European Union
European Regional
Development Fund

Investing
in your future

Keemilise ja
Bioloogilise Füüsika Instituut
National Institute of Chemical Physics and Biophysics

Testing Bell Inequalities and entanglement with di-boson final states

Luca Marzola
luca.marzola@cern.ch

Based on: "*Bell inequalities and quantum entanglement in weak gauge bosons production at the LHC and future colliders*",
M. Fabbrichesi, R. Floreanini, E. Gabrielli, LM. — *arXiv: 2302.00683*

Rencontres de Moriond 2023 — Electroweak Interactions & Unified Theories

Tired of waiting for S**Y? Buy this now!

I am selling WW, WZ and ZZ final states because they can be used to study interesting physics.

Tired of waiting for S**Y? Buy this now!

I am selling WW, WZ and ZZ final states because they can be used to study interesting physics.

- Why these? Massive spin-1 objects have 3 polarization modes, hence are quantum 3-level systems — *qutrits*. These are rarer than qubits which are commonly used in these studies (e, γ, t…)

Tired of waiting for S**Y? Buy this now!

I am selling WW, WZ and ZZ final states because they can be used to study interesting physics.

- Why these? Massive spin-1 objects have 3 polarization modes, hence are quantum 3-level systems — *qutrits*. These are rarer than qubits which are commonly used in these studies (e, γ, t…)
- What can you do with these? At least 3 things:
  - Test quantum mechanics with *Bell inequalities* in a novel setting

Tired of waiting for S**Y? Buy this now!

I am selling WW, WZ and ZZ final states because they can be used to study interesting physics.

- Why these? Massive spin-1 objects have 3 polarization modes, hence are quantum 3-level systems — *qutrits*. These are rarer than qubits which are commonly used in these studies (e, γ, t…)
- What can you do with these? At least 3 things:
  - Test quantum mechanics with *Bell inequalities* in a novel setting
  - Test the presence of/quantify *entanglement* in a novel setting

Tired of waiting for S**Y? Buy this now!

I am selling WW, WZ and ZZ final states because they can be used to study interesting physics.

- Why these? Massive spin-1 objects have 3 polarization modes, hence are quantum 3-level systems — *qutrits*. These are rarer than qubits which are commonly used in these studies (e, γ, t…)

- What can you do with these? At least 3 things:
  - Test quantum mechanics with *Bell inequalities* in a novel setting
  - Test the presence of/quantify *entanglement* in a novel setting
  - Test the *Standard Model* through new observables*

**preliminary results, use at your own risk*

Tired of waiting for S**Y? Buy this now!

I am selling WW, WZ and ZZ final states because they can be used to study interesting physics.

- Why these? Massive spin-1 objects have 3 polarization modes, hence are quantum 3-level systems — *qutrits*. These are rarer than qubits which are commonly used in these studies (e, γ, t…)

- What can you do with these? At least 3 things:
  - Test quantum mechanics with *Bell inequalities* in a novel setting
  - Test the presence of/quantify *entanglement* in a novel setting
  - Test the *Standard Model* through new observables*

- How much does it cost?
  - Experimentalists: a precise measurement of W and Z polarizations
  - Theoreticians: pen(s), paper(s) and patience

**preliminary results, use at your own risk*

What are you talking about anyway?!?

- The *density operator* ρ is a positive operator such that Tr($\boldsymbol{\rho}$)=1. We can always write

$$\rho = \sum_i p_i \, |\Psi_i\rangle\langle\Psi_i| \ , \qquad p_i \geq 0 \ , \qquad \sum_i p_i = 1$$

with $|\Psi_i\rangle$ being a state of the Hilbert space $\mathcal{H}$ on which ρ acts.

What are you talking about anyway?!?

- The *density operator* ρ is a positive operator such that Tr(**ρ**)=1. We can always write

$$\rho = \sum_i p_i \, |\Psi_i\rangle\langle\Psi_i| \ , \qquad p_i \geq 0 \ , \qquad \sum_i p_i = 1$$

with $|\Psi_i\rangle$ being a state of the Hilbert space $\mathcal{H}$ on which ρ acts.

- Given a density operator ρ and an observable O, we compute *expectation values/ensemble averages* as: $\langle O \rangle = \mathrm{Tr}(\rho \, O)$

What are you talking about anyway?!?

- The *density operator* ρ is a positive operator such that Tr($\boldsymbol{\rho}$)=1. We can always write

$$\rho = \sum_i p_i \, |\Psi_i\rangle\langle\Psi_i| \;, \qquad p_i \geq 0 \;, \qquad \sum_i p_i = 1$$

  with $|\Psi_i\rangle$ being a state of the Hilbert space $\mathcal{H}$ on which ρ acts.

- Given a density operator ρ and an observable O, we compute *expectation values/ensemble averages* as: $\langle O \rangle = \mathrm{Tr}(\rho\, O)$

- Now, $\rho_i = |\Psi_i\rangle\langle\Psi_i|$ is by itself the density operator of the pure state $|\Psi_i\rangle$ (and also a projector), so:
  - The *convex combination* of density matrices is a density matrix
  - We can tell apart pure states from mixtures by looking at the *purity*:

$$\frac{1}{\dim(\mathcal{H})} \leq \mathrm{Tr}\left(\rho^2\right) \leq 1$$

What are you talking about anyway?!?

- The *density operator* ρ is a positive operator such that Tr(**ρ**)=1. We can always write

$$\rho = \sum_i p_i \, |\Psi_i\rangle\langle\Psi_i| \ , \qquad p_i \geq 0 \ , \qquad \sum_i p_i = 1$$

with $|\Psi_i\rangle$ being a state of the Hilbert space $\mathcal{H}$ on which ρ acts.

- Given a density operator ρ and an observable O, we compute *expectation values/ensemble averages* as: $\langle O \rangle = \mathrm{Tr}(\rho\, O)$

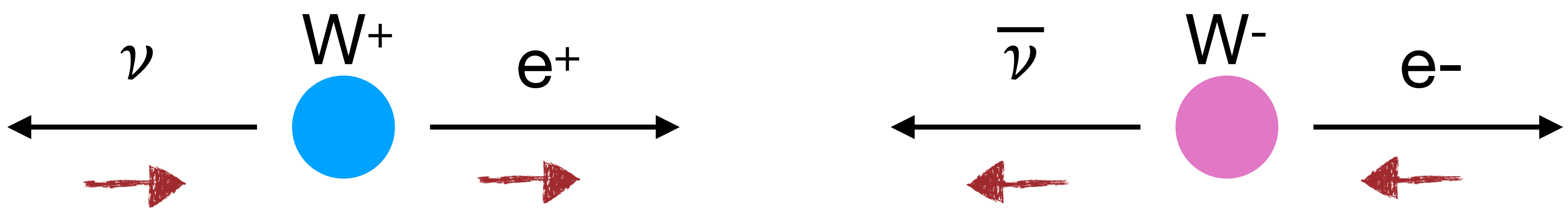
- Now, $\rho_i = |\Psi_i\rangle\langle\Psi_i|$ is by itself the density operator of the pure state $|\Psi_i\rangle$ (and also a projector), so:
  - The *convex combination* of density matrices is a density matrix
  - We can tell apart pure states from mixtures by looking at the *purity*:

$$\frac{1}{\dim(\mathcal{H})} \leq \mathrm{Tr}\left(\rho^2\right) \leq 1$$

- The density operator of a qubit is a 2x2 matrix, of a qutrit a 3x3 and for *2 qutrits a 9x9* —plenty to compute, plenty to measure.

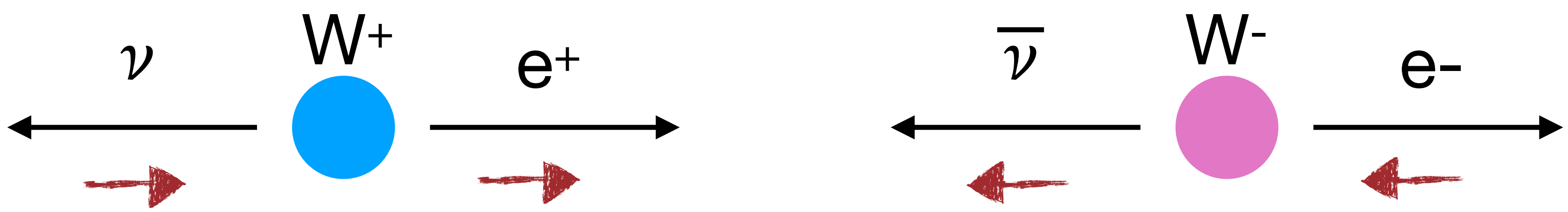
Real life problems

- How do we measure ρ? W and Z are kind enough to act as their own polarimeters when looking at *leptonic decays*. For example, WW:



Real life problems

- How do we measure ρ? W and Z are kind enough to act as their own polarimeters when looking at *leptonic decays*. For example, WW:



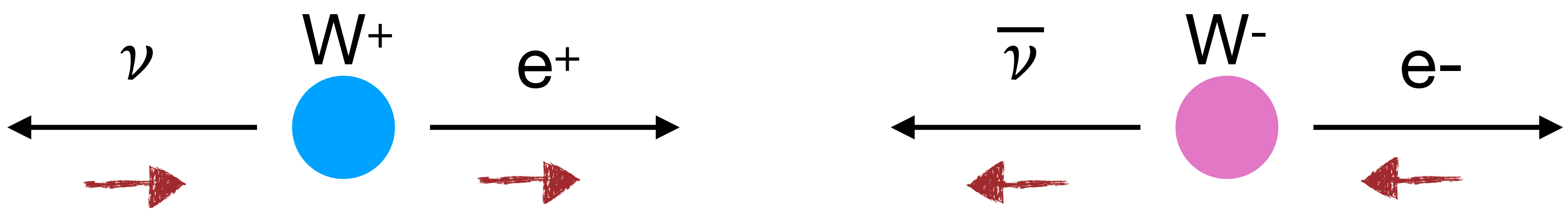
- So from the processes $p\,p \to V_1 + V_2 + X \to \ell^+\ell^- + \text{jets} + E_T^{\text{miss}}$ with cross section σ, we need to measure

$$f_a = \frac{1}{\sigma}\int \frac{d\sigma}{d\Omega^+}\,\mathfrak{p}_+^a\,d\Omega^+ \quad g_a = \frac{1}{\sigma}\int \frac{d\sigma}{d\Omega^-}\,\mathfrak{p}_-^a\,d\Omega^- \quad h_{ab} = \frac{1}{\sigma}\int\int \frac{d\sigma}{d\Omega^+\,d\Omega^-}\,\mathfrak{p}_+^a\,\mathfrak{p}_-^b\,d\Omega^+ d\Omega^-$$

where $d\Omega^\pm = \sin\theta^\pm d\theta^\pm\,d\phi^\pm$ are the spherical angles with polar axes defined by the charged lepton momenta in the rest frames of the W^+ and W^-. The indices a,b=1…8 and $\mathfrak{p}^{a(b)}$ are known functions of the angles

Real life problems

- How do we measure ρ? W and Z are kind enough to act as their own polarimeters when looking at *leptonic decays*. For example, WW:



- So from the processes $p\,p \to V_1 + V_2 + X \to \ell^+\ell^- + \text{jets} + E_T^{\text{miss}}$ with cross section σ, we need to measure

$$f_a = \frac{1}{\sigma}\int \frac{d\sigma}{d\Omega^+}\,\mathfrak{p}_+^a\,d\Omega^+ \qquad g_a = \frac{1}{\sigma}\int \frac{d\sigma}{d\Omega^-}\,\mathfrak{p}_-^a\,d\Omega^- \qquad h_{ab} = \frac{1}{\sigma}\int\int \frac{d\sigma}{d\Omega^+\,d\Omega^-}\,\mathfrak{p}_+^a\,\mathfrak{p}_-^b\,d\Omega^+ d\Omega^-$$

where $d\Omega^\pm = \sin\theta^\pm d\theta^\pm\,d\phi^\pm$ are the spherical angles with polar axes defined by the charged lepton momenta in the rest frames of the W^+ and W^-. The indices a,b=1…8 and $\mathfrak{p}^{a(b)}$ are known functions of the angles

- Once f_a, g_a, and h_{ab} are inferred from data:

$$\rho(\lambda_1,\lambda_1',\lambda_2,\lambda_2') = \left(\frac{1}{9}\left[\mathbb{1}\otimes\mathbb{1}\right] + \sum_a f_a\left[\mathbb{1}\otimes T^a\right] + \sum_a g_a\left[T^a\otimes\mathbb{1}\right] + \sum_{ab} h_{ab}\left[T^a\otimes T^b\right]\right)_{\lambda_1\lambda_1',\lambda_2\lambda_2'}$$

where $T^{a(b)}$ are the Gell-Mann matrices

How do you compute this?

- *Wave vector* of a spin-boson of mass *M* momentum *p* and helicity *λ*:

$$\varepsilon^{\mu}(p,\lambda) = -\frac{1}{\sqrt{2}}|\lambda|\,(\lambda\, n_1^{\mu} + i\, n_2^{\mu}) + \Big(1 - |\lambda|\Big) n_3^{\mu}\,, \qquad (\lambda=0,+1,-1)$$

where n_i^{μ} track the 3 polarization directions as seen from the CoM frame of the process.

How do you compute this?

- *Wave vector* of a spin-boson of mass *M* momentum *p* and helicity *λ*:

$$\varepsilon^\mu(p,\lambda) = -\frac{1}{\sqrt{2}}|\lambda|\,(\lambda\, n_1^\mu + i\, n_2^\mu) + \Big(1-|\lambda|\Big)n_3^\mu\,, \qquad (\lambda=0,+1,-1)$$

  where n_i^μ track the 3 polarization directions as seen from the CoM frame of the process.

- The *density matrix of one spin-boson* is then

$$\mathscr{P}^{\mu\nu}_{\lambda\lambda'}(p) \;=\; \varepsilon^\mu(p,\lambda)^\star\varepsilon^\nu(p,\lambda') \;=\; \frac{1}{3}\left(-g^{\mu\nu}+\frac{p^\mu p^\nu}{M^2}\right)\delta_{\lambda\lambda'} - \frac{i}{2M}\epsilon^{\mu\nu\alpha\beta}p_\alpha n^i_\beta\,(S_i)_{\lambda\lambda'} - \frac{1}{2}n_i^\mu n_j^\nu\,(S_{ij})_{\lambda\lambda'}$$

  with S_i being the 3x3 spin matrices and $S_{ij} = S_iS_j + S_jS_i - \frac{4}{3}\mathbb{1}\,\delta_{ij}$

How do you compute this?

- *Wave vector* of a spin-boson of mass *M* momentum *p* and helicity *λ*:

$$\varepsilon^\mu(p,\lambda) = -\frac{1}{\sqrt{2}}|\lambda|\,(\lambda\, n_1^\mu + i\, n_2^\mu) + \Big(1-|\lambda|\Big) n_3^\mu\,, \qquad (\lambda=0,+1,-1)$$

  where n_i^μ track the 3 polarization directions as seen from the CoM frame of the process.

- The *density matrix of one spin-boson* is then

$$\mathscr{P}^{\mu\nu}_{\lambda\lambda'}(p) \;=\; \varepsilon^\mu(p,\lambda)^\star \varepsilon^\nu(p,\lambda') \;=\; \frac{1}{3}\left(-g^{\mu\nu} + \frac{p^\mu p^\nu}{M^2}\right)\delta_{\lambda\lambda'} - \frac{i}{2M}\epsilon^{\mu\nu\alpha\beta} p_\alpha n^i_\beta \,(S_i)_{\lambda\lambda'} - \frac{1}{2} n_i^\mu n_j^\nu \,(S_{ij})_{\lambda\lambda'}$$

  with S_i being the 3x3 spin matrices and $S_{ij} = S_i S_j + S_j S_i - \frac{4}{3}\mathbb{1}\,\delta_{ij}$

- To obtain the *density matrix for a sample of gauge boson pairs* produced in a repeated interaction with amplitude $\mathcal{M}$ use

$$\rho(\lambda_1,\lambda_1',\lambda_2,\lambda_2') = \sum_{\mu,\mu',\nu,\nu'} \frac{\mathcal{M}_{\mu\nu}\mathcal{M}^\dagger_{\mu'\nu'}}{|\bar{\mathcal{M}}^2|} \mathscr{P}^{\mu\mu'}_{\lambda_1\lambda_1'}(p_1)\mathscr{P}^{\nu\nu'}_{\lambda_2\lambda_2'}(p_2) \quad \text{(convex combination of 2 } \rho\text{)}$$

  where $|\bar{\mathcal{M}}^2|$ is the squared amplitude summed over the polarizations

How do you compute this?

