# Results and Prospects of T2K

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# Outline

I $\nu$ oscillation

II T2K experiment

- General intro
- Beyond neutrino oscillation physics

III T2K 2022 $\nu$ oscillation analysis

- General strategy
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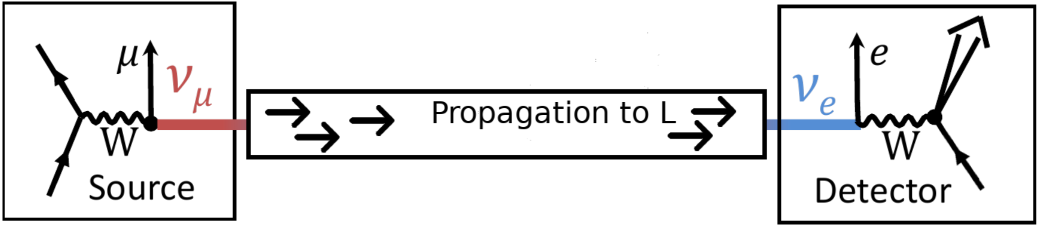
- 2022 $\nu$ oscillation results

IV The rosy future of T2K

- T2K+NOvA, T2K+SK joint analyses
- SK-Gd
- Beam upgrade
- ND280 upgrade

# $\nu$ Oscillation in $3\nu$-paradigm

# $\nu$ Oscillation




- Source producing neutrinos of certain flavor – e.g. $\nu_\mu$
- Detector (at a certain distance $L$) observes reduction in the flux of neutrinos of the flavor

  $\Rightarrow$ **NEUTRINO DISAPPEARANCE:** $\nu_\mu \longrightarrow \nu_\mu$
- Detector observes increase in the flux of neutrinos of different flavors from the one produced

  $\Rightarrow$ **NEUTRINO APPEARANCE:** $\nu_\mu \longrightarrow \nu_e$

- Described by $\nu$-mixing, each **flavor state** $\nu_\alpha$ (weak int.) is **a superposition of mass states** $\nu_i$ (free particle)
- The (dis)appearance of $\nu$ has an oscillatory pattern as a function of distance/energy $\Rightarrow$

NEUTRINO OSCILLATION

# $\nu$ Oscillation Parameters of the $3\nu$-paradigm

NuFIT global analysis *JHEP 09, 178 (2020)*

| | Normal ordering (best fit) | | Inverted ordering | |
|---|---|---|---|---|
| | Best fit $\pm 1\sigma$ | $3\sigma$ range | Best fit $\pm 1\sigma$ | $3\sigma$ range |
| $\sin^2\theta_{12}$ | $0.304 \pm 0.012$ | $0.269 - 0.343$ | $0.304^{+0.013}_{-0.012}$ | $0.269 - 0.343$ |
| $\sin^2\theta_{23}$ | $0.573^{+0.016}_{-0.020}$ | $0.415 - 0.616$ | $0.575^{+0.016}_{-0.019}$ | $0.419 - 0.617$ |
| $\sin^2\theta_{13}$ | $0.02219^{+0.00062}_{-0.00063}$ | $0.02032 - 0.02410$ | $0.02238^{+0.00063}_{-0.00062}$ | $0.02052 - 0.02428$ |
| $\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$ | $7.42^{+0.21}_{-0.20}$ | $6.82 - 8.04$ | $7.42^{+0.21}_{-0.20}$ | $6.82 - 8.04$ |
| $\frac{\Delta m^2_{3l}}{10^{-3}\ \mathrm{eV}^2}$ | $2.517^{+0.026}_{-0.028}$ | $2.435 - 2.598$ | $-2.498 \pm 0.028$ | $-2.581 - -2.414$ |
| $\frac{\delta_{CP}}{\pi}$ | $1.09^{+0.15}_{-0.13}$ | $0.67 - 2.05$ | $1.57^{+0.14}_{-0.17}$ | $1.07 - 1.96$ |

**What is there to measure, anyway?**

- Ordering of the mass states (mass ordering or hierarchy), is $\nu_3$ the heaviest or the lightest: **NORMAL** vs. **INVERTED**?
- $\theta_{23}$ =, > (UO), < (LO) 45°? 23, $\mu\tau$ symmetry?
- CP violation in lepton sector, $\delta_{CP}$?
- Tests of unitarity, $3\nu$-paradigm completeness, sterile $\nu$ etc.?

**Long-baseline accelerator experiments** $L/E \sim 10^{2-3}$ km/GeV are sensitive to

NO/IO, $\theta_{23}$ and $\delta_{CP}$
(also $\theta_{13}$)

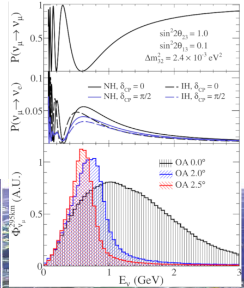
T2K (Japan) 295 km / 0.6 GeV
NOvA (USA) 810 km / 2 GeV

# T2K Experiment

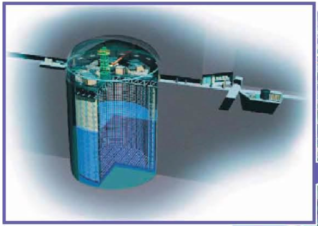
# T2K Overview

**579 people**

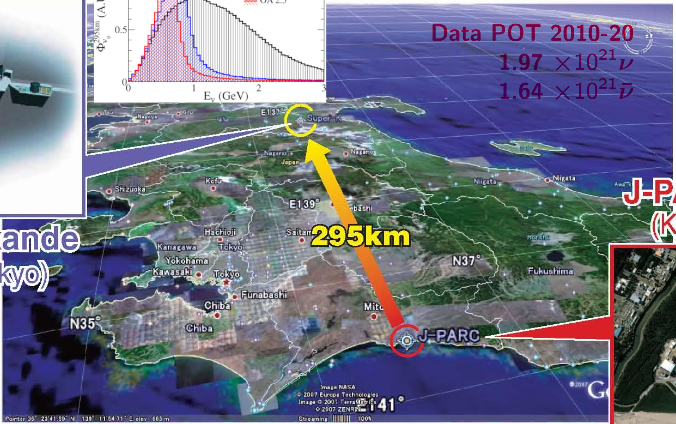
14 countries
78 institutions




- Long-baseline $\nu$ oscillation experiment ≈490 km/GeV
- $\nu_\mu$-beam, $\nu/\bar{\nu}$ modes, 2.5° off-axis, $E$ around 0.6 GeV
- $\nu_\mu$ disappearance: $\sin^2 2\theta_{23}, |\Delta m^2_{32}|$
- $\nu_e$ appearance: $\sin^2 \theta_{23}$, $\delta_{CP}$, $\Delta m^2_{32}$



**Super-Kamiokande**
(ICRR, Univ. Tokyo)




**Powerful $\nu$ source**

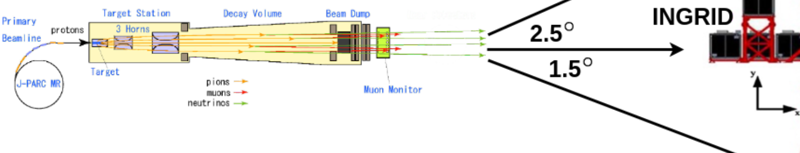
**J-PARC Main Ring**
(KEK-JAEA, Tokai)

