Resummation in gg→h via light quarks

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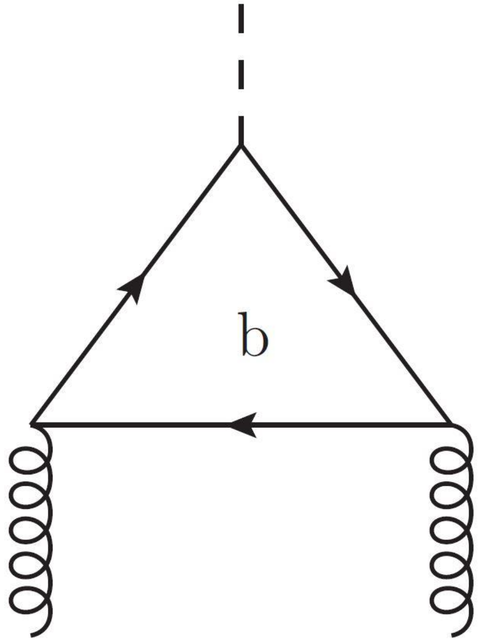
Motivation and Introduction

- Gluon fusion is dominant Higgs production channel
- Light quarks contribute ~15% to amplitude



Motivation and Introduction

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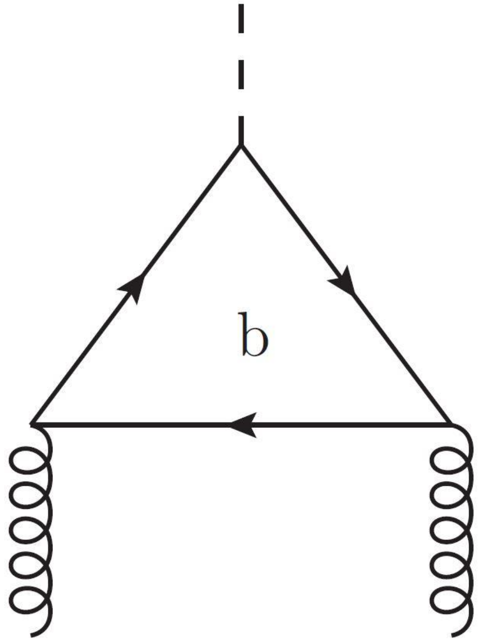
- N-loop amplitude is $\propto \alpha_s^n L^{2n}\,, \quad L = \ln\left(\frac{-M_h^2}{m_b^2}\right)$

➢ $\alpha_s L^2 > 1$ → perturbation series does not converge

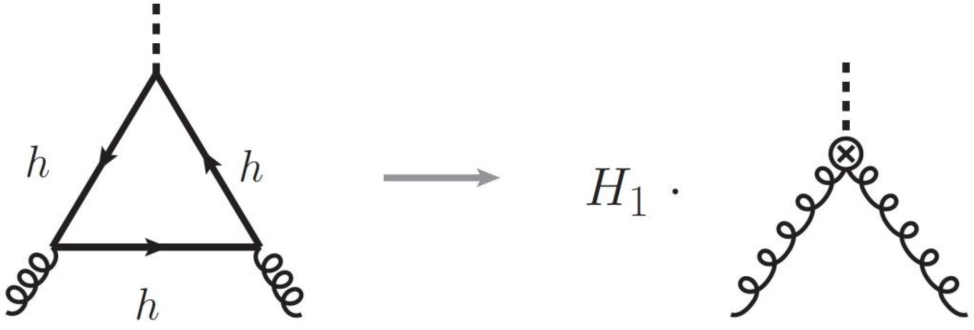
➢ Need to resum logarithmic corrections to all order of perturbation theory

