

# Relativistic effects on large scale structures\*: a leap into the non-linear regime with RayGal simulations

*Yann RASERA (LUTH/Paris Cité Univ./Paris Obs./PSL/CNRS/IUF)*

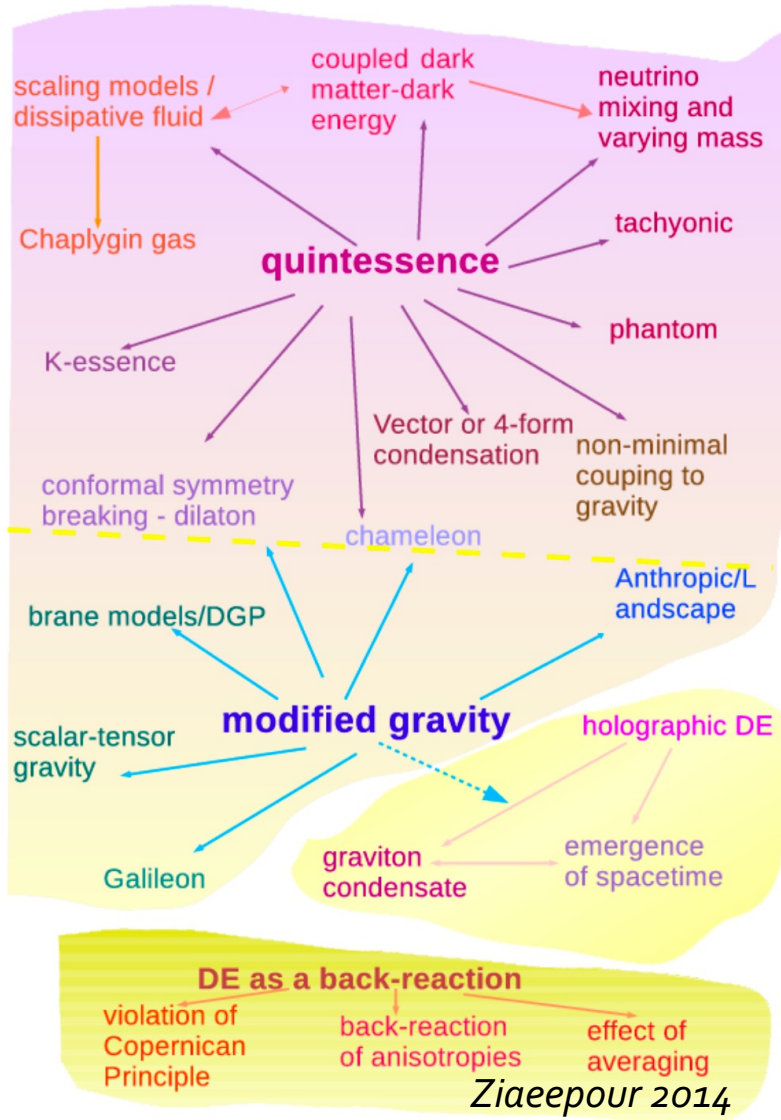
*Work with: Michel-Andrès Breton (ICE), P-S. Corasaniti (LUTH), J.Allingham (Technion), F.Roy (LUTH), V.Reverdy (LAPP), T.Pellegrin (LPP), S.Saga (IAP), A.Taruya (YITP), J-M Alimi (LUTH), S.Agarwal (AIMS), S.Anselmi (INFN)*

\* I will mostly skip the introduction, please check C.Bonvin & S.Castello talks

ADE 2023

# WHAT IS THE NATURE OF THE DARK SECTOR?

## • VARIOUS POSSIBILITIES FOR DE



## • SEARCH FOR NEW OR REFINED PROBES IN THE NON-LINEAR REGIME OF STRUCTURE FORMATION



Non linear imprints of DARK SECTOR on COSMIC STRUCTURES ?  
How to probe DARK SECTOR with COSMIC STRUCTURES?

# THE COSMIC PROGRAM

(beyond  $a(t)$ )

CAN WE POSSIBLY TEST ALL THE MAIN HYPOTHESIS AT COSMOLOGICAL SCALES ?

$$ds^2 = -(1 + 2\Psi)dt^2 + a^2(t)(1 - 2\Phi)\delta_{ab}dx^a dx^b$$

From number count

Overdensity  $\delta$

Continuity

From quadrupole/hexadecapole RSD

Velocity  $v$

Poisson

Euler or Jeans

**STANDARD LENSING**  
(i.e. Born)

**STANDARD RSD**  
(i.e. Doppler)

Potential  $\Phi$

Potential  $\Psi$

How ?

$\Phi + \Psi$

From LENSING

Geodesics

Observer

Adapted from  
Bonvin&Fleury 2018

Simplified view  
Assume  $a(t)$  known

# THE COSMIC PROGRAM

(beyond  $a(t)$ )

CAN WE POSSIBLY TEST ALL THE MAIN HYPOTHESIS AT COSMOLOGICAL SCALES ?

$$ds^2 = -(1 + 2\Psi)dt^2 + a^2(t)(1 - 2\Phi)\delta_{ab}dx^a dx^b$$

From number count

(relativistic contribution ~0.5-5%)

Overdensity  $\delta$

Continuity



From quadrupole/hexadecapole RSD

(relativistic contribution ~0.5-5%)

Velocity  $v$



RELATIVISTIC LENSING



RELATIVISTIC RSD

Potential  $\Phi$

Potential  $\Psi$

FROM DIPOLE !