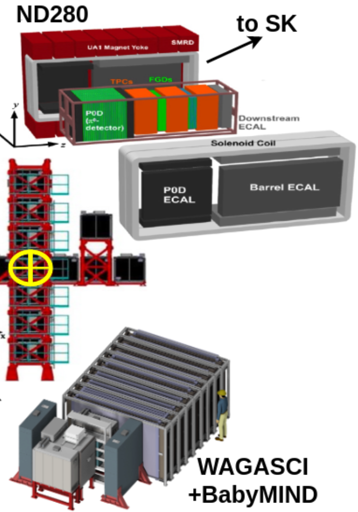
**Dominating $\delta_{CP}$ effect**

# Beamline and Near Detectors

- Switching between $\nu$ and $\bar{\nu}$ beam by inverting the horns' polarities
- Almost pure $\nu_\mu/\bar{\nu}_\mu$ beam with $< 1\%$ of $\nu_e/\bar{\nu}_e$ at peak
- Reached operating power $\approx$500 kW

- **INGRID** monitors beam direction and stability, tracking calorimeter, on-axis, Fe + plastic scintillator



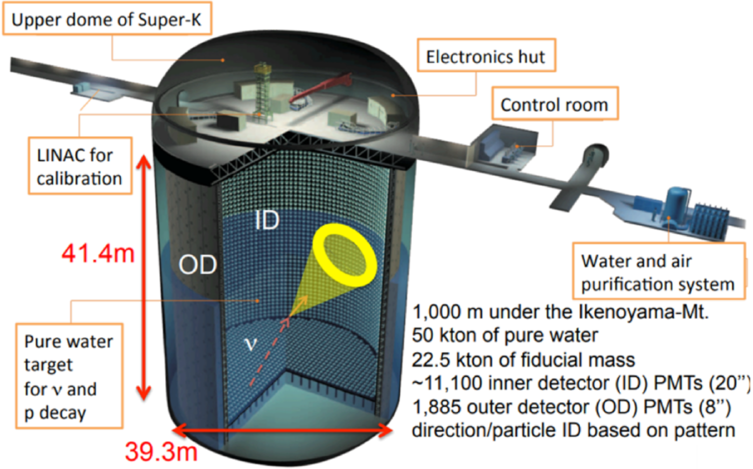

- **WAGASCI+BM** $H_2O$ target, magnetized, off-axis 1.5°, probing spectrum variations with the off-axis angle

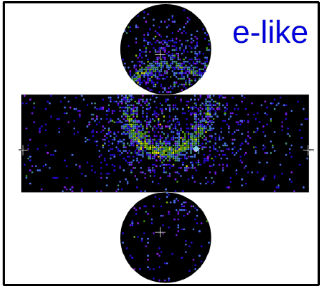
**ND280**

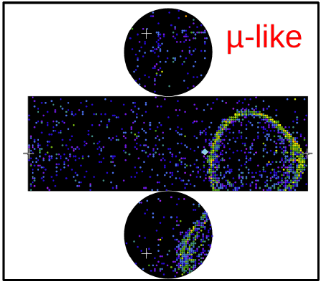
- 0.2 T magnet, trackers+calorimeters, $H_2O$/CH targets, off-axis 2.5°
- Directly used in the analysis to constrain XSecs, beam $\nu_e/\bar{\nu}_e$, etc.

# Super-Kamiokande, SK – The Far Detector

**Excellent $\mu/e$-like rings separation** ($\nu_\mu/\nu_e$ CC int.)



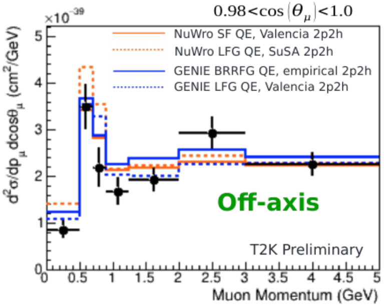



# Physics Program Beyond $\nu$ Oscillation

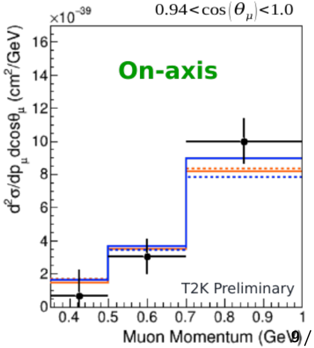
**What might interest you from the current results...**

**...but there's much more coming!**




### Joint XSec Measurements

- Joint On/Off axis $\nu_\mu$ CC0$\pi$ measurement
- Simultaneous measurement of the $\nu_\mu$ CC0$\pi$ cross-section on O and C
  - FGD2 of ND280 has several modules with $H_2O$




### Low rate measurements

- $\nu_\mu$ and $\bar{\nu}_\mu$ CC coherent $\pi$ production on C

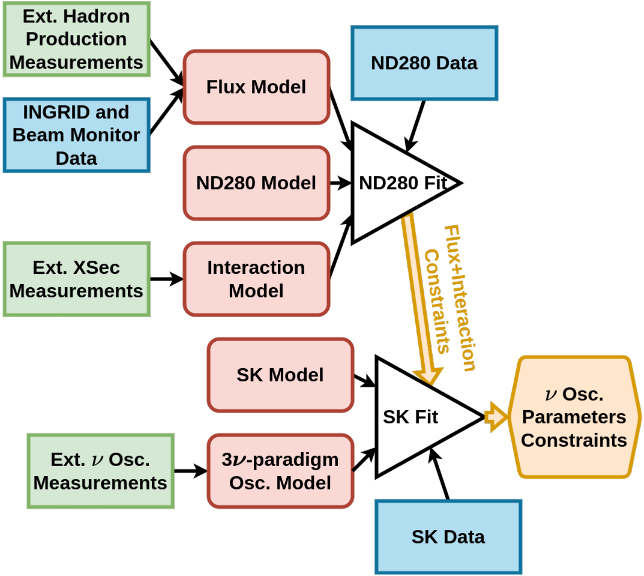
### SK measurements

- Neutron multiplicities at SK

# T2K 2022 $\nu$ Oscillation Analysis

A follow-up on 2020 analysis *arXiv:2303.03222*

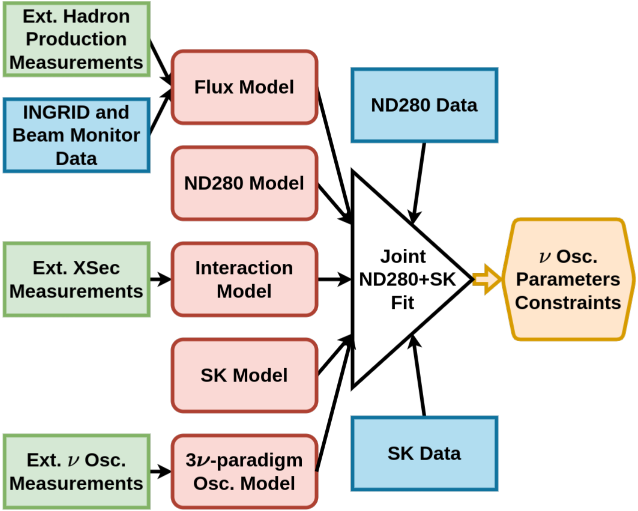
# Frequentist ND280 to SK Consequential Analysis




