GR effects on large-scale structure

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Overview

- How relativistic effects contribute to **galaxy clustering**

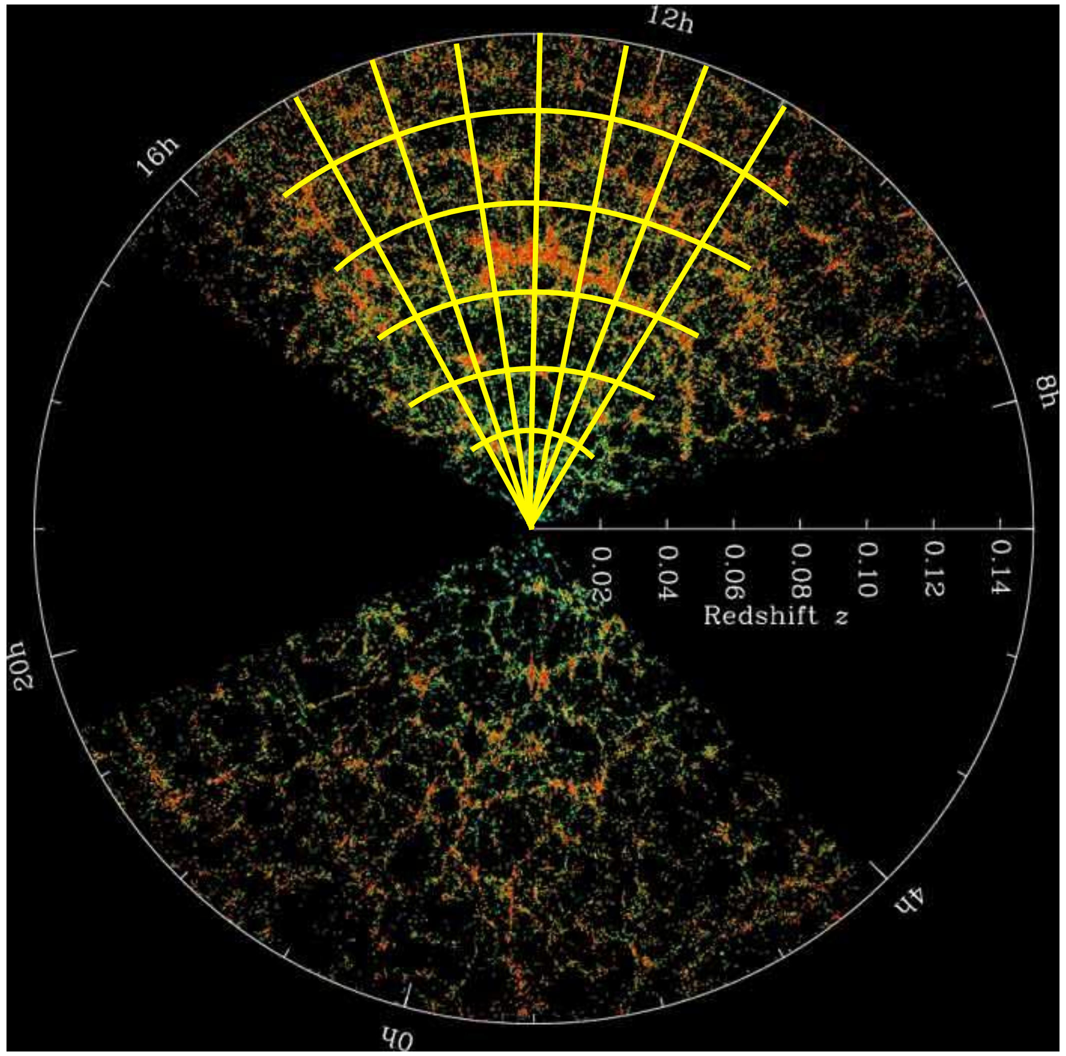
- How well can we measure them with DESI and SKA2

- How they can help us to test **gravity** and **dark matter**

Galaxy clustering

We count the number of **galaxies** per **pixel**: $\Delta = \dfrac{N - \bar{N}}{\bar{N}}$

Credit: M. Blanton, SDSS

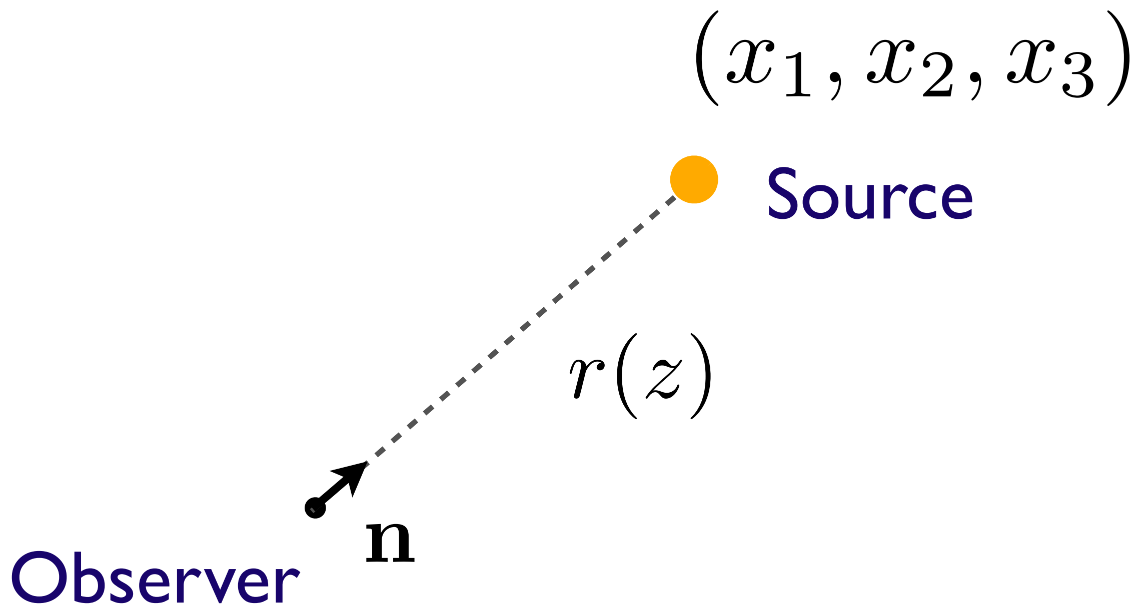


- Galaxies follow the distribution of matter $\Delta = b \cdot \delta$

- We never observe directly the position of galaxies, we observe the **redshift** z and the **direction** of incoming photons $\mathbf{n}$

In a **homogeneous** universe:

- we calculate the distance $r(z)$
- light propagates on straight lines



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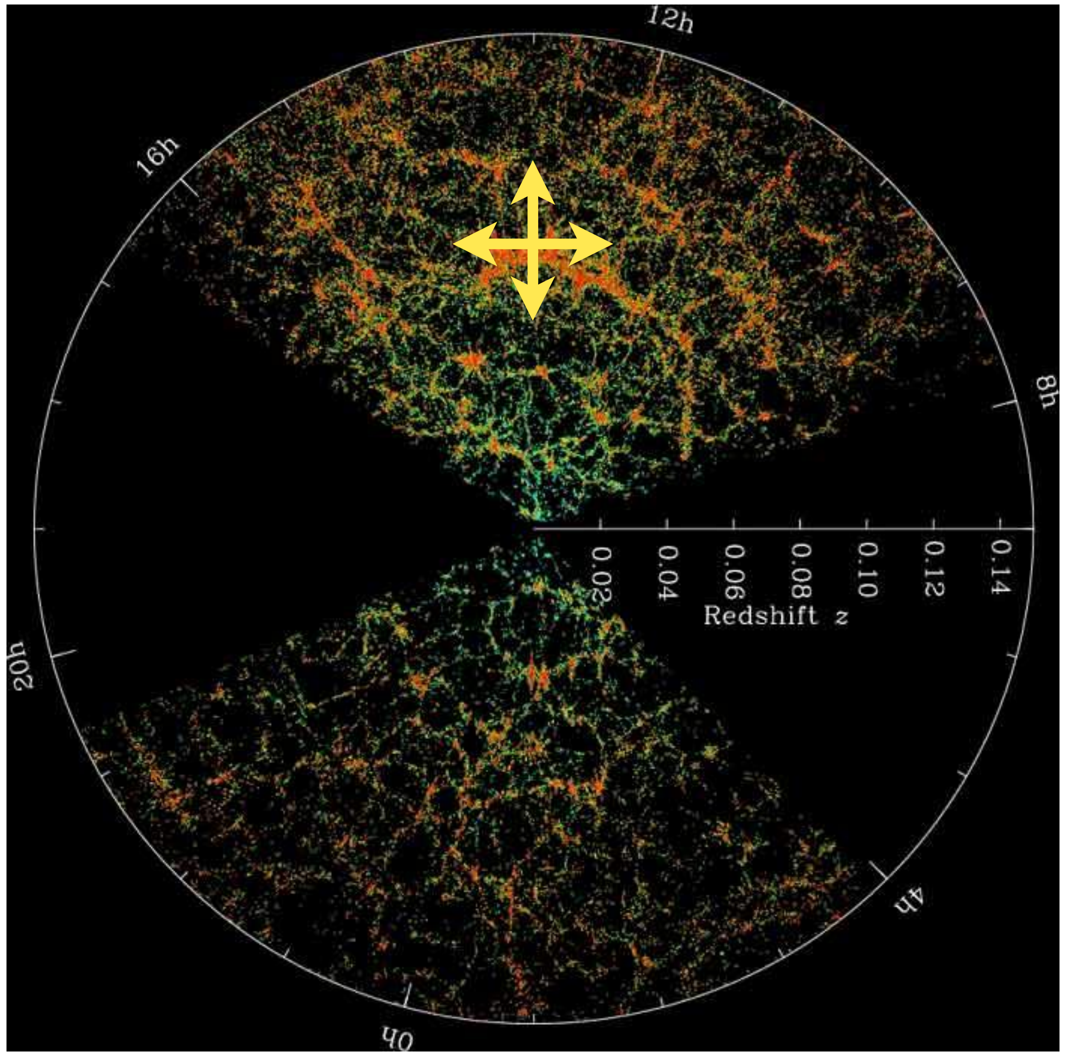

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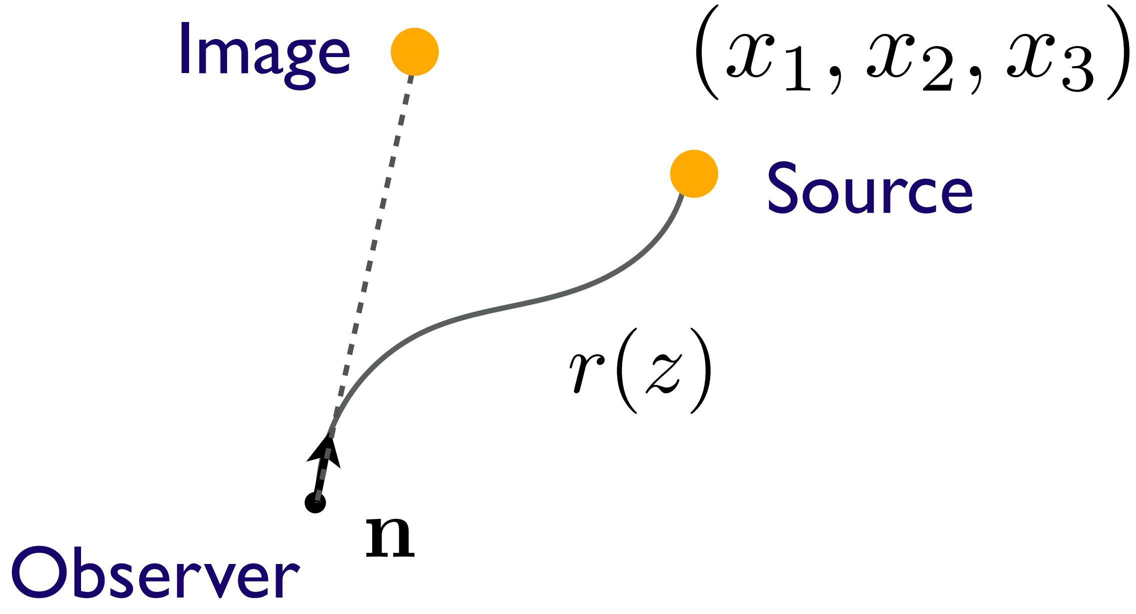

Inhomogeneities modify:

- distance-redshift relation
- angular position of the image

What we really observe

Yoo et al (2010)
CB and Durrer (2011)
Challinor and Lewis (2011)

$$\Delta(z,\mathbf{n}) = b\cdot\delta - \frac{1}{\mathcal{H}}\partial_r(\mathbf{V}\cdot\mathbf{n})$$
$$+ (5s-2)\int_0^r dr'\frac{r-r'}{2rr'}\Delta_\Omega(\Phi+\Psi)$$
$$+\left(1-\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}+\frac{5s-2}{r\mathcal{H}}-5s+f^{\rm evol}\right)\mathbf{V}\cdot\mathbf{n}+\frac{1}{\mathcal{H}}\dot{\mathbf{V}}\cdot\mathbf{n}+\frac{1}{\mathcal{H}}\partial_r\Psi$$
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Redshift-space distortion

$$
\begin{aligned}
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Current standard analyses

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Lensing: measured with quasars, relevant at high redshift

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What we really observe

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Lensing: measured with quasars, relevant at high redshift

$$+ \left(1-\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}+\frac{5s-2}{r\mathcal{H}}-5s+f^{\rm evol}\right)\mathbf{V}\cdot\mathbf{n} + \frac{1}{\mathcal{H}}\dot{\mathbf{V}}\cdot\mathbf{n} + \frac{1}{\mathcal{H}}\partial_r\Psi$$

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Relativistic effects: measured in clusters, never detected in linear regime

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Current standard analyses

$$+\,(5s-2)\int_0^r dr'\frac{r-r'}{2rr'}\Delta_\Omega(\Phi+\Psi)$$

Lensing: measured with quasars, relevant at high redshift

$$+\left(1-\ \cdots\right.\ \cdots\ +\frac{1}{\mathcal{H}}\partial_r\Psi$$

$$+\,\frac{2-5s}{r}\int_0\ \cdots\ 2)\Phi$$

$$+\,\frac{1}{\mathcal{H}}\dot\Phi+\left(\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}+\frac{2-5s}{r\mathcal{H}}+5s-f^{\rm evol}\right)\left[\Psi+\int_0^r dr'(\dot\Phi+\dot\Psi)\right]$$

Can we detect relativistic effects?