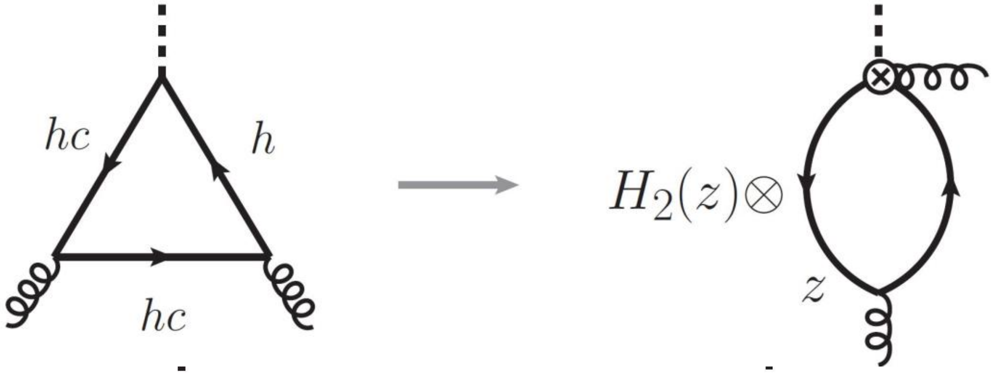
➢ Apply soft-collinear effective theory (SCET)

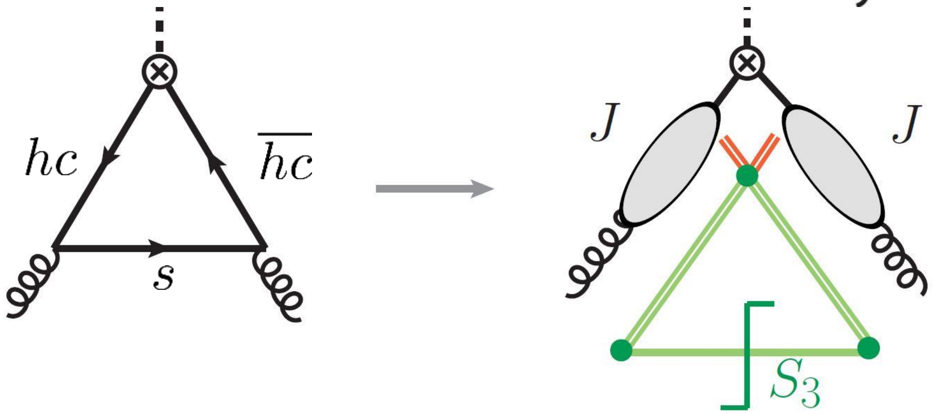


Factorisation theorem

- Amplitude factorises into three terms







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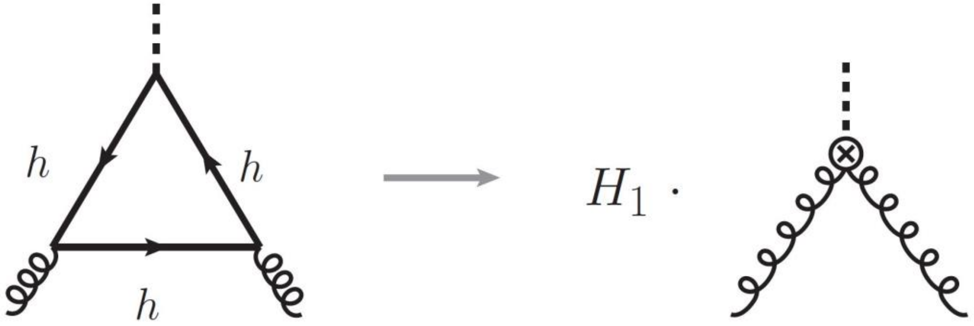

