Novel lattice insights into heavy-light meson decays

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Electroweak Moriond

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Outline

Exclusive semileptonic $B_s \to K\ell\nu$ decays on the lattice

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Semileptonic $B_s \to K\ell\nu$ decays provide an alternative b-decay channel to determine the CKM matrix element $|V_{ub}|$, and to obtain a R-ratio to investigate lepton-flavor-universality violations. Results for the CKM matrix element may also shed light on the discrepancies seen between analyses of inclusive or exclusive decays. We calculate the decay form factors using lattice QCD with domain-wall light quarks and a relativistic b-quark. We analyze data at three lattice spacings with unitary pion masses down to 268 MeV. Our numerical results are interpolated/extrapolated to physical quark masses and to the continuum to obtain the vector and scalar form factors $f_+(q^2)$ and $f_0(q^2)$ with full error budgets at q^2 values spanning the range accessible in our simulations. We provide a possible explanation of tensions found between results for the form factor from different lattice collaborations. Model- and truncation-independent z-parameterization fits following a recently proposed Bayesian-inference approach extend our results to the entire allowed kinematic range. Our results can be combined with experimental measurements of $B_s \to D_s$ and $B_s \to K$ semileptonic decays to determine $|V_{ub}| = 3.8(6) \times 10^{-3}$. The error is currently dominated by experiment. We compute differential branching fractions and two types of R ratios, the one commonly used as well as a variant better suited to test lepton-flavor universality.

Bayesian inference for form-factor fits regulated by unitarity and analyticity

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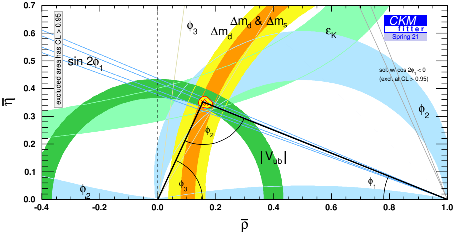
ABSTRACT: We propose a model-independent framework for fitting hadronic form-factor data, which is often only available at discrete kinematical points, to unitarity- and analyticity-based parameterisations. In this novel approach the latter two properties of quantum-field theory regulate this ill-posed problem and allow to determine model-independent predictions over the entire physical range. Kinematical constraints, which exist for example for the vector and scalar form factor of semileptonic meson decays can be imposed exactly. The core formulae are straight-forward to implement with standard math libraries and we provide one such implementation as supplementary material. As well as proposing a generalised version of the Boyd Grinstein Lebed (BGL) unitarity constraint for form factors, we demonstrate the novel method for the case of exclusive semileptonic decay $B_s \to K\ell\nu$, for which we make a number of phenomenologically relevant predictions, such as for instance the CKM matrix element $|V_{ub}|$.

Where to find New Physics?

1. Direct searches:
   $\Rightarrow$ *Bump in the spectrum*
2. Indirect searches:
   Precision tests of SM:
   $\rightarrow$ Quantum corrections due to new particles modify SM predictions
   $\rightarrow$ NP shows as discrepancy between experiment and theory

$\Rightarrow$ **Over-constrain SM**

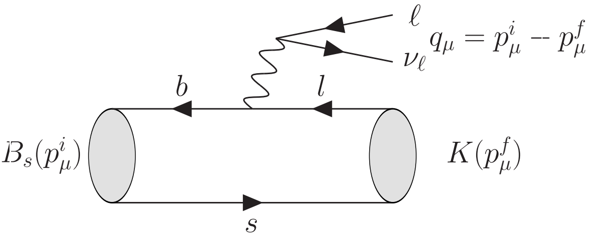
$$|V_{ub}|^{\text{FLAG21}}_{\text{exclusive}} = 3.74(17) \times 10^{-3}$$
$$|V_{ub}|^{\text{PDG22}}_{\text{inclusive}} = 4.13(26) \times 10^{-3}$$



e.g. $|V_{ub}|$ can be determined from different, i.e. **complementary** channels:

- $B \to \pi \ell \nu$
- $B_s \to K \ell \nu$
- $\Lambda_b \to p \ell \nu$
- ...

