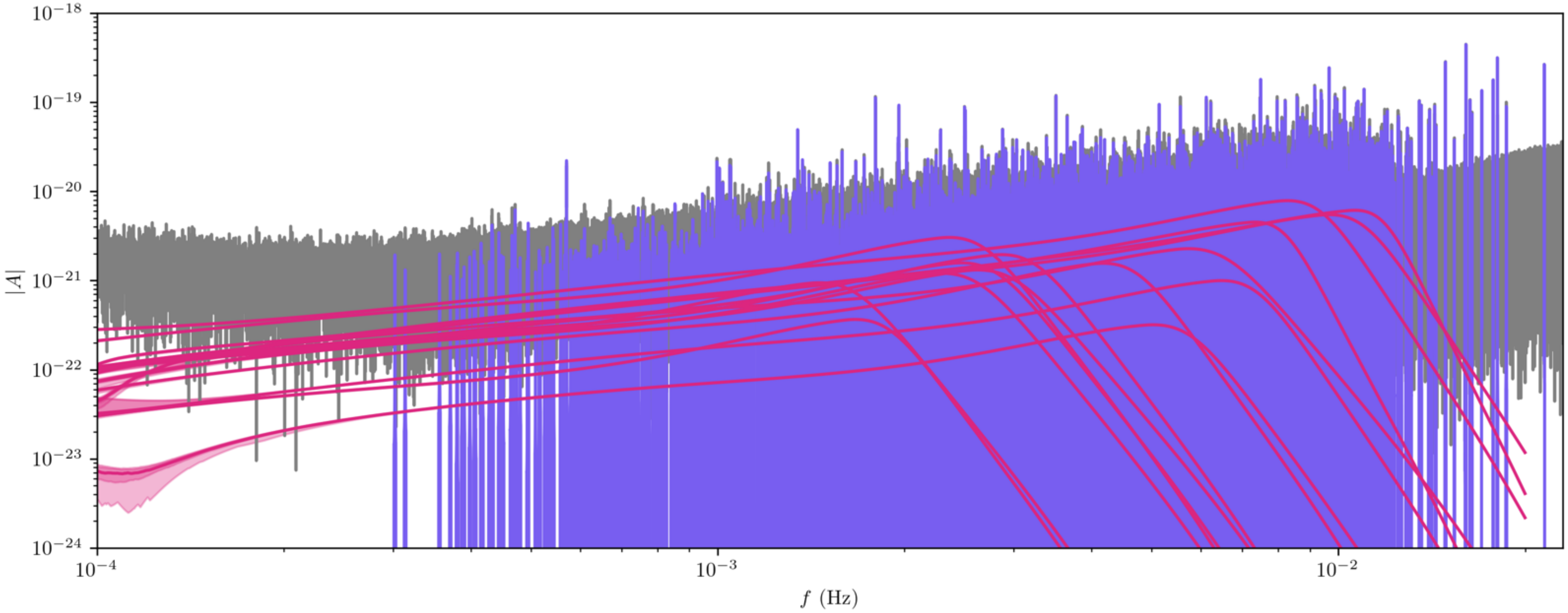

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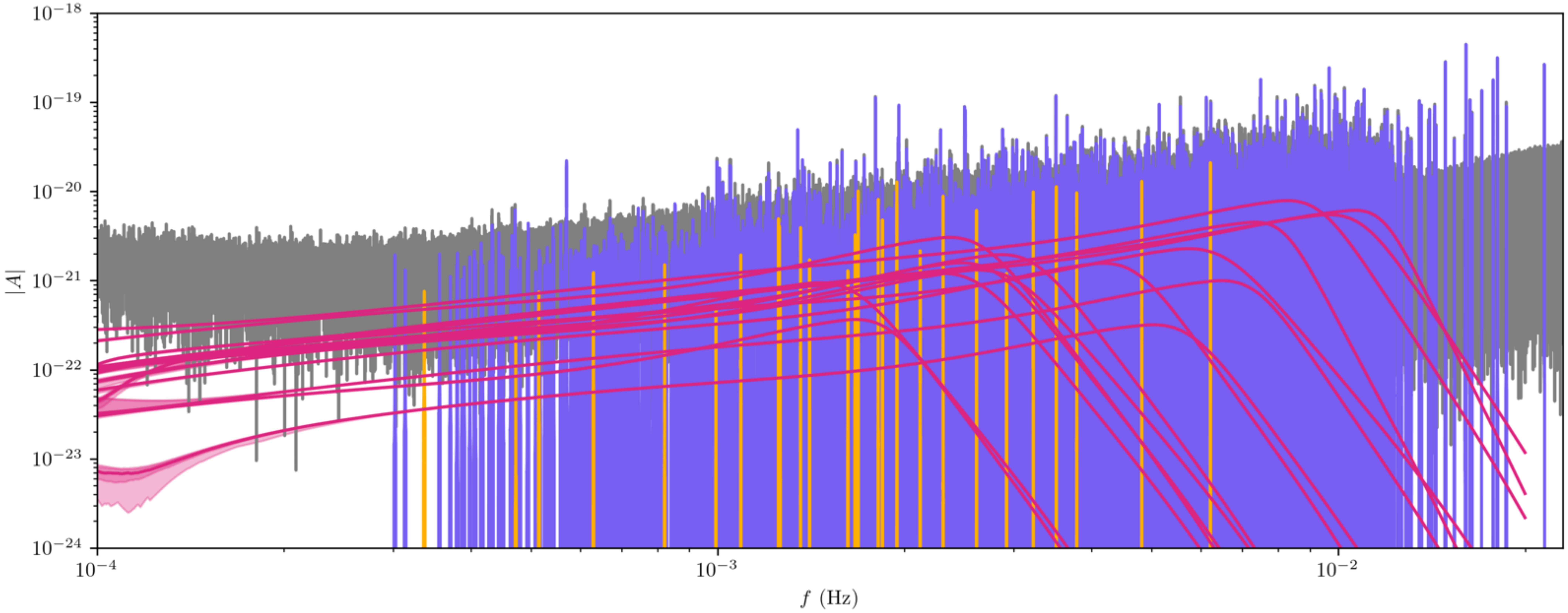

