New Physics, Fundamental Constants & Muon g-2

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□ Phys.Rev.Lett. 127 (2021) 25, 251801 with Ohayon (ETHZ→Technion) and Soreq (Technion)

□ hep-ph/2210.10056, to appear in *PRL* with Karr (LKB), Kitahara (Nagoya), Koelemeij (Amsterdam), Soreq (Technion) and Zupan (Cincinnati)

Moriond EW | Mar. 21st, 2023

Outline

□ Accurate muon g-2 from **muonium**

- Spin-precession measurements disagree with the SM
- Is the SM prediction well under control?
- New test with forthcoming precision muonium spectroscopy

Do we truly understand **fundamental constants**?

- New physics below $\sim 1\,{\rm GeV}$ affects the determination of the FCs
- Modified CODATA makes associated measurements a sensitive NP search
- Several observed inconsistencies, explained by a single new particle

Muon(ium) g-2

muon magnetic moment **below 1ppm** from $\mu^+ e^-$ spectroscopy



Is this really an evidence of **BSM** Physics? $a_{\mu}^{\rm BSM} = 251(59) \times 10^{-11}$



Is this really an evidence of **BSM** Physics? $a_{\mu}^{\rm BSM} = 251(59) \times 10^{-11}$ Do we really **control** the SM prediction? **R-ratio** method: [Bouchiat-Michel 1961] $a_{\mu}^{\rm HVP-LO} = \frac{\alpha^2}{3\pi^2} \int_{m_{\pi}^2}^{\infty} ds \frac{K(s)}{s} R(s)$ $R(s) = \frac{3s}{4\pi\alpha^2} \sigma(e^+e^- \to \text{hadrons})$ $\pi\pi \sim 70\%$



New lattice results cast **doubts**

[BMW coll. Nature 593 (2021) 7857]

$$a_{\mu}^{\rm HVP-LO} = 7075(55) \times 10^{-11}$$



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Recent $e^+e^- \to \pi\pi$ VEPP data also [CMD-3 coll. hep-ex/2302.08834] $a_{\mu}^{\text{HVP}-\text{LO}}[\pi\pi] = 3793(30) \times 10^{-11}$ $(0.6 < \sqrt{s} < 0.9 \,\text{GeV})$



Towards solving the puzzle



New experimental determinations of a_{μ} are more than welcome!

JPARC is coming up, but like BNL/FNAL it could be affected by « environmental » NP effects,

e.g. [Davoudiasl-Szafron hep-ph/2210.14959] [Agrawal et al. hep-ph/2210.17547]

MUonE will measure HVP directly,

should be clean from NP, see *e.g.* [Masiero-Paradisi-Passera PRD 2020]

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Muonium spectroscopy in <10yrs will offer another test at 1ppm!

Muonium energy levels

full angular momentum: $\vec{F}=\vec{L}+\vec{S}_e+\vec{S}_\mu$

The electron spin flips in the (static) magnetic field sourced by the muon, which lifts the degeneracy of the S state.

The 1S-HFS in muonium is **very precisely** measured: $\nu_{\rm HFS}^{\rm exp} = 4\,463\,302\,765(53)\,{\rm Hz}\,\,(12{\rm ppb})$ [Liu et al. PRL 1999]



Ground-state HFS theory



Ground-state HFS theory



Ground-state HFS theory



Alternative muon mass determination

To extract a_{μ} from muonium HFS, **another observable** is needed to fix the muon mass

The second best determination of m_e/m_μ is provided by the muonium 1S-2S transition

The muon mass enters as a recoil correction to all Mu energies through the reduced mass $m_r\equiv m_e m_\mu/(m_e+m_\mu)$

$$E_n^{\text{Mu}} \simeq -\alpha^2 \frac{m_r c^2}{2n^2} = -\frac{R_\infty ch}{1+m_e/m_\mu} \frac{1}{n^2}$$

The 1S-2S is the best measured Mu transition between different n's:

 $\nu_{1S-2S}^{exp} = 2\,455\,528\,941.0(9.8)\,MHz$ (4ppb)

[Meyer et al. PRL 2000]



1S-2S theory

$$\begin{array}{l} \text{nonrelativistic energy} \\ \text{(including recoil)} \\ \nu_{1\mathrm{S}-2\mathrm{S}} = \frac{3}{4} \frac{R_{\infty}c}{(1+m_e/m_{\mu})} \big[1+\delta_{1\mathrm{S}-2\mathrm{S}} \big] \\ \end{array} \begin{array}{l} \mathcal{O}(\alpha^2) \text{ correction} \\ \text{(CODATA 2018 + refs therein]} \\ \text{(scaling hydrogen formulae} \\ \text{with the muon mass and removing} \\ \text{nuclear finite size and pol. effects} \end{array}$$

1S-2S theory



Least-square adjustment of muonium data

Following the CODATA procedure [see CODATA 1998] we construct a least-square fit of the Mu HFS and 1S-2S transitions to extract both m_e/m_μ and a_μ from spectroscopy