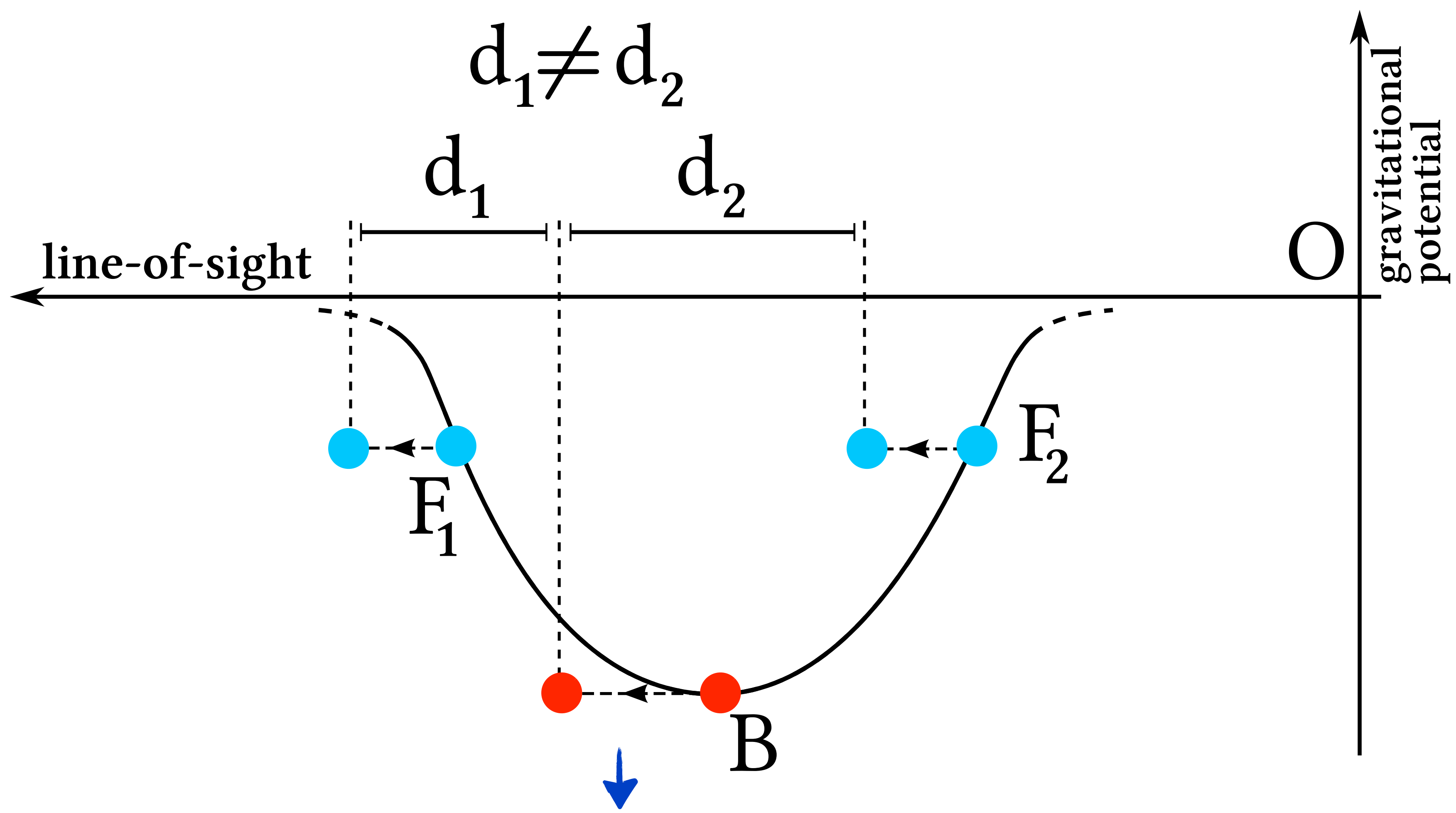
Can we learn something from them?

Relativistic effects: measured in clusters, never detected in linear regime

Asymmetric correlation function

- Relativistic effects are negligible in the even multipoles
- They generate **asymmetries** in the correlation function, that can be targeted by cross-correlating two populations

Gravitational redshift

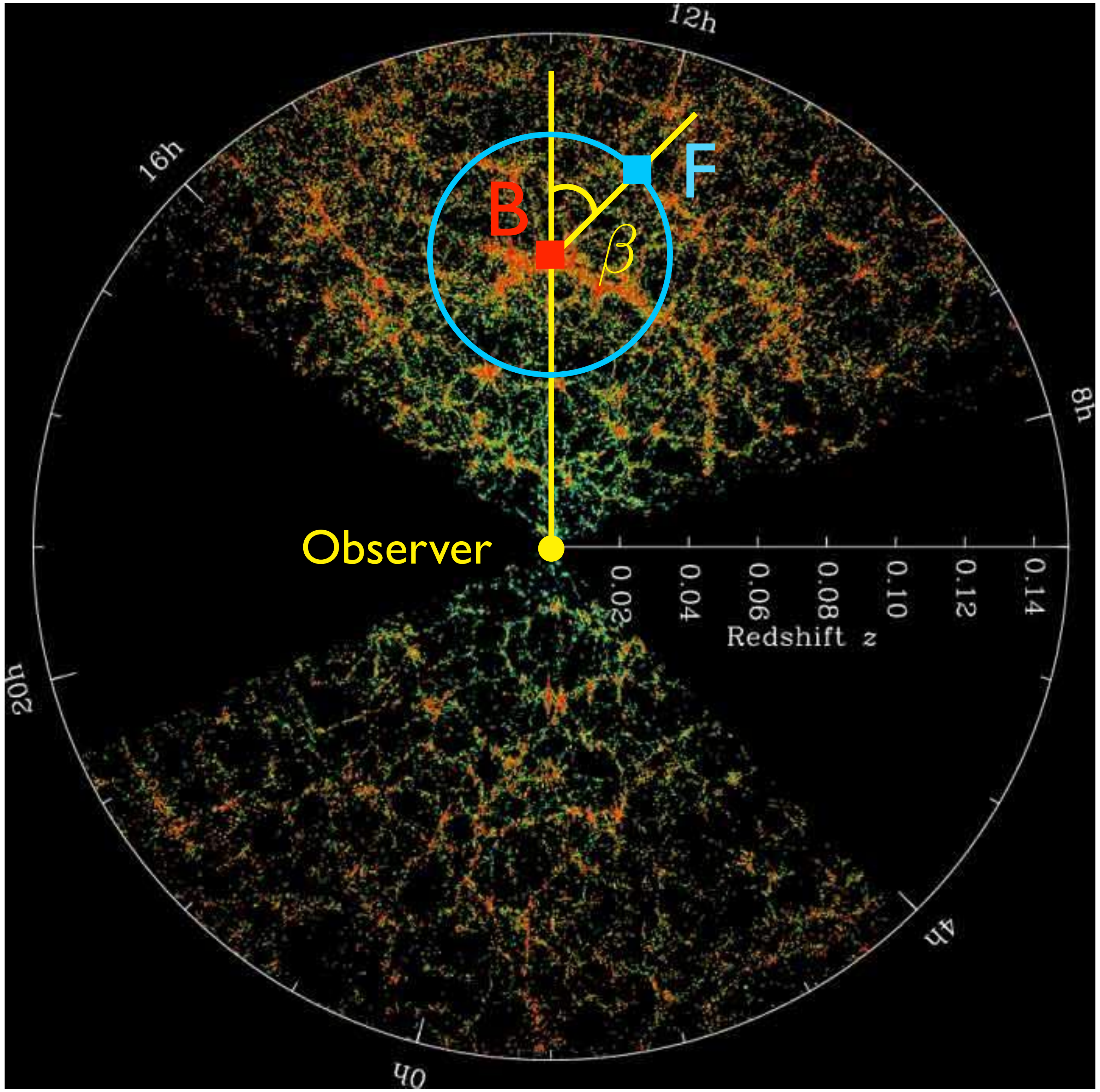


shift in position due to gravitational redshift

Combining all pairs

CB, Hui & Gaztanaga (2014)

