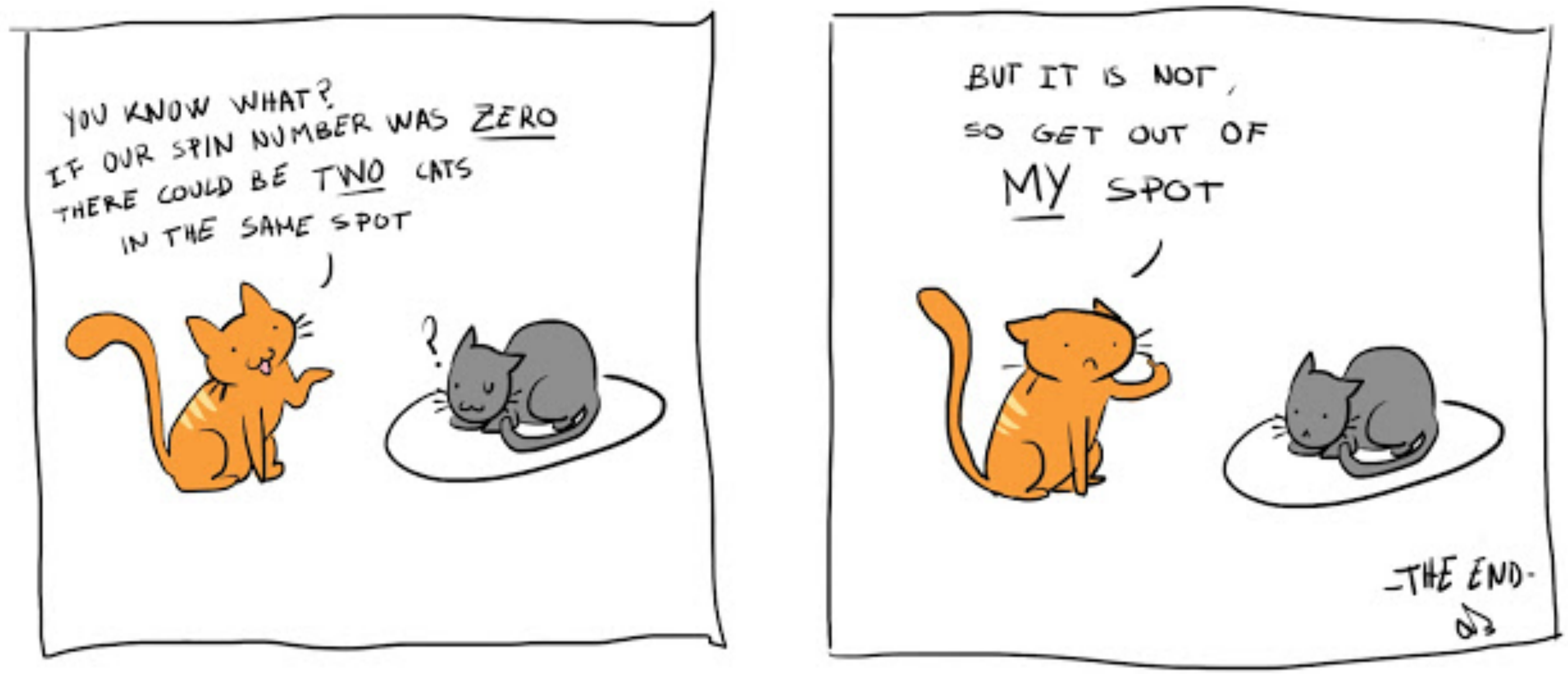
Dark Matter scattering in low threshold detectors

Simon Knapen

LBNL



http://dingercatadventures.blogspot.com/2012/08/

SK, J. Kozaczuk, T. Lin: arXiv 2104.12786, 2101.08275, 2011.09496
B. Campbell-Deem, SK, T. Lin, E. Villarama: arXiv 2205.02250

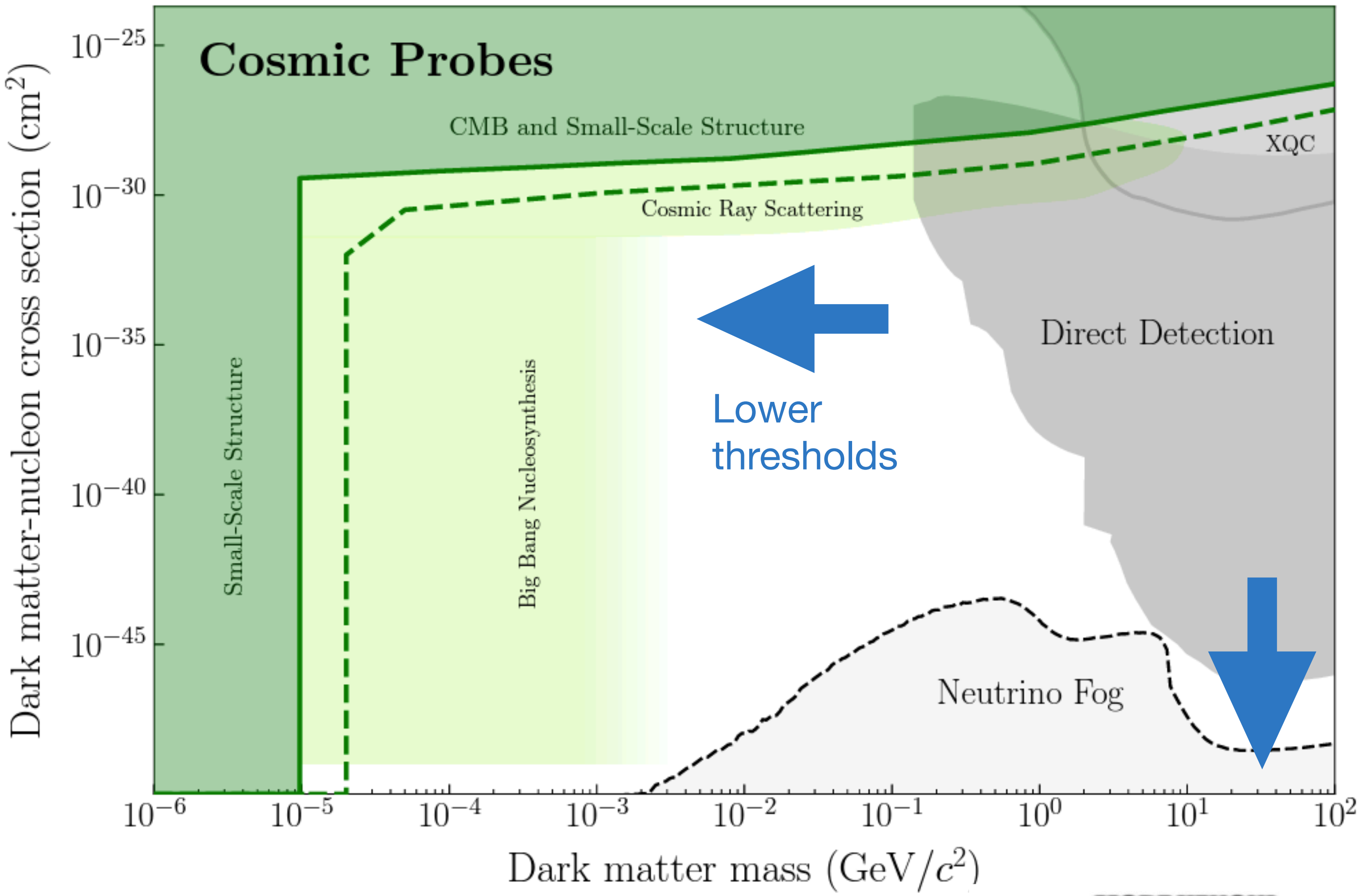
BERKELEY LAB

Light dark matter direct detection

What do we need?

Experiment:*

1. Ultra-low threshold calorimeters
   (Spice/HeRALD, …)
2. Single electron detectors
   (SENSEI, DAMIC, superCDMS, …)



* Gross oversimplification, but focusing on theory for this talk
More on experiment & references in back-up slides

Plot from arXiv 2211.09978

Light dark matter direct detection

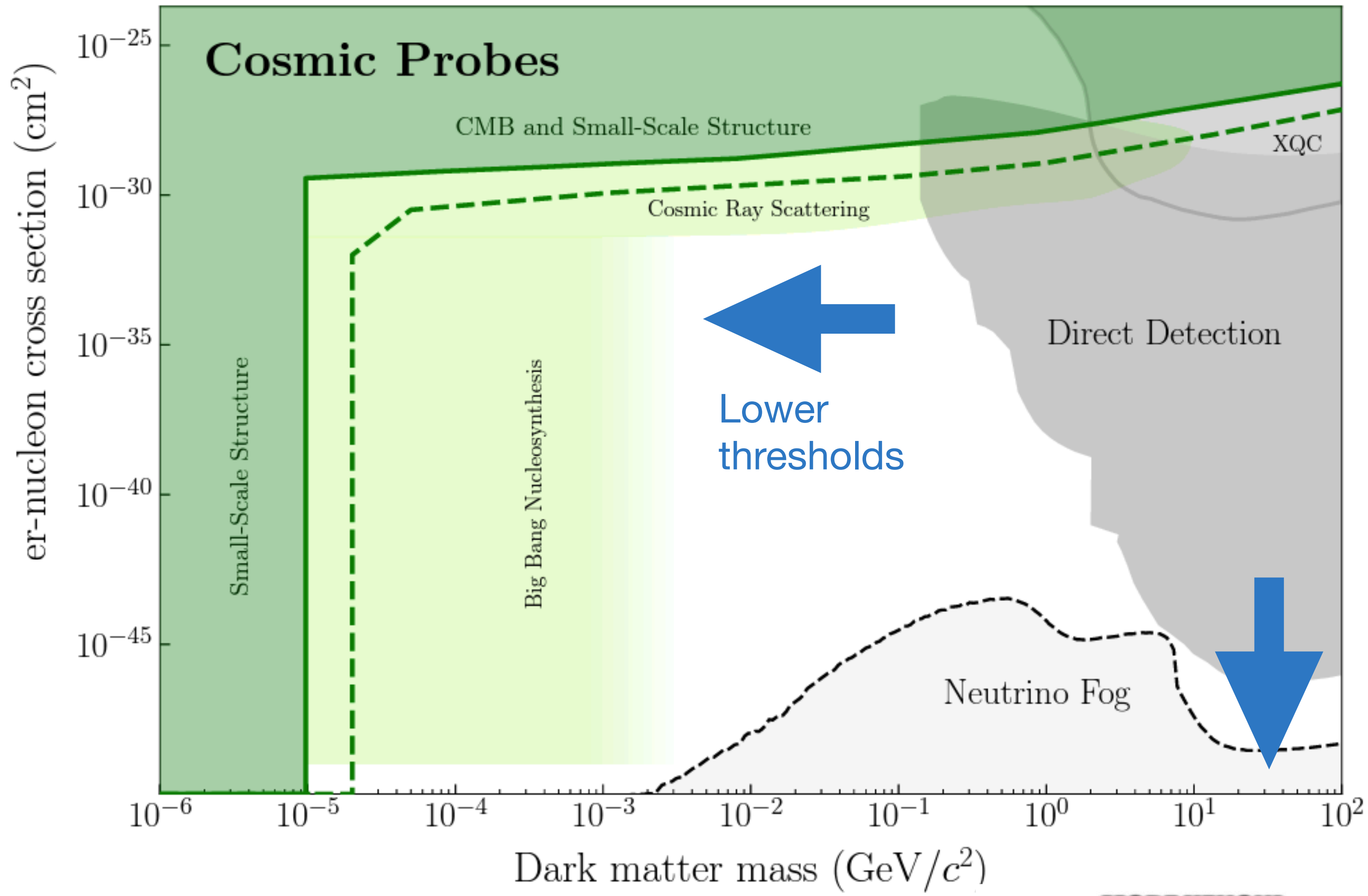
What do we need?

Experiment:*

1. Ultra-low threshold calorimeters
   (Spice/HeRALD, …)
2. Single electron detectors
   (SENSEI, DAMIC, superCDMS, …)

Theory:

1. Models
   (constraints are complicated)
2. Rate calculations
   (Collective effects important)
3. Background predictions



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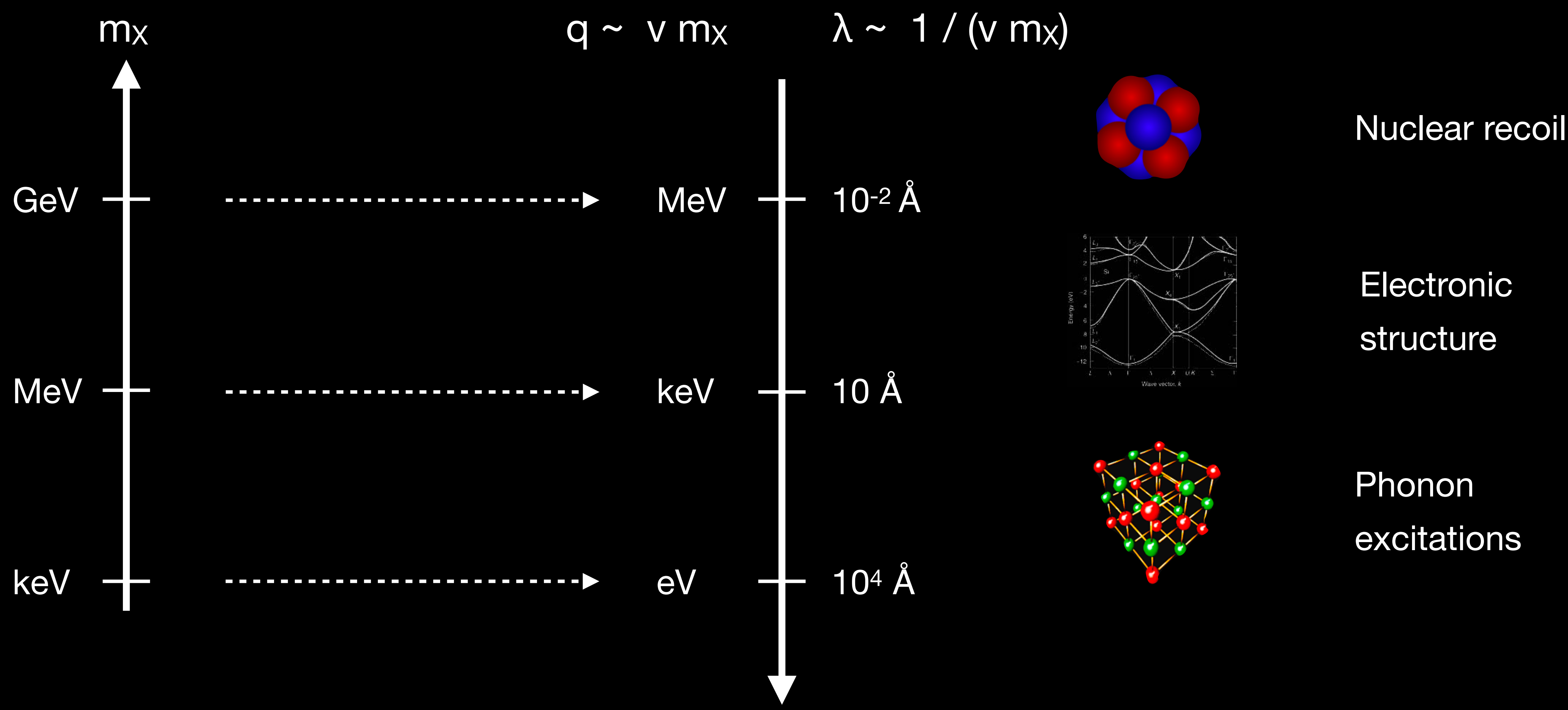
The need for theory



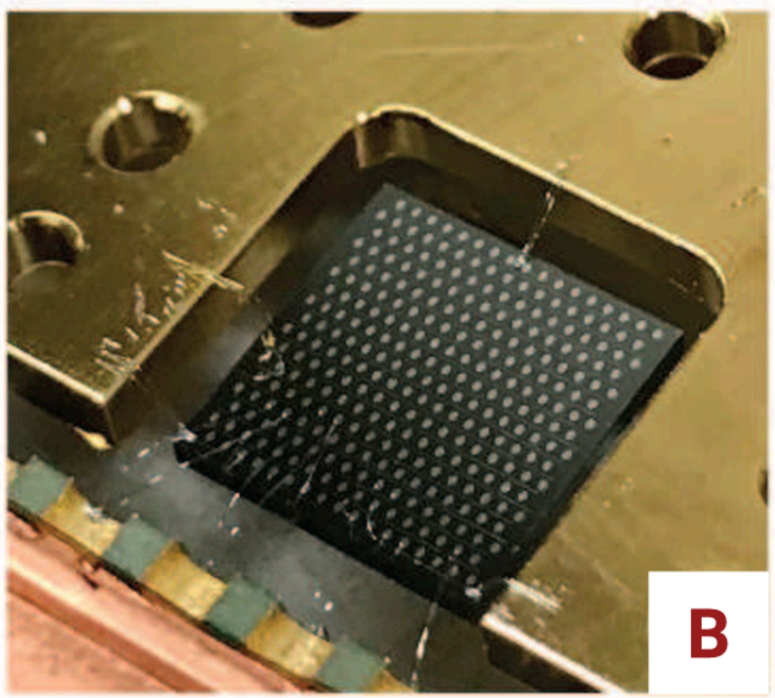
For light DM deBroglie wavelength >> interatomic spacing

↓

"Billiard ball" nuclear recoil not applicable!

