HIDDeν
Hunting Invisibles: Dark sectors, Dark matter and Neutrinos

University of Southampton

# Twin Pati-Salam theory for new and old puzzles in *B*-physics and the SM

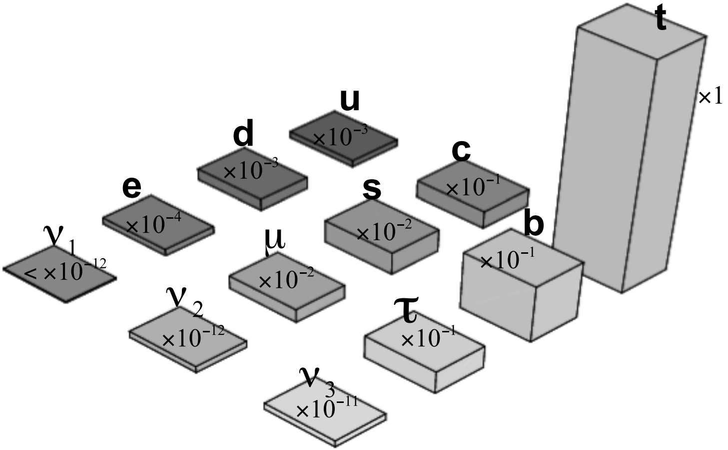
Mario Fernández Navarro[†]

57th Rencontres de Moriond (EW), 20th March 2023

Based on JHEP 02 (2023) 188, [2209.00276] [hep-ph] in collaboration with Steve King

[†] M.F.Navarro@soton.ac.uk

# An old puzzle in the flavour sector of the SM




- Suggests BSM dynamics behind the origin of Yukawa couplings.

# A new puzzle in $B$-physics

- Theoretically clean observables:
  Deviations in the $R_{K^{(*)}}$ ratios are gone LHCb [2212.09152], and the combination of $\mathcal{B}(B_s \to \mu\mu)$ measurements (see talk by Ben Allanach!) is closer to the SM after new data by CMS [CMS PAS BPH-21-006].

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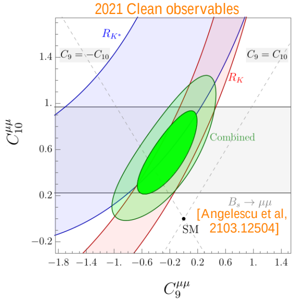


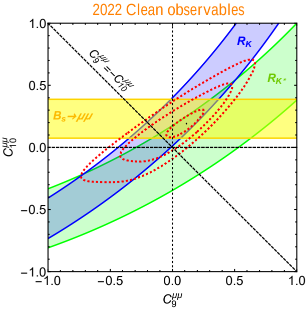


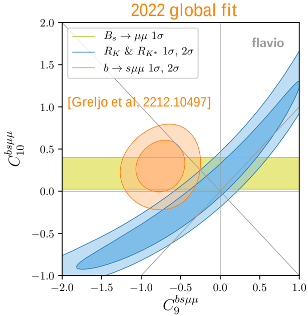

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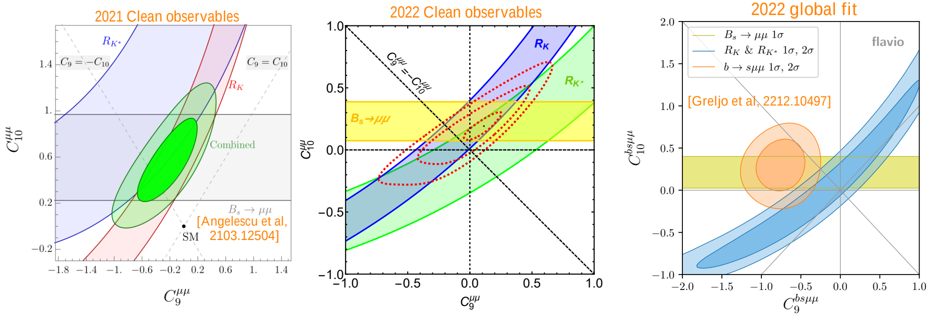


- However, now an important tension exists between the $R_{K^{(*)}}$ ratios and the remaining $b \to s\mu\mu$ data (caveat: hadronic uncertainties Gubernari et al, [2206.03797] and fit mostly driven by LHCb).

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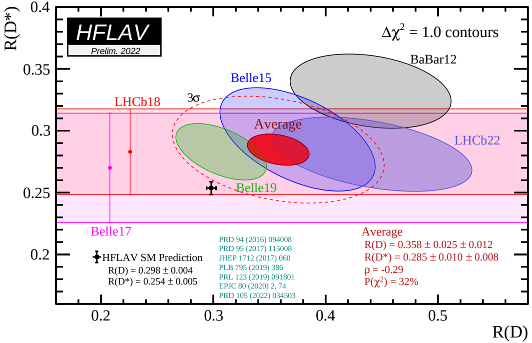
- Deviations in $R_{D^{(*)}}$ remain at $> 3\sigma$.



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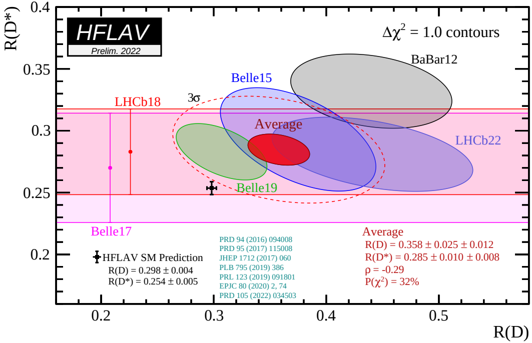


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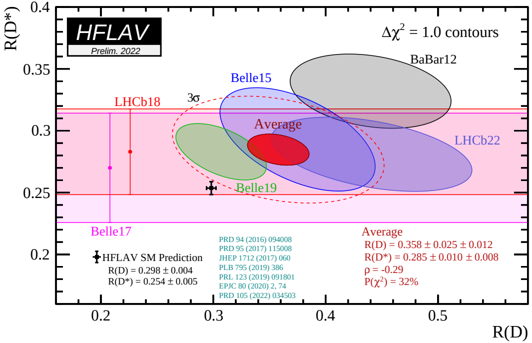
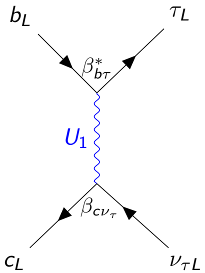
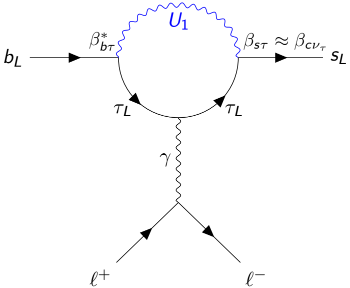



- Measurements by meson factories reinforced by LHCb in late 2022 (plus new LHCb update on $R_{D^*}$ coming tomorrow!).
- BSM interpretation? Strong NP coupled to the third family, competing with SM charged currents.

# $U_1$ vector leptoquark?

- $U_1(\mathbf{3},\mathbf{1},2/3)$ can explain $R_{D^{(*)}}$ at tree-level, and provides an important LFU vector contribution to $b \to s\ell\ell$ at 1-loop ($C_9^U$) Crivellin et al, [1807.02068]



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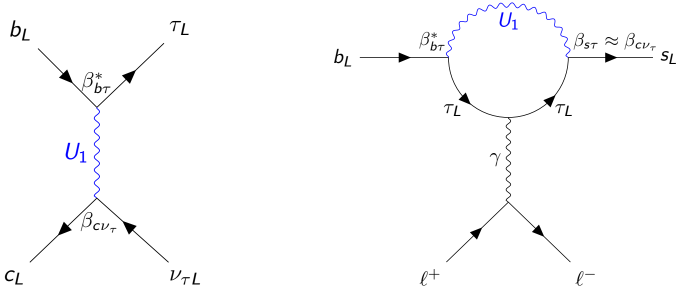


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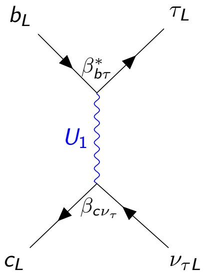


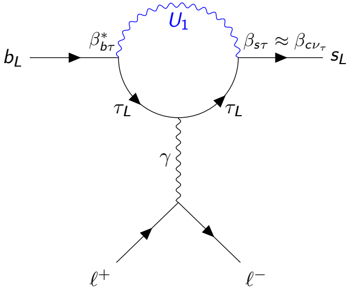


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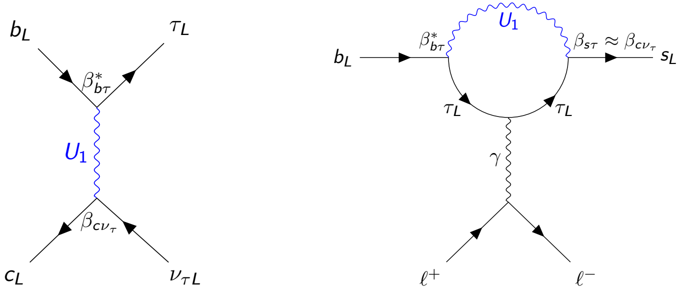


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- Gauge origin of $U_1$ at the TeV scale? UV completion?
  $\Rightarrow$ perfect opportunity for model builders
  $\Rightarrow$ Relate $B$-physics with theoretical questions like quark-lepton unification or the origin of flavour hierarchies.