(relativistic contribution ~100%)

$\Phi + \Psi$

Geodesics

From LENSING

(relativistic contribution ~0.1-50%)

Observer

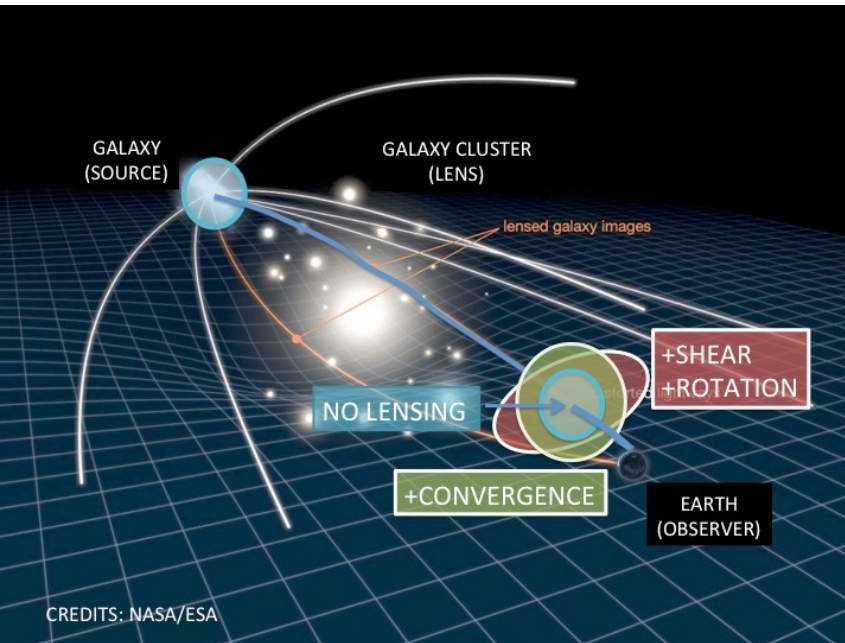
Adapted from  
Bonvin&Fleury 2018

Simplified view  
Assume  $a(t)$  known



# Usual approach: compute WL maps or RSD catalogs

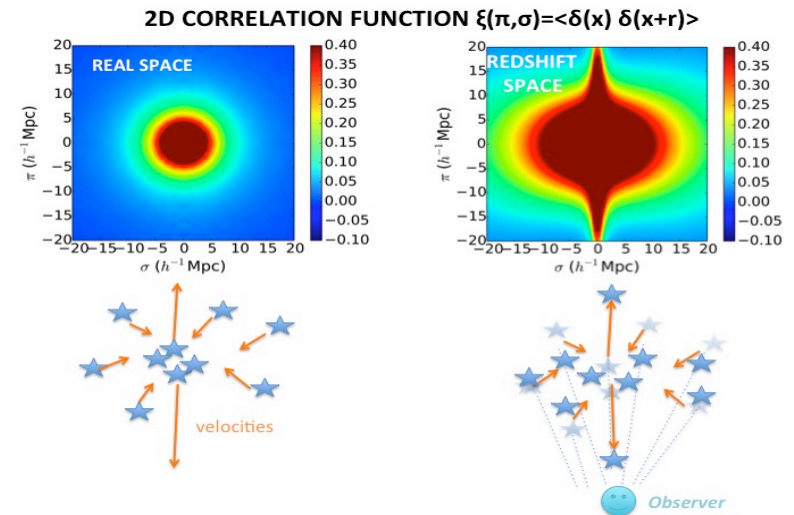
## LENSING



Many approximations-> Example of approximations: no-RSD, flat sky, Born, multiple-lens, replications

OR

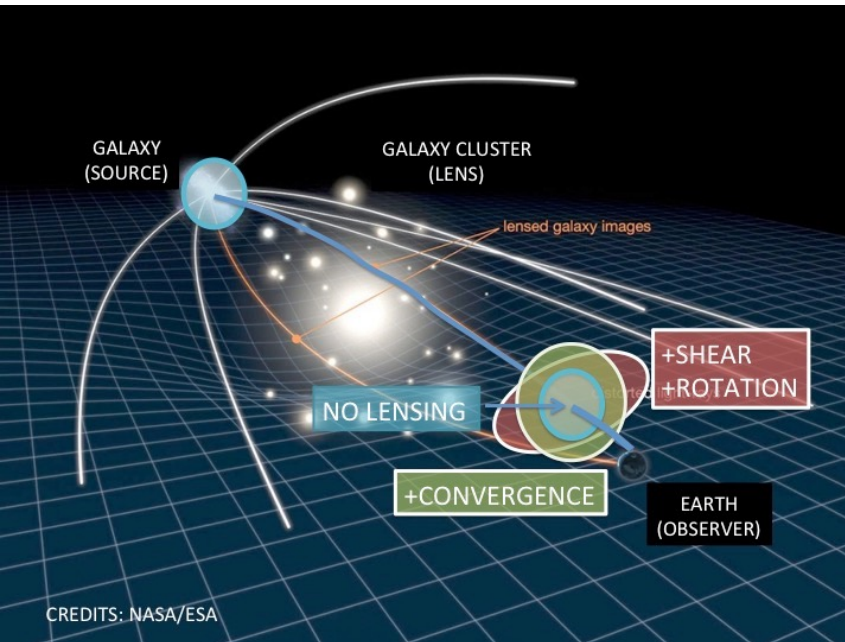
## Redshift-Space Distortions (RSD)



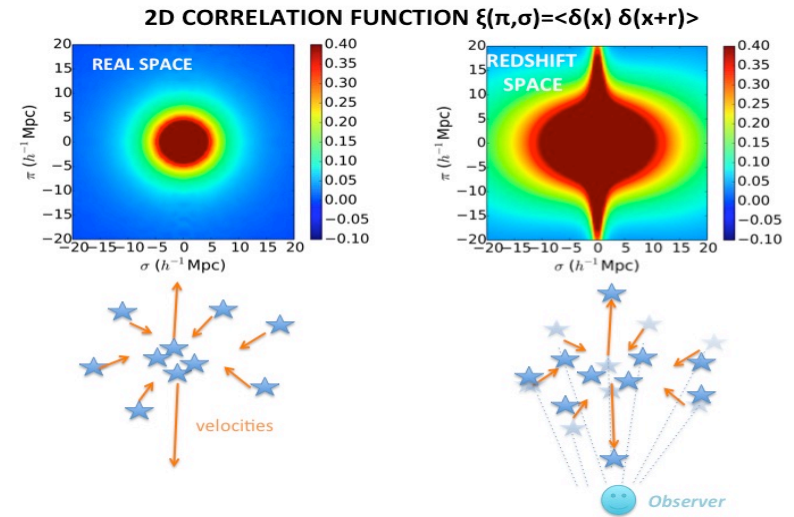
Many approximations-> Example of approximations: no-lensing, distant observer, no gravitational redshift (i.e. Doppler only), no light-cone effect