Calculations needed

Phonon signals



Electronic signals

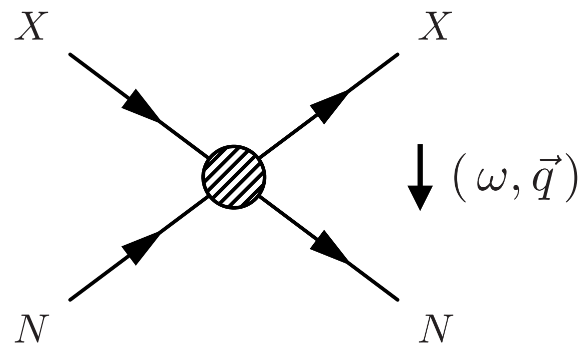
SPICE prototype detector
R. Anthony-Petersen et. al. arXiv 2208.02790

SENSEI detector
L. Barak et. al. arXiv 2004.11378

Phonon Effective Theory

Nuclear recoil

$$\omega = \frac{q^2}{2m_N}$$



SK, T. Lin, M. Pyle, K. Zurek: arXiv 1712.06598
S. Griffin, SK, T. Lin, M. Pyle, K. Zurek: arXiv 1807.10291

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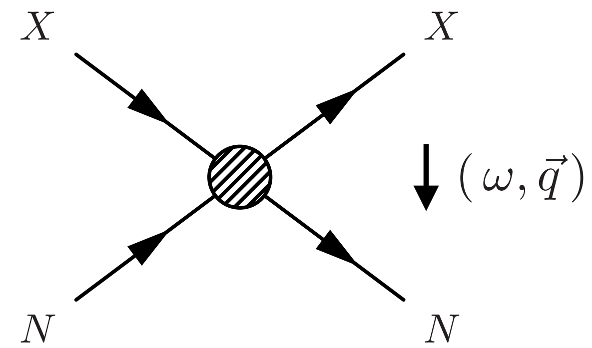
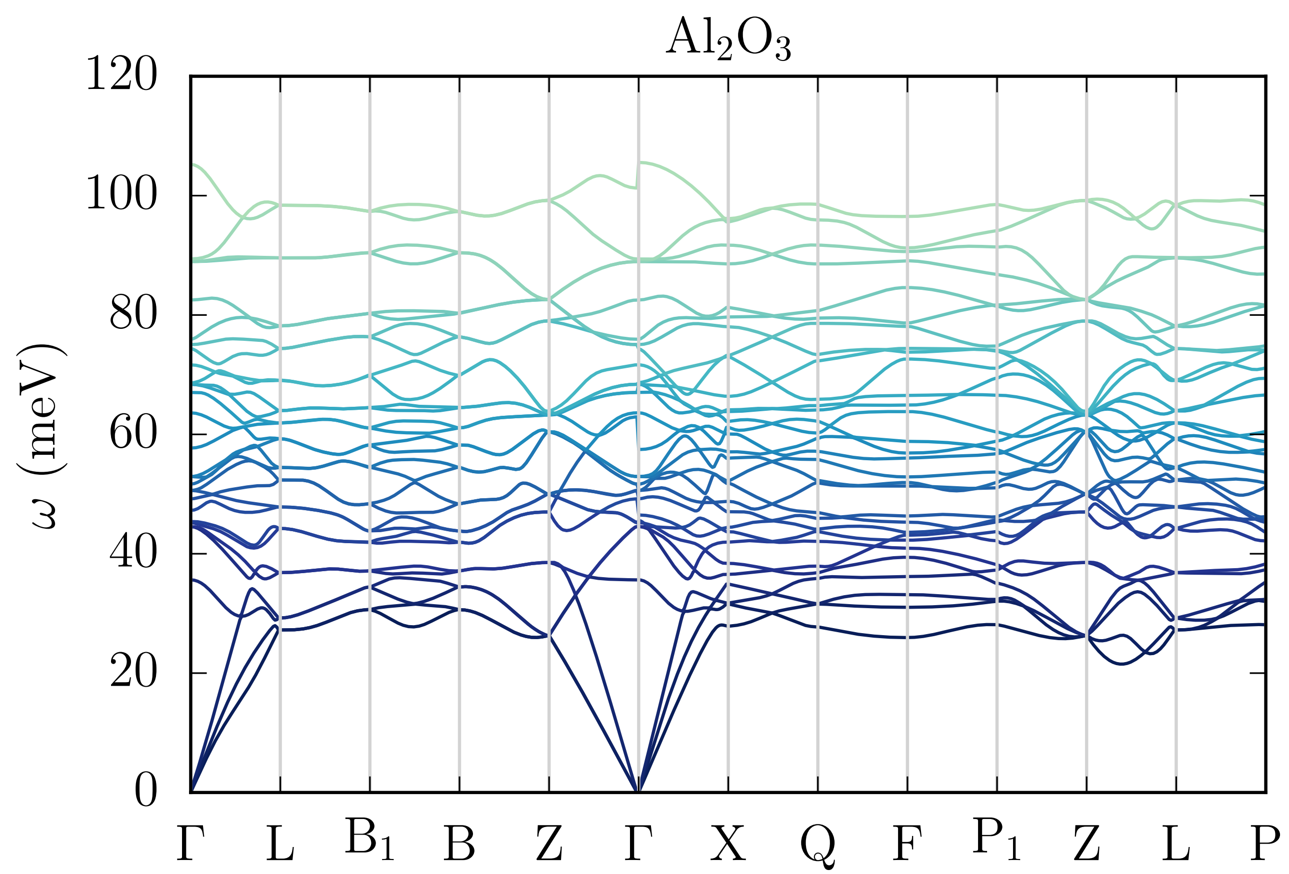


<u>Phonon regime</u>

$q \ll \sqrt{2m_N\omega}$ → **Momentum exchange is a good expansion parameter**
(phonons are goldstones, similar to chiral perturbation theory)



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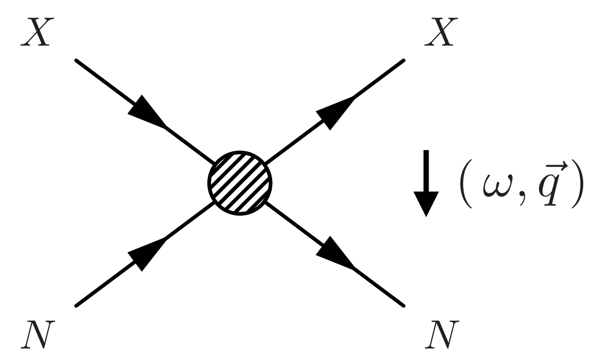

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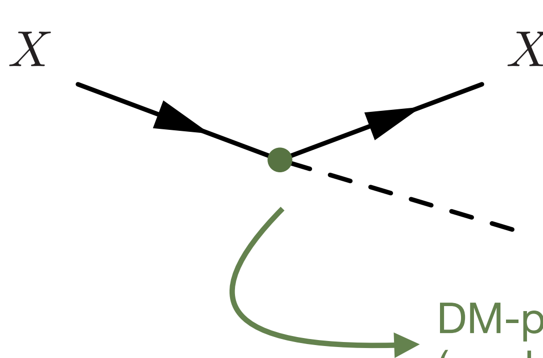
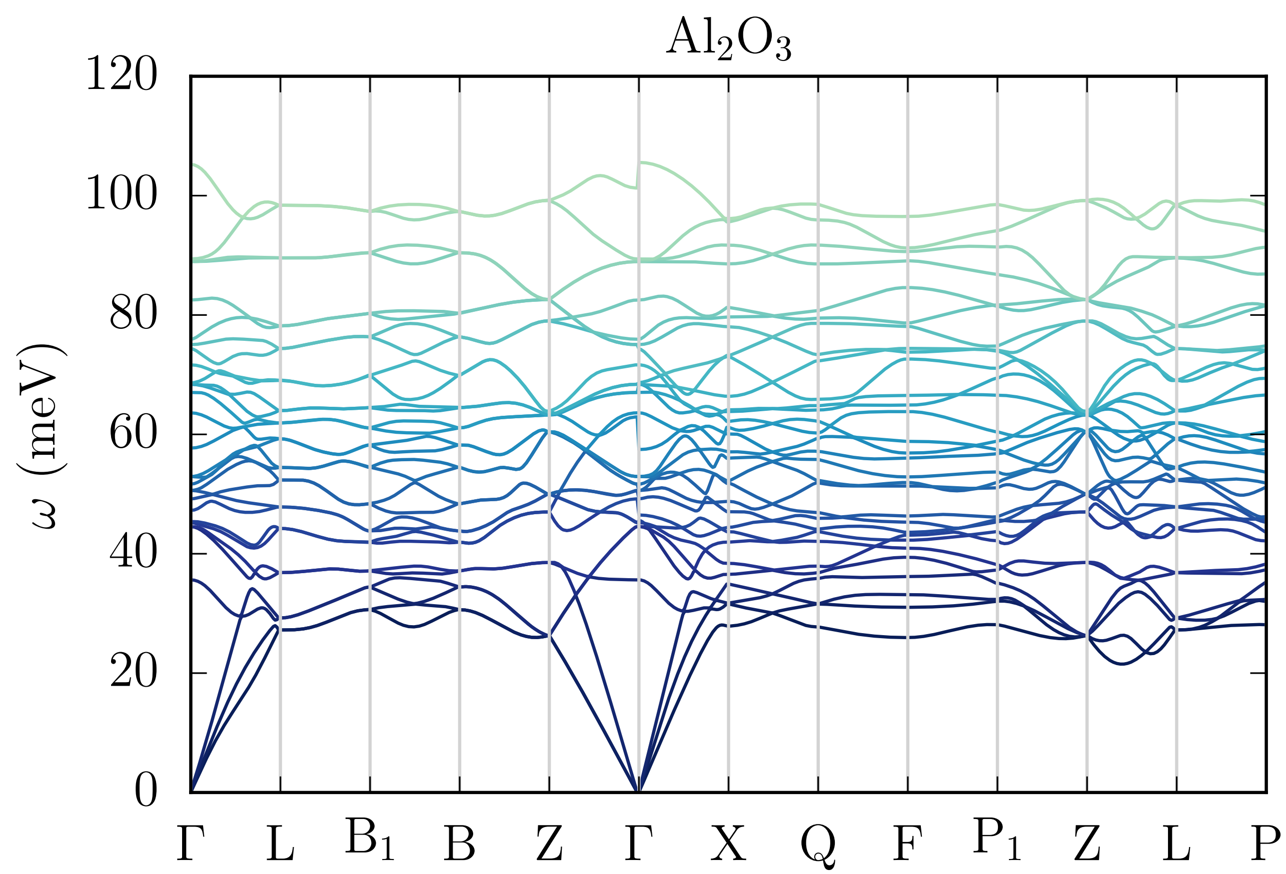
Leading Order



$$\mathcal{O}(q),\ \mathcal{O}(q^2) \text{ or } \mathcal{O}(q^4)$$

(Depends on DM model & phonon branch)



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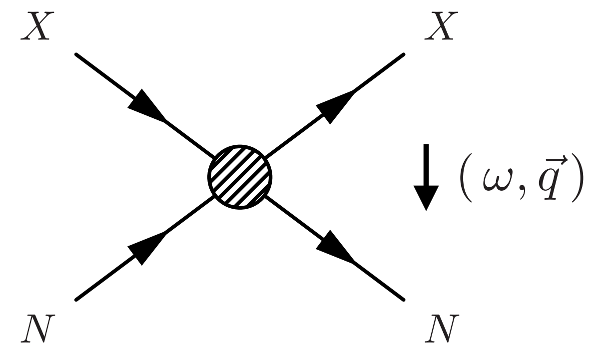

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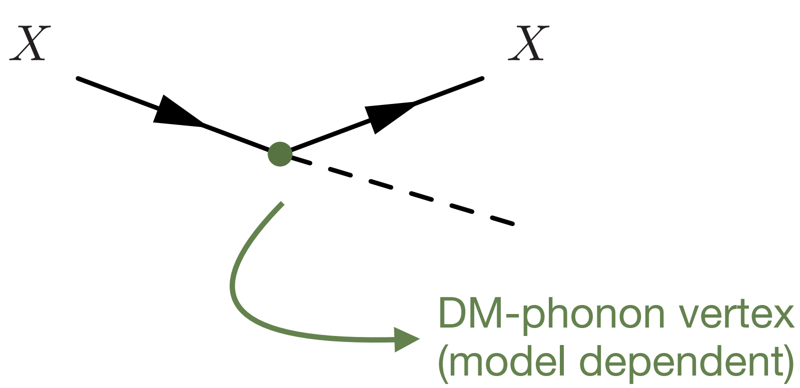

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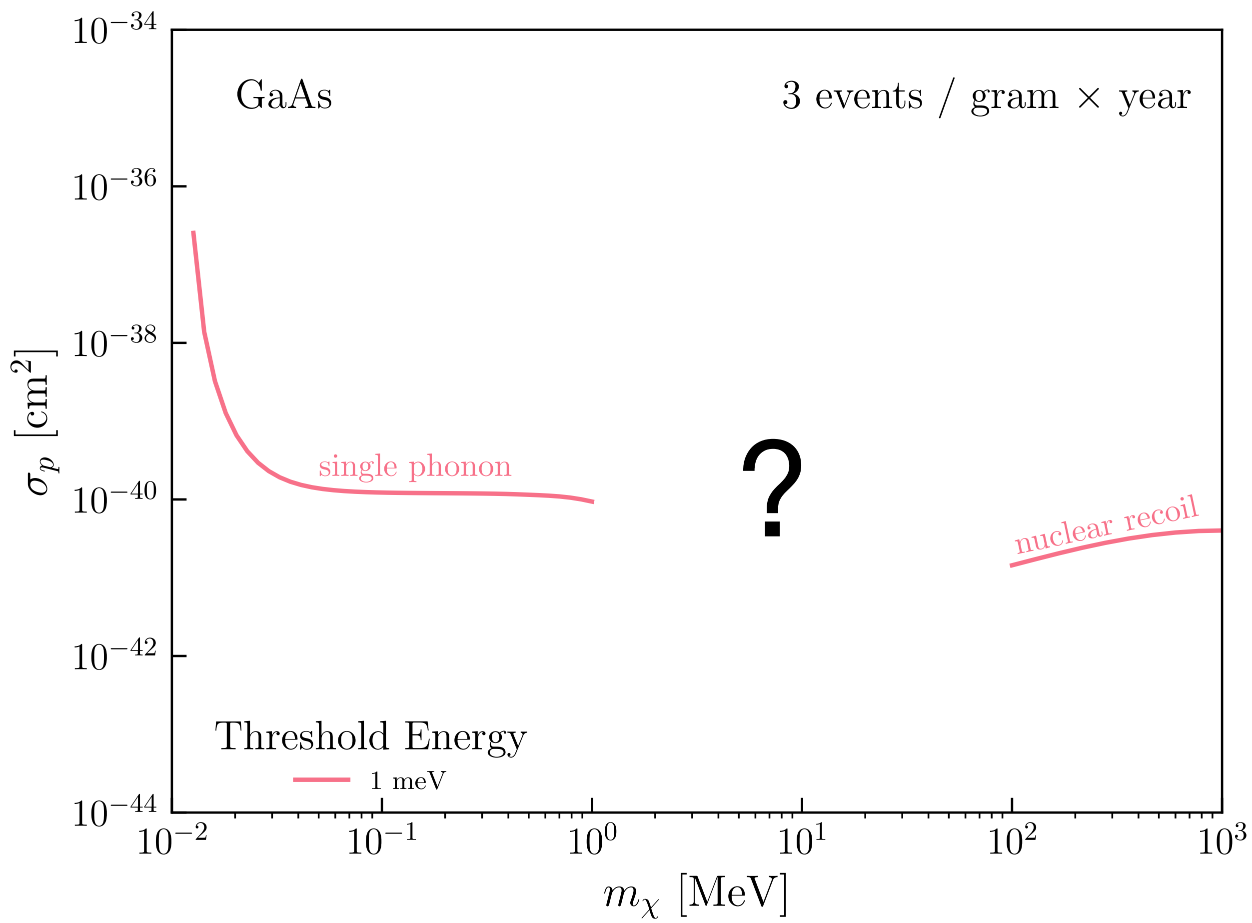

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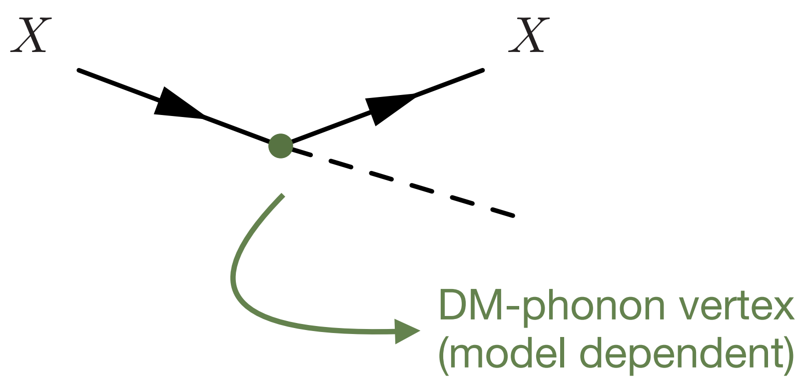
LO insufficient for m_χ > 1 MeV!