- *Wave vector* of a spin-boson of mass M momentum p and helicity λ:

$$\varepsilon^{\mu}(p,\lambda) = -\frac{1}{\sqrt{2}}|\lambda|\,(\lambda\, n_1^{\mu} + i\, n_2^{\mu}) + \Big(1-|\lambda|\Big) n_3^{\mu}\,, \qquad (\lambda=0,+1,-1)$$

  where n_i^{μ} track the 3 polarization directions as seen from the CoM frame of the process.

- The *density matrix of one spin-boson* is then

$$\mathscr{P}^{\mu\nu}_{\lambda\lambda'}(p) \;=\; \varepsilon^{\mu}(p,\lambda)^{\star}\varepsilon^{\nu}(p,\lambda') \;=\; \frac{1}{3}\left(-g^{\mu\nu}+\frac{p^{\mu}p^{\nu}}{M^2}\right)\delta_{\lambda\lambda'} - \frac{i}{2M}\epsilon^{\mu\nu\alpha\beta}p_{\alpha}n^{i}_{\beta}\,(S_i)_{\lambda\lambda'} - \frac{1}{2}n_i^{\mu}n_j^{\nu}\,(S_{ij})_{\lambda\lambda'}$$

  with S_i being the 3x3 spin matrices and $S_{ij} = S_iS_j + S_jS_i - \frac{4}{3}\mathbb{1}\,\delta_{ij}$

- To obtain the *density matrix for a sample of gauge boson pairs* produced in a repeated interaction with amplitude $\mathcal{M}$ use

$$\rho(\lambda_1,\lambda_1',\lambda_2,\lambda_2') = \sum_{\mu,\mu',\nu,\nu'} \frac{\mathcal{M}_{\mu\nu}\mathcal{M}^{\dagger}_{\mu'\nu'}}{|\bar{\mathcal{M}}^2|}\mathscr{P}^{\mu\mu'}_{\lambda_1\lambda_1'}(p_1)\mathscr{P}^{\nu\nu'}_{\lambda_2\lambda_2'}(p_2) \quad \text{(convex combination of 2 } \rho\text{)}$$

  where $|\bar{\mathcal{M}}^2|$ is the squared amplitude summed over the polarizations

- Finally: $f_a = \frac{1}{6}\,\mathrm{Tr}\left[\rho\,(\mathbb{1}\otimes T^a)\right]\,,\quad g_a = \frac{1}{6}\,\mathrm{Tr}\left[\rho\,(T^a\otimes\mathbb{1})\right]\,,\quad h_{ab} = \frac{1}{4}\,\mathrm{Tr}\left[\rho\left(T^a\otimes T^b\right)\right]$

Ok, and then?

- We test a suitable instance of Bell inequality tailored to qutrits—*Collins-Gisin-Linden-Massar-Popescu inequality*

D.Collins, N. Gisin, N. Linden, S. Massar and S. Popescu, *Phys. Rev. Lett. 88, 040404 (2002)*

$$\mathcal{I}_3 = \mathrm{Tr}\left[\rho\,\mathcal{B}\right] \qquad \mathcal{B} \to (U \otimes V)^\dagger \cdot \mathcal{B} \cdot (U \otimes V)$$

(we optimize $\mathcal{B}$ by using unitary matrices *U*, *V*)

$$\mathcal{B} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 2 & 0 & 0 \\ 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 \\ 0 & 0 & 2 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

A. Acin, T. Durt, N. Gisin and J. I. Latorre, Phys. Rev. A 65, 052325 (2002)

Ok, and then?

- We test a suitable instance of Bell inequality tailored to qutrits—*Collins-Gisin-Linden-Massar-Popescu inequality*

  D.Collins, N. Gisin, N. Linden, S. Massar and S. Popescu, Phys. Rev. Lett. 88, 040404 (2002)

$$\mathcal{I}_3 = \mathrm{Tr}\left[\rho\,\mathcal{B}\right] \qquad \mathcal{B} \to (U \otimes V)^\dagger \cdot \mathcal{B} \cdot (U \otimes V)$$

(we optimize $\mathcal{B}$ by using unitary matrices *U*, *V*)

$$\mathcal{B} = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 2 & 0 & 0 \\
0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 \\
0 & 0 & 2 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

Ok, and then?

- We test a suitable instance of Bell inequality tailored to qutrits—*Collins-Gisin-Linden-Massar-Popescu inequality*

  D.Collins, N. Gisin, N. Linden, S. Massar and S. Popescu, Phys. Rev. Lett. 88, 040404 (2002)

$$\mathcal{I}_3 = \mathrm{Tr}\left[\rho\,\mathcal{B}\right] \qquad \mathcal{B} \rightarrow (U \otimes V)^{\dagger} \cdot \mathcal{B} \cdot (U \otimes V)$$

(we optimize $\mathcal{B}$ by using unitary matrices U, V)

  - Deterministic local theory: *always* $\mathcal{I}_3 \leq 2$
  - Quantum mechanics: *it can be* $4 \geq \mathcal{I}_3 > 2$

$$\mathcal{B} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 2 & 0 & 0 \\ 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 \\ 0 & 0 & 2 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

A. Acin, T. Durt, N. Gisin and J. I. Latorre, Phys. Rev. A 65, 052325 (2002)

Ok, and then?

- We test a suitable instance of Bell inequality tailored to qutrits—*Collins-Gisin-Linden-Massar-Popescu inequality*

D.Collins, N. Gisin, N. Linden, S. Massar and S. Popescu, Phys. Rev. Lett. 88, 040404 (2002)

$$\mathcal{I}_3 = \mathrm{Tr}\left[\rho\,\mathcal{B}\right] \qquad \mathcal{B} \to (U \otimes V)^\dagger \cdot \mathcal{B} \cdot (U \otimes V)$$

(we optimize $\mathcal{B}$ by using unitary matrices *U*, *V*)

  - Deterministic local theory: *always* $\mathcal{I}_3 \leq 2$
  - Quantum mechanics: *it can be* $4 \geq \mathcal{I}_3 > 2$

$$\mathcal{B} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 2 & 0 & 0 \\ 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 \\ 0 & 0 & 2 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

A. Acin, T. Durt, N. Gisin and J. I. Latorre, Phys. Rev. A 65, 052325 (2002)

- As for entanglement: what is entanglement?

  Entanglement is a property of *entangled states: states that are NOT separable*. For instance, bipartite qubit states:

$$|\psi\rangle = \frac{1}{2}\left(|0\,0\rangle - |0\,1\rangle + |1\,0\rangle - |1\,1\rangle\right) = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) \otimes \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)$$ *not entangled*

$$|\psi'\rangle = \frac{1}{\sqrt{2}}\left(|0\,0\rangle + |1\,1\rangle\right)$$ *very entangled*

Measuring the entanglement is generally very complicated!

- For *bipartite pure states* it's easy: use the *entropy of entanglement*

$$\mathscr{E}[\rho] = -\mathrm{Tr}\left[\rho_A \log \rho_A\right] = -\mathrm{Tr}\left[\rho_B \log \rho_B\right]$$

(A, B are the two subsystems with reduced density matrices ρ_A and ρ_B)

The *entropy of entanglement is an entanglement measure*

- For *bipartite pure states* it's easy: use the *entropy of entanglement*

$$\mathscr{E}[\rho] = -\mathrm{Tr}\left[\rho_A \log \rho_A\right] = -\mathrm{Tr}\left[\rho_B \log \rho_B\right]$$

(A, B are the two subsystems with reduced density matrices $\boldsymbol{\rho}_A$ and $\boldsymbol{\rho}_{\mathrm{B}}$)

The *entropy of entanglement is an entanglement measure*

- For *bipartite mixed states*: use the *concurrence*, if you can…
  - This is only an *entanglement witness* (says yes/no but not how much)

- For *bipartite pure states* it's easy: use the *entropy of entanglement*

$$\mathscr{E}[\rho] = -\mathrm{Tr}\left[\rho_A \log \rho_A\right] = -\mathrm{Tr}\left[\rho_B \log \rho_B\right]$$

(A, B are the two subsystems with reduced density matrices ρ_A and ρ_B)

The *entropy of entanglement is an entanglement measure*

- For *bipartite mixed states*: use the *concurrence*, if you can…
  - This is only an *entanglement witness* (says yes/no but not how much)
  - Finding the concurrence of a mixed state involves an optimization problem

$$\rho = \sum_i p_i \left|\Psi_i\right\rangle\!\left\langle\Psi_i\right| \qquad \mathcal{C}[\rho] = \inf_{\{|\Psi\rangle\}} \sum_i p_i \, \mathcal{C}[|\Psi_i\rangle] \qquad \mathcal{C}[|\Psi\rangle] = \sqrt{2\left(1 - \mathrm{Tr}\left[(\rho_r)^2\right]\right)}$$

$r = A$ or B

- For *bipartite pure states* it's easy: use the *entropy of entanglement*

$$\mathscr{E}[\rho] = -\mathrm{Tr}\left[\rho_A \log \rho_A\right] = -\mathrm{Tr}\left[\rho_B \log \rho_B\right]$$

(A, B are the two subsystems with reduced density matrices ρ_A and ρ_B)

The *entropy of entanglement is an entanglement measure*

- For *bipartite mixed states*: use the *concurrence*, if you can…
  - This is only an *entanglement witness* (says yes/no but not how much)
  - Finding the concurrence of a mixed state involves an optimization problem

$$\rho = \sum_i p_i \, |\Psi_i\rangle\langle\Psi_i| \qquad \mathcal{C}[\rho] = \inf_{\{|\Psi\rangle\}} \sum_i p_i \, \mathcal{C}[|\Psi_i\rangle] \qquad \mathcal{C}[|\Psi\rangle] = \sqrt{2\left(1 - \mathrm{Tr}\left[(\rho_r)^2\right]\right)} \quad r = A \text{ or } B$$

too complicated to compute this for qutrits. We use instead the lower bound $\mathscr{C}_2$

$$\left(\mathcal{C}[\rho]\right)^2 \geq \mathscr{C}_2[\rho] \qquad \mathscr{C}_2[\rho] = 2\max\left(0,\ \mathrm{Tr}\left[\rho^2\right] - \mathrm{Tr}\left[(\rho_A)^2\right],\ \mathrm{Tr}\left[\rho^2\right] - \mathrm{Tr}\left[(\rho_B)^2\right]\right) \leq \frac{4}{3}$$

- For *bipartite pure states* it's easy: use the *entropy of entanglement*

$$\mathscr{E}[\rho] = -\mathrm{Tr}\left[\rho_A \log \rho_A\right] = -\mathrm{Tr}\left[\rho_B \log \rho_B\right]$$

(A, B are the two subsystems with reduced density matrices ρ_A and ρ_B)

The *entropy of entanglement is an entanglement measure*

- For *bipartite mixed states*: use the *concurrence*, if you can…
  - This is only an *entanglement witness* (says yes/no but not how much)
  - Finding the concurrence of a mixed state involves an optimization problem

$$\rho = \sum_i p_i \left|\Psi_i\right\rangle\left\langle\Psi_i\right| \qquad \mathcal{C}[\rho] = \inf_{\{|\Psi\rangle\}} \sum_i p_i\, \mathcal{C}[|\Psi_i\rangle] \qquad \mathcal{C}[|\Psi\rangle] = \sqrt{2\left(1 - \mathrm{Tr}\left[(\rho_r)^2\right]\right)}$$

$$r = A \text{ or } B$$

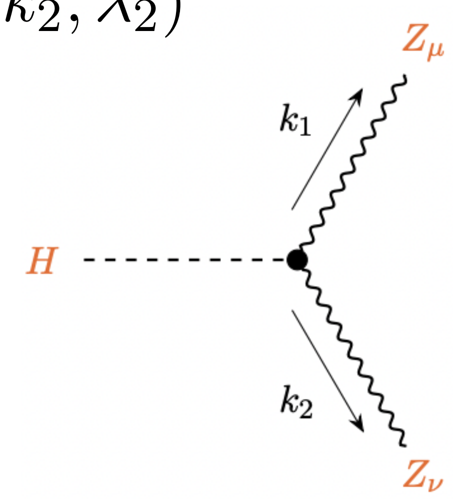
too complicated to compute this for qutrits. We use instead the lower bound $\mathscr{C}_2$

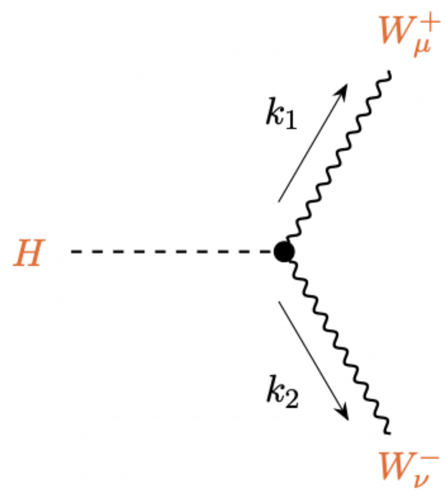
$$\left(\mathcal{C}[\rho]\right)^2 \geq \mathscr{C}_2[\rho] \qquad \mathscr{C}_2[\rho] = 2\max\left(0,\ \mathrm{Tr}\left[\rho^2\right] - \mathrm{Tr}\left[(\rho_A)^2\right],\ \mathrm{Tr}\left[\rho^2\right] - \mathrm{Tr}\left[(\rho_B)^2\right]\right) \leq \frac{4}{3}$$

- To cut it short:
  - If $\mathscr{E}$ >0 or $\mathscr{C}_2$ >0 the two subsystems (i.e. bosons) are entangled
  - If $\mathcal{I}_3$ >2 we disprove local deterministic models

A Higgs boson walks into a bar and…

…decays to WW* or ZZ*: $H \to V(k_1, \lambda_1)\, V^*(k_2, \lambda_2)$





We model the off-shell fields as having a mass

$$M_V^* = f M_V$$

where 0<*f*<1 and M_V is the on-shell mass

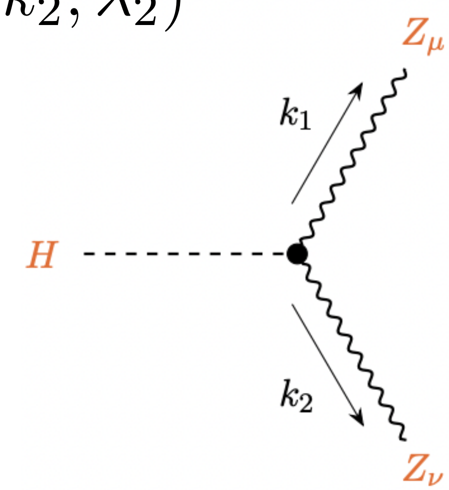
A Higgs boson walks into a bar and…

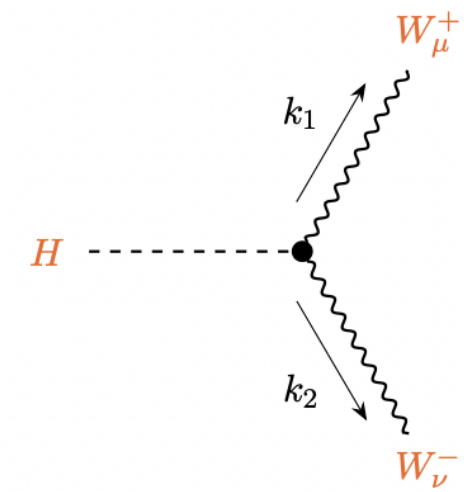
…decays to WW* or ZZ*: $H \to V(k_1, \lambda_1)\, V^*(k_2, \lambda_2)$

We model the off-shell fields as having a mass

$$M_V^* = f M_V$$

where 0<f<1 and M_V is the on-shell mass





- We grind through the computations and obtain in both the cases something like:

$$\rho_H = 2 \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_{44} & 0 & h_{16} & 0 & h_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_{16} & 0 & 2\,h_{33} & 0 & h_{16} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_{44} & 0 & h_{16} & 0 & h_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$h_{16} = \frac{f M_V^2 \Big[- m_H^2 + (1+f^2) M_V^2 \Big]}{m_H^4 - 2(1+f^2) m_H^2 M_V^2 + (1 + 10 f^2 + f^4) M_V^4}$$

$$h_{33} = \frac{1}{4} \frac{\Big[m_H^2 - (1+f^2) M_V^2 \Big]^2}{m_H^4 - 2(1+f^2) m_H^2 M_V^2 + (1 + 10 f^2 + f^4) M_V^4}$$

$$h_{44} = \frac{2 f^2 M_V^4}{m_H^4 - 2(1+f^2) m_H^2 M_V^2 + (1 + 10 f^2 + f^4) M_V^4}$$