Using CODATA 2018 recommended values for $R_\infty\,$ and $\,\alpha$, current Mu data yield:

$$m_e/m_\mu = 4\,836\,329(4) \times 10^{-9}$$

$$a_{\mu}^{\mathrm{Mu}} = 116\,637(82) \times 10^{-8}(700\mathrm{ppm})$$

larger value than Muon g-2 coll. result $a_{\mu}^{\rm Mu} - a_{\mu}^{
m exp} \simeq 4.5 imes 10^{-7}$ but consistent w/in uncertainties

input datum	value	relative uncertainty	identification	reference
$\nu_{1\mathrm{S}-2\mathrm{S}}$	2455528941.0(9.8) MHz	4.0×10^{-9}	RAL-99	[40]
$ u_{ m HFS}$	$4463302776(51)\mathrm{Hz}$	1.2×10^{-8}	LAMPF-99	[38]
$ u_{ m HFS}$	$4463302.88(16){ m kHz}$	$3.6 imes 10^{-8}$	LAMPF-82	[55]
$\delta E(1{ m S})/h$	$0.000(14)\mathrm{MHz}$	4.3×10^{-12}	theory	[43]
$\delta E(2{ m S})/h$	$0.0(1.8)\mathrm{kHz}$	2.2×10^{-12}	theory	[43]
$\delta E(\mathrm{HFS})/h$	$0.000(70)\mathrm{kHz}$	1.6×10^{-8}	theory	[52]

input datum	observational equation				
$ u_{ m 1S-2S}$	$\nu_{1S-2S} = [E_M(2S; m_e/m_\mu) + \delta_{2S}^{th} - E_M(1S; m_e/m_\mu) - \delta_{1S}^{th}]/h$				
$ u_{ m HFS}$	$ u_{ m HFS} \doteq u_{ m HFS}^{ m th}(m_e/m_\mu,a_\mu) + \delta_{ m HFS}^{ m th}/h$				
$\delta E(1{ m S})/h$	$\delta E(1S) \doteq \delta_{1S}^{th}$				
$\delta E(2{ m S})/h$	$\delta E(2{ m S})\doteq\delta^{ m th}_{2{ m S}}$				
$\delta E({ m HFS})/h$	$\delta E(\mathrm{HFS}) \doteq \delta^{\mathrm{th}}_{\mathrm{HFS}}$				

very large uncertainty (Muon g-2 coll. result is $\sim 0.35 \mathrm{ppm}$)
 dominated by the 1S-2S measurement uncertainty

However, there is room for **improvement** in muonium spectroscopy!¹¹

Big improvements coming up!

The Mu-MASS experiment at PSI plans to reduce the 1S-2S uncertainty to [Crivelli Hyperfine Interact. 2018] $\sim 10 \, \mathrm{kHz}(4 \mathrm{ppt})$ $\sim 10^3$ improvement!

This could be further reduced to $\sim few \, kHz$ after the High-Intensity Muon Beam upgrade at PSI

[Kiselev et al. J-PARC symbosium 2019]

The **MuSEUM** experiment using a high-intensity pulsed muon beam at **J-PARC** will reduce the HFS uncertainty to [Tanaka et al. 2021] $\sim 10 \,\mathrm{Hz}(2.2 \mathrm{ppb})$

~ 10 improvement!

The linewidth can be reduced by selecting the « old muonium » tail (if statistics is high enough) which could bring down the HFS uncertainty to $\sim 4 \text{ Hz}(1\text{ppb})$

Theory is expected to also improve with a complete calculation of the 3-loop contribution in bound-state QED [Eides 2018]

Expected a_{μ} uncertainty from muonium

parameter	quantity	u_r			
(unit)		current	ongoing	ultimate	
	$\nu_{1S-2S}(exp)$	825	0.84	0.34	
m_e/m_μ	QED(1S-2S)	1.7	1.2	0.1	
(ppb)	R_∞	0.40	0.13		
	total	825	1.5	0.37	
	$\nu_{1S-2S}(exp)$	708	0.73	0.29	
	$ u_{ m HFS}(m exp)$	10	1.9	0.77	
	QED(1S-2S)	1.4	1.0	0.07	
a_{μ}	QED(HFS)	14	1.9	0.2	
(ppm)	HVP(HFS)	0.29	0.16		
	R_{∞}	0.35	0.13		
	α	0.26	0.14	/	
	total	708	3.0	0.88	

$\mathcal{O}(1\text{ppm})$

assuming plausible future improvements

with official goals of Mu-MASS/MuSEUM

Shedding light on Muon g-2 puzzle

A value of a_{μ}^{Mu} at $\mathcal{O}(1ppm)$ is not competitive to current spin-precession measurements

However, it may help to understand the origin of the $\sim 2ppm$ difference between (R-ratio) SM and experiment

* Committee on Data for Science and Technology

BSN CODATA*

Light new particles affect the determination of fundamental constants

The precision atomic frontier

Measurements of atomic lines in **hydrogen** are very precise:

 $\nu_{1S-2S} = 2\,466\,061\,413\,187\,035(10)\,\text{Hz}$ $u_{\nu} = 4.2 \times 10^{-15} \qquad \text{Parthey et al. (2011)}$

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QED prediction is even better: $R_{\infty} \equiv \alpha^{2} m_{e} c/2h$ $\nu_{1S-2S} = \frac{3}{4} \frac{R_{\infty} c}{(1+m_{e}/m_{p})} [1 + \delta_{1S-2S}^{QED}(\alpha) + \delta_{1S-2S}^{FNS}(r_{p})]$ TH uncertainty ~ 2 Hz

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TH uncertainty ~ 2Hz

limited by proton radius

Direct comparison fixes the Rydberg constant:

Tiesinga et al. [CODATA 2018]

Q

 $R_{\infty}c = 3.289\,841\,960\,2508(64) \times 10^{15}\,\mathrm{Hz}$

 $u_{R_{\infty}} = 1.9 \times 10^{-12}$

most precisely known fundamental constant in physics! 16

 R_∞ is *interconnected* with other constants

Rydberg constant

hydrogen $u_{\rm 1S-2S}$ (+22 other lines)

 $A_{\mathbf{r}}(p) \quad A_{\mathbf{r}}(d)$

 $A_{\rm r}(e)$

 r_p

 r_d

 R_{∞}

 R_{∞} is *interconnected* with other constants

Rydberg constant

hydrogen $u_{\rm 1S-2S}$ (+22 other lines)

fine structure constant

electron g-2 Fan et al. [2023] or atomic recoil

$$\alpha^2 = 2R_{\infty}/c \times \frac{m}{m_e} \times \frac{h}{m}$$

 87 Rb **Morel** et al. [2020] 133 Cs **Parker** et al. [2018]

There are **tensions**...