Phonon Effective Theory

LO



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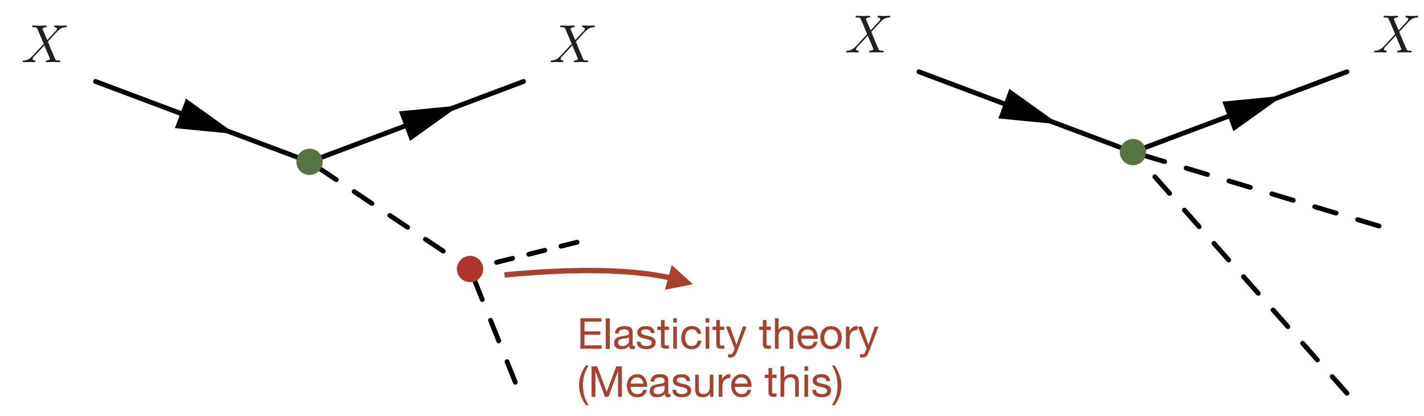
NLO

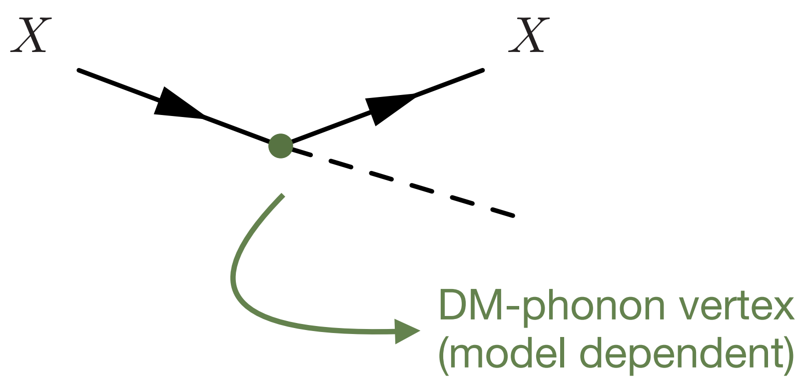


$\mathcal{O}(q^4)$

B. Campbell-Deem, P. Cox, SK, T. Lin, T. Melia : 1911.03482

Phonon Effective Theory

LO



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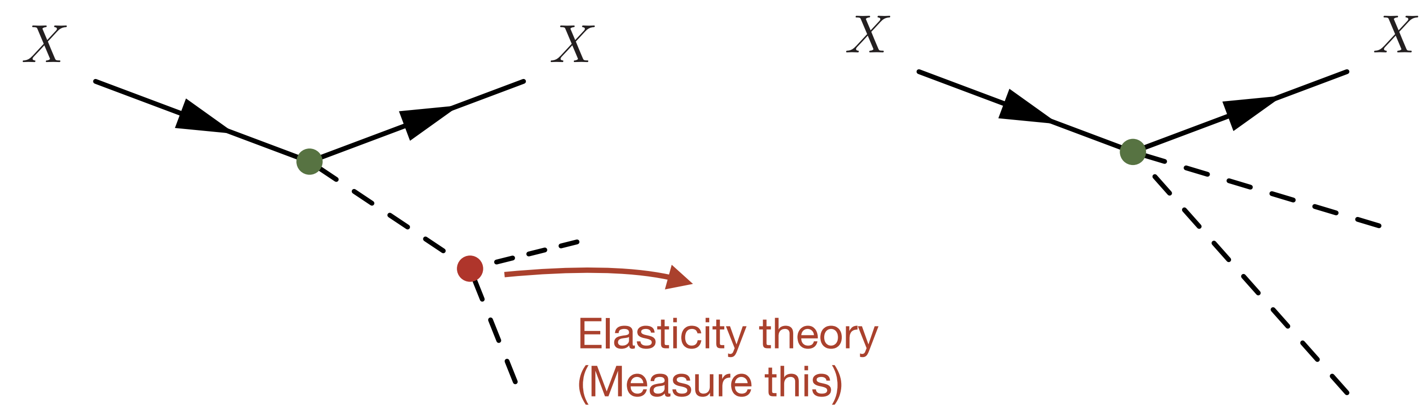

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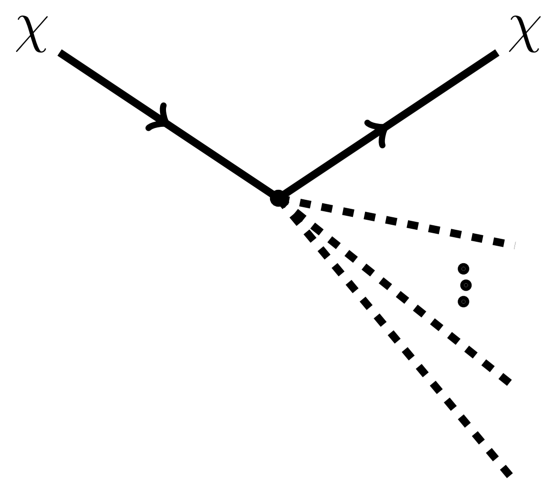
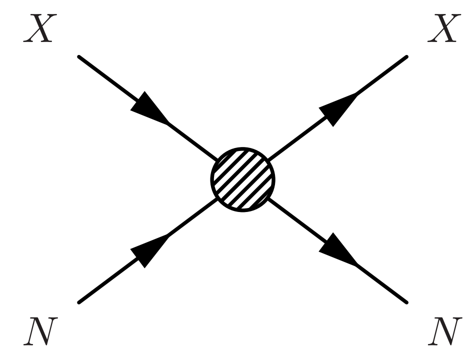
N^{n}LO



$\mathcal{O}(q^{2n})$

N$^\infty$LO = nuclear recoil



$$\sim \delta\left(\omega - \frac{q^2}{2m_N}\right)$$

Brian Campbell-Deem (UCSD)

Ethan Villarama (UCSD)

B. Campbell-Deem, SK, T. Lin, E. Villarama: arXiv 2205.02250

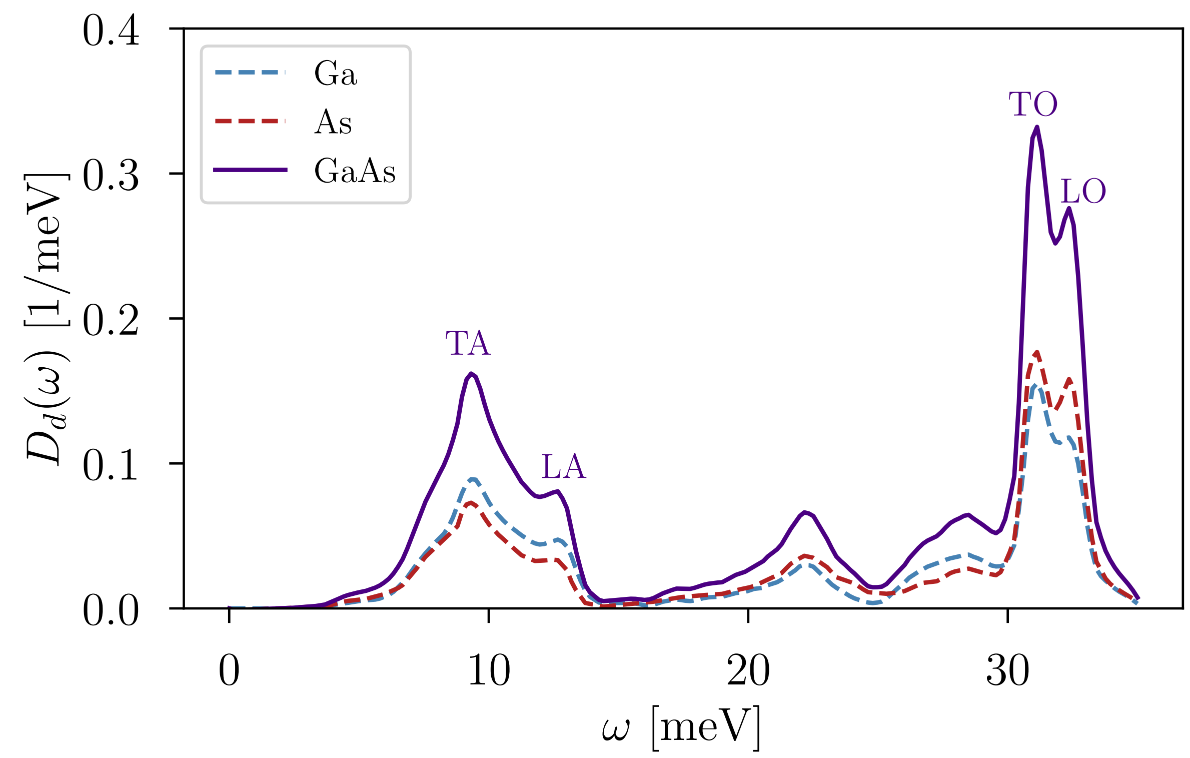
Results

d: labels atoms (e.g. Ga and As)
n: number of phonons

Inclusive all orders result

$$\frac{d\sigma}{d^3\mathbf{q}d\omega} \sim \sum_d^{\mathfrak{n}} A_d^2 e^{-2W_d(\mathbf{q})} \sum_n \left(\frac{q^2}{2m_d}\right)^n \frac{1}{n!} \left(\prod_{i=1}^n \int d\omega_i \frac{D_d(\omega_i)}{\omega_i}\right) \delta\left(\sum_j \omega_j - \omega\right).$$

Partial density of states



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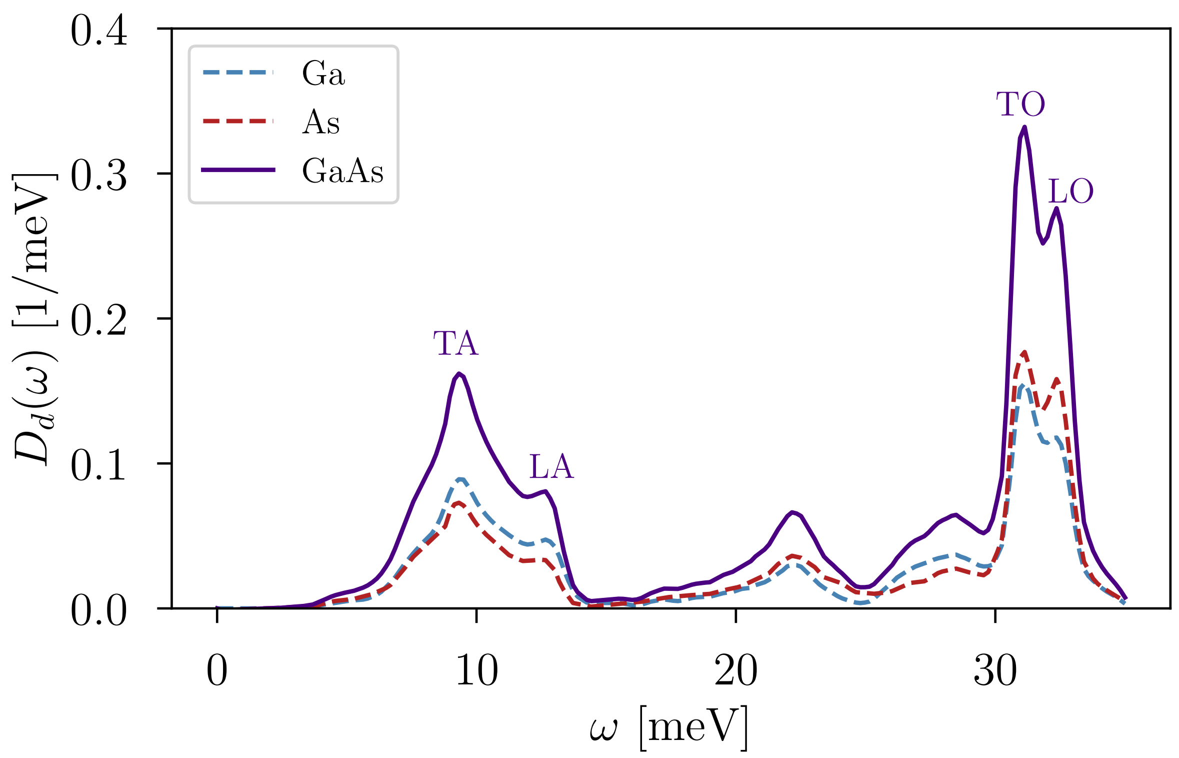
Partial density of states

$q \gg \sqrt{2\omega m_d}$

$$\frac{d\sigma}{d^3\mathbf{q}d\omega} \sim \sum_d^{\mathfrak{n}} A_d^2 \sqrt{\frac{2\pi}{\Delta_d^2}} \exp\left(-\frac{\left(\omega - \frac{q^2}{2m_d}\right)^2}{2\Delta_d^2}\right)$$



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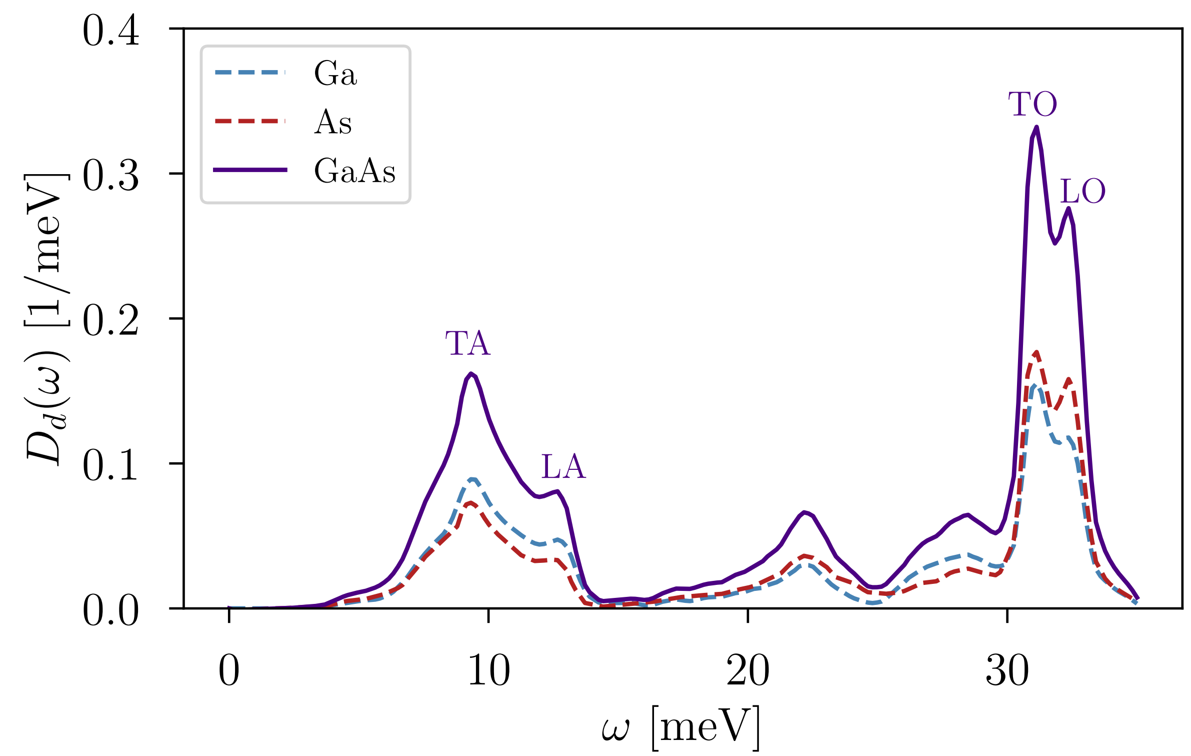


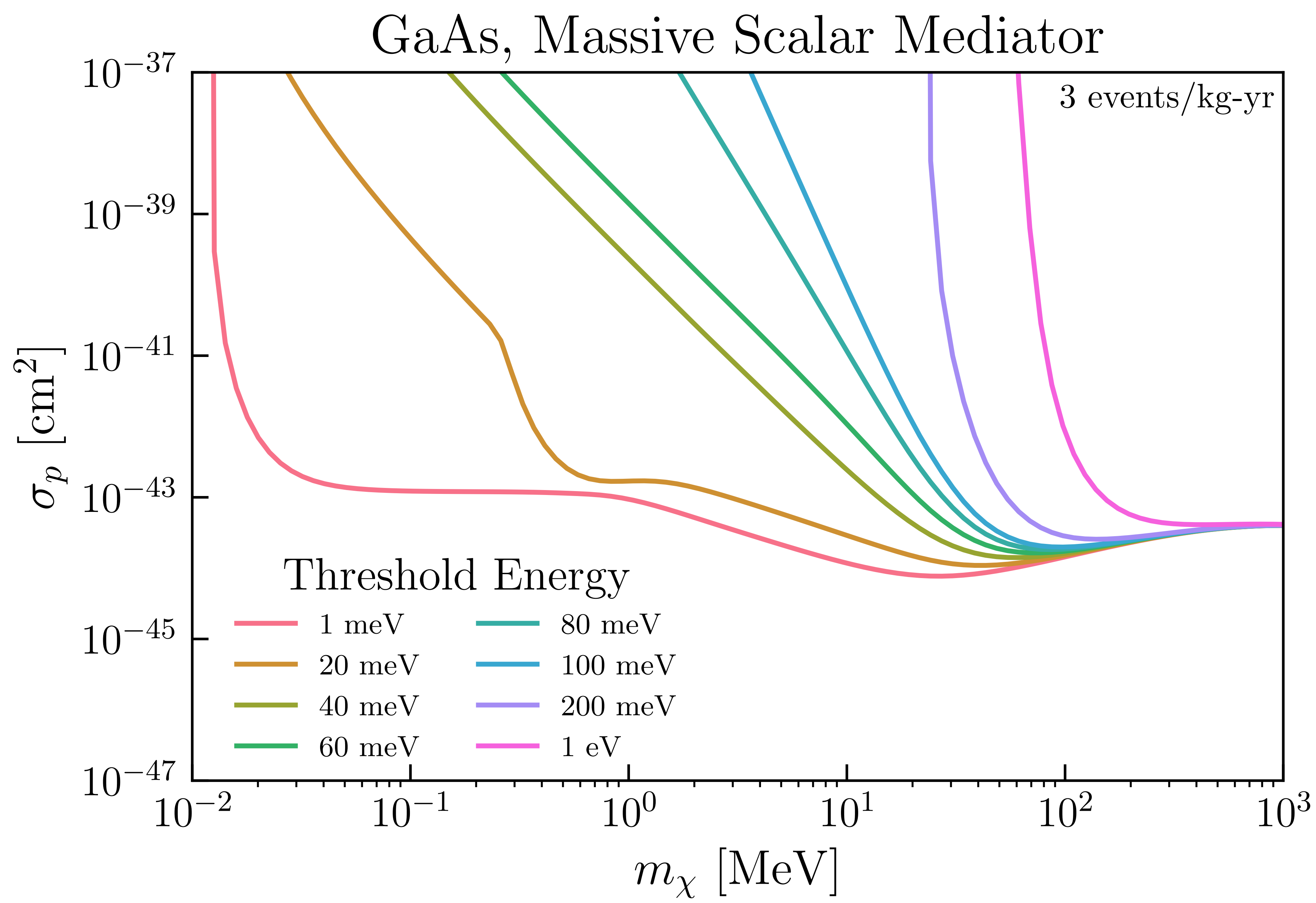
$q \ggg \sqrt{2\omega m_d}$

$$\frac{d\sigma}{d^3\mathbf{q}d\omega} \sim \sum_d^{\mathfrak{n}} A_d^2 \times \delta\left(\omega - \frac{q^2}{2m_d}\right)$$

Free nuclear recoil limit

B. Campbell-Deem, SK, T. Lin, E. Villarama: arXiv 2205.02250

Results



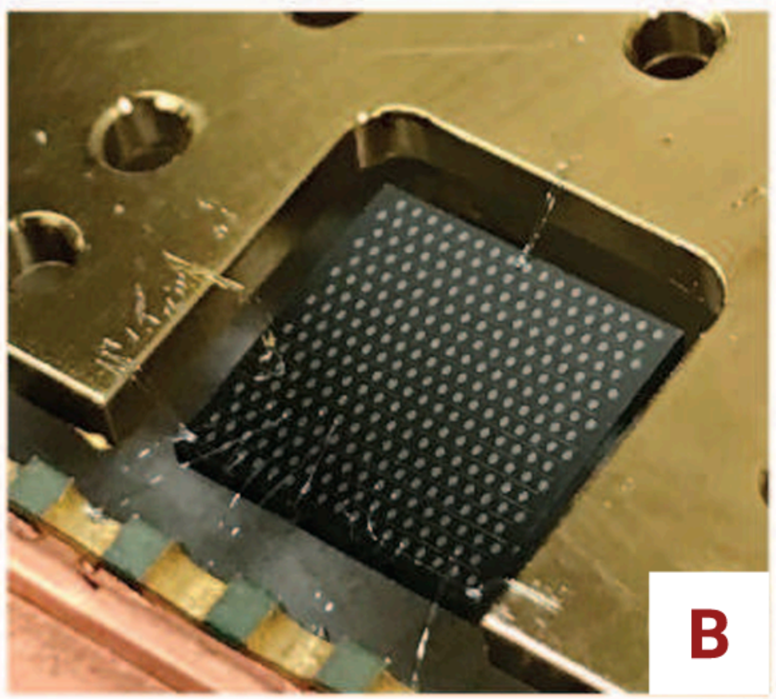
First calculation for all m_χ and arbitrary energy threshold!

