FAST GALAXIES IN THE COSMIC MICROWAVE

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Based on https://arxiv.org/abs/2301.08381 (LG, C. Howlett, R. Ruggeri, F. Bianchini, K. Said, T.M. Davis)

"Cross-correlating radial peculiar velocities and CMB lensing convergence"

Introduction

- ΛCDM in a nutshell
- What are Cross Correlations?

$<$ PV, κ $>$: Theory

- CMB convergence
- What's peculiar about velocities?
- Let's Bet!
- Physics of this cross-correlation

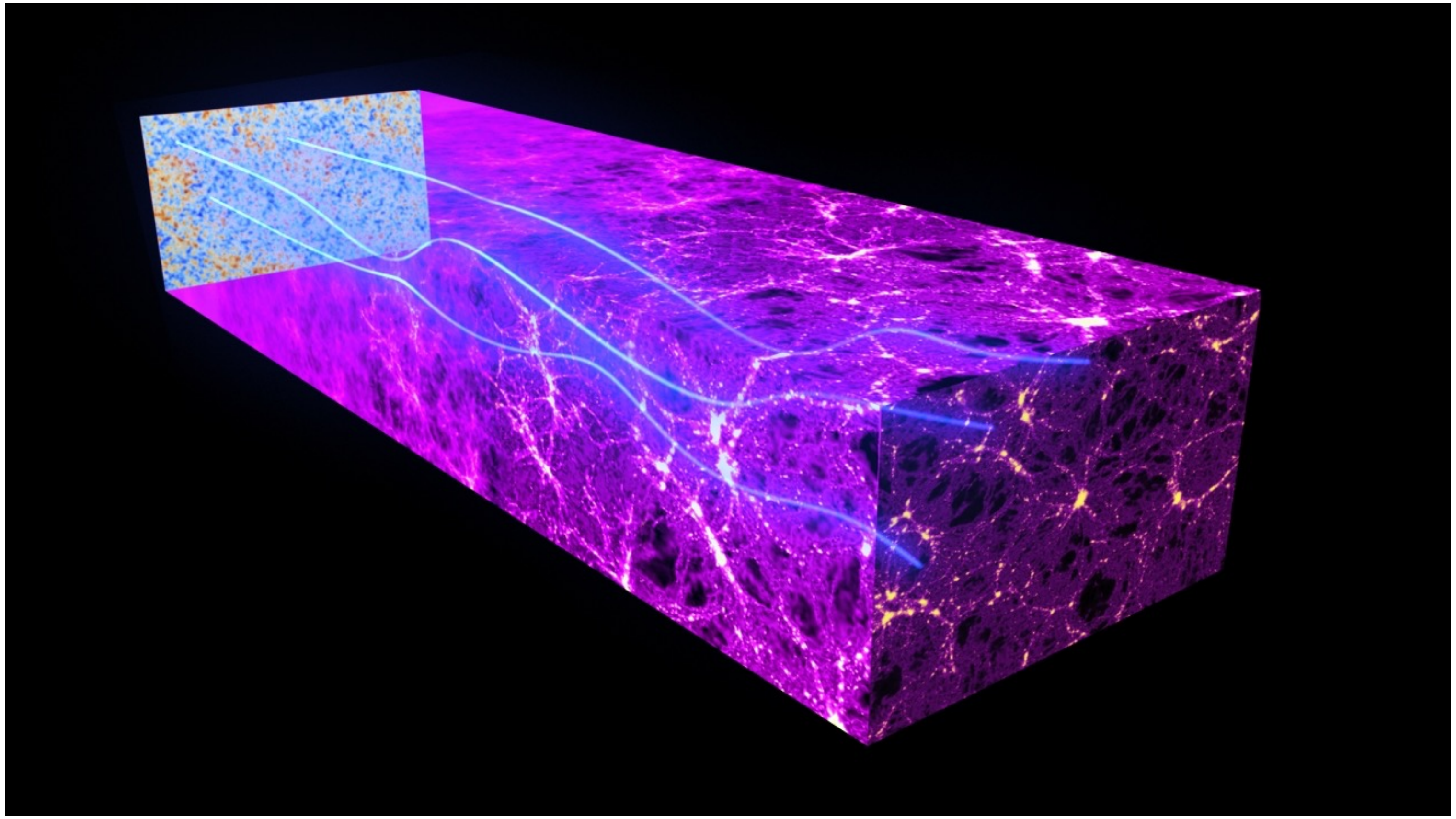
$<$ PV, κ $>$: Practice

- What's all that noise?
- Direct PV: a dead end?
- Reconstructed PV: a new hope?

Pillars of the ΛCDM model

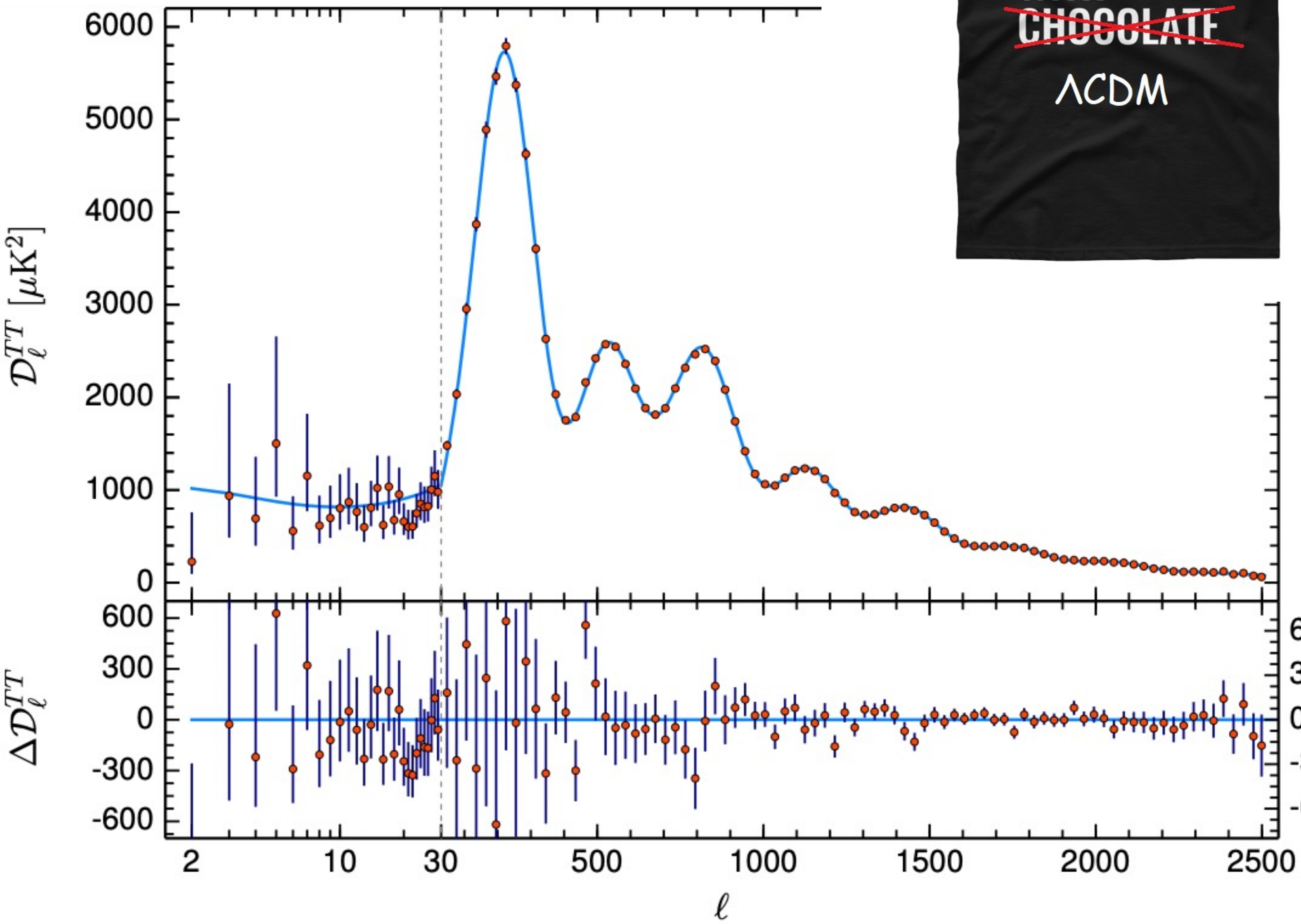
- Cold Dark Matter (CDM) and a Cosmological Constant Λ
- General Relativity is the correct theory of Gravity
- Cosmological Principle realized on sufficiently large scales

Simple, cheap and effective!



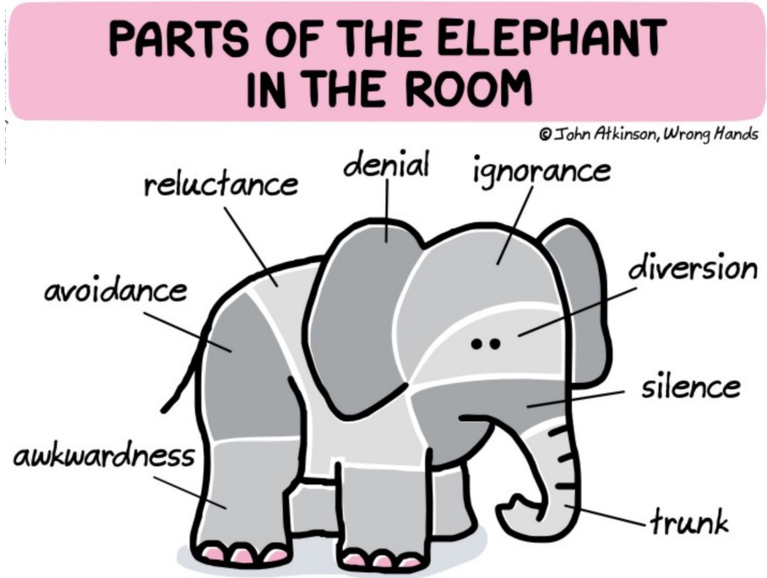
Describes with a bunch of parameters the evolution of the Universe across the last ∽13.5 Gyr…

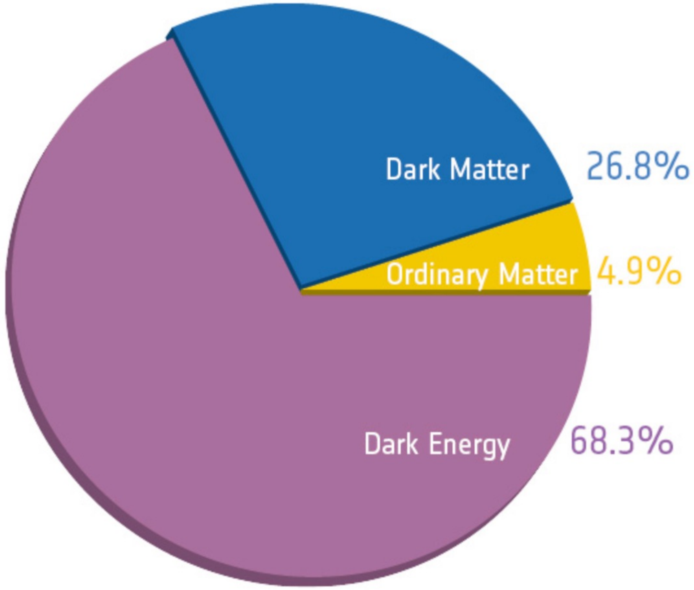
..and does that very well!