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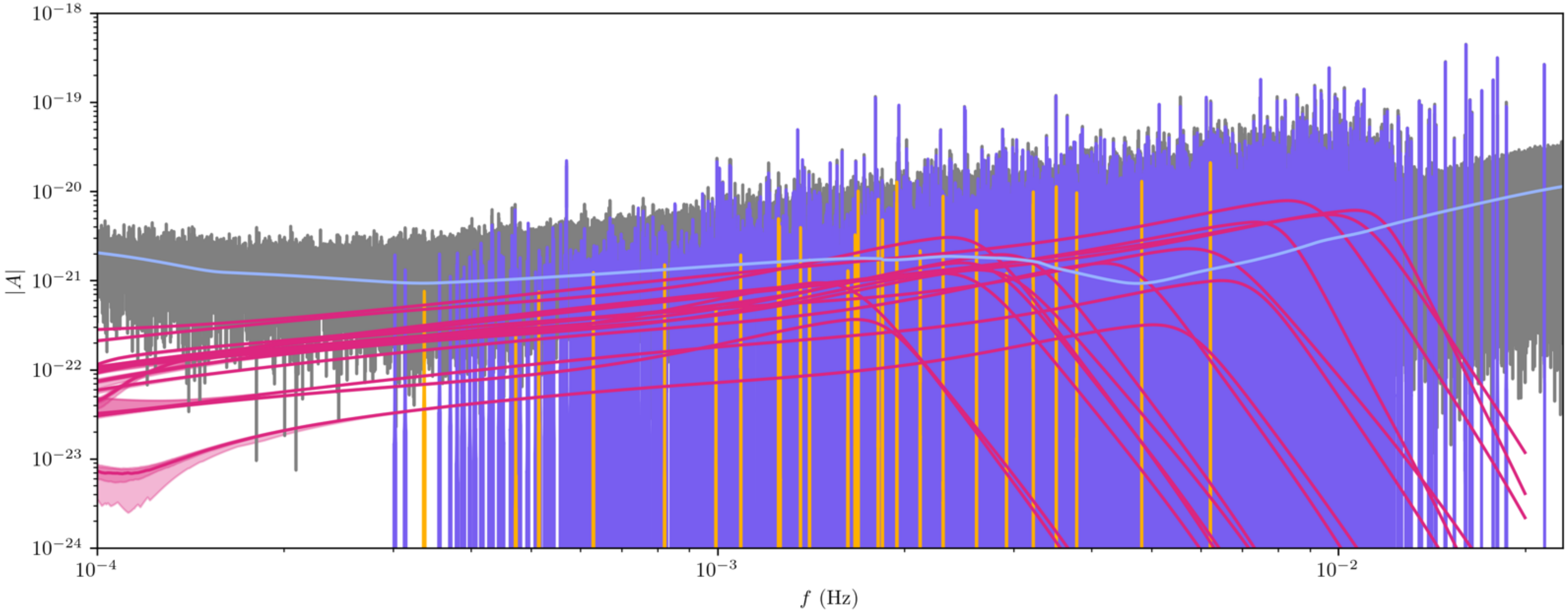
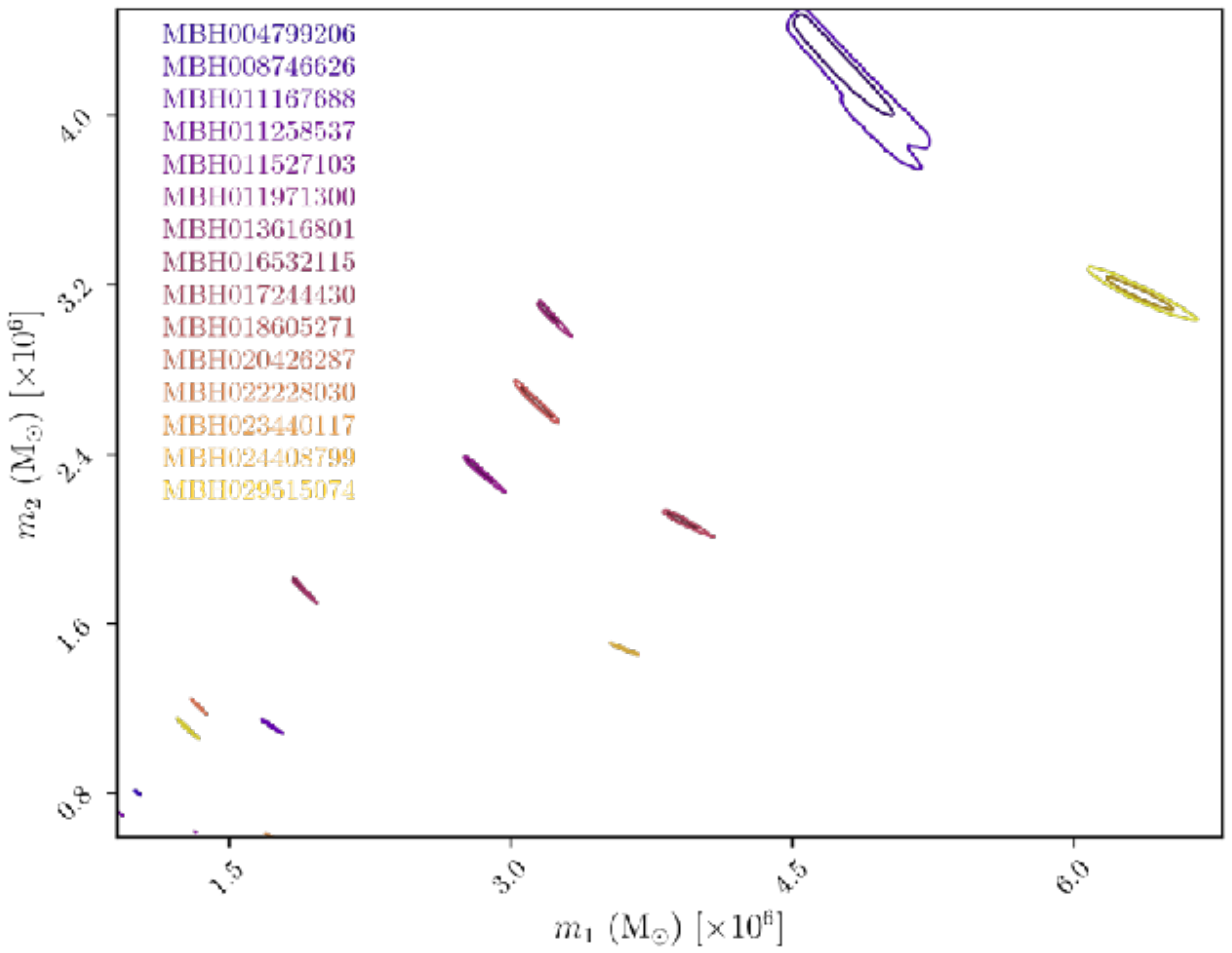
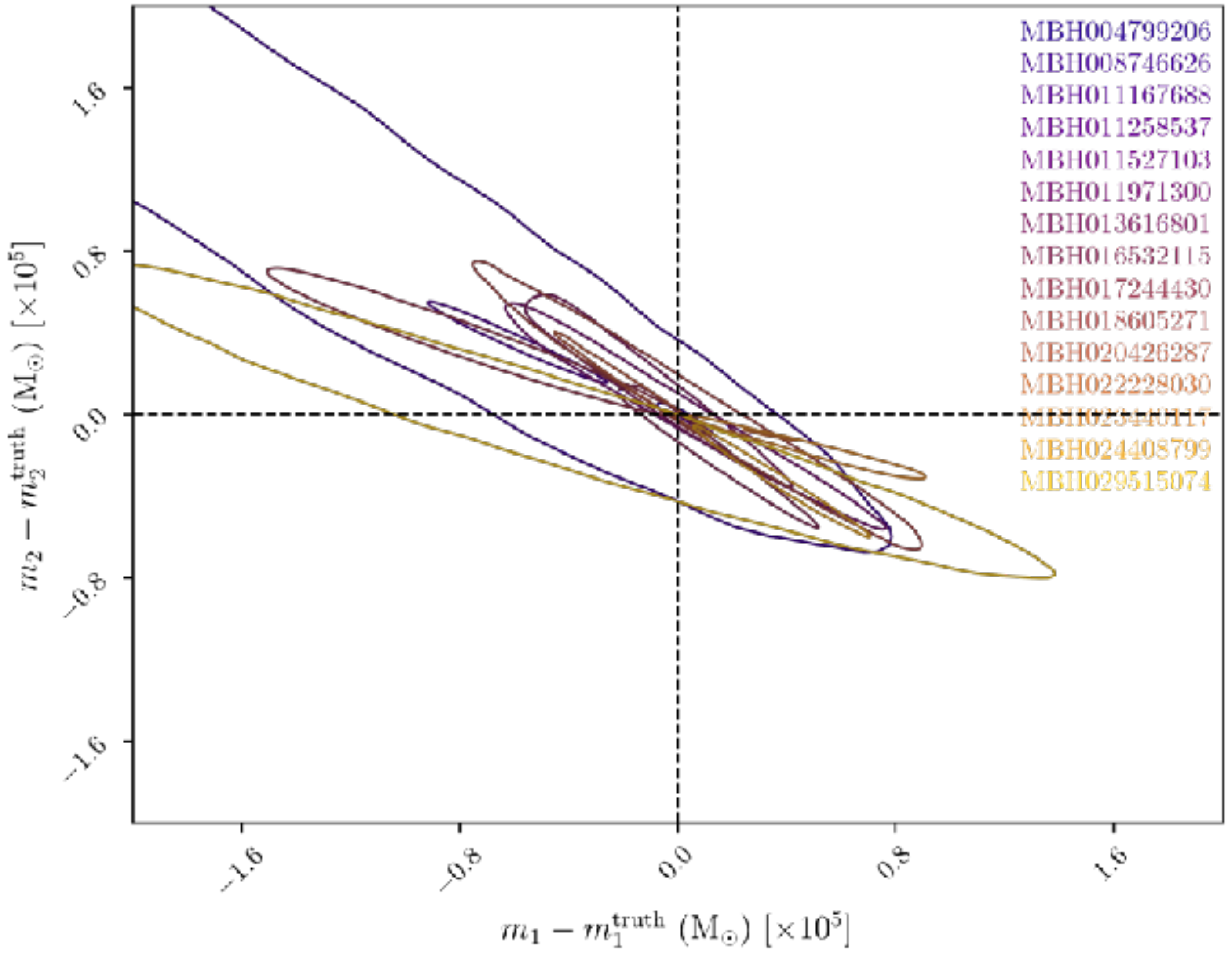
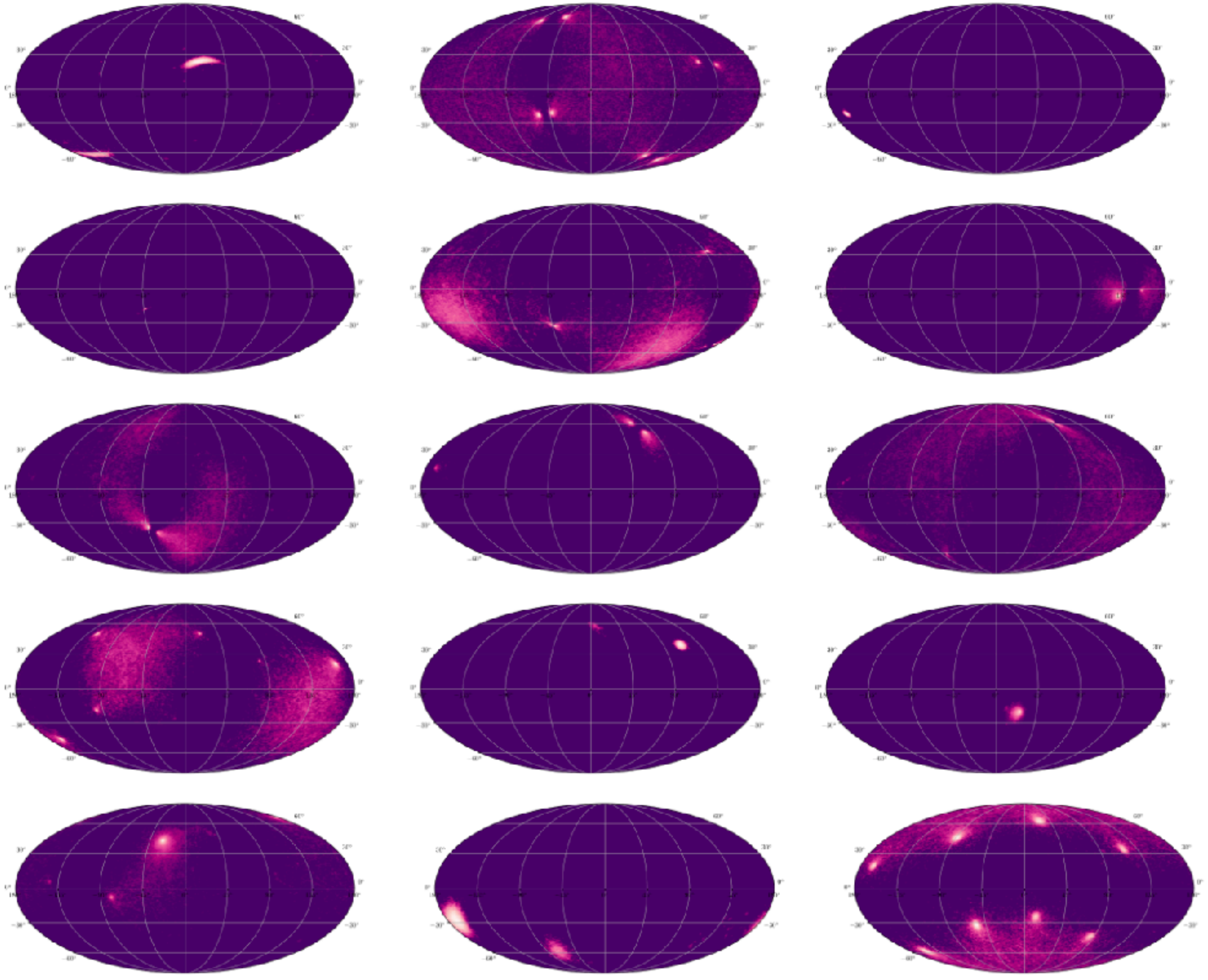