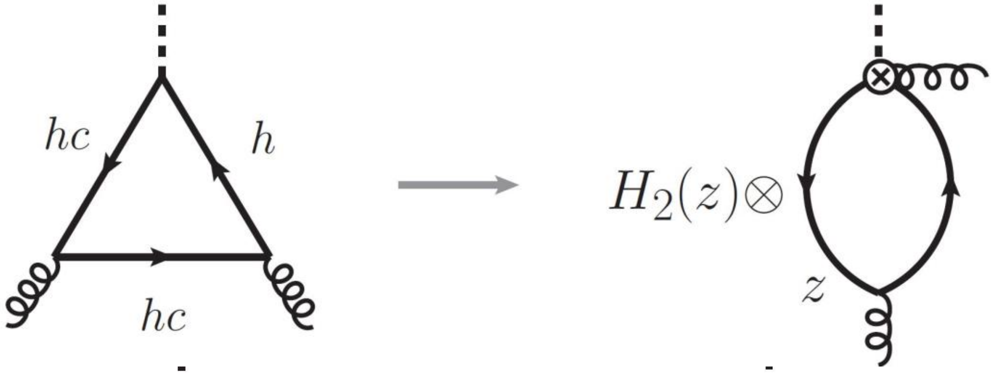

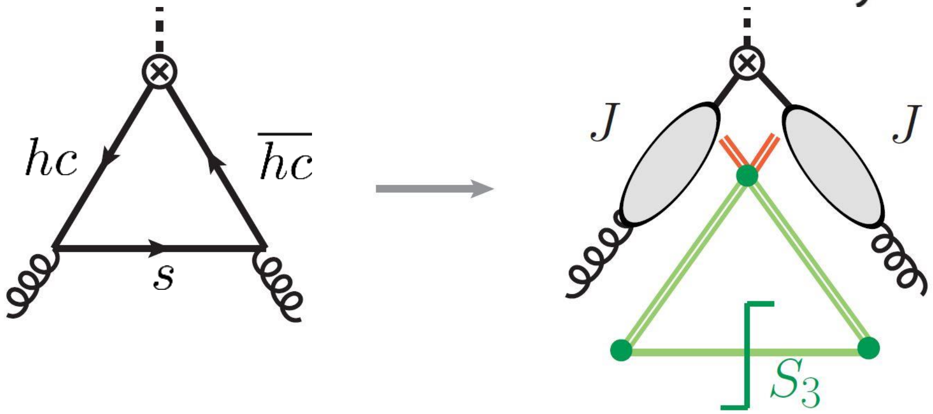

$$F_{gg}^{(0)} = \Big(H_1^{(0)} \qquad \Big) S_1 + 4\int_0^1 \frac{\mathrm{d}z}{z}\Big(\bar{H}_2^{(0)}(z)\, S_2^{(0)}(z) \qquad \Big)$$

$$+ \quad H_3^{(0)} \int_0^\infty \frac{\mathrm{d}\ell_-}{\ell_-}\int_0^\infty \frac{\mathrm{d}\ell_+}{\ell_+} J^{(0)}(-M_h\ell_-)\,J^{(0)}(M_h\ell_+)\,S_3^{(0)}(\ell_-\ell_+)\Big|_{\text{leading power}}$$

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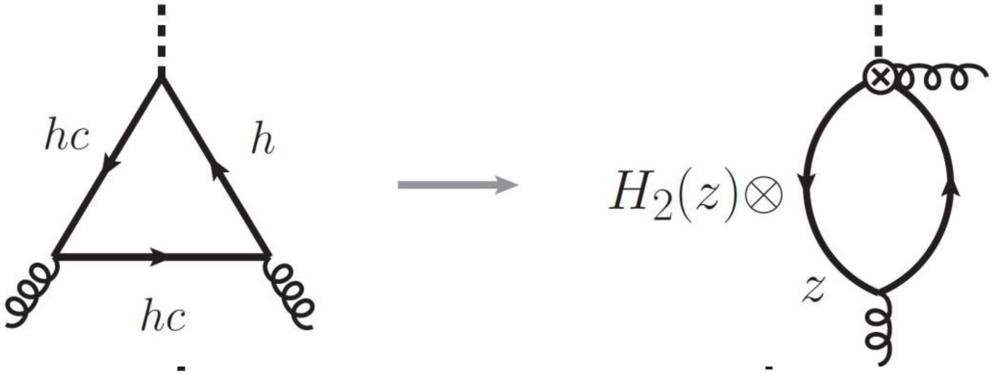