- Frequentist analysis proceeds in two consecutive fits
- Fit to ND280 samples provides a constraint to flux and interaction model
- Results of ND280 fit are used as input to fit SK data to constrain $\nu$ oscillation parameters

- Frequentist analysis uses Poisson likelihood with gaussian penalty terms to account for systematic nuisance parameters and Barlow-Beeston approach to account for ND280 MC stat uncertainty
- Gradient descent algorithm / grid search
- Feldman-Cousins method to construct CL intervals

# Bayesian Joint ND280+SK Analysis




- Based on Bayes' theorem:

Posterior, Likelihood, Prior

$$P(\boldsymbol{a}|D) \propto P(D|\boldsymbol{a}) \times \pi(\boldsymbol{a})$$

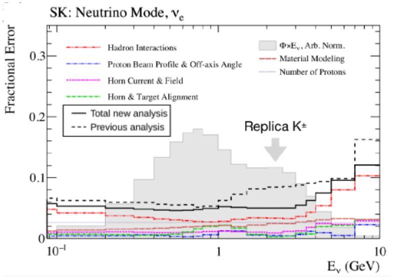
$D$ = Data
$\boldsymbol{a}$ = Parameters

- Also uses Poisson likelihood
- Markov Chain Monte Carlo (MCMC) with Metropolis-Hastings algorithm
- Systematic nuisance parameters may have different priors
- Fitting all ND280 samples and SK samples at once

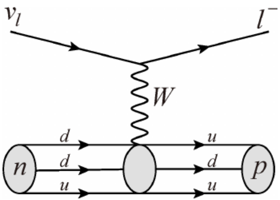
# The Novelties of 2022 – Flux and Interaction Modeling

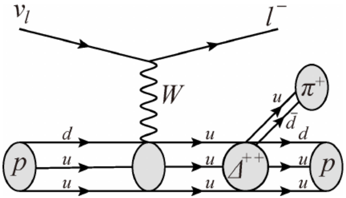
## Flux Model

- Updated tune to new NA61/SHINE T2K replica target measurement *Eur.Phys.J.C 79 2, 100*
- More $\pi^{\pm}$ stats, new $K^{\pm}$ and $p$ data
- Significant reduction of hadron production uncertainties




**Most dominant interaction at 0.6 GeV CCQE and CC RES**




## CCQE (Spectral Function)

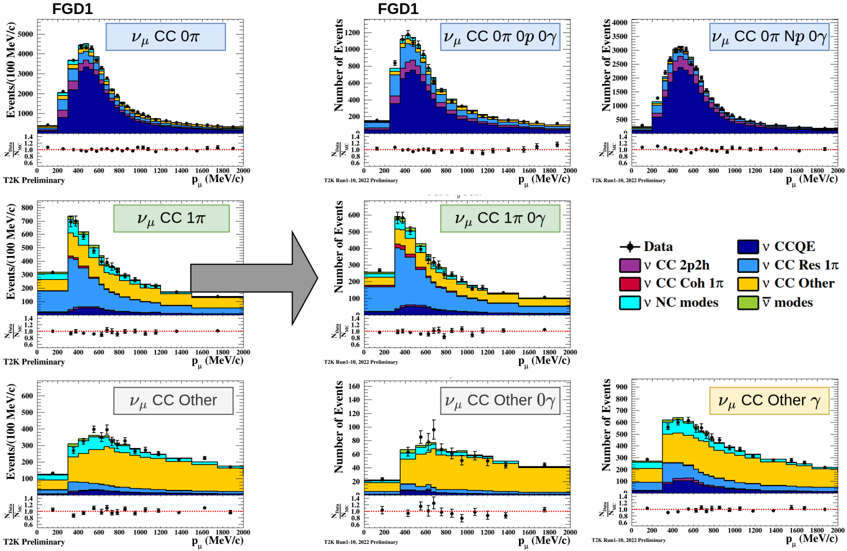
- More theory-driven uncerts
- Normalization of each nuclear shell for Mean Field
- Short Range Correlations and Pauli Blocking

## CC Resonant (Rein-Sehgal)

- New bubble chamber tune
- New resonance decay uncerts
- Eff. inclusion of binding energy
- New uncert in $\pi^{\pm}$ vs $\pi^0$ production

# The Novelties of 2022 – ND280 Samples

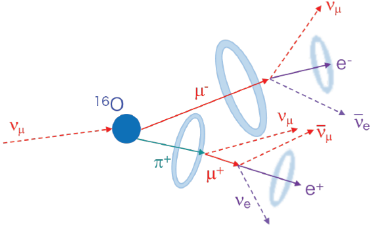
- 22 ND280 samples based on reconstructed topology
- **New ND280 $\nu$ samples with $p$ and $\gamma$ tagging**
- $p$: better access to nuclear effects, lower $\pi^+$ bkg.
- $\gamma$: filters out DIS and COH $\pi^0$ production
- No change in $\bar{\nu}$ samples



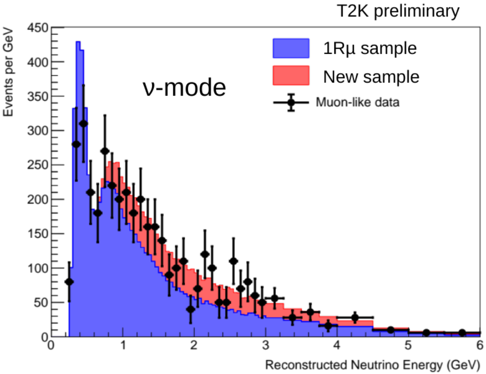

2020

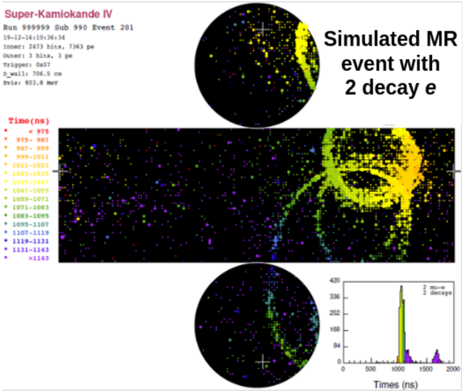
2022

# The Novelties of 2022 – SK $\nu_\mu$ Multi-ring Sample MR$\mu$ CC1$\pi^+$




- $\mu^-$ ring and ($\pi^+$ ring + 1-2 Michel $e$) or (2 Michel $e$)
- Targeting $\nu_\mu$CC1$\pi^+$ interactions in $\nu$-mode
- +30% stats $\mu$-like $\nu$-mode, sensitive to $\nu$ oscillation
- Model cross-checks and background constraints