(Just an example plot, have results for most relevant models and target materials)

B. Campbell-Deem, SK, T. Lin, E. Villarama: arXiv 2205.02250

Calculations needed

Phonon signals



Electronic signals

SPICE prototype detector
R. Anthony-Petersen et. al. arXiv 2208.02790

SENSEI detector
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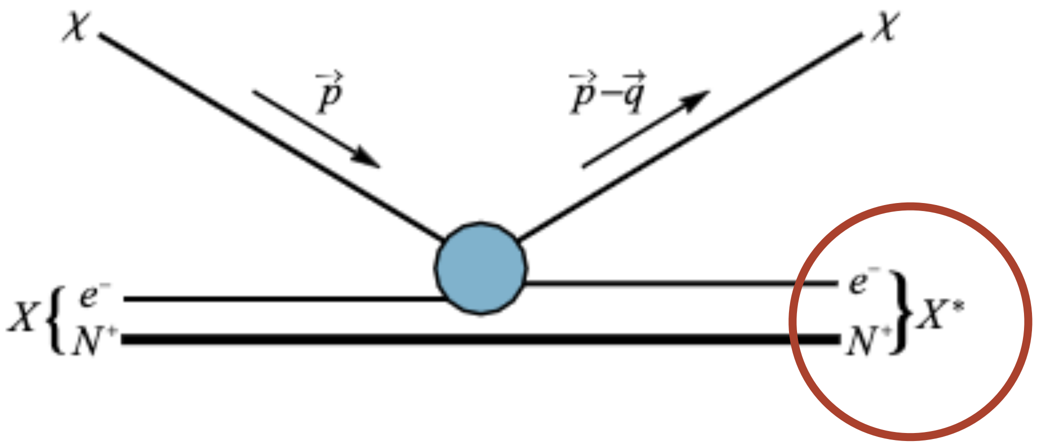
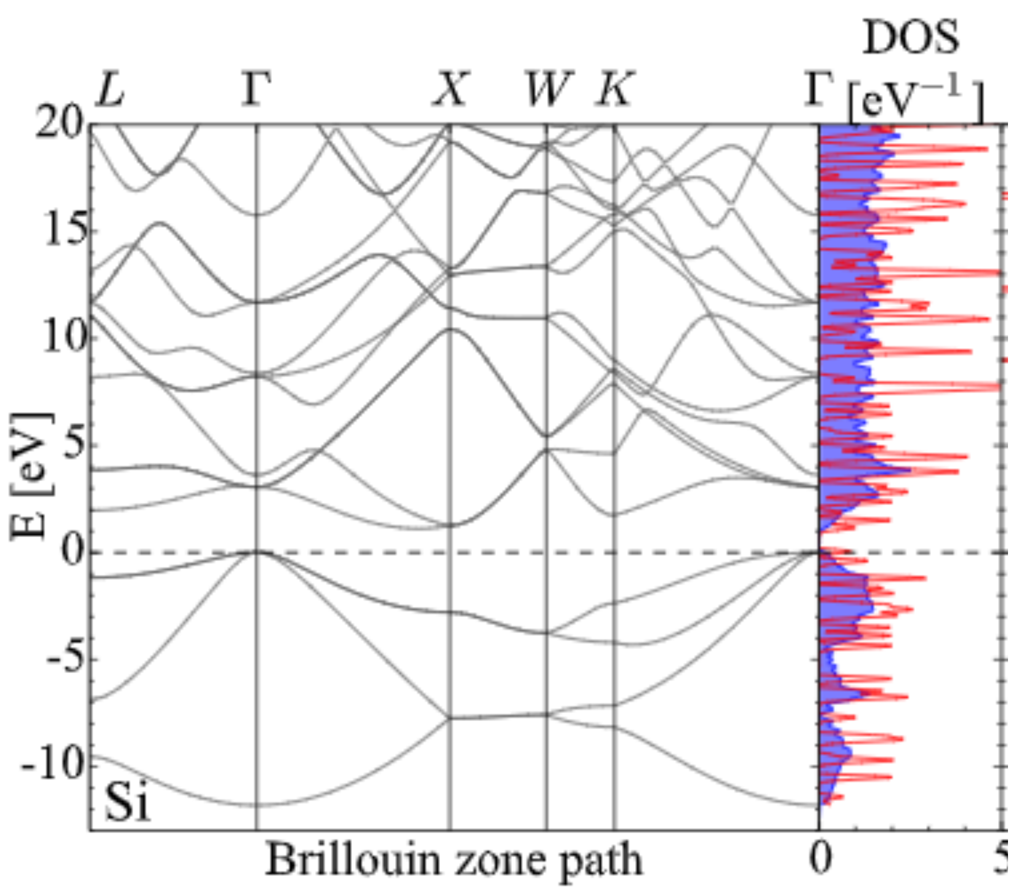
Electrons are complicated





Essig et. al. arXiv 1509.01598

e⁻ are not free

e⁻ are not at rest

e⁻ are not localized

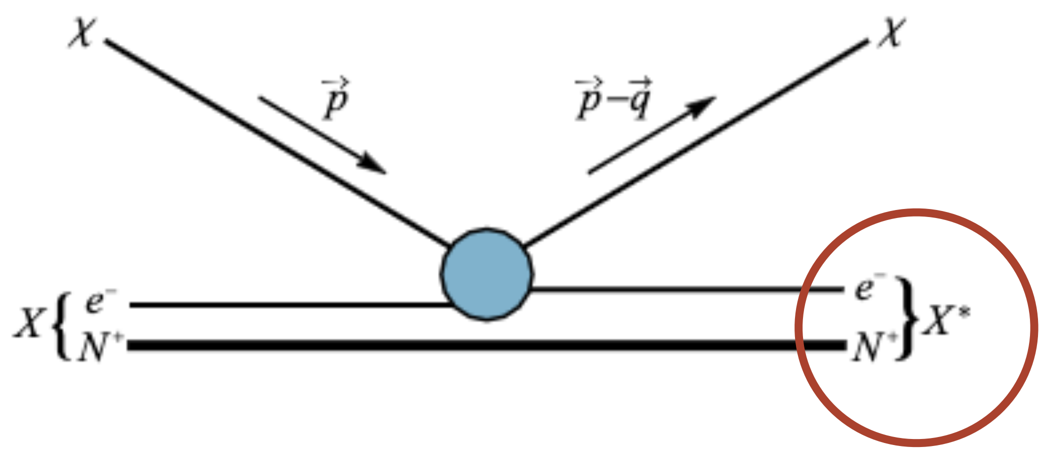
e⁻ are not alone

→ screening

Problem: Calculate wave functions & stick them into matrix element calculation

Essig et. al. arXiv 1509.01598

Electrons are complicated



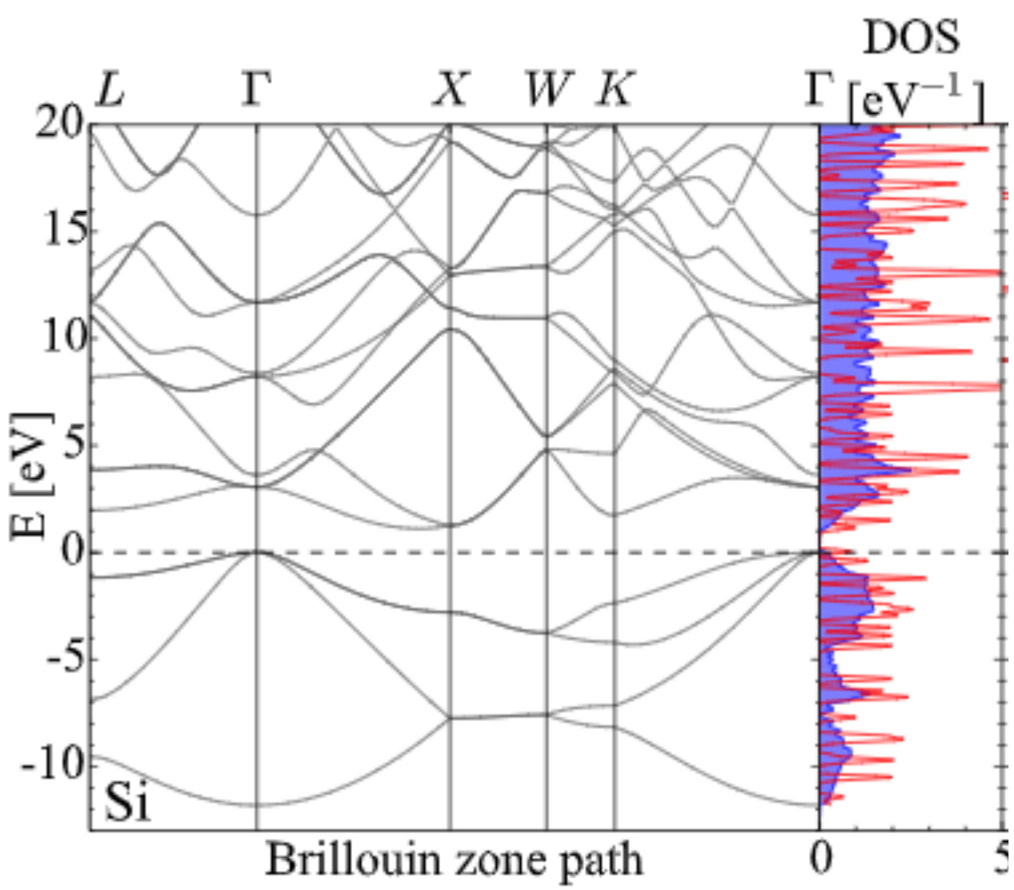
e^- are not free

e^- are not at rest

e^- are not localized

e^- are not alone

→ screening



Essig et. al. arXiv 1509.01598

Problem: Calculate wave functions & stick them into matrix element calculation

Essig et. al. arXiv 1509.01598

Equivalent problem: Calculate rate of energy dissipation in the crystal

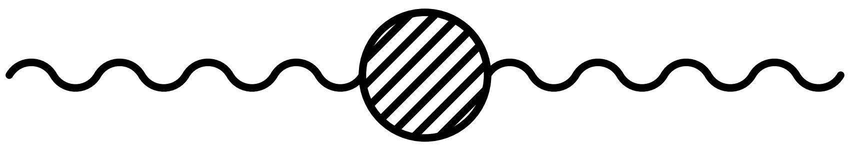
SK, J. Kozaczuk, T. Lin: arXiv 2101.08275
Y. Hochberg et. al.: arXiv 2101.08263

Schematic argument

Coulomb potential in a dielectric:

$$H = eQ_\chi \int \frac{d^3\mathbf{k}}{(2\pi)^2} \frac{1}{\epsilon(\mathbf{k},\omega)} \frac{e^{i\mathbf{k}\cdot\mathbf{x}}}{k^2}$$

In QFT language:



$$\sim \frac{1}{\epsilon(\mathbf{k},\omega)} \frac{1}{k^2}$$

(Non-relativistic limit)

(Exact derivation in the back-up slides)

Schematic argument

Coulomb potential in a dielectric:

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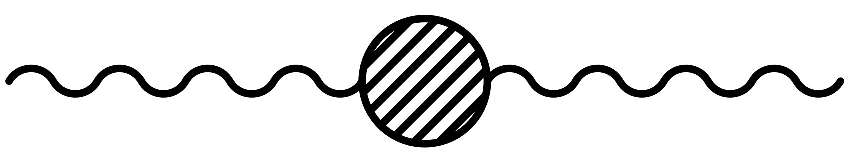
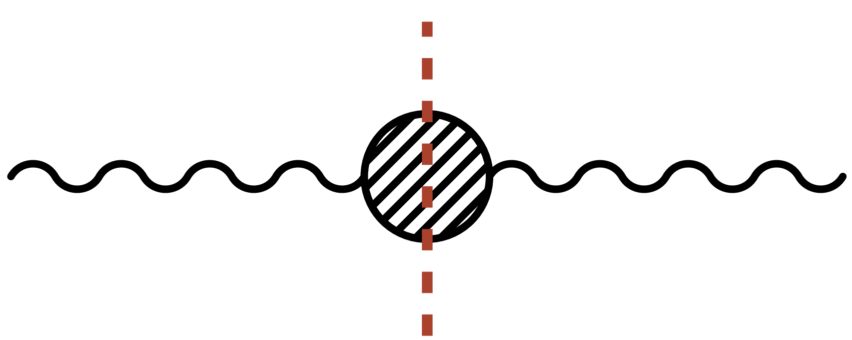


$$\sim \frac{1}{\epsilon(\mathbf{k},\omega)} \frac{1}{k^2}$$

(Non-relativistic limit)

We are interested in energy dissipation:



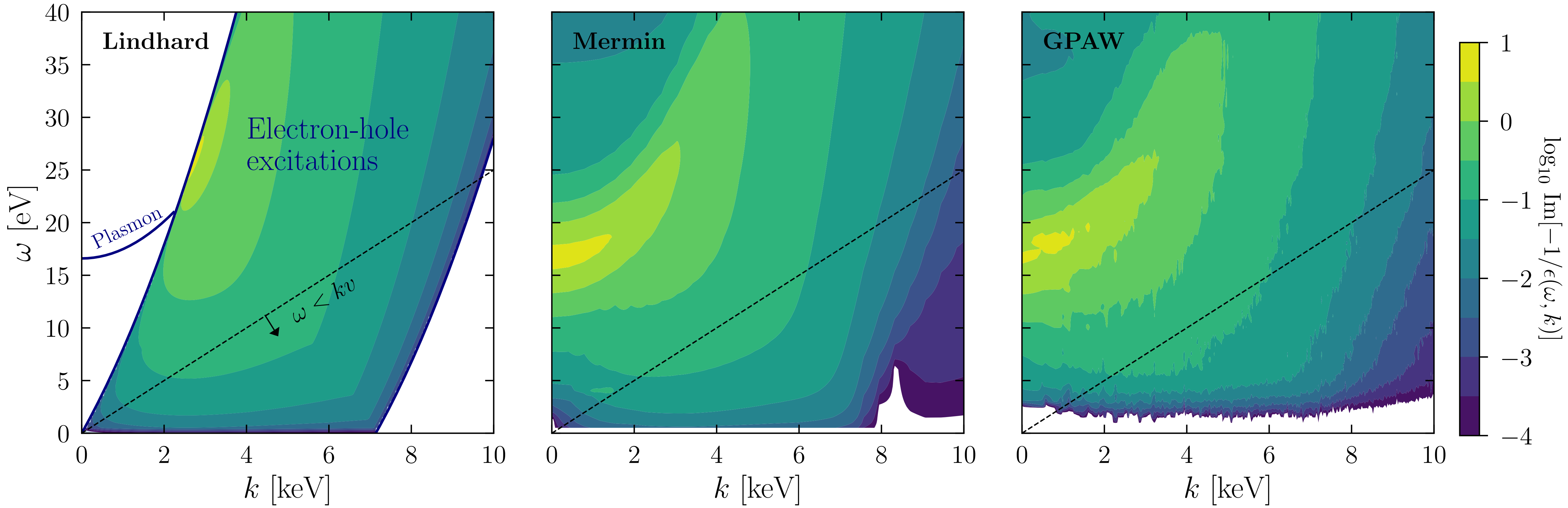
$$\sim \mathrm{Im}\left[\frac{-1}{\epsilon(\mathbf{k},\omega)}\right]$$

"Energy Loss Function" (ELF)

(Exact derivation in the back-up slides)

Calculating the ELF

Simple → Sophisticated



Free electron gas approximation	Phenomenological model fit to data	First principles DFT calculation
100% analytic	semi-analytic	fully numerical

(Details in back-up slides)