Extracting $|V_{ub}|$ from $B_s \to K\ell\nu$



Experiment $\approx$ *CKM* $\times$ non-perturbative $\times$ (PT+kinematics)

$$\frac{d\Gamma(B_s \to K\ell\nu_\ell)}{dq^2} \approx |V_{ub}|^2 \times \left[\left| f_+(q^2) \right|^2 \mathcal{K}_1 + \left| f_0(q^2) \right|^2 \mathcal{K}_2 \right]$$

- L.H.S. has been measured by LHCb
- $\mathcal{K}_1$ and $\mathcal{K}_2$ contain η_{EW}, G_F, m_ℓ, M_{B_s}, M_K, E_K - all known!
- Compute non-perturbative form factors $f_+(q^2)$ and $f_0(q^2)$
  $\Rightarrow$ e.g. from Lattice QCD.

Lattice QCD in a nutshell

Based on the **Path Integral** formulation.

$$\langle \mathcal{O} \rangle_M = \frac{1}{\mathcal{Z}} \int \mathcal{D}[\psi, \overline{\psi}, U]\, \mathcal{O}[\psi, \overline{\psi}, U]\, e^{iS[\psi, \overline{\psi}, U]}$$

Minkowski: Highly oscillatory, infinite dimensional integral. ✗

⇒ Wick rotate to Euclidean (i.e. imaginary) time ($t \to i\tau$).

$$\langle \mathcal{O} \rangle_E = \frac{1}{\mathcal{Z}} \int \mathcal{D}[\psi, \overline{\psi}, U]\, \mathcal{O}_E[\psi, \overline{\psi}, U]\, e^{-S_E[\psi, \overline{\psi}, U]}$$

Euclidean: Exponentially decaying, infinite dimensional integral. ✗

⇒ Discretise space-time and **interpret as a probability distribution**.

- Lattice spacing a (UV regulator)
- Box of length L (IR regulator)
- $\int \to \sum$, $\partial \to$ finite differences
- Evaluate **stochastically**

Lattice: Exponentially decaying and finite dimensional ✓

Set up of our calculation and required extrapolations

Simulations

- 3 $a \in [0.07, 0.11]\,\mathrm{fm}$.
- finite volume $L \Rightarrow$ discrete momenta $\boldsymbol{p} = \frac{2\pi}{L}\boldsymbol{n}$.
- heavier-than-physical $M_\pi^{\mathrm{sim}} \in [270, 430]\,\mathrm{MeV}$.

Physics

- Need to recover $a \to 0$ limit.
- Need continuous description of $q^2 = M_{B_s}^2 + M_K^2 - 2M_{B_s}E_K$.
- Recover physical light quark masses.

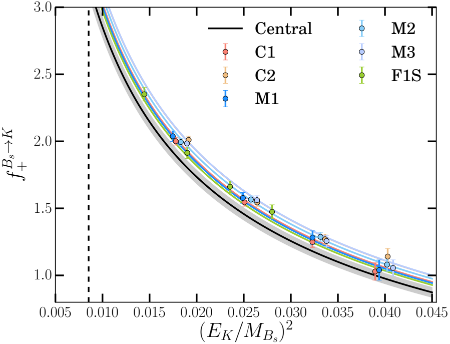
Use HMχPT [NP **B812** 64,NP **840** 54,PRD **67** 054010] for extrapolation:

$$f_X^{B_s\to K}(M_\pi, E_K, a^2) = \frac{\Lambda}{E_K + \Delta_X}\left[c_{X,0}\left(1 + \frac{\delta f(M_\pi^s) - \delta f(M_\pi^p)}{(4\pi f_\pi)^2}\right) + c_{X,1}\frac{\Delta M_\pi^2}{\Lambda^2} + c_{X,2}\frac{E_K}{\Lambda} + c_{X,3}\frac{E_K^2}{\Lambda^2} + c_{X,4}(a\Lambda)^2\right]$$

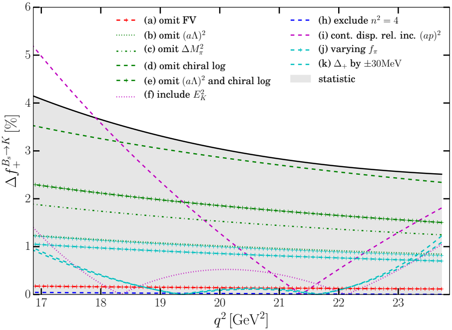
(+vary fit ansatz + estimate missing/H.O. terms)