"I only know one thing, and that is I know nothing"

(Socrates et al, Private Comm. 420 BC)



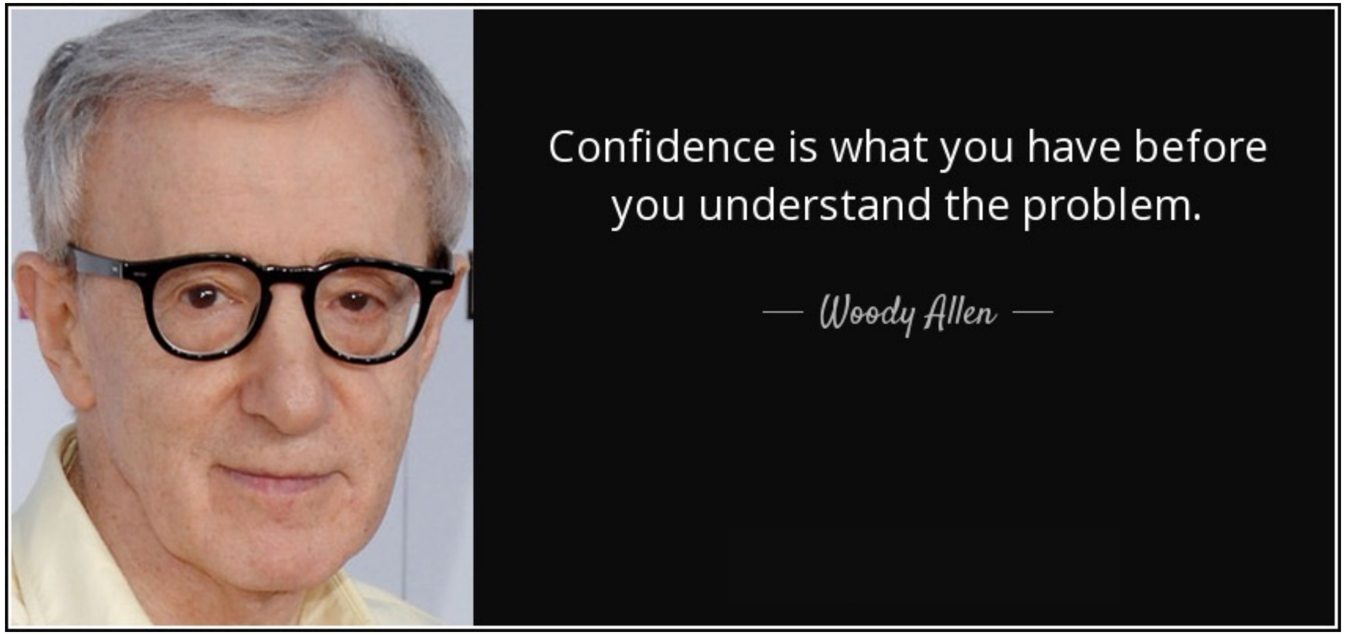


The incompatibility of our two "Standard Models" ranges from (at least) 55 to 120 orders of magnitude.

Λ behave as vacuum energy, but SM QFT vacuum is overweight

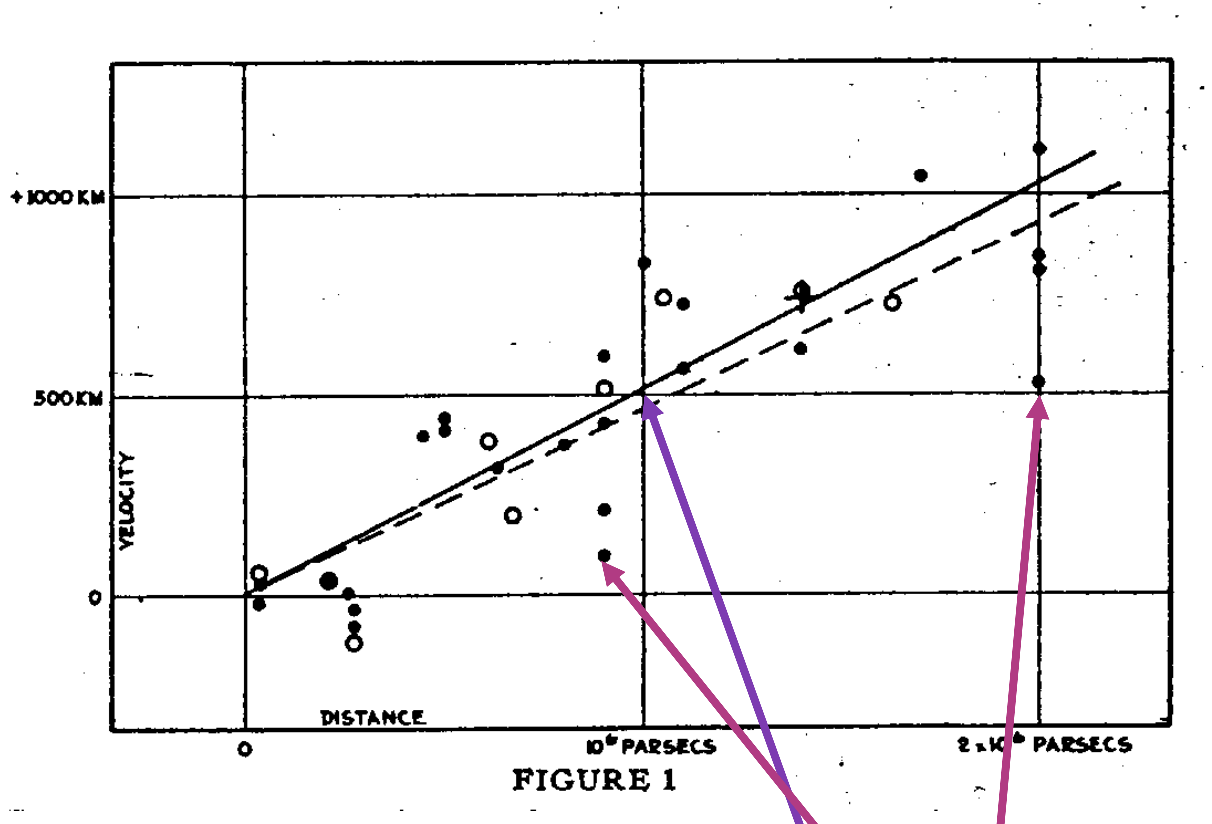
Problems Sourced by our Knowledge

… The curse of living in the age of high precision cosmology..



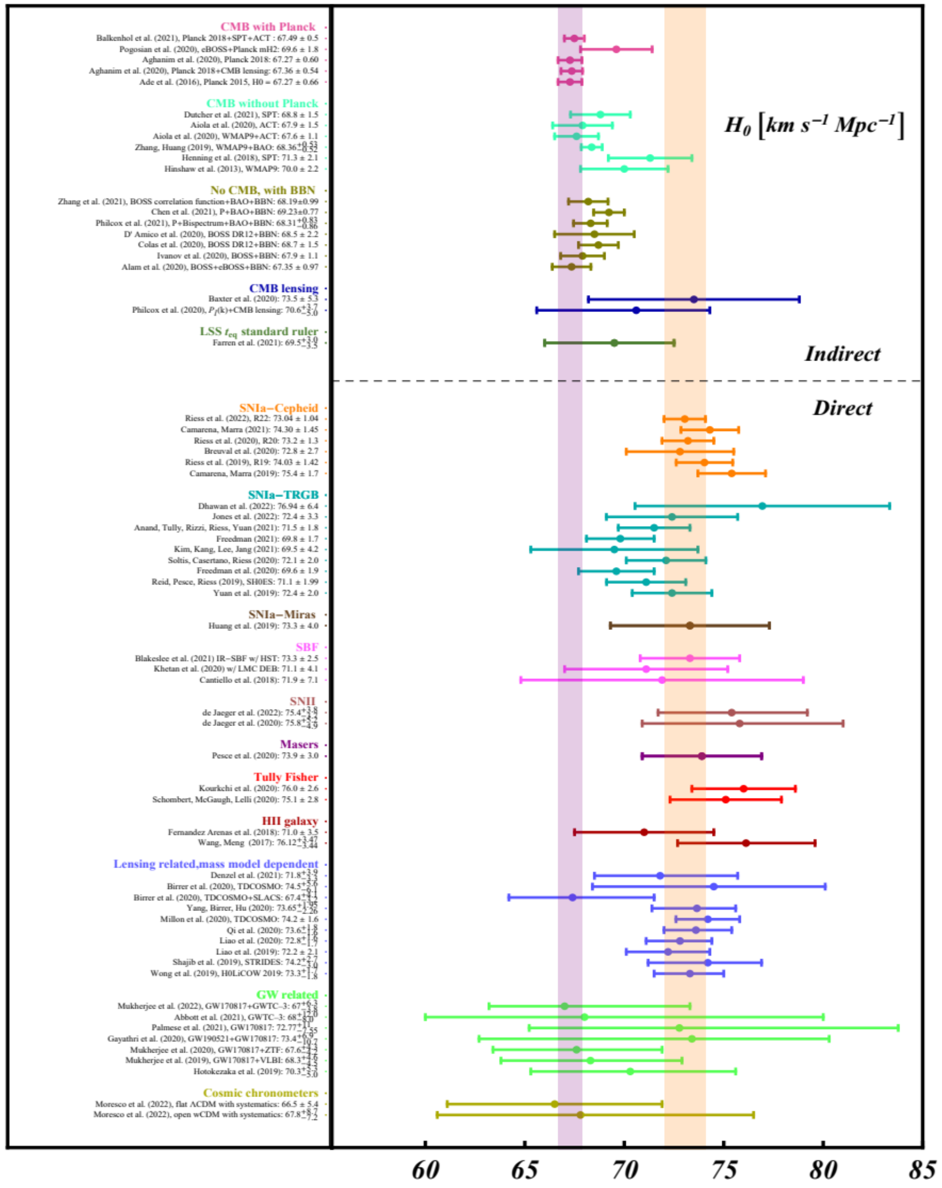
- Cosmic Dipole tension (Where are we going?)
- Hubble Tension (How fast are we going?)
- S8 Tension (How strong is gravity?)
- Curvature tension (Flat, open, closed on weekends?)