 R_∞ is interconnected with other constants

Rydberg constant

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still a **proton size puzzle**...

proton|deuteron (charge) radius

muonic hydrogen | deuterium Lamb shifts *or* ordinary hydrogen | deuterium lines *or* e-proton | e-deuteron scattering data

 $A_{\mathbf{r}}(p) A_{\mathbf{r}}(d)$

 $A_{\rm r}(e)$

 r_p

 r_d

 R_{∞}

 R_{∞} is *interconnected* with other constants

Rydberg constant

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atomic mass

constants $m_X = A_r(X)m_u$

cyclotron motion $[A_r(^{12}C) \equiv 12]$ or HD+ molecular lines Patra et al. [2020]

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 r_d

 R_{∞}

 R_∞ is interconnected with other constants

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$$\alpha^2 = 2R_\infty/c \times \frac{m}{m_e} \times \frac{h}{m}$$

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to be determined together in a global fit \rightarrow **CODATA** recommended values

https://pml.nist.gov/cuu/Constants/

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proton|deuteron (charge) radius

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or ordinary hydrogen|deuterium lines
or e-proton|e-deuteron scattering data

CODATA 2018 (selected) values

Quantity	Symbol	Numerical value	Unit	Relative std. uncert. u_r
Rydberg frequency $\alpha^2 m_{\rm e} c^2/2h = E_{\rm h}/2h$	cR_{∞}	$3.2898419602508(64)\times 10^{15}$	Hz	$1.9 imes 10^{-12}$
deuteron mass	$m_{\rm d}$	$\begin{array}{c} 3.3435837724(10)\times10^{-27}\\ 2.013553212745(40)\end{array}$	kg u	$\begin{array}{c} 3.0 \times 10^{-10} \\ 2.0 \times 10^{-11} \end{array}$
electron mass	m _e	$\begin{array}{l} 9.1093837015(28)\times10^{-31} \\ 5.48579909065(16)\times10^{-4} \end{array}$	kg u	3.0×10^{-10} 2.9×10^{-11}
proton mass	$m_{ m p}$	$\begin{array}{c} 1.67262192369(51)\times10^{-27}\\ 1.007276466621(53) \end{array}$	kg u	3.1×10^{-10} 5.3×10^{-11}
fine-structure constant $e^2/4\pi\epsilon_0\hbar c$ inverse fine-structure constant	$\alpha \atop \alpha^{-1}$	$\begin{array}{c} 7.2973525693(11)\times10^{-3}\\ 137.035999084(21) \end{array}$		$\begin{array}{c} 1.5\times 10^{-10} \\ 1.5\times 10^{-10} \end{array}$
deuteron rms charge radius	r _d	$2.12799(74) \times 10^{-15}$	m	$3.5 imes 10^{-4}$
proton rms charge radius	$r_{ m p}$	$8.414(19) \times 10^{-16}$	m	2.2×10^{-3}

TABLE XXXI. The CODATA recommended values of the fundamental constants of physics and chemistry based on the 2018 adjustment.

This accuracy relies on **assuming** the SM! How robust is it to BSM? 22

New particles below $\sim GeV$ will affect the observables used to determine the fundamental constants:

$$\mathcal{L}_{ ext{int}} = \sum_{\psi \in e, \, \mu, \, p, \, n} g_{\psi} ar{\psi} [\Gamma \cdot \phi] \psi \ = e, \mu, p, n \ \phi ext{ scalar} \ \gamma^{\mu} \phi_{\mu} ext{ vector}$$

New particles below $\sim {\rm GeV}$ will affect the observables used to determine the fundamental constants:

spin $= \frac{|g_e g_p|}{4\pi} \ge 0$ $V_{\rm NP}^{ij} = (-1)^{s+1} \alpha_{\phi} q_i q_j \frac{e^{-m_{\phi}r}}{r}$ $q_i \equiv \frac{g_i}{\sqrt{|g_i g_j|}}$ Yukawa potentials contributing to spectral lines hydrogen $~\sim \alpha_{\phi}q_eq_p = \pm \alpha_{\phi}$ deuterium $\sim \alpha_{\phi} [1 + q_e q_n]$ μ H/ μ D $\sim lpha_{\phi} q_{\mu} q_{p/n}$ $\mathrm{HD^{+}} \sim \alpha_{\phi} [1, q_e q_n, q_p q_n]$

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$$\mathcal{L}_{ ext{int}} = \sum_{\psi = e, \, \mu, \, p, \, n} g_{\psi} ar{\psi} [\Gamma \cdot \phi] \psi \ | egin{array}{c} \phi & ext{scalar} \ \gamma^{\mu} \phi_{\mu} & ext{vector} \end{array}$$