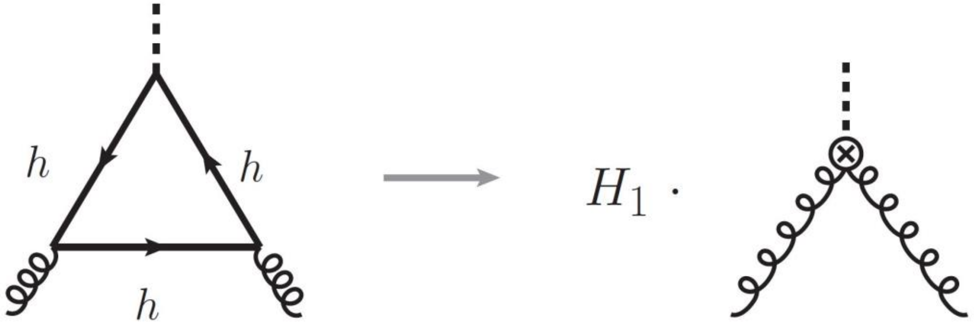

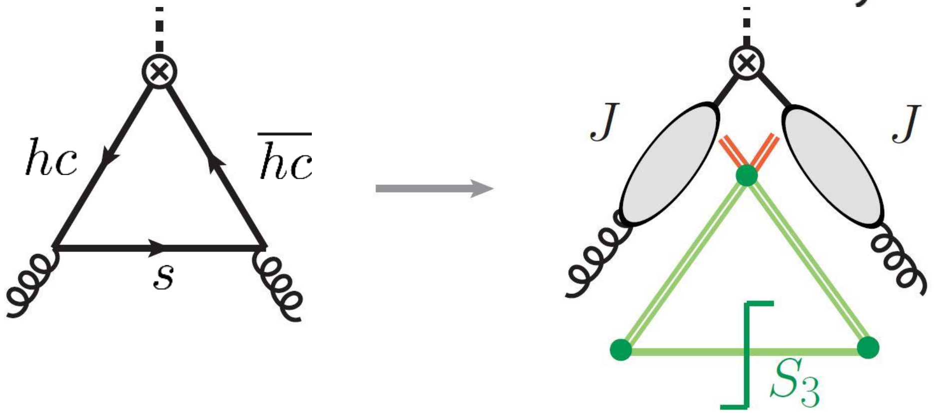

$$F_{gg}^{(0)} = \left(H_1^{(0)} + \Delta H_1^{(0)}\right) S_1 + 4\int_0^1 \frac{\mathrm{d}z}{z}\left(\bar{H}_2^{(0)}(z) S_2^{(0)}(z) - [\![\bar{H}_2^{(0)}(z)]\!][\![S_2^{(0)}(z)]\!]\right)$$

$$+ \lim_{\sigma\to -1} H_3^{(0)} \int_0^{M_h} \frac{\mathrm{d}\ell_-}{\ell_-} \int_0^{\sigma M_h} \frac{\mathrm{d}\ell_+}{\ell_+} J^{(0)}(-M_h \ell_-) J^{(0)}(M_h \ell_+) S_3^{(0)}(\ell_- \ell_+) \Big|_{\text{leading power}}$$

Resummation of large logarithms

- Renormalisation is non-trivial (operator mixing, regularisation and renormalisation do not commute)
- Derive RG equations and solve them to derive resummed formula

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special functions

$$F_{gg}(gg \to h)^{\text{RGi-LO}} = \mathcal{M}_0(\mu_h)\frac{L^2}{2}\Bigg\{F_1(\rho) + \frac{1}{L}\left[\left(\frac{23}{3}\rho - 18\right)F_3(\rho) - \frac{27}{10}\right]$$

$$+ \frac{1}{L^2}\left[\frac{529}{1600}\rho^2 + \frac{25}{16}\rho - \left(\frac{11}{60}\pi^2 + \frac{1577}{300}\right)\rho F_1(\rho) - \frac{327}{40}\rho F_2(\rho)\right.$$

$$\left. + \left(-\frac{529}{1600}\rho^2 + \frac{1909}{480}\rho + \frac{3\pi^2}{5} + \frac{8243}{600}\right)\rho F_3(\rho)\right]\Bigg\}$$

$$\rho = \frac{\alpha_s}{2\pi}\left(C_F - C_A\right)L^2$$

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Known from trad. QCD

Infer from h→γγ

New!

$$\rho=\frac{\alpha_s}{2\pi}\left(C_F-C_A\right)L^2$$

Thank you for your attention!