1929



$$H_0 \approx 500 \pm 480 \quad km\, s^{-1} Mpc^{-1}$$

2022



Angular Cross Correlations of Cosmological observables

Based on Schöneberg et al. 1807.09540

Any observable in the sky can be expanded in Spherical Harmonics:

$$\mathcal{O}^\alpha(\hat{\mathbf{n}}, z) = \sum_{\ell m} Y_{\ell m}(\hat{\mathbf{n}}) a^\alpha_{\ell m}(z) \,, \qquad a^\alpha_{\ell m}(z) = \int d\Omega_{\hat{\mathbf{n}}} \, Y^*_{\ell m}(\hat{\mathbf{n}}) \mathcal{O}^\alpha(\hat{\mathbf{n}}, z) \,.$$

The correlation function between any two of them (α, β) is just:

$$C_\ell^{\alpha\beta}(z_1, z_2) \equiv \left\langle a^{\alpha,*}_{\ell m}(z_1), a^\beta_{\ell m}(z_2) \right\rangle$$

What is a correlation Function?

0th order definition: Probability of finding a certain value of α in z_1 ***and*** a certain value of β in z_2

Law of nature seems to be based on 2nd order differential equations.

Unfortunately, I never learnt how to solve them.

Fourier-transform everything:

$$\mathcal{O}^\alpha(\hat{\mathbf{n}}, z) = \int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{x}} \mathcal{O}^\alpha(\mathbf{k}, z)$$

Then some black magic to massage the plane waves:

$$e^{i\mathbf{k}\cdot\mathbf{x}} = 4\pi \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} i^l j_\ell(kx)\, Y_{\ell m}(\hat{\mathbf{k}}) Y^*_{\ell m}(\hat{\mathbf{n}})$$

Exploiting orthogonality of spherical harmonics, finally:

$$a^{\alpha}_{\ell m}(z) = 4\pi i^{\ell} \int \frac{d^3k}{(2\pi)^3} j_{\ell}\left(k\chi(z)\right) \mathcal{O}^{\alpha}(\mathbf{k}, z) Y^{*}_{\ell m}(\hat{\mathbf{k}})$$

Until now, no Cosmology has been involved.
(even though many cosmologists have been harmed)

In the SM the very early Universe was a rather homogeneous inflating soup (and even if not, inflation *soupify* very quickly).

↓

- The only inhomogeneities in the broth are the quantum fluctuations of the inflaton field, whatever that is.

↓

We assume that all the Cosmological observables we are interested in have evolved from these *primordial fluctuations*, whose correlation function is the primordial Power Spectrum:

$$\langle \mathcal{R}(\mathbf{k}), \mathcal{R}(\mathbf{k}') \rangle = \frac{2\pi^2}{k^3} P_{\mathcal{R}}(k) \delta\left(\mathbf{k} - \mathbf{k}'\right) \ ,$$

Disclaimer:

It is usually assumed that the primordial perturbations are *adiabatic* (no entropic ones), and *Gaussian*.

According to Planck, these hypotheses are very reasonable and observationally justified

Single scalar field models of inflation (including Starobinksy's f(R)) and similar models predicts these features, but **it is not** a general property of inflation

However, compensated isocurvature perturbations are not very well constrained by Planck (Barreira et. Al. 2302.01927)

We thus write any cosmological observable as the product of the Primordial perturbation with a certain transfer function:

$$\mathcal{O}^\alpha(\mathbf{k}, z) = T^\alpha(k, z)\mathcal{R}(\mathbf{k})$$

Combining everything we can rewrite the angular cross-correlation as:

$$C_\ell^{\alpha\beta}(z_1, z_2) \equiv \left\langle a_{\ell m}^{\alpha,*}(z_1), a_{\ell m}^{\beta}(z_2) \right\rangle$$

$$\Downarrow$$

$$C_\ell^{\alpha\beta}(z_1, z_2) = 4\pi \int_0^\infty \frac{dk}{k} P_{\mathcal{R}}(k) T^\alpha(k, z_1) T^\beta(k, z_2) j_\ell(k\chi(z_1)) j_\ell(k\chi(z_2))$$

(Stay with me one more slide, we are almost done)

In practice one does not measure continuously the observable, but a weighted average over some redshift bin:

$$\bigstar C_\ell^{\alpha\beta} = \int W^\alpha(z_1) W^\beta(z_2) C_\ell^{\alpha\beta}(z_1, z_2) dz_1 dz_2 = 4\pi \int_0^\infty \frac{dk}{k} P_{\mathcal{R}}(k) \Delta_\ell^\alpha(k) \Delta_\ell^\beta(k)$$

Kernels

$$\Delta_\ell^\alpha(k) \equiv \int_0^\infty dz\, T^\alpha(k, z) j_\ell(k\chi(z)) W^\alpha(z)$$

CMB Lensing (Convergence)

$$\varphi(\hat{\mathbf{n}}) = -\int_0^{\chi_*} d\chi \, \frac{\chi_* - \chi}{\chi_* \chi} \, [\Phi + \Psi] \, (\chi \hat{\mathbf{n}}, \eta_0 - \chi) \, ,$$

(Lewis and Challinor, astro-ph/0601594)

CMB lensing potential along the LOS $\hat{n}$.
Using Poisson Eq. we can relate the gravitational potential to the Matter distribution

$$\kappa(\hat{\mathbf{n}}) = \frac{3}{2c^2} \Omega_m H_0^2 \int_0^{\chi_{\mathrm{CMB}}} d\chi \frac{\chi}{a(\chi)} \frac{\chi_{\mathrm{CMB}} - \chi}{\chi_{\mathrm{CMB}}} \delta \left(\chi \hat{\mathbf{n}}, \eta_0 - \chi \right)$$

Convergence, basically the trace of the Jacobi Matrix (obtained linearizing the lensing eq.)

$$\mathcal{A}_{ij} = \frac{\partial \beta_i}{\partial \theta_j} = \delta_{ij} - \psi_{ij} \, ,$$

(Bartelmann and Maturi, 1612.06535)

CMB Lensing (Convergence)

Fourier-transforming ecc. ecc. we are left with:

$$\Delta^{\kappa}_{\ell}(k) \equiv \int_0^{\chi_{\rm CMB}} d\chi \, W^{\kappa}(\chi) j_{\ell}\left(k\chi\right) D(\chi) \; ,$$

$$W^{\kappa}(\chi) \equiv \frac{3}{2c}\Omega_m H_0^2 \, \chi \frac{1+z\left(\chi\right)}{H\left(\chi\right)} \frac{\chi_{\rm CMB}-\chi}{\chi_{\rm CMB}} \; .$$

Peculiar Velocities

- What's peculiar about velocities?

For a given galaxy, (neglecting magnification and beaming effects) we have: (e.g. Davis and Scrimgeour, 1405.0105)

$$(1+\ z_{obs}) = (1+z_{cos})(1+z_{pec})(1+z_{CMB})$$

z_{pec} is due to the motion of the galaxy induced by the local gravitational field.
At first order in linear theory, combining the Euler and continuity equation, the local velocity field obeys:

$$\nabla \cdot v(\mathbf{r}) = aHf\delta(\mathbf{r}) \longleftrightarrow v(\mathbf{k}) = -iaHf\frac{\delta(\mathbf{k})}{\mathbf{k}}$$

And since I am are not very good with diff. equations

However, only the radial component of the velocity field contributes to z_{pec} :

$$u(\hat{\mathbf{r}}, \mathbf{k}) \equiv v(\mathbf{k}) \cdot \hat{\mathbf{r}} = -iaHf \frac{\delta\,(\mathbf{k})}{k^2} \mathbf{k} \cdot \hat{\mathbf{r}}$$

$$u(\mathbf{r}) = -i \int \frac{d^3k}{(2\pi)^3} aHf \, \frac{\delta\,(\mathbf{k})}{k^2} \mathbf{k} \cdot \hat{\mathbf{r}} \; e^{i\mathbf{k}\cdot\mathbf{r}}$$

(black magic to get rid of the annoying i)

$$a^u_{\ell m}(\chi) = (4\pi i^\ell) aHf \int \frac{d^3k}{(2\pi)^3} \frac{\delta(\mathbf{k})}{k} j'_\ell\,(k\chi)\, Y^*_{\ell m}(\hat{\mathbf{k}}) \leftarrow \hat{\mathbf{k}} \cdot \hat{\mathbf{r}} \; e^{i\mathbf{k}\cdot\mathbf{r}} = -i \frac{d}{d(kr)} e^{i\mathbf{k}\cdot\mathbf{r}}$$

Peculiar Velocities

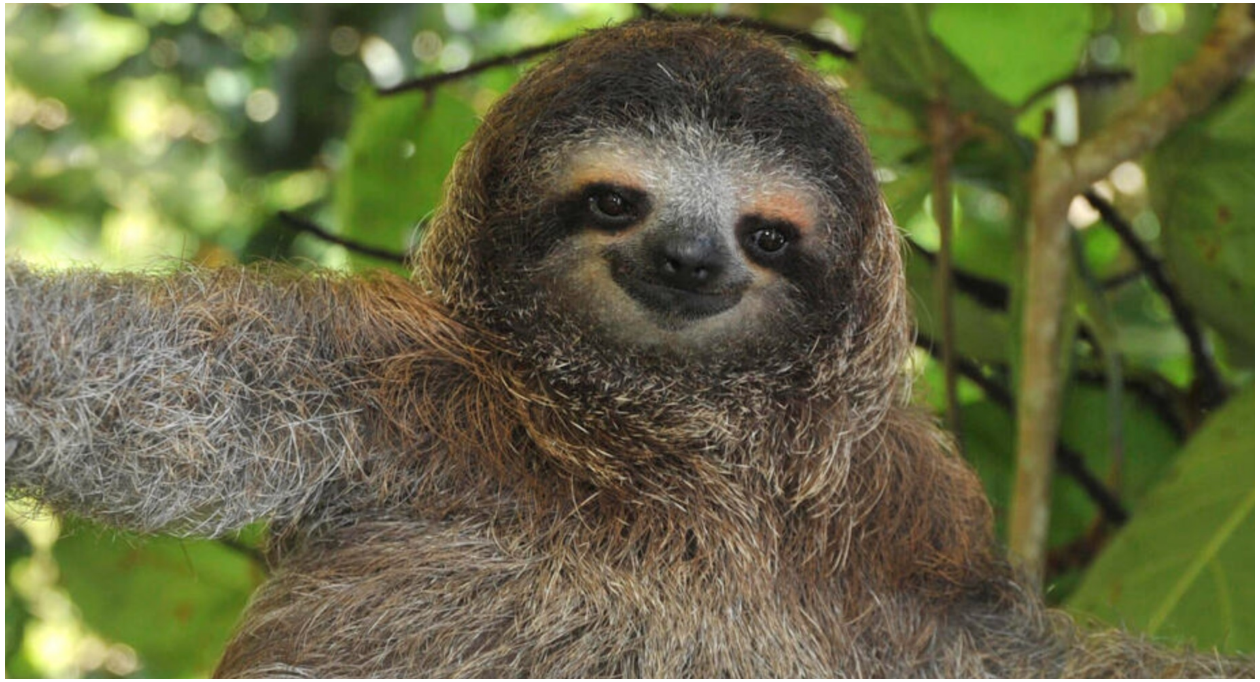
Finally:

$$\Delta_\ell^u(k) \equiv \frac{1}{k}\int_0^\infty d\chi\, W^u(\chi) j'_\ell(k\chi)\, D(\chi)$$

$$W^u(\chi) = H f a \frac{dn}{d\chi}$$

Cross Correlation < PV, κ >

$$C_\ell^{u\kappa} \equiv \frac{2}{\pi} \int dk\, d\chi_1\, d\chi_2\, k\, W^u(\chi_1) W^\kappa(\chi_2) j_\ell'(k\chi_1)\, j_\ell(k\chi_2)\, P_m(k, \chi_1, \chi_2)$$



As obscure (and trivial) as it is, the above formula took me roughly 3 weeks of work
(Does not mean that it is hard to derive, just that I am slow. However, smiles and slow peace appear to be correlated!)