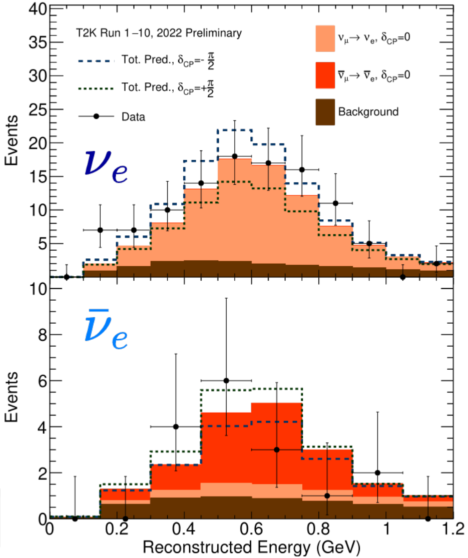
# Results 2022 – SK Data

| | | BF Prediction and $\delta_{\text{CP}} =$ | | | |
|---|---|---|---|---|---|
| | Sample | $-\pi/2$ | $\pi/2$ | **-2.18** | Data |
| $\nu$ | 1R$e$ | 99.1 | 68.6 | **96.5** | 94 |
| | 1R$e$1$\pi^+$ | 10.9 | 7.7 | **10.5** | 14 |
| | 1R$\mu$ | 358.7 | 358.6 | **359.1** | 318 |
| | MR$\mu$1$\pi^+$ | 118.5 | 118.5 | **118.8** | 134 |
| $\bar{\nu}$ | 1R$e$ | 17.0 | 21.4 | **17.3** | 16 |
| | 1R$\mu$ | 139.4 | 139.4 | **139.6** | 137 |

$\nu_e$-like samples:
- 1R$e$ = 1 ring $e$
- 1R$e$1$\pi^+$ = 1 ring $e$ + 1 M. $e$

$\nu_\mu$-like samples:
- 1R$\mu$ = 1 ring $\mu$ + 0/1 M. $e$
- MR$\mu$1$\pi^+$ = 1 ring $\mu$ + (1 ring $\pi$ + 1-2 M. $e$) or 2 M. $e$

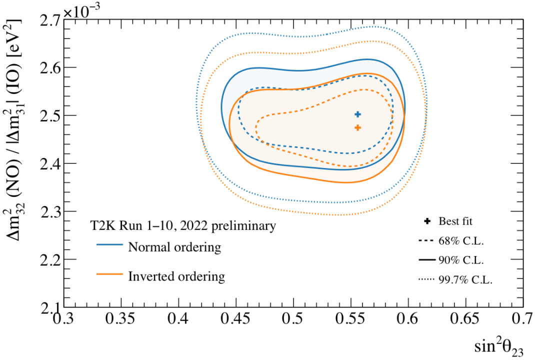
Good energy estimation from lepton momentum and angle
**SK cannot separate ± charged particles event by event, i.e. $\nu/\bar{\nu}$**

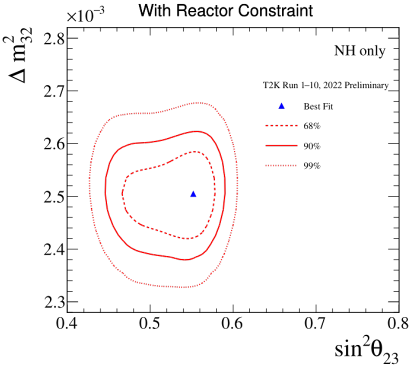


# Results 2022 – $\Delta m^2_{32}$ vs. $\sin^2\theta_{23}$

- Leading measurements of $\theta_{23}$ and $\Delta m^2_{32}$
- Excellent agreement of both frequentist and bayesian analyses

**Frequentist**




**Bayesian**

| | $\sin^2\theta_{23} < 0.5$ | $\sin^2\theta_{23} > 0.5$ | Line total |
|---|---|---|---|
| Normal ordering | 0.236 | 0.540 | 0.776 |
| Inverted ordering | 0.049 | 0.174 | 0.224 |
| Column total | 0.285 | 0.715 | 1.000 |

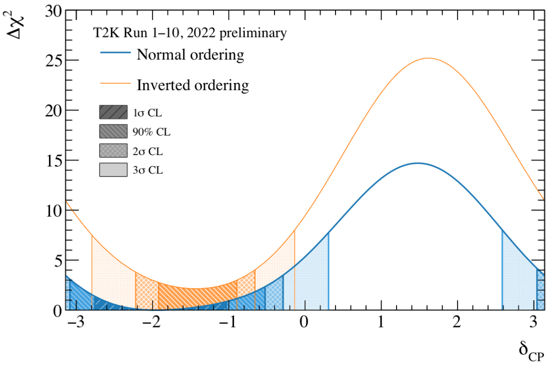
T2K Run 1-10, preliminary

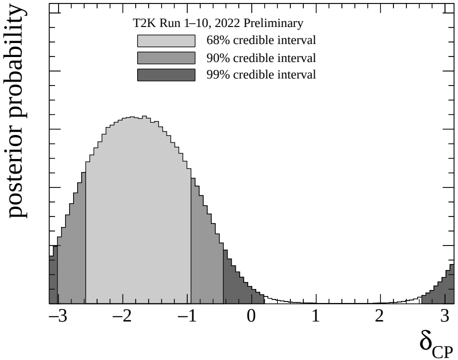
Slight preference for UO of $\theta_{23}$ (2.51) and NO (3.46) of $\nu$ mass states (from Bayes factors)

Using reactor constraints for $\theta_{13}$

# Results 2022 – $\delta_{CP}$

Frequentist




Bayesian

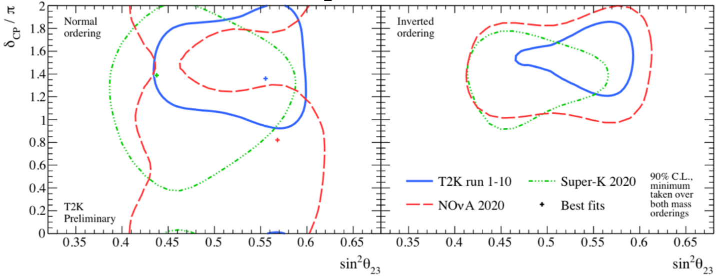
$\delta_{CP}$ **best fit at -2.18** (-0.694$\pi$), CP conserving values 0 and $\pi$ are outside of 90% CL intervals

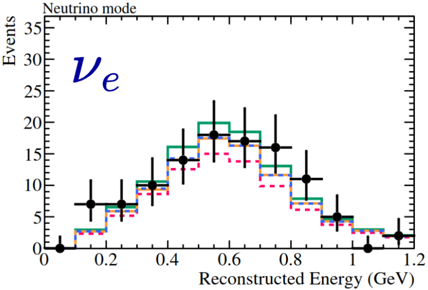
Using reactor constraints for $\theta_{13}$

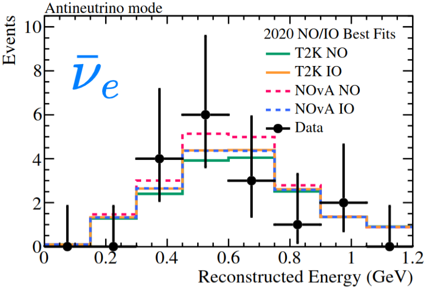
- Frequentist p-value 0.35
- Bayesian posterior predictive p-value 85%
- No biases to undermine the statements found in a test with an alternative interaction model