Run time ~ 2 days on AWS, $12K

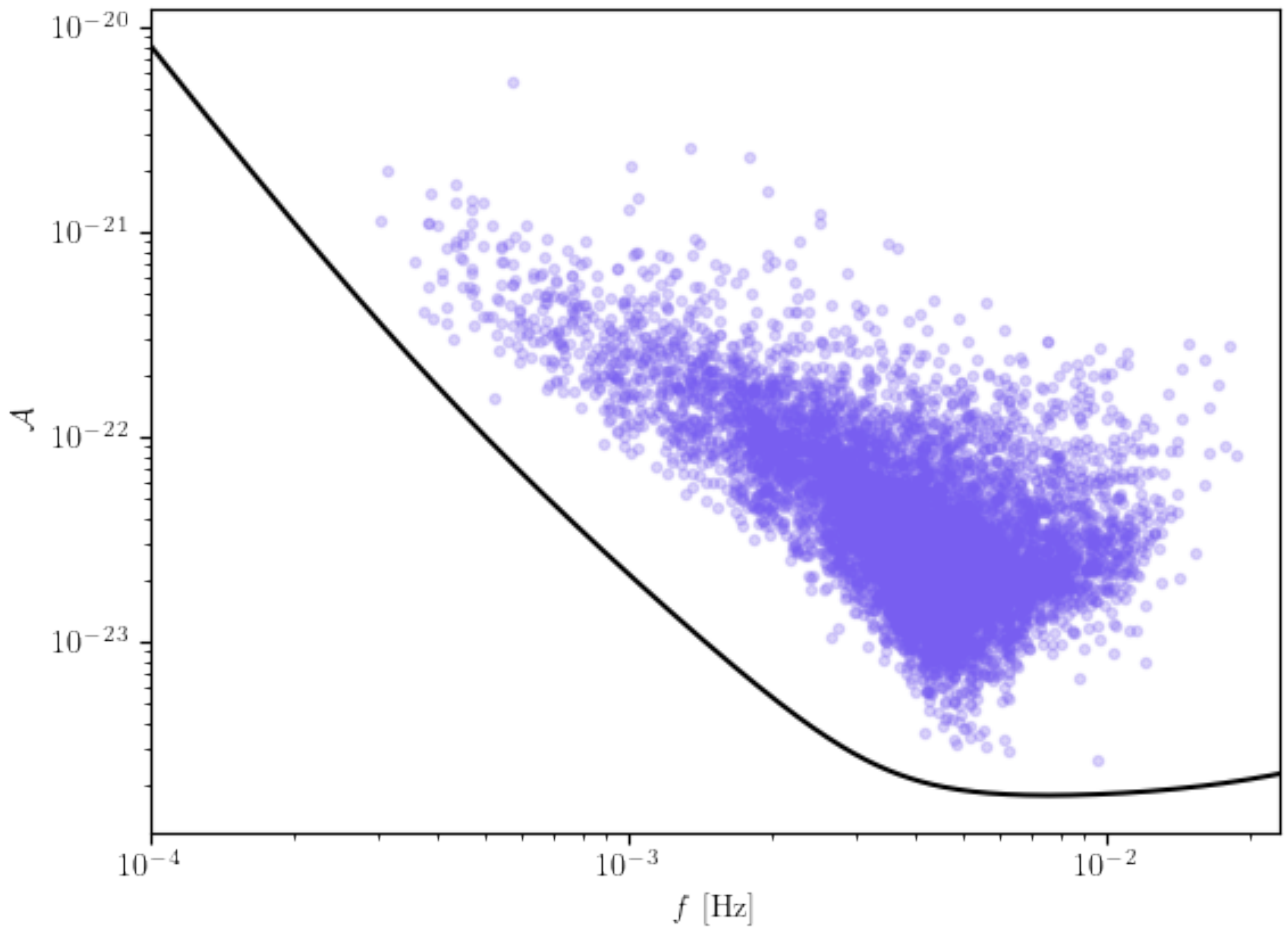
12 months of Sangria data - MBHBs



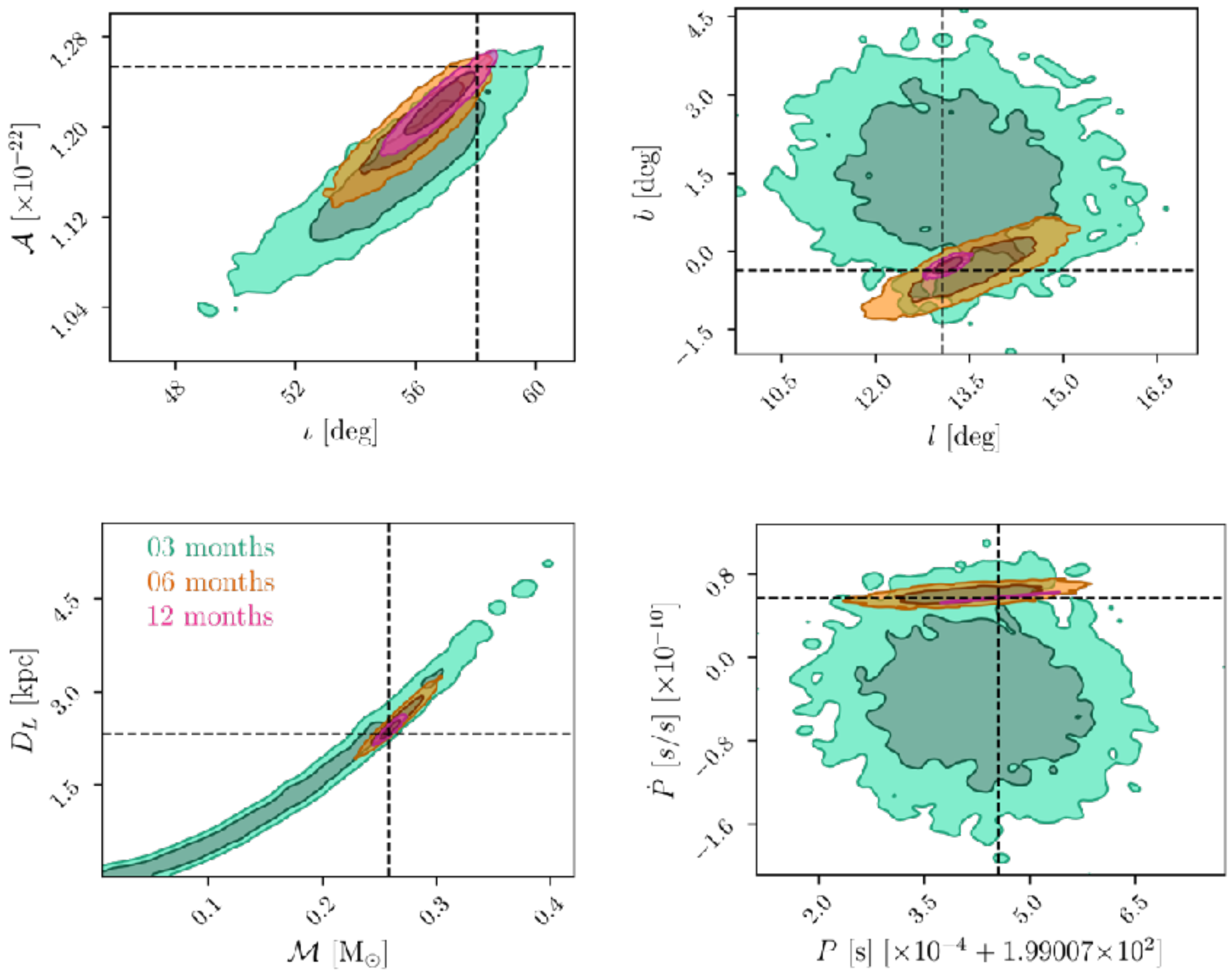




Sangria data - Galactic Binaries