Credit: M. Blanton, SDSS



$$\xi_{\rm BF} = A(d)\cos\beta$$

$$\downarrow$$

$$\Psi$$

By fitting for a dipole, we **isolate** relativistic effects

Number counts

$$
\begin{aligned}
\Delta(z,\mathbf{n}) =\; & \cancel{b\cdot\delta - \frac{1}{\mathcal{H}}\partial_r(\mathbf{V}\cdot\mathbf{n})} \\
& + \cancel{(5s-2)\int_0^r dr'\frac{r-r'}{2rr'}\Delta_\Omega(\Phi+\Psi)} \\
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\end{aligned}
$$

Dipole

Forecasts for DESI

- Theoretical forecasts indicate that the **Bright Galaxy Sample** is optimal to detect the dipole

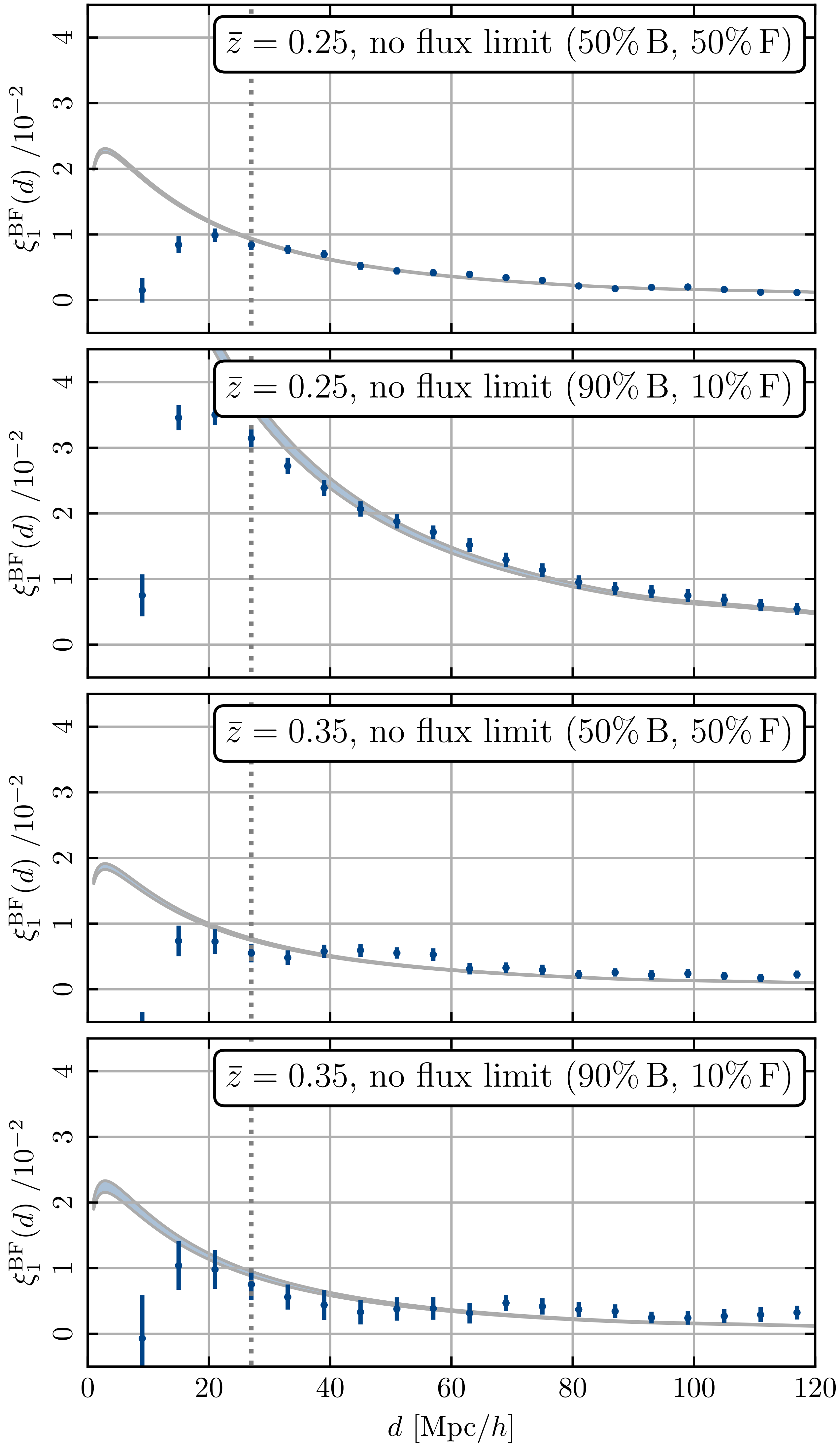
- We build halo catalogues from the numerical simulation **gevolution** → same number density, bias and sky coverage

- We reproduce the expected catalogues in 3 bins $z \in [0.2, 0.5]$

- We **measure** the dipole and **compare** with theoretical predictions in the linear regime: bias and magnification bias as input

Mock **DESI** catalogues

CB, Lepori et al (2023)



detection at $19\,\sigma$

dominated by **Doppler** effects

consistency check with RSD

Isolating gravitational redshift with SKA

- The **cumulative SNR** over redshift reaches 80
- **Gravitational redshift** is large enough to be measured
  - Dipole → Ψ and V
  - Redshift-space distortions → V and δ

Forecasts for Ψ for SKA2

Redshift	0.35	0.45	0.55	0.65	0.75	0.85	0.95
Constraints	23%	24%	28%	33%	40%	48%	60%

Interest of relativistic effects

Crucial to distinguish between **modified gravity** and non-standard **dark matter** affected by a **fifth force**

General relativity and cold non-interacting dark matter

- Poisson equation $-k^2\Phi = 4\pi G a^2 \rho\delta$
- No anisotropic stress $\Phi = \Psi$
- Euler equation for baryons and dark matter

$$\dot{V}_{\rm dm} + \mathcal{H}V_{\rm dm} + \partial_r\Psi = 0$$

$$\dot{V}_b + \mathcal{H}V_b + \partial_r\Psi = 0$$

Beyond GR and non-interacting CDM

Castello, Grimm and CB (2022)
CB and Pogosian (2022)

Modified gravity

$$\Phi = \eta \Psi \qquad\qquad -k^2\Psi = 4\pi G a^2 \rho_{\rm m} \mu \delta$$

$$\dot{V}_{\rm dm} + \mathcal{H} V_{\rm dm} + \partial_r \Psi = 0$$

Dark fifth force

$$\Phi = \Psi \qquad\qquad -k^2\Psi = 4\pi G a^2 \rho_{\rm m} \delta$$

$$\dot{V}_{\rm dm} + \mathcal{H}(1+\Theta) V_{\rm dm} + (1+\Gamma)\partial_r \Psi = 0$$

Beyond GR and non-interacting CDM

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Modified gravity

Can we measure and test these two types of modifications with LSS?