RBC/UKQCD 23 Fit results f_+

HMχPT fit to lattice data

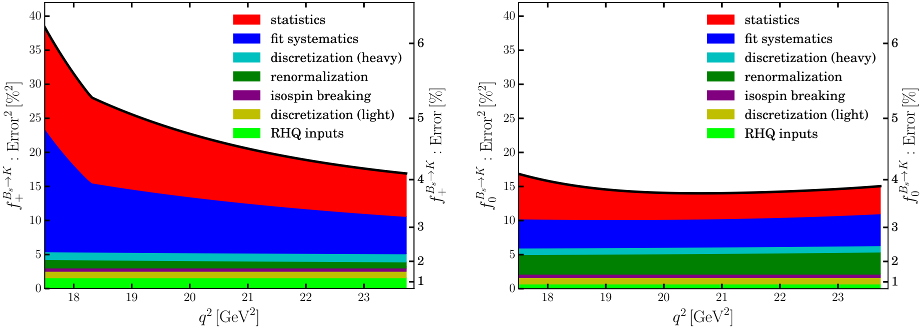


Fit systematics



⇒ take maximal deviation between the chosen fit and fit variation as fit-systematic value.

Assembling the error budget



- Dominated by statistical and fit systematic uncertainties ⇒ both improvable!
- Most precise near $q^2_{\max}$
- Data covers range $q^2 > 17\,\mathrm{GeV}^2$

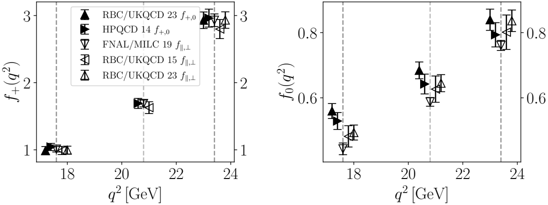
Caveat: HMχPT in terms of f_+, f_0 or $f_\parallel, f_\perp$?

Recall $1/(E_K + \Delta_X)$-term in HMχPT

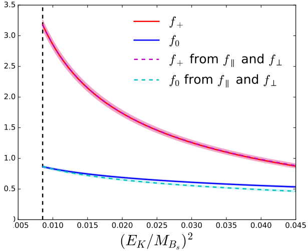
$$\Delta_+ = M_{B^*(1^-)} - M_{B_s}, M_{B^*(1^-)} = 5.32471\,\mathrm{GeV} \quad \text{(exp.)}$$
$$\Delta_0 = M_{B^*(0^+)} - M_{B_s}, M_{B^*(0^+)} = 5.63\,\mathrm{GeV} \quad \text{(the.)}$$

$f_\parallel, f_\perp$ directly accessible from lattice.

$$f_0(q^2) = \frac{\sqrt{2M_{B_s}}}{M_{B_s}^2 - M_K^2}\left[(M_{B_s} - E_K)f_\parallel(E_K) + (E_K^2 - M_K^2)f_\perp(E_K)\right]$$
$$f_+(q^2) = \frac{1}{\sqrt{2M_{B_s}}}\left[f_\parallel(E_K) + (M_{B_s} - E_K)f_\perp(E_K)\right],$$



Is the assumption $\Delta_\parallel \sim \Delta_0$, $\Delta_\perp \sim \Delta_+$ and extrapolation in $f_\perp$, $f_\parallel$, <u>before</u> converting to f_+, f_0 justified?



- Might explain tensions
- Not unique to $B_s \to K\ell\nu$

Extracting $|V_{ub}|$ from $B_s \to K\ell\nu$ – in practice

$$\frac{d\Gamma(B_s \to K\ell\nu_\ell)}{dq^2} \approx |V_{ub}|^2 \times \left[\left|f_+(q^2)\right|^2 \mathcal{K}_1 + \left|f_0(q^2)\right|^2 \mathcal{K}_2\right]$$

- Two bins for LHCb measurement

$$R_{BF} \equiv \frac{\mathcal{B}(B_s^0 \to K^- \mu^+ \nu_\mu)}{\mathcal{B}(B_s^0 \to D_s^- \mu^+ \nu_\mu)}$$

$$q^2 \leq 7\,\mathrm{GeV}^2: \quad R_{BF}^{\mathrm{low}} = 1.66(80)(86) \times 10^{-3}\,,$$
$$q^2 \geq 7\,\mathrm{GeV}^2: \quad R_{BF}^{\mathrm{high}} = 3.25(21)(^{+18}_{-19}) \times 10^{-3}\,.$$

- Lattice: Controlled uncertainties for $17\,\mathrm{GeV}^2 \lesssim q^2$.