Special functions in the resummed amplitude

$$F_1(z) = {}_2F_2\left(1, 1; \frac{3}{2}, 2; -\frac{z}{4}\right)$$

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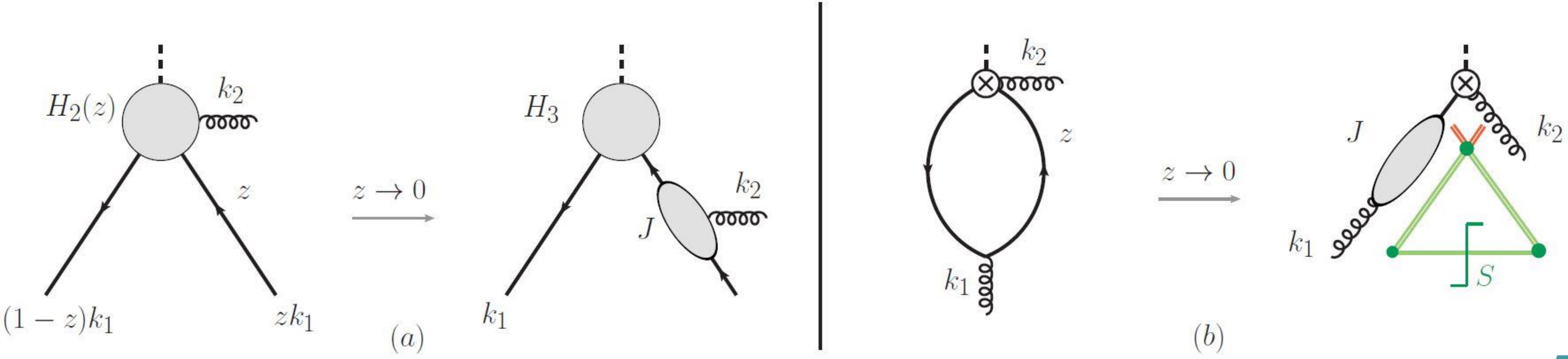
hypergeometric functions

$$F_3(z) = \frac{1}{\sqrt{z}} D(z) = \frac{1}{\sqrt{z}} e^{-z^2} \int_0^z \mathrm{d}x e^{x^2}$$