Dark fifth force

$$\Phi = \Psi \qquad\qquad -k^2\Psi = 4\pi G a^2 \rho_{\rm m}\delta$$

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Growth of structure

Castello, Grimm and CB (2022)
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Modified gravity

$$\Phi = \eta \Psi \qquad\qquad -k^2\Psi = 4\pi G a^2 \rho_{\rm m} \mu \delta$$

$$\ddot{\delta} + \mathcal{H}\dot{\delta} = 4\pi a^2 \rho_{\rm m} G \mu \delta$$ growth of structure

Dark fifth force

$$\Phi = \Psi \qquad\qquad -k^2\Psi = 4\pi G a^2 \rho_{\rm m} \delta$$

$$\ddot{\delta} + \mathcal{H}(1+\Theta)\dot{\delta} = 4\pi a^2 \rho_{\rm m} G(1 + x\Gamma)\delta$$ growth of structure

Growth of structure

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growth of structure

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$$\ddot{\delta} + \mathcal{H}(1+\underbrace{\cancel{\Theta}}_{\text{negligible}})\dot{\delta} = 4\pi a^2 \rho_{\rm m} \underbrace{G(1+x\Gamma)}_{G_{\rm eff}}\delta$$

growth of structure

Enhanced growth
Undistinguishable with RSD

Modified gravity

$$\Phi = \eta\,\Psi \qquad -k^2\Psi = 4\pi G a^2 \rho_{\rm m} \mu \delta$$

$$\ddot{\delta} + \mathcal{H}\dot{\delta} = 4\pi a^2 \rho_{\rm m} \underbrace{G\mu}_{G_{\rm eff}} \delta$$ growth of structure

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Anisotropic stress

Castello, Grimm and CB (2022)
CB and Pogosian (2022)

Modified gravity

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Anisotropic stress

$\boxed{\eta \neq 1 \rightarrow \text{Smoking gun for modified gravity}}$

Modified gravity

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$$\ddot{\delta} + \mathcal{H}\dot{\delta} = 4\pi a^2 \rho_\mathrm{m} \underbrace{G\mu}_{G_\mathrm{eff}} \delta$$

growth of structure

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growth of structure

negligible

Measurements

Castello, Grimm and CB (2022)
CB and Pogosian (2022)

- Lensing $\Phi + \Psi$
- Redshift-space distortion $V_{\rm dm}$

Modified gravity $\partial_r \Psi = -\dot{V}_{\rm dm} - \mathcal{H} V_{\rm dm}$

$$\frac{\Phi + \Psi}{\Psi} = 1 + \eta \neq 2$$

Dark fifth force $\underbrace{\partial_r (1 + \Gamma)\Psi}_{\Psi^{\rm eff} > \Psi} = -\dot{V}_{\rm dm} - \mathcal{H} V_{\rm dm}$

$$\frac{\Phi + \Psi}{\Psi^{\rm eff}} = 1 + \eta^{\rm eff} \neq 2 = \frac{\Phi + \Psi}{\Psi}$$

Measurements

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Modified gravity $\partial_r \Psi = -\dot{V}_{\rm dm} - \mathcal{H} V_{\rm dm}$

$\boxed{\eta \neq 1 \text{ Not a smoking gun!}}$

Dark fifth force $\partial_r \underbrace{(1+\Gamma)\Psi}_{\Psi_{\rm eff} > \Psi} = -\dot{V}_{\rm dm} - \mathcal{H} V_{\rm dm}$

$$\frac{\Phi + \Psi}{\Psi^{\rm eff}} = 1 + \eta^{\rm eff} \neq 2 = \frac{\Phi + \Psi}{\Psi}$$

Adding gravitational redshift

- We have a new variable Ψ which **breaks** the **degeneracy**

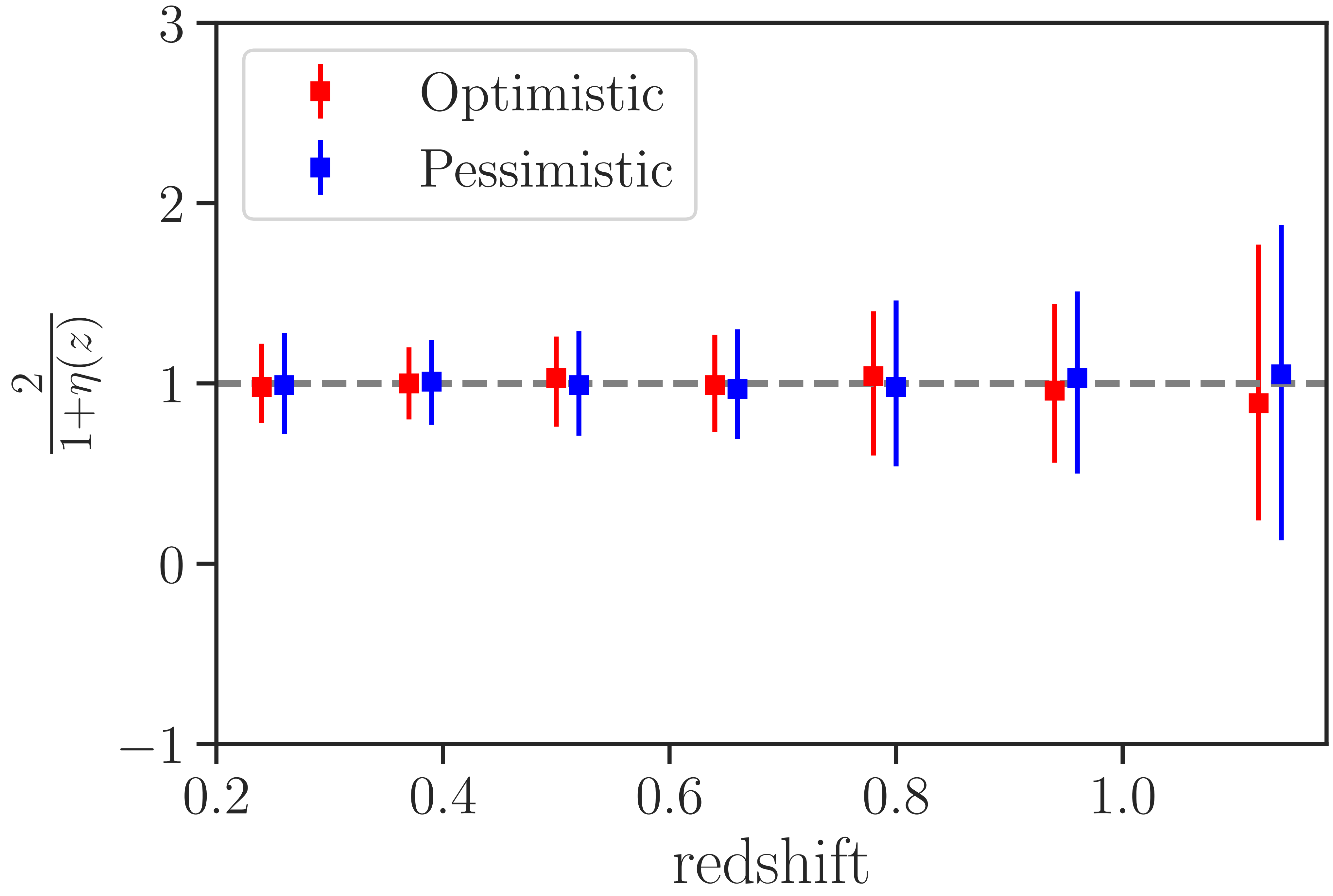
- Comparing Ψ with $\Phi + \Psi$: measure true **anisotropic stress**