# 4321 embedding and beyond

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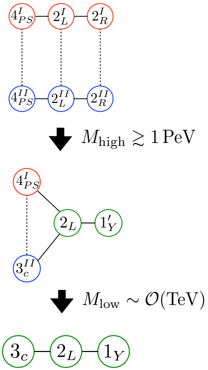
  - We will present here a theory of flavour featuring a fermiophobic 4321 at TeV scale, with extended phenomenology

# Twin Pati-Salam theory of flavour

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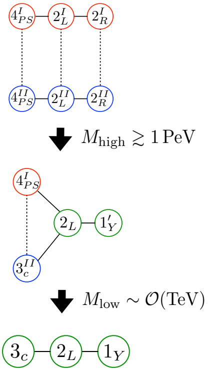


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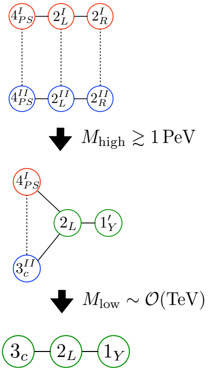


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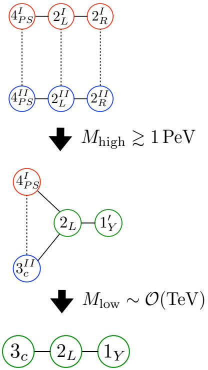


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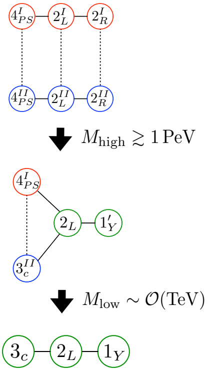


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- Low scale PS for vector-like fermions: TeV scale $U_1$ coupled to vector-like fermions
- Choice of the scalar sector and twin PS symmetry forbids SM-like Yukawa couplings
- Scalar sector is a link between PS groups, generates VL-chiral fermion mixing $\Rightarrow$ Leads to 2nd-3rd family SM Yukawas, also to $U_1$ couplings for $B$-anomalies

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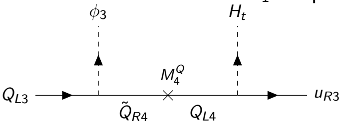


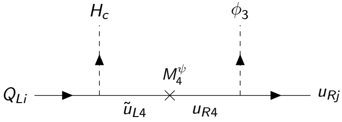


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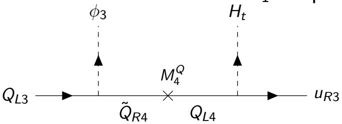


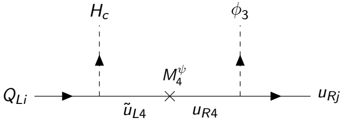


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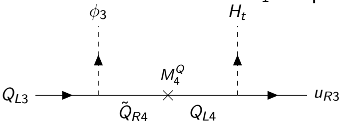


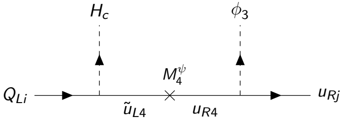


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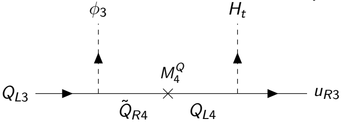


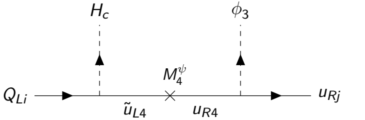


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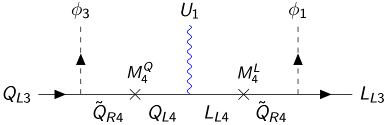


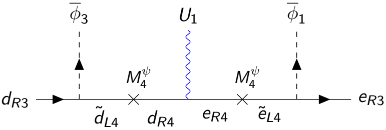


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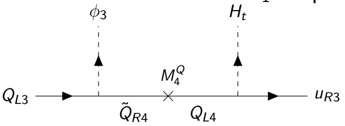


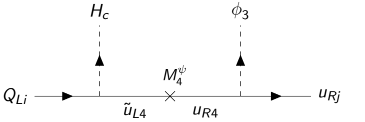


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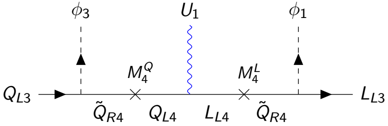


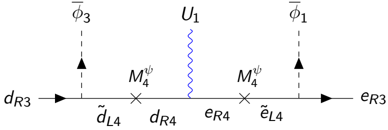


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- Extended Higgs sector not to be discussed here (but feel free to ask!)
- Predicting the flavour hierarchies naturally leads to dominantly left-handed $U_1$ couplings!

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$$\left.\begin{aligned} \beta_{b\tau} &= c_{\theta_{LQ}} s_{34}^Q s_{34}^L \\ \beta_{s\tau} &= s_{\theta_{LQ}} s_{25}^Q s_{34}^L \approx \beta_{c\nu_\tau} \end{aligned}\right\} \Rightarrow R_{D^{(*)}} \propto \beta_{b\tau}^* \beta_{s\tau}$$

$s_{\theta_{LQ}} \approx 1/\sqrt{2}$ allows for the largest contribution to $R_{D^{(*)}}$
$\Rightarrow \beta_{b\tau} \approx 1/\sqrt{2} < 1$ protects from tight constraints at high-$p_T$.

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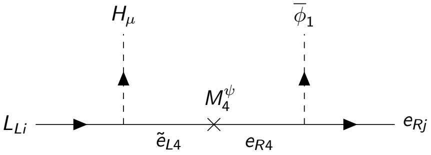


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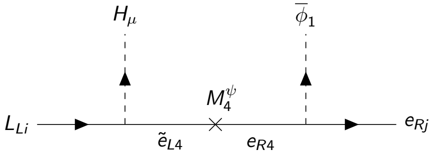


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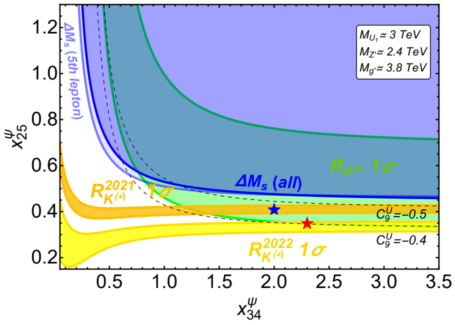
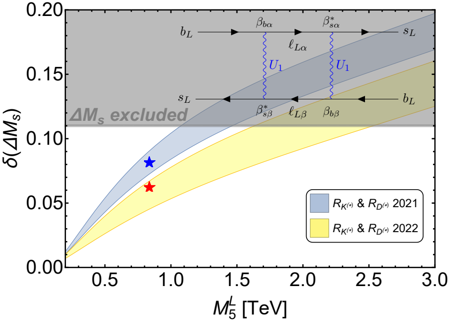
- We now fix all VEVs and VL masses to explore the parameter space of $s_{34}^{Q,L} \propto x_{34}^\psi$ and $s_{25}^{Q,L} \propto x_{25}^\psi$.