# Relativistic approach: compute what is really observed following (weak-field) GR

## LENSING



## Redshift-Space Distortions (RSD)



AND

AND OTHERS (gravitational redshift, ISW effect, transverse Doppler, etc)

- Relativistic approach at large scales: Yoo+ 2010; Bonvin&Durrer 2011; Yoo 2011; Lewis&Challinor 2011 (SEE CAMILLE BONVIN AND SVEVA CASTELLO'S TALKS)

Use similar formalism as for CMB (i.e. weak field GR) but applied to galaxies

-> LIMITATION OF ORIGINAL WORKS: LINEAR REGIME

- Relativistic approach at cluster scale and around: Kaiser2013, Zhao2013, Croft2013, Cai+2017

-> LIMITATION: How to connect with linear predictions ?

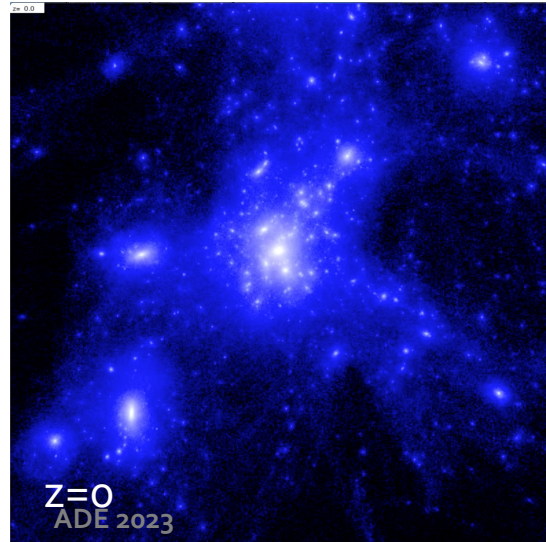
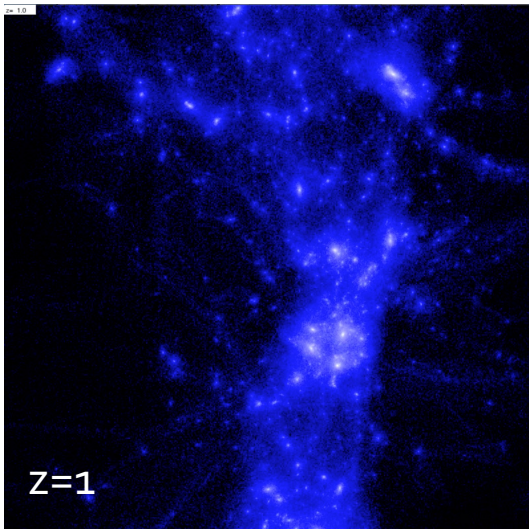
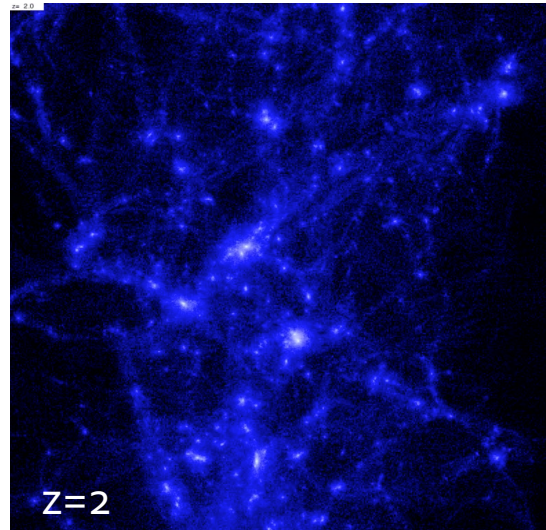
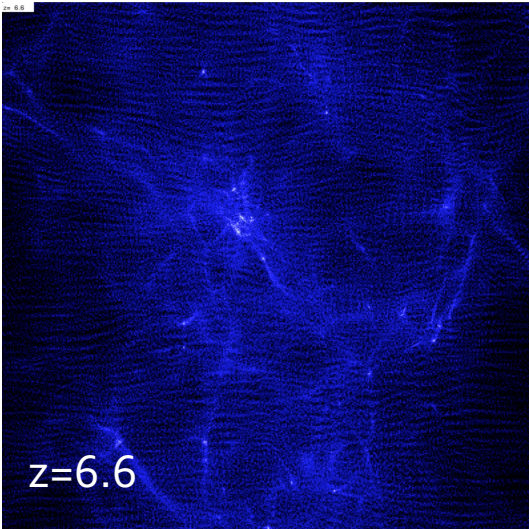
=> GR effects WITH SIM IS A HOT TOPIC: Killedar12, Reverdy14, Adamek16, Giblin17, Borzyszkowski17, Breton19, Adamek19, Leporizo, Guandalin21, Leporiz21, Raser22, ... 6