- Comparing Ψ with V_{dm} : test **Euler equation** for dark matter

We can distinguish a **dark fifth force** from a modification of **gravity**

Tutusaus, Sobral-Blanco and CB (2022)

Forecasts with LSST and SKA



Restore η as **smoking gun** for modified gravity

Conclusion

- Relativistic effects will be **measurable** with DESI and SKA2 → **dipole** in cross-correlation

- Among them, **gravitational redshift** is essential to **distinguish** modified gravity from a dark fifth force

- It allows us to measure directly the **anisotropic stress** → smoking gun for modified gravity

- We can also test the validity of **Euler equation:** test of the weak equivalence principle (Sveva Castello's talk)

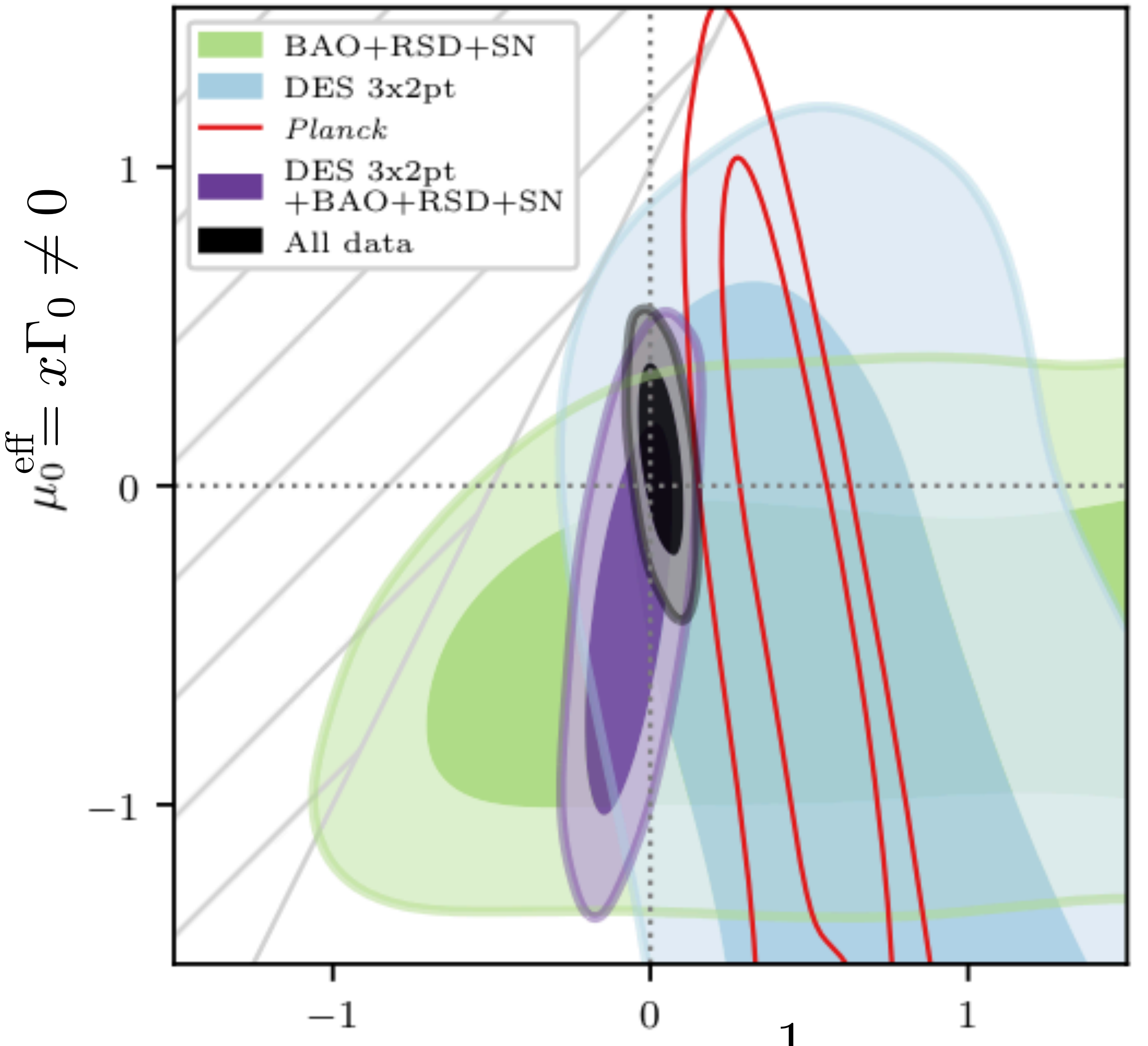
Backup slides

Exemple from DES and eBOSS

$$\left.\begin{aligned} \Phi &= \eta\,\Psi \\ -k^2\Psi &= 4\pi G a^2 \rho_\mathrm{m} \mu \delta \end{aligned}\right\} \qquad -k^2(\Phi+\Psi) = 8\pi G a^2 \rho_\mathrm{m} \Sigma\,\delta$$

$$\Sigma \searrow \frac{1}{2}\mu(1+\eta)$$

Fifth force

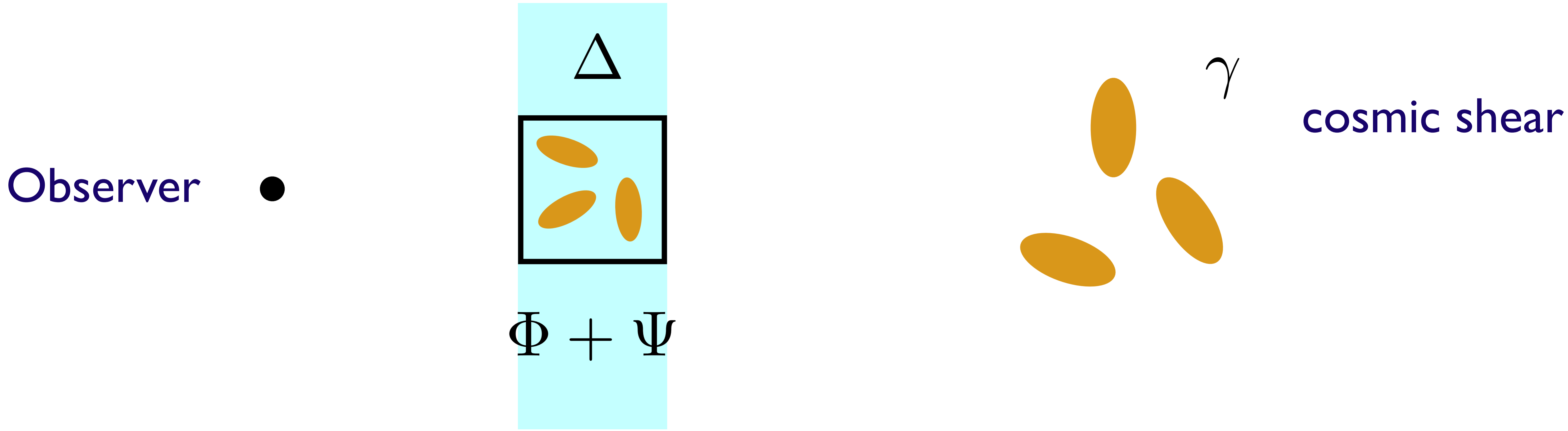


Abbott et al. (DES coll.) arXiv:2207.05766

Combine with lensing

Tutusaus, Sobral-Blanco and CB (2022)