# $R_{K^{(*)}}$ and $R_{D^{(*)}}$, $B_s - \bar{B}_s$ mixing

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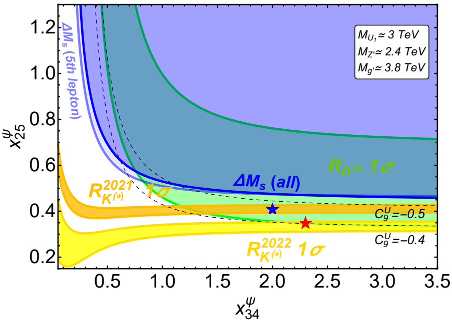


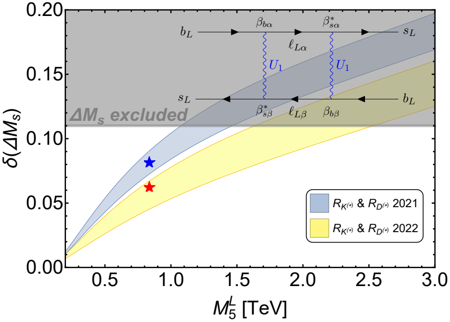


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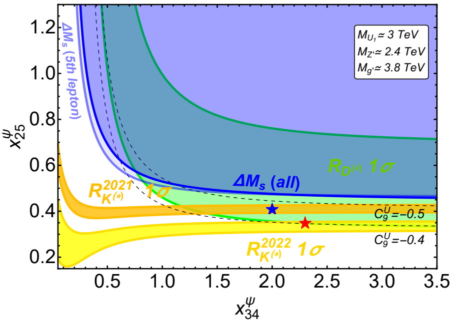


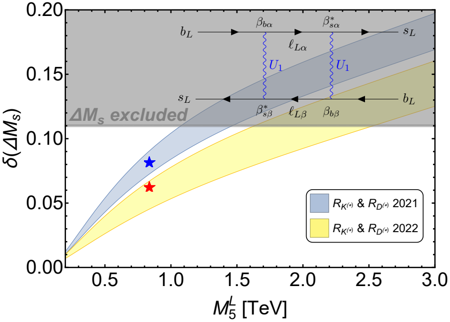


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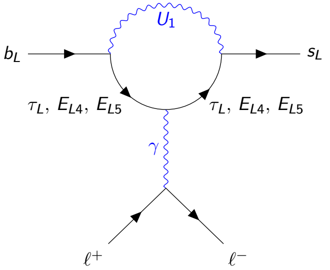
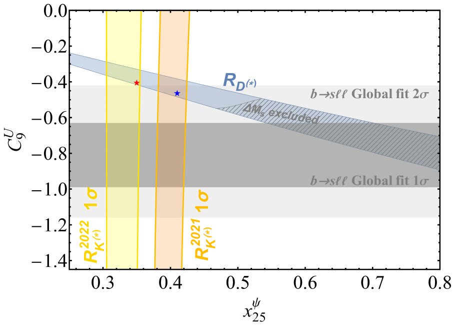
- Light VL lepton required to relax $\Delta M_s$, $2.8\sigma$ excess at CMS searches of a VL lepton with these features [2208.09700]

# Universal contribution to $b \to s\ell\ell$

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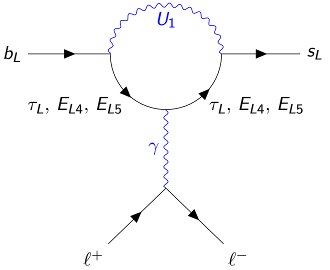


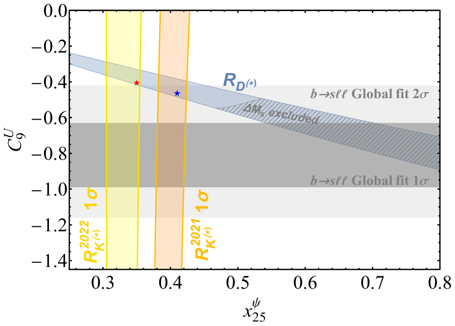


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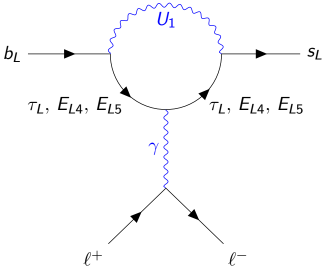


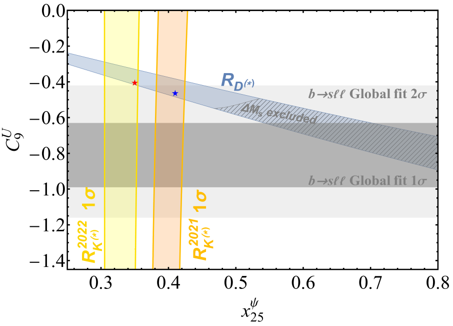


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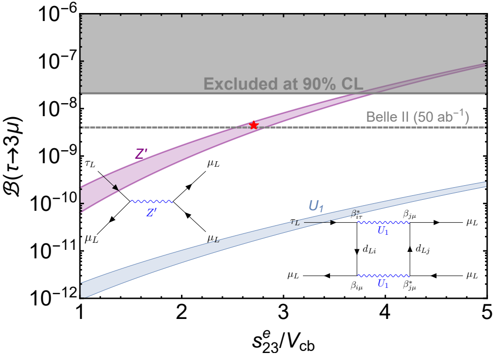
- Reduces the tension in $b \to s\ell\ell$ data, but cannot reach the values preferred by the global fits.
- However, the fit to $b \to s\ell\ell$ data can change in the future (hadronic uncertainties and/or experimental numbers can shift).

# LFV processes

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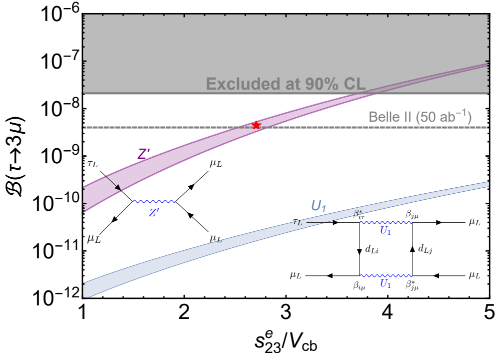


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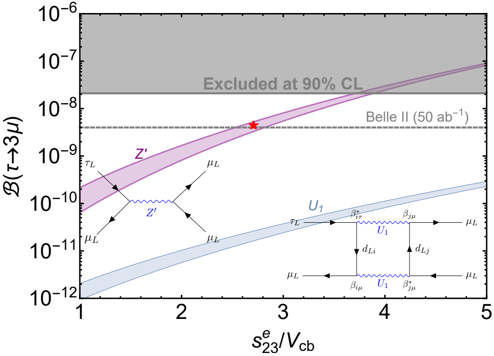


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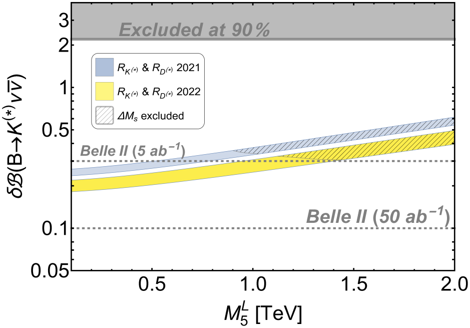
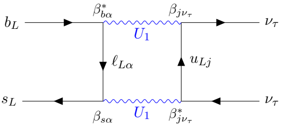
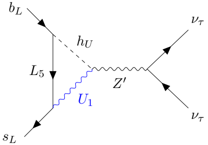
- Signals predicted also in $\tau \to \mu\gamma$ $B_s \to \tau\mu$, $B \to K\tau\mu$, $\tau \to \mu\phi$ for LHCb and Belle II searches.

# Rare decays




$$\delta\mathcal{B}(B\to K^{(*)}\nu\bar{\nu}) = \frac{\mathcal{B}(B\to K^{(*)}\nu\bar{\nu})}{\mathcal{B}(B\to K^{(*)}\nu\bar{\nu})_{\rm SM}} - 1$$

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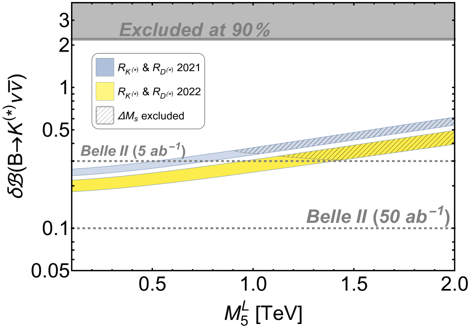


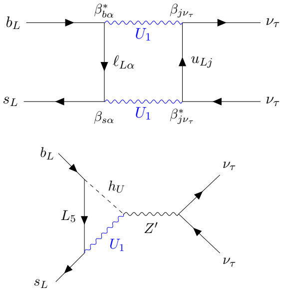


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- $B \to K^{(*)}\nu\nu$ enhanced, all parameter space whithin the reach of Belle II.