**HOW TO ADDRESS  
THE NON-LINEAR REGIME?  
The RayGal way...**



# STEP 1: RUN RAYGAL N-BODY SIMULATIONS

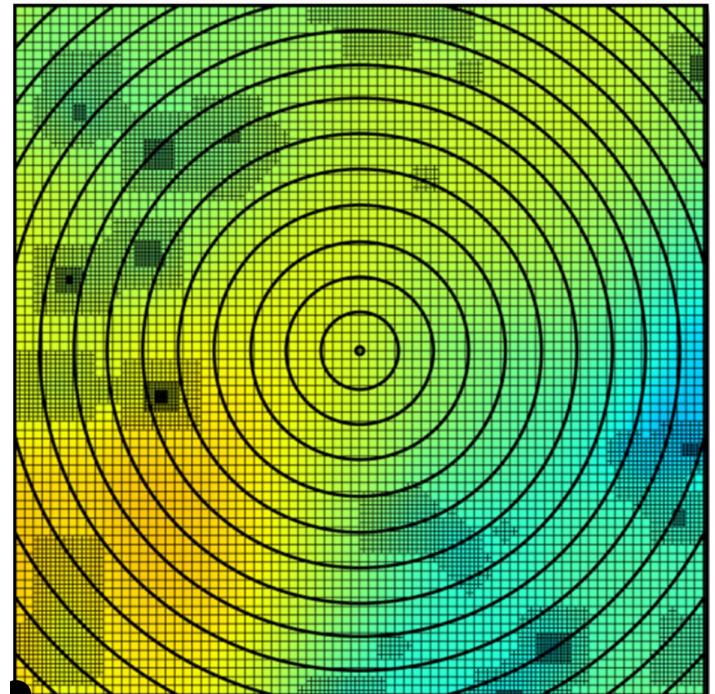
- Goal: Build a (virtual) « real » universe by running N-Body sims
- N-body solver: RAMSES Particle-Mesh with Adaptive Mesh Refinement (PM-AMR)
- Specs:  $4096^3$  particles,  $(2.6 \text{ Gpc}/h)^3$ ,  $\Lambda\text{CDM}$  &  $w\text{CDM}$  ( $w=-1.2$ )
- #halos: >10 millions of halos from Milky-Way size to cluster size



*Illustrative  
example of  
the formation  
of one large  
halo in a  
simulation*

## STEP 2: BACKUP A GRAVITY LIGHTCONE

- What: Gravity (and particles) lightcone
- Where: At light-travel distance from the observer (center of the box)
- Remark: also backup in the vicinity of the null-FLRW lightcone (called « thick » light-cone)
- Which quantities: Potential (i.e. metric), gradient of the potential (i.e. gravitational field), time derivative of the potential
- Type of light-cone: wide (fullsky,  $z_{\max}=0.5$ ), deep ( $2500 \text{ deg}^2, z_{\max}=2$ ), very deep ( $400 \text{ deg}^2, z_{\max}=10$ )





# STEP 3 : DIRECT INTEGRATION OF BILLION WEAK-FIELD GEODESICS EQUATIONS IN PERTURBED FLRW WITHIN AMR GRID

- Geodesic equations:

$$\frac{d^2 x^\alpha}{d\lambda^2} = -\Gamma_{\beta\gamma}^\alpha \frac{dx^\beta}{d\lambda} \frac{dx^\gamma}{d\lambda}$$

- Redshift definition:

$$1 + z = \frac{\nu_s}{\nu_o} = \frac{(g_{\mu\nu} k^\mu k^\nu)_s}{(g_{\mu\nu} k^\mu k^\nu)_o}$$

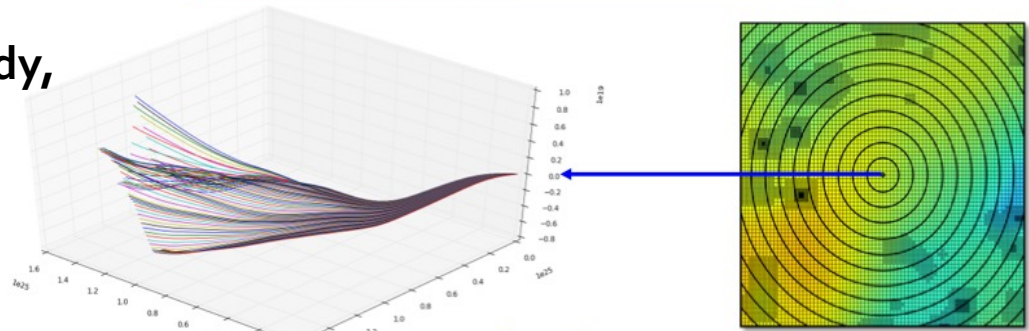
- **MAGRATHEA library (Reverdy 2014):**  
optimized/light AMR (MPI+p-threads)

- **MAGRATHEA-PATHFINDER: ray-tracing, WL, RSD (Breton&Reverdy, 2021)**

- Self-consistent calculation of WL AND RSD AND other relativistic effects

- Little number of controlled assumptions: weak-field GR + neglect horizon-scale GR effects on DM dynamics (Chisari & Zaldarriaga, 2011, Adamek et al. 2016)

## 3D backward raytracing



For free...

**Weak lensing**  
(convergence & shear)

**Integrated Sachs-Wolfe**

**Luminosity distance**  
**Angular distance**  
**Redshift distortions**  
**Time delays**

...

V. Reverdy thesis

# STEP 4: MAGRATHEA-PATHFINDER'S ITERATIVE GEODESICS FINDER

## Find null geodesics

Find the connection between  
Observer  $O$  and Source  $S$   
Using Newton's method :

$$x = (x_1, \dots, x_n)$$

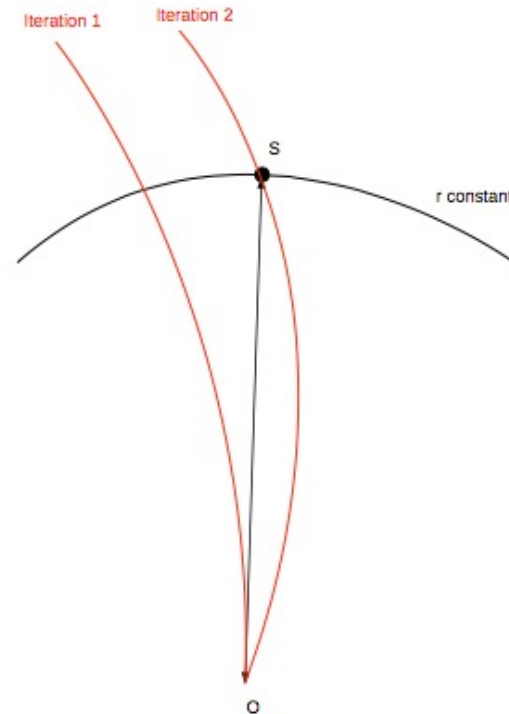
$$x_{k+1} = x_k - F(x_k)/F'(x_k)$$

## Output

« NEW » : Catalogs of sources  
taking into account weak  
lensing effects and redshift  
space distortions