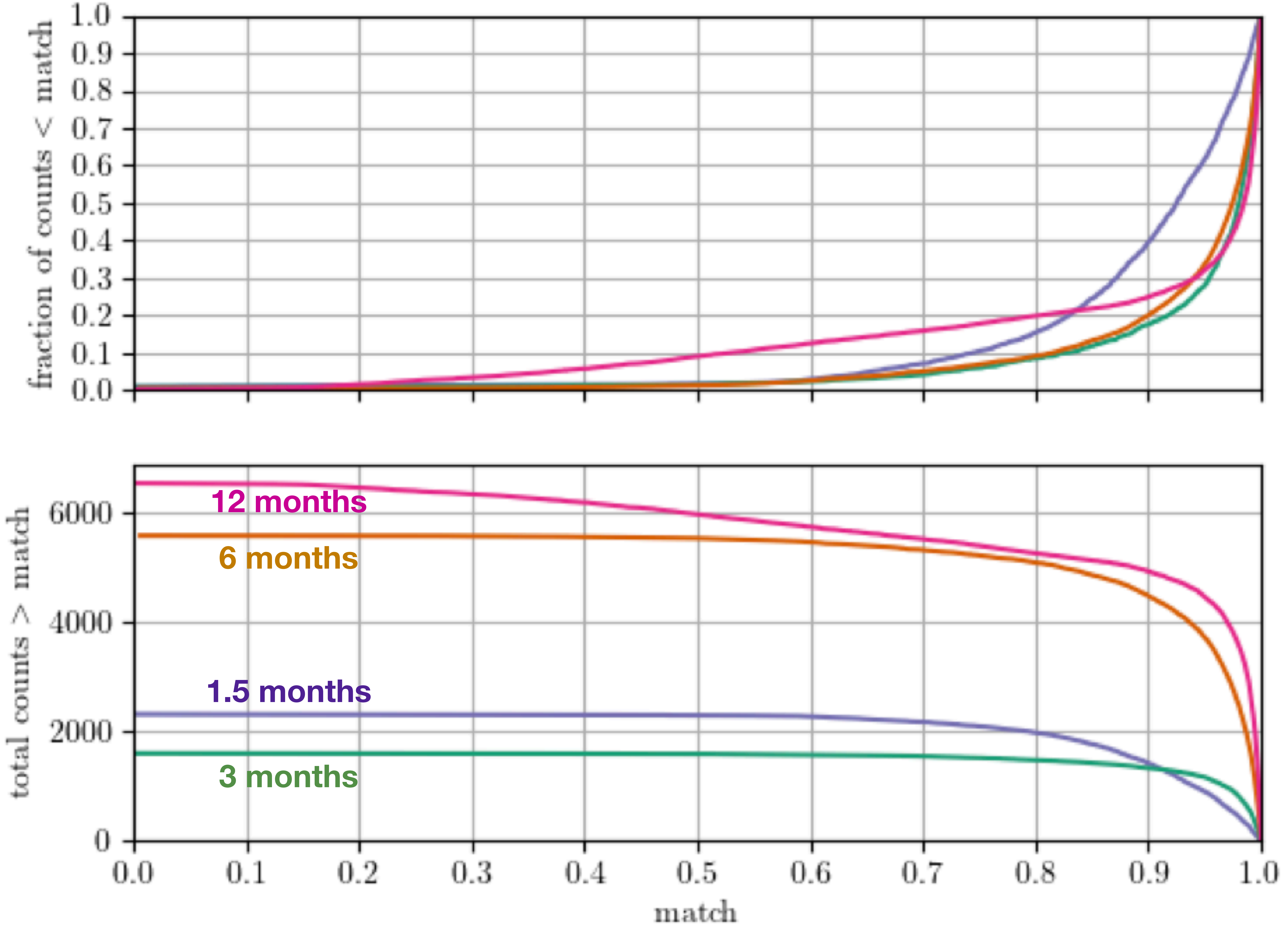


All candidate sources at 12 months



Example of how a source resolves with time

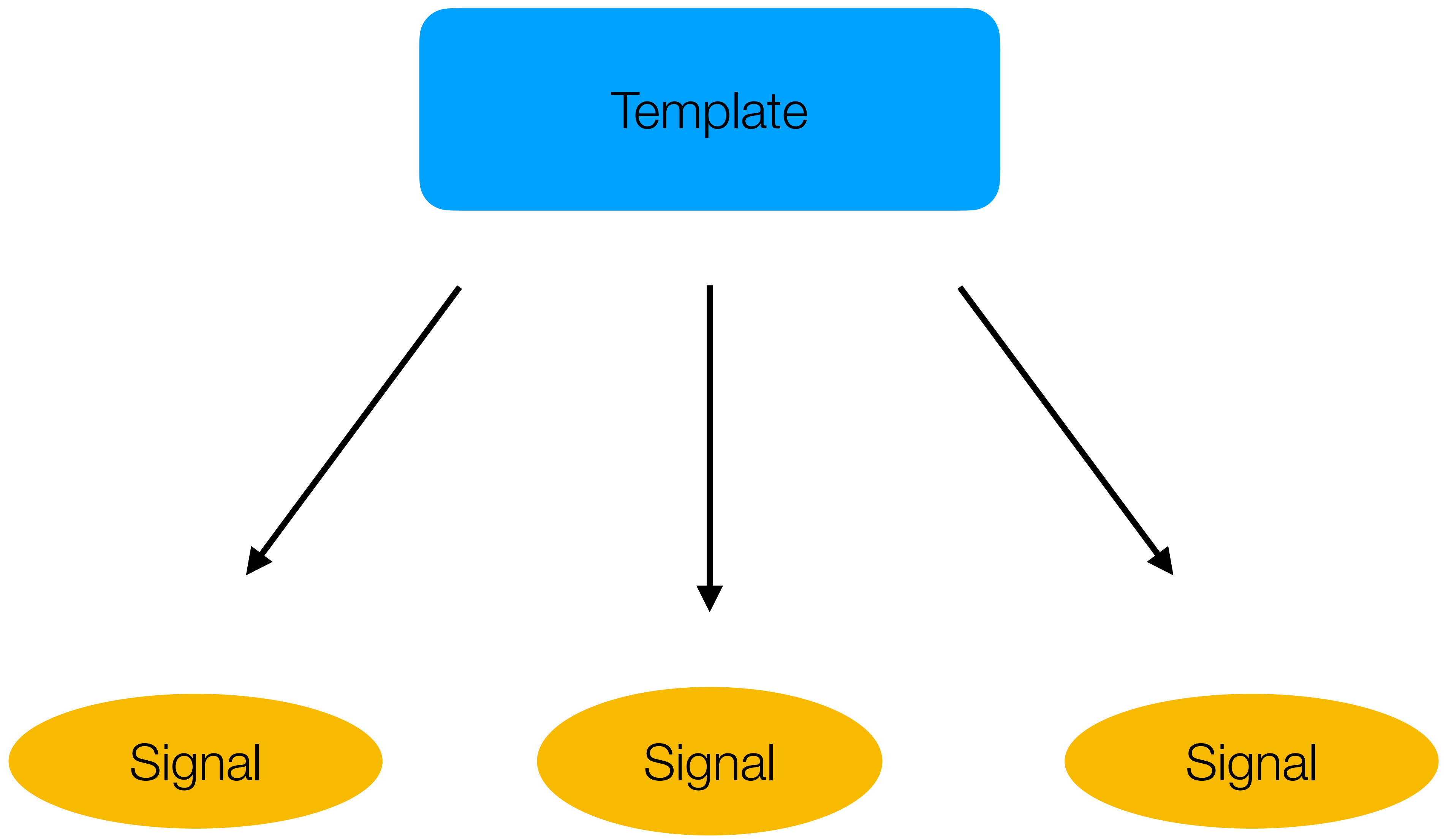
GB matches over time for 90+% confident detections