Let’s bet! How much signal from this Cross Correlation?

-CMB Lensing is peaked around $z \sim 1.5$!

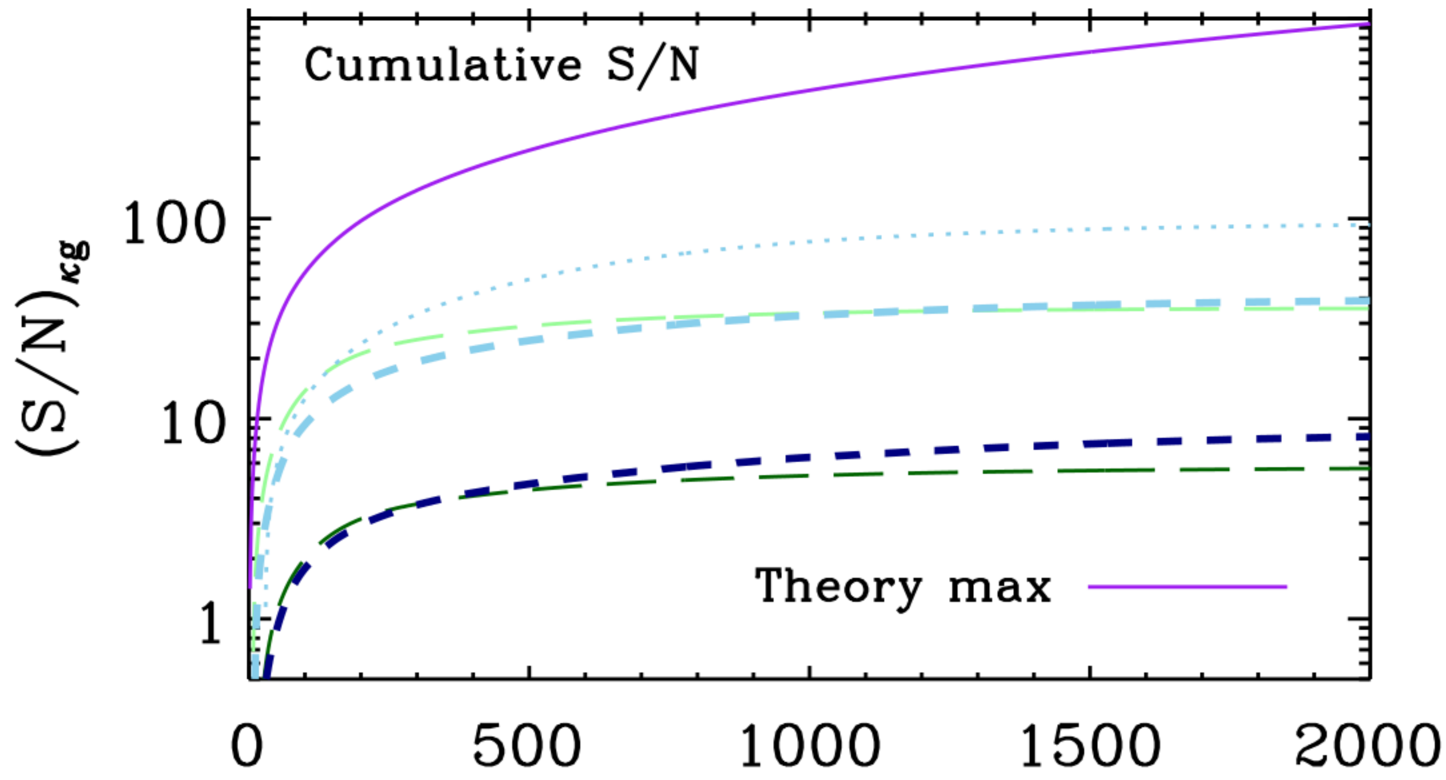
-Usually $v_{pec} \approx 200 - 300$ km/s, and extremely noisy at higher z.
Hubble law($v_{cos} \approx H_0 \, d$) , at redshift $z \sim 0.1$ ($d \approx 400 \, Mpc$) gives

$$v_{cos} \approx 70 * 400 \approx 30000 \text{ km/s}$$

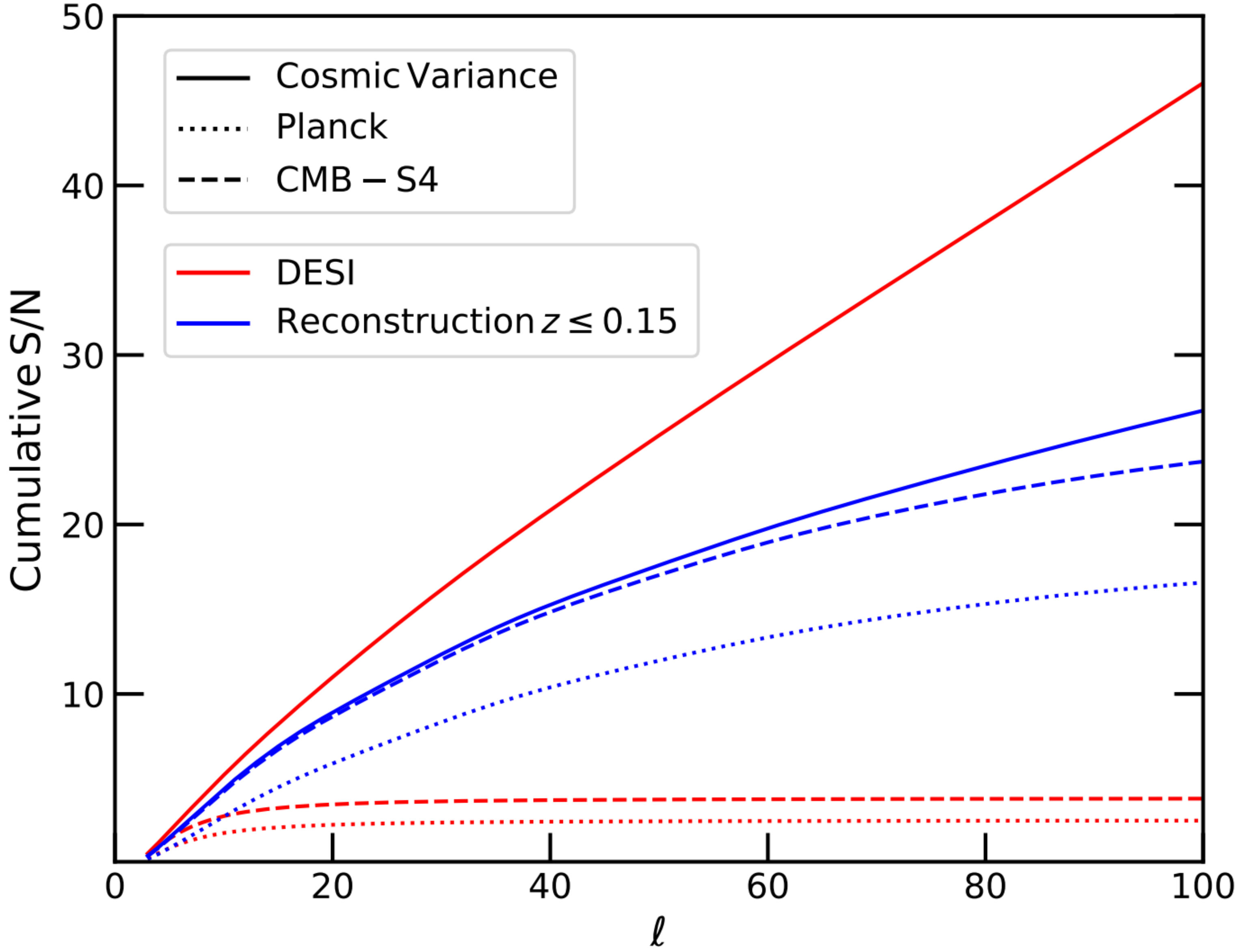
2 orders of magnitude difference in velocity! since $c \sim 10^5$, PVs at $z \sim 0.1$ cause $\Delta z \approx 10^{-3}$

What cumulative signal-to-noise you expect from combining e.g. DESI and Planck at cosmic variance limit?

For reference, the one below is the one obtained combining redshifts (density field) and CMB Lensing
(Giannantonio et. al 1507.05551)



Our Results!



Cosmic Variance
Planck
CMB – S4
DESI
Reconstruction $z \leq 0.15$
Cumulative S/N
50
40
30
20
10
0
20
40
60
80
100
ℓ

At Cosmic variance limit there is a lot of signal in the cross correlation! Why is that, despite such a huge redshift difference in the peaks of the Kernels?