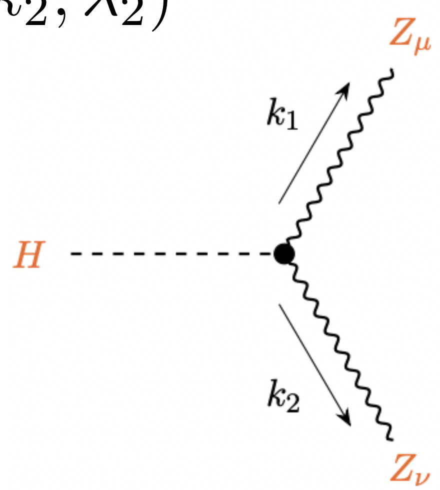
A Higgs boson walks into a bar and…

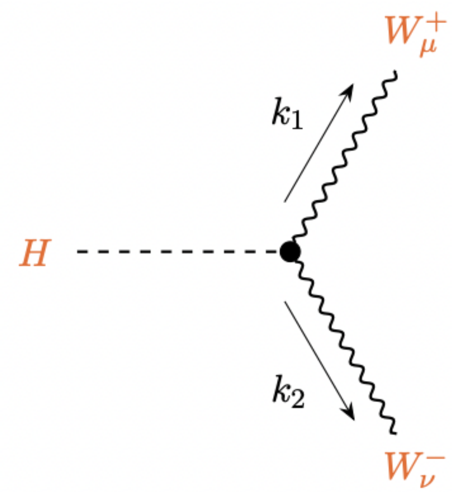
…decays to WW* or ZZ*: $H \to V(k_1, \lambda_1)\, V^*(k_2, \lambda_2)$

We model the off-shell fields as having a mass

$$M_V^* = f M_V$$

where 0<*f*<1 and M_V is the on-shell mass





- We grind through the computations and obtain in both the cases something like:

$$\rho_H = 2 \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_{44} & 0 & h_{16} & 0 & h_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_{16} & 0 & 2\,h_{33} & 0 & h_{16} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_{44} & 0 & h_{16} & 0 & h_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$h_{16} = \frac{f M_V^2 \Big[- m_H^2 + (1+f^2) M_V^2 \Big]}{m_H^4 - 2(1+f^2) m_H^2 M_V^2 + (1 + 10 f^2 + f^4) M_V^4}$$

$$h_{33} = \frac{1}{4} \frac{\Big[m_H^2 - (1+f^2) M_V^2 \Big]^2}{m_H^4 - 2(1+f^2) m_H^2 M_V^2 + (1 + 10 f^2 + f^4) M_V^4}$$

$$h_{44} = \frac{2 f^2 M_V^4}{m_H^4 - 2(1+f^2) m_H^2 M_V^2 + (1 + 10 f^2 + f^4) M_V^4}$$

- Check purity: $\mathrm{Tr}(\rho_H^2) = 1 \implies \rho_H = |\Psi_H\rangle\langle\Psi_H|$, *the bosons form a pure state!*

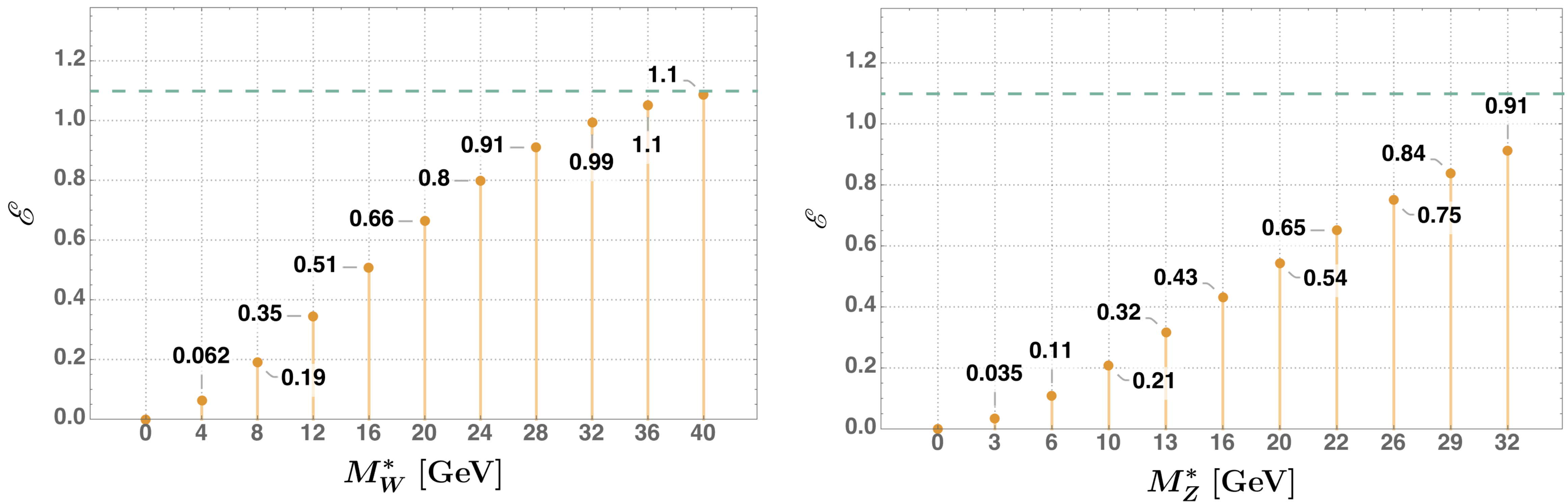
…and this pure state explicitly is:

$$|\Psi_H\rangle = \frac{1}{\sqrt{2+\varkappa^2}}\left[|{+}{-}\rangle - \varkappa\,|0\,0\rangle + |{-}{+}\rangle\right] \qquad \varkappa = 1 + \frac{m_H^2 - (1+f)^2 M_V^2}{2 f M_V^2}$$

...and this pure state explicitly is:

$$|\Psi_H\rangle = \frac{1}{\sqrt{2+\varkappa^2}}\left[|{+}{-}\rangle - \varkappa\,|0\,0\rangle + |{-}{+}\rangle\right] \qquad \varkappa = 1 + \frac{m_H^2 - (1+f)^2 M_V^2}{2 f M_V^2}$$

- For pure states we can use *entropy* to quantify entanglement



We can see that *the polarizations of the two gauge bosons are entangled* barring for the case where the off-shell boson is effectively massless. The entanglement reaches the *maximally allowed value (ln3) for gauge bosons produced at rest*.

- A touch of *new physics*: allow for anomalous HWW couplings

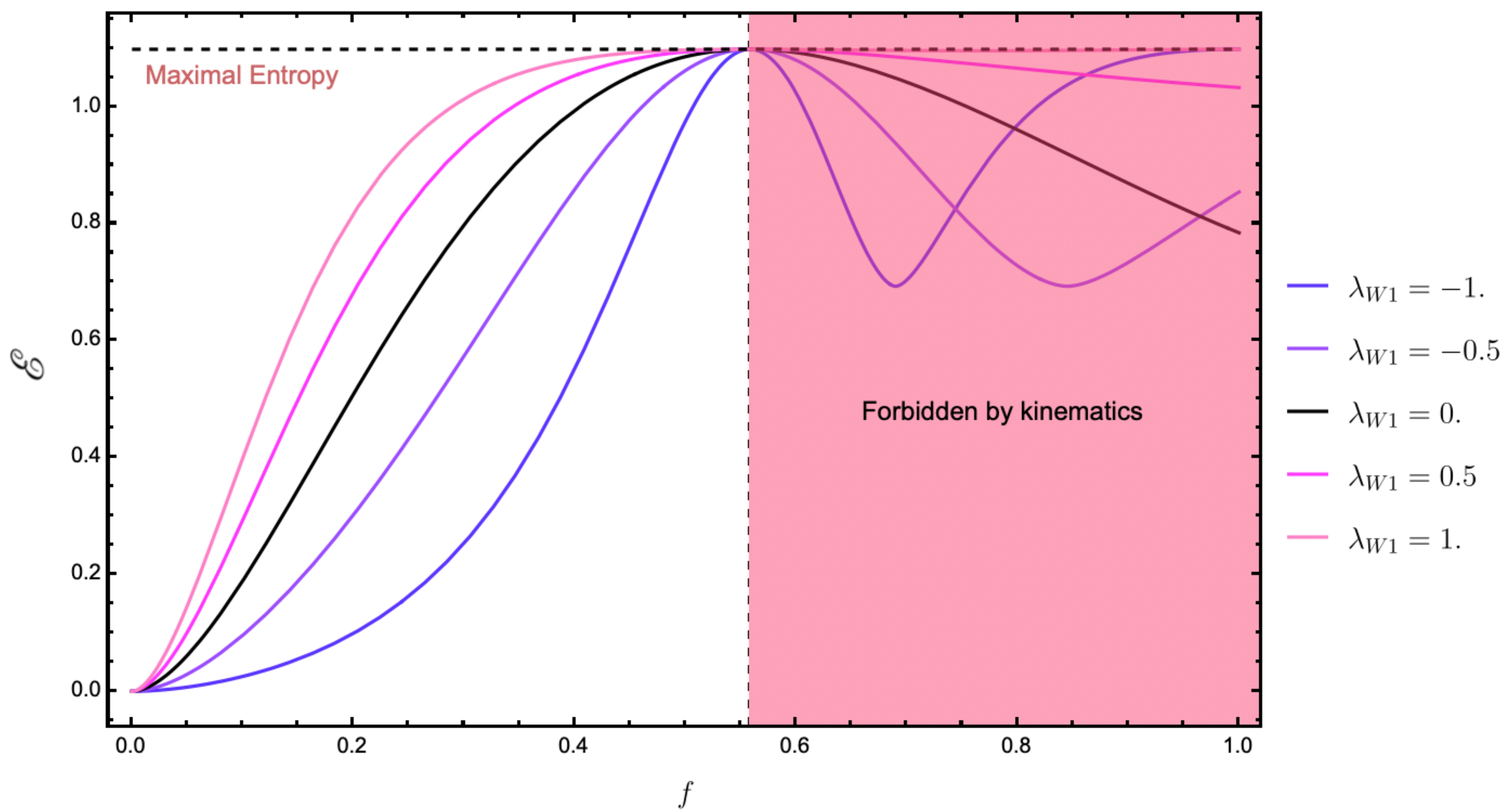
$$\mathcal{L}_{HVV} = g\, m_W W^+_\mu W^{-\mu} H + \frac{g}{2\cos\theta_W} m_Z Z_\mu Z^\mu H - \frac{g}{m_W}\left[\frac{\lambda_1^W}{2} W^+_{\mu\nu} W^{-\mu\nu} + \lambda_2^W \left(W^{+\nu}\partial^\mu W^-_{\mu\nu} + \text{H.c.}\right) + \frac{\widetilde{\lambda}^W_{CP}}{4} W^+_{\mu\nu}\widetilde{W}^{-\mu\nu} \right.$$
$$\left. + \frac{\lambda_1^Z}{2} Z_{\mu\nu} Z^{\mu\nu} + \lambda_2^Z Z^\nu \partial^\mu Z_{\mu\nu} + \frac{\widetilde{\lambda}^Z_{CP}}{4} Z_{\mu\nu}\widetilde{Z}^{\mu\nu}\right] H$$

and then check how the entanglement entropy changes. For instance let λ_1^W vary and set the remaining deformations of the SM to vanish

- A touch of *new physics*: allow for anomalous HWW couplings

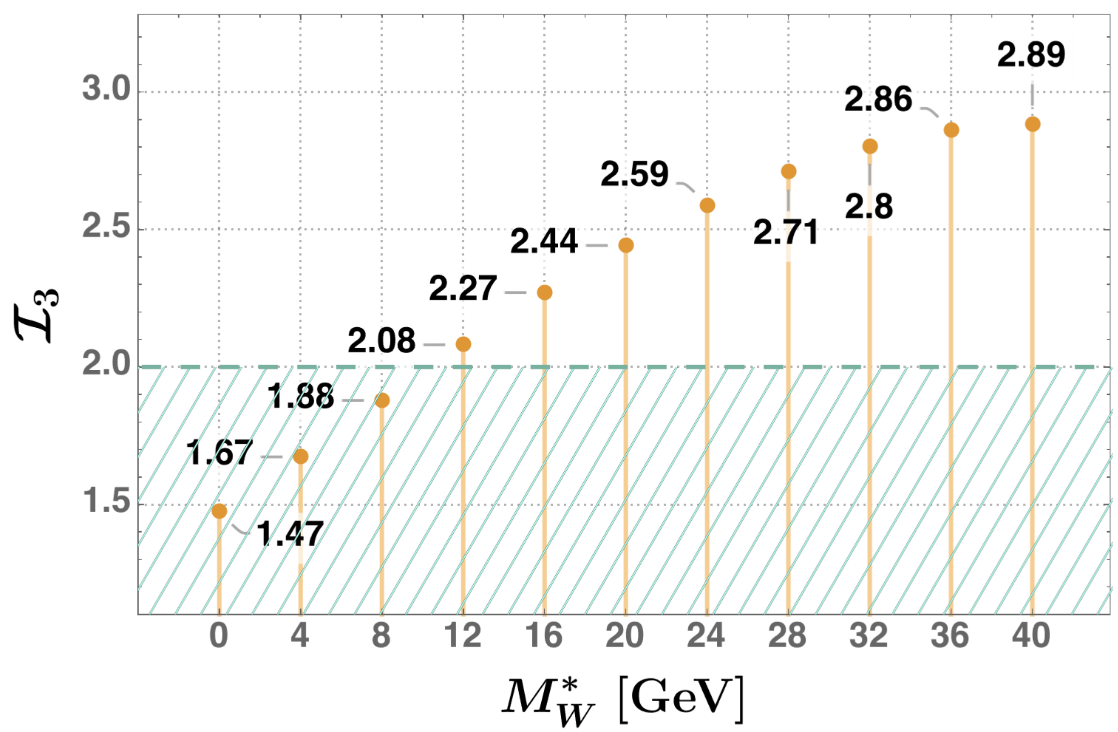
$$\mathcal{L}_{HVV} = g\, m_W W^+_\mu W^{-\mu} H + \frac{g}{2\cos\theta_W} m_Z Z_\mu Z^\mu H - \frac{g}{m_W}\Big[\frac{\lambda_1^W}{2} W^+_{\mu\nu} W^{-\mu\nu} + \lambda_2^W \left(W^{+\nu}\partial^\mu W^-_{\mu\nu} + \text{H.c.}\right) + \frac{\widetilde{\lambda}^W_{CP}}{4} W^+_{\mu\nu}\widetilde{W}^{-\mu\nu}$$
$$+ \frac{\lambda_1^Z}{2} Z_{\mu\nu} Z^{\mu\nu} + \lambda_2^Z Z^\nu \partial^\mu Z_{\mu\nu} + \frac{\widetilde{\lambda}^Z_{CP}}{4} Z_{\mu\nu}\widetilde{Z}^{\mu\nu}\Big] H$$

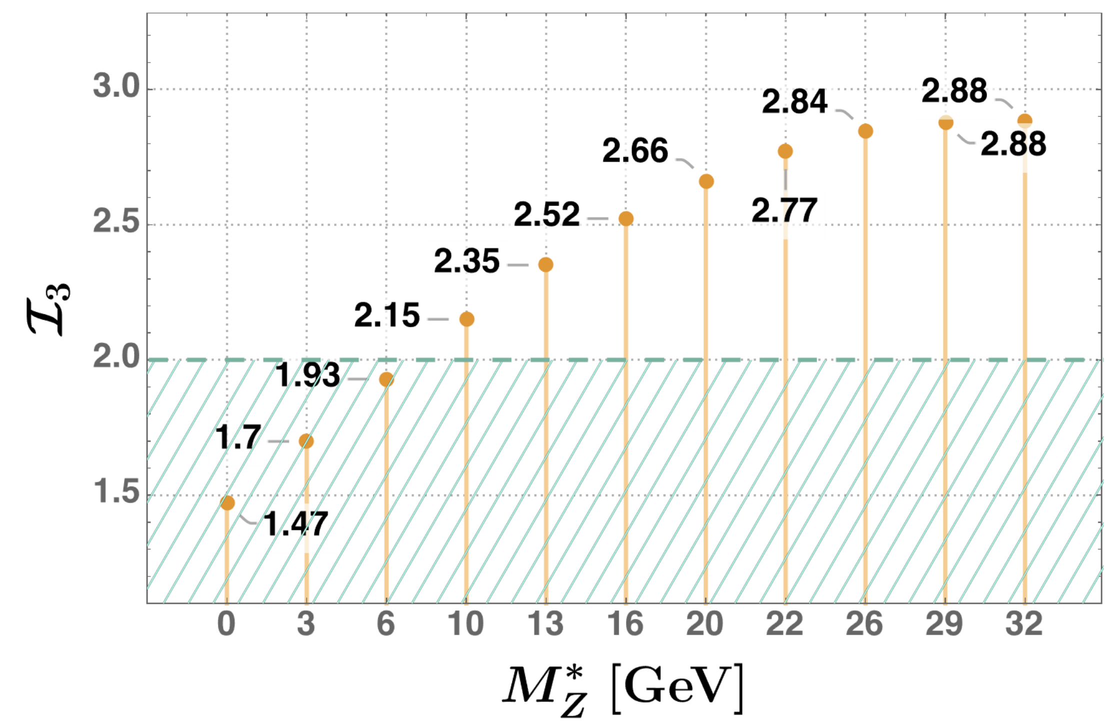
and then check how the entanglement entropy changes. For instance let λ_1^W vary and set the remaining deformations of the SM to vanish



entanglement (and ρ in general) as a new observable for new physics???