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  - Tree-level LFV via $Z'$ associated to charged lepton mixing, $B \to K^{(*)}\nu\nu$ to be tested in Belle II, naturally light VL leptons below TeV (and more in the paper).

# Acknowledgements

The author acknowledges support from the European Union's Horizon 2020 Research and Innovation programme under the Marie Skłodowska-Curie grant agreement No. 860881-HIDDeN.

# Backup: Gauge boson masses

$$M_{U_1} = \frac{1}{2} g_4 \sqrt{3v_1^2 + 3v_3^2 + \frac{4}{3} v_{15}^2}\,,$$

$$M_{g'} = \frac{\sqrt{3}}{\sqrt{2}} \sqrt{g_4^2 + g_3^2}\, v_3\,,$$

$$M_{Z'} = \frac{1}{2} \sqrt{\frac{3}{2}} \sqrt{g_4^2 + \frac{2}{3} g_1^2} \sqrt{3v_1^2 + v_3^2}\,.$$

with

$$\langle \phi_3 \rangle = \begin{pmatrix} \frac{v_3}{\sqrt{2}} & 0 & 0 \\ 0 & \frac{v_3}{\sqrt{2}} & 0 \\ 0 & 0 & \frac{v_3}{\sqrt{2}} \\ 0 & 0 & 0 \end{pmatrix}, \quad \langle \phi_1 \rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{v_1}{\sqrt{2}} \end{pmatrix}, \quad \langle \Omega_{15} \rangle = \frac{1}{2\sqrt{6}} \mathrm{diag}(1,1,1,-3) v_{15}$$

The choice $v_3 \gg v_1$ leads to $M_{g'} \approx \sqrt{2} M_{U_1}$ (phenomenologically motivated).

# Backup: Personal Higgses

$$H(\bar{4},\overline{2},1;4,2,1) \to H_t(\bar{4},3,\overline{2},2/3),\; H_b(\bar{4},3,\overline{2},-1/3),\; H_\tau(\bar{4},1,\overline{2},-1),\; H_{\nu_\tau}(\bar{4},1,\overline{2},0)\,,$$

$$\overline{H}(\bar{4},\overline{2},1;\bar{4},1,\overline{2}) \to H_c(4,\overline{3},\overline{2},1/3),\; H_s(4,\overline{3},\overline{2},-2/3),\; H_\mu(\bar{4},1,\overline{2},0),\; H_{\nu_\mu}(\bar{4},1,\overline{2},1)\,,$$

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- FCNCs in the Higgs basis? $\Rightarrow$ we assume that only one pair of Higgs doublets, $H_u$ and $H_d$ are light, given by linear combinations of the personal Higgs,

$$H_u=\widetilde{\alpha}_u H_t+\widetilde{\beta}_u H_c+\widetilde{\gamma}_u H_{\nu_\tau}+\widetilde{\delta}_u H_{\nu_\mu}\,,\ H_d=\widetilde{\alpha}_d H_b+\widetilde{\beta}_d H_s+\widetilde{\gamma}_d H_\tau+\widetilde{\delta}_d H_\mu$$

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- The orthogonal linear combinations are very heavy, only the light Higgs doublets get VEVs for EW SSB, $\langle H_u\rangle=v_u$, $\langle H_d\rangle=v_d$,

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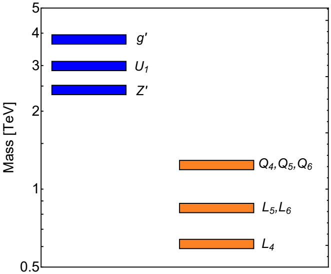
- The orthogonal linear combinations are very heavy, only the light Higgs doublets get VEVs for EW SSB, $\langle H_u\rangle = v_u$, $\langle H_d\rangle = v_d$,
- Familiar from $SO(10)$ models, where 6 Higgs doublets arise as $H_{10}$, $H_{120}$, $H_{\overline{126}}$, two from each, but below the $SO(10)$ scale only two Higgs are light. We invert the unitary transformations

$$\begin{aligned} H_t &= \alpha_u H_u + \ldots\,, & H_b &= \alpha_d H_d + \ldots\,, & H_\tau &= \gamma_d H_d + \ldots\,, & H_{\nu_\tau} &= \gamma_u H_u + \ldots\,,\\ H_c &= \beta_u H_u + \ldots\,, & H_s &= \beta_d H_d + \ldots\,, & H_\mu &= \delta_d H_d + \ldots\,, & H_{\nu_\mu} &= \delta_u H_u + \ldots\,, \end{aligned}$$

the personal Higgses in the original basis can be thought of as gaining VEVs $\langle H_t\rangle = \alpha_u v_u = v_{\rm SM}/\sqrt{2}$, etc and $\langle H_{b,s,\tau,\mu}\rangle \sim \mathcal{O}(\mathrm{GeV})$, $\langle H_{t,c}\rangle \sim v_{\rm SM}/\sqrt{2}$

# High-$p_T$ signatures




| Particle | Decay mode | $\mathcal{B}$(benchmark) |
|---|---|---|
| $U_1$ | $Q_3L_5 + Q_5L_3$ | $\sim 0.47$ |
| | $Q_3L_3$ | $\sim 0.22$ |
| | $Q_5L_5$ | $\sim 0.24$ |
| | $Q_iL_a + Q_aL_i$ | $\sim 0.07$ |
| $g'$ | $Q_3Q_3$ | $\sim 0.3$ |
| | $Q_5Q_5$ | $\sim 0.3$ |
| | $Q_6Q_6$ | $\sim 0.3$ |
| | $Q_1Q_6+Q_2Q_5 + Q_3Q_4$ | $\sim 0.1$ |
| $Z'$ | $L_5L_5$ | $\sim 0.29$ |
| | $L_6L_6$ | $\sim 0.29$ |
| | $L_3L_3$ | $\sim 0.27$ |
| | $Q_3Q_3+Q_5Q_5 + Q_6Q_6$ | $\sim 0.09$ |
| | $L_1L_6 + L_2L_5 + L_3L_4$ | $\sim 0.06$ |

- VL fermions with physical masses

$$\tilde{M}_a^Q = \sqrt{\left(x_{ia}^{\psi}\,\langle\phi_3\rangle\right)^2 + \left(M_a^Q\right)^2}\,, \quad \tilde{M}_a^L = \sqrt{\left(x_{ia}^{\psi}\,\langle\phi_1\rangle\right)^2 + (M_a^L)^2}\,,$$

with VL leptons naturally predicted below 1 TeV (excess at CMS [2208.09700]).

# Backup: VL-chiral mixing

$$s_{34}^{Q} = \frac{x_{34}^{\psi}\langle\phi_3\rangle}{\sqrt{\left(x_{34}^{\psi}\langle\phi_3\rangle\right)^2 + \left(M_{44}^{Q}\right)^2}}\,, \qquad s_{34}^{L} = \frac{x_{34}^{\psi}\langle\phi_1\rangle}{\sqrt{\left(x_{34}^{\psi}\langle\phi_1\rangle\right)^2 + \left(M_{44}^{L}\right)^2}}\,,$$

$$s_{25}^{Q} = \frac{x_{25}^{\psi}\langle\phi_3\rangle}{\sqrt{\left(x_{25}^{\psi}\langle\phi_3\rangle\right)^2 + \left(M_{55}^{Q}\right)^2}}\,, \qquad s_{25}^{L} = \frac{x_{25}^{\psi}\langle\phi_1\rangle}{\sqrt{\left(x_{25}^{\psi}\langle\phi_1\rangle\right)^2 + \left(M_{55}^{L}\right)^2}}\,,$$

$$s_{16}^{Q} = \frac{x_{16}^{\psi}\langle\phi_3\rangle}{\sqrt{\left(x_{16}^{\psi}\langle\phi_3\rangle\right)^2 + \left(M_{66}^{Q}\right)^2}}\,, \qquad s_{16}^{L} = \frac{x_{16}^{\psi}\langle\phi_1\rangle}{\sqrt{\left(x_{16}^{\psi}\langle\phi_1\rangle\right)^2 + \left(M_{66}^{L}\right)^2}}\,,$$

$$\tilde{M}_4^{Q} = \sqrt{\left(x_{34}^{\psi}\langle\phi_3\rangle\right)^2 + \left(M_{44}^{Q}\right)^2}\,, \quad \tilde{M}_4^{L} = \sqrt{\left(x_{34}^{\psi}\langle\phi_1\rangle\right)^2 + \left(M_{44}^{L}\right)^2}\,,$$

$$\tilde{M}_5^{Q} = \sqrt{\left(x_{25}^{\psi}\langle\phi_3\rangle\right)^2 + \left(x_{35}^{\psi}\langle\phi_3\rangle\right)^2 + \left(M_{55}^{Q}\right)^2}\,, \quad \tilde{M}_5^{L} = \sqrt{\left(x_{25}^{\psi}\langle\phi_1\rangle\right)^2 + \left(x_{35}^{\psi}\langle\phi_1\rangle\right)^2 + \left(M_{55}^{L}\right)^2}\,,$$

$$\tilde{M}_6^{Q} = \sqrt{\left(x_{16}^{\psi}\langle\phi_3\rangle\right)^2 + \left(M_{66}^{Q}\right)^2}\,, \quad \tilde{M}_6^{L} = \sqrt{\left(x_{16}^{\psi}\langle\phi_1\rangle\right)^2 + \left(M_{66}^{L}\right)^2}\,.$$