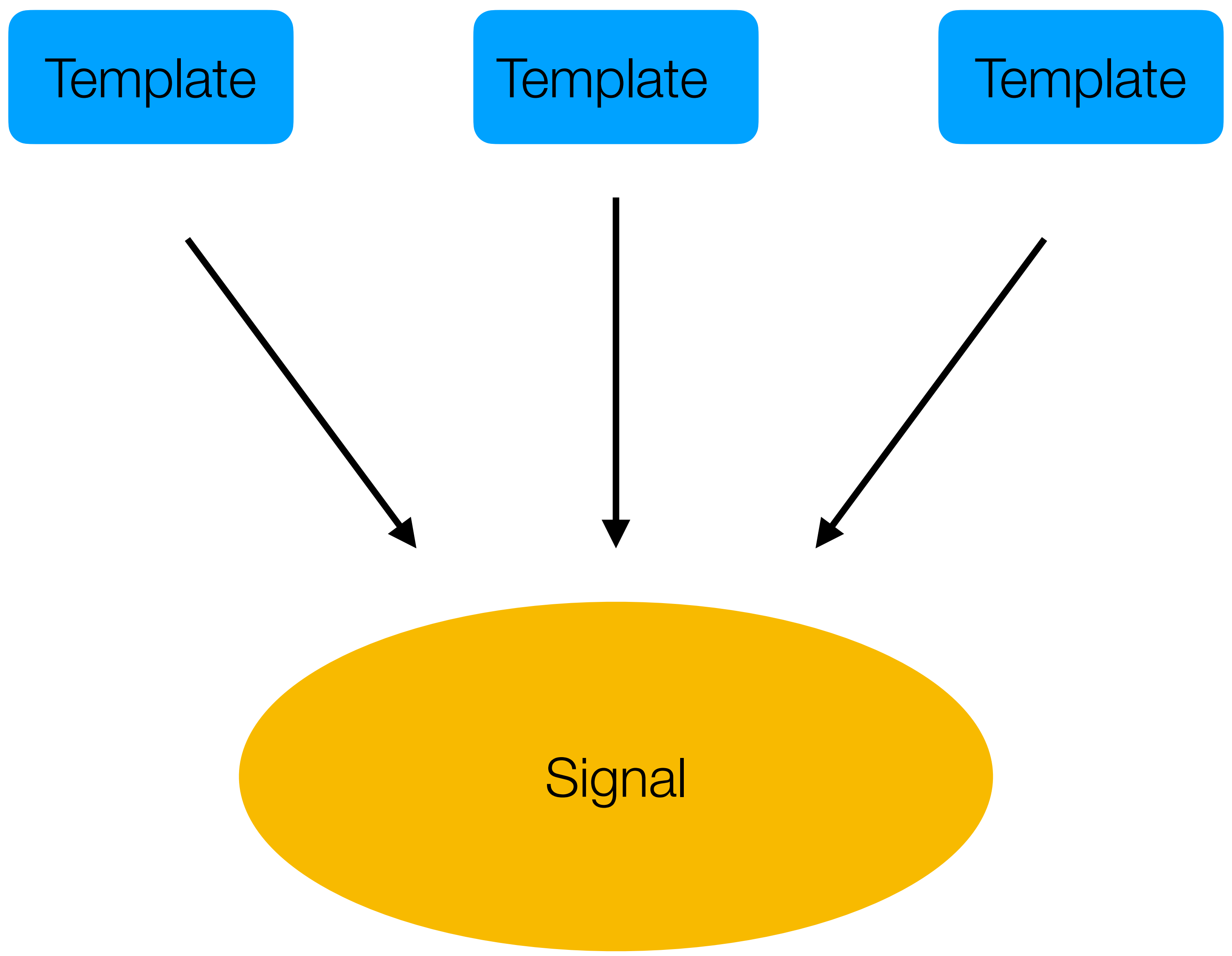
Theory: $M \approx 1 - \frac{D}{2\,\mathrm{SNR}^2}$

Galactic Binaries - what went wrong at 12 months?

One to Many



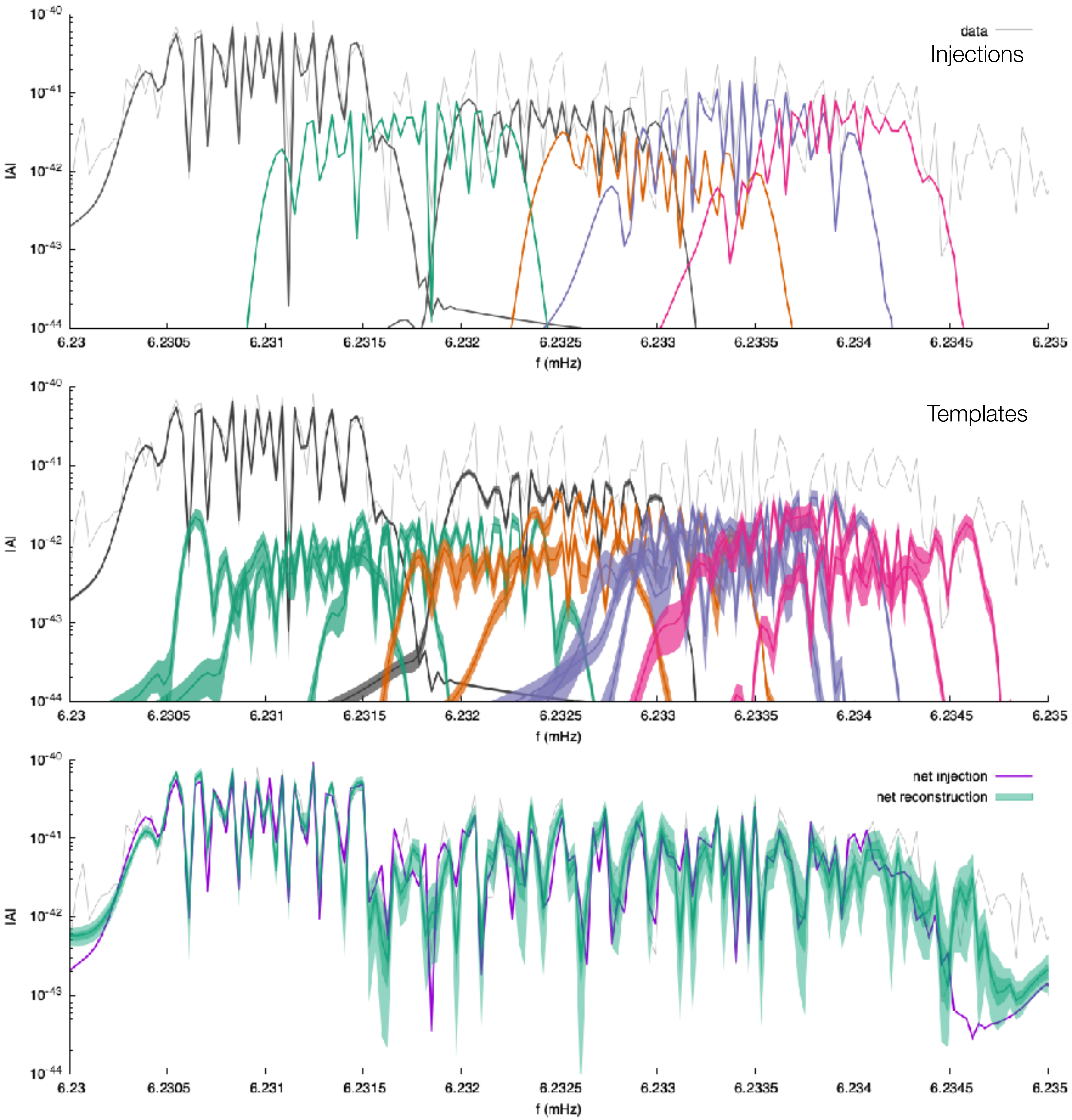
Many to One

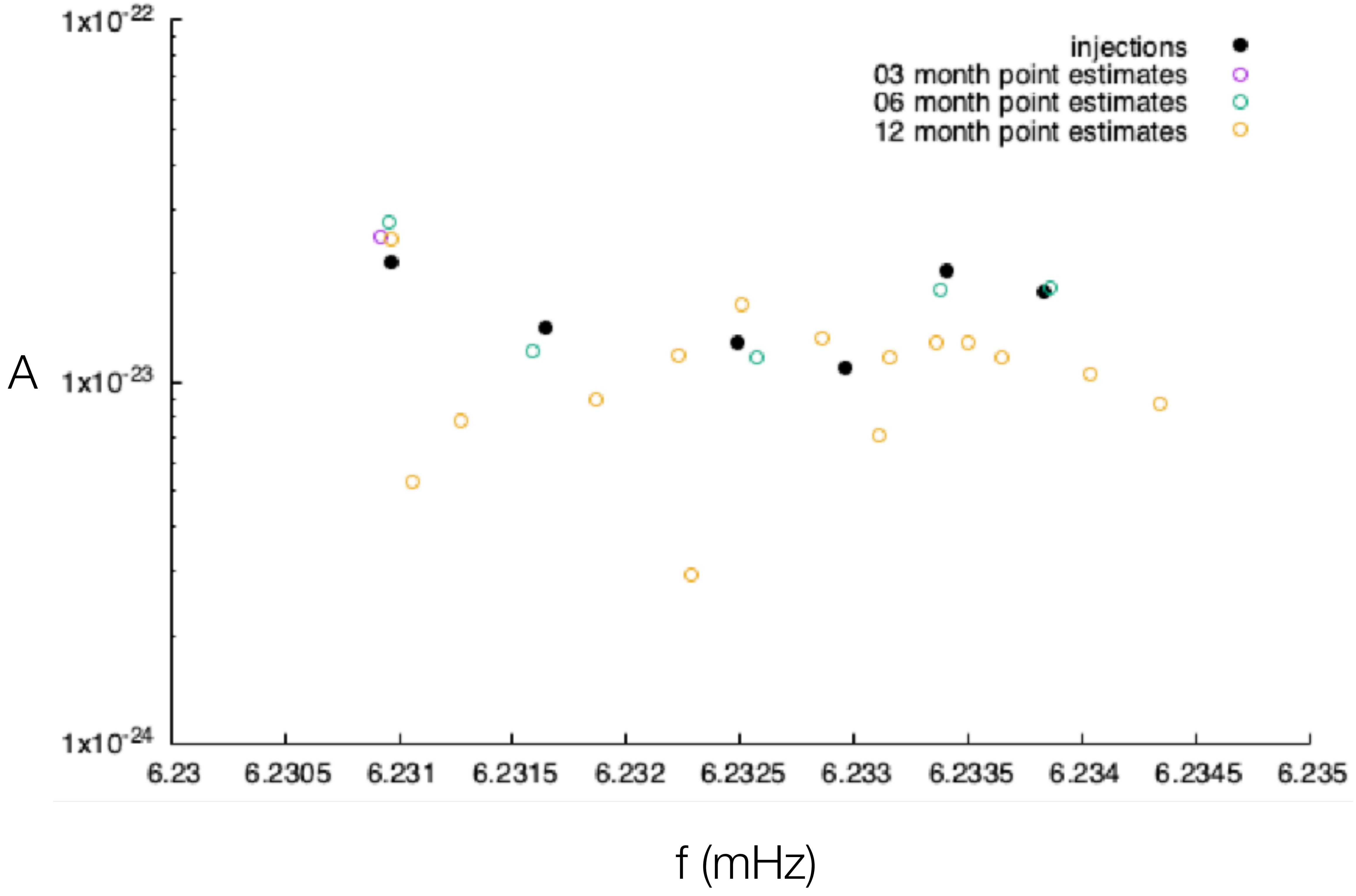


Can be the right answer in a Bayesian sense

Never the right answer - poor sampling

Galactic Binaries - what went wrong at 12 months?