# The Rosy Future of T2K

# T2K+NOvA Analysis



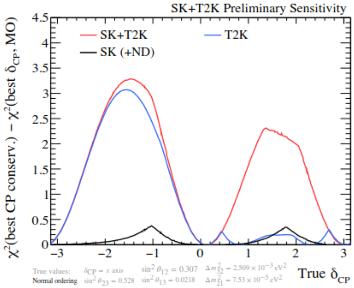



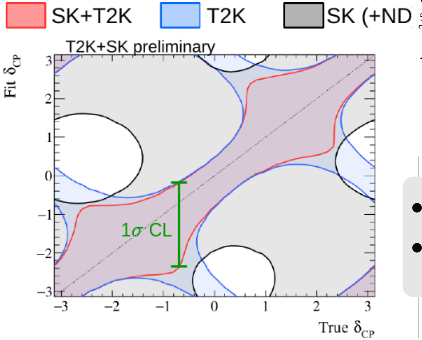

- Two **complementary** long-baseline $\nu$ oscillation experiments (baselines, $\nu$ energy and underlying interaction modes, detector technology, analysis techniques and more), increased sensitivity to NO/IO, $\delta_{\mathrm{CP}}$, and $\theta_{23}$ thanks to resolving degeneracies
- One of the most important results on $\nu$ oscillation in $3\nu$-paradigm before the age of the next generation oscillation experiments
- Based on 2020 analyses (*arXiv:2303.03222*, *PRD 106 032004*)
- Expecting preliminary results soon

# T2K+SK

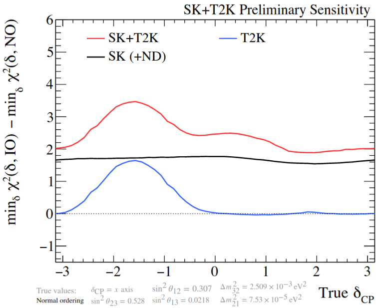
- Combining T2K $\nu/\bar{\nu}$ beam data and SK atmospheric data
- Wider range of baselines and energies in comparison to T2K-only
- Significantly increased sensitivity to $\delta_{CP}$ and NO/IO in respective regions of parametric space
- Based on T2K 2020 analysis (*arXiv:2303.03222*)

**True NO**



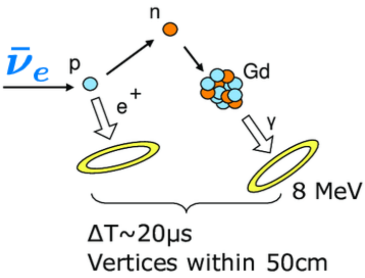



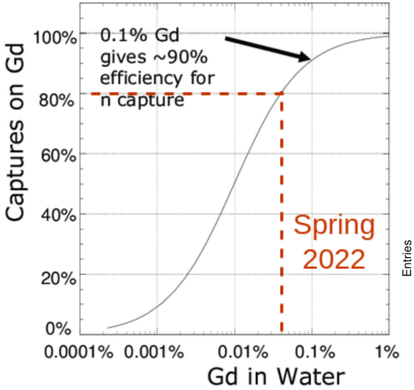

**True NO**



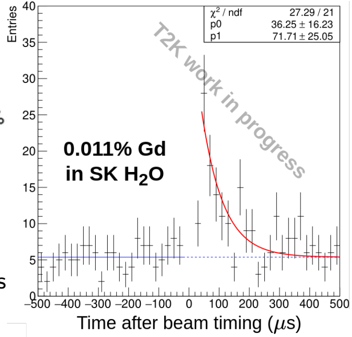

- Coherent detector MC
- Unified interaction model for T2K and similar SK samples

# SK-Gd Phase



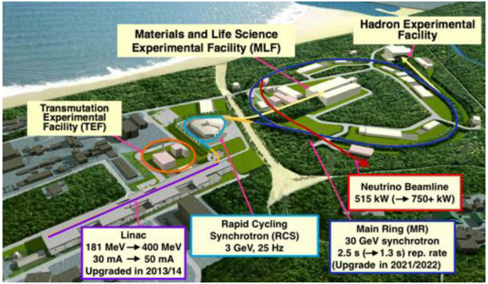



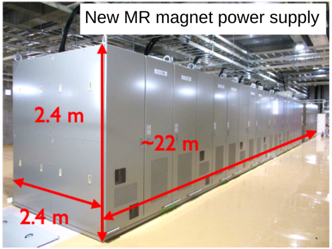

$\bar{\nu}_e$ can be identified through $n$ captures which would help to **separate $\nu_e/\bar{\nu}_e$ in the $\nu_e$-like samples**



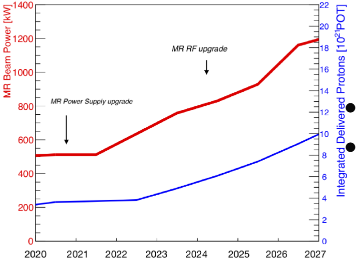

- SK was loaded with Gd sulfate to improve neutron tagging
- Already acquired data from SK-Gd phase, but not analyzed
- Development and optimization in progress, implementation into T2K analysis in progress
- Increase in the time-delayed signals indicates the presence of Gd in SK

# Beam Upgrade



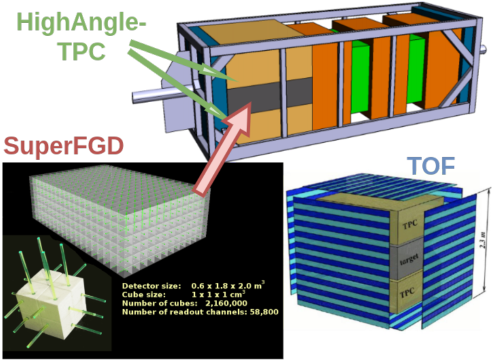




T2K Projected POT (Protons-On-Target)




- Main Ring (MR) reached ≈515 kW stable operation during the last runs
- Upgrades during the 2-year shutdown:
  - New MR fast extraction magnets and power supply
  - New proton focusing magnet
  - New beam monitors and $\mu$ monitor
  - New target and target cooling system
  - New focusing horns and power supplies (250 → 320 kA)
- Scheduled for spring 2023
- Targeting over 1 MW with future MR RF systems upgrade (down to 1.16 s rep. rate)

# ND280 Upgrade

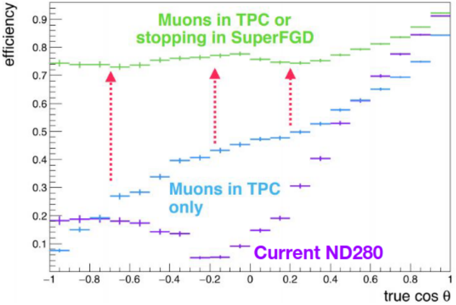


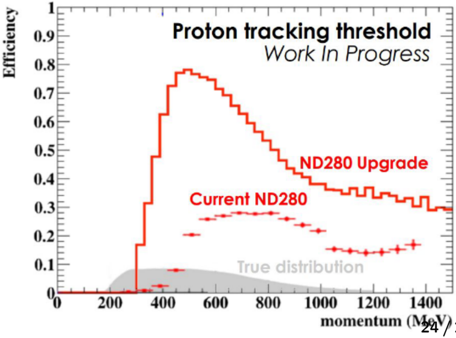

Replacing P0D with a set of new high performance detectors

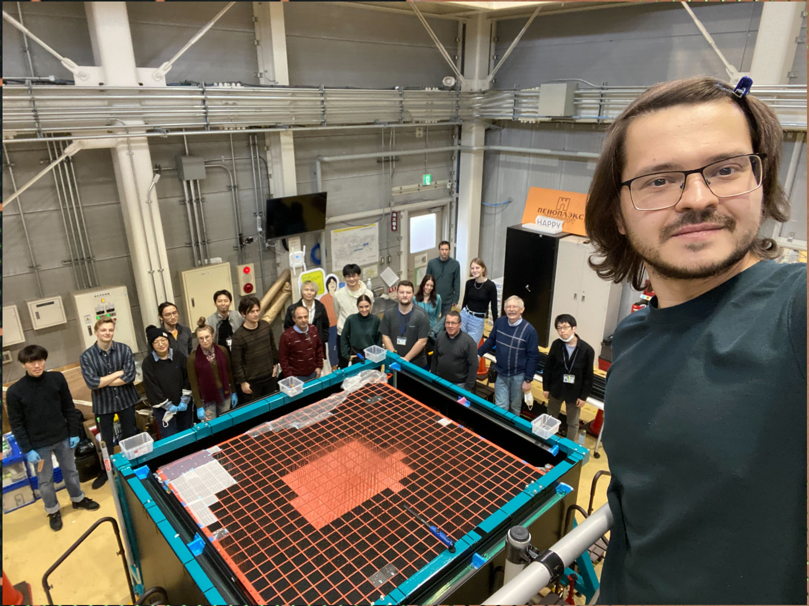
New test beam for HATPC in 2022

2 million cubes, ≈60000 fibers for SFGD installed, attaching electr.