Galaxy-galaxy lensing allows us to measure directly $\Phi + \Psi$



Measurements with **DES** (preliminary)

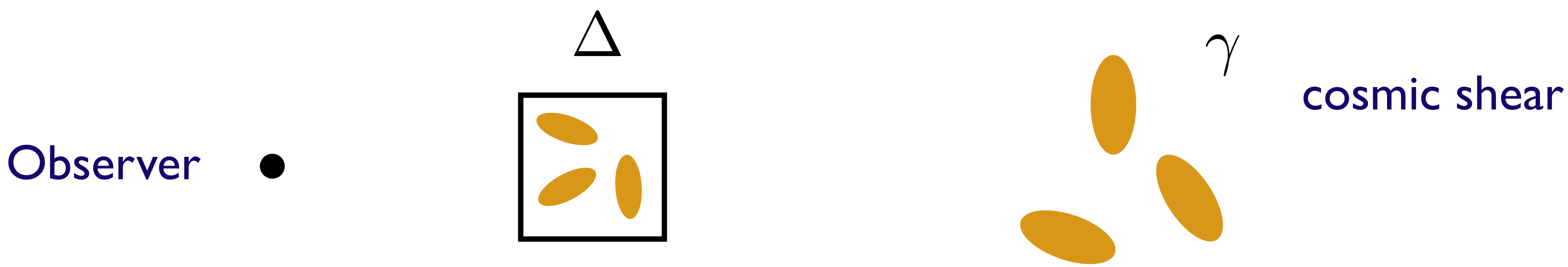
Tutusaus, CB and Grimm-Thieme (in prep)

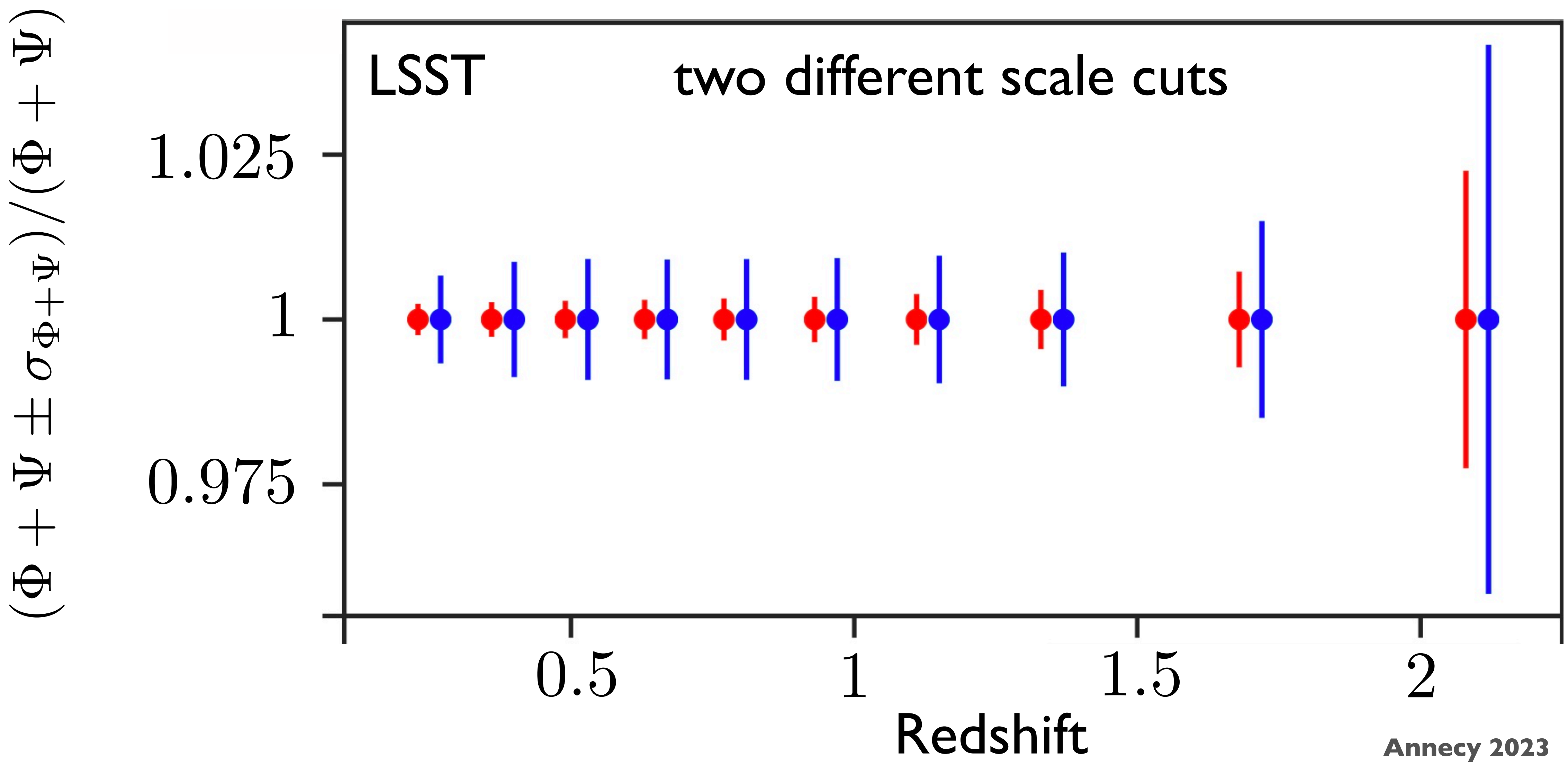
Measure the evolution of $\Phi + \Psi$ in four redshift bins with a precision of **4 to 9 percent**

Combine with lensing

Tutusaus, Sobral-Blanco and CB (2022)

Galaxy-galaxy lensing allows us to measure directly $\Phi+\Psi$





$$S^{\mathrm{GBD}} = \int \mathrm{d}^4\sqrt{-g}\left[\frac{A^{-2}(\phi)}{16\pi G}R - \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - V(\phi) + \mathcal{L}_{\mathrm{m}}(\psi_{\mathrm{DM}}, \psi_{\mathrm{SM}}, g_{\mu\nu})\right];$$

$$S^{\mathrm{CQ}} = \int \mathrm{d}^4\sqrt{-g}\left[\frac{1}{16\pi G}R - \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - V(\phi) + \mathcal{L}_{\mathrm{SM}}(\psi_{\mathrm{SM}}, g_{\mu\nu}) + \mathcal{L}_{\mathrm{DM}}(\psi_{\mathrm{DM}}, A^2(\phi)g_{\mu\nu})\right]$$