How to do better?

- Better proposals - easiest fix is to increment by smaller amounts in time
- Time-frequency modeling of signals and noise
- Include all three data channels, A, E & T
- Treat the unresolved binaries as a stochastic background (signal), and model the noise component by component
- Include a galaxy shape prior with hyper-parameters for bulge radius, disk radius, disk height etc

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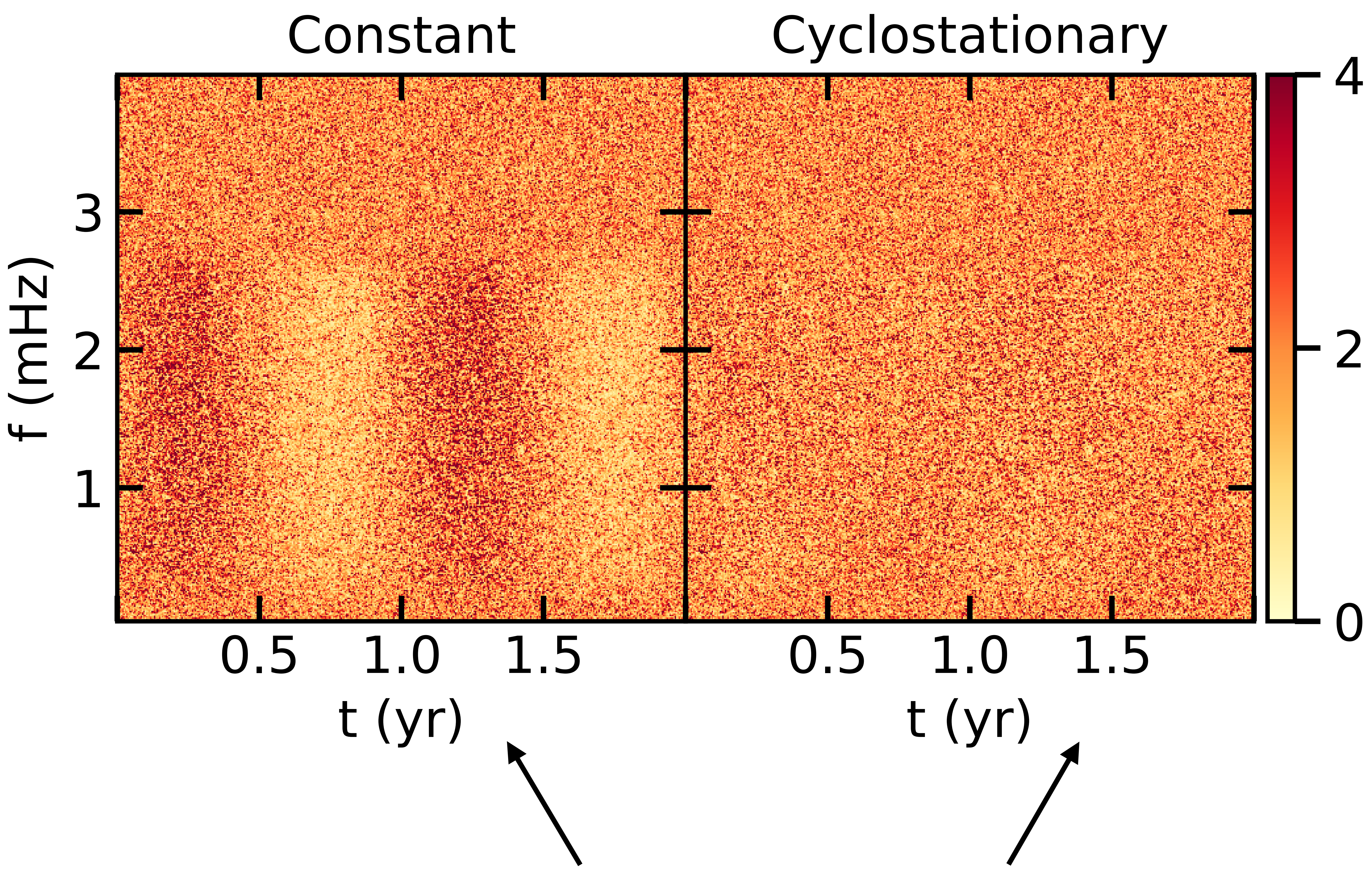
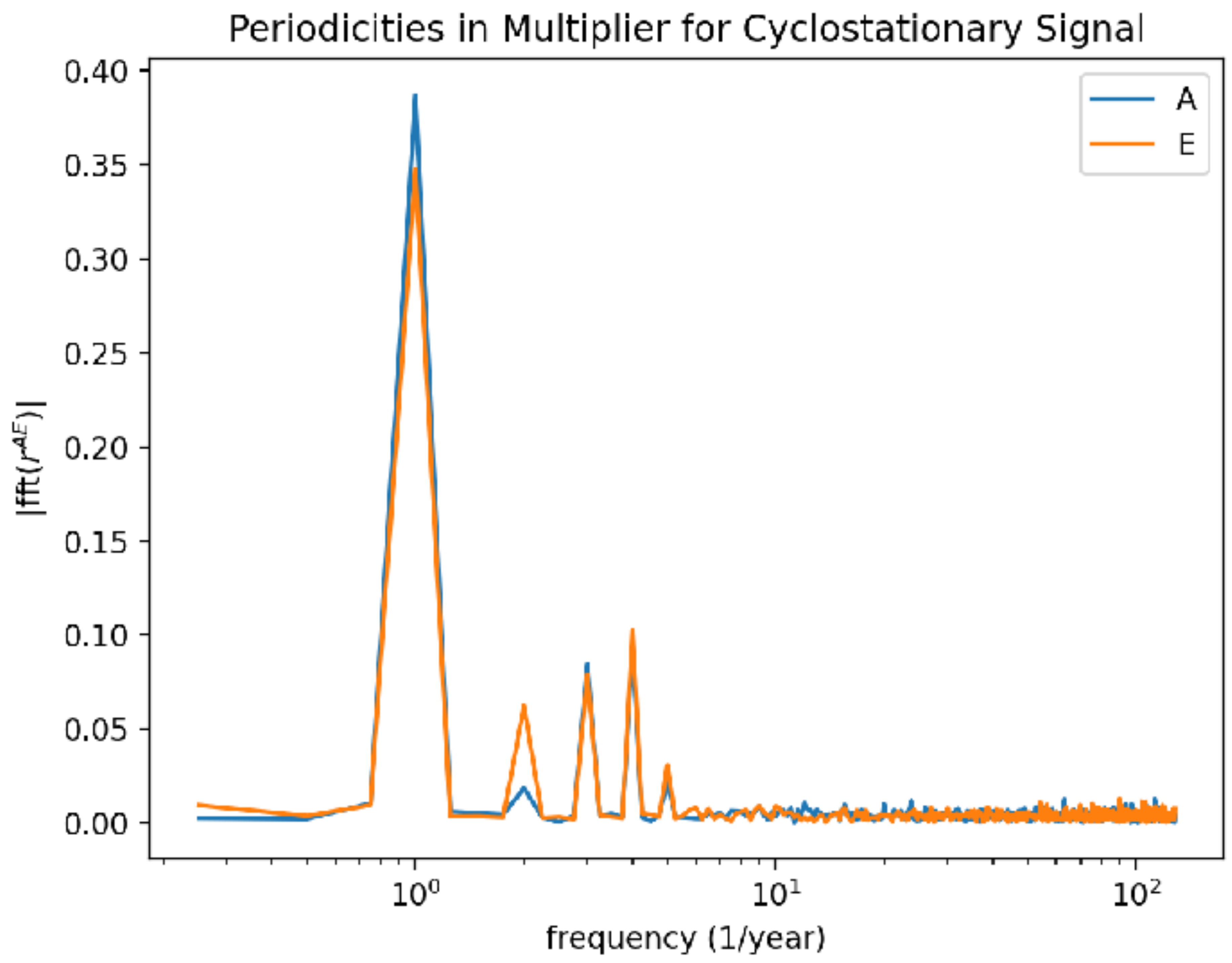
Galactic Confusion - Cyclostationary Noise



