Naturalness, Gegenbauer polynomials, and the Higgs self-coupling

Gauthier Durieux
(CERN)

Gegenbauer Goldstones, JHEP 01 (2022) 076, [2110.06941]
Gegenbauer's Twin, JHEP 05 (2022) 140, [2202.01228]
Charting the Higgs self-coupling boundaries, JHEP 12 (2022) 148, [2209.00666]

with Matthew McCullough and Ennio Salvioni

Moriond EW – 23 March 2023

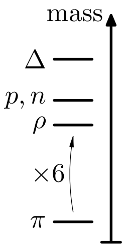
The Higgs at 10

Ten years of LHC measurements
exacerbated the naturalness puzzle.

Small mass?

SM-like couplings?

Composite Higgs



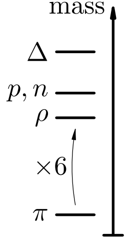
realise the Higgs as the
pseudo-Nambu-Goldstone boson (pNGB)
of a new strong sector

e.g. global SO(5) $\rightarrow$ SO(4) spontaneous breaking
at scale f

small mass obtained from the
explicit breaking of SO(5)
by e.g. the SM

SM-like couplings require $v^2/f^2 \ll 1$
which requires tuning in minimal models

Composite Higgs



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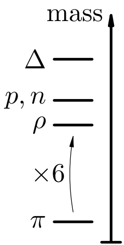

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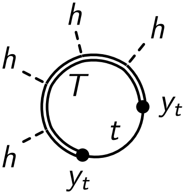
small vev?

Minimal composite Higgs

$V(h) \sim$

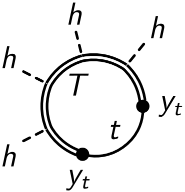


$$+ \cdots \sim \boldsymbol{\kappa}\, \frac{y_t^2 N_c}{16\pi^2} f^2 M_T^2 \left(-\sin^2\frac{h}{f} + \boldsymbol{\delta}\, \sin^4\frac{h}{f} \right)$$

$$\rightarrow \frac{v^2}{f^2} = \sin^2\frac{\langle h \rangle}{f} = \frac{1}{2\delta} \qquad \text{vs.} \qquad \frac{c_{hVV}}{c_{hVV}^{\rm SM}} = \sqrt{1-\frac{v^2}{f^2}} \gtrsim 0.94$$

$$\rightarrow m_h^2 = \kappa\, \frac{y_t^2 N_c}{4\pi^2} M_T^2 \left(1 - \frac{1}{2\delta}\right) \qquad \text{vs.} \qquad M_T \gtrsim 1.3\,\text{TeV}$$

Minimal composite Higgs

$V(h) \sim$ 

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$1/\delta \lesssim 0.23$

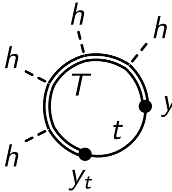
$\rightarrow m_h^2 = \kappa\, \frac{y_t^2 N_c}{4\pi^2} M_T^2 \left(1 - \frac{1}{2\delta}\right)$

$\kappa \lesssim 0.14$

$M_T \gtrsim 1.3\,\text{TeV}$

Few percent tuning wrt $1/\delta \gtrsim 1$, $\kappa \simeq 1$ expectation

Minimal composite Higgs

$V(h) \sim$ 

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Gegenbauer fix

$1/\delta \lesssim 0.23$

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$\kappa \lesssim 0.14$

Twin-Higgs fix

Few percent tuning wrt $1/\delta \gtrsim 1$, $\kappa \simeq 1$ expectation

Naturally small vev

$$=$$

radiatively stable potential
with deepest minimum close to the origin

agnostic of the UV

considering the SO(5) $\to$ SO(4) pNGB EFT

with $\vec{\phi} = (\frac{\vec{h}}{h} \sin \frac{h}{f},\ \cos \frac{h}{f})$ where $h \equiv |\vec{h}|$

Radiatively stable potentials

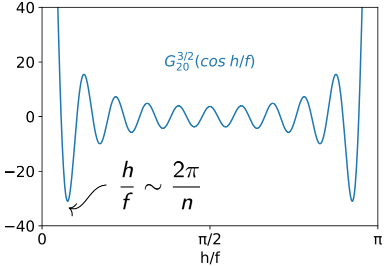
Explicit SO(5) $\rightarrow$ SO(4) breaking by an irrep spurion
(symmetric traceless)

$$K^{i_1 \dots i_n} \, \phi_{i_1} \cdots \phi_{i_n}$$

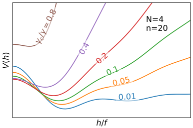
No other invariant, linear in K, can be constructed,
so all-loop linear renormalisation can only be multiplicative.