Generalized Brans-Dicke (GBD)

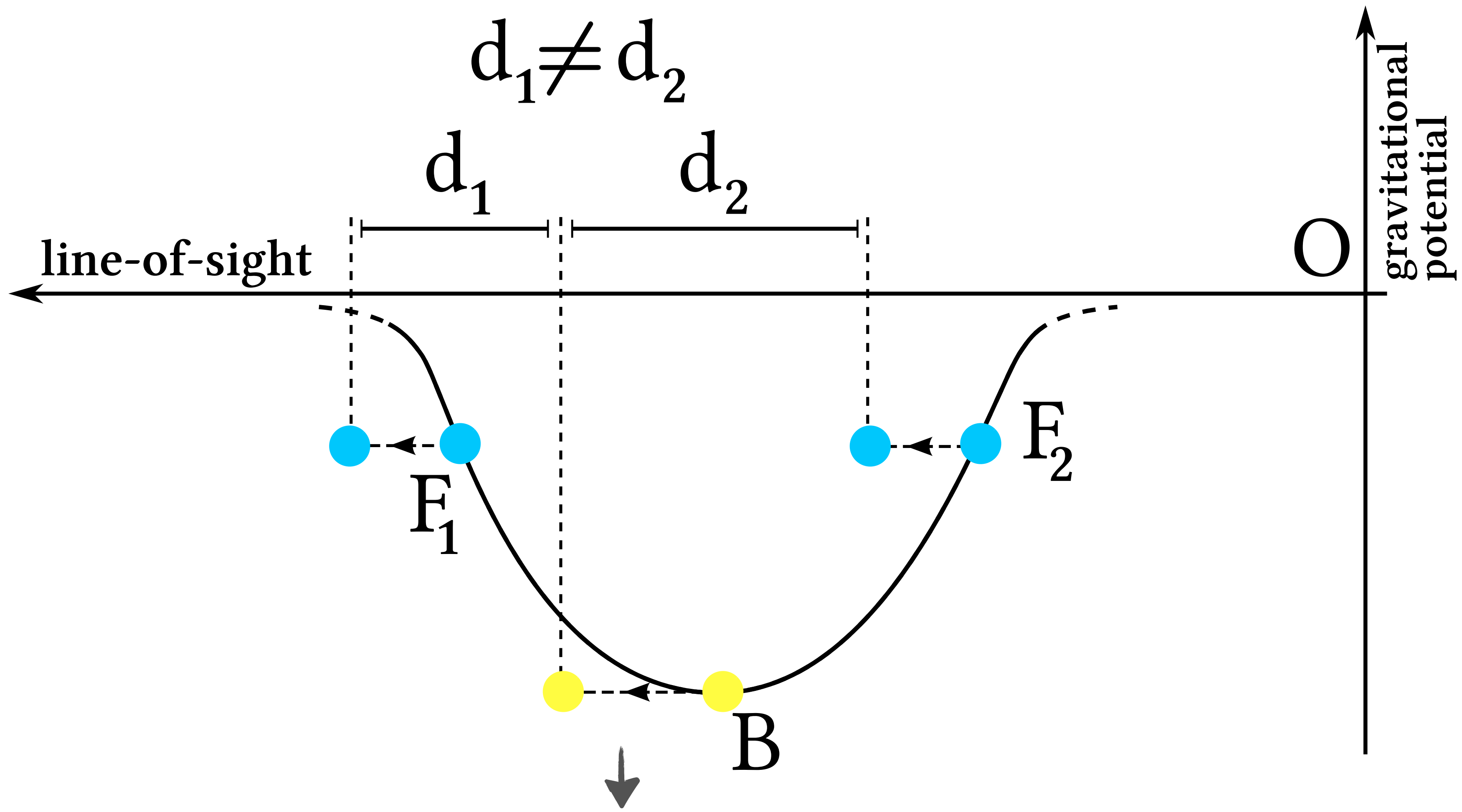
$$k^2\Phi = -4\pi G a^2\left(\rho_b\delta_b + \rho_c\delta_c\right) - \beta k^2\delta\phi \tag{4}$$
$$k^2(\Phi - \Psi) = -2\beta k^2\delta\phi \tag{5}$$
$$\dot{\delta}_b + \theta_b = 0 \tag{6}$$
$$\dot{\theta}_b + \mathcal{H}\theta_b = k^2\Psi \tag{7}$$
$$\dot{\delta}_c + \theta_c = 0 \tag{8}$$
$$\dot{\theta}_c + \mathcal{H}\theta_c = k^2\Psi \tag{9}$$
$$\delta\phi = -\frac{\beta(\rho_c\delta_c + \rho_b\delta_b)}{m^2 + k^2/a^2} \tag{10}$$
$$\Box\phi = V_{,\phi} + \beta(\rho_c + \rho_b) \equiv V^{\mathrm{eff}}{}_{,\phi} \tag{11}$$
$$\ddot{\delta}_m + \mathcal{H}\dot{\delta}_m = 4\pi G a^2\rho_m\delta_m\left[1 + \frac{2\tilde{\beta}^2k^2}{a^2m^2 + k^2}\right] \tag{12}$$

Coupled Quintessence (CQ)

$$k^2\Phi = -4\pi G a^2\left(\rho_b\delta_b + \rho_c\delta_c\right) \tag{13}$$
$$k^2(\Phi - \Psi) = 0 \tag{14}$$
$$\dot{\delta}_b + \theta_b = 0 \tag{15}$$
$$\dot{\theta}_b + \mathcal{H}\theta_b = k^2\Psi \tag{16}$$
$$\dot{\delta}_c + \theta_c = 0 \tag{17}$$
$$\dot{\theta}_c + (\mathcal{H} + \beta\dot{\phi})\theta_c = k^2\Psi + k^2\beta\delta\phi \tag{18}$$
$$\delta\phi = -\frac{\beta\rho_c\delta_c}{m^2 + k^2/a^2} \tag{19}$$
$$\Box\phi = V_{,\phi} + \beta\rho_c \equiv V^{\mathrm{eff}}{}_{,\phi} \tag{20}$$
$$\ddot{\delta}_m + \mathcal{H}\dot{\delta}_m = 4\pi G a^2\rho_m\delta_m\left[1 + \frac{2\tilde{\beta}^2k^2}{a^2m^2 + k^2}\left(\frac{\rho_c}{\rho_m}\right)^2\left(\frac{\delta_c}{\delta_m}\right)\right] \tag{21}$$

Breaking of symmetry from gravitational redshift

CB, Hui & Gaztanaga (2014)



shift in position due to gravitational redshift

$$\xi = \frac{\mathcal{H}}{\mathcal{H}_0}\left(\frac{D_1}{D_{10}}\right)^2 \Big[(b_\mathrm{B}-b_\mathrm{F})\left(\frac{2}{r\mathcal{H}}+\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}\right)+3(s_\mathrm{F}-s_\mathrm{B})f^2\left(1-\frac{1}{r\mathcal{H}}\right)$$
$$+5(b_\mathrm{B}s_\mathrm{F}-b_\mathrm{F}s_\mathrm{B})f\left(1-\frac{1}{r\mathcal{H}}\right)\Big]\nu_1(d)\cos(\beta)$$