Way out: Model Independent z-expansion!

Extrapolating over the full kinematic range: z-expansion

- Lattice data typically limited to $q^2 \in [q^2_{\mathrm{min,\,sim}}, q^2_{\mathrm{max}}]$.
- Want form factors over **full** range $[0, q^2_{\mathrm{max}}]$.
- Ff's satisfy kinematic constraint $f_+(0) = f_0(0)$.
- Map $q^2 \in [0, q^2_{\mathrm{max}}]$ to $z \in [z_{\mathrm{min}}, z_{\mathrm{max}}]$ with $|z| < 1$ and branch cut t_*.

$$z(q^2; t_0) = \frac{\sqrt{t_* - q^2} - \sqrt{t_* - t_0}}{\sqrt{t_* - q^2} + \sqrt{t_* - t_0}}$$

- Form factor is a polynomial in z after poles have been removed

e.g. BGL: Boyd, Grinstein, Lebed [PRL **74** 4603]:

$$f_X(q^2) = \left(\prod_{\mathrm{poles}} \frac{1}{B_X(q^2)}\right) \frac{1}{\phi_X(q^2)} \sum_{n\geq 0} a_{X,n} z^n$$

Goal: Determine some un-truncated number of coefficients $a_{X,n}$ to obtain model independent parameterisation

Unitarity constraint

Substituting BGL into the unitarity constraint

$$\frac{1}{2\pi i}\oint_C \frac{dz}{z}|B_X(q^2)\phi_X(q^2,t_0)f_X(q^2)|^2 \leq 1\,,$$

gives

$$|\boldsymbol{a}_X|^2 \leq 1$$

(see Paper 2 for details of a modified version of this constraint)

- **MUST** be satisfied for any believable fit!
- Any z-expansion fit must necessarily be truncated.
- Strong constraint on allowed size of coefficients $a_{X,i}$.
- How can we best make use of this?

Frequentist z-expansion fit: pro's and con's

Small number of coefficients “survive” in chiral-continuum limit

$$f_X^{B_s\to K} = \frac{\Lambda}{E_K + \Delta_X}\left(c_{X,0} + c_{X,2}\frac{E_K}{\Lambda} + c_{X,3}\frac{E_K^2}{\Lambda^2}\right)$$

$\Rightarrow$ limited number of independent data points N_X - typically $\approx$ 2-4 per ff.

$\Rightarrow$ Maximum meaningful truncation K_X of the z-expansion requires

$$N_+ + N_0 - K_+ - K_0 + 1 = N_{\mathrm{dof}} \geq 1$$

Goal: Determine $\boldsymbol{a}$, given data $\boldsymbol{f}$, covariance $C_{\boldsymbol{f}}$ and Z (encoding ϕ_X, B_X, kinematic constraint)

$$\chi^2(\boldsymbol{a},\boldsymbol{f}) = [\boldsymbol{f} - Z\boldsymbol{a}]^T\, C_{\boldsymbol{f}}^{-1}\, [\boldsymbol{f} - Z\boldsymbol{a}]^T \ .$$

pro Clear measure to assess quality of fit (p-value).

con No satisfactory way to assess systematic effect of truncation.

con Can only check unitarity constraint *a posteriori*.

Our *prior knowledge*

Our prior knowledge about probability distribution for

1. data points $\pi_{\boldsymbol{f}}(\boldsymbol{f}|\boldsymbol{f}_p, C_{\boldsymbol{f}_p}) \propto \exp\left(-\frac{1}{2}(\boldsymbol{f}-\boldsymbol{f}_p)^T C_{\boldsymbol{f}_p}^{-1}(\boldsymbol{f}-\boldsymbol{f}_p)\right)$
2. BGL ansatz: $\pi_{\boldsymbol{f}}(\boldsymbol{f}|\boldsymbol{a}, Z) \propto \delta\left(|\boldsymbol{f} - Z\boldsymbol{a}|\right)$
3. unitarity (and possible extra knowledge $\boldsymbol{a}_p, M$):
$\pi_{\boldsymbol{a}}(\boldsymbol{a}|\boldsymbol{a}_p, M) \propto \theta(1-|\boldsymbol{a}_+|^2)\,\theta(1-|\boldsymbol{a}_0|^2)\exp\left(-\frac{1}{2}(\boldsymbol{a}-\boldsymbol{a}_p)^T M(\boldsymbol{a}-\boldsymbol{a}_p)\right)$