Dawson integral function

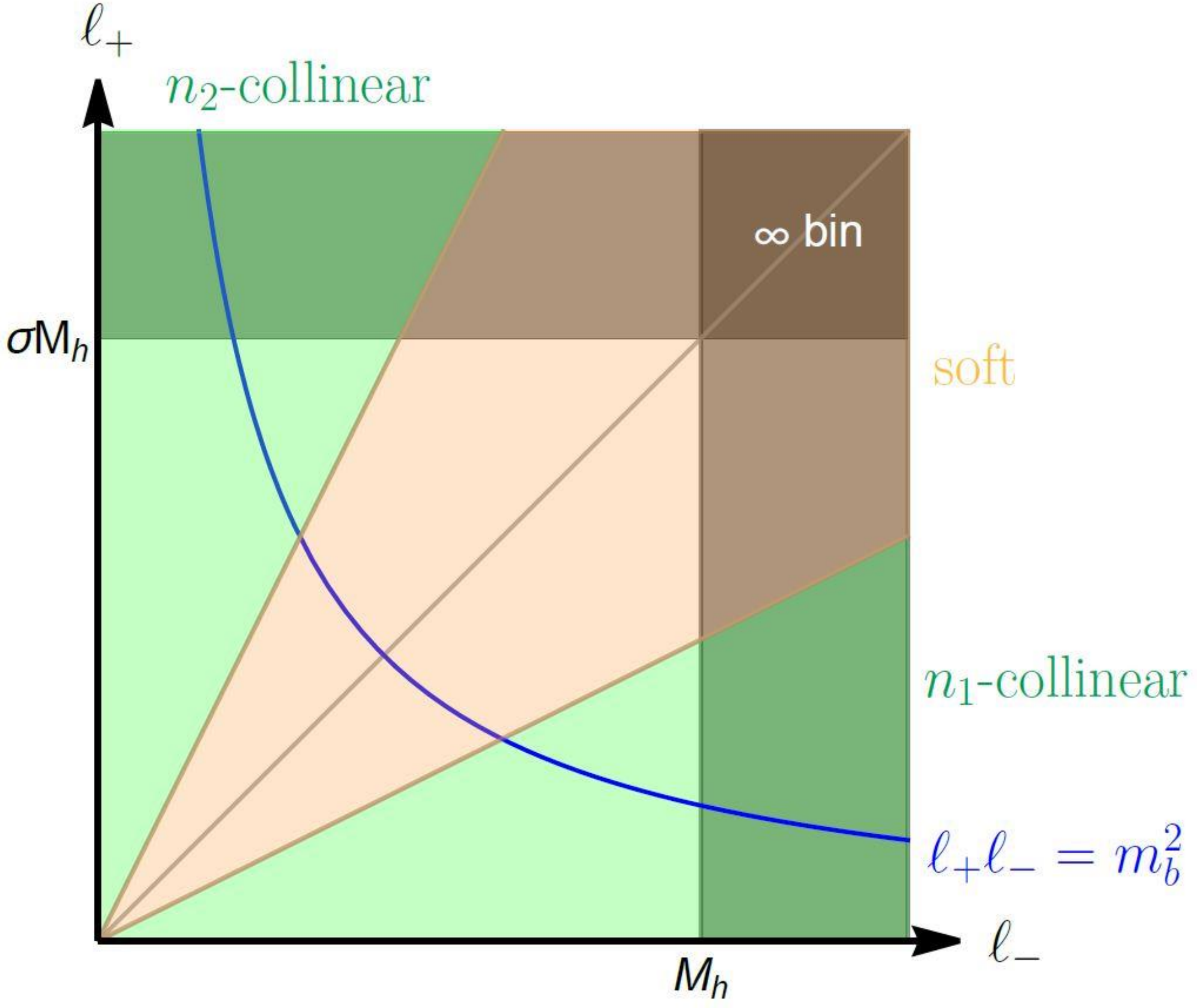
Refactorisation theorems



$$(a) \quad [\![\bar{H}_2(z)]\!] = H_3^{(0)} J^{(0)}(z M_h^2)$$

$$(b) \quad [\![S_2^{(0)}(z)]\!] = -\frac{1}{2} \int_0^\infty \frac{\mathrm{d}\ell_+}{\ell_+} J^{(0)}(-M_h \ell_+) S_3^{(0)}(z M_h \ell_+)$$

Regularisation of endpoint divergences



Prediction of large logs at higher loops

- Solving the RGEs iteratively, we can predict the leading logs in the 3-loop amplitude:

$$
\begin{aligned}
F_{gg}(\hat{\mu}_h) = T_F \delta_{ab} \frac{\alpha_s(\hat{\mu}_h)}{\pi} \frac{m_b^2}{v} \Bigg\{ -2 + \frac{L^2}{2} + \frac{\alpha_s(\hat{\mu}_h)}{4\pi} \Bigg[\frac{C_A - C_F}{12} L^4 - C_F L^3 \\
+ \left(\left(1 + \frac{5\pi^2}{12} \right) C_A - \frac{2\pi^2}{3} C_F \right) L^2 + \left(\left(12 + \frac{2\pi^2}{3} + 16\zeta_3 \right) C_F - 12\zeta_3 C_A \right) L \\
+ \left(4\zeta_3 - \frac{\pi^4}{5} - 20 \right) C_F + \left(12\zeta_3 + \frac{8\pi^4}{45} - \frac{\pi^2}{3} - 12 \right) C_A \Bigg] \\
+ \left(\frac{\alpha_s(\hat{\mu}_h)}{4\pi} \right)^2 \Bigg[\frac{(C_A - C_F)^2}{90} L^6 + (C_A - C_F) \left(\frac{\beta_0}{30} - \frac{C_F}{10} \right) L^5 \\
+ d_4^{\rm OS} L^4 + d_3^{\rm OS} L^3 + \cdots \Bigg] \Bigg\},
\end{aligned}
$$