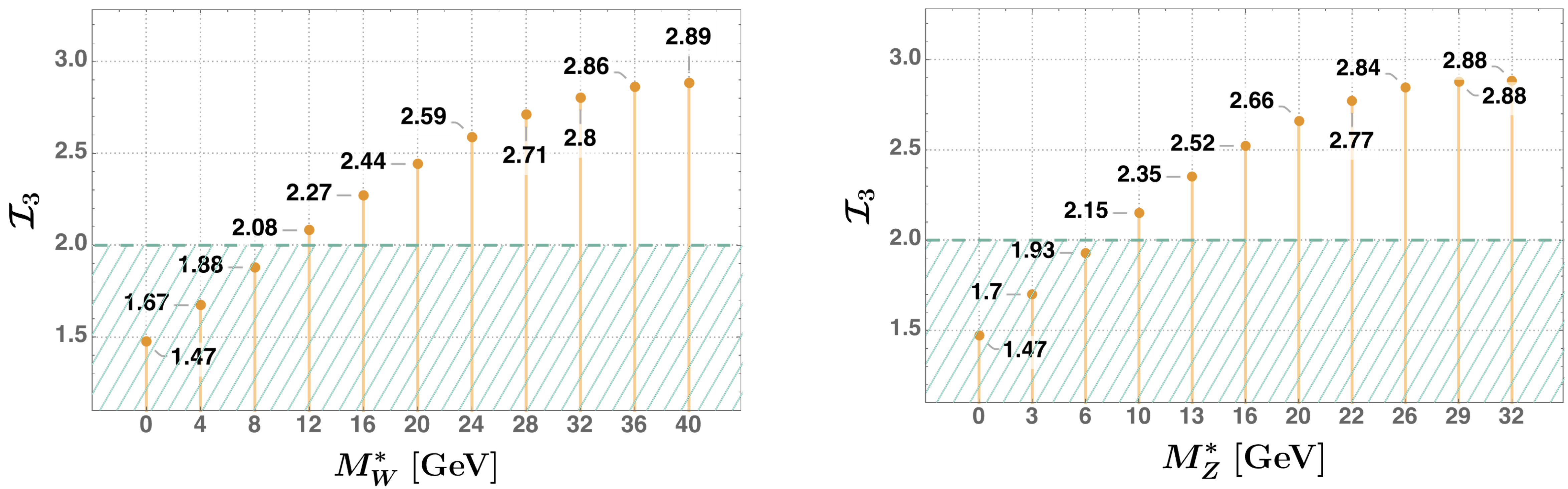
- Back to reality: Bell inequalities





Violation of the CGLMP inequality in both channels!

- Back to reality: Bell inequalities



Violation of the CGLMP inequality in both channels!

- Let's see if we see it. *Madgraph* tells us the cross sections of

$$p\,p \to H \to W^+ \ell^- \bar{\nu}_\ell \qquad\qquad p\,p \to H \to Z \ell^+ \ell^-$$

and with a *70% efficiency for the identification of each charged lepton* we have:

		$\ell^+ \nu_\ell\, \ell^- \bar{\nu}_\ell$	$\ell^- \ell^+ \ell^- \ell^+$
LHC run2	$(\mathcal{L} = 140\ \text{fb}^{-1})$	4571	28
Hi-Lumi	$(\mathcal{L} = 3\ \text{ab}^{-1})$	9.8×10^3	589

- *Statistical error*: modeled in a *gaussian distribution* of the di-boson invariant mass (meaning f) with dispersion controlled by *#events*

- *Statistical error*: modeled in a *gaussian distribution* of the di-boson invariant mass (meaning f) with dispersion controlled by *#events*
- For the *WW* channel, we add *a 5 GeV systematic error* in the "reconstruction" of *ν momenta* and propagate this to the di-boson invariant mass

- *Statistical error*: modeled in a *gaussian distribution* of the di-boson invariant mass (meaning f) with dispersion controlled by *#events*
- For the *WW* channel, we add *a 5 GeV systematic error* in the "reconstruction" of *ν momenta* and propagate this to the di-boson invariant mass
- We then sample $\mathcal{I}_3$ and compute the *significance* to reject $\mathcal{I}_3 \leq 2$

~~$\mathcal{I}_3 \leq 2$~~	LHC run2	Hi-Lumi
*WW**	*1*	*1*
*ZZ**	*1.3*	*5.6*

*The systematic error dominates WW**

ZZ quite promising!*

In line with the results obtained from MC analyses

Aguilar-Saavedra, Bernal, Casas, Moreno 2209.13441 [hep-ph]

A. Barr, PLB 825 (2022)

- *Statistical error*: modeled in a *gaussian distribution* of the di-boson invariant mass (meaning f) with dispersion controlled by *#events*
- For the *WW* channel, we add *a 5 GeV systematic error* in the "reconstruction" of ν *momenta* and propagate this to the di-boson invariant mass
- We then sample $\mathcal{I}_3$ and compute the *significance* to reject $\mathcal{I}_3 \leq 2$

~~$\mathcal{I}_3 \leq 2$~~	LHC run2	Hi-Lumi
*WW**	*1*	*1*
*ZZ**	*1.3*	*5.6*

*The systematic error dominates WW**

ZZ quite promising!*

In line with the results obtained from MC analyses

Aguilar-Saavedra, Bernal, Casas, Moreno 2209.13441 [hep-ph]

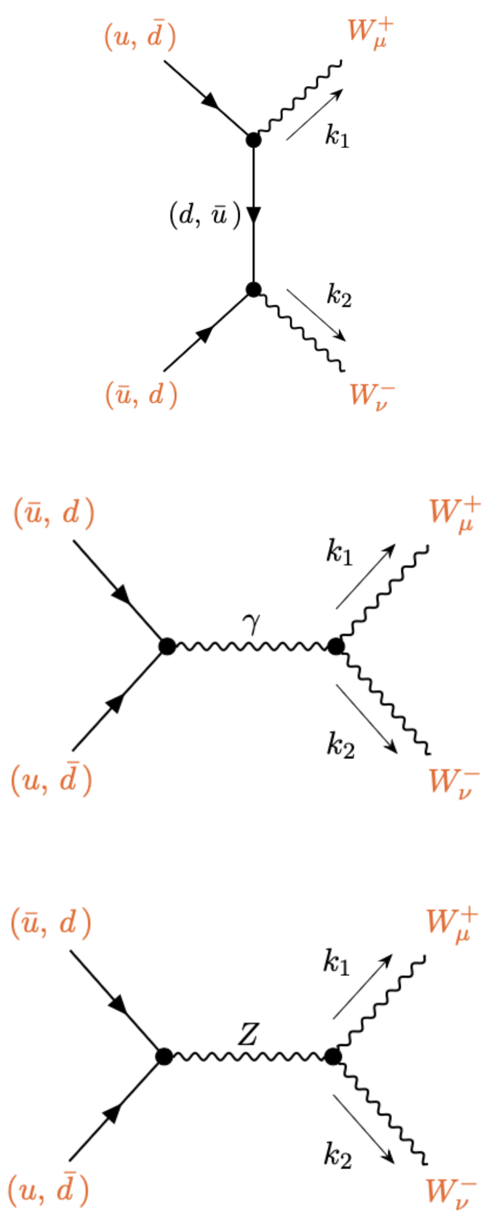
A. Barr, PLB 825 (2022)

- All very nice but *a true estimate of the significance requires a dedicated full simulation! ….just sayin'….*

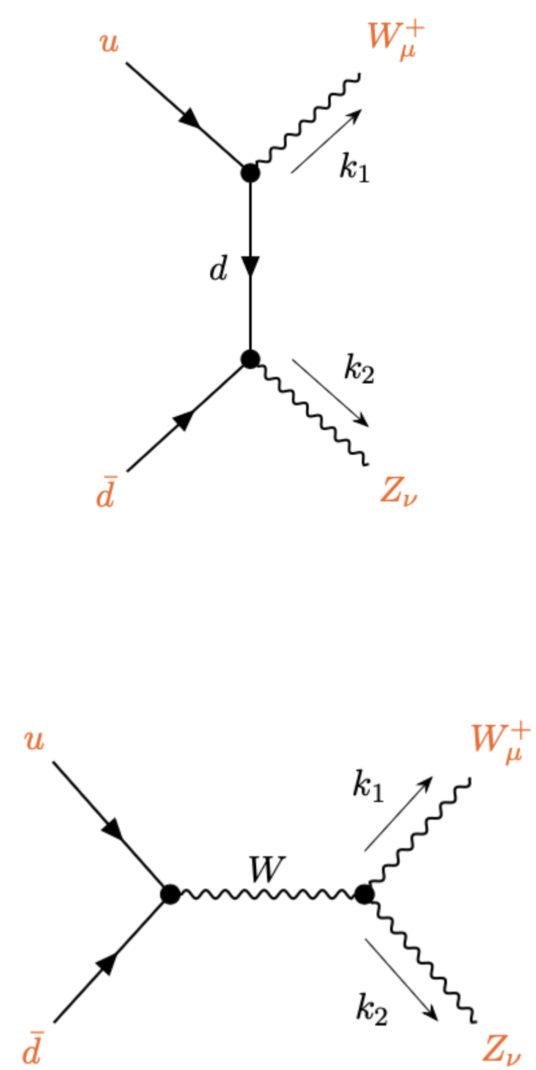
Other processes?

- We looked also at pp→…

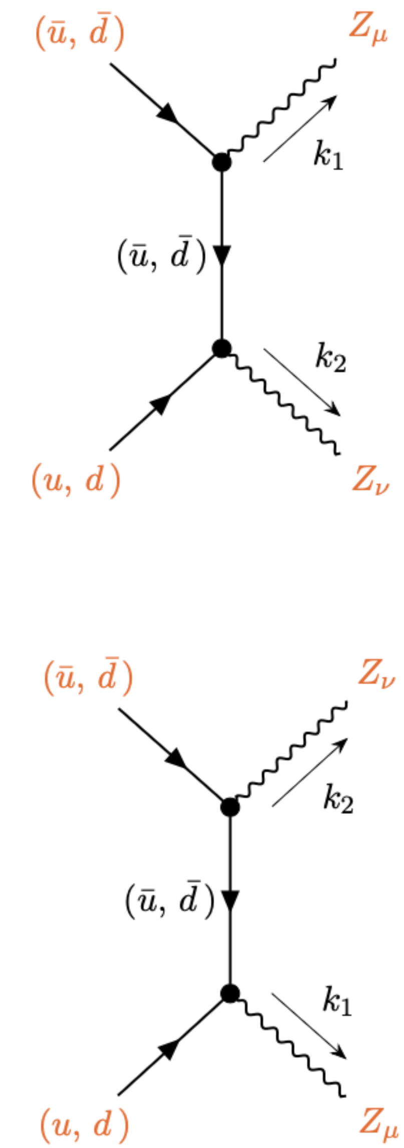
WW



WZ



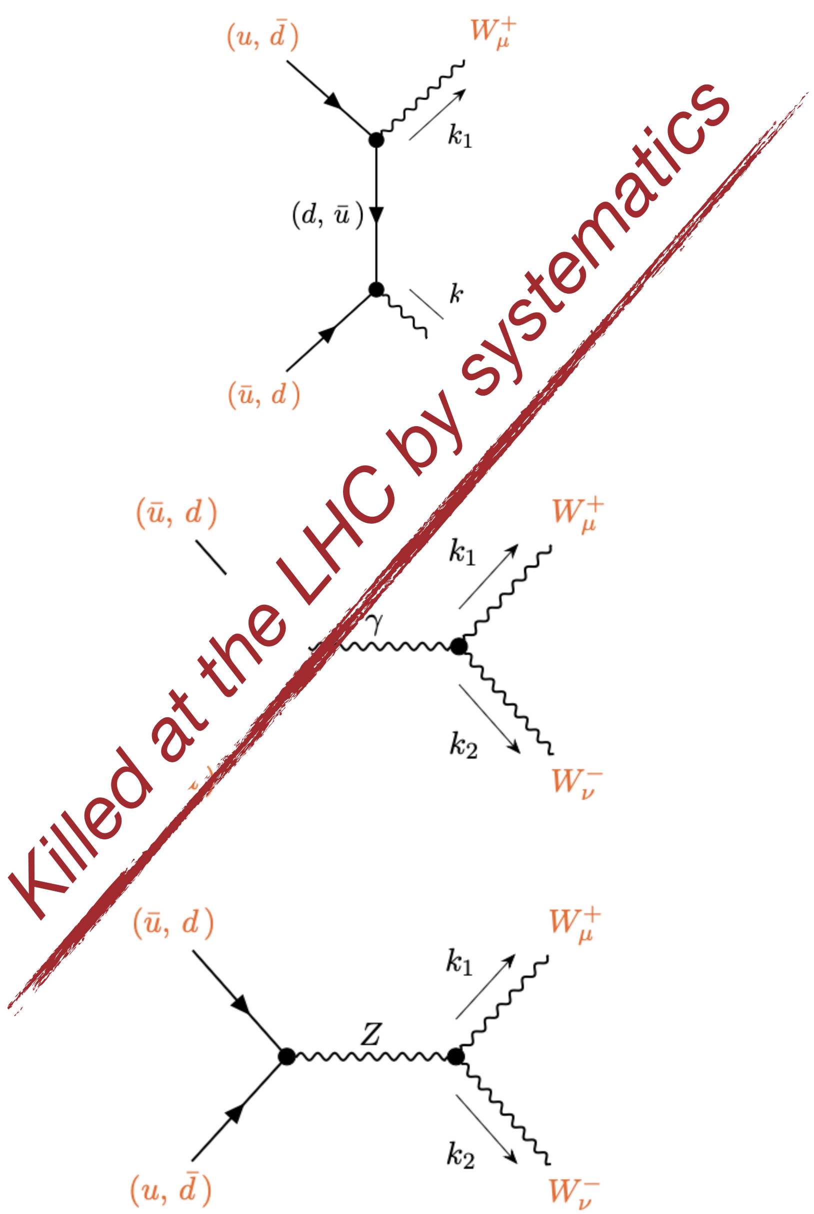
ZZ



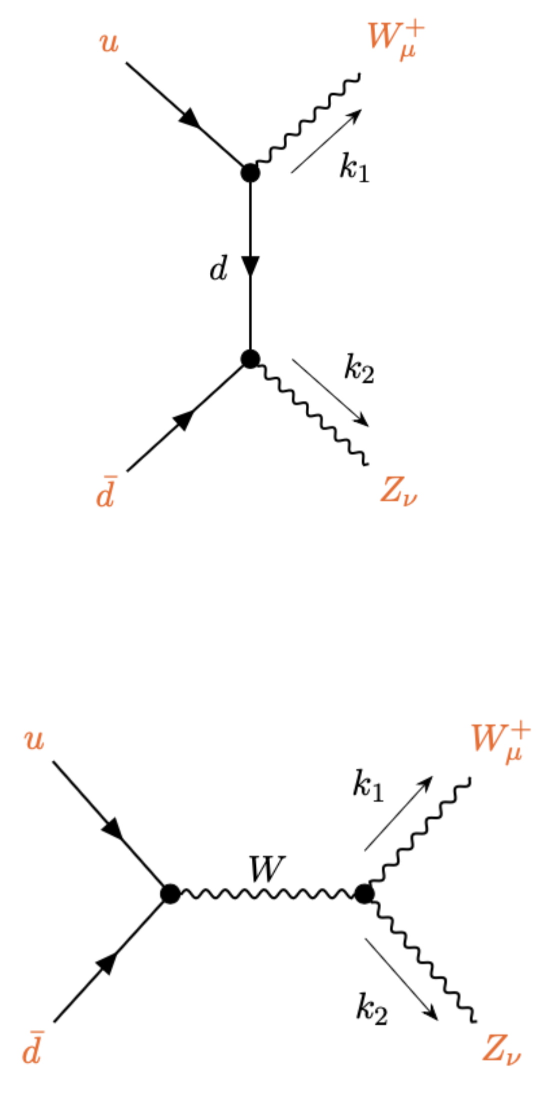
Other processes?