Marginalising over $\boldsymbol{f}$ (using 1. and 2.) gives

$$\pi_{\boldsymbol{a}}(\boldsymbol{a}|\boldsymbol{f}_p, C_{\boldsymbol{f}_p}) \propto \exp\left(-\frac{1}{2}\chi^2(\boldsymbol{a}, \boldsymbol{f}_p)\right)$$

Now combining these

$$\begin{aligned}\pi_{\boldsymbol{a}}(\boldsymbol{a}|\boldsymbol{f}_p, C_{\boldsymbol{f}_p})\pi_{\boldsymbol{a}}(\boldsymbol{a}|\boldsymbol{a}_p, M) &\propto \theta(\boldsymbol{a})\exp\left(-\frac{1}{2}\chi^2(\boldsymbol{a}, \boldsymbol{f}_p) - \frac{1}{2}(\boldsymbol{a}-\boldsymbol{a}_p)^T M(\boldsymbol{a}-\boldsymbol{a}_p)\right)\\ &= \theta(\boldsymbol{a})\exp\left(-\frac{1}{2}(\boldsymbol{a}-\tilde{\boldsymbol{a}})^T C_{\tilde{\boldsymbol{a}}}^{-1}(\boldsymbol{a}-\tilde{\boldsymbol{a}})\right)\end{aligned}$$

with $\tilde{\boldsymbol{a}} = \tilde{C}_{\boldsymbol{f}_p}(Z^T C_{\boldsymbol{f}_p}^{-1}\boldsymbol{f}_p + M\boldsymbol{a}_p)$ and $C_{\tilde{\boldsymbol{a}}}^{-1} = Z^T C_{\boldsymbol{f}_p}^{-1} Z + M$

Bayesian Inference regulated by unitarity and analyticity

Compute the expectation value $g(\boldsymbol{a})$ in the presence of "*prior knowledge* B" via Bayes' theorem as (with normalisation $\mathcal{Z} = \int d\boldsymbol{a}\, \pi(\boldsymbol{a}|B)$)

$$\begin{aligned}\langle g(\boldsymbol{a})\rangle &= \frac{1}{\mathcal{Z}} \int d\boldsymbol{a}\, g(\boldsymbol{a})\, \pi(\boldsymbol{a}|B)\,, \\ &= \frac{1}{\mathcal{Z}} \int d\boldsymbol{a}\, g(\boldsymbol{a})\, \theta(1 - |\boldsymbol{a}_+|^2)\, \theta(1 - |\boldsymbol{a}_0|^2) \exp\left(-\frac{1}{2}(\boldsymbol{a} - \tilde{\boldsymbol{a}})^T C_{\tilde{\boldsymbol{a}}}^{-1} (\boldsymbol{a} - \tilde{\boldsymbol{a}})\right)\end{aligned}$$

⇒ Computable via Monte-Carlo integration by drawing from multivariate normal distribution $\mathcal{N}(\tilde{\boldsymbol{a}}, C_{\tilde{\boldsymbol{a}}})$[1].

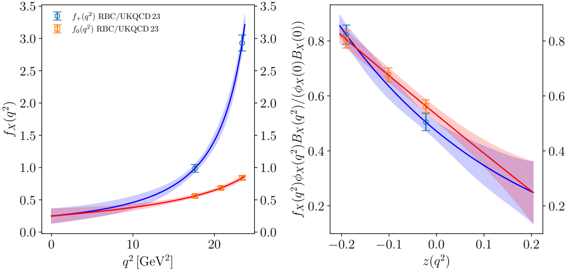
- con No clear measure to assess quality of fit (p-value).
- pro Unitarity constraint automatically satisfied
- pro Remove truncation error by computing as many terms as relevant