Obtain Gegenbauer polynomials:

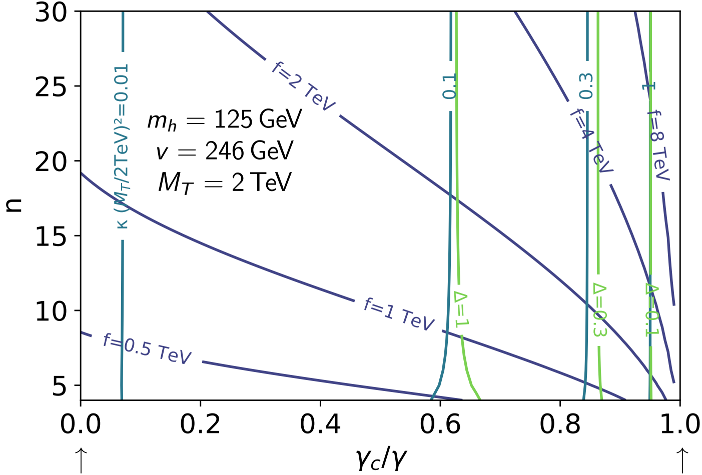
$$K^{i_1 \dots i_n} \, \phi_{i_1} \cdots \phi_{i_n} \;=\; G_n^{3/2}(\cos \tfrac{h}{f})$$



The Gegenbauer Higgs



$$V(h) = \boldsymbol{\kappa} \frac{N_c y_t^2}{16\pi^2} f^2 M_T^2 \left[\sin^2 \frac{h}{f} + \gamma\, G_n^{3/2}\left(\cos \frac{h}{f}\right) \right]$$



↑ pure Gegenbauer $v/f \approx 5.1/n$

↑ no EWSB $v/f \to 0$

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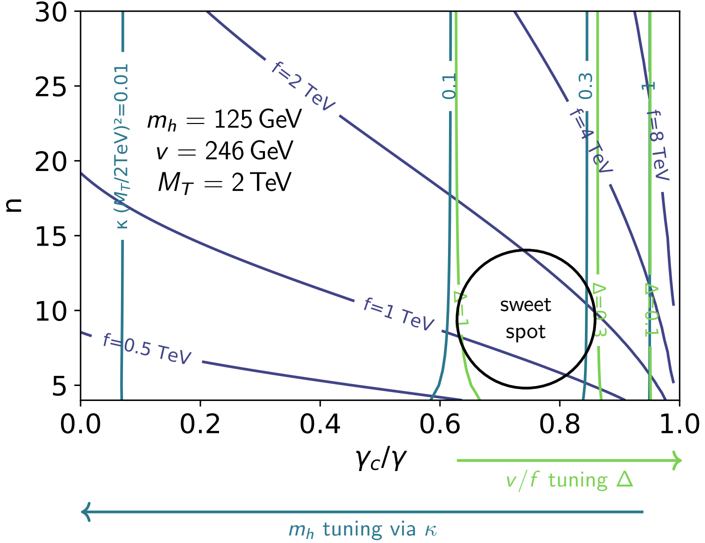