ND280 upgrade essentially enhances the detector acceptance to access new regions of phase space when constraining $\nu$ and $\bar{\nu}$ interactions for $\nu$ oscillation analysis and XSecs




HAPPY

# SUMMARY

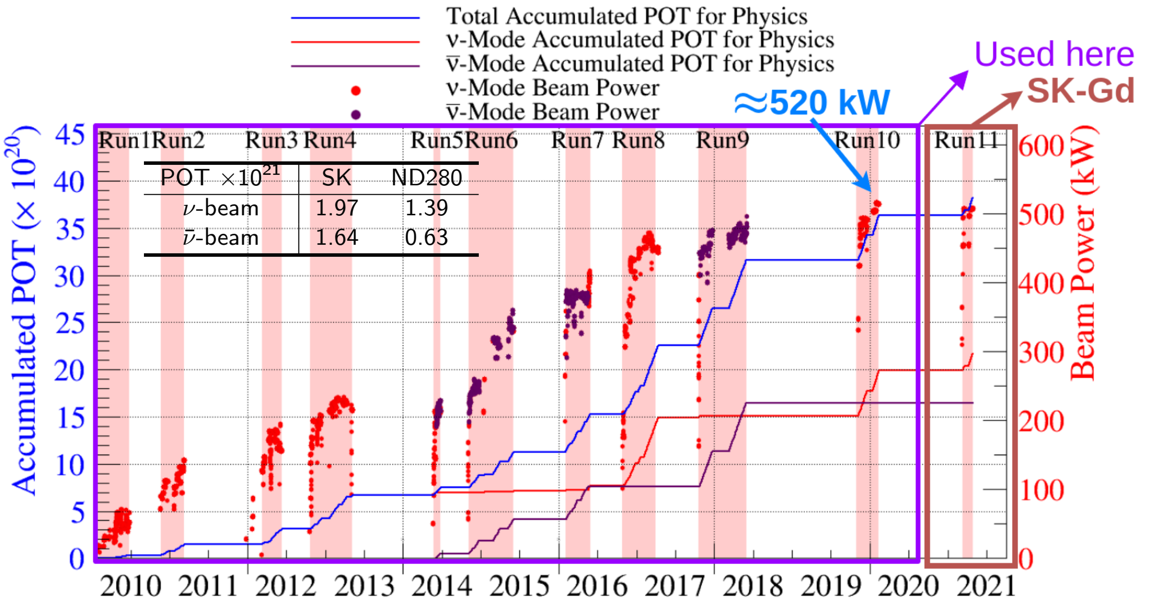
- 2022 re-analysis of $\nu$ oscillation data with new features and data samples, consistent with previous results
- **CP conservation excluded at 90% CL**
- Slight preference for NO of $\nu$ mass states and $\theta_{23} > 45^\circ$

**Bright future ahead!**

- T2K+NOvA and T2K+SK joint analyses to be presumably the most impactful analyses on $\nu$ oscillation before the dawn of the next generation experiments
- SK-Gd phase already began, data acquired but not analyzed
- Beam upgrades to reach $> 1$ MW power in a near future
- ND280 upgrade ongoing, installation already happening

# BACKUP

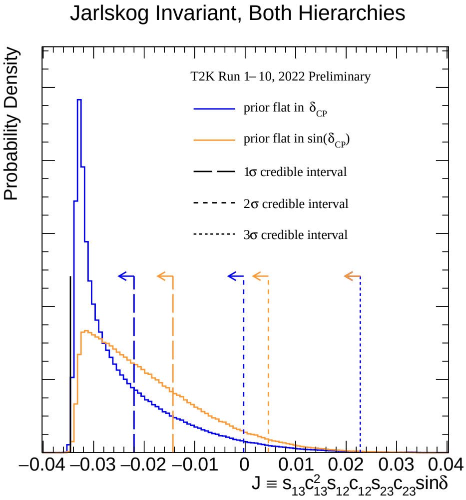
# Detectors Exposure, Accumulated POT

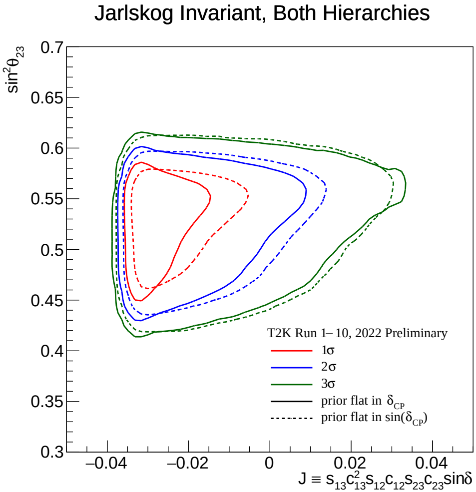



| POT $\times 10^{21}$ | SK | ND280 |
|---|---|---|
| $\nu$-beam | 1.97 | 1.39 |
| $\bar{\nu}$-beam | 1.64 | 0.63 |

# Jarlskog Invariant

$$J_{\mathrm{CP}} = \sin\theta_{13}\cos^2\theta_{13}\sin\theta_{12}\cos\theta_{12}\sin\theta_{23}\cos\theta_{23}\sin\delta_{\mathrm{CP}}$$



# Systematic Uncertainties SK

| Error source (units: %) | 1R FHC | 1R RHC | MR FHC CC1$\pi^+$ | 1R$e$ FHC | 1R$e$ RHC | 1R$e$ FHC CC1$\pi^+$ | 1R$e$ FHC/RHC |
|---|---|---|---|---|---|---|---|
| Flux | 2.8 | 2.9 | 2.8 | 2.8 | 3.0 | 2.8 | 2.2 |
| Xsec (ND constr) | 3.7 | 3.5 | 3.0 | 3.8 | 3.5 | 4.1 | 2.4 |
| Flux+Xsec (ND constr) | 2.7 | 2.6 | 2.2 | 2.8 | 2.7 | 3.4 | 2.3 |
| Xsec (ND unconstr) | 0.7 | 2.4 | 1.4 | 2.9 | 3.3 | 2.8 | 3.7 |
| SK+SI+PN | 2.0 | 1.7 | 4.1 | 3.1 | 3.8 | 13.6 | 1.2 |
| **Total** All | 3.4 | 3.9 | 4.9 | 5.2 | 5.8 | 14.3 | 4.5 |