- We looked also at pp→…

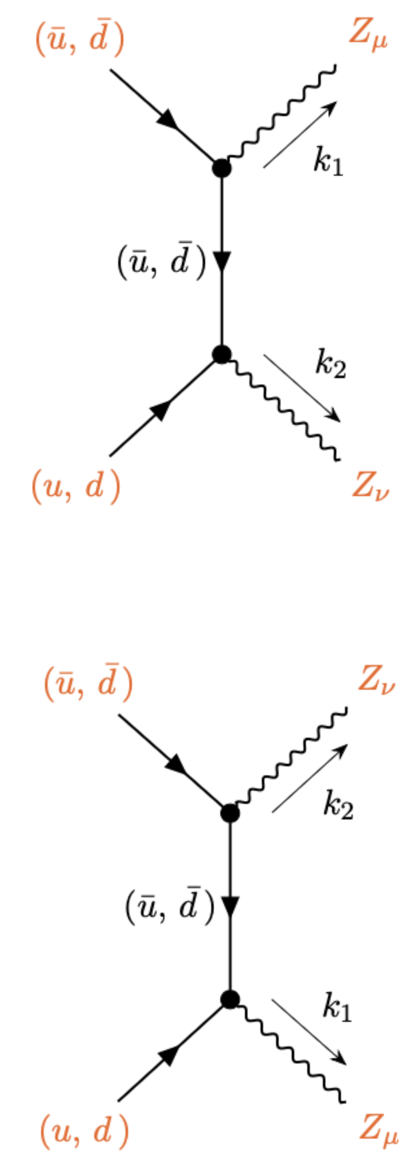
WW



WZ



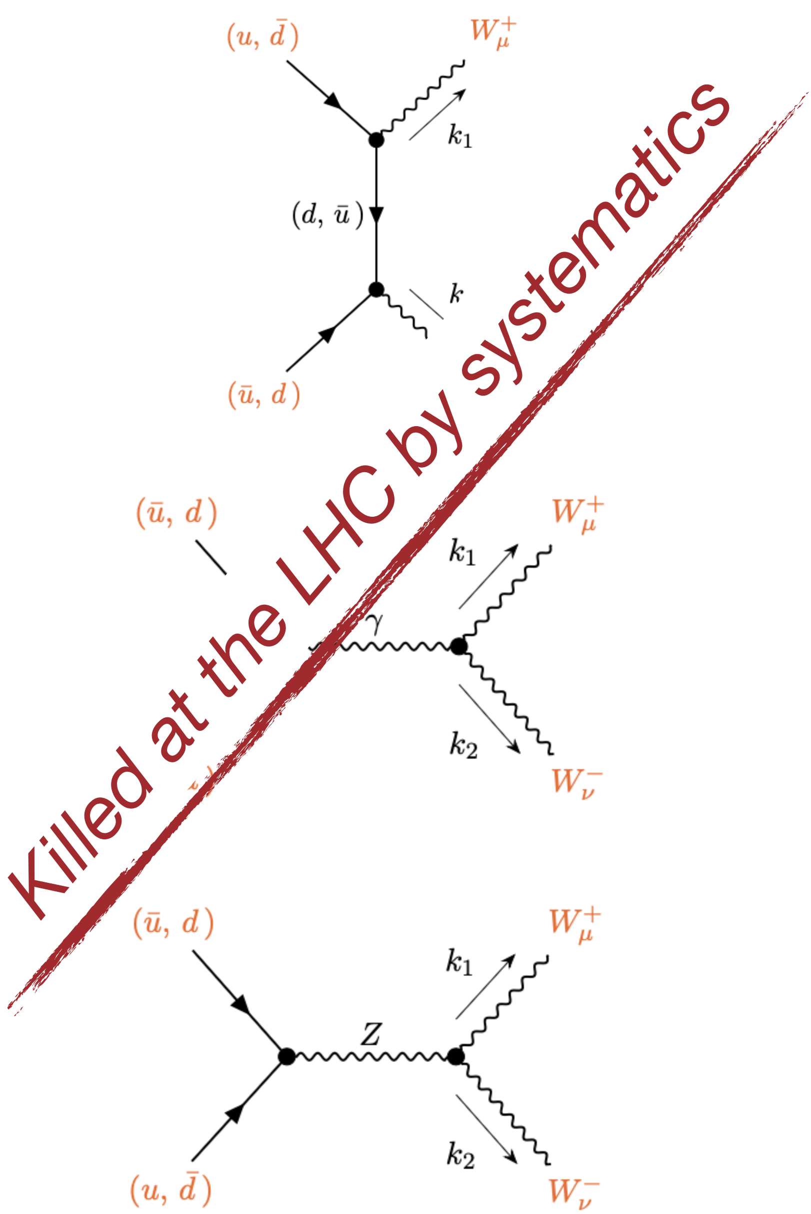
ZZ



Other processes?

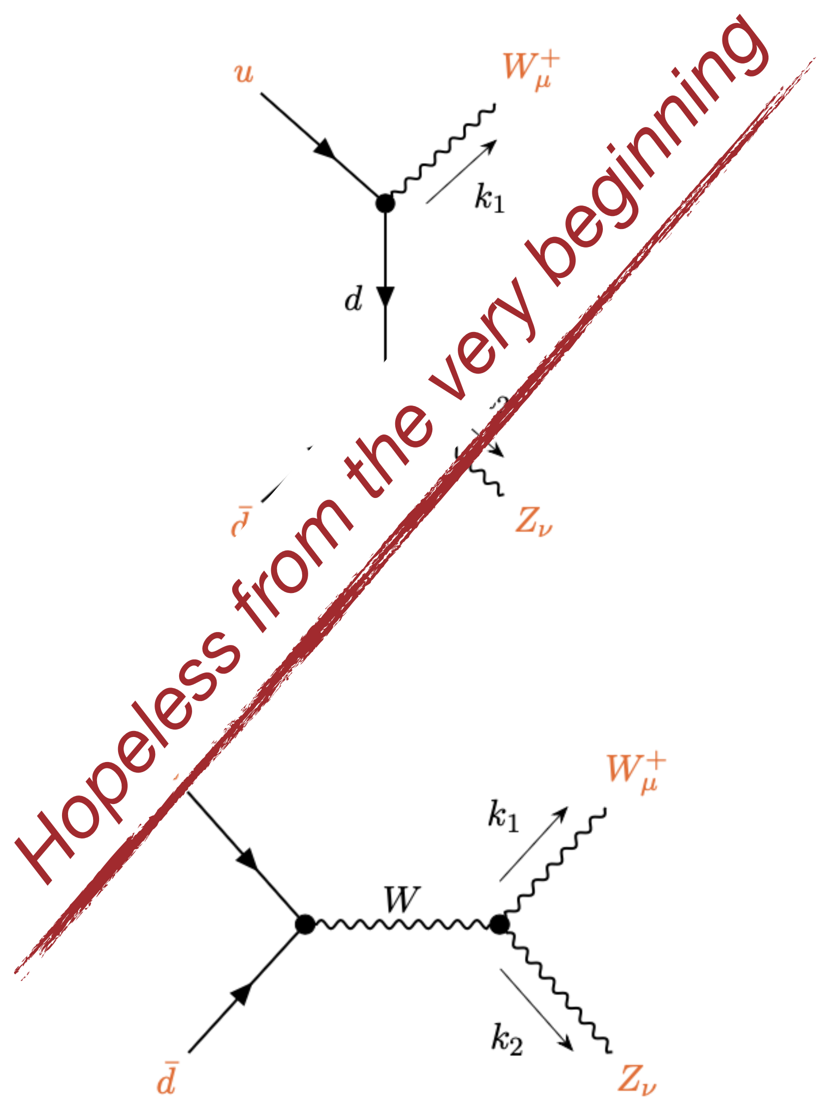
- We looked also at pp→…

WW



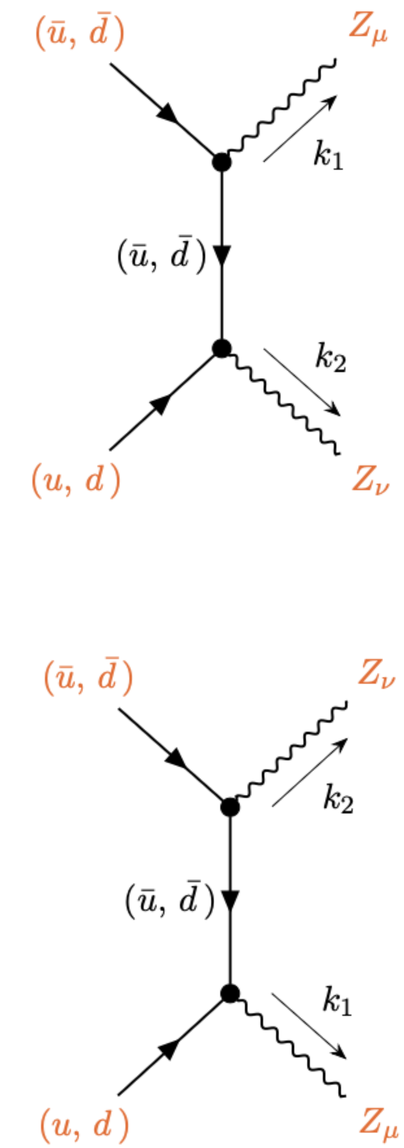
Killed at the LHC by systematics

WZ



Hopeless from the very beginning

ZZ

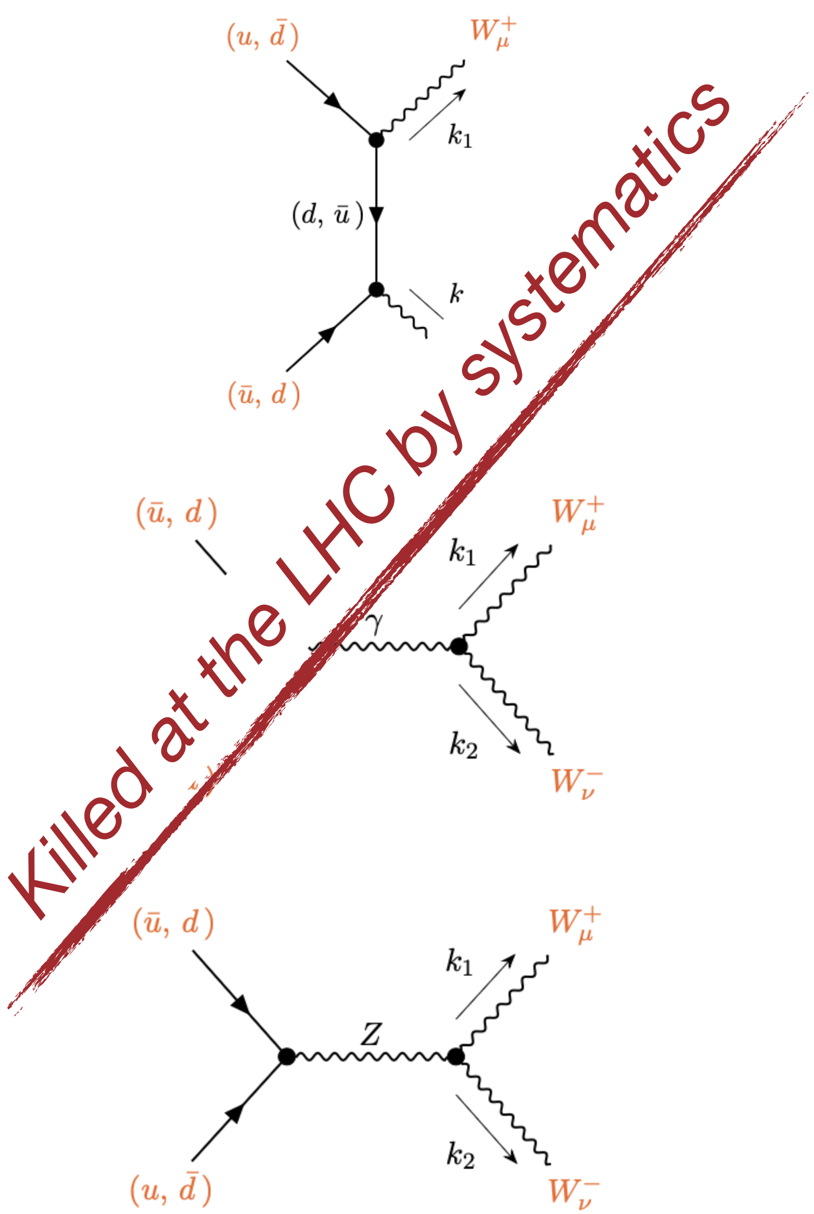


Other processes?