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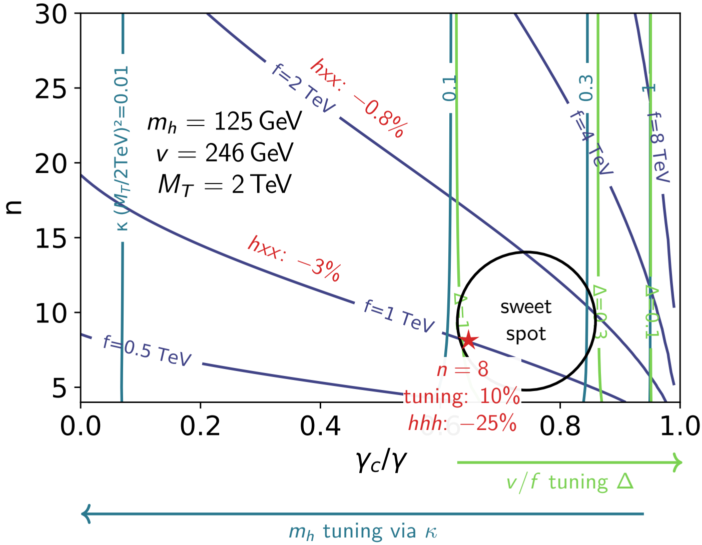

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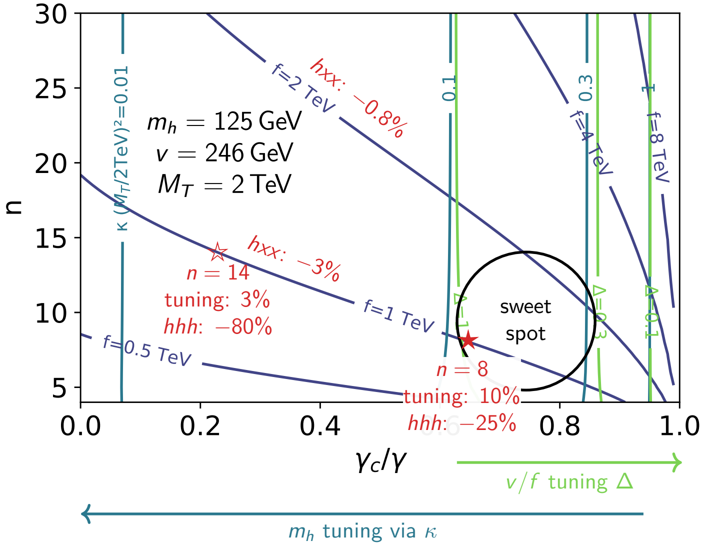
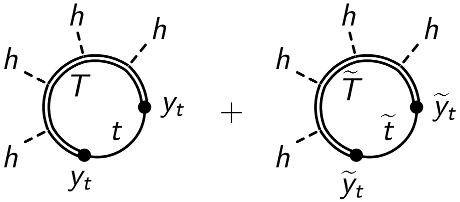


Adding twins

[Chacko, Goh, Harnik '05]
[Barbieri, Greco, Rattazzi, Wulzer '15]



$$\frac{N_c y_t^2}{16\pi^2}\, f^2 M_T^2\, \sin^2\frac{h}{f} \quad + \quad \frac{N_{\widetilde{c}} \widetilde{y}_t^2}{16\pi^2}\, f^2 M_{\widetilde{T}}^2\, \cos^2\frac{h}{f}$$

no M_T^2 sensitivity
if twin parity enforces $y_t = \widetilde{y}_t$ and $M_T = M_{\widetilde{T}}$

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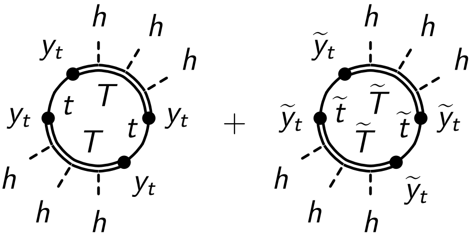


$$\frac{N_c y_t^4}{16\pi^2}\; f^4\; \sin^4\frac{h}{f}\;\; \log M_T \quad + \quad \frac{N_c \widetilde{y}_t^4}{16\pi^2}\; f^4\; \cos^4\frac{h}{f}\;\; \log M_{\widetilde{T}}$$

retaining $\log M_T$ sensitivity only

Gegenbauer's Twin

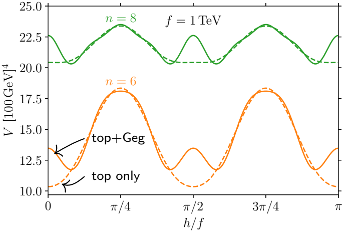
L.B. Gegenbauer
1849–1903

- global $\mathrm{SO}(8) \supset \mathrm{SO}(4) \times \widetilde{\mathrm{SO}(4)}$