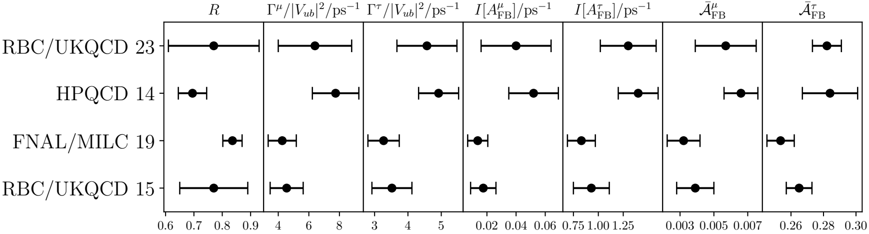
[1] See paper 2 for details about an efficient algorithm without any assumption of prior knowledge M

Results I: RBC/UKQCD

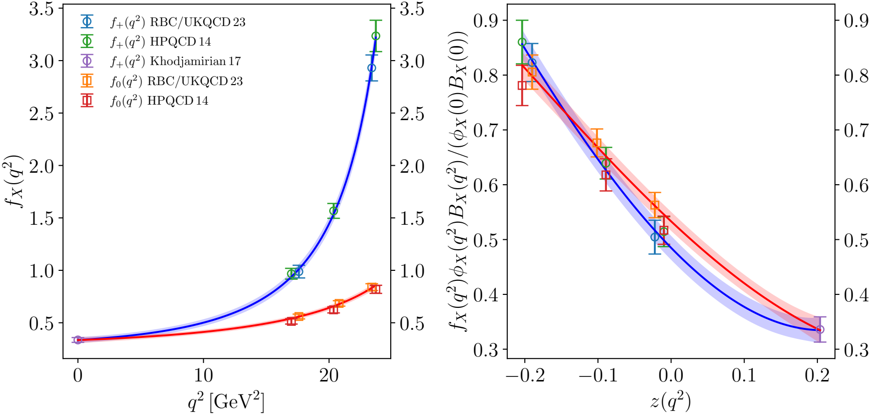
- B.I. converges from $(K_+, K_0) \sim (5, 5)$
- $10^3 \times |V_{ub}| = 3.78(61)$
- $R_{B_s \to K} = 0.77(16)$
- $R^{\mathrm{impr}}_{B_s \to K} = 1.72(11)$

$\Rightarrow$ more pheno in paper 1





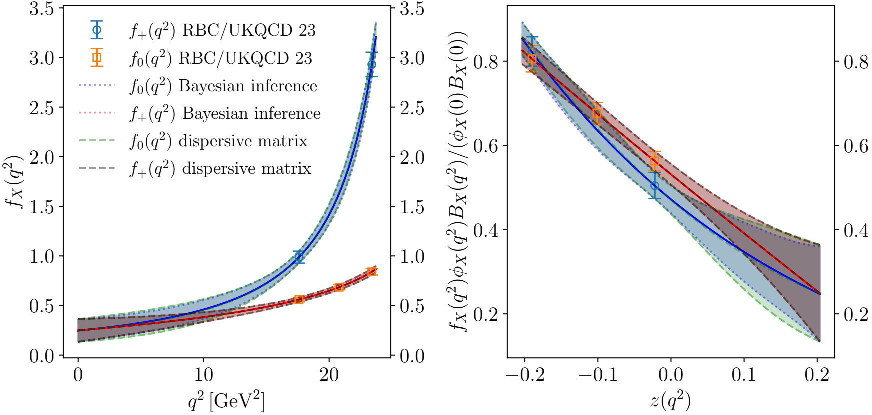
Results II: Joint analysis to multiple datasets



- Easy to jointly fit multiple datasets without any truncation systematic

$\Rightarrow$ more pheno in paper 2

Results III: Bayesian Inference vs Dispersive Matrix Method



- BI and DM produce equivalent results
- BI simpler to implement + easily to combine with multiple data sets
- BI provides numerical value of the coefficients

Conclusions and Outlook

Paper 1

- RBC/UKQCD 23 update for $B_s \to K\ell\nu$
- Identified possible reason for tensions between lattice results
- Removed systematic due to truncation of z-expansion via Bayesian Inference procedure
- Proposed improved R-ratio to better test Lepton Flavour Universality (see paper)
- Pheno: $|V_{ub}|$, R, R^{imp}, FB and polarisation asym's, decay rates ... (see paper)