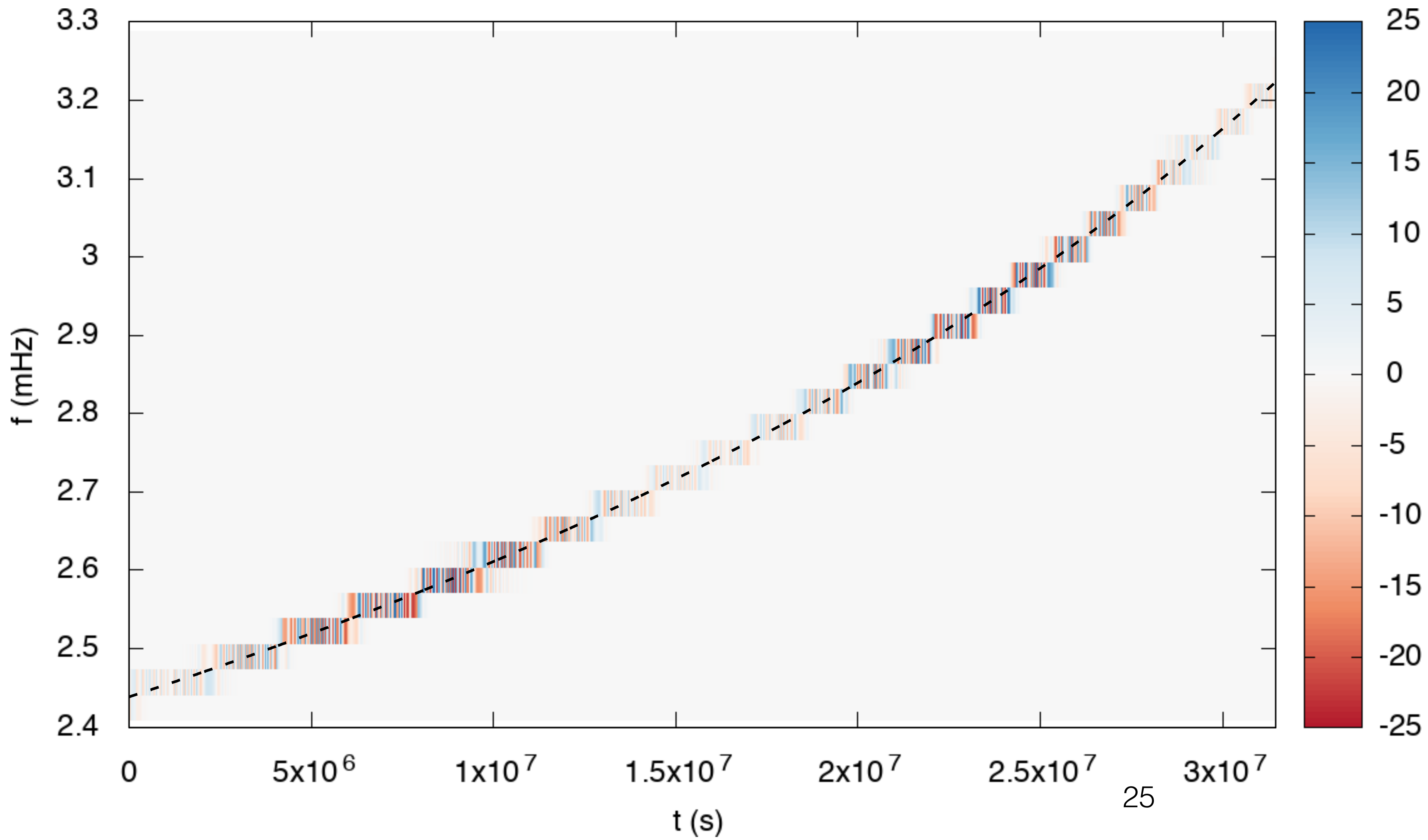


Whitening using constant PSD and dynamic PSD

[Digman & Cornish, arXiv: 2206.148132]

Wavelet domain waveforms

$$p(\mathbf{d}|\mathbf{h}) = \frac{1}{\sqrt{\det(2\pi\mathbf{C})}} e^{-\frac{1}{2}(\mathbf{d}-\mathbf{h})^\dagger \mathbf{C}^{-1}(\mathbf{d}-\mathbf{h})}$$



Fast wavelet transforms of the signals for computational efficiency

Faster than frequency domain, $\sqrt{N}$ scaling

[Cornish, Phys Rev D **102,** 124038 (2020)]

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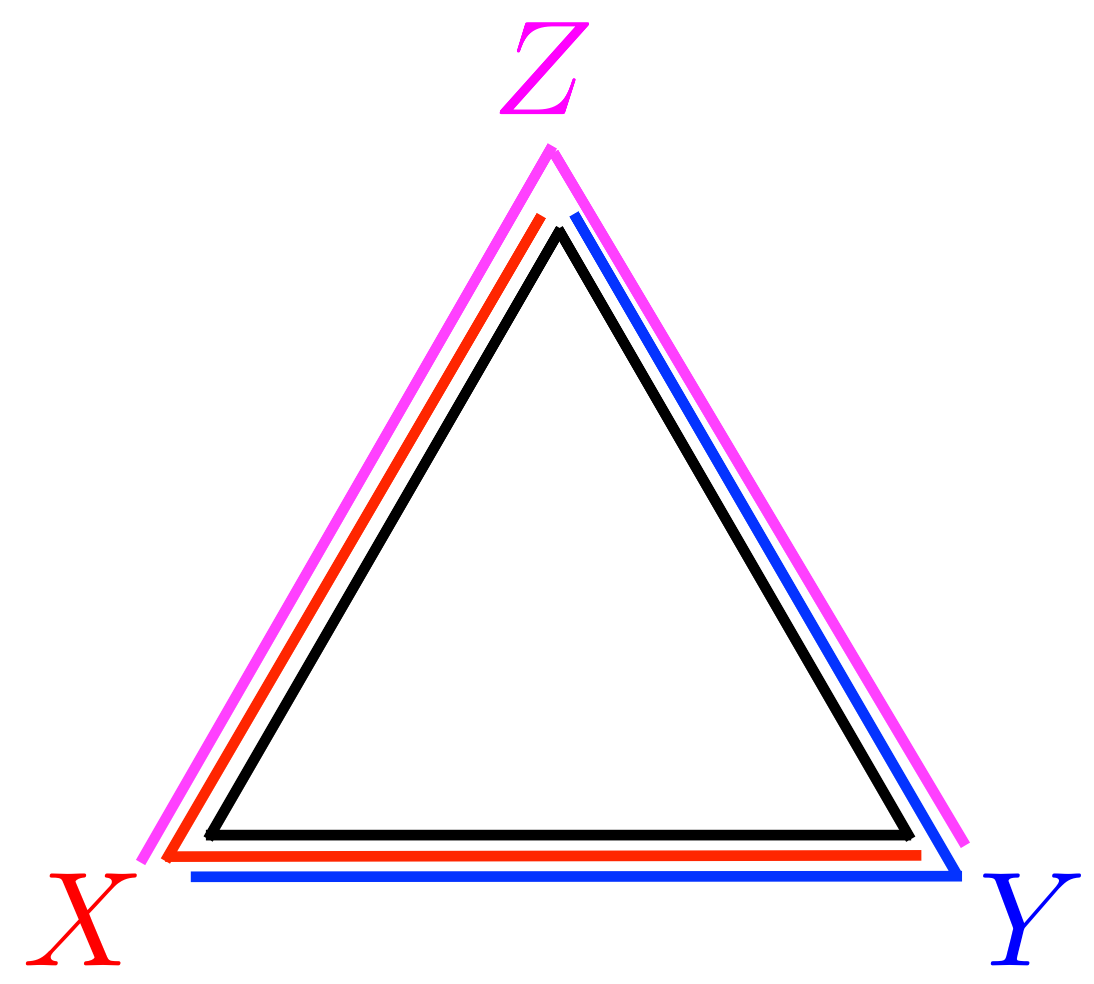
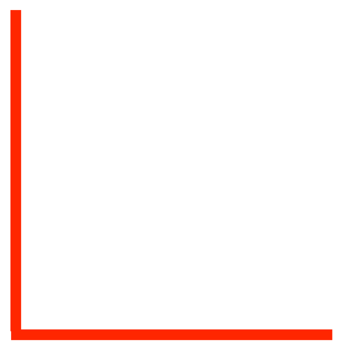
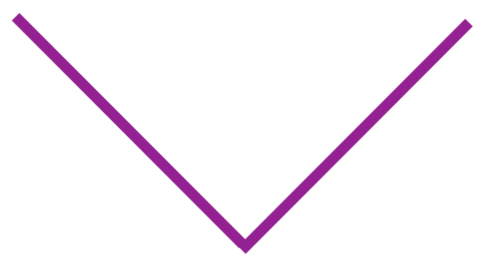
Multiple Data LISA Channels