- spontaneous $\mathrm{SO}(8) \to \mathrm{SO}(7)$
  7 NGBs
  6 eaten by $W^{\pm}, Z$ and $\widetilde{W}^{\pm}, \widetilde{Z}$
  1 Higgs: $\vec{\phi} = (\vec{0}_3,\ \sin\frac{h}{f};\ \ \vec{0}_3,\ \cos\frac{h}{f})^T$ in unitary gauge

- minimal explicit breaking is insufficient

- explicit $\mathrm{SO}(8) \to \mathrm{SO}(4) \times \widetilde{\mathrm{SO}(4)}$
  radiative stability from irrep spurion
  $G_n^{3/2}(\cos\frac{2h}{f})$ potential



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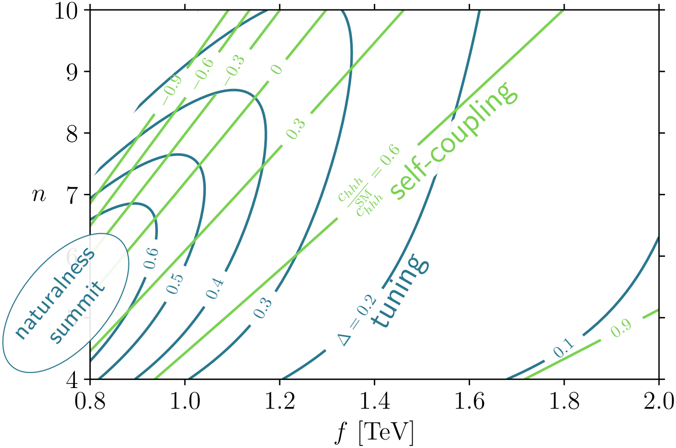

HL-LHC 2σ reach: hVV: 2.6%

hhh: 100%

[ECFA report '19]

(and possibly large M_T, with unitarity violating H scattering towards 6 TeV)

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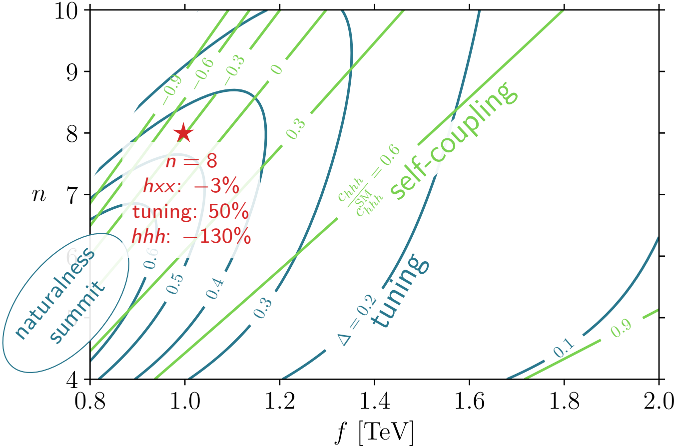

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Structurally large *hhh*/*hxx*

see also: [Di Luzio, Gröber, Spannowsky '17]
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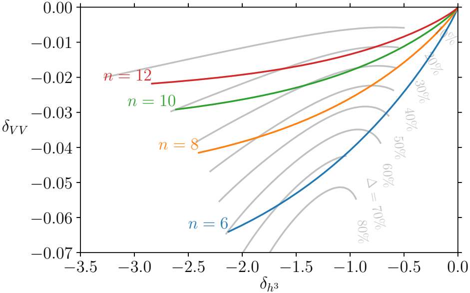
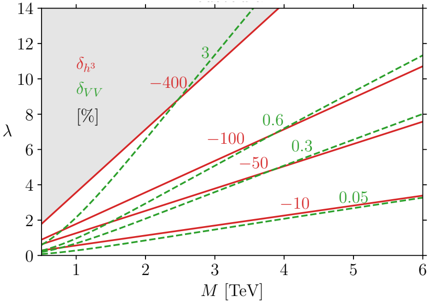


custodial weak-quadruplet scalar



$$\lambda\, H^* H^* (\epsilon H)\Phi + \lambda \frac{1}{\sqrt{3}} H^* H^* H^* \widetilde{\Phi}$$