It is easier to understand why using the Limber approximation: **Looking Beyond the Survey effect!**

$$\lim_{\epsilon\to 0}\int dx\,\sqrt{\frac{2x}{\pi}}e^{\epsilon(x-\ell)}j_{\ell-\frac{1}{2}}(x)f(x)\approx f(\ell)+\mathcal{O}\left(\ell^{-2}\right)$$

$$C_\ell^{u\kappa}\equiv\frac{2}{\pi}\int dk\,d\chi_1\,d\chi_2\,k\,W^u(\chi_1)W^\kappa(\chi_2)j'_\ell(k\chi_1)\,j_\ell\left(k\chi_2\right)P_m\left(k,\chi_1,\chi_2\right)$$

$$C_\ell^{u\kappa}\approx\int\frac{dk}{k}\sqrt{\frac{1}{\ell+\frac{1}{2}}}\left[\frac{W^u\left(\frac{\ell-\frac{1}{2}}{k}\right)W^\kappa\left(\frac{\ell+\frac{1}{2}}{k}\right)}{\sqrt{\ell-\frac{1}{2}}}P_m\left(k,\frac{\ell-\frac{1}{2}}{k},\frac{\ell+\frac{1}{2}}{k}\right)\right.$$
$$\left.-\frac{(\ell+1)\,W^u\left(\frac{\ell+\frac{1}{2}}{k}\right)W^\kappa\left(\frac{\ell+\frac{1}{2}}{k}\right)}{\sqrt{\ell+\frac{1}{2}}\left(\ell+\frac{1}{2}\right)}P_m\left(k,\frac{\ell+\frac{1}{2}}{k},\frac{\ell+\frac{1}{2}}{k}\right)\right]$$

Derivatives of the Bessel functions are written in terms of other Bessels with different ℓ

Looking Beyond the Survey effect!

$$\chi_{\pm} = (\ell \pm 1/2)/k$$

In practice the probe combines the fields at two different distances because of the derivatives of the Bessel functions

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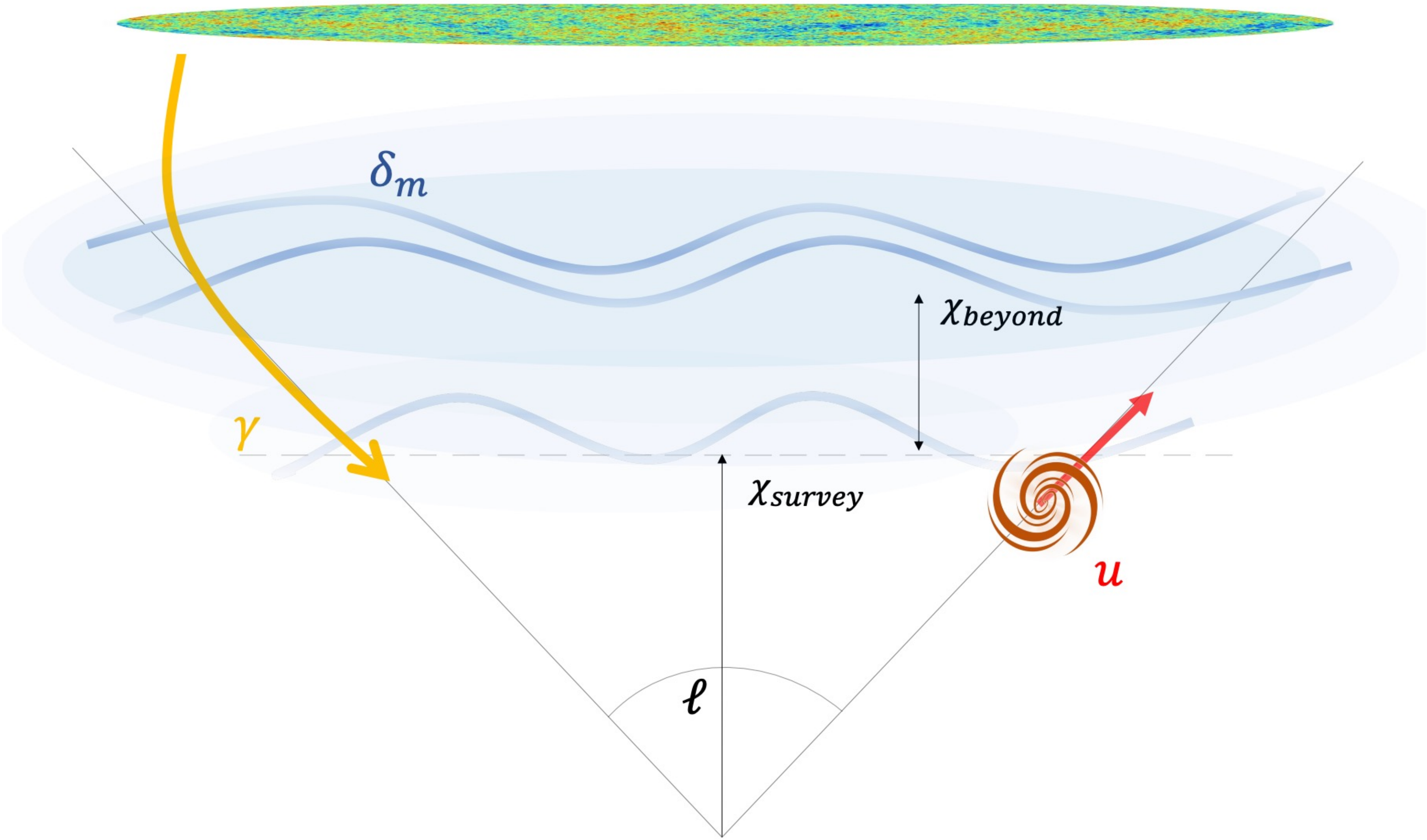
Edge of the PV Survey!

For large ℓ they almost coincide, but for smaller ones they can be very distant! E.g. for $\ell = 2$:

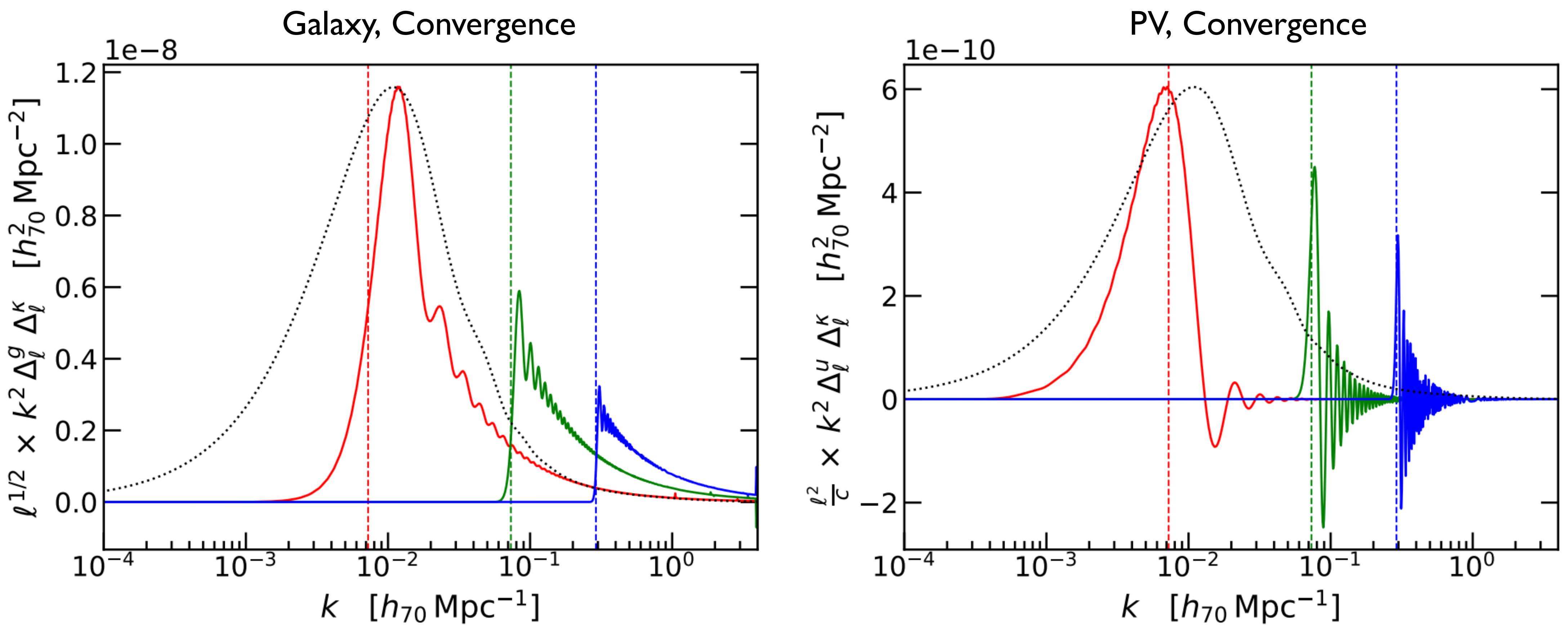
PV $$\chi_{-} \approx 300h^{-1}\mathrm{Mpc}$$

κ $$\chi_{+} \approx 500h^{-1}\mathrm{Mpc}$$

In practice PVs are sensitive to matter distribution which is beyond the PV survey itself, allowing to look *Beyond* it!



The signal is, indeed, all at the edge!



$$\ell = 4, 45, 180$$

Dashed lines are $k = \ell/\chi(z_{max})$,
with $z_{max} = 0.15$ for PV Surveys

Look at the fluctuations after the peaks!

The signal is, indeed, all at the edge!