# Backup: Mass matrix, block-diagonalisation

$$
M^\psi = \left(\begin{array}{c|ccccccccc}
 & \psi_1^c & \psi_2^c & \psi_3^c & \psi_4^c & \psi_5^c & \psi_6^c & \overline{\psi_4} & \overline{\psi_5} & \overline{\psi_6} \\
\hline
\psi_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & x_{16}^{\psi}\phi \\
\psi_2 & 0 & 0 & 0 & y_{24}^{\psi}\overline{H} & y_{25}^{\psi}\overline{H} & 0 & 0 & x_{25}^{\psi}\phi & 0 \\
\psi_3 & 0 & 0 & 0 & y_{34}^{\psi}\overline{H} & y_{35}^{\psi}\overline{H} & 0 & x_{34}^{\psi}\phi & x_{35}^{\psi}\phi & 0 \\
\psi_4 & 0 & 0 & y_{43}^{\psi}H & 0 & 0 & 0 & \bar{M}_{44}^{\psi} & M_{45}^{\psi} & 0 \\
\psi_5 & 0 & 0 & y_{53}^{\psi}H & 0 & 0 & 0 & M_{54}^{\psi} & \bar{M}_{55}^{\psi} & 0 \\
\psi_6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \bar{M}_{66}^{\psi} \\
\overline{\psi_4^c} & 0 & x_{42}^{\psi^c}\overline{\phi'} & x_{43}^{\psi^c}\overline{\phi} & \bar{M}_{44}^{\psi^c} & M_{45}^{\psi^c} & 0 & 0 & 0 & 0 \\
\overline{\psi_5^c} & 0 & x_{52}^{\psi^c}\overline{\phi'} & x_{53}^{\psi^c}\overline{\phi} & M_{54}^{\psi^c} & \bar{M}_{55}^{\psi^c} & 0 & 0 & 0 & 0 \\
\overline{\psi_6^c} & x_{61}^{\psi^c}\overline{\phi} & 0 & 0 & 0 & 0 & \bar{M}_{66}^{\psi^c} & 0 & 0 & 0
\end{array}\right),
$$

$$
\begin{pmatrix} M_{44}^{Q} & 0 & 0 \\ 0 & M_{55}^{Q} & 0 \\ 0 & 0 & M_{66}^{Q} \end{pmatrix} = V_{45}^{Q} \begin{pmatrix} \bar{M}_{44}^{Q} & M_{45}^{\psi} & 0 \\ M_{54}^{\psi} & \bar{M}_{55}^{Q} & 0 \\ 0 & 0 & \bar{M}_{66}^{Q} \end{pmatrix} V_{45}^{\bar{Q}\dagger}, \qquad \bar{M}_{aa}^{Q} \equiv M_{aa}^{\psi} + \frac{\lambda_{15}^{aa}\langle\Omega_{15}\rangle}{2\sqrt{6}},
$$

$$
\begin{pmatrix} M_{44}^{L} & 0 & 0 \\ 0 & M_{55}^{L} & 0 \\ 0 & 0 & M_{66}^{L} \end{pmatrix} = V_{45}^{L} \begin{pmatrix} \bar{M}_{44}^{L} & M_{45}^{\psi} & 0 \\ M_{54}^{\psi} & \bar{M}_{55}^{L} & 0 \\ 0 & 0 & \bar{M}_{66}^{L} \end{pmatrix} V_{45}^{\bar{L}\dagger}, \qquad \bar{M}_{aa}^{L} \equiv M_{aa}^{\psi} - 3\frac{\lambda_{15}^{aa}\langle\Omega_{15}\rangle}{2\sqrt{6}},
$$

# Backup: Mass matrix, block-diagonalisation

Block-diagonalised via (effective mass matrices arise):

$$V_\psi = V_{16}^\psi V_{35}^\psi V_{25}^\psi V_{34}^\psi V_{45}^\psi V_{45}^{\overline{\psi^c}} \,,$$

$$V_{\psi^c} = V_{16}^{\psi^c} V_{35}^{\psi^c} V_{25}^{\psi^c} V_{34}^{\psi^c} V_{24}^{\psi^c} V_{45}^{\psi^c} V_{45}^{\bar\psi} \,.$$

$$M^{\psi\prime} = \left(\begin{array}{c|ccccccccc}
 & \psi_1^{c\,\prime} & \psi_2^{c\,\prime} & \psi_3^{c\,\prime} & \psi_4^{c\,\prime} & \psi_5^{c\,\prime} & \psi_6^{c\,\prime} & \overline{\psi_4^\prime} & \overline{\psi_5^\prime} & \overline{\psi_6^\prime} \\
\hline
\psi_1^\prime & & & & & & & 0 & 0 & 0 \\
\psi_2^\prime & & & & & & & 0 & 0 & 0 \\
\psi_3^\prime & & & & & & & 0 & 0 & 0 \\
\psi_4^\prime & & & & \tilde{y}^{\prime\psi}_{\alpha\beta} & & & \tilde{M}_4^\psi & 0 & 0 \\
\psi_5^\prime & & & & & & & 0 & \tilde{M}_5^\psi & 0 \\
\psi_6^\prime & & & & & & & 0 & 0 & \tilde{M}_6^\psi \\
\overline{\psi_4^{c\,\prime}} & 0 & 0 & 0 & \tilde{M}_4^{\psi^c} & 0 & 0 & 0 & 0 & 0 \\
\overline{\psi_5^{c\,\prime}} & 0 & 0 & 0 & 0 & \tilde{M}_5^{\psi^c} & 0 & 0 & 0 & 0 \\
\overline{\psi_6^{c\,\prime}} & 0 & 0 & 0 & 0 & 0 & \tilde{M}_6^{\psi^c} & 0 & 0 & 0
\end{array}\right) \,,$$

# Backup: Effective Yukawa couplings (mass matrices)

- Zeroes enforced by $Z_4$:

$$M^u_{\rm eff} = \begin{pmatrix} & u_1^{c\,\prime} & u_2^{c\,\prime} & u_3^{c\,\prime} \\ \hline Q_1^\prime | & 0 & 0 & 0 \\ Q_2^\prime | & 0 & 0 & s_{25}^Q y_{53}^\psi \\ Q_3^\prime | & 0 & 0 & s_{34}^Q y_{43}^\psi \end{pmatrix} \langle H_t \rangle + \begin{pmatrix} & u_1^{c\,\prime} & u_2^{c\,\prime} & u_3^{c\,\prime} \\ \hline Q_1^\prime | & 0 & 0 & 0 \\ Q_2^\prime | & 0 & c_{25}^Q s_{24}^{q^c} y_{24}^\psi & c_{25}^Q s_{34}^{q^c} y_{24}^\psi \\ Q_3^\prime | & 0 & c_{34}^Q s_{24}^{q^c} y_{34}^\psi & c_{34}^Q s_{34}^{q^c} y_{34}^\psi \end{pmatrix} \langle H_c \rangle + \text{h.c.}\,,$$

$$M^d_{\rm eff} = \begin{pmatrix} & d_1^{c\,\prime} & d_2^{c\,\prime} & d_3^{c\,\prime} \\ \hline Q_1^\prime | & 0 & 0 & 0 \\ Q_2^\prime | & 0 & 0 & s_{25}^Q y_{53}^\psi \\ Q_3^\prime | & 0 & 0 & s_{34}^Q y_{43}^\psi \end{pmatrix} \langle H_b \rangle + \begin{pmatrix} & d_1^{c\,\prime} & d_2^{c\,\prime} & d_3^{c\,\prime} \\ \hline Q_1^\prime | & 0 & 0 & 0 \\ Q_2^\prime | & 0 & c_{25}^Q s_{24}^{q^c} y_{24}^\psi & c_{25}^Q s_{34}^{q^c} y_{24}^\psi \\ Q_3^\prime | & 0 & c_{34}^Q s_{24}^{q^c} y_{34}^\psi & c_{34}^Q s_{34}^{q^c} y_{34}^\psi \end{pmatrix} \langle H_s \rangle + \text{h.c.}\,,$$