A $S_{+} = \frac{\sqrt{3}}{2} X \Rightarrow$



E $S_{\times} = \frac{1}{2}(X + 2Y) \Rightarrow$



Sensitive to GWs

T $S_{\odot} = \frac{1}{3}(X + Y + Z) \Rightarrow \cdot$

Insensitive to GWs

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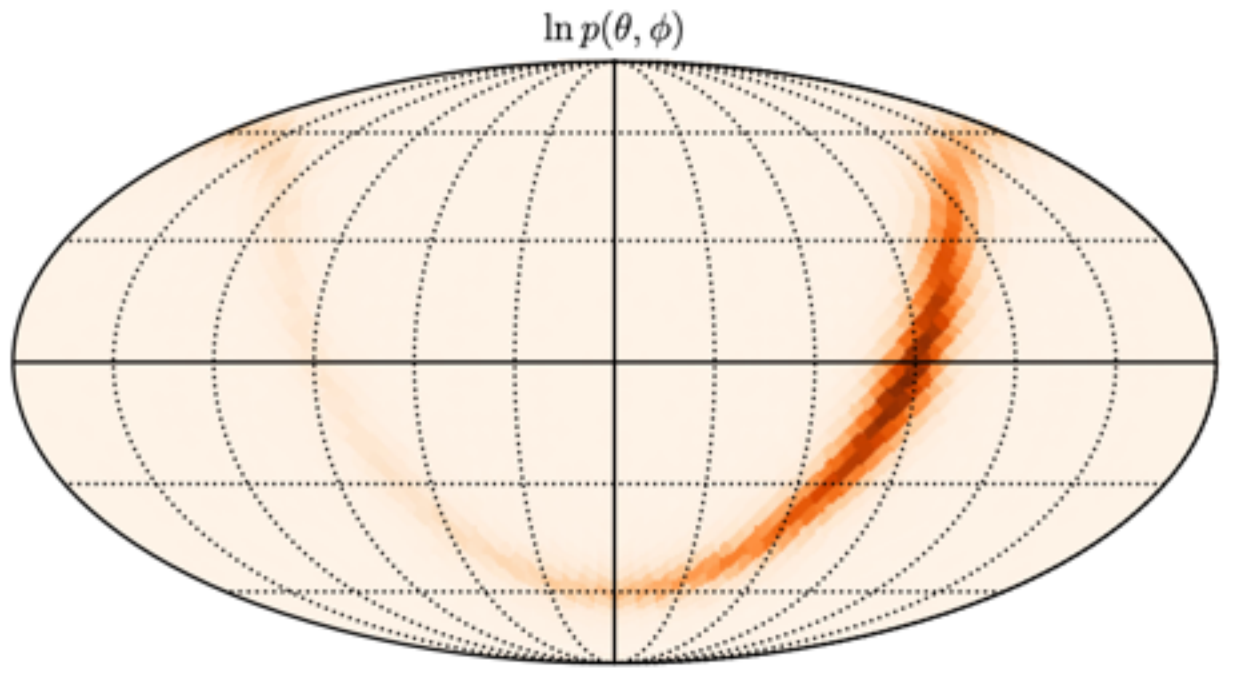
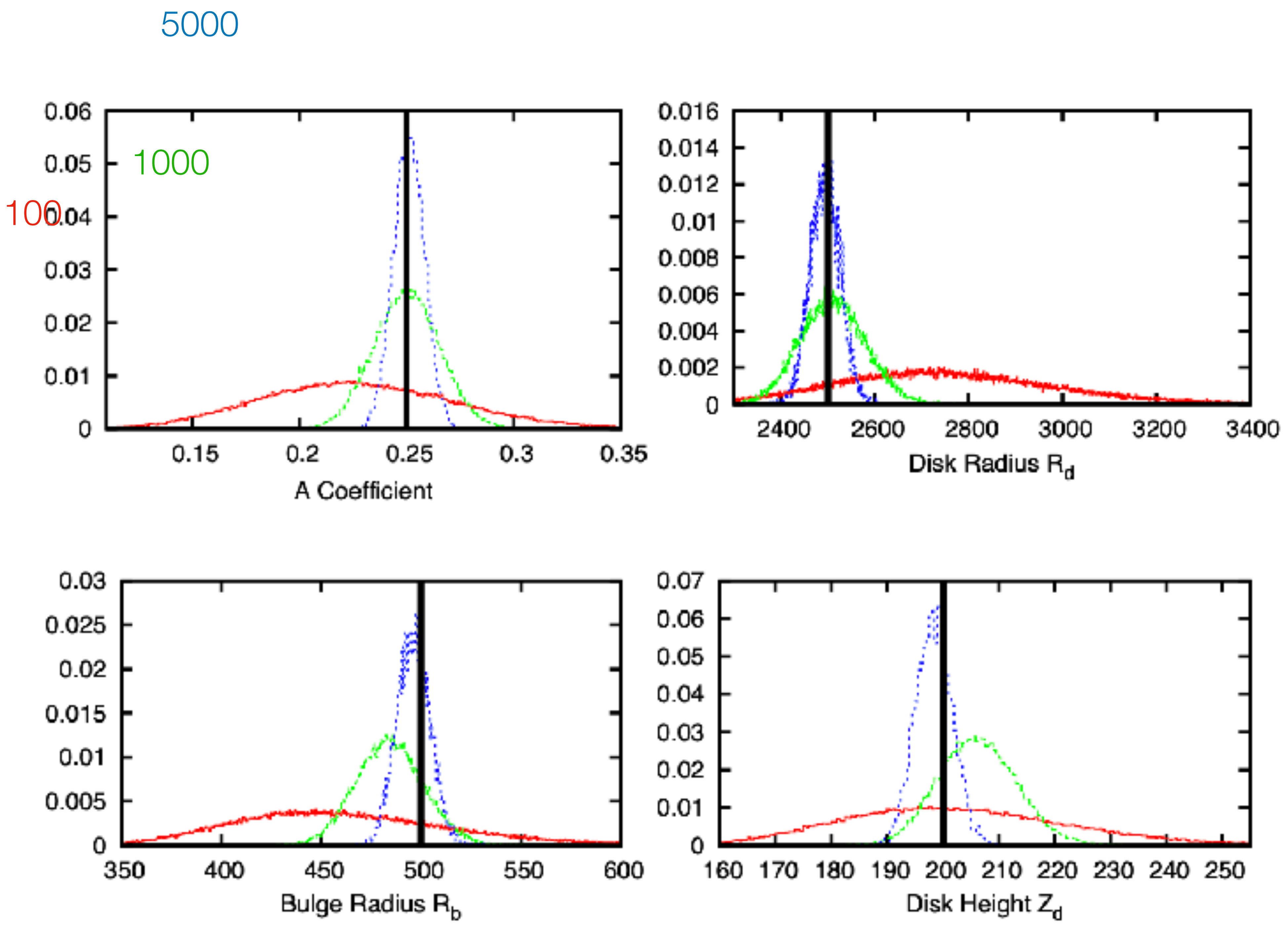
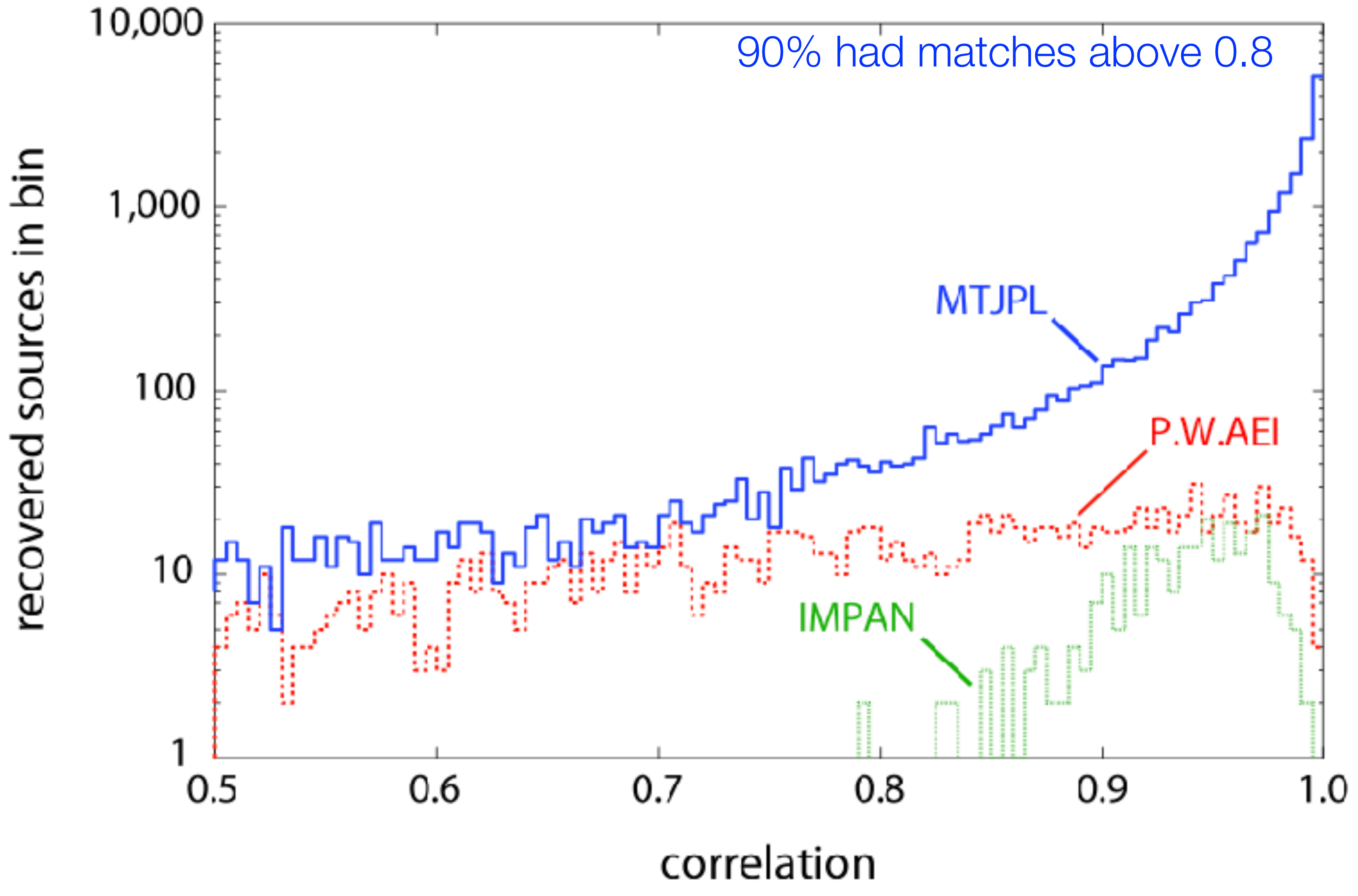
Galaxy shape prior with hyper-parameters

$$\rho(x,y,z) = \rho_0 \left(A_b e^{-r^2/R_b^2} + (1-A_b) e^{-\sqrt{x^2+y^2}/R_d} \operatorname{sech}^2(z/Z_d) \right)$$







[Adams, Cornish & Littenberg, PRD 86, 124032 (2012)]

Crowder & Cornish (2007)