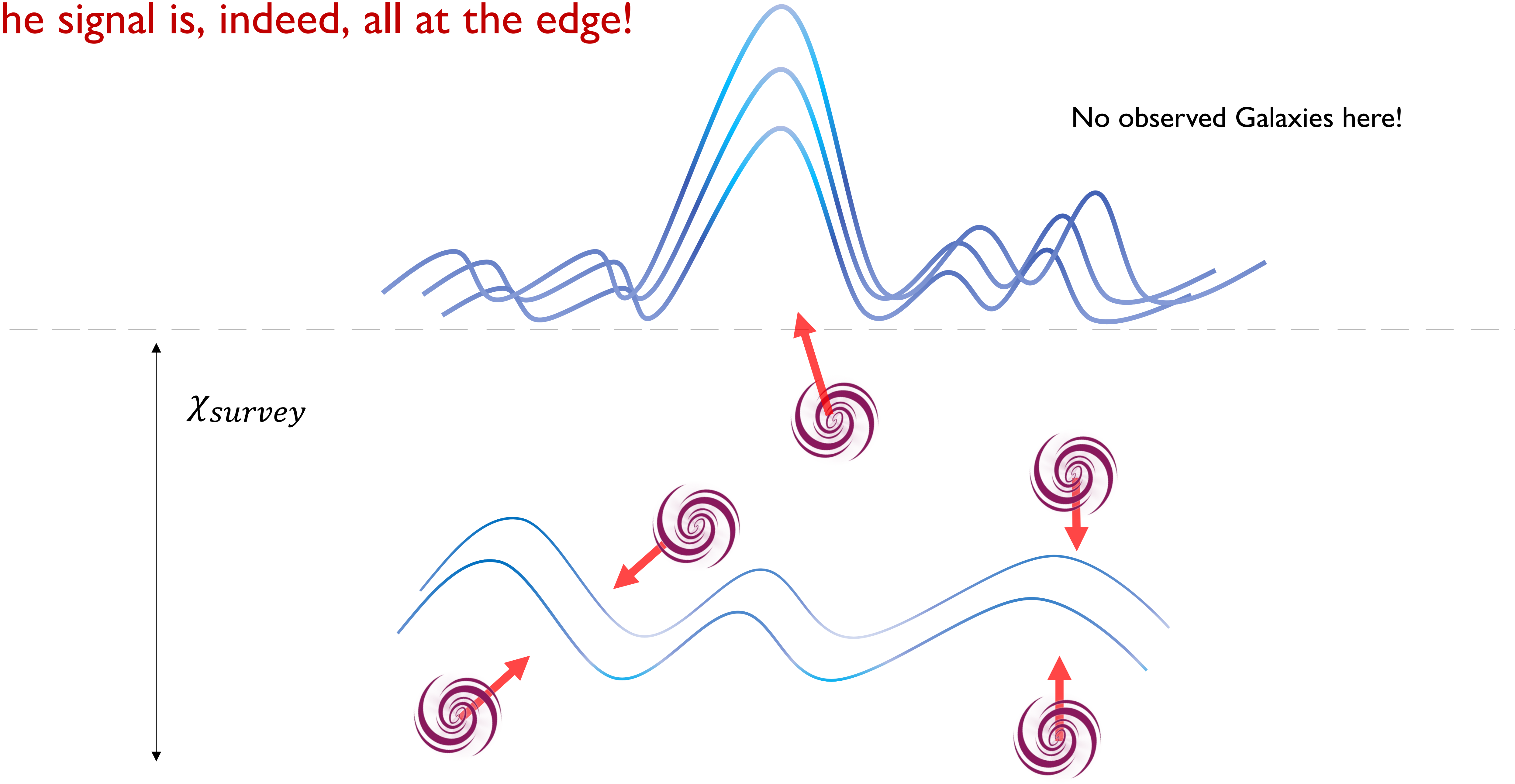
$$C_\ell^{u\kappa} \approx \int \frac{dk}{k}\sqrt{\frac{1}{\ell+\frac{1}{2}}}\left[\frac{W^u\left(\frac{\ell-\frac{1}{2}}{k}\right)W^\kappa\left(\frac{\ell+\frac{1}{2}}{k}\right)}{\sqrt{\ell-\frac{1}{2}}}P_m\left(k,\frac{\ell-\frac{1}{2}}{k},\frac{\ell+\frac{1}{2}}{k}\right)\right.$$

$$\left.-\frac{(\ell+1)\,W^u\left(\frac{\ell+\frac{1}{2}}{k}\right)W^\kappa\left(\frac{\ell+\frac{1}{2}}{k}\right)}{\sqrt{\ell+\frac{1}{2}}\left(\ell+\frac{1}{2}\right)}P_m\left(k,\frac{\ell+\frac{1}{2}}{k},\frac{\ell+\frac{1}{2}}{k}\right)\right]$$

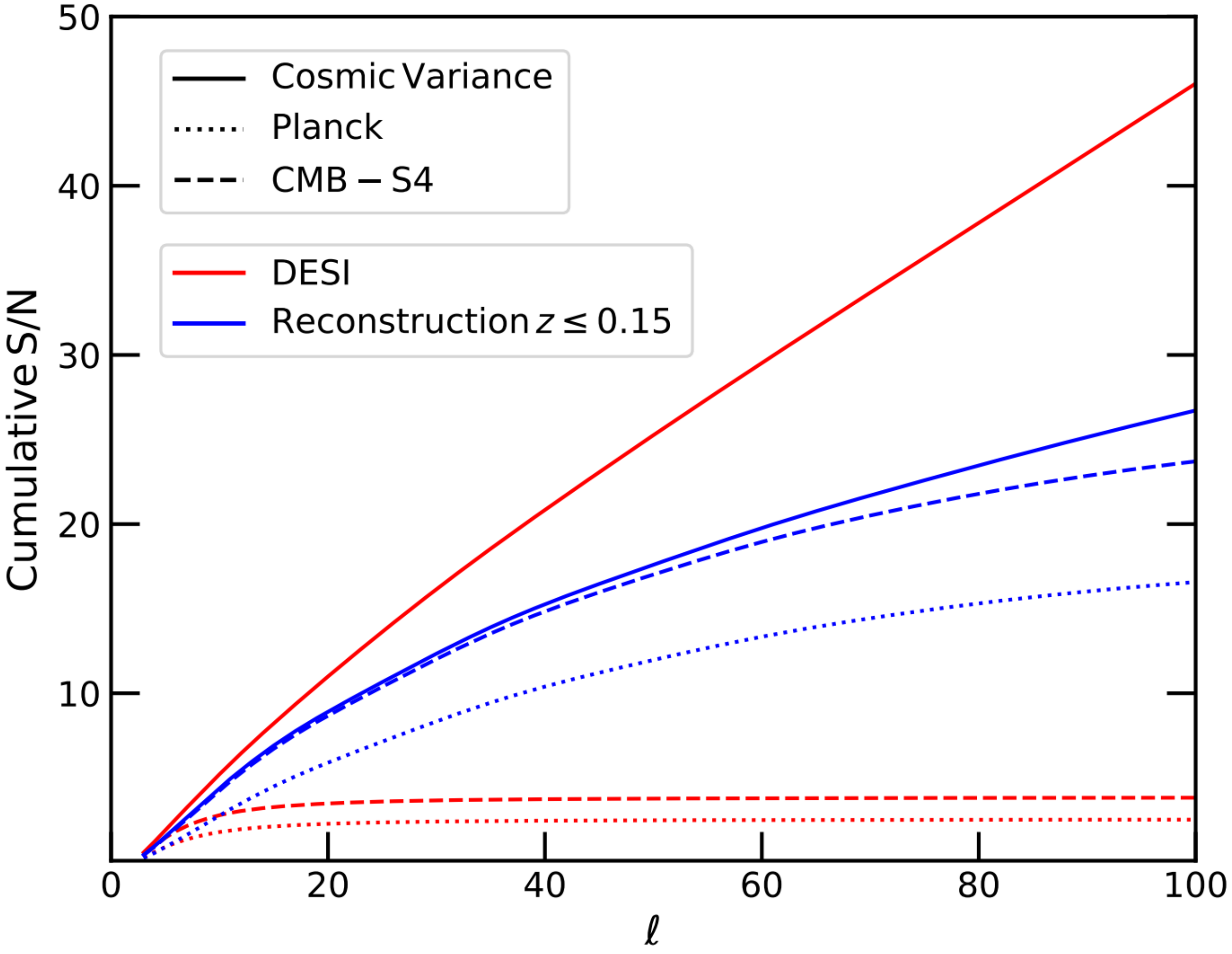
If $(\ell+\frac{1}{2})/k$ is outside the PV Survey, $W^u = 0$ and the second term disappears.

Otherwise the two terms in square brackets are close and tend to 0 (apart from very small ℓ)

The signal is, indeed, all at the edge!



Detectability

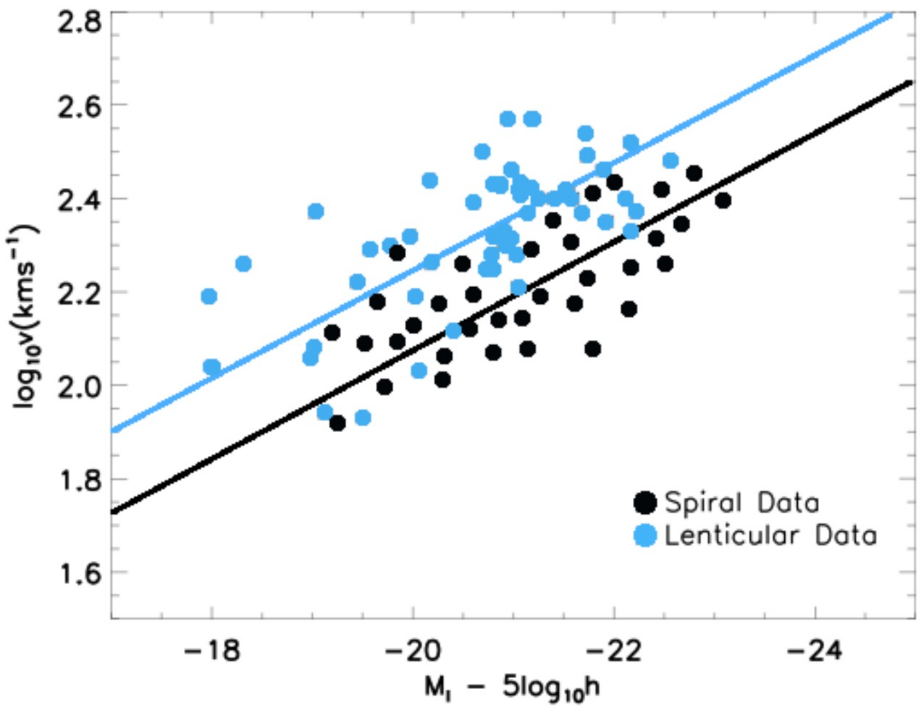


- What's all that noise?

Directly observed PV are very noisy

$$(1+\ z_{obs}) = (1+z_{cos})(1+z_{pec})(1+z_{CMB})$$

Unfortunately, to measure z_{pec} one needs to know z_{cos}, i.e. the distance to the Source with very good precision



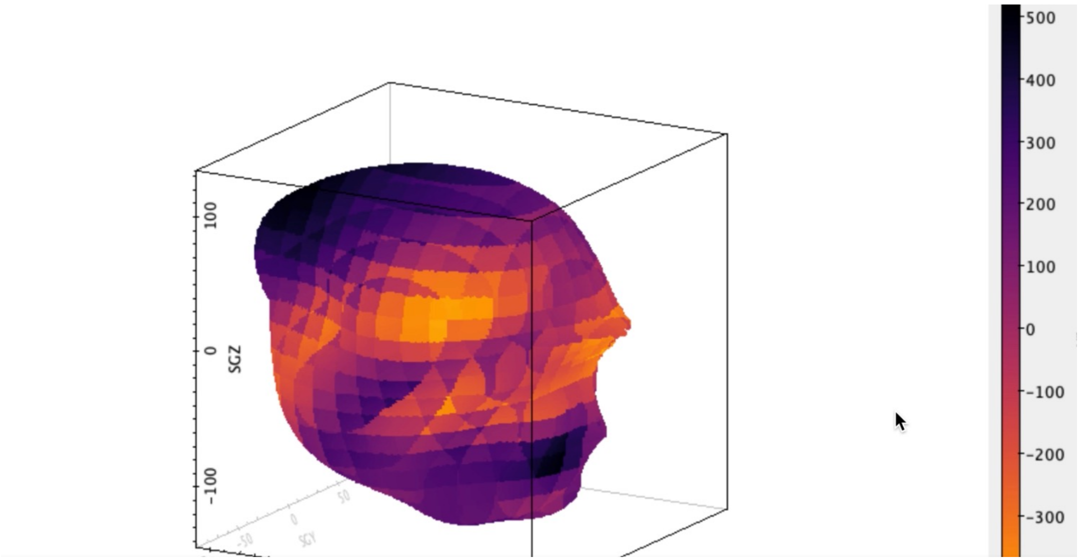
Current methods use scaling relations (Tully Fisher, Fundamental plane), which are empirical relations relating some galaxy properties to their distance.