$$M^e_{\rm eff} = \begin{pmatrix} & e_1^{c\,\prime} & e_2^{c\,\prime} & e_3^{c\,\prime} \\ \hline L_1^\prime | & 0 & 0 & 0 \\ L_2^\prime | & 0 & 0 & s_{25}^L y_{53}^\psi \\ L_3^\prime | & 0 & 0 & s_{34}^L y_{43}^\psi \end{pmatrix} \langle H_\tau \rangle + \begin{pmatrix} & e_1^{c\,\prime} & e_2^{c\,\prime} & u_3^{c\,\prime} \\ \hline L_1^\prime | & 0 & 0 & 0 \\ L_2^\prime | & 0 & c_{25}^L s_{24}^{e^c} y_{24}^\psi & c_{25}^L s_{34}^{e^c} y_{24}^\psi \\ L_3^\prime | & 0 & c_{34}^L s_{24}^{e^c} y_{34}^\psi & c_{34}^L s_{34}^{e^c} y_{34}^\psi \end{pmatrix} \langle H_\mu \rangle + \text{h.c.}\,,$$

- CKM down alligned if

$$s_{25}^Q y_{53}^\psi \langle H_b \rangle + c_{25}^Q s_{34}^{q^c} y_{24}^\psi \langle H_s \rangle \approx 0 \Rightarrow y_{53}^\psi \approx (-)\mathcal{O}(0.1 - 0.5) \Rightarrow \theta_{23}^d \approx 0$$

$$\Rightarrow \begin{cases} \theta_{23}^u \approx \dfrac{-s_{25}^Q \left|y_{53}^\psi\right| \langle H_t \rangle + c_{25}^Q s_{34}^{q^c} y_{24}^\psi \langle H_c \rangle}{s_{34}^Q y_{43}^\psi \langle H_t \rangle} \approx s_{25}^Q \left|y_{53}^\psi\right| \approx \mathcal{O}(V_{cb})\,, \\ \theta_{23}^e \approx \dfrac{-s_{25}^L \left|y_{53}^\psi\right| \langle H_\tau \rangle + c_{25}^L s_{34}^{e^c} y_{24}^\psi \langle H_\mu \rangle}{s_{34}^L y_{43}^\psi \langle H_\tau \rangle} \approx \mathcal{O}(V_{cb} - 4V_{cb}) \end{cases}$$

# Backup: Top mass

In good approximation, the mass of the top quark is given by the (3,3) entry in $M^u_{\rm eff}$, i.e.

$$m_t \approx s^Q_{34} y^\psi_{43} \langle H_t \rangle = s^Q_{34} y^\psi_{43} \alpha_u \frac{1}{\sqrt{1+\tan^{-2}\beta}} \frac{v_{\rm SM}}{\sqrt{2}} \,,$$

where $v_{\rm SM} = 246\,{\rm GeV}$ and we have applied $\langle H_t \rangle = \alpha_u v_u$, where

$$v_u = \frac{v_{\rm SM}}{\sqrt{2}} \sin\beta = \frac{1}{\sqrt{1+\tan^{-2}\beta}} \frac{v_{\rm SM}}{\sqrt{2}} \,,$$

as in usual 2HDM. If we consider $\tan\beta \approx 10$ and $\alpha_u \approx 1$[1], then we obtain

$$m_t \approx s^Q_{34} y^\psi_{43} \frac{v_{\rm SM}}{\sqrt{2}} \equiv y_t \frac{v_{\rm SM}}{\sqrt{2}} \,.$$

[1] This choice preserves $\langle H_t \rangle$ at the EW scale, larger values would break the decoupling approximation that we have assumed during the diagonalisation of the full mass matrix.

# Backup: 1st family masses

- Add one VL family split across both PS groups, take advantage of scalars performing high scale breaking, $Z_4$ still provides flavour structure

| Field | $SU(4)^I_{PS}$ | $SU(2)^I_L$ | $SU(2)^I_R$ | $SU(4)^{II}_{PS}$ | $SU(2)^{II}_L$ | $SU(2)^{II}_R$ | $Z_4$ |
|---|---|---|---|---|---|---|---|
| $\psi_7$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{1}$ | $\mathbf{4}$ | $\mathbf{1}$ | $\mathbf{1}$ | 1 |
| $\overline{\psi}_7$ | $\mathbf{1}$ | $\overline{\mathbf{2}}$ | $\mathbf{1}$ | $\overline{\mathbf{4}}$ | $\mathbf{1}$ | $\mathbf{1}$ | 1 |
| $\psi_7^c$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{1}$ | $\overline{\mathbf{4}}$ | $\overline{\mathbf{2}}$ | $\overline{\mathbf{2}}$ | 1 |
| $\overline{\psi_7^c}$ | $\mathbf{1}$ | $\overline{\mathbf{2}}$ | $\mathbf{1}$ | $\mathbf{4}$ | $\mathbf{2}$ | $\mathbf{2}$ | 1 |
| $h$ | $\mathbf{1}$ | $\overline{\mathbf{2}}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\alpha^3$ |
| $H'$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{4}$ | $\mathbf{1}$ | $\mathbf{2}$ | 1 |
| $\overline{H}'$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\overline{\mathbf{4}}$ | $\mathbf{1}$ | $\overline{\mathbf{2}}$ | 1 |
| $\Phi$, $\Phi'$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\overline{\mathbf{2}}$ | $\mathbf{1}$ | 1, $\alpha^2$ |

$$M^\psi = \begin{pmatrix} & \psi_1^c & \psi_2^c & \psi_3^c & \psi_7^c & \overline{\psi_7} \\ \hline \psi_1 | & 0 & 0 & 0 & y_{17}^\psi h & 0 \\ \psi_2 | & 0 & 0 & 0 & 0 & x_{27}^\psi \Phi \\ \psi_3 | & 0 & 0 & 0 & 0 & x_{37}^\psi \Phi \\ \psi_7 | & y_{71}^\psi h & 0 & 0 & 0 & M_7^\psi \\ \overline{\psi_7^c} | & 0 & x_{72}^{\psi^c} \Phi' & x_{73}^{\psi^c} \Phi & M_7^{\psi^c} & 0 \end{pmatrix}$$

$$M_{\mathrm{eff}} = \begin{pmatrix} & \psi_1^{c\prime} & \psi_2^{c\prime} & \psi_3^{c\prime} \\ \hline \psi_1' | & 0 & 0 & 0 \\ \psi_2' | & y_{71}^\psi x_{27}^\psi & 0 & 0 \\ \psi_3' | & y_{71}^\psi x_{37}^\psi & 0 & 0 \end{pmatrix} \frac{\langle \Phi \rangle}{M_7^\psi} \langle h \rangle + \begin{pmatrix} & \psi_1^{c\prime} & \psi_2^{c\prime} & \psi_3^{c\prime} \\ \hline \psi_1' | & 0 & y_{17}^\psi x_{27}^{\psi^c} & y_{17}^\psi x_{37}^{\psi^c} \\ \psi_2' | & 0 & 0 & 0 \\ \psi_3' | & 0 & 0 & 0 \end{pmatrix} \frac{\langle \Phi \rangle}{M_7^{\psi^c}} \langle h \rangle + \mathrm{h.c.}$$

- VL masses splitted via $\Omega_{15}$. Texture zero and up-alligned structure in the (1-2) CKM sector.