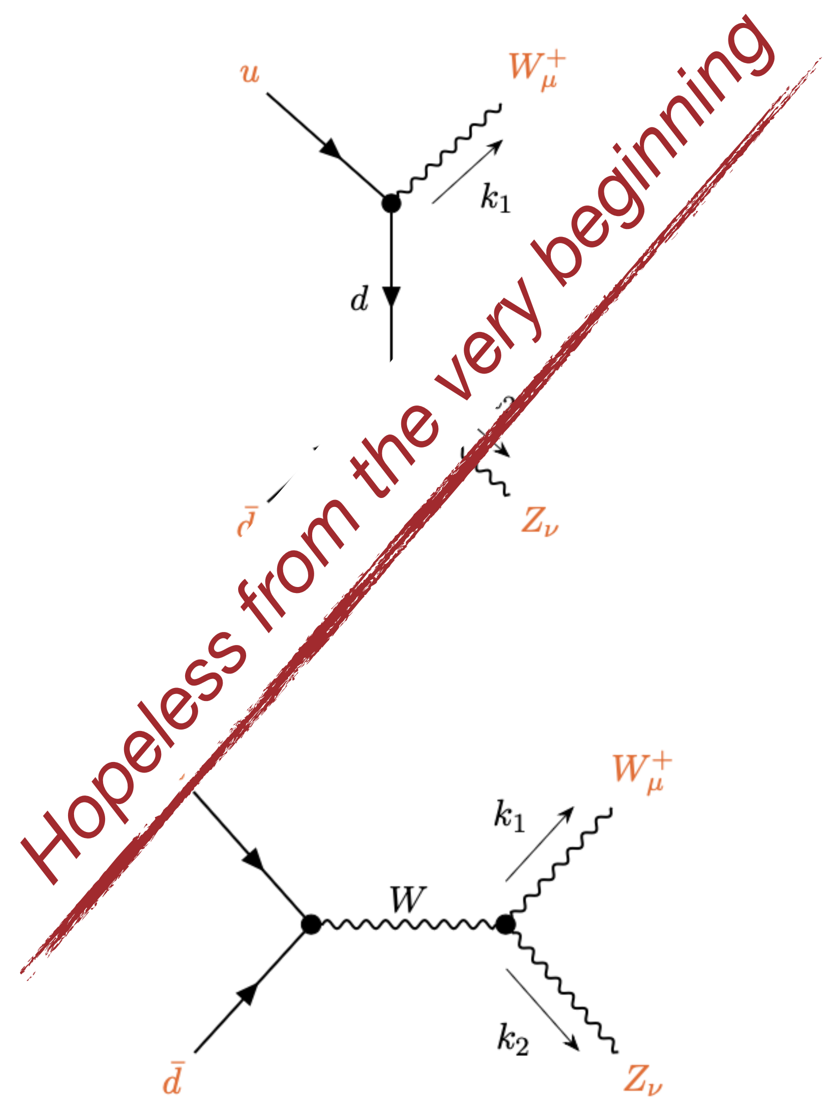
- We looked also at pp→…

Let's see…

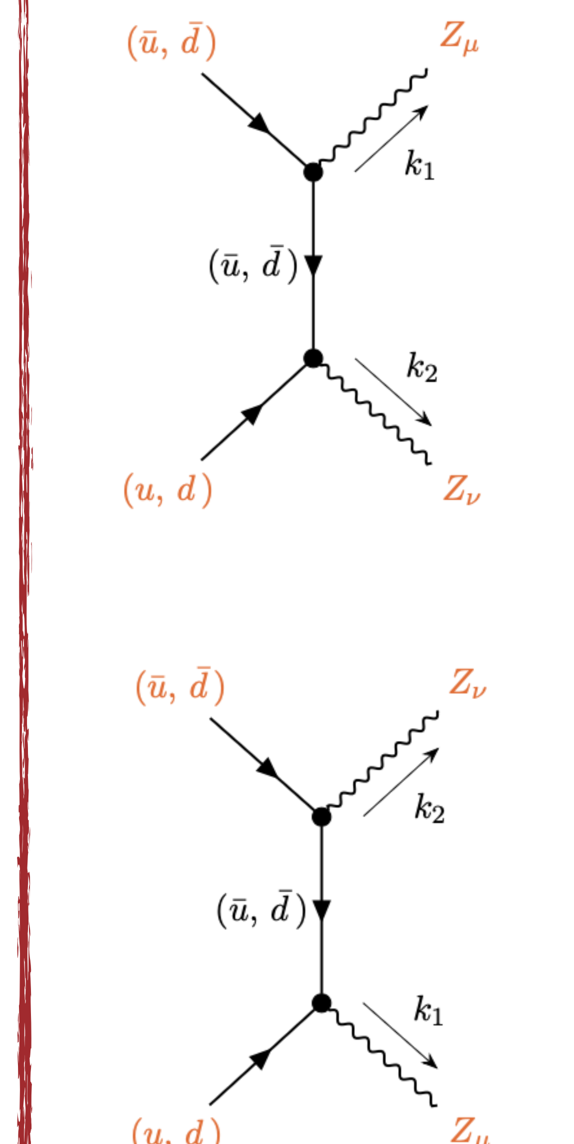
WW

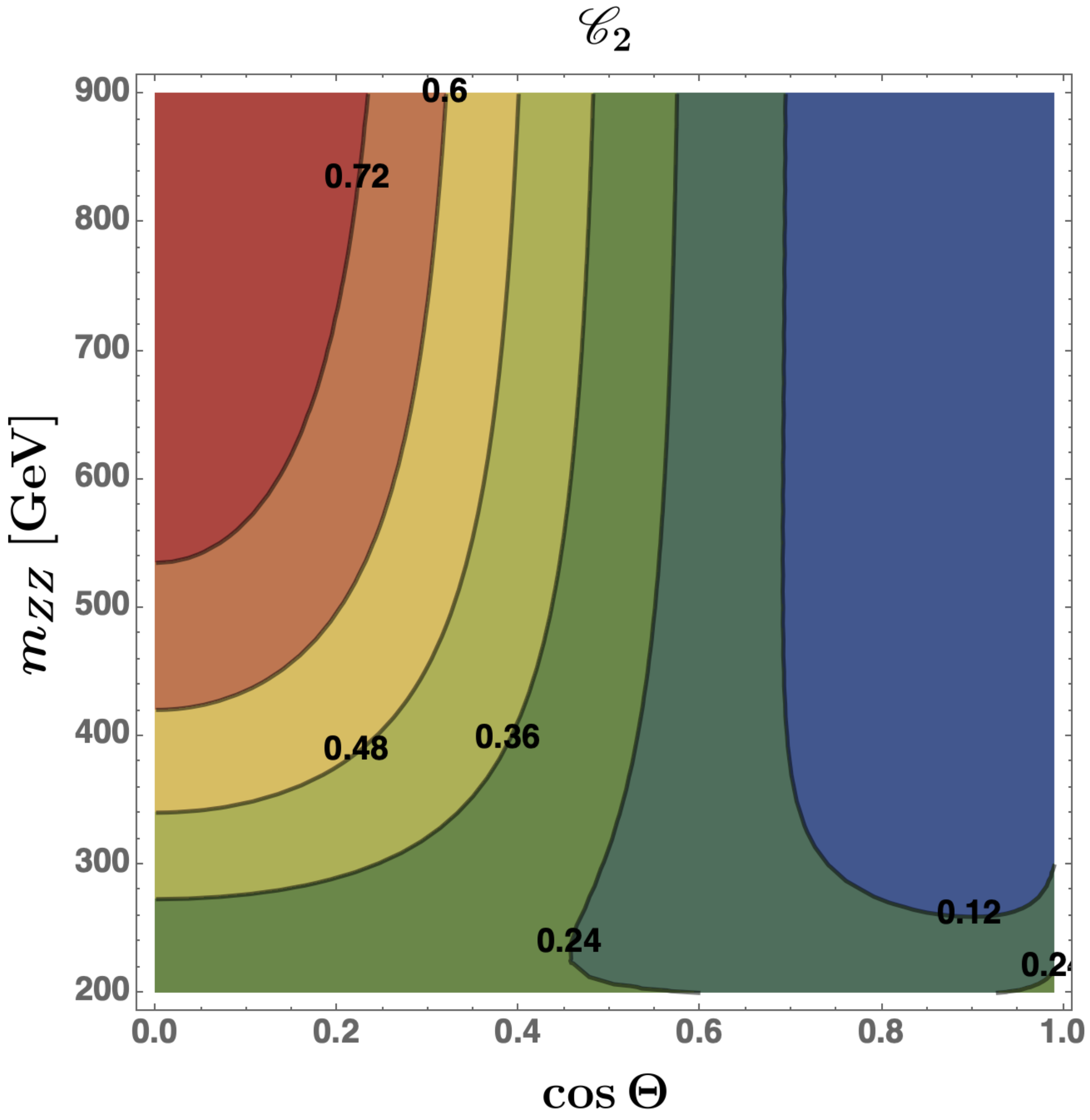


WZ



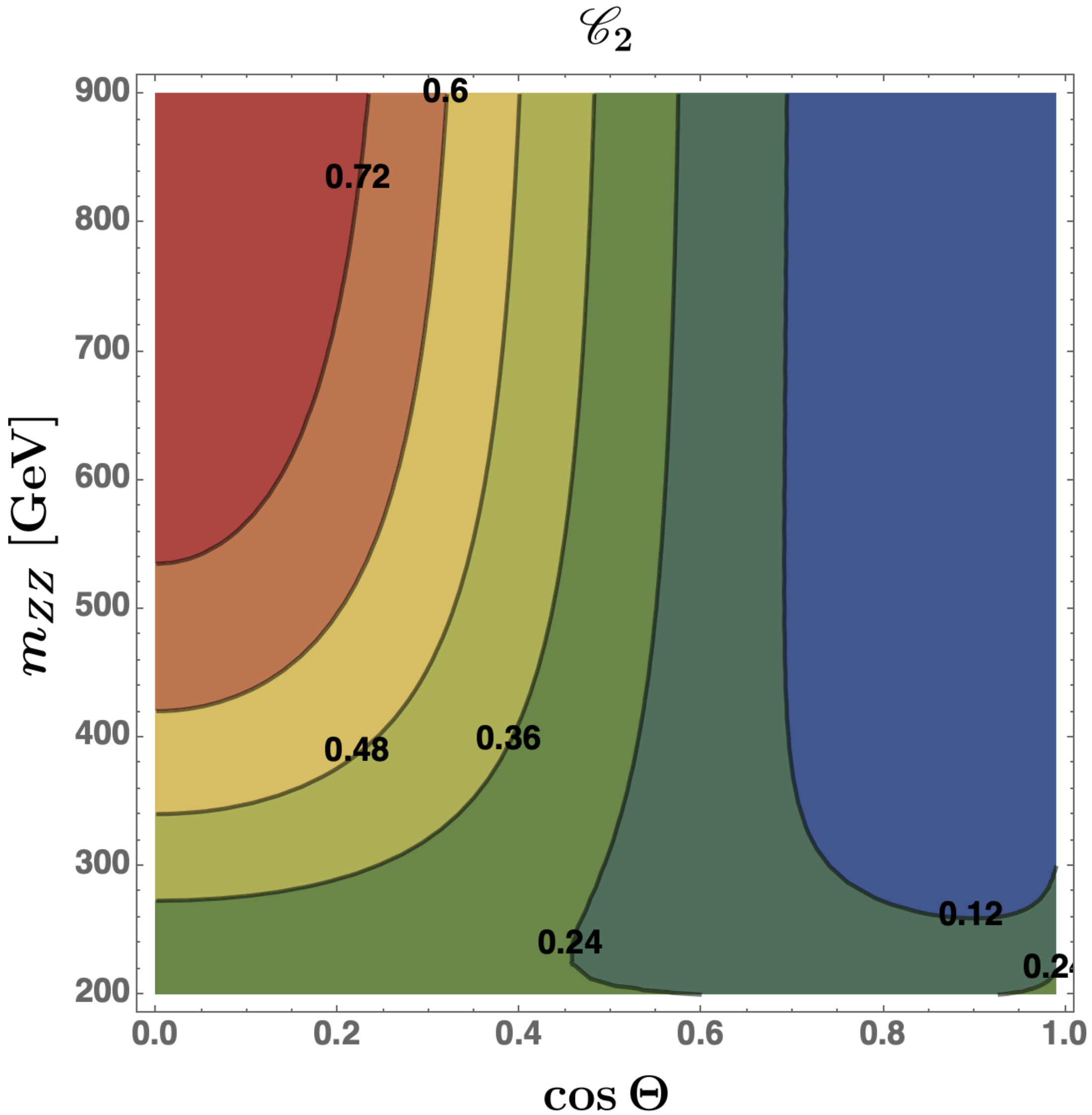
ZZ





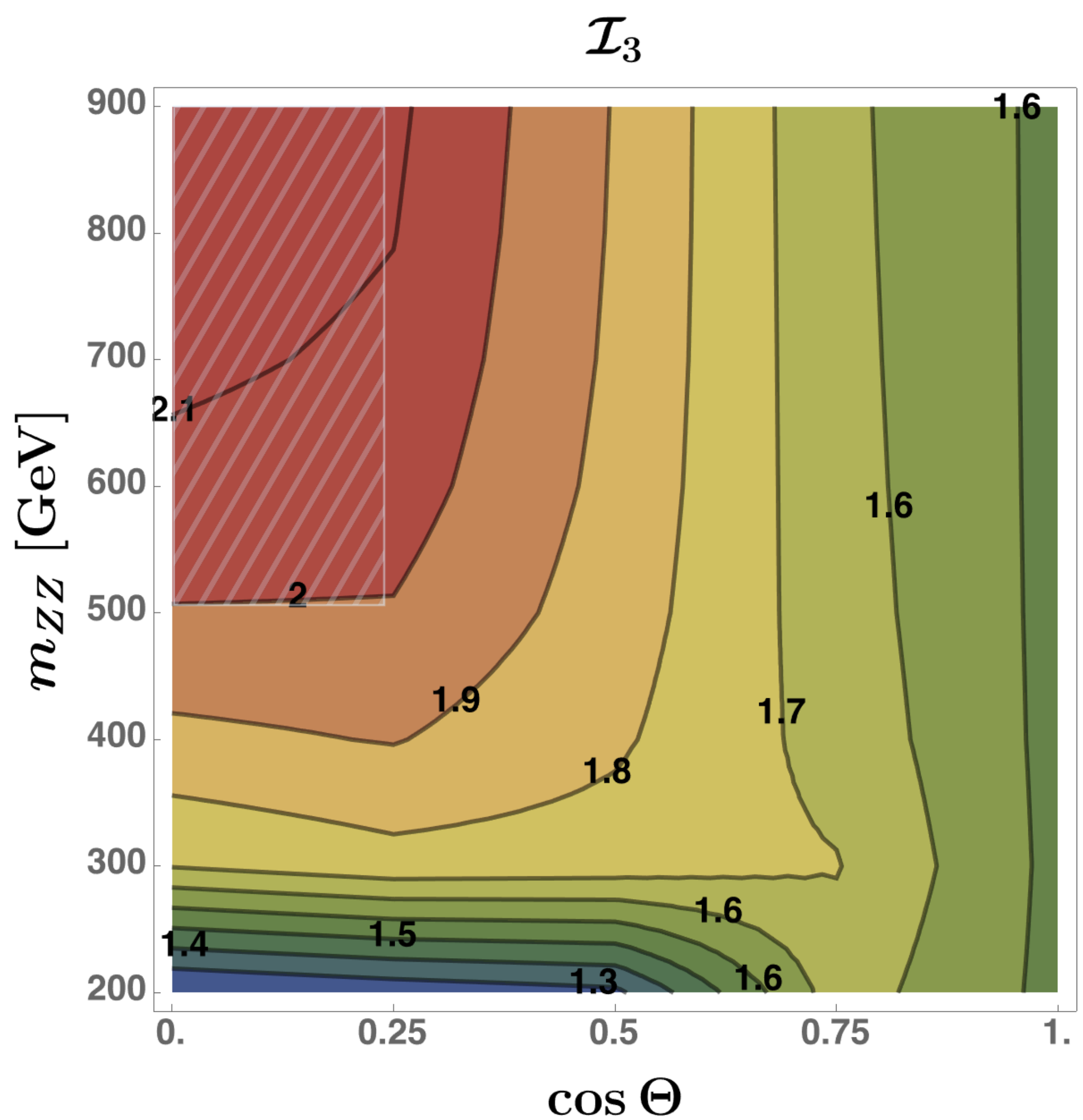
- The ZZ pair is always produced in a *mixed state* ($\text{Tr}(\rho^2)<1$)
- We find *$\mathcal{C}_2>0$* over all of the considered parameter space, witnessing that *the two Z bosons are always entangled*

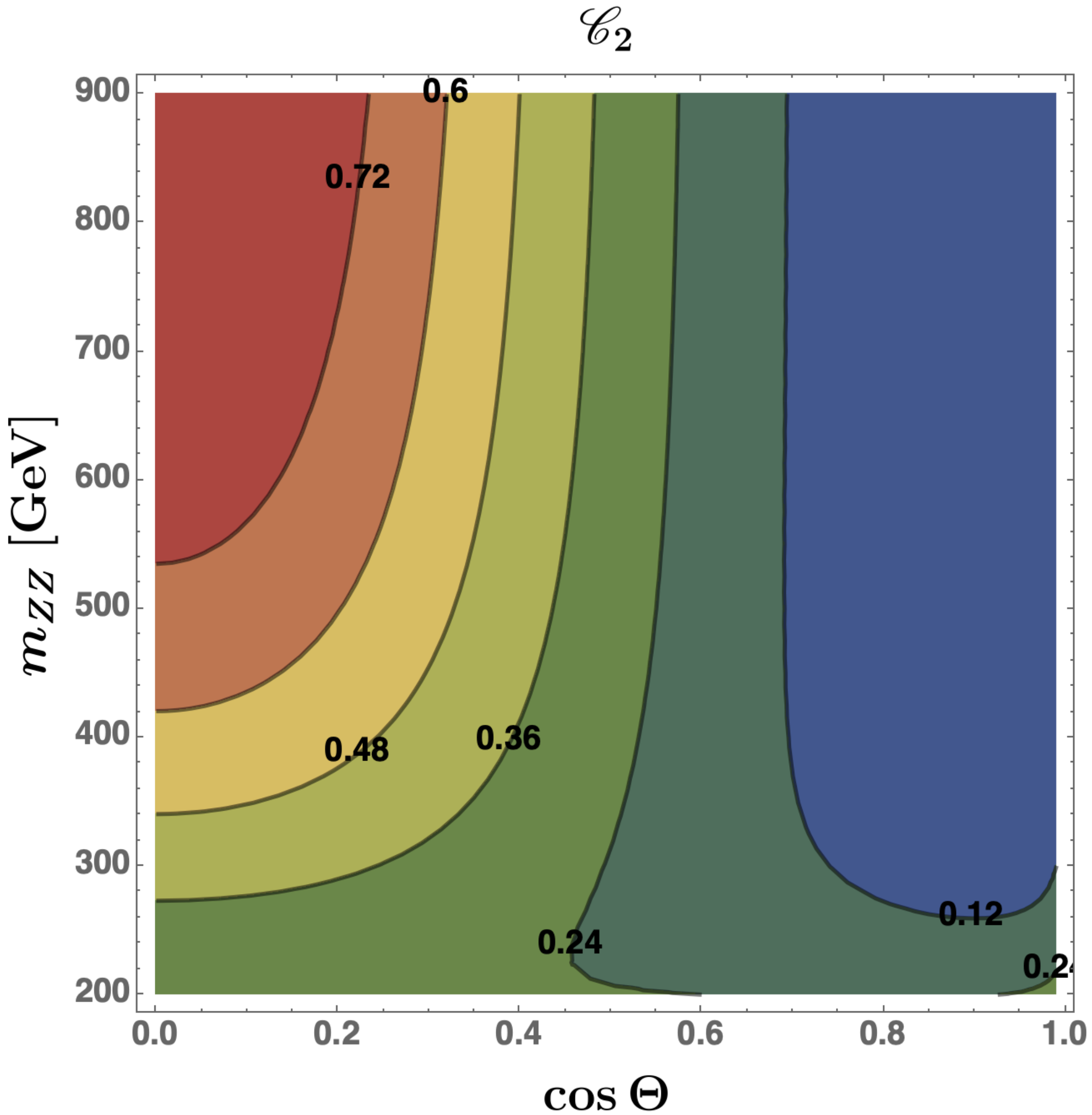
(all true also for WW and WZ)