The noise grows proportionally to (roughly 5% of) the distance to the source!

Reconstructed PV are constantly noisy

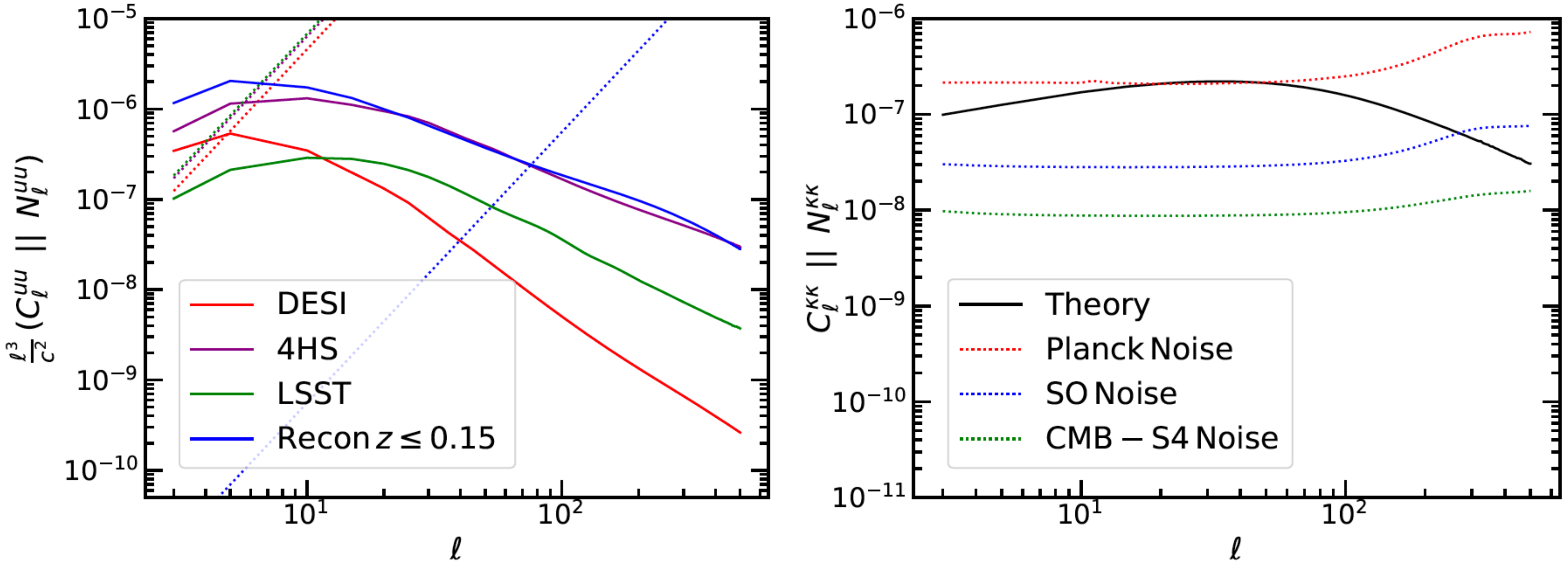
If one knows (roughly) the underlying density field, and has some directly observed PV, they can reconstruct a map for the PV field whose noise is extremely smaller (which however introduces some dependence from the galaxy bias b(z)).

The final result is a velocity map where all the volume has been divided in cells, and every cell has a velocity vector assigned, with a constant dispersion depending on the reconstruction technique (The "constant" dispersion may vary spatially if the directly observed set of PV used for the reconstruction is inhomogeneous).



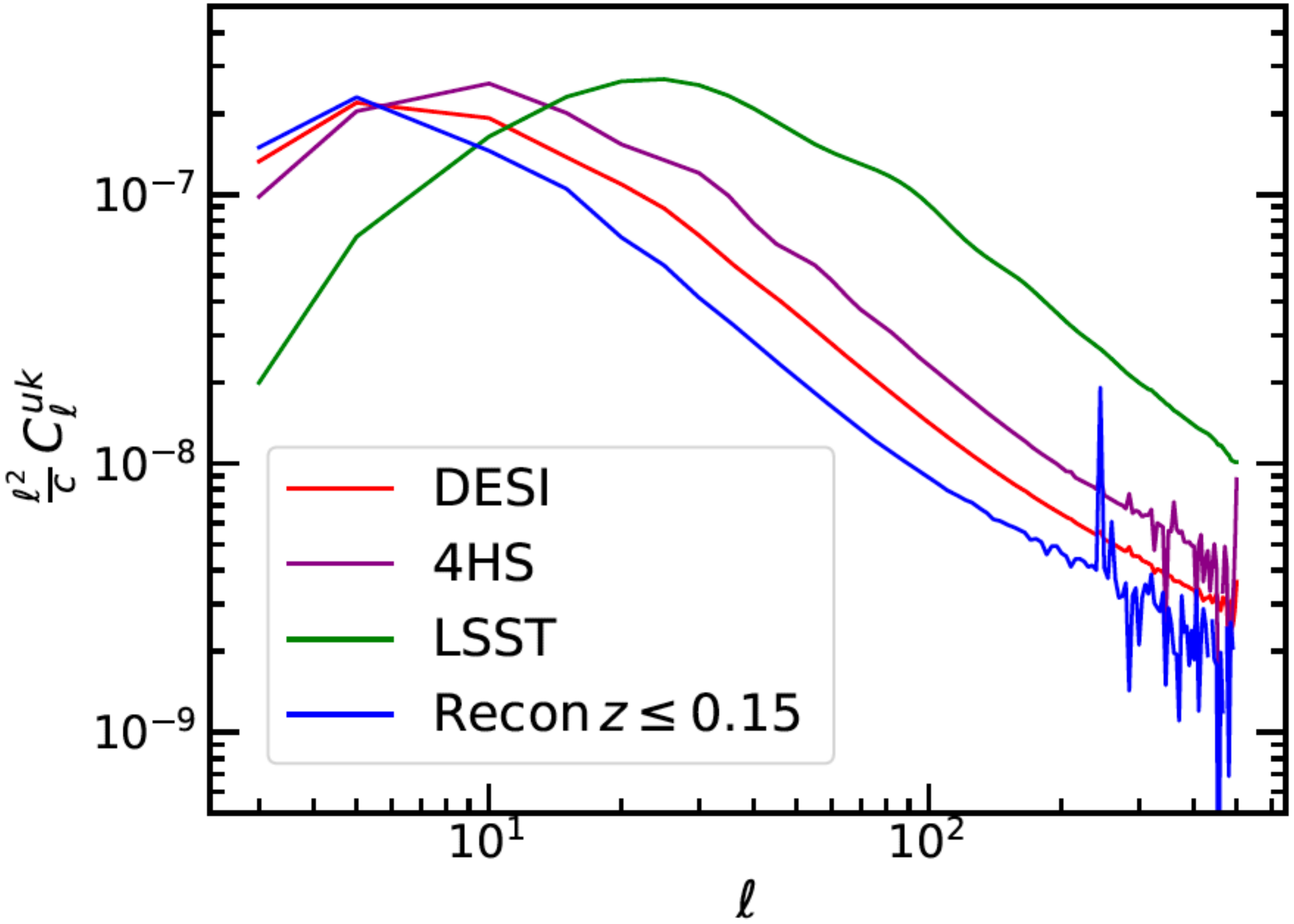
Example:
The Laniakea supercluster (our home in the cosmos) reconstructed radial velocity field.

Signals and noises for different Surveys



We see that reconstructed PV's at small ℓ have much less noise than directly observed ones.
On the other hand, CMB experiments are "Roughly" comparable

Signals and noises for different Surveys



Signal from the cross correlation for different surveys (without noise). Notice that LSST, 4HS and DESI all have more signal at large ℓ than the reconstruction (at smaller angular scales reconstructed cells are just too big)

Short (very short) summary and Conclusion:

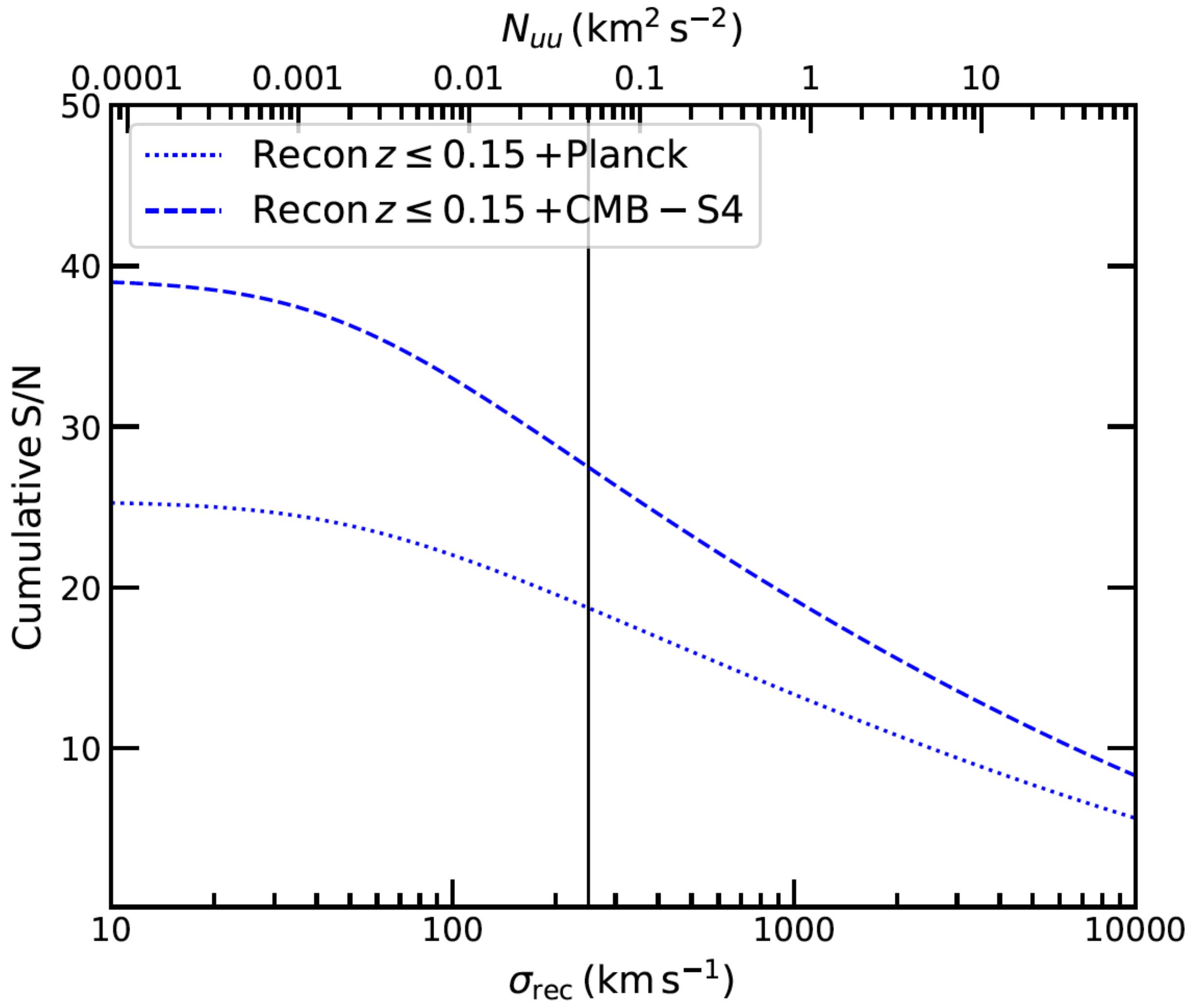
Once we get rid of the scale-dependent noise, the <PV, k> probe is a very cool one!