One-loop correction to a_e

 $\bigwedge \sim \alpha_{\phi} q_e^2 / 4\pi$

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$$\mathcal{L}_{\text{int}} = \sum_{\psi = e, \mu, p, n} g_{\psi} \bar{\psi} [\Gamma \cdot \phi] \psi \longrightarrow V_{\text{NP}}^{ij} = (-1)^{s+1} \alpha_{\phi} q_i q_j \frac{e^{-m_{\phi}r}}{r} + q_i \equiv \frac{q_i}{\sqrt{|g_e g_p|}}$$

$$(\gamma^{\mu} \phi_{\mu} \text{ vector}) \longrightarrow V_{\text{NP}}^{ij} = (-1)^{s+1} \alpha_{\phi} q_i q_j \frac{e^{-m_{\phi}r}}{r} + q_i \equiv \frac{q_i}{\sqrt{|g_e g_p|}} + q_i \equiv \frac{q_i}{\sqrt{|g_e g_p|}}$$

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One-loop correction to a_e

 $\sum_{m} \sim \alpha_{\phi} q_e^2 / 4\pi$

Theoretical prediction for any observable:

 $-\equiv \frac{|g_e g_p|}{4\pi} \ge 0$

Datasets

*g*_e-2,masses...

Hydrogon/Doutorium

	/ui uuci // Dculc	IIMIII						
	<u> </u>		Label	Input datum	Value	e	Rel. uncer	rt.
Label	Input datum	Value (kHz)	D1	$a_e \equiv \frac{1}{2}(g-2)_e$	1.15965218073	$(28) \times 10^{-3}$	2.4×10^{-3}	10
A1	$\nu_{\rm H}(2S_{1/2} - 4S_{1/2}) - \frac{1}{4}\nu_{\rm H}(1S_{1/2} - 2S_{1/2})$	4797338(10)	D2	δ_e	$0.000(18) \times$	10^{-12}	$1.5 imes 10^{-3}$	11
A2	$\nu_{\rm H}(2S_{1/2} - 4D_{5/2}) - \frac{1}{4}\nu_{\rm H}(1S_{1/2} - 2S_{1/2})$	6490144(24)	D3	$h/m_{ m Rb}(^{87}{ m Rb})$	4.5913592729(57)	$\times 10^{-9}{\rm m}^{2}{\rm s}^{-1}$		n
A3	$\nu_{\rm D}(2S_{1/2} - 4S_{1/2}) - \frac{1}{4}\nu_{\rm D}(1S_{1/2} - 2S_{1/2})$	4801693(20)	D4	$h/m_{\rm Cs}(^{133}{ m Cs})$	3.0023694721(12)	$\times 10^{-9}{\rm m}^{2}{\rm s}^{-1}$	Label	Input datum
A4	$\nu_{\rm D}(2S_{1/2} - 4D_{5/2}) - \frac{1}{4}\nu_{\rm D}(1S_{1/2} - 2S_{1/2})$	6494841(41)	D5	$A_{\rm r}(^{87}{\rm Rb})$	86.90918053	312(65)	·	
A5	$\nu_{\rm D}(1S_{1/2} - 2S_{1/2}) - \nu_{\rm H}(1S_{1/2} - 2S_{1/2})$	670994334.606(15)	D6	$A_{\rm r}(^{133}{\rm Cs})$	132.9054519	610(86)	A30	$ u_{ m H}(1S_{1/2} - 3S_{1/2}) $
A6	$ u_{ m H}(1S_{1/2} - 2S_{1/2}) $	2466061413187.035(10	D7	$\omega_{ m s}/\omega_{ m c}(^{12}{ m C}^{5+})$	4376.210500	0.87(12)	A31	$ u_{ m H}(2S_{1/2} - 8D_{5/2}) $
A7	$ u_{ m H}(1S_{1/2}-2S_{1/2})$	2466061413187.018(11	D8	$\Delta E_{\rm B}(^{12}{\rm C}^{5+})/hc$	c = 43.563233(25)	$ imes 10^7 \mathrm{m}^{-1}$	D1	$a_e \equiv \frac{1}{2}(g-2)_e$
A8	$ u_{ m H}(1S_{1/2} - 3S_{1/2}) $	2922743278659(17)	D9	$\delta_{ m C}$	$0.0(2.5) \times$	10^{-11}	D3	$h/m_{\rm Rb}(^{87}{\rm Rb})$
A9	$\nu_{\rm H}(2S_{1/2}-4P)$	616520931626.8(2.3)	D10	$\omega_{\rm s}/\omega_{\rm c}(^{28}{\rm Si}^{13+})$	3912.866 064	484(19)	' D5	$A_{\rm r}(^{87}{\rm Bb})$
A10	$ u_{ m H}(2S_{1/2}-8S_{1/2}) $	770649350012.0(8.6)	D11	$A_{\rm r}(^{28}{ m Si})$	27.97692653	499(52)	De	$A_{\rm r}(^{133}C_{\rm s})$