- The ZZ pair is always produced in a *mixed state* (Tr(ρ^2)<1)
- We find $\mathcal{C}_2 > 0$ over all of the considered parameter space, witnessing that *the two Z bosons are always entangled*

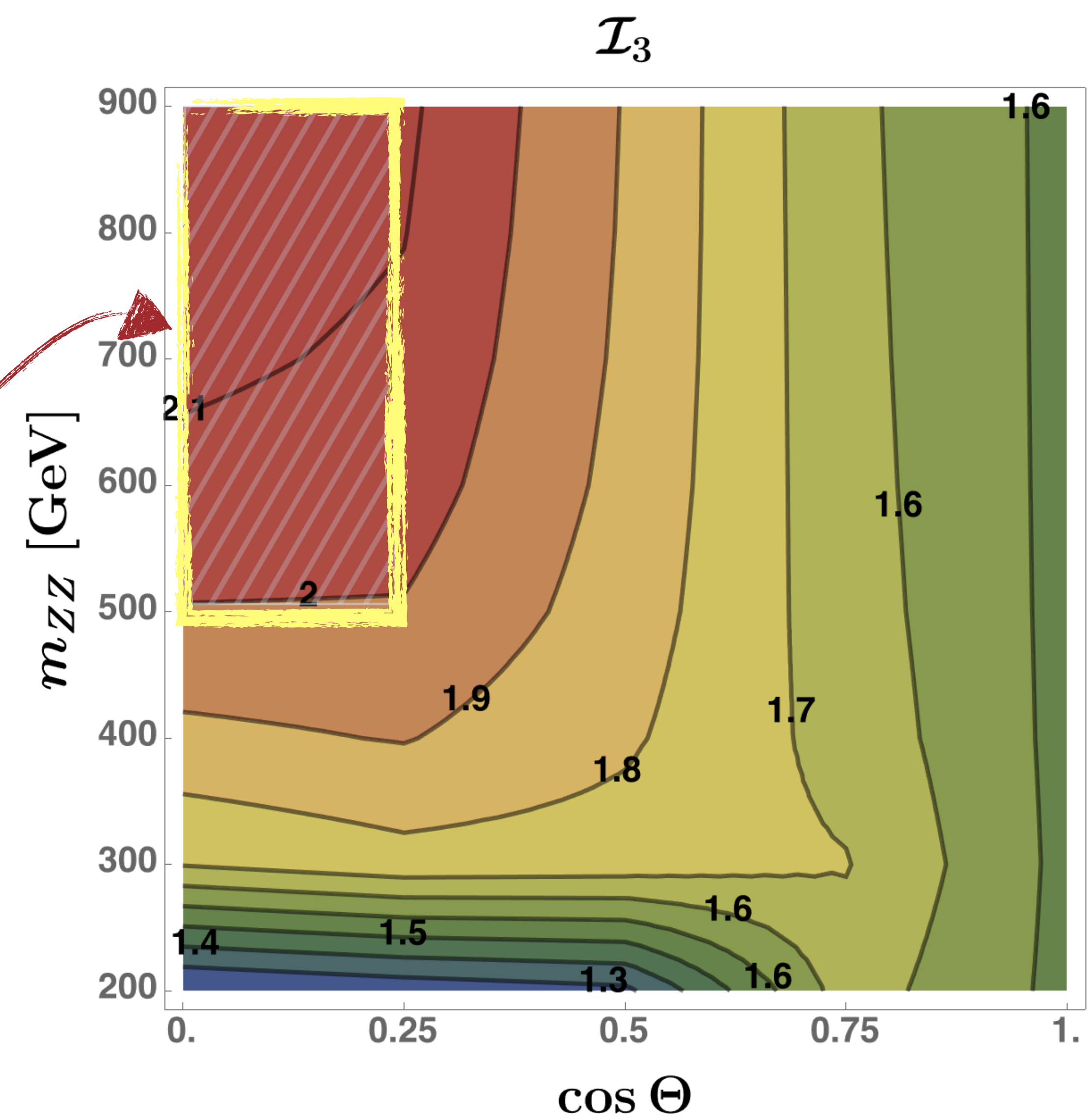
(all true also for WW and WZ)





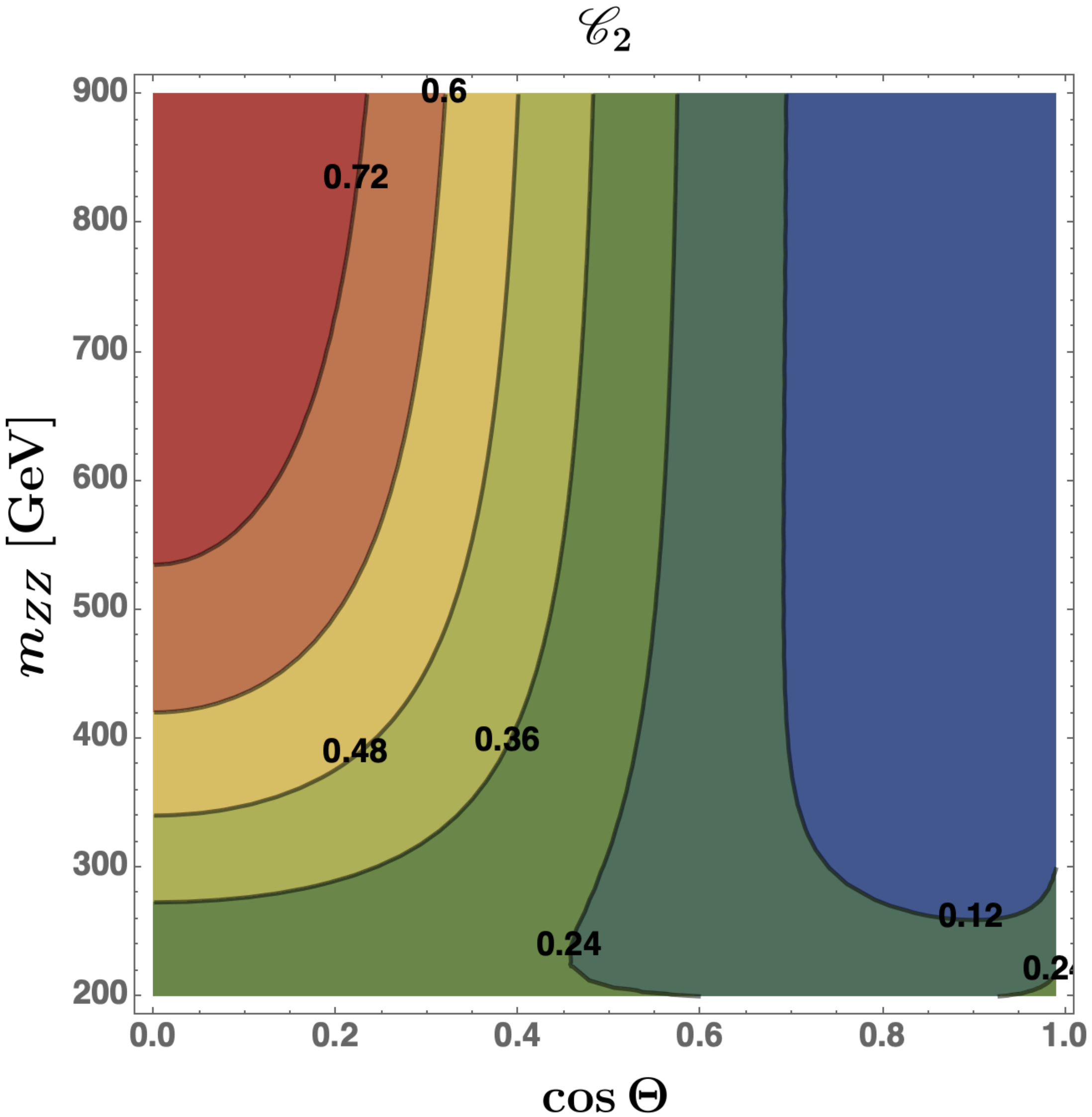
- The ZZ pair is always produced in a *mixed state* (Tr(ρ^2)<1)
- We find *$\mathcal{C}_2$>0* over all of the considered parameter space, witnessing that *the two Z bosons are always entangled*

(all true also for WW and WZ)



(#events computed with Madgraph assuming 70% efficiency for the identification of charged leptons and cuts as shown)

	(run2) $\mathcal{L} = 140\ \text{fb}^{-1}$	(Hi-Lumi) $\mathcal{L} = 3\ \text{ab}^{-1}$
events	4	77



- The ZZ pair is always produced in a *mixed state* (Tr(ρ^2)<1)
- We find *$\mathcal{C}_2$>0* over all of the considered parameter space, witnessing that *the two Z bosons are always entangled*

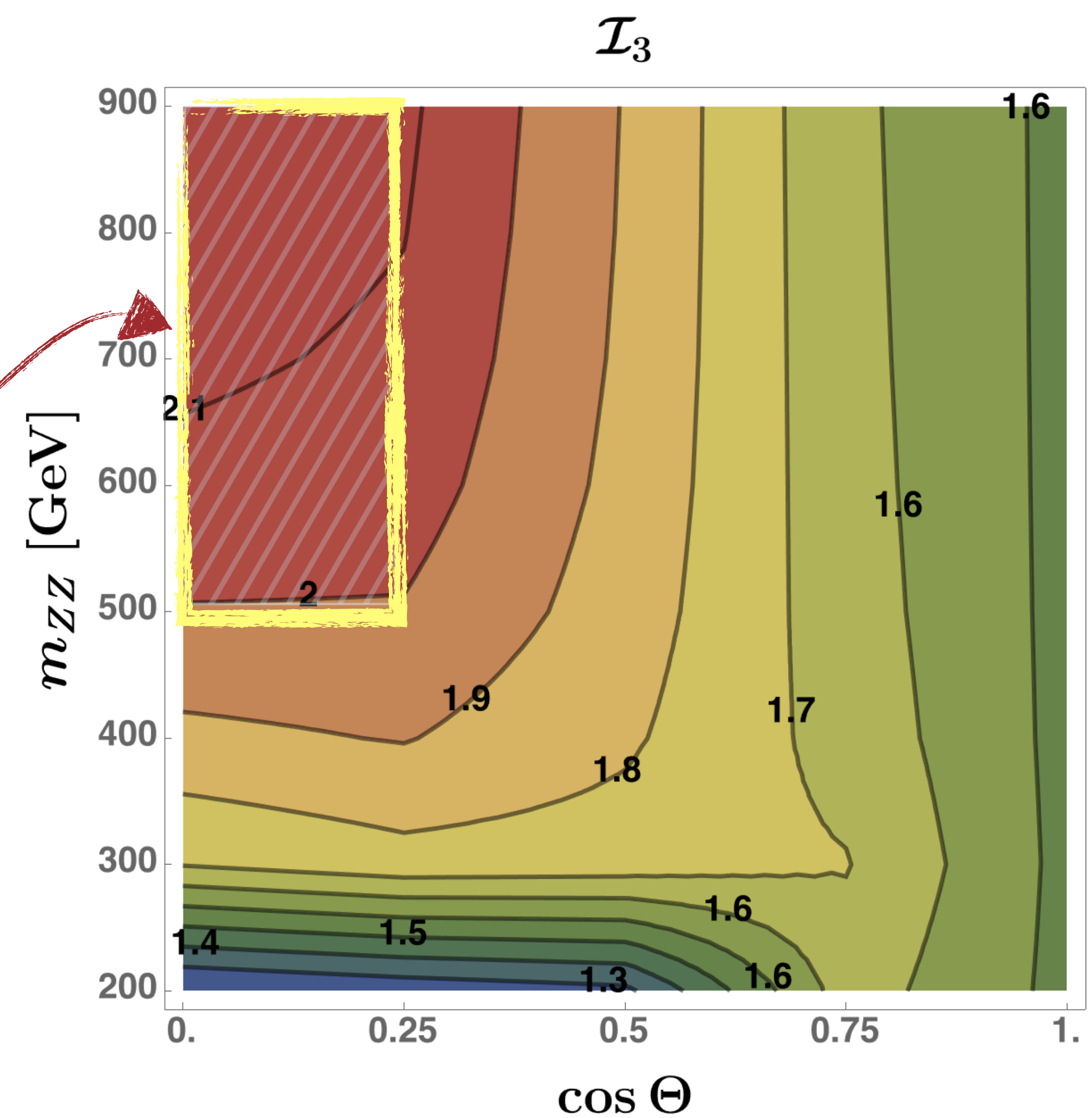
(all true also for WW and WZ)

(#events computed with Madgraph assuming 70% efficiency for the identification of charged leptons and cuts as shown)

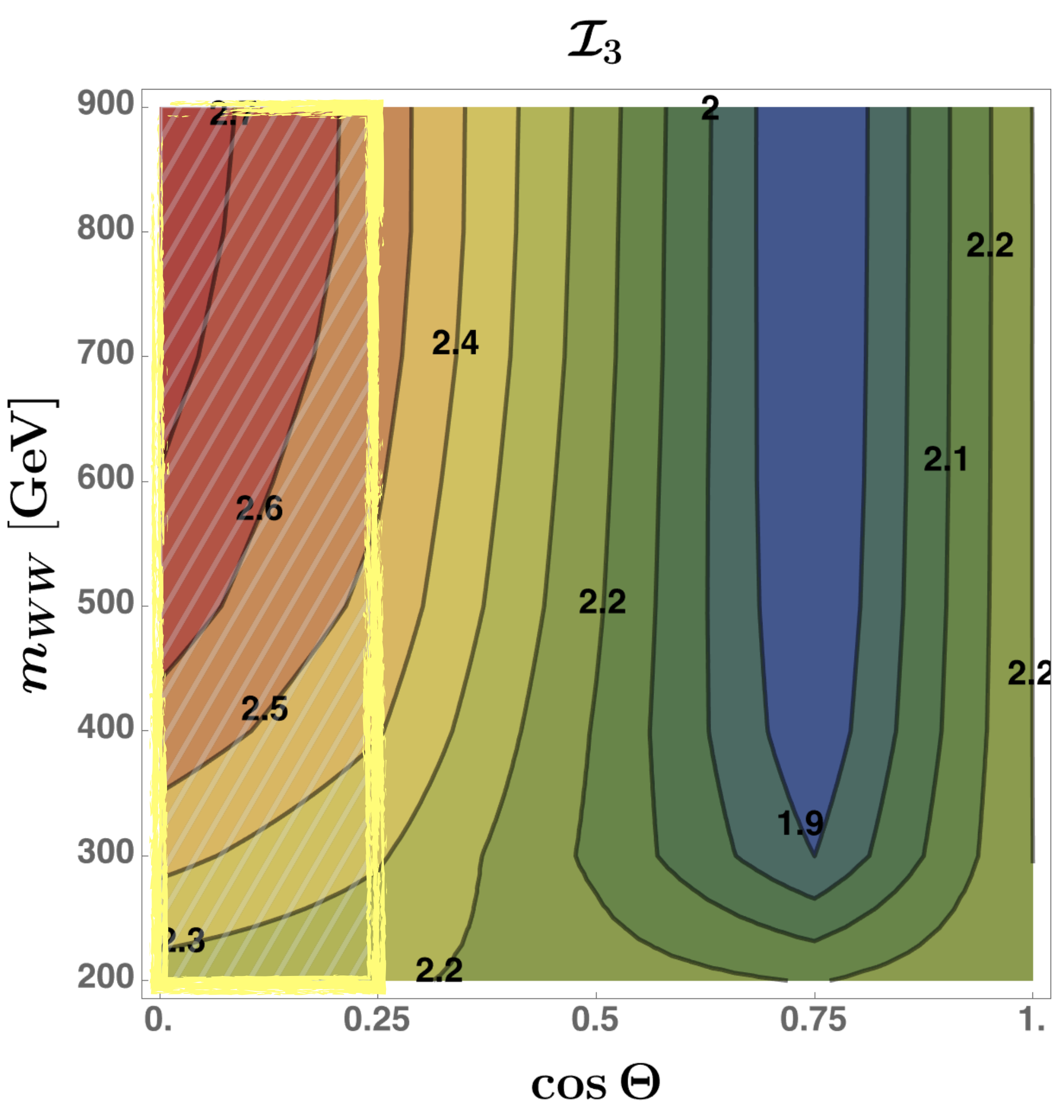
	(run2) $\mathcal{L} = 140\ \text{fb}^{-1}$	(Hi-Lumi) $\mathcal{L} = 3\ \text{ab}^{-1}$
events	4	77