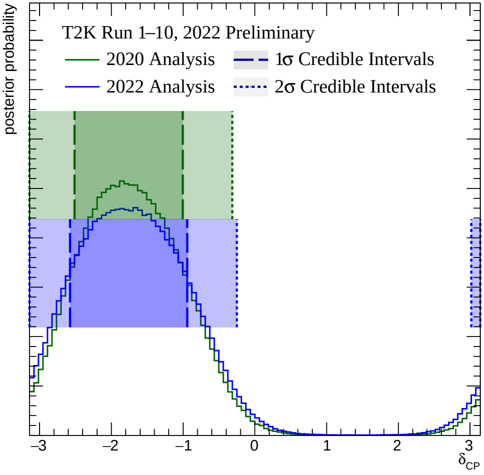
- Numbers quoted are the RMS of the predicted numbers of events in the far detector sample obtained when varying systematic parameters according to their prior distribution
- Some systematic parameters do not have a prior constraint, and can end up having larger effect than estimated with this method in a fit

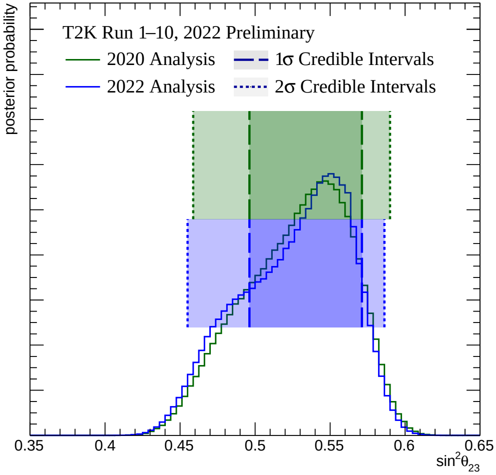
# Systematic Uncertainties ND280

| Sample | δN/N(%) | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Flux | | Xsec | | ND280 | | Total | |
| | pri. | post. | pri. | post. | pri. | post. | pri. | post. |
| FGD1 FHC CC0$\pi$-0p-0$\gamma$ | 5.0 | 2.7 | 11.8 | 2.8 | 1.8 | 1.2 | 12.8 | 0.6 |
| FGD1 FHC CC0$\pi$-Np-0$\gamma$ | 5.5 | 2.8 | 11.7 | 3.2 | 3.5 | 2.2 | 12.9 | 0.9 |
| FGD1 FHC CC1$\pi$-0$\gamma$ | 5.2 | 2.7 | 9.1 | 2.7 | 3.0 | 1.4 | 10.6 | 1.0 |
| FGD1 FHC CC-Other-0$\gamma$ | 5.4 | 2.8 | 8.0 | 2.8 | 5.2 | 2.3 | 11.0 | 1.6 |
| FGD1 FHC CC-Photon | 5.5 | 2.8 | 8.5 | 2.8 | 2.8 | 1.8 | 10.5 | 0.8 |
| FGD2 FHC CC0$\pi$-0p-0$\gamma$ | 5.1 | 2.7 | 11.2 | 2.8 | 2.1 | 1.1 | 11.5 | 0.6 |
| FGD2 FHC CC0$\pi$-Np-0$\gamma$ | 5.5 | 2.8 | 11.3 | 3.3 | 3.9 | 2.4 | 12.2 | 1.0 |
| FGD2 FHC CC1$\pi$-0$\gamma$ | 5.2 | 2.7 | 9.0 | 2.7 | 3.6 | 1.6 | 10.5 | 1.0 |
| FGD2 FHC CC-Other-0$\gamma$ | 5.6 | 2.8 | 8.0 | 2.8 | 6.3 | 2.7 | 11.5 | 1.9 |
| FGD2 FHC CC-Photon | 5.4 | 2.8 | 8.3 | 2.8 | 2.5 | 1.6 | 10.4 | 0.8 |
| FGD1 RHC CC0$\pi$ | 4.9 | 3.2 | 11.3 | 3.2 | 1.9 | 1.2 | 12.2 | 0.9 |
| FGD1 RHC CC1$\pi$ | 4.6 | 3.1 | 10.3 | 3.0 | 4.2 | 2.6 | 11.4 | 1.9 |
| FGD1 RHC CC-Other | 4.5 | 2.9 | 9.3 | 3.0 | 3.5 | 2.0 | 10.5 | 1.5 |
| FGD2 RHC CC0$\pi$ | 4.8 | 3.2 | 10.4 | 3.0 | 2.1 | 1.2 | 13.8 | 0.9 |
| FGD2 RHC CC1$\pi$ | 4.6 | 3.0 | 9.9 | 3.2 | 3.9 | 2.3 | 10.9 | 1.9 |
| FGD2 RHC CC-Other | 4.6 | 2.9 | 9.7 | 3.1 | 2.9 | 1.8 | 11.3 | 1.4 |
| FGD1 RHC BKG CC0$\pi$ | 5.8 | 2.8 | 10.1 | 2.8 | 2.2 | 1.1 | 10.6 | 1.1 |
| FGD1 RHC BKG CC1$\pi$ | 5.6 | 2.8 | 8.0 | 2.5 | 3.3 | 1.6 | 11.2 | 1.3 |
| FGD1 RHC BKG CC-Other | 5.9 | 2.9 | 8.6 | 2.7 | 2.6 | 1.4 | 10.1 | 1.4 |
| FGD2 RHC BKG CC0$\pi$ | 5.8 | 2.8 | 9.5 | 2.8 | 2.2 | 1.1 | 10.4 | 1.1 |
| FGD2 RHC BKG CC1$\pi$ | 5.6 | 2.8 | 8.2 | 2.5 | 3.2 | 1.6 | 10.7 | 1.3 |
| FGD2 RHC BKG CC-Other | 5.9 | 2.9 | 8.6 | 2.7 | 2.5 | 1.4 | 10.6 | 1.4 |
| Total | 4.5 | 2.7 | 8.0 | 2.6 | 2.1 | 1.2 | 9.1 | 0.3 |

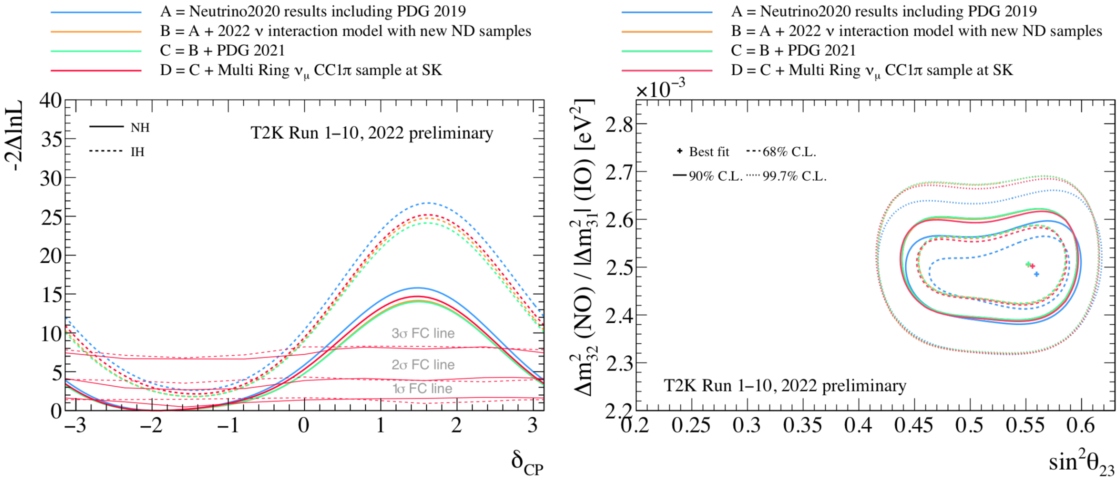
# Results 2022 vs. 2020

- Very good consistency with 2020 results which used the very same beam exposure



# Effect of Analysis Change



# SK Energy Reconstruction

- Assuming quasi-elastic interaction with a single bound nucleon
- Known beam direction allows one to calculate reconstructed neutrino energy

$$E_{rec}^{\nu\text{CCQE}-\text{like}} = \frac{2E_l(M_n - E_b) - M_l^2 + 2M_nE_b - E_b^2 + M_p^2 - M_n^2}{2(M_n - E_b - E_l + P_l \cos\theta_l)}$$