SK, J. Kozaczuk, T. Lin: arXiv 2101.08275, 2104.12786

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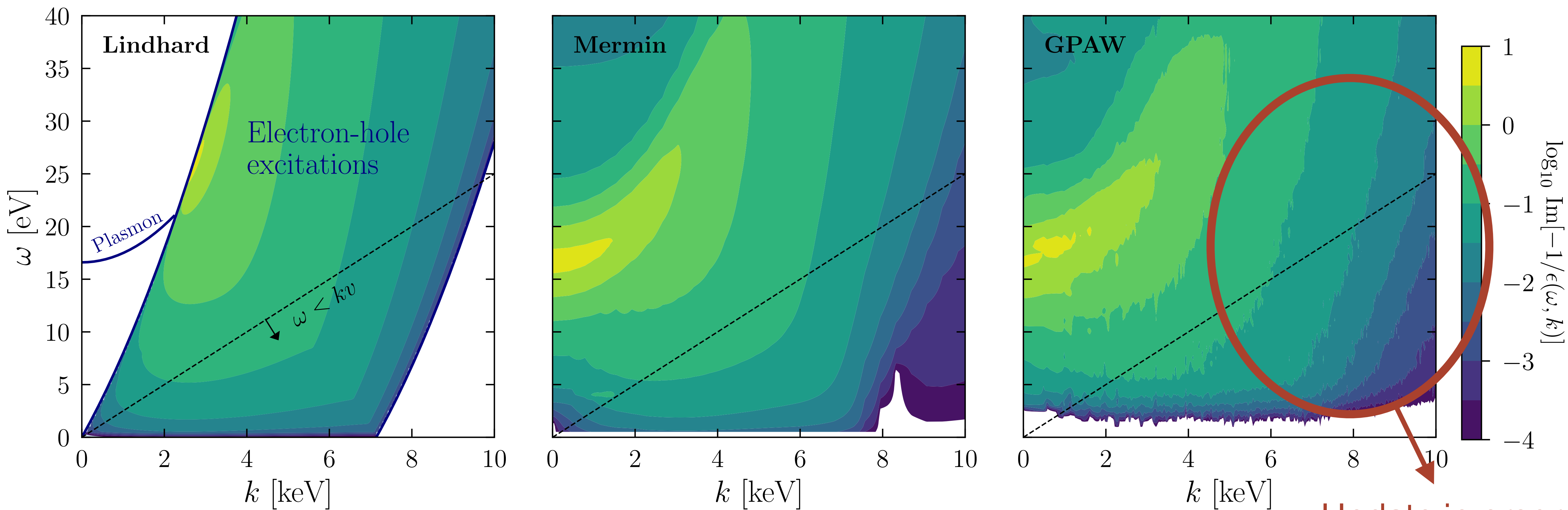


Update in progress

Free electron gas approximation	Phenomenological model fit to data	First principles DFT calculation
100% analytic	semi-analytic	fully numerical

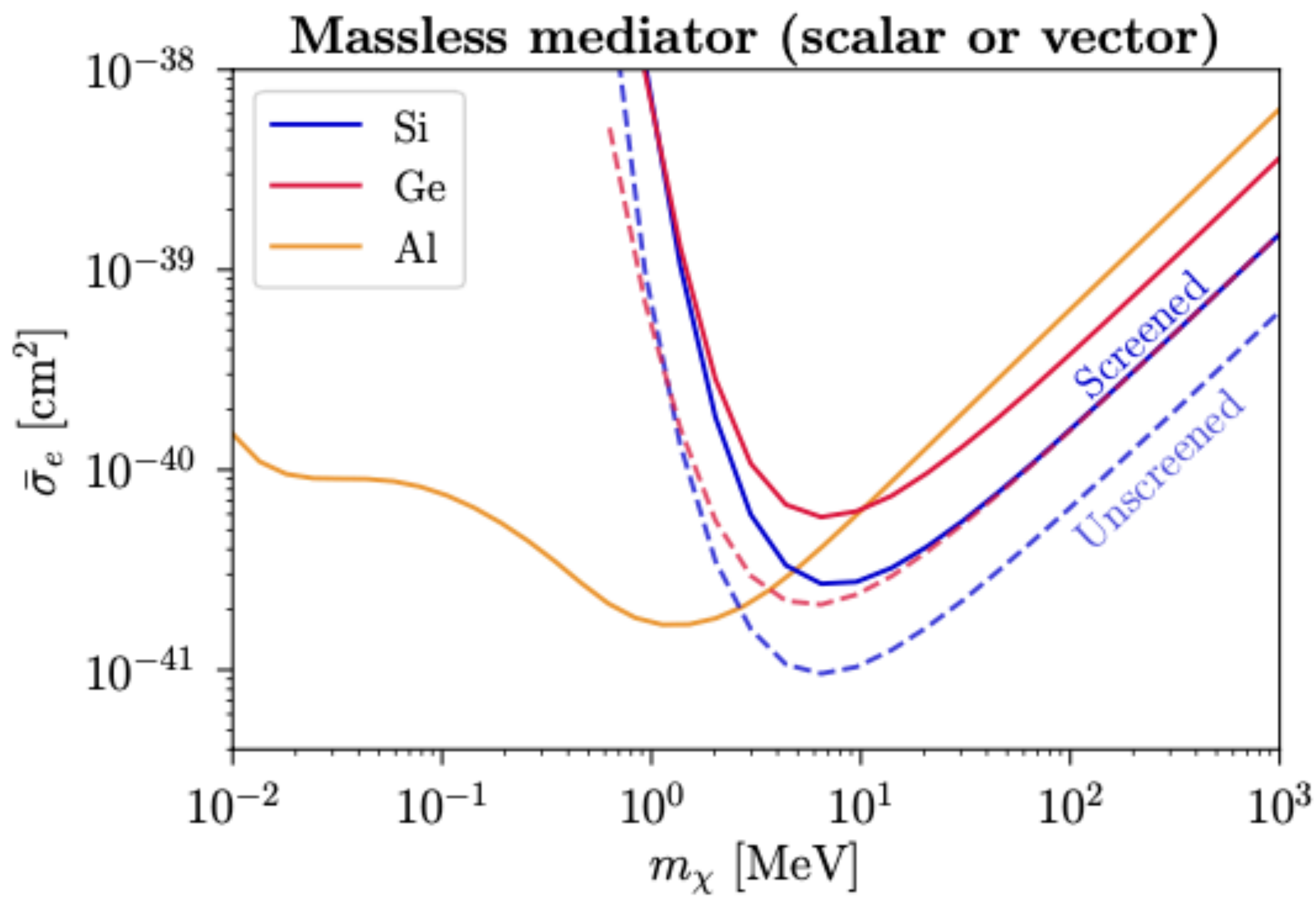
Bashi Mandava
UC Berkeley

(Details in back-up slides)

SK, J. Kozaczuk, T. Lin: arXiv 2101.08275, 2104.12786

Results

Screening has O(1) effect on integrated rate

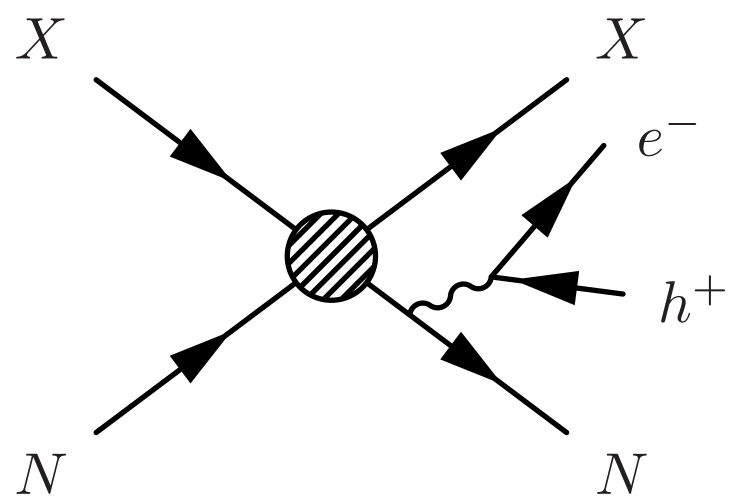


Applicable to *any* mediator that couples to e^- density
(e.g. scalar mediator and dark photon mediator yield *identical* scattering rate)

SK, J. Kozaczuk, T. Lin: arXiv 2101.08275, 2104.12786

Bonus: The Migdal effect

A hard nuclear recoil can shake some electrons into the conduction band



(Analogous to internal conversion process in collider physics)

Had been calculated for isolated atoms, but not yet for realistic crystals

Ibe et. al. arXiv 1707.07258

In a real material, spectator ions must be accounted for!

SK, J. Kozaczuk, T. Lin: arXiv 2011.09496
Liang et.al. : arXiv 2011.13352

Bonus: The Migdal effect

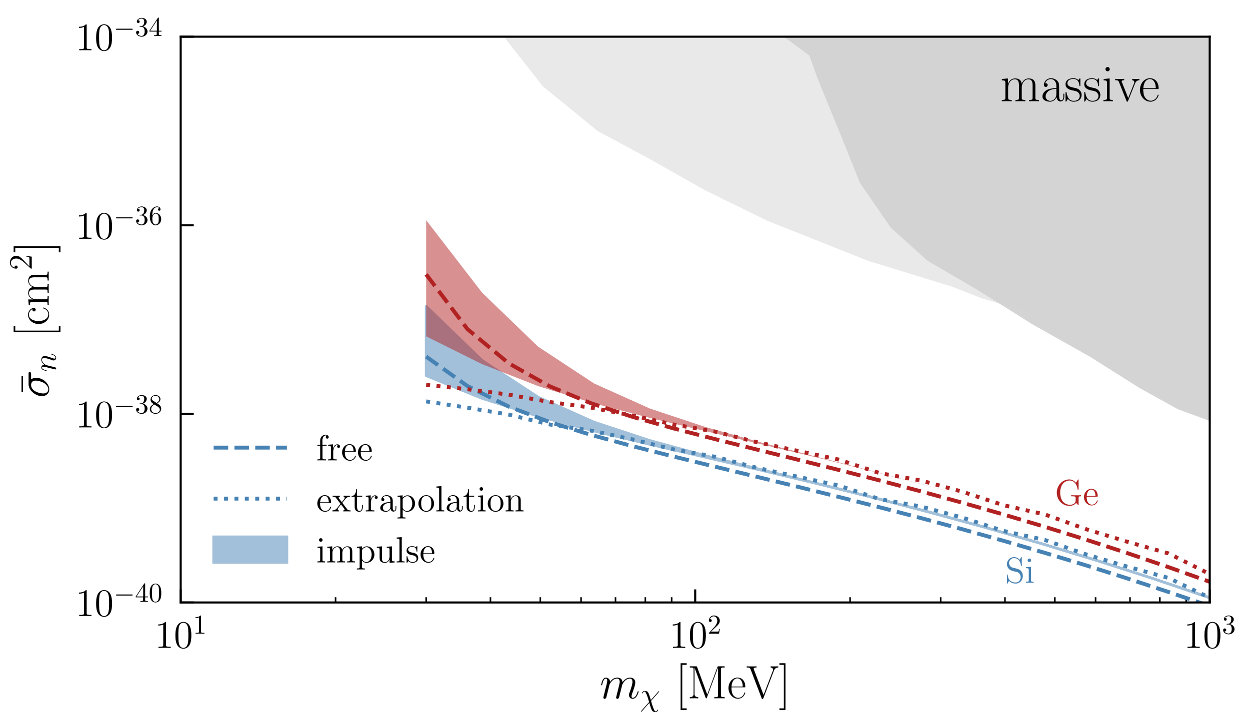
<u>Result:</u>

$$R = \frac{8\pi^2 Z_{\rm ion}^2 \alpha A^2 \rho_\chi \bar{\sigma}_n}{m_N m_\chi \mu_{\chi n}^2} \int d^3v f_\chi(v) \int d\omega \int \frac{d^3\mathbf{q}_N}{(2\pi)^3} \int \frac{d^3\mathbf{p}_f}{(2\pi)^3} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{1}{k^2} \underbrace{\mathrm{Im}\left[\frac{-1}{\epsilon(\mathbf{k},\omega)}\right]}_{\text{ELF}} \underbrace{\left[\frac{1}{\omega - \frac{\mathbf{q}_N\cdot\mathbf{k}}{m_N}} - \frac{1}{\omega}\right]^2}_{\text{Nucleus propagator}}$$

$$\times \underbrace{|F_{DM}(\mathbf{p}_i - \mathbf{p}_f)|^2}_{\text{DM form factor}} \underbrace{|F(\mathbf{p}_i - \mathbf{p}_f - \mathbf{q}_N - \mathbf{k})|^2}_{\text{Crystal form factor}} \delta\left(E_i - E_f - E_N - \omega\right).$$

Nucleus propagator $\sim \dfrac{1}{\omega^4}$

Crystal form factor ↑ Nucleus is not a free particle!



SK, J. Kozaczuk, T. Lin: arXiv 2011.09496

Bonus: The Migdal effect

<u>Result:</u>

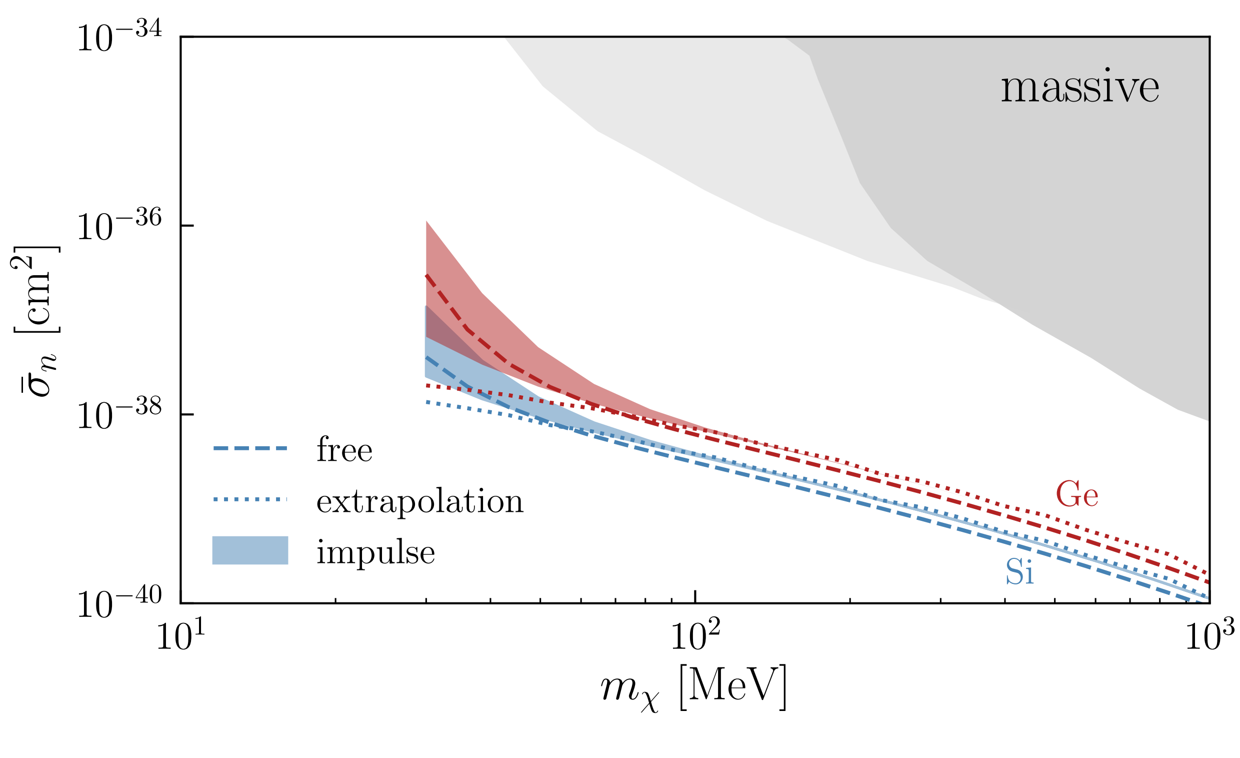
ELF — Nucleus propagator

$$R = \frac{8\pi^2 Z_{\text{ion}}^2 \alpha A^2 \rho_\chi \bar{\sigma}_n}{m_N m_\chi \mu_{\chi n}^2} \int d^3v f_\chi(v) \int d\omega \int \frac{d^3\mathbf{q}_N}{(2\pi)^3} \int \frac{d^3\mathbf{p}_f}{(2\pi)^3} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{1}{k^2} \text{Im}\left[\frac{-1}{\epsilon(\mathbf{k},\omega)}\right] \left[\frac{1}{\omega - \frac{\mathbf{q}_N\cdot\mathbf{k}}{m_N}} - \frac{1}{\omega}\right]^2$$
$$\times |F_{DM}(\mathbf{p}_i - \mathbf{p}_f)|^2 |F(\mathbf{p}_i - \mathbf{p}_f - \mathbf{q}_N - \mathbf{k})|^2 \delta(E_i - E_f - E_N - \omega).$$

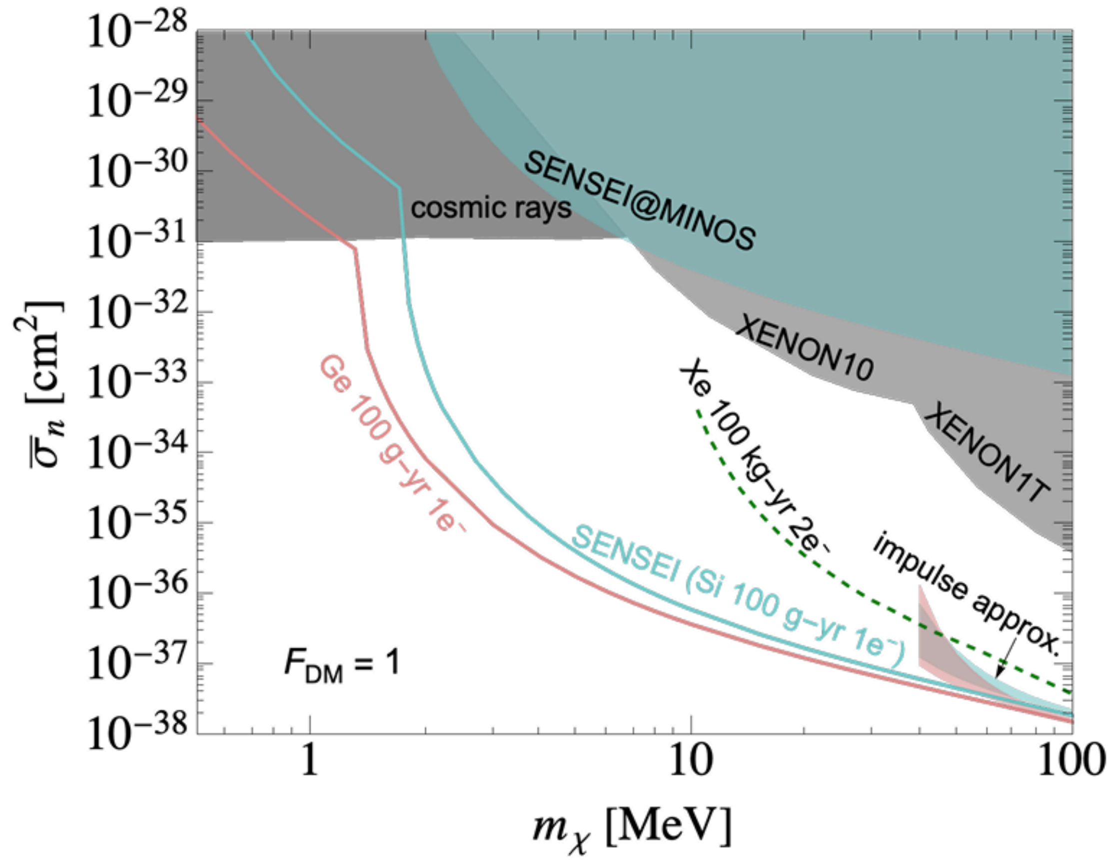
DM form factor — Crystal form factor

Nucleus propagator $\sim \frac{1}{\omega^4}$

Nucleus is not a free particle!