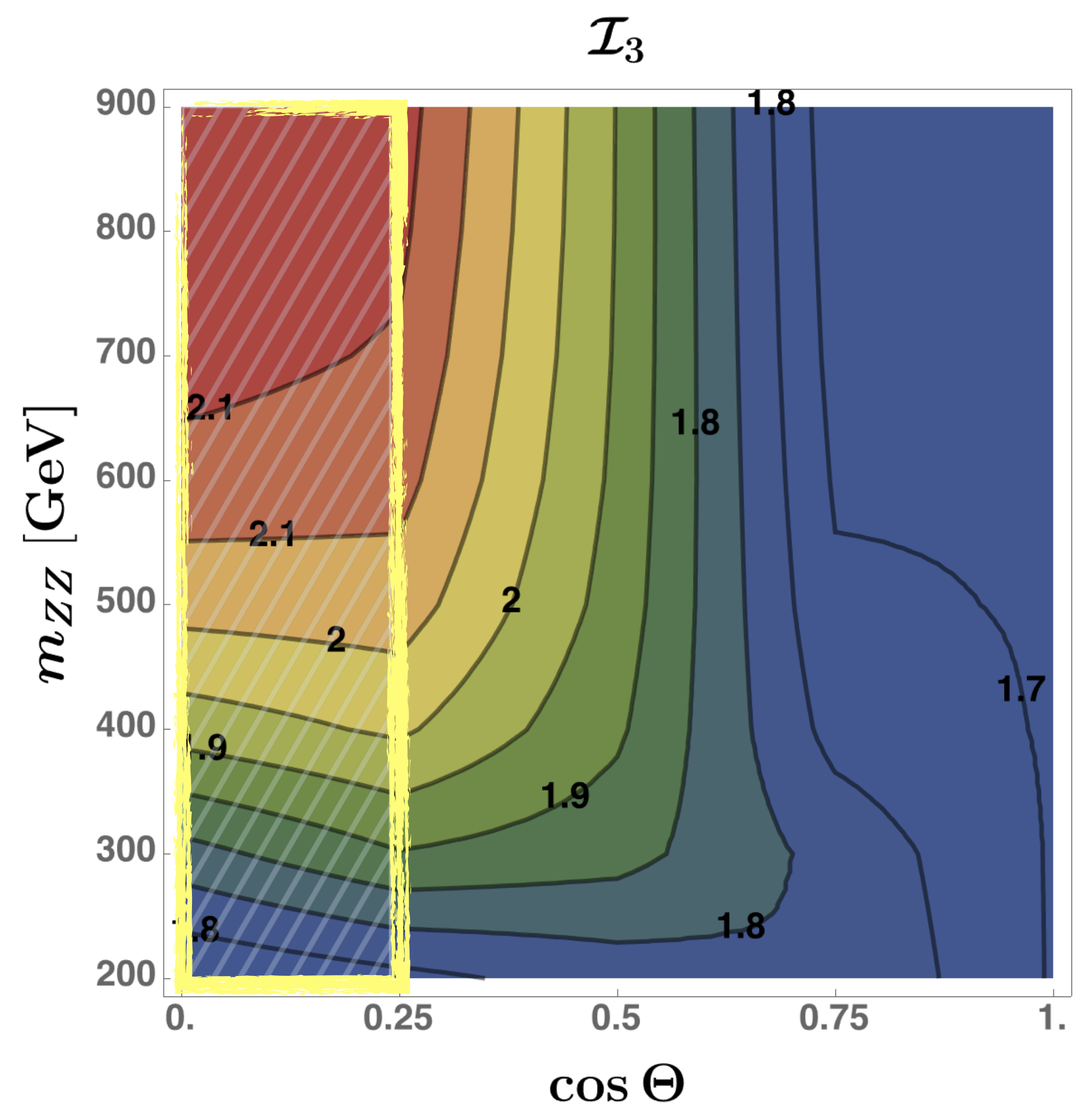
forget it

significance > 2 for rejecting $\mathcal{I}_3 \leq 2$ at Hi-Lumi via ZZ channel



Bell inequalities @ future lepton colliders

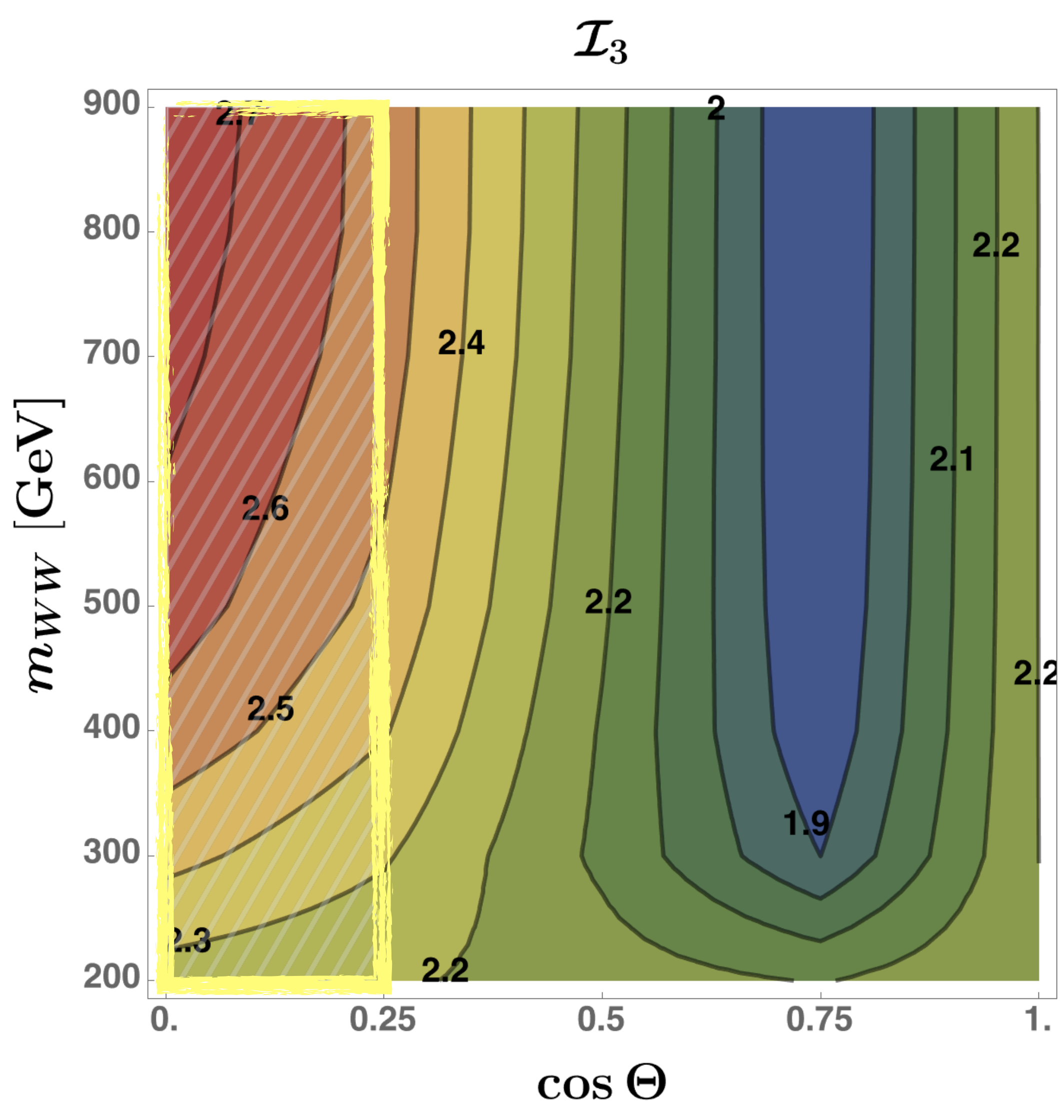


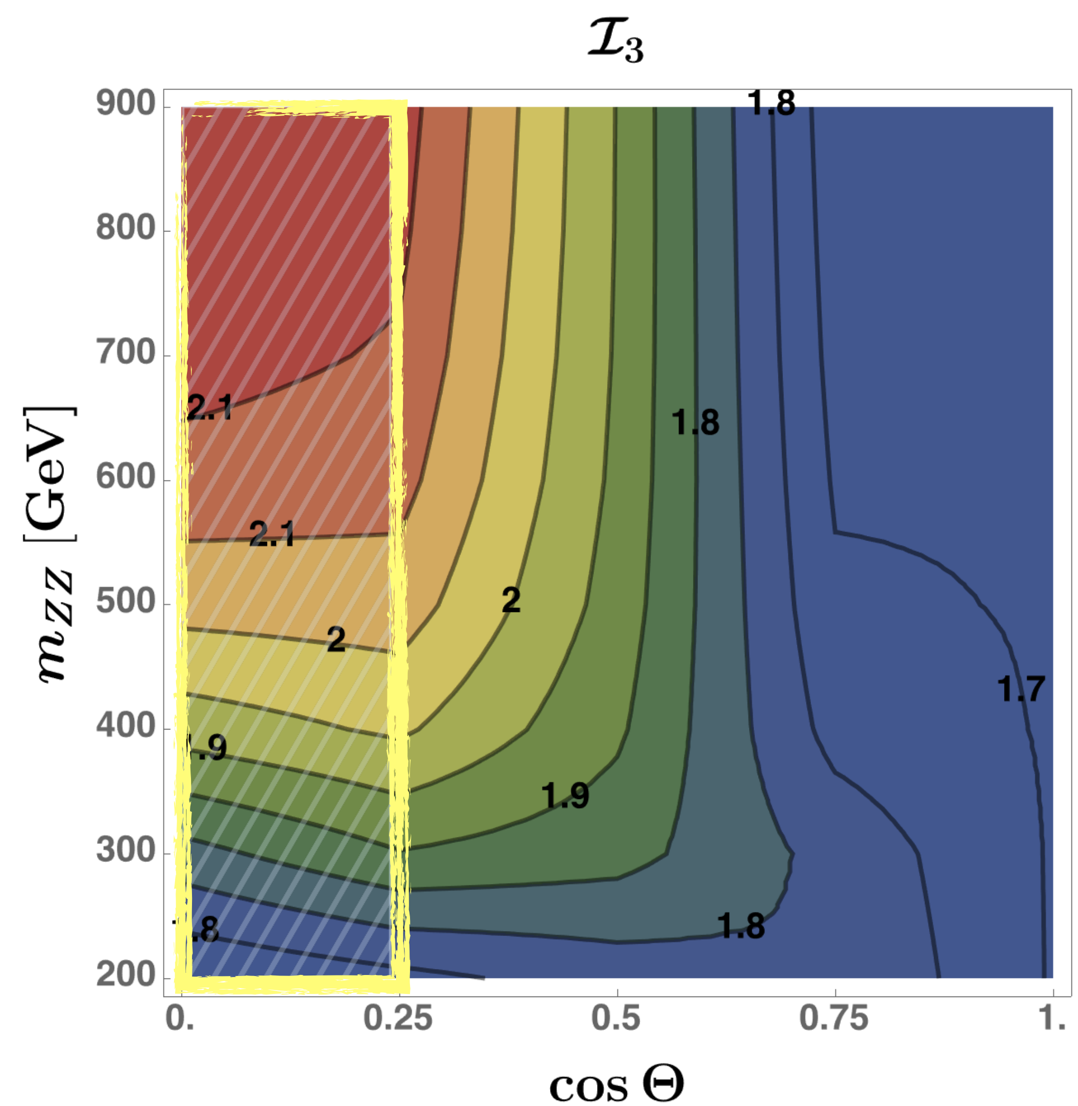


Number of events after cuts and efficiency

		$\ell^+\nu_\ell\,\ell^-\bar{\nu}_\ell$	$\ell^-\ell^+\ell^-\ell^+$
muon	($\mathcal{L} = 1$ ab^{-1}) 1 TeV	3.6×10^3	44
FCC	($\mathcal{L} = 1.5$ ab^{-1}) 364 GeV	5.8×10^4	748

Bell inequalities @ future lepton colliders





Number of events after cuts and efficiency

		$\ell^+\nu_\ell\,\ell^-\bar{\nu}_\ell$	$\ell^-\ell^+\ell^-\ell^+$
muon	($\mathcal{L} = 1$ ab^{-1}) 1 TeV	3.6×10^3	44
FCC	($\mathcal{L} = 1.5$ ab^{-1}) 364 GeV	5.8×10^4	748

…leading to a significance of…

~~$\mathcal{I}_3 \leq 2$~~	FCC	Muon
WW	*2*	*2*
ZZ	*4*	*2*

What to take home?

- The most promising process for *testing Bell inequalities* with qutrits is the *resonant Higgs boson decay* $p\,p \to H \to ZZ$. The *WW channel* could become competitive if *progress in the reconstruction of neutrino momenta* is made

What to take home?

- The most promising process for *testing Bell inequalities* with qutrits is the *resonant Higgs boson decay* $p\,p \to H \to ZZ$. The *WW channel* could become competitive if *progress in the reconstruction of neutrino momenta* is made
- *Electroweak quark fusion* yielding ZZ final states will become competitive *only with LHC Hi-Lumi* data. *Future lepton colliders* are able to probe Bell inequalities with *WW and ZZ final states*

What to take home?

- The most promising process for *testing Bell inequalities* with qutrits is the *resonant Higgs boson decay* $p\,p \to H \to ZZ$. The *WW channel* could become competitive if *progress in the reconstruction of neutrino momenta* is made
- *Electroweak quark fusion* yielding ZZ final states will become competitive *only with LHC Hi-Lumi* data. *Future lepton colliders* are able to probe Bell inequalities with *WW and ZZ final states*
- The entanglement content of a di-boson state can be effectively probed by quantifying the *entanglement entropy* or the *concurrence* of the system. These observables could be employed for *BSM searches*.

What to take home?

- The most promising process for *testing Bell inequalities* with qutrits is the *resonant Higgs boson decay* $p\,p \to H \to ZZ$. The *WW channel* could become competitive if *progress in the reconstruction of neutrino momenta* is made
- *Electroweak quark fusion* yielding ZZ final states will become competitive *only with LHC Hi-Lumi* data. *Future lepton colliders* are able to probe Bell inequalities with *WW and ZZ final states*
- The entanglement content of a di-boson state can be effectively probed by quantifying the *entanglement entropy* or the *concurrence* of the system. These observables could be employed for *BSM searches*.
- Dear *experimentalists*, please:
  - look into reconstructing the density matrix of these di-boson systems
  - get better at dealing with neutrino momenta
  - please please please perform *full simulations* of these processes so we learn the *actual significances*.

Cheers!