In the catalogs :

$\vec{\beta}$ ,  $\vec{\theta}$ ,  $\bar{z}$ ,  $z$ , errors,  $A_{ij}$

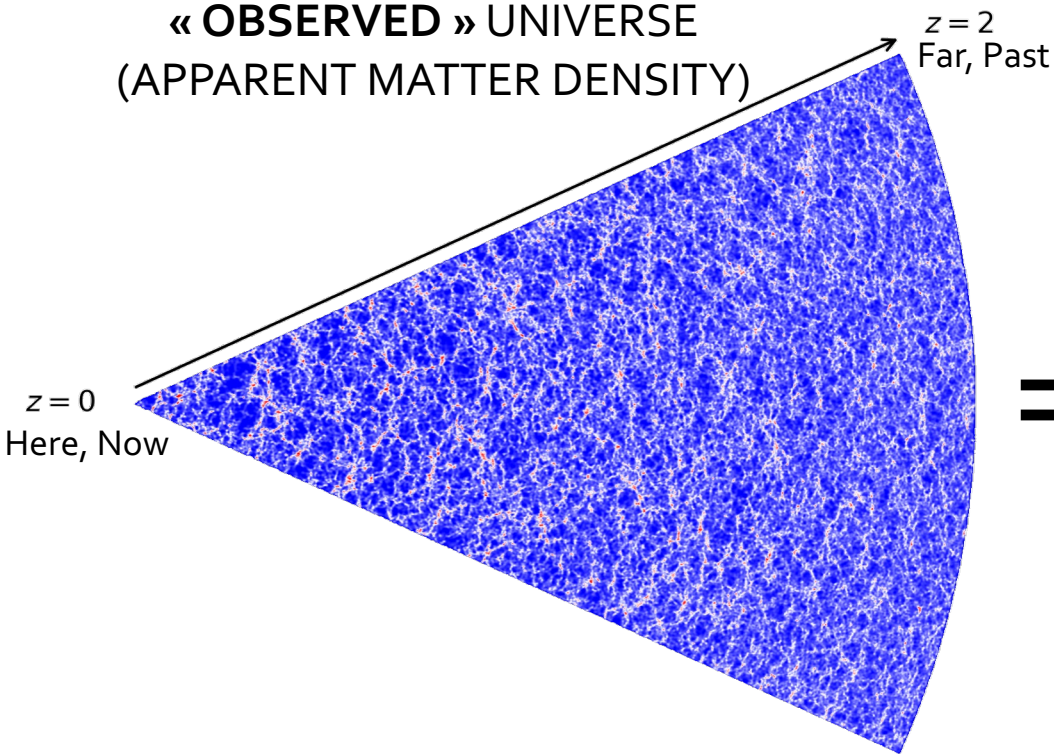


Breton et al, 2019

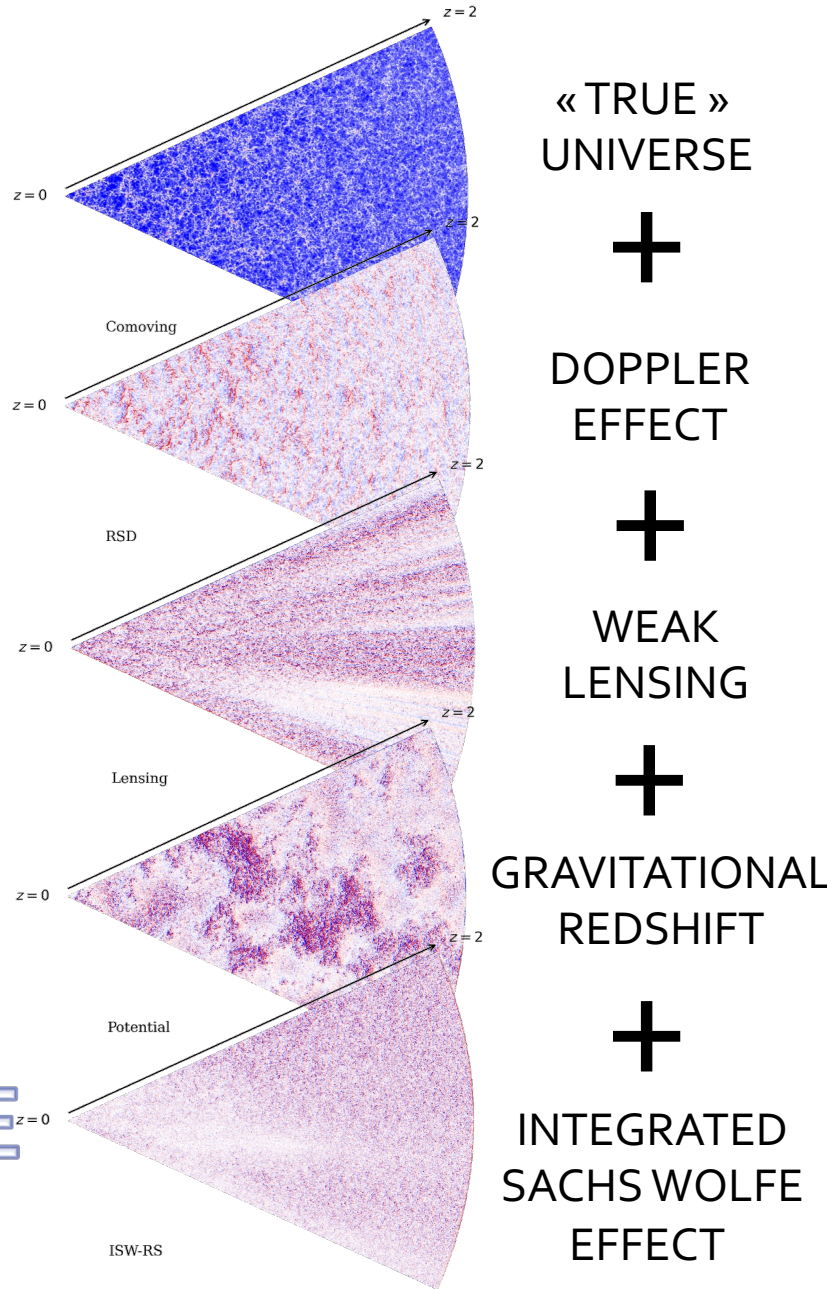
- Weak-lensing using the ray-bundle approach
- Launch a beam of photons and directly compute its distortion (i.e. distortion matrix)
- Account for finite beam effect (i.e. the size of a galaxy is not zero as in the usual WL formalism)