Energy of outgoing muon

$\Delta_{++}$ mass

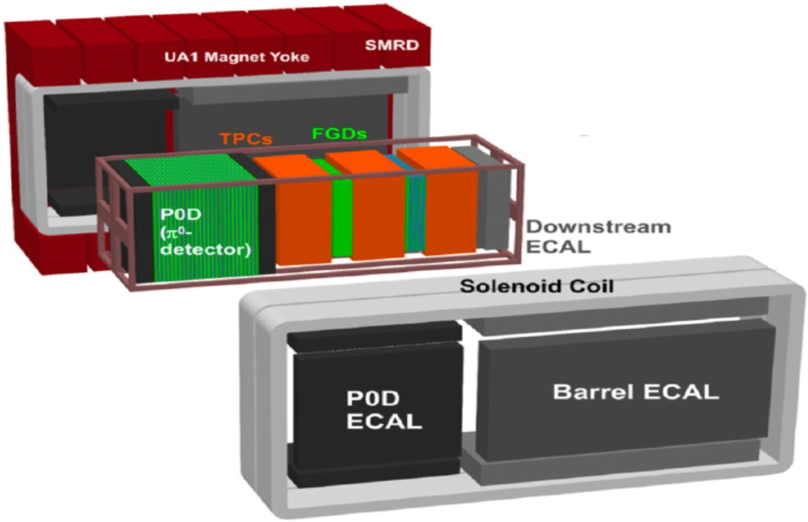
Proton mass

Muon mass

$$E_{\text{rec}}^{\nu_\mu \text{CC}\Delta^{++}} = \frac{2M_pE_\mu + M_{\Delta^{++}}^2 - M_p^2 - M_\mu^2}{2\left(M_p - E_\mu + |p_\mu| \cos\theta_\mu\right)}$$

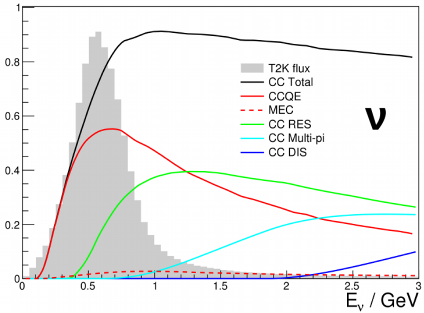
Momentum and angle of outgoing muon

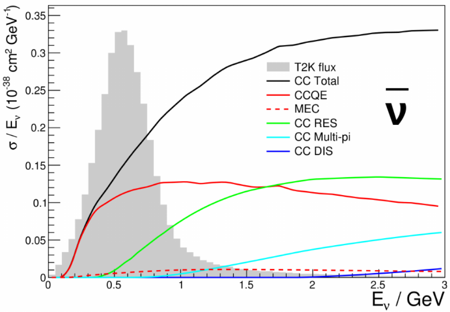
# ND280

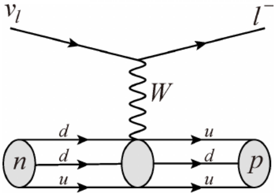



- UA1 magnet
- SMRD
- P0D
- FGDs
- TPCs
- ECAL

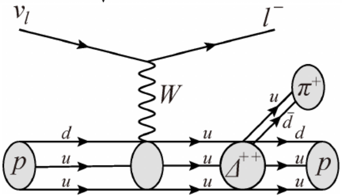
# $\nu$ Interactions



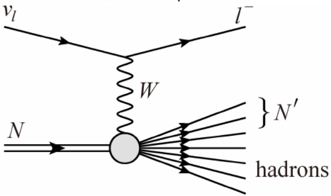




CCQE




CC RES




CC DIS / Multi $\pi$

# Simulated Momentum Thresholds for Counting Visible Particles

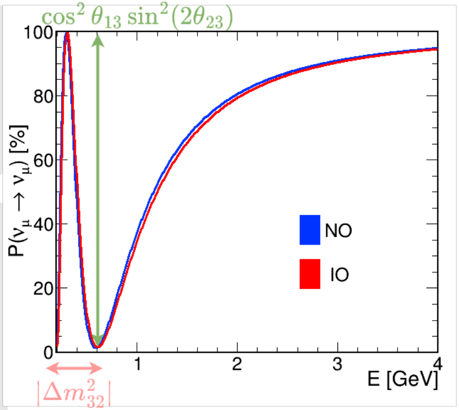
| Particle | Momentum [MeV/$c$] |
|---|---|
| $e^{\pm}$ | 10 |
| $\gamma$ | 20 |
| $\mu^{\pm}$ | 120.495 |
| $\pi^{+}$ | 159.169 |
| $\pi^{0}$ | 0 |
| Proton | 1070.03 |
| Other charged particles | Cherenkov threshold |

# $\nu$ Oscillation Probabilities

Small parameters $\varepsilon \equiv \frac{\Delta m^2_{21}}{\Delta m^2_{31}} \approx 0.03$ and $\sin^2\theta_{13} \approx 0.02$

**Disappearance channel $\nu_\mu \to \nu_\mu$**

$$P(\nu_\mu \to \nu_\mu; L, E) \approx 1 - \cos^2\theta_{13}\sin^2(2\theta_{23})\sin^2\frac{\Delta m^2_{32} L}{4E} + \\ + \mathcal{O}(\varepsilon, \sin^2\theta_{13})$$




**Appearance channel $\nu_\mu \to \nu_e$**

$+\nu_e$ coherent forward scattering on pseudo-free $e$ (matter effect), $N_e$ density of $e$

$$P(\nu_\mu \to \nu_e; L, E, A) \approx 4 s^2_{13} s^2_{23} \frac{\sin^2\Delta}{(1-A)^2} + \varepsilon^2 \sin^2 2\theta_{12} c^2_{23} \frac{\sin^2 A\Delta}{A^2} + \\ +8\,\varepsilon\; c_{12} s_{12} c_{23} s_{23} c^2_{13} s_{13} \cos(\Delta + \delta_{\mathrm{CP}}) \frac{\sin A\Delta}{A} \frac{\sin\Delta(1-A)}{1-A}$$

$$\Delta \equiv \frac{\Delta m^2_{31} L}{4E} \qquad A \equiv \sqrt{2} G_{\mathrm{F}} N_e \frac{2E}{\Delta m^2_{31}}$$

# $\nu$ Oscillation Probabilities $\nu_\mu \to \nu_e$