A11	$ u_{ m H}(2S_{1/2} - 8D_{3/2}) $	770649504450.0(8.3)	D12	$\Delta E_{\rm B}(^{28}{\rm Si}^{13+})/h$	c 420.6467(85)	$\times 10^{7} {\rm m}^{-1}$	DO	$A_{\rm r}(-{\rm OS})$
A12	$ u_{ m H}(2S_{1/2} - 8D_{5/2}) $	770649561584.2(6.4)	D13	$\delta_{ m Si}$	0.0(1.7) ×	10^{-9}	D9	0C
A13	$\nu_{\rm D}(2S_{1/2} - 8S_{1/2})$	770859041245.7(6.9)	D14	$\omega_{\rm c}({\rm d})/\omega_{\rm c}(^{12}{\rm C}^{6+})$) 0.992 996 654	743(20)	D13	$\delta_{ m Si}$
A14	$ u_{ m D}(2S_{1/2}-8D_{3/2})$	770859195701.8(6.3)	D15	$\omega_{\rm c}(^{12}{\rm C}^{6+})/\omega_{\rm c}(p)$	0.503776367	662(17)	D11	$A_{ m r}(^{28}{ m Si})$
A15	$ u_{ m D}(2S_{1/2} - 8D_{5/2}) $	770859252849.5(5.9)	D19	$A_{\rm r}(^{1}{\rm H})$	1.007825032	241(94)	D14	$A_{\rm r}(^2{\rm H})$
A16	$\nu_{ m H}(2S_{1/2}-12D_{3/2})$	799191710472.7(9.4)	D21	$\Delta E_{\rm B}(^{1}{\rm H^{+}})/hc$	1.0967877174307($(10) \times 10^7 \mathrm{m}^{-1}$	D15	$\Delta E_{\rm B}(^2{\rm H^+})/hc$
A17	$ u_{ m H}(2S_{1/2}-12D_{5/2})$	799191727403.7(7.0)	D23	$\Delta E_{\rm B}(^{12}{\rm C}^{6+})/hc$	c = 83.083850(25)	$ imes 10^7 \mathrm{m^{-1}}$	D19	$4_{-}(^{1}\mathrm{H})$
A18	$ u_{ m D}(2S_{1/2} - 12D_{3/2})$	799409168038.0(8.6)					D19	$\Lambda E_{-} ({}^{12}C^{6+}) / h$
A19	$\nu_{\rm D}(2S_{1/2} - 12D_{5/2})$	799409184966.8(6.8)	8.5×1	0^{-12}			D25	$\Delta E_{\rm B}(-C^{-1})/m$
A20	$\nu_{\rm H}(2S_{1/2} - 6S_{1/2}) - \frac{1}{4}\nu_{\rm H}(1S_{1/2} - 3S_{1/2})$	4197604(21)	4.9×1	0^{-6}			$\mathbf{E1}$	$\nu_{\rm HD^+}((0,0)-(0,0))$
A21	$\nu_{\rm H}(2S_{1/2} - 6D_{5/2}) - \frac{1}{4}\nu_{\rm H}(1S_{1/2} - 4S_{1/2})$	4699099(10)	2.2×1	0^{-6}		_	E2	$ u_{ m HD^+}((0,0) - (1,1)) $
A22	$ u_{ m H}(1S_{1/2} - 3S_{1/2}) $	2922743278678(13)	4.4×1	0^{-12}	uH/u	D	E3	$\nu_{\rm HD^+}((0,3)-(9,3))$
A23	$\nu_{ m H}(1S_{1/2} - 3S_{1/2})$	2922743278671.5(2.6)					G1	$\nu_{\bar{p}^4 He}((32, 31) - (31)$
A24	$\nu_{\rm H}(2S_{1/2} - 4P_{1/2}) - \frac{1}{4}\nu_{\rm H}(1S_{1/2} - 2S_{1/2})$	4664269(15)	Labe	l Input datum	Value	Rel. uncert.	G2	$\nu_{=4}$ $\mu_{=4}$ $((33, 32) - (31)$
A25	$\nu_{\rm H}(2S_{1/2} - 4P_{3/2}) - \frac{1}{4}\nu_{\rm H}(1S_{1/2} - 2S_{1/2})$	6035373(10)	<u>C1</u>	$E_{\rm c}$	200.2706(02) = V	1.1×10^{-5}	C3	ν_{-2} ((32, 31) – (31)
A26	$\nu_{\rm H}(2S_{1/2} - 2P_{3/2})$	9911200(12)		$E_{\rm LS}(\mu {\rm H})$	202.3706(23) meV	1.1×10^{-5}	C1	$\nu_{\bar{p}^3He}((02,01))$ (01)
A27	$ u_{ m H}(2P_{1/2}-2S_{1/2}) $	1057862(20)	C2	$E_{\rm LS}(\mu {\rm D})$	202.8785(34) meV	1.7×10^{-5}	G4	$\nu_{\bar{p}^3 \text{He}}((35, 35) - (35))$
A28	$ u_{ m H}(2P_{1/2}-2S_{1/2})$	1057845.0(9.0)	C7	$\delta E_{\rm LS}(\mu {\rm H})$	0.0000(129) meV	6.4×10^{-3}	11	$E_{\rm LS}(\mu^*{\rm He})$
A29	$ u_{ m H}(2P_{1/2}-2S_{1/2}) $	1057829.8(3.2)	C8	$\delta E_{\rm LS}(\mu { m D})$	0.0000(210) meV	1.0×10^{-4}	I2	$E_{\rm LS}(\mu^3{ m He})$
			C9	r_p	0.880(20) fm	2.3×10^{-2}		
			C10	r_d	2.111(19) fm	9.0×10^{-3}		