SIMULATION  
OF A SLICE OF THE  
« **OBSERVED** » UNIVERSE  
(APPARENT MATTER DENSITY)



=



# THE RAYGAL UNIVERSE

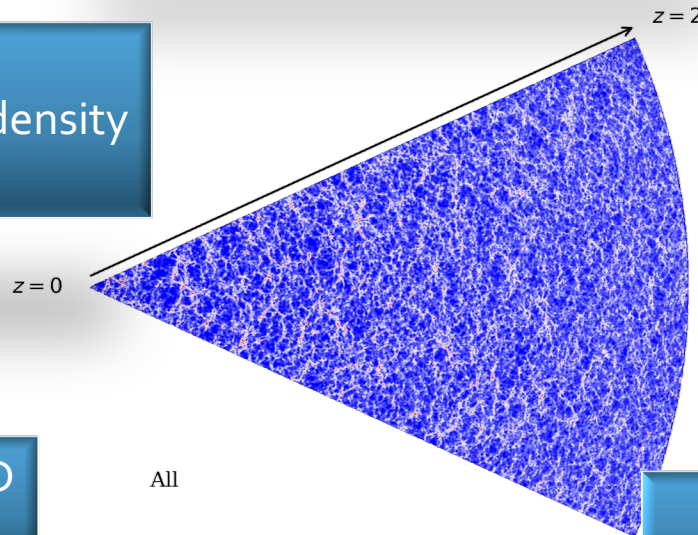
70 billion particles  $\Rightarrow$  cosmic structure formation  
1 billion photons  $\Rightarrow$  general relativistic effects

OPEN DATA: <https://cosmo.obspm.fr/public-datasets/>  
(or type « **RayGal data** » on any search engine)

# Very generic, built from 1st principles=> many applications

Bias of distance-redshift relation  
Breton&Fleury, 2021

3x2pts in WL  
Convergence-matter overdensity  
Rasera et al. 2022



Dipole in RSD  
Breton et al. 2019  
Taruya et al. 2020  
Saga et al. 2020, 2021

Magnification bias in RSD  
Breton et al. 2022

Gravitational redshift or WL in clusters

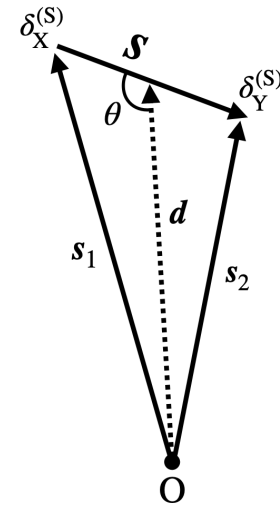
CMB-galaxies cross-correlations

**I am pretty sure  
you have an idea :  
the (cross-)correlation of  
(several) cosmological observables!**

# **Application 1: Relativistic Redshift Space Distortions**

# GEOMETRY AND QUANTITIES

- Geometry: Center of pair of halos + average in spherical shells
- Quantity : halo density
- Statistic: halo-halo cross-correlation (i.e. multi-population)
- Projection: Multipoles

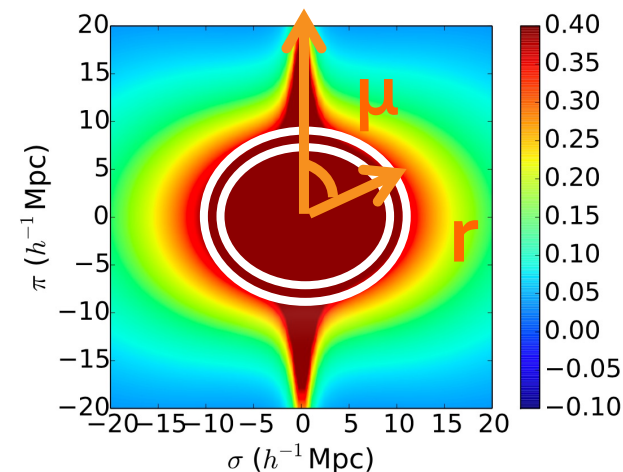


$$\xi_l(r) = \langle \delta_1(\mathbf{x}) \delta_2(\mathbf{x} + \mathbf{r}) P_l(\mu) \rangle$$

- DEFINITION:

**NON-TRIVIAL RELATIVISTIC EFFECT =  
BEYOND STANDARD RSD (IE. DOPPLER + DISTANT OBSERVER)**

- THEORY (for comparison): Linear (Bonvin & Durrer 2011; Bonvin et al, 2014)