- a loop factor allowed dimensionally (or v^2/M^2 if dim-6/dim-8)
- dim≫6 operators may be very relevant
- vacuum stability constrains large self-couplings

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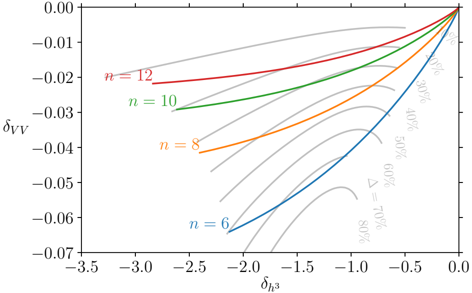
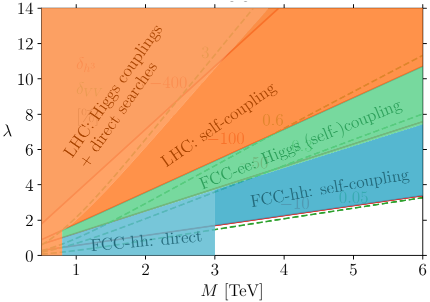


custodial weak-quadruplet scalar



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Naturalness, Gegenbauer polynomials, and the Higgs self-coupling

Gegenbauer potentials are eigenfunctions of linear renorm.
for $\mathrm{SO}(N+1) \to \mathrm{SO}(N)$ pNGBs
(on spherical manifolds, more generally).

Explicit global symmetry breakings in non-minimal irreps
lead to naturally small pNGB Higgs vev's.

Structurally large hhh/hxx ratios in coupling deviations occur,
making self-coupling measurements essential probes.

Extras

Radiatively stable potentials (II)

Linear one-loop correction to $V(\frac{h}{f})$:

$$\frac{\Lambda^2}{32\pi^2 f^2}\left(\underline{V'' + 3\cot\frac{h}{f}\ V'}\right)$$

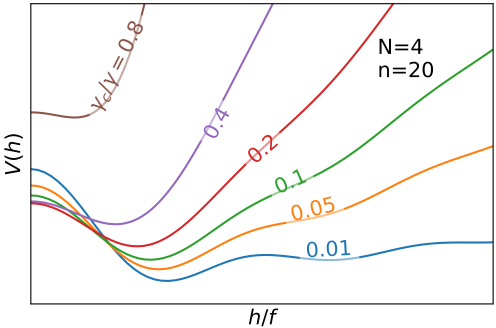
Radiative stability at one-loop and linear order order if $\propto V$

Differential equation of Gegenbauer polynomials

$$V(\tfrac{h}{f}) \propto G_n^{3/2}(\cos\tfrac{h}{f})$$

Gegenbauer Higgs potential

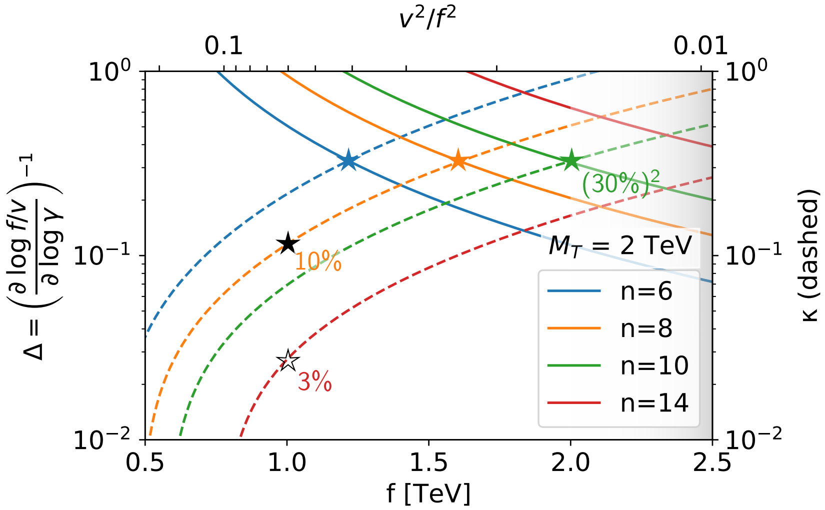
$$V(h) = \kappa \frac{N_c y_t^2}{16\pi^2} f^2 M_T^2 \left[\sin^2 \frac{h}{f} + \gamma\, G_n^{3/2}\left(\cos \frac{h}{f} \right) \right]$$