# Backup: Neutrino masses

Single right-handed neutrino dominance

$$M_\nu^M = \begin{pmatrix} \tilde{\xi}^2 & \tilde{\xi}^5 & \tilde{\xi}^4 \\ \tilde{\xi}^5 & \tilde{\xi}^2 & \tilde{\xi} \\ \tilde{\xi}^4 & \tilde{\xi} & 1 \end{pmatrix} \frac{\langle H' \rangle \langle H' \rangle}{\Lambda} \simeq \begin{pmatrix} M_1^M & 0 & 0 \\ 0 & M_2^M & \tilde{\xi} \\ 0 & \tilde{\xi} & M_3^M \end{pmatrix}, \tag{1}$$

$$M_1^M \simeq M_2^M \simeq \tilde{\xi}^2 M_3^M , \tag{2}$$

$$M_3^M = \frac{\langle H' \rangle \langle H' \rangle}{\Lambda} . \tag{3}$$

$$M_\nu^D = \begin{pmatrix} 0 & a & a' \\ e & b & b' \\ f & c & c' \end{pmatrix} . \tag{4}$$

Now we apply the seesaw formula:

$$m_\nu = M_\nu^D \left(M_\nu^M\right)^{-1} \left(M_\nu^D\right)^T . \tag{5}$$

If we neglect the off-diagonal $\tilde{\xi}$ terms

$$m_\nu = \begin{pmatrix} 0 & 0 & 0 \\ 0 & e^2 & ef \\ 0 & ef & f^2 \end{pmatrix} \frac{1}{M_1^M} + \begin{pmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{pmatrix} \frac{1}{M_2^M} + \begin{pmatrix} a'^2 & a'b' & a'c' \\ a'b' & b'^2 & b'c' \\ a'c' & b'c' & c'^2 \end{pmatrix} \frac{1}{M_3^M} . \tag{6}$$

# Backup: $R_{K^{(*)}}$ and $R_{D^{(*)}}$

$$\mathcal{L}_{\rm eff} \supset C^{U_1}_{bs\mu\mu}\,(\bar{s}_L\gamma_\mu b_L)\,(\bar{\mu}_L\gamma^\mu \mu_L) + C^{U_1}_{bc\tau\nu}\,(\bar{c}_L\gamma_\mu b_L)\,(\bar{\tau}_L\gamma^\mu \nu_{\tau L}) + \text{h.c.}\,,$$

$$C^{U_1}_{bs\mu\mu} = -\frac{g_4^2}{2M_{U_1}^2}\beta^*_{b\mu}\beta_{s\mu} = \frac{g_4^2}{2M_{U_1}^2} c_{\theta_{LQ}} s_{\theta_{LQ}} s^Q_{25} s^Q_{34}\left(s^L_{25}\right)^2\,,$$

$$C^{U_1}_{bc\tau\nu_\tau} = -\frac{g_4^2}{2M_{U_1}^2}\beta^*_{b\tau}\beta_{c\nu_\tau} = -\frac{g_4^2}{2M_{U_1}^2} c_{\theta_{LQ}} s_{\theta_{LQ}} s^Q_{25} s^Q_{34}\left(s^L_{34}\right)^2\,,$$

in order to fit

$$C^{U_1}_{bs\mu\mu} = \frac{4G_F}{\sqrt{2}} V_{tb} V^*_{ts} \frac{\alpha}{4\pi} 2\delta C^\mu_L\,, \qquad \delta C^\mu_L = -0.7^{+0.07}_{-0.07}\,,$$

$$C^{U_1}_{cb\tau\nu_\tau} = -2\sqrt{2} G_F V_{cb} g_{V_L}\,, \qquad g_{V_L} = 0.07 \pm 0.02\,,$$

# Backup: $\Delta M_s$

$$\delta(\Delta M_s) \equiv \frac{\Delta M_s - \Delta M_s^{\rm SM}}{\Delta M_s^{\rm SM}} = \left|1 + \frac{C_{bs}^{\rm NP}}{C_{bs}^{\rm SM}}\right| - 1 = \frac{C_{bs}^{\rm NP}}{C_{bs}^{\rm SM}} \lesssim 0.11 \tag{7}$$

$$C_{bs}^{\rm NP-loop} = \frac{g_4^4}{(8\pi M_{U_1})^2} \sum_{\alpha,\beta} \left(\beta_{s\alpha}^* \beta_{b\alpha}\right) \left(\beta_{s\beta}^* \beta_{b\beta}\right) F(x_\alpha, x_\beta) \tag{8}$$

where $\alpha, \beta = \mu, \tau, E_4, E_5$ run for all charged leptons, including the vector-like partners, and $x_\alpha = (m_\alpha / M_U)^2$. We have generalised the loop function in [Fuentes-Martin et al, 2009.11296] to the case of more than one VL families,

$$F(x_\alpha, x_\beta) = \left(1 + \frac{x_\alpha x_\beta}{4}\right) B(x_\alpha, x_\beta)\,, \tag{9}$$

where

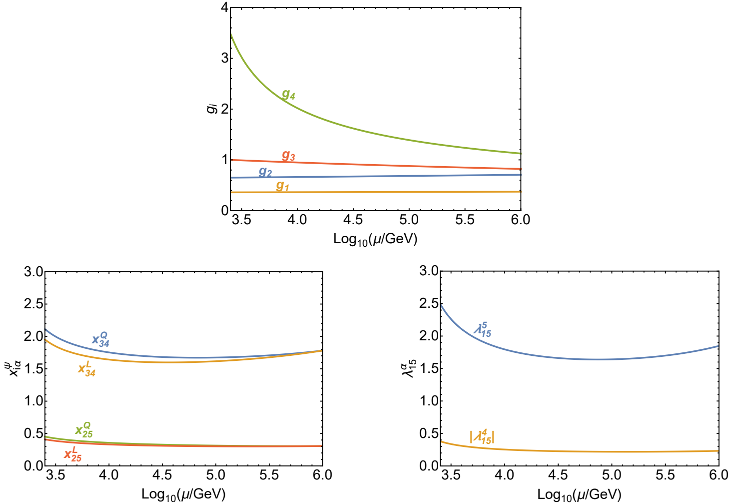
$$B(x_\alpha, x_\beta) = \frac{1}{(1 - x_\alpha)(1 - x_\beta)} + \frac{x_\alpha^2 \log x_\alpha}{(x_\beta - x_\alpha)(1 - x_\alpha^2)} + \frac{x_\beta^2 \log x_\beta}{(x_\alpha - x_\beta)\left(1 - x_\beta^2\right)}\,. \tag{10}$$

The product of couplings $\beta_{s\alpha}^* \beta_{b\alpha}$ has the fundamental property

$$\sum_\alpha \beta_{s\alpha}^* \beta_{b\alpha} = 0\,. \tag{11}$$

# Backup: Perturbativity

The low-energy 4321 theory must remain perturbative until the high scale of the twin Pati-Salam symmetry.



# Backup: Gauge bosons couplings

$$\mathcal{L}_{U_1}^{\text{gauge}} = \frac{g_4}{\sqrt{2}} Q_i^{\dagger\,\prime} \gamma_\mu \begin{pmatrix} s_{16}^Q s_{16}^L \epsilon & 0 & 0 \\ 0 & c_{\theta_{LQ}} s_{25}^Q s_{25}^L & s_{\theta_{LQ}} s_{25}^Q s_{34}^L \\ 0 & -s_{\theta_{LQ}} s_{34}^Q s_{25}^L & c_{\theta_{LQ}} s_{34}^Q s_{34}^L \end{pmatrix} L_j^\prime U_1^\mu + \text{h.c.}\,,$$

$$\mathcal{L}_{g'}^{\text{gauge}} = \frac{g_4 g_s}{g_3} Q_i^{\dagger\,\prime} \gamma^\mu T^a \begin{pmatrix} \left(s_{16}^Q\right)^2 - \left(c_{16}^Q\right)^2 \frac{g_3^2}{g_4^2} & 0 & 0 \\ 0 & \left(s_{25}^Q\right)^2 - \left(c_{25}^Q\right)^2 \frac{g_3^2}{g_4^2} & 0 \\ 0 & 0 & \left(s_{34}^Q\right)^2 - \left(c_{34}^Q\right)^2 \frac{g_3^2}{g_4^2} \end{pmatrix} Q_j^\prime g_\mu^{a\,\prime}\,.$$

$$\mathcal{L}_{Z',\ell}^{\text{gauge}} = -\frac{\sqrt{3}}{\sqrt{2}} \frac{g_4 g_Y}{g_1} L_i^{\dagger\,\prime} \gamma^\mu \begin{pmatrix} \frac{1}{2}\left(s_{16}^L\right)^2 - \left(c_{16}^L\right)^2 \frac{g_1^2}{3g_4^2} & 0 & 0 \\ 0 & \frac{1}{2}\left(s_{25}^L\right)^2 - \left(c_{25}^L\right)^2 \frac{g_1^2}{3g_4^2} & 0 \\ 0 & 0 & \frac{1}{2}\left(s_{34}^L\right)^2 - \left(c_{34}^L\right)^2 \frac{g_1^2}{3g_4^2} \end{pmatrix} L_j^\prime Z_\mu^\prime\,.$$