PV Survey \ CMB Survey	Planck ($27500\,\mathrm{deg}^2$)	SO ($16500\,\mathrm{deg}^2$)	CMB-S4 ($27500\,\mathrm{deg}^2$)
DESI ($14000\,\mathrm{deg}^2$)	2.5	3.6	3.8
4HS ($17000\,\mathrm{deg}^2$)	2.5	3.5	3.8
LSST ($18000\,\mathrm{deg}^2$)	2.4	3.2	3.5
Reconstruction $z \leq 0.15$ (Full Sky)	18.7	20.1	27.4

PV Survey \ CMB Survey	Planck ($27500\,\mathrm{deg}^2$)	SO ($16500\,\mathrm{deg}^2$)	CMB-S4 ($27500\,\mathrm{deg}^2$)
DESI ($14000\,\mathrm{deg}^2$)	1.2	1.6	1.7
4HS ($17000\,\mathrm{deg}^2$)	1.5	2.0	2.1
LSST ($18000\,\mathrm{deg}^2$)	1.9	2.5	2.7
Reconstruction $z \leq 0.15$ (Full Sky)	17.0	18.2	24.8

Short (very short) summary and Conclusion:

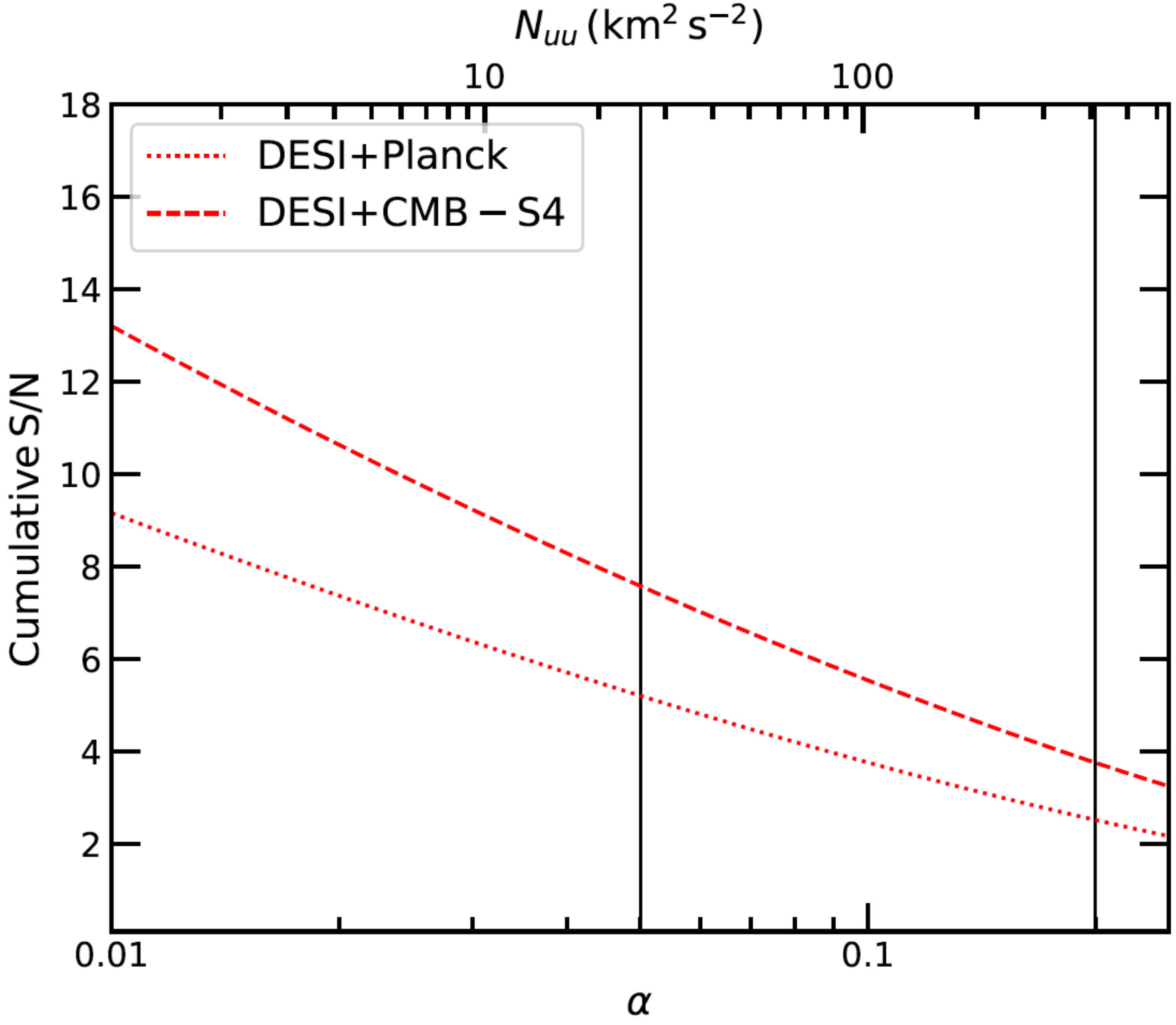
Can we do better improving on the Reconstruction technique?



Solid line is 250 km/s, used in the paper

Short (very short) summary and Conclusion:

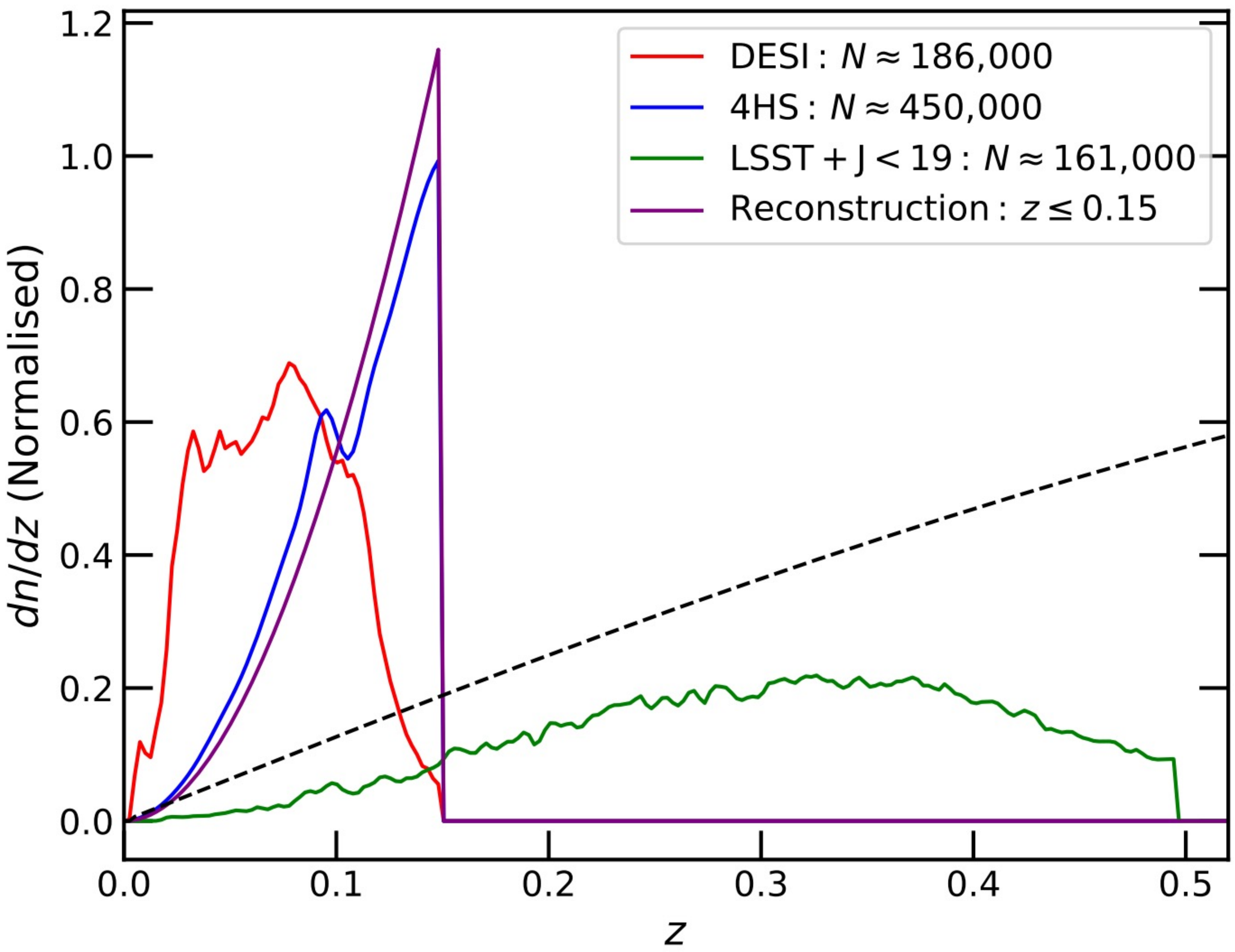
Can we do better improving on directly observed PV?



Solid line to the left is 0.2 for DESI and 4HS, right is 0.05 for LSST

Thank You!

QUESTIONS???



DESI : $N \approx 186{,}000$
4HS : $N \approx 450{,}000$
LSST + J < 19 : $N \approx 161{,}000$
Reconstruction : $z \leq 0.15$
dn/dz (Normalised)
z
0.0
0.1
0.2
0.3
0.4
0.5
0.0
0.2
0.4
0.6
0.8
1.0
1.2