DATA22 ← used for NP analysis including post-CODATA18 improvements from

Hydrogen, HD+, pbar-He μ He, g_e -2 and masses

bel	Input datum	Value	Rel. uncert.	Reference
80	$ u_{ m H}(1S_{1/2} - 3S_{1/2}) $	2922743278665.79(72) kHz	2.5×10^{-13}	Grinin et al. [21]
31	$ u_{ m H}(2S_{1/2} - 8D_{5/2}) $	$770649561570.9(2.0)~\mathrm{kHz}$	2.6×10^{-12}	Brandt $et al. [20]$
1	$a_e \equiv \frac{1}{2}(g-2)_e$	$1.15965218059(13) imes 10^{-3}$	1.1×10^{-10}	Fan <i>et al.</i> [70]
3	$h/m_{ m Rb}(^{87}{ m Rb})$	$4.59135925890(65)\times 10^{-9}{\rm m^2s^{-1}}$	1.4×10^{-10}	Morel $et al.$ [69]
5	$A_{ m r}(^{87}{ m Rb})$	86.909180529(6)	6.9×10^{-11}	AME 2020 [73]
6	$A_{\rm r}(^{133}{\rm Cs})$	132.905 451 958(8)	6.0×10^{-11}	AME 2020 [73]
9	$\delta_{ m C}$	$0.0(9.4) \times 10^{-12}$	4.9×10^{-12}	Czarnecki et al. [71]
3	$\delta_{ m Si}$	$0.0(5.8) \times 10^{-10}$	2.8×10^{-10}	Czarnecki et al. [71]
1	$A_{\rm r}(^{28}{ m Si})$	27.97692653442(55)	2.0×10^{-11}	AME 2020 [73]
4	$A_{\rm r}(^2{ m H})$	2.014101777844(15)	7.4×10^{-12}	AME 2020 [73]
5	$\Delta E_{\rm B}(^2{\rm H^+})/hc$	$1.0970861455299(10)\times 10^{7}\mathrm{m^{-1}}$	9.1×10^{-13}	NIST ASD 2021 [62]
9	$A_{ m r}(^1{ m H})$	1.007825031898(14)	1.4×10^{-11}	AME 2020 [73]
23	$\Delta E_{\rm B}(^{12}{\rm C^{6+}})/hc$			
1	$ u_{ m HD^+}((0,0)-(0,1)) $	1314925752.910(17) kHz	1.3×10^{-11}	Alighanbari et al. [33]
2	$ u_{ m HD^+}((0,0)-(1,1)) $	$58605052164.24(86)~\rm kHz$	1.5×10^{-11}	Kortunov et al. [35]
3	$ u_{ m HD^+}((0,3)-(9,3)) $	$415264925501.8(1.3)~\rm kHz$	3.1×10^{-12}	Patra et al. [34] + Germann et al. [14]
1	$\nu_{\rm \bar{p}^4He}((32,31)-(31,30))$	1132609226.7(4.0) MHz	3.5×10^{-9}	Hori <i>et al.</i> [37]
2	$\nu_{\bar{p}^4He}((33, 32) - (31, 30))$	2145054858(7) MHz	3.4×10^{-9}	Hori <i>et al.</i> [36]
3	$\nu_{\bar{\rm p}^{3}{\rm He}}((32,31)-(31,30))$	1043128581(6) MHz	6.2×10^{-9}	Hori <i>et al.</i> [37]
4	$\nu_{\bar{\rm p}^{3}{\rm He}}((35,33)-(33,31))$	1553643100(10) MHz	6.7×10^{-9}	Hori <i>et al.</i> [36]
L	$E_{\rm LS}(\mu^4{ m He})$	1378.521(48) meV	3.5×10^{-5}	Krauth et al. [78]
2	$E_{\rm LS}(\mu^3{ m He})$	1258.586(49) meV	$3.9 imes 10^{-5}$	Krauth [79]

Benchmark NP models

Dark photon $\mathcal{L}_{int} = -\frac{\epsilon}{2} F^{\mu\nu} F'_{\mu\nu}$ $\alpha_{\phi} = \alpha \epsilon^{2}$ $q_{\ell} = -q_{p} = -1$ $q_{n} = 0$

$$\begin{array}{l} \textbf{U(1)}_{\textbf{B-L}} \quad \alpha_{\phi} = g_{\text{B-L}}^2 / 4\pi \\ q_{\ell} = -q_p = -1 \\ q_n = 1 \leftarrow \text{highlights deuterium} \end{array}$$

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Higgs portal $\alpha_{\phi} = \sin \theta^{2} m_{e} \kappa_{p} m_{p} / (4\pi v^{2})$
 $\kappa_{p} = 0.306(14), \ \kappa_{n} = 0.308(14) \leftarrow \text{from nucleon form-factors}$
 $q_{\ell} = m_{\ell} / \sqrt{m_{e} \kappa_{p} m_{p}} \leftarrow \text{larger effects in muonic atoms and molecules}$

$\begin{aligned} & \text{Hadrophilic scalar} \\ & \alpha_{\phi} = \sin \theta^2 m_e \kappa_p m_p / (4\pi v^2) \\ & q_{\ell} = 0 \quad \leftarrow \text{highlights molecules} \\ & q_{p,n} = \kappa_{p,n} m_{p,n} / \sqrt{m_e \kappa_p m_p} \end{aligned}$

U(1), $\alpha_{\pm} = a_{\rm D}^2 \pm /4\pi$

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Up-Lepto-Darko-philic (ULD) scalar $\alpha_{\phi} = k^2 m_e \kappa'_p m_p / (4\pi v^2)$ $q_{\ell} = m_{\ell} / \sqrt{m_e \kappa'_p m_p}, \ q_{p,n} = \kappa'_{p,n} m_{p,n} / \sqrt{m_e \kappa'_p m_p}$ $\kappa'_p = 0.018(5), \ \kappa'_n = 0.016(5) \leftarrow \text{couples only to up-quark}$ + dominant ϕ decay to invisible states (see later) ²⁶