SK, J. Kozaczuk, T. Lin: arXiv 2011.09496



K. Berghaus et.al. : arXiv 2210.06490

DarkELF

You'd like to quickly calculate DM scattering rates, but don't want to learn Density Functional Theory?

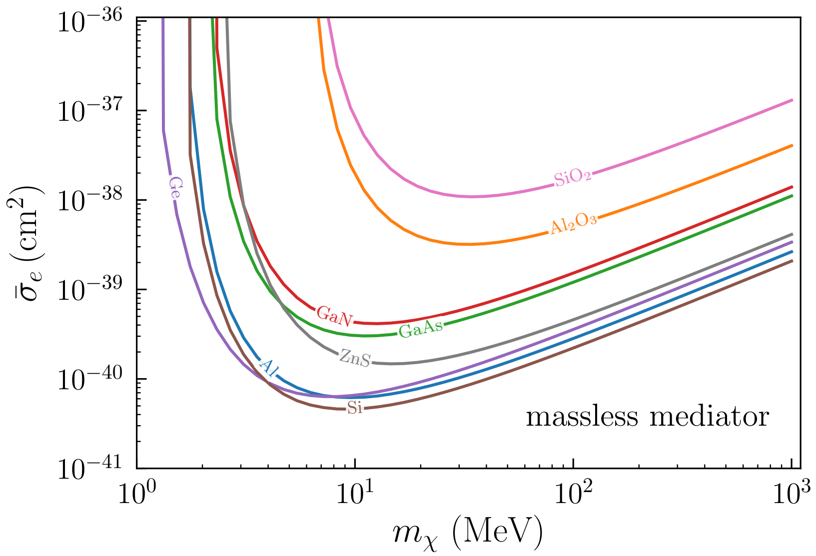
Then DarkELF is the answer for you!

https://github.com/tongylin/DarkELF

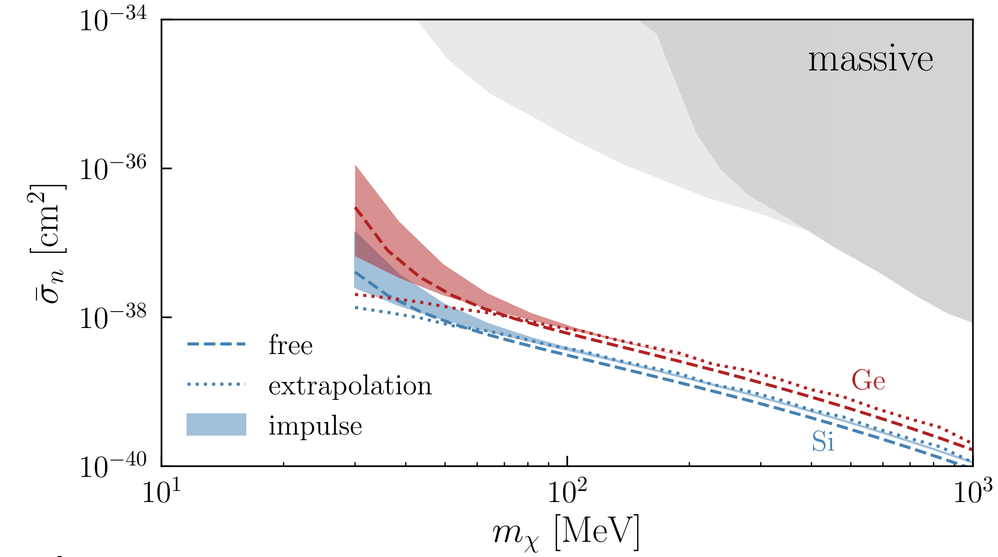
- Python 3 package with lots of example Jupyter notebooks
- No dependencies other than numpy, scipy and Vegas
- All DFT results included as look-up tables, no DFT code necessary
- Library of most common materials (Si, Ge, GaAs, diamond, sapphire, etc etc)

DarkELF processes

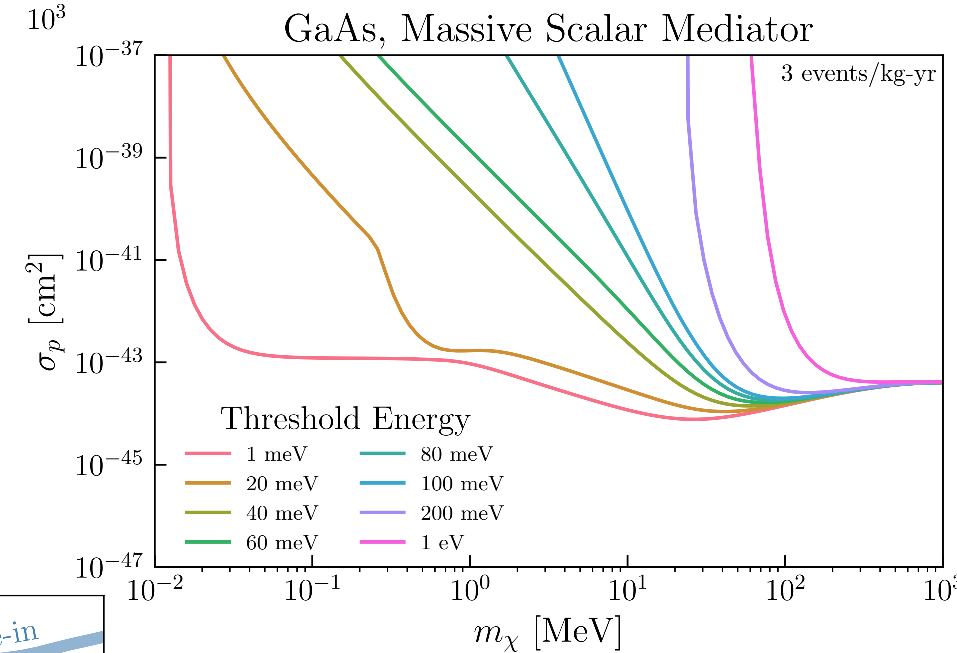
DM - electron scattering



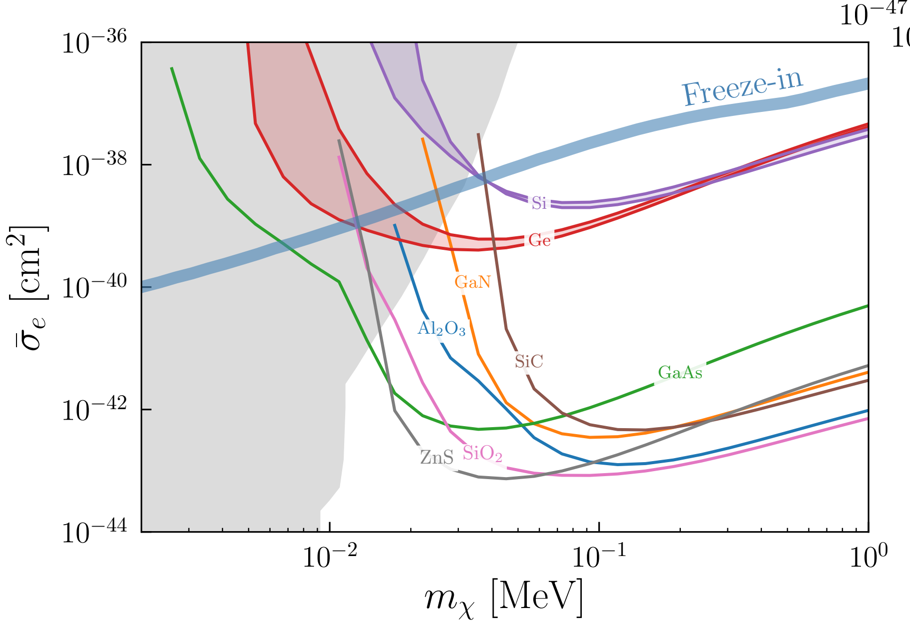
Migdal effect



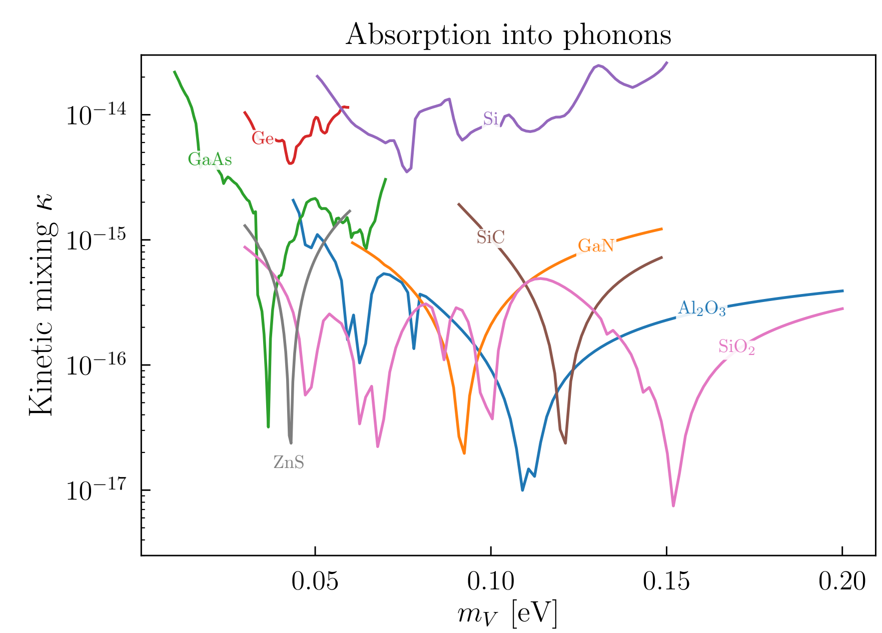
DM - multiphonon scattering



DM - phonon scattering



Dark photon absorption



Summary

We calculated:

- DM-phonon scattering to *all orders* in the multiphonon expansion
- DM-electron scattering, *including screening*
- The Migdal effect in semi-conductors

All calculations publicly available in our DarkELF package

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Some future / ongoing work:

- Background processes, e.g. Frenkel pair recombination (ongoing)
- Include core and semi-core electrons in our DFT calculations (ongoing)
- Quenching factor calculation for low energy recoils (very hard)
- Going beyond the isotropic limit (future)

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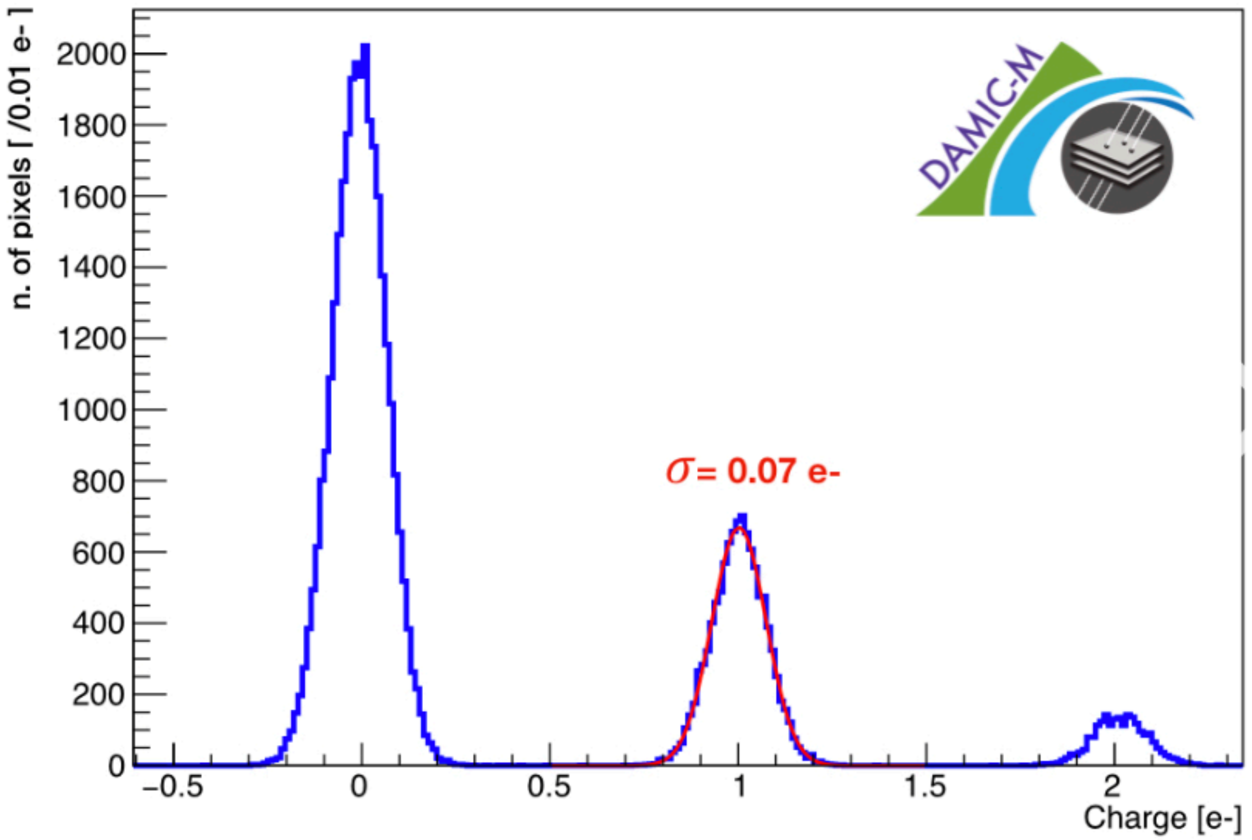
Extra slides

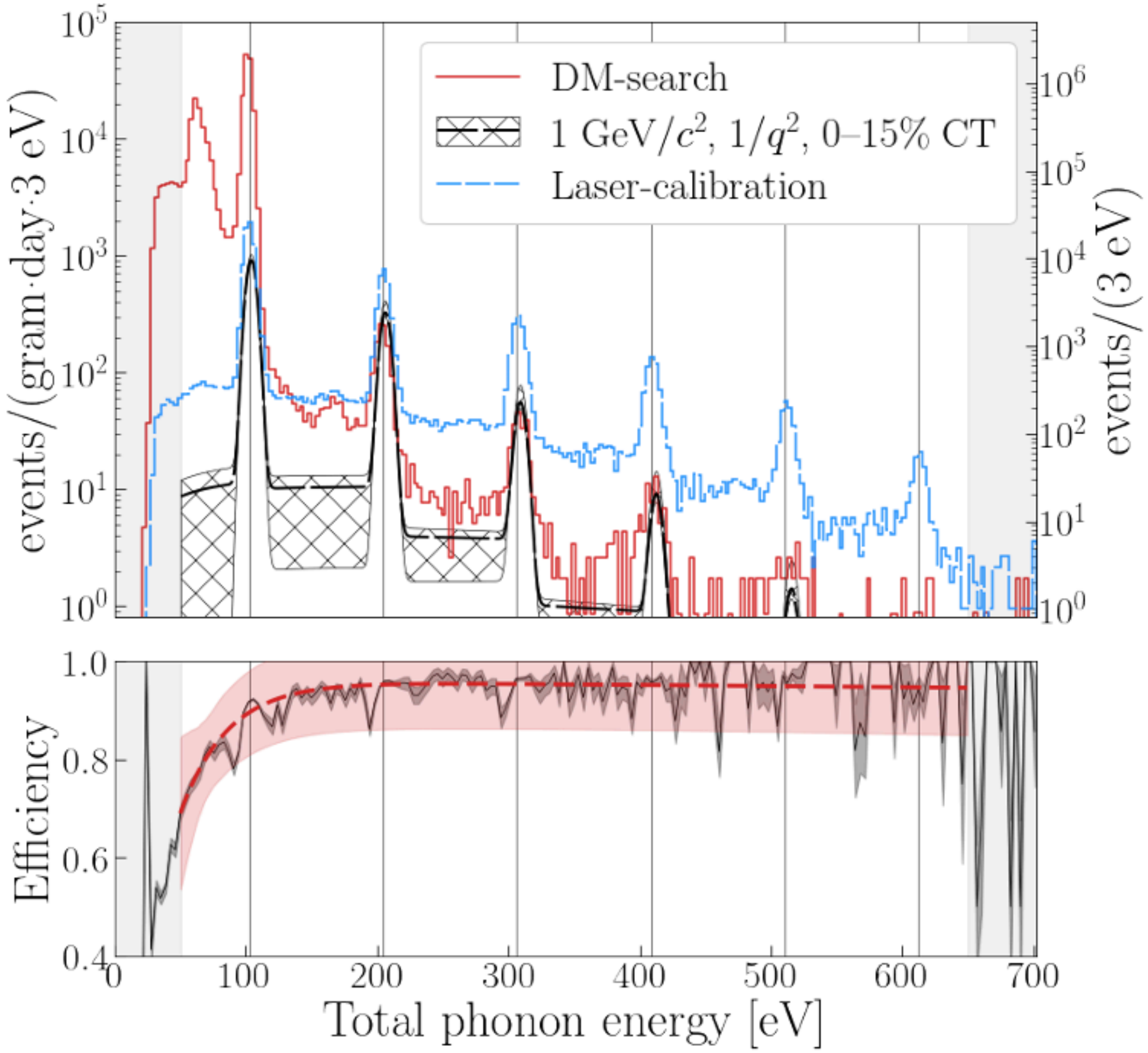
Electron Detectors

Sensitivity to single e- excitations has already been demonstrated



Skipper CCDs

(SENSEI, DAMIC)