$$v/f \searrow 0 \quad \text{as} \quad \gamma \searrow \gamma_c$$

$$\frac{m_h^2}{\kappa \frac{N_c y_t^2}{16\pi^2} M_T^2} \searrow \quad \text{as} \quad \gamma \searrow \gamma_c \quad \text{relaxing} \quad \kappa \nearrow 1$$

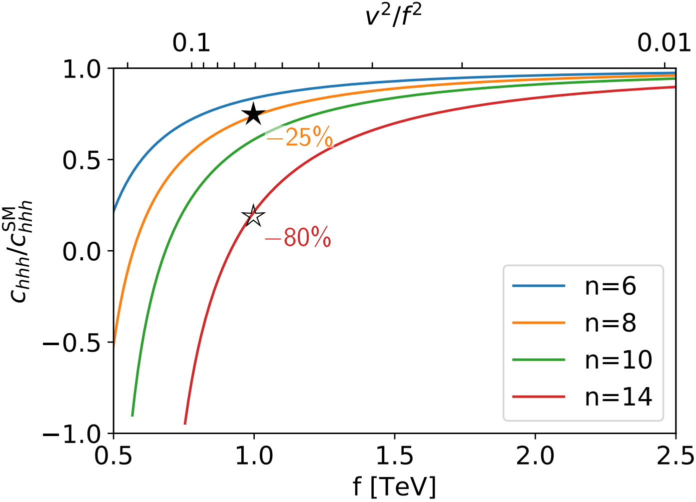
Gegenbauer Higgs tunings



$(M_T \gtrsim y_t f/\sqrt{2}$ since $m_t \sim M_T v/f$ for $y_t f \gg M_T)$

$$\Delta \approx 30\% \left(\frac{f}{4v}\frac{5.1}{n}\right)^{-2.1} \qquad \kappa \approx 30\% \left(\frac{f}{4v}\frac{5.1}{n}\frac{2\,\text{TeV}}{M_T}\right)^{2}$$

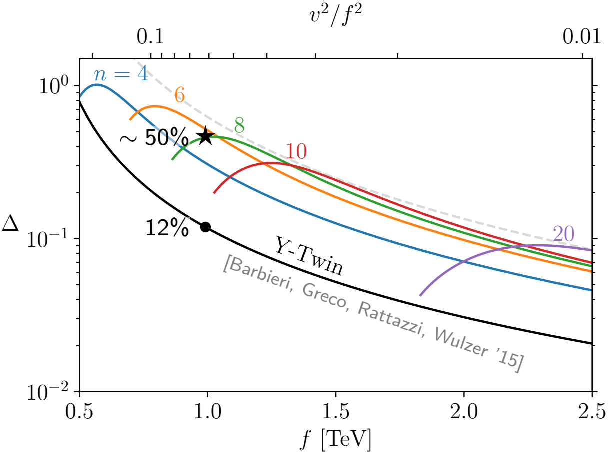
Gegenbauer Higgs self-coupling



$$\frac{c_{hhh}}{c_{hhh}^{\mathrm{SM}}} \approx 1 - 1.2 \left(\frac{f}{v} \frac{5.1}{n + \lambda} \right)^{-2}$$ when close to 1

Gegenbauer's Twin tunings

- conservative definition RMS(eig. log-derivative matrix)
- dominated by top-sector dependence of v/f
- about 4 times better than usual $\Delta \approx 2v^2/f^2$ minimum



Gegenbauer's Twin self-coupling