 $U(1)_{-}$, $\alpha_{+} = a_{\rm D}^{2} + 4\pi$

Vectors with $m_{\phi} \ll \alpha m_e \simeq 4 \text{ keV}$ induce a long-range force Then, effects are suppressed for couplings aligned with QED ($q_i \simeq Q_i$) because:

inverse Bohr radius

$$\mathcal{L}_{\text{QED}}(\alpha) + \mathcal{L}_{A'_{\mu}}(\alpha', m_{A'} \to 0) \to \mathcal{L}_{\text{QED}}(\alpha + \alpha')$$

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This behavior is only manifest for $\mathcal{O}_{NP}(\alpha')$ and $\mathcal{O}_{SM}(\alpha)$ calculated at the <u>same</u> order in couplings. Otherwise:

$$\mathcal{O} \to \mathcal{O}_{\mathrm{SM}}^{\mathrm{LO}}(\alpha + \alpha') + \mathcal{O}_{\mathrm{SM}}^{\mathrm{NLO}}(\alpha)$$

would distinguish the photon from massless DP

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inverse Bohr radius

Instead, we use a simple *prescription*:

$$\begin{split} V_{\rm NP}^{ij} &= \alpha_{\phi} \frac{Q_i Q_j}{r} + \tilde{V}_{\rm NP}^{ij} & \text{ with } \tilde{V}_{\rm NP}^{ij} \equiv \alpha_{\phi} (q_i q_j e^{-m_{\phi}r} - Q_i Q_j)/r \\ \text{included to all orders} & & \\ \text{by shifting } \alpha \to \alpha + \alpha_{\phi} & \text{ deviations from either } m_{\phi} \neq 0 \text{ or } q_i \neq Q_i \\ \text{in } \mathcal{O}_{\rm SM} & \text{ deviations at LO} \end{split}$$

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Hence:
$$\mathcal{O} = \mathcal{O}_{SM}(\alpha + \alpha_{\phi}) + \tilde{\mathcal{O}}_{NP}(\alpha + \alpha_{\phi}, \alpha_{\phi}, m_{\phi}) + \delta \mathcal{O}_{th}$$

 $\sum \propto m_{\phi}^2 \text{ or } \delta q_i Q_j + Q_i \delta q_j$

28

 10^{-} 10^{-8} $\propto m_{\phi}^2$ 10^{-9} $\propto m_{\phi}^0$ thanks to 95% CL exclusion 10^{-10} **Deuterium** data 10^{-11} 10^{-12} 10^{-13} $U(1)_{B-L}$ 10^{-14} $\overset{\circ}{\upsilon}$ 10⁻¹⁵ ULD scalar stronger sensitivity $n_\mu/m_e \ a_0^{-1}$ 10^{-16} from internuclear forces Higgs portal 10^{-17} hadrophilic scalar in **molecules** in models 10^{-18} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 1 $10 \ 10^2 \ 10^3 \ 10^4 \ 10^5$ where $q_N/q_e \sim m_N/m_e \sim 10^3$ m_{ϕ} [keV]

stronger sensitivity from internuclear forces in **molecules** in models where $q_N/q_e \sim m_N/m_e \sim 10^3$

stronger sensitivity from **muonic** atoms in models where $q_{\mu}/q_e \sim m_{\mu}/m_e \sim 200$

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Data favors $\alpha_{\phi} \neq 0$ for **Higgs portal** and **ULD** scalars

New physics significance $=\sqrt{\chi^2_{SM}/\nu_{dof}-\chi^2_{NP}/\nu_{dof}}$

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 $\sim 4.5\sigma$ pull for scalar masses around $300-600 \, {\rm keV}$

> solving several tensions between data and SM with a *single* NP state:

- ${\it g}_{\rm e}$ -2 vs. atomic recoil $~\sim 2\sigma$
- μ H vs. H (w/in CODATA18) $\sim 3\sigma$
- CODATA18 vs. H 2S-8D $\sim 3\sigma$

Brandt et al. [2022]

Non-zero Higgs Portal?

Best-fit point $\begin{vmatrix} \sin \theta \simeq 0.35 \\ m_{\phi} \simeq 400 \text{ keV} \end{vmatrix}$ is largely **excluded** by $K^+ \to \pi^+ X_{inv}$ searches

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The **NA62** bound is driven by coupling to heavy quarks through one-loop penguins

The E137 beam-dump bound relies on scalars dominantly decaying to $\phi \rightarrow e^+e^-$

Evidence for a ULD-philic scalar?

Best-fit point $egin{array}{c} lpha_{\phi} \simeq 6.7 imes 10^{-11} \ m_{\phi} \simeq 300 \, {\rm keV} \end{array}$

<u>evades</u> the **NA62** bound by coupling only to up quarks

The E137 bound does not apply assuming invisible decay dominantes ($\phi \rightarrow DMDM$?)