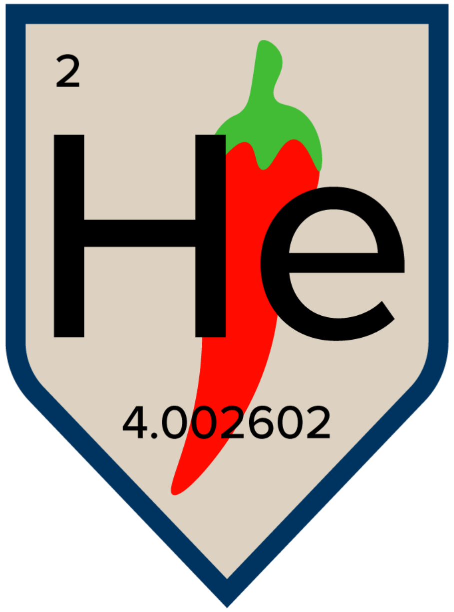


Luke-Neganov effect

(superCDMS)

SENSEI already has 50g-day exposure in shallow underground site

Phonon Detectors



SPICE / HeRALD

SPICE conceptual design

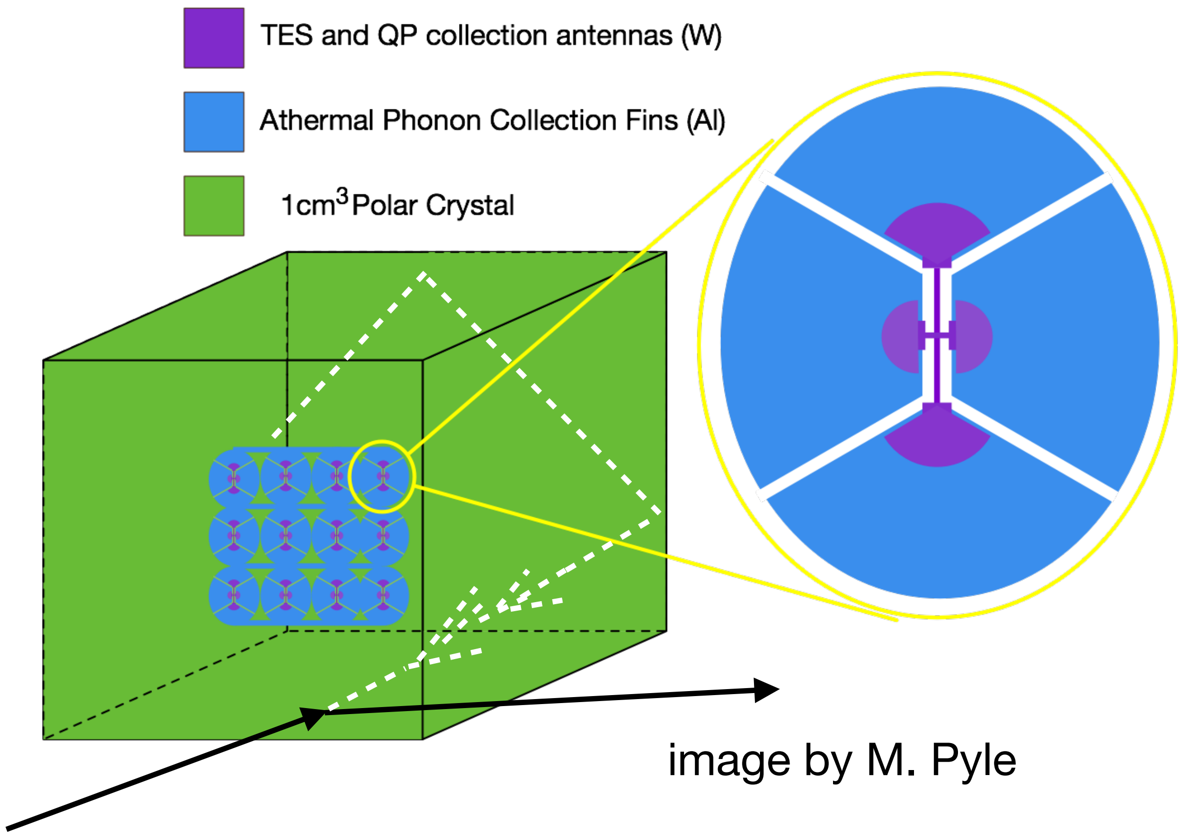
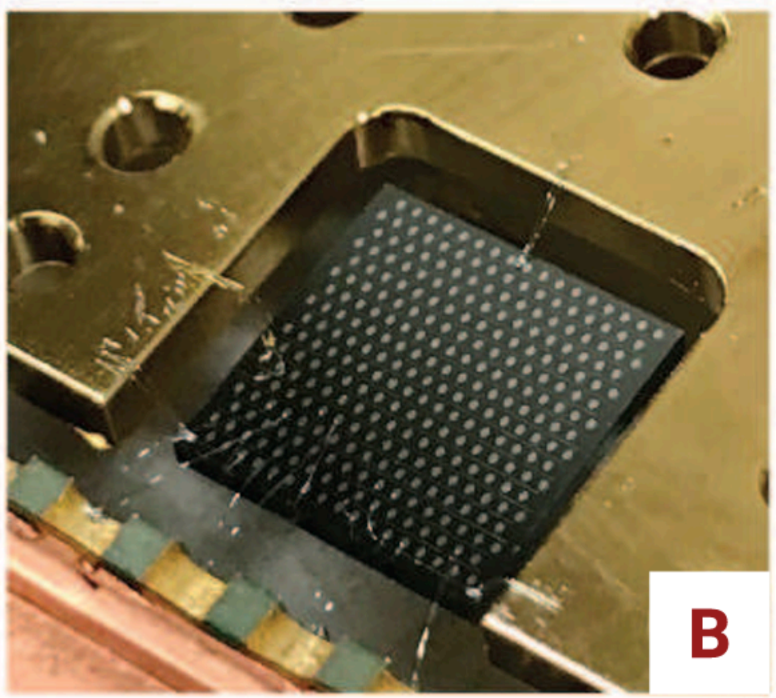
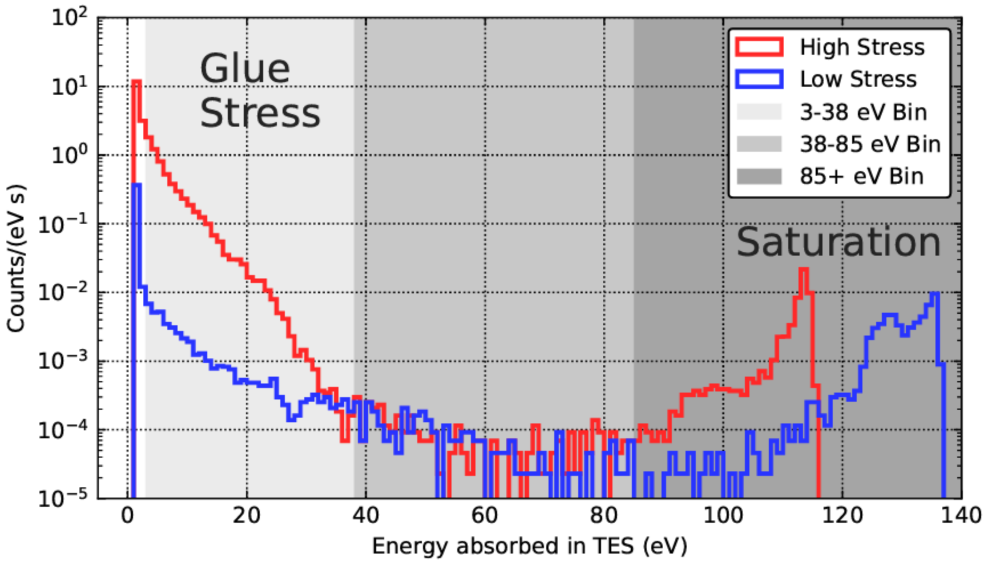


image by M. Pyle



detector hanging on aluminium wire bonds

Qualitative progress in background mitigation

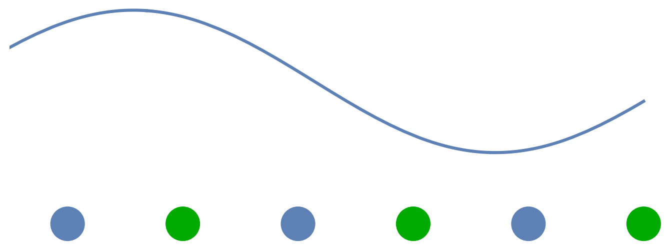


R. Anthony-Petersen et. al. arXiv 2208.02790

Approximations

1. Incoherent approximation

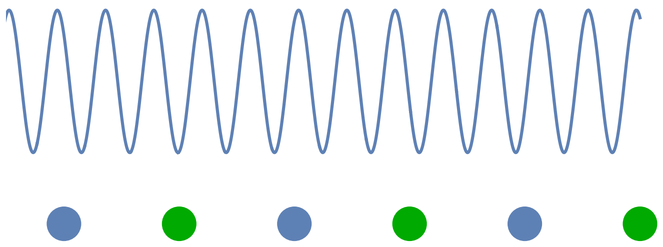
Low q



Interference is critical

(e.g. Bragg diffraction)

High q



Interference can be neglected

Approximations

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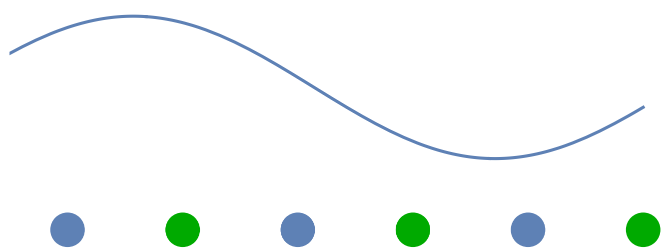

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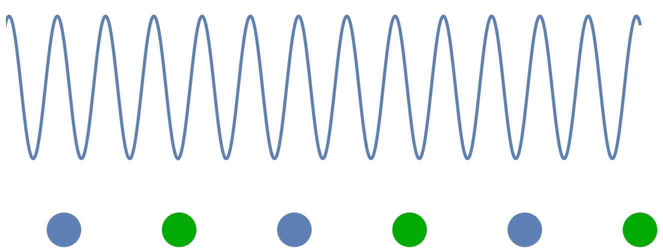
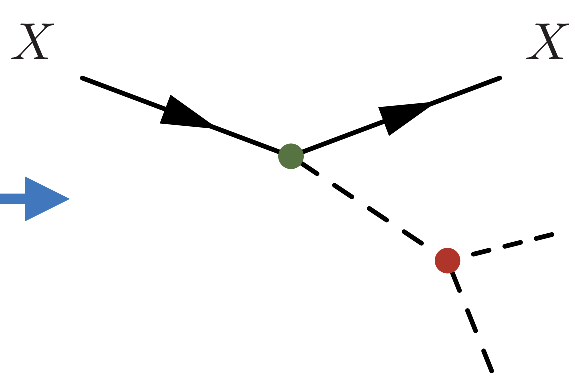


Interference can be neglected

2. Anharmonic approximation

Neglect these beyond leading order →



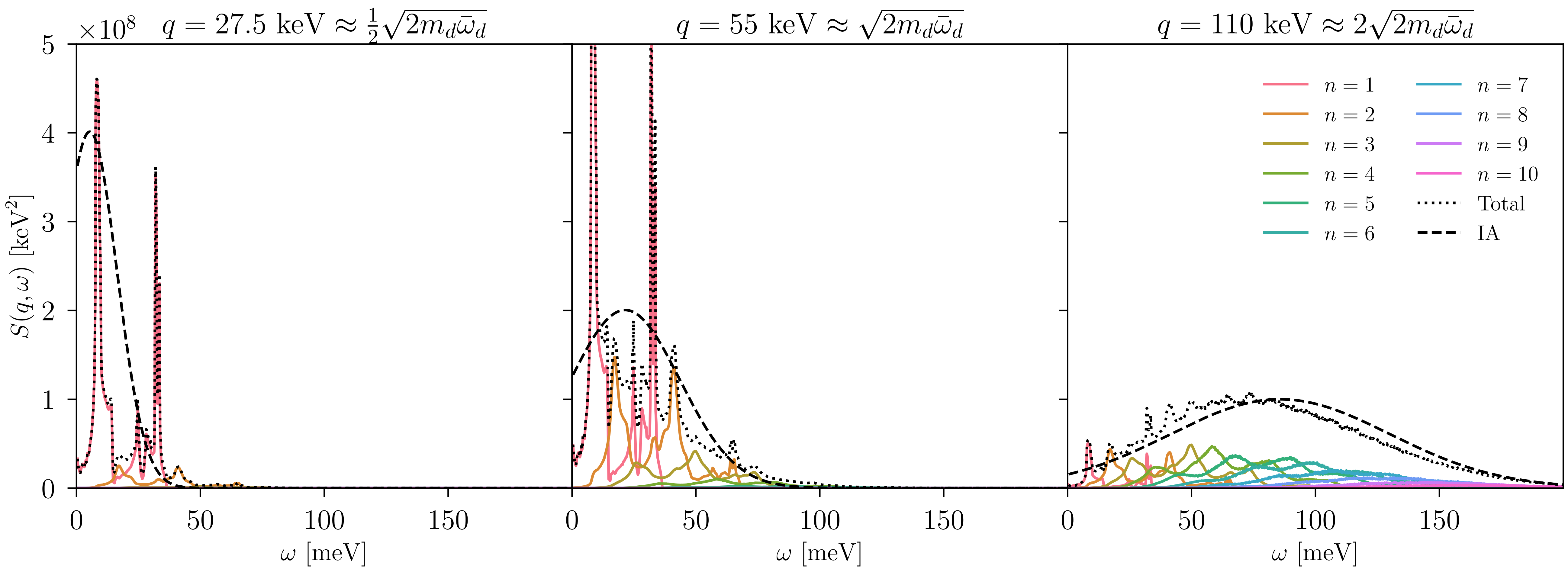
3. Isotropic approximation

Multiphonon results

Numerics

Low q

High q



1 phonon dominates

1,2 and 3 phonons

Multiphonons

B. Campbell-Deem, SK, T. Lin, E. Villarama: arXiv 2205.02250

ELF Derivation (I)

Need to use *linear response theory*, essentially non-relativistic QFT

Susceptibility: how does the crystal respond to a density perturbation?

$$\chi(\omega, \mathbf{k}) = -\frac{i}{V} \int_0^\infty dt\, e^{i\omega t} \langle [n_{\mathbf{k}}(t), n_{-\mathbf{k}}(0)] \rangle$$

V → Crystal volume

$n_{\mathbf{k}}$ → Electron number density operator

This is the non-relativistic, retarded Green's function (fully dressed)

P. Nozières, D. Pines (1958)

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Now we use the fluctuation-dissipation theorem

$$\mathrm{Im}\chi(\omega, \mathbf{k}) = -\frac{1}{2}(1 - e^{-\beta\omega})S(\omega, \mathbf{k}) \qquad \beta \equiv \frac{1}{k_B T}$$

With the dynamical structure factor defined as

$$S(\omega, \mathbf{k}) \equiv \frac{2\pi}{V}\sum_{i,f}\frac{e^{-\beta E_i}}{Z}|\langle f|n_{-\mathbf{k}}|i\rangle|^2\delta(\omega + E_i - E_f)$$

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Fermi's golden rule

P. Nozières, D. Pines (1958)

ELF Derivation (II)

Now consider the response to an external electromagnetic perturbation.
The induced electron number density is

$$\begin{aligned}\langle \delta n(\mathbf{k},\omega)\rangle &= \langle n(\mathbf{k},\omega) H_{coul}\rangle \\ &= -\frac{e}{k^2}\chi(\mathbf{k},\omega)\rho_{ext}(\mathbf{k},\omega)\end{aligned}$$

with

$$H_{coul} = -e\int \frac{d^3\mathbf{k}}{(2\pi)^2}\frac{e^{i\mathbf{k}\cdot\mathbf{x}}}{k^2} n(-\mathbf{k},\omega)\rho_{ext}(\mathbf{k},\omega)$$

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Now plugging this into the fluctuation-dissipation theorem

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Energy Loss Function (ELF)

P. Nozières, D. Pines (1958)

DM-electron scattering rate

Full formula

$$R = \frac{1}{\rho_T}\frac{\rho_\chi}{m_\chi}\frac{\bar{\sigma}_e}{\mu_{\chi e}^2}\frac{\pi}{\alpha_{em}}\int d^3v\, f_\chi(v) \int \frac{d^3\mathbf{k}}{(2\pi)^3} k^2 |F_{DM}(k)|^2 \int \frac{d\omega}{2\pi}\frac{1}{1-e^{-\beta\omega}}\,\mathrm{Im}\left[\frac{-1}{\epsilon(\omega,\mathbf{k})}\right]\delta\left(\omega + \frac{k^2}{2m_\chi} - \mathbf{k}\cdot\mathbf{v}\right).$$