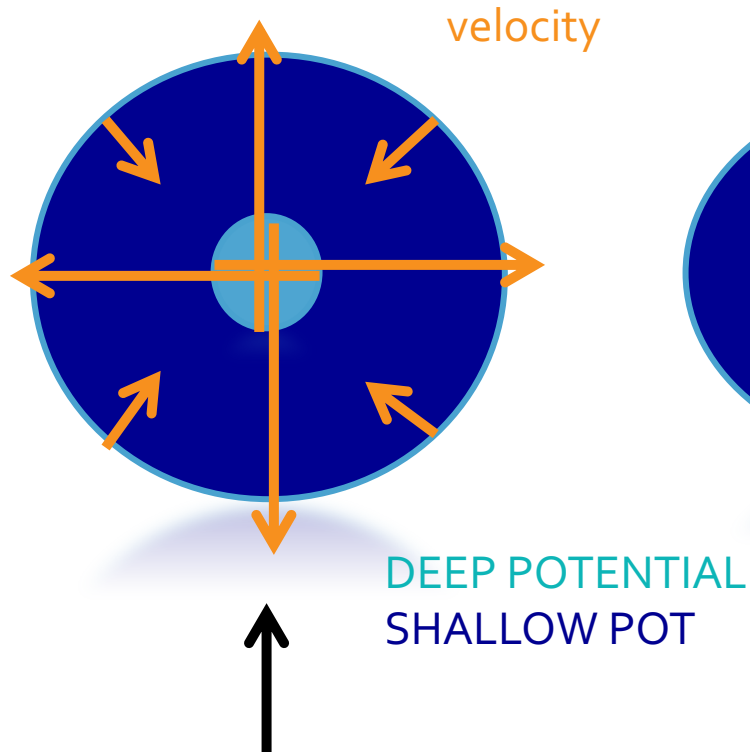


# IMPORTANT TAKE HOME MESSAGE

THE DIPOLE IS A PROBE OF THE GRAVITATIONAL POTENTIAL IN THE NL REGIME

## MONOPOLE

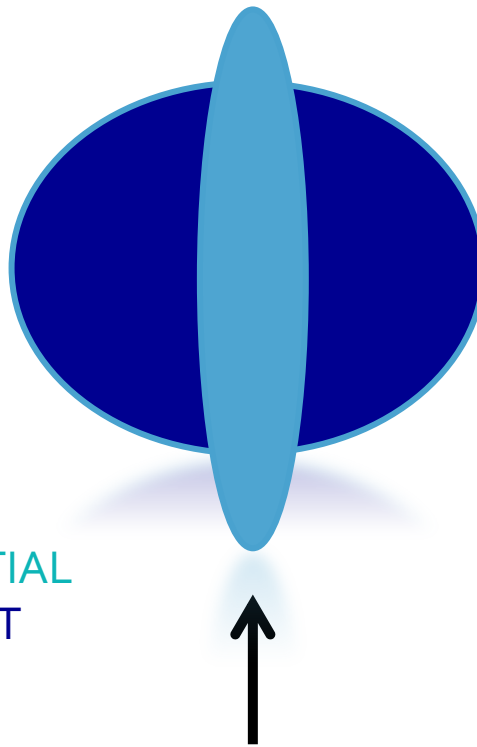
REAL SPACE



## EVEN MULTIPOLES

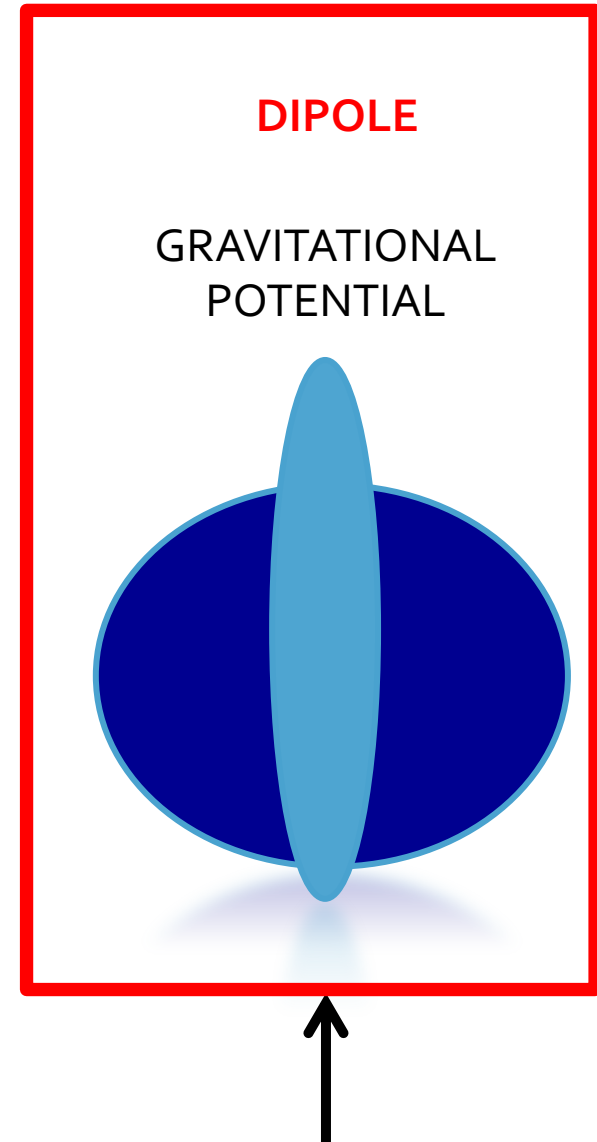
STANDARD RSD

KAISER EFFECT-LARGE SCALE  
FINGERS OF GOD-SMALL SCALE



## DIPOLE

GRAVITATIONAL  
POTENTIAL



DISTANT OBSERVER

DISTANT OBSERVER

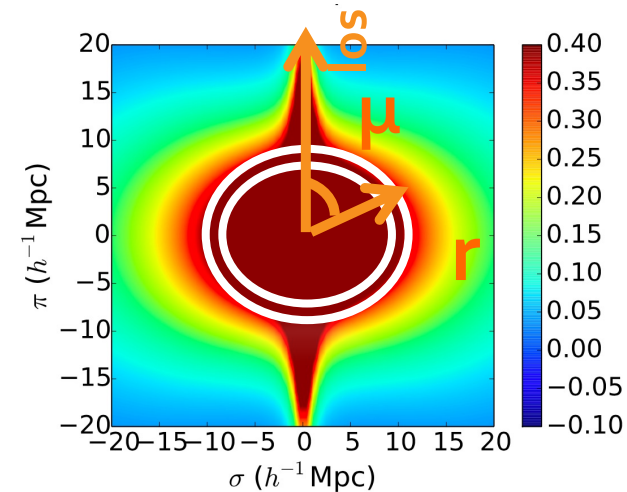
DISTANT OBSERVER

# Relativistic RSD: Even multipoles

- Multipole

$$\xi_l(r) = \langle \delta_1(\mathbf{x}) \delta_2(\mathbf{x}+\mathbf{r}) P_l(\mu) \rangle$$