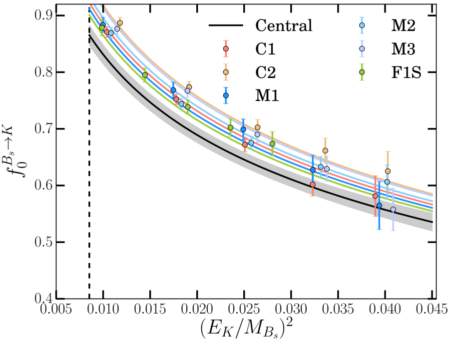
Paper 2

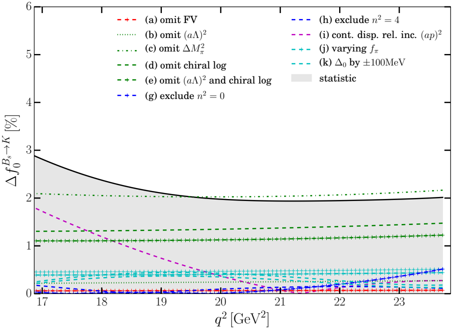
- Model-independent Bayesian Inference procedure based only on analyticity and unitarity
- easy to include multiple data sets
- modified unitarity constraint for $B_s \to K\ell\nu$ (see paper)
- proposed efficient algorithm and implementation (see paper)
- compatible with Dispersive Matrix method

$\Rightarrow$ **More broadly applicable – we keep on working on other decays...**

ADDITIONAL SLIDES

RBC/UKQCD 23 Fit results f_0





Improving lepton flavour universality tests

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2} = \underbrace{\frac{\eta_{EW}\,|V_{qb}|^2\,G_F^2}{24\pi^3}}_{\Phi}\underbrace{\left(1-\frac{m_\ell^2}{q^2}\right)^2\left[\left(1+\frac{m_\ell^2}{2q^2}\right)\right.}_{\omega_\ell}\underbrace{|\vec{p}|^3\,|f_+(q^2)|^2}_{F_V^2} + \left.\underbrace{\frac{3(m_{B_s}^2-m_K^2)^2}{8q^2m_{B_s}^2}\,|\vec{p}|\,m_\ell^2|f_0(q^2)|^2}_{(F_S^\ell)^2/(1+m_\ell^2/2q^2)}\right]$$

can be rewritten so that only ω_ℓ and F_S^ℓ depend on the lepton mass, i.e.

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2} = \Phi\,\omega_\ell\left[F_V^2 + (F_S^\ell)^2\right]$$

Define $R_{\mathrm{imp}}^{\tau/\mu}(K)$ as (changes in red) [motivated by Isidori, Sumensari'20]

$$R_{\mathrm{imp}}^{\tau/\mu}(K) \equiv \frac{\int_{q^2_{\mathrm{min}}}^{q^2_{\mathrm{max}}}\mathrm{d}q^2\,\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2}(B_s\to K\tau\nu_\tau)}{\int_{q^2_{\mathrm{min}}}^{q^2_{\mathrm{max}}}\mathrm{d}q^2\left[\frac{\omega_\tau(q^2)}{\omega_\mu(q^2)}\right]\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2}(B_s\to K\mu\nu_\mu)}$$

Noting that $(F_S^\ell)^2 \propto m_\ell^2 \sim 0$ for $\ell=\mu,e$ the SM prediction becomes

$$R^{\tau/\mu}(K) = \frac{\int_{q^2_{\mathrm{min}}}^{q^2_{\mathrm{max}}}\mathrm{d}q^2\Phi\,\omega_\tau(q^2)\left[F_V^2+(F_S^\tau)^2\right]}{\int_{q^2_{\mathrm{min}}}^{q^2_{\mathrm{max}}}\mathrm{d}q^2\Phi\,\omega_\tau(q^2)F_V^2} \approx 1 + \frac{\int_{q^2_{\mathrm{min}}}^{q^2_{\mathrm{max}}}\mathrm{d}q^2\Phi\,\omega_\tau(q^2)(F_S^\tau)^2}{\int_{q^2_{\mathrm{min}}}^{q^2_{\mathrm{max}}}\mathrm{d}q^2\Phi\,\omega_\tau(q^2)F_V^2}$$

$\Rightarrow$ **Experiment and theory might profit from cancellations!**