In that case NA64 is relevant $e^- Z \rightarrow e^- Z \phi$ Andreev et al. [2021] yielding a weaker bound but NP sensitivity not clear below MeV

Impact on fundamental constants

FCs can undergo **huge shifts** in the presence of NP

Impact on fundamental constants

FCs can undergo **huge shifts** in the presence of NP

and their uncertainty *significantly* inflates relative to the SM-only hypothesis 33

Do we truly know $\alpha?$

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...which could hide the presence of a dark photon with $m_{A^{'}} \ll {\rm keV}$

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$$u_r(\alpha)/u_r^{\rm SM} \sim (m_{A'}/{\rm keV})^2$$

($u_r^{
m max}(lpha)\sim 1$ for $~m_{A^{'}}\lesssim 10\,{
m meV}$) $_{_{34}}$

Conclusions

\Box **Muonium** could provide a new measurement of a_{μ} at $\sim 1 \, \text{ppm}$

- Improved measurements coming up (Mu-MASS@PSI+MuSEUM@JPARC)
- This requires completing the 3-loop QED calculation in Mu (underway)

□Fundamental constants are affected by light NP states

- We proposed a consistent way to modify the CODATA adjustment
- This turns high-precision spectroscopy into a new sensitive search for NP!
- Interesting interplay with kaon physics (work in progress)

backups

Improving 1S-2S measurement

The 1S-2S is a two-photon transition ($\Delta L = 0$) with low excitation efficiency.

To increase the transition probability, a high-power pulsed laser was used in previous experiments. The price to pay was a broadening of the linewidth from $\simeq 145 \, \mathrm{kHz}$ (muon lifetime) to $\sim 20 \, \mathrm{MHz}$ and an extra $\sim 10 \, \mathrm{MHz}$ systematic uncertainty from « chirping »

The **Mu-MASS** experiment at PSI proposed to circumvent this limitation by using cavity-enhanced continuous-wave excitation, together with an intense low-energy muon beam, thus planning to reduce the 1S-2S uncertainty to $\sim 10 \, \text{kHz}(4 \text{ppt})$ [Crivelli Hyperfine Interact. 2018]

This could be further reduced to $\sim \text{few kHz}$ after the High-Intensity Muon Beam upgrade at PSI [Kiselev et al. J-PARC symbosium 2019]

Improving HFS measurement

The a_{μ}^{Mu} uncertainty can be further reduced by improving the HFS measurement

Previous measurements at LAMPF were statistics limited. The **MuSEUM** experiment using a high-intensity pulsed muon beam at J-PARC is expected to bring down the statistics uncertainty to $\sim 10 \text{ Hz}(2.2 \text{ ppb})$ [Tanaka et al. 2021]

A reduction of systematics is also needed at this level of uncertainty. The dominant one is due to pressure shift from the finite gas density in the experiment. [Kanda et al. 2021] which could be reduced by measuring the HFS in vacuum or in a gas admixture with opposite shifts.

Further improvements are <u>very</u> challenging.

A $10 \,\text{Hz}$ uncertainty already requires resolving the line to 10^{-4} of the linewidth (from muon decay), only done once in spectroscopy: the 2S-4P transition in hydrogen [Beyer et al. Science 2017]

The linewidth can be reduced by selecting the « old muonium » tail (if statistics is high enough) which could bring down the HFS uncertainty to $\sim 4 \text{ Hz}(1\text{ppb})$

Improving 1S-2S theory

Once experimental uncertainty is down to $\sim {
m few}\,{
m kHz}$, the theory must be improved by a factor ~ 10

The main theory uncertainty comes from the **uncalculated radiative-recoil** terms at three-loop QED of $\mathcal{O}[(m_e/m_\mu)\alpha(Z\alpha)^6]$

There is extra insentive to calculate them: Once the proton radius puzzle is fully resolved, such terms will become the limiting factor to further improvements of R_{∞} in hydrogen. Subleading uncertainty from **uncalculated recoil** terms of $\mathcal{O}[(m_e/m_{\mu})^2(Z\alpha)^6]$ at three-loop QED should also be reduced.

 R_{∞} should also improve by a factor few. The QED uncertainty in hydrogen was recently reduced to $\sim 1 \, \mathrm{kHz}$ [Karshenboim et al. PLB 2019] meaning that a three-fold improvement is already possible relative to CODATA 2018.

All of the above would then allow to determine the **electron-muon mass ratio** to $\sim 0.37 \text{ppb}$ thus making it a subleading source of uncertainty for a_{μ}^{Mu}

Improving HFS theory

The HFS theory should improve in the meantime by a factor ~ 20 .

To this level the uncertainty is only limited by **uncalculated terms in QED**. (The HVP uncertainty is $\sim 1 \text{ Hz}$, still subdominant.)

The required QED calculation is currently being done, with a goal of $\sim {\rm few \, Hz}$. [Eides 2018] This is motivated by the upcoming MuSEUM measurement, aiming at a reduced uncertainty of m_e/m_μ and thus of a_μ in future Fermilab/J-PARC runs.

New physics contamination

The extraction of a_{μ}^{Mu} from spectroscopy is somehow **indirect** since it assumes that the muonium theory follows from QED.

As one popular interpretation of the current puzzle is the **existence of new physics**, an immediate question is:

Could NP *contaminate* the muonium lines used to extract a_{μ} ?

We addressed the question by assuming the existence of the new boson (scalar or vector) with a muon-coupling that resolves the muon g-2 puzzle and a free coupling to electrons

> If NP *only* to muons, muonium theory is *unchanged*. An additional coupling to electrons is *constrained* by the th/exp agreement for **electron g-2**, and astrophysics from **stellar cooling**.

Maximal NP effects in muonium

Except a small range of NP mass around $\sim 1 MeV$

possible NP effects are sufficiently constrainted to below the expected Mu-MASS/MuSEUM uncertainty.