- Monopole:  $l=0 \Rightarrow$  density
- Quadrupole/hexadecapole:  $l=2,4 \Rightarrow$  velocity



*Expected Impact of relativistic effects in a Euclid like spectroscopic survey: even multipoles*

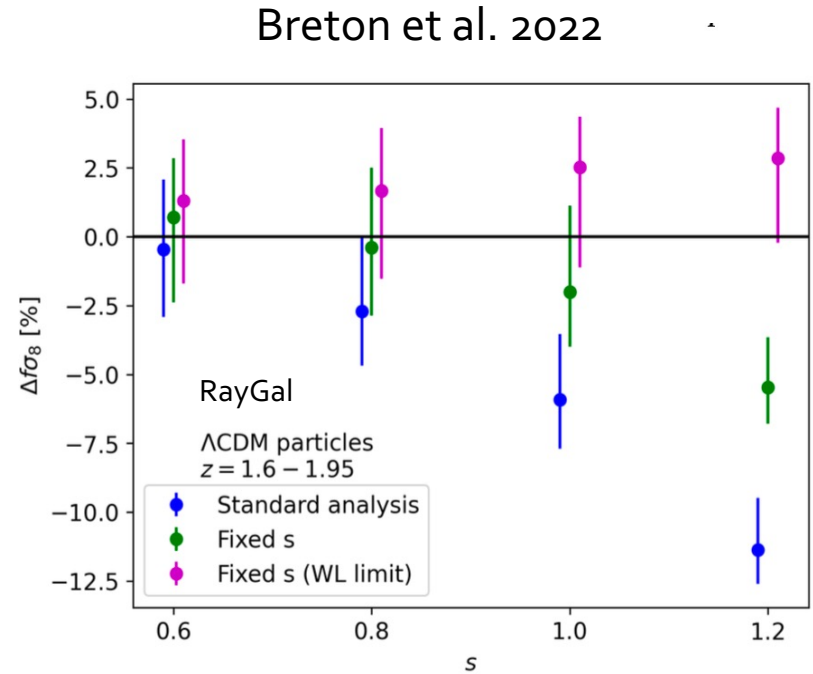
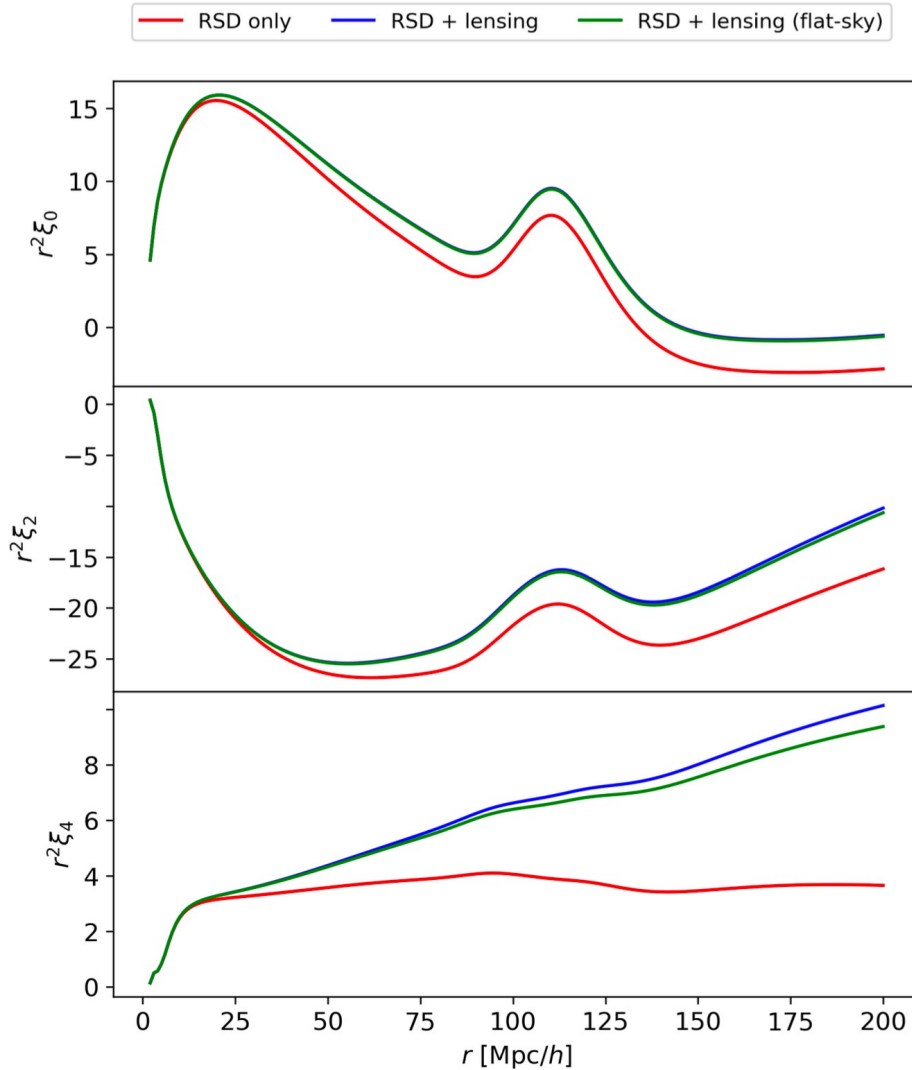
| $\xi_l$ | Doppler | $v_o$ | Grav. redshift | Lensing* | T. Doppler | ISW  |
|---------|---------|-------|----------------|----------|------------|------|
| $\xi_0$ | > 20%   | 3%    | < 1%           | 1 – 10%  | < 1%       | < 1% |
| $\xi_2$ | > 20%   | 2%    | < 1%           | 2%       | < 1%       | < 1% |
| $\xi_4$ | > 20%   | -     | < 1%           | 1 – 10%  | < 1%       | < 1% |

Breton et al. 2022

Courtesy: M-A Breton



# Relativistic RSD: Even multipoles effect