# Backup: $B \to K\nu\bar{\nu}$

$$\mathcal{L}_{b\to s\nu\nu} = -C_{\nu}^{\tau\tau}\,(\bar{s}_L\gamma_\mu b_L)\,(\bar{\nu}_{L\tau}\gamma^\mu\nu_{L\tau})\;,\; C_{\nu}^{\tau\tau} = C_{\nu,\mathrm{NP}}^{\tau\tau} + C_{\nu,\mathrm{SM}}\,.$$

We parameterise corrections to the SM branching fraction as

$$\delta\mathcal{B}(B\to K^{(*)}\nu\bar{\nu}) = \frac{\mathcal{B}(B\to K^{(*)}\nu\bar{\nu})}{\mathcal{B}(B\to K^{(*)}\nu\bar{\nu})_{\mathrm{SM}}} - 1 \approx \frac{1}{3}\left|\frac{C_{\nu\nu}^{\mathrm{NP}} - C_{\nu\nu}^{\mathrm{SM}}}{C_{\nu\nu}^{\mathrm{SM}}}\right|^2 - \frac{1}{3}\,.$$

We split the NP effects into $Z'$-mediated and $U_1$-mediated contributions as follows

$$C_{\nu,\mathrm{NP}}^{\tau\tau} = C_{\nu,Z'}^{\tau\tau} + C_{\nu,U}^{\tau\tau}\,.$$

The $U_1$ contribution at NLO accuracy reads

$$C_{\nu,U}^{\tau\tau} \approx C_{\nu,U}^{\mathrm{RGE}} + \frac{g_4^4}{32\pi^2 M_{U_1}^2}\sum_{\alpha,j}\left(\beta_{s\alpha}^*\beta_{b\alpha}\right)\left(\beta_{j\nu_\tau}\right)^2 F(x_\alpha, x_j)\,,$$

where the second term arises from the semileptonic box diagram and the first term encodes the RGE-induced contribution from the tree-level leptoquark-mediated operator $(\bar{s}_L\gamma_\mu b_L)\,(\bar{\tau}_L\gamma^\mu\tau_L)$,

$$C_{\nu,U}^{\mathrm{RGE}} = -0.047\,\frac{g_4^2}{2M_{U_1}^2}\,\beta_{b\tau}\beta_{s\tau}\,.$$

$$C_{\nu,Z'}^{\tau\tau} \approx \frac{3g_4^2}{2M_{Z'}^2}\left[\xi_{bs}\xi_{\nu_\tau\nu_\tau}\left(1+\frac{3}{2}\frac{g_4^2}{16\pi^2}\xi_{\nu_\tau\nu_\tau}^2\right) + \frac{g_4^2}{16\pi^2}\beta_{sE_5}^*\beta_{bE_5}\,G_{\Delta Q=1}(x_{E_5}, x_{Z'}, x_R)\right]\,,$$

where $x_{E_5} \equiv (M_5^L)^2/M_U^2$, $x_{Z'} \equiv M_{Z'}^2/M_U^2$ and $x_R \equiv M_R^2/M_U^2$ with $M_R$ being a scale associated to the radial mode $h_U(3,1,2/3)$ arising from $\phi_{3,1}$. The loop function [Fuentes-Martin et al, 2009.11296]

$$G_{\Delta Q=1}(x_1, x_2, x_3) \approx \frac{5}{4}x_1 + \frac{x_1}{2}\left(x_2 - \frac{3}{2}\right)\left(\ln x_3 - \frac{5}{2}\right)\,,$$

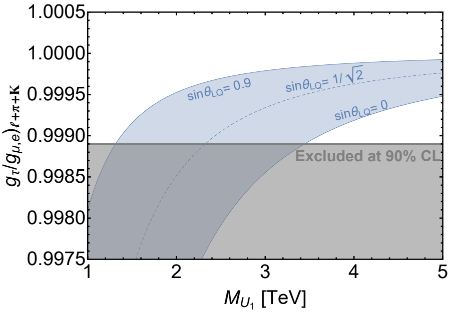
# Backup: Tests of universality in leptonic $\tau$ decays

$$\left(\frac{g_\tau}{g_\mu}\right)_\ell = 1 + \frac{9}{12} C_{Z'} \left(|\xi_{\tau e}|^2 - |\xi_{\mu e}|^2\right) - \eta C_U \left(|\beta_{b\tau}|^2 - |\beta_{b\mu}|^2\right) , \tag{12}$$

$$\left(\frac{g_\tau}{g_e}\right)_\ell = 1 + \frac{9}{12} C_{Z'} \left(|\xi_{\tau \mu}|^2 - |\xi_{\mu e}|^2\right) - \eta C_U \left(|\beta_{b\tau}|^2 - |\beta_{be}|^2\right) , \tag{13}$$

where $\eta = 0.079$ parameterises the running from $\Lambda = 2\,\mathrm{TeV}$. Due to the hierarchy in leptoquark couplings, we find $\beta_{b\tau} \gg \beta_{b\mu}$ and $\beta_{be} \approx 0$, hence in good approximation both ratios receive the same contribution proportional to $\beta_{b\tau}$, so we can approximate

$$\left(\frac{g_\tau}{g_{\mu,e}}\right)_{\ell+\pi+K} \approx 1 - \eta C_U \, |\beta_{b\tau}|^2 , \tag{14}$$

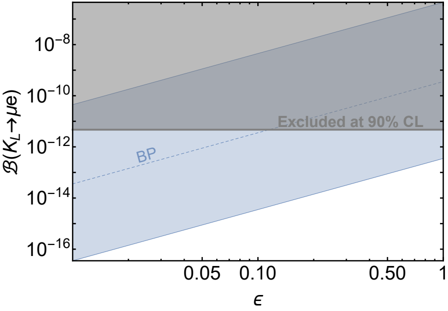


# Backup: $K_L \to \mu e$

The LFV process $K_L \to \mu e$ sets a strong constraint over all models featuring a vector leptoquark $U_1$ with first and second family couplings,

$$\mathcal{B}\left(K_L \to \mu e\right) = \frac{\tau_{K_L} G_F^2 f_K^2 m_\mu^2 m_K}{8\pi} \left(1 - \frac{m_\mu^2}{m_K^2}\right)^2 C_U^2 \left|\beta_{de}\beta_{s\mu}^*\right|^2 .$$

The first family coupling $\beta_{de}$ can be diluted via mixing with vector-like fermions, which we parameterised via the effective parameter $\epsilon$, so that $\beta_{se} \approx s_{16}^Q s_{16}^L \epsilon$.



# Backup: $K_L \to \mu e$ (cont.)

| field | $Z_2$ |
|---|---|
| $\bar{\psi}_6,\ \psi_6$ | 1, 1 |
| $\bar{\psi}'_6,\ \psi'_6$ | -1, -1 |
| $\chi$ | -1 |

$$\mathcal{L}_{\rm mix} = x_{66}\chi\bar{\psi}_6\psi'_6 + x'_{66}\chi^*\bar{\psi}'_6\psi_6 \ + {\rm h.c.} \tag{15}$$

$$\mathcal{L}_{\rm mass} = (M^{\psi}_{66} + \lambda^{66}_{15}T_{15}\Omega_{15})\bar{\psi}_6\psi_6 + (M^{\psi}_{66'} + \lambda^{66'}_{15}T_{15}\Omega_{15})\bar{\psi}'_6\psi'_6 + {\rm h.c.} \tag{16}$$

Then for LQ couplings

$$\mathcal{L}_{U_1} = \frac{g_4}{\sqrt{2}}\begin{pmatrix} Q^\dagger_6 & Q^{\dagger\prime}_6 \end{pmatrix}\gamma_\mu V^Q_{66'}\,{\rm diag}(1,1)V^{L\dagger}_{66'}\begin{pmatrix} L_6 \\ L'_6 \end{pmatrix} U^\mu_1 + {\rm h.c.} \tag{17}$$

If we define

$$V^Q_{66'}, V^{L\dagger}_{66'} \equiv \begin{pmatrix} \cos\theta_6 & \sin\theta_6 \\ -\sin\theta_6 & \cos\theta_6 \end{pmatrix}, \tag{18}$$

then the $Q^\dagger_6 L_6 U_1$ coupling receives a suppression via $\cos\theta_6$ as

$$\beta_{de} = s^Q_{16}s^L_{16}\cos\theta_6\ . \tag{19}$$

which is identified with the suppression parameter $\epsilon$,

$$\epsilon \equiv \cos\theta_6\,. \tag{20}$$