$f_\chi(v)$ → DM velocity distribution

$|F_{DM}(k)|^2$ → DM form factor

$\mathrm{Im}\left[\frac{-1}{\epsilon(\omega,\mathbf{k})}\right]$ → ELF

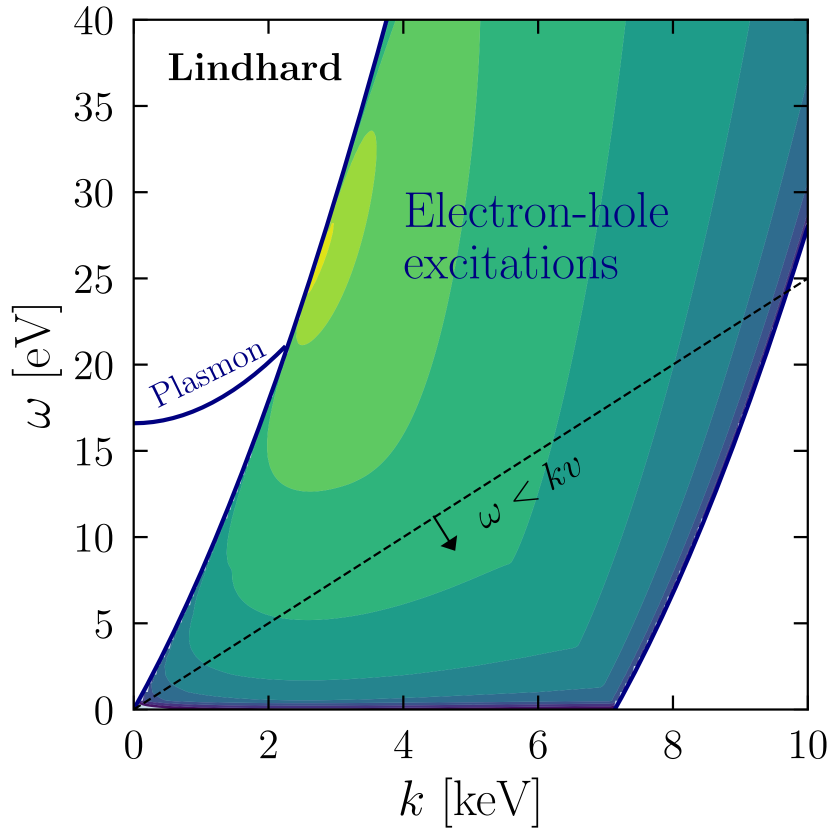
Advantages of using the ELF:

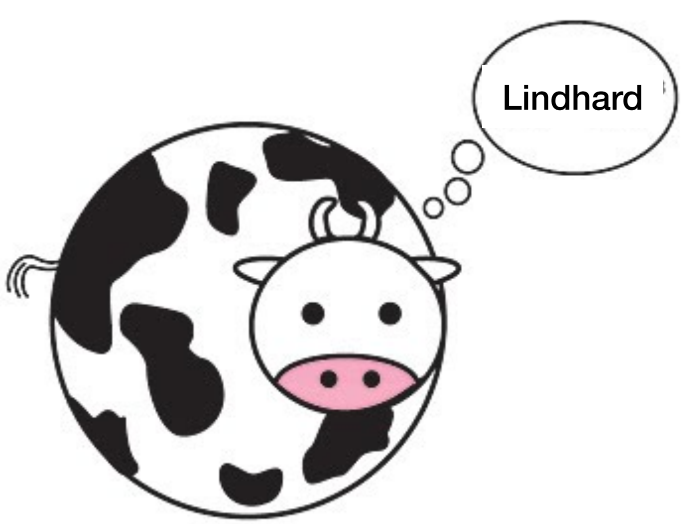
- Screening included automatically
- ELF has been measured and calculated extensively in the condensed matter literature

Applicable to *any* mediator that couples to e^- density
(e.g. scalar mediator and dark photon mediator yield *identical* scattering rate)

SK, J. Kozaczuk, T. Lin: arXiv 2101.08275, 2104.12786

Lindhard model





Homogenous, free electron gas:

$$\epsilon_{\rm Lin}(\omega, k) = 1 + \frac{3\omega_p^2}{k^2 v_F^2} \lim_{\eta \to 0} \left[f\left(\frac{\omega + i\eta}{k v_F}, \frac{k}{2 m_e v_F} \right) \right]$$

with

$$v_F = \left(\frac{3\pi\omega_p^2}{4\alpha m_e^2} \right)^{1/3}$$

Plasmon frequency

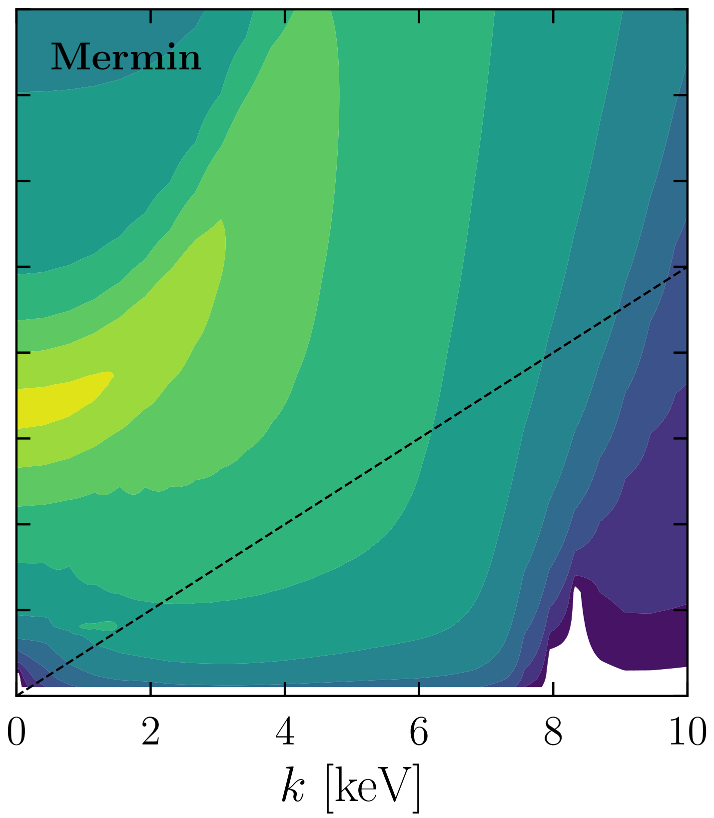
$$f(u, z) = \frac{1}{2} + \frac{1}{8z} \left[g(z - u) + g(z + u) \right]$$

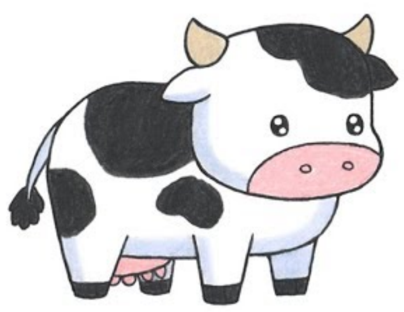
$$g(x) = (1 - x^2) \log\left(\frac{1 + x}{1 - x} \right)$$

<u>Features:</u>

- [x] Pauli blocking
- [x] e-h pair continuum
- [ ] Plasmon width
- [ ] Low k region
- [ ] Bandgap

Mermin model





M. Vos, P. Grande: chapidif package
Data from Y. Sun et. al. Chinese Journal of Chemical Physics 9, 663 (2016)

Homogenous, free electron gas with dissipation (Γ)

$$\epsilon_{\mathrm{Mer}}(\omega,k) = 1 + \frac{(1+i\frac{\Gamma}{\omega})(\epsilon_{\mathrm{Lin}}(\omega+i\Gamma,k)-1)}{1+(i\frac{\Gamma}{\omega})\frac{\epsilon_{\mathrm{Lin}}(\omega+i\Gamma,k)-1}{\epsilon_{\mathrm{Lin}}(0,k)-1}}.$$

Fit a linear combination of Mermin oscillators to optical data:

$$\mathrm{Im}\left[\frac{-1}{\epsilon(\omega,k)}\right] = \sum_i A_i(k)\mathrm{Im}\left[\frac{-1}{\epsilon_{\mathrm{Mer}}(\omega,k;\omega_{p,i},\Gamma_i)}\right]$$

Features:

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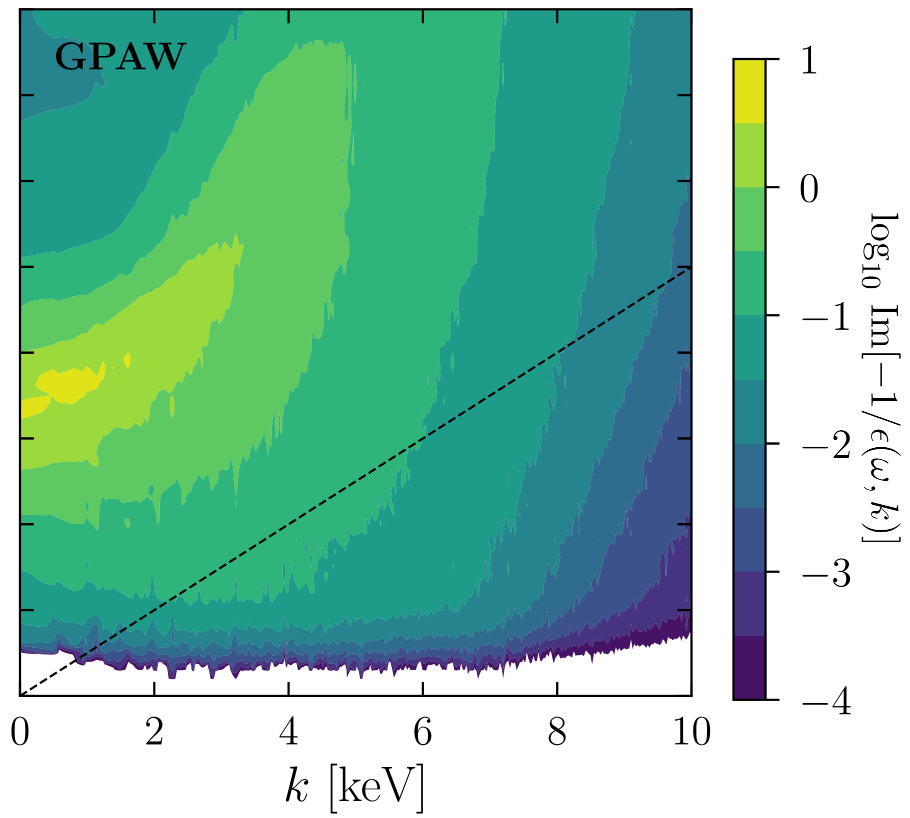
GPAW method



Compute the ELF from first principles with time-dependent Density Functional Theory methods (TD-DFT)

Puts atoms on periodic lattice and model interacting e^- as non-interacting e^- + effective external potential (Kohn-Sham method)

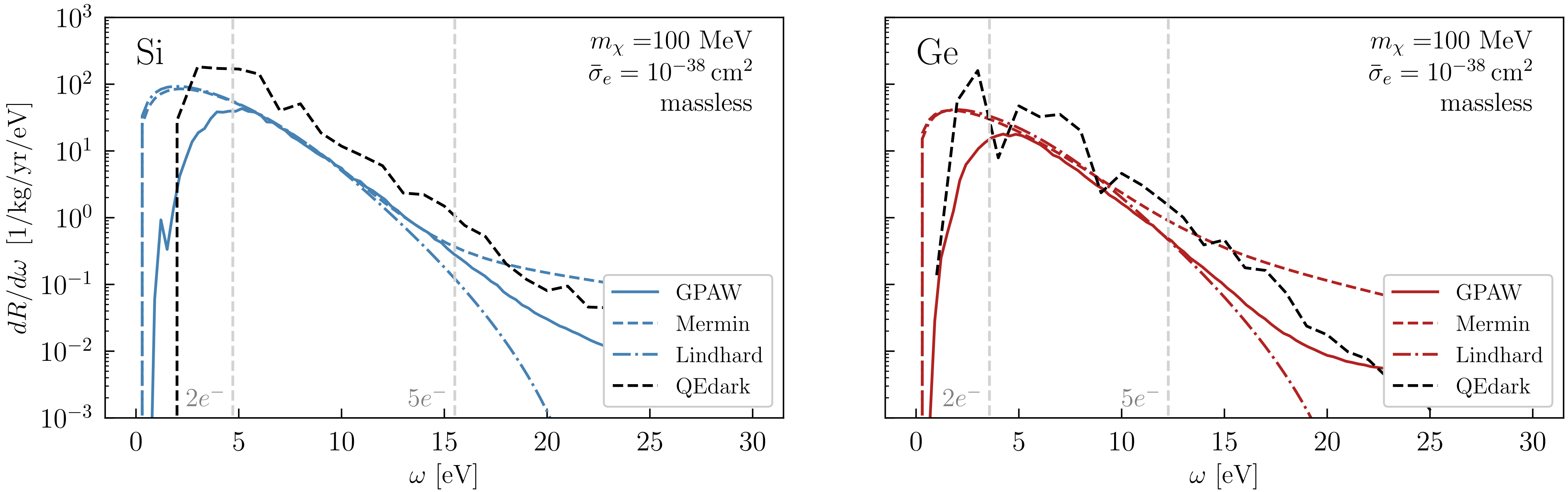
Inner shell e^- are treated as part of the ion (frozen core approximation)

Features:

- ☑ Pauli blocking
- ☑ e-h pair continuum
- ☑ Plasmon width
- ☑ Low k region
- ☑ Bandgap

GPAW: https://wiki.fysik.dtu.dk/gpaw/

Differential rate



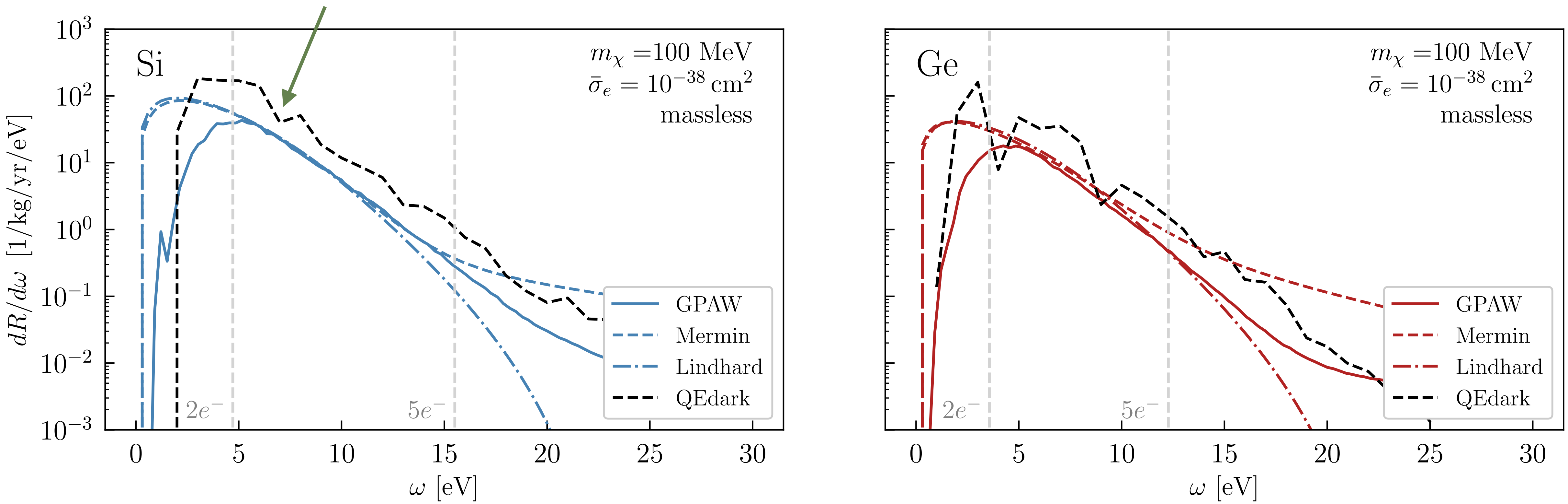
Mermin & GPAW in very good agreement except:

- Single ionization e- region (background dominated)
- High energy region (subdominant)

(Agreement is less good in massive mediator case; work in progress)

Differential rate

Does not include screening



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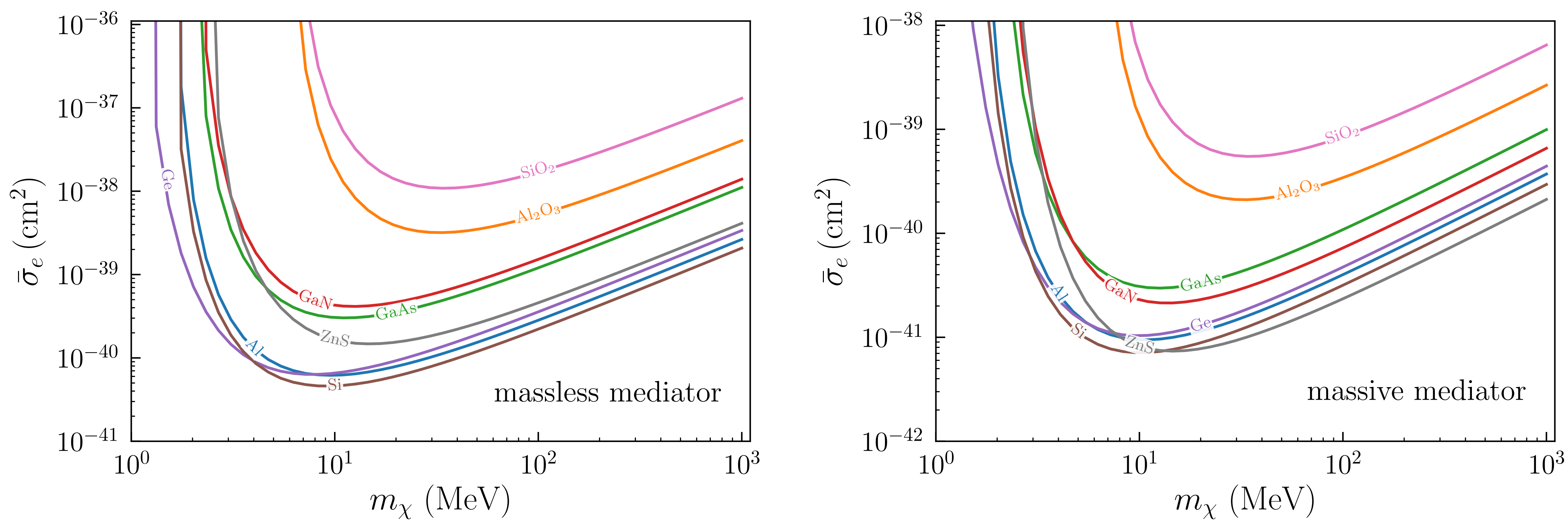
Integrated rate: Other materials

Using the *Mermin* method we can easily scan over many possible targets:



So far only *GPAW* results for Ge and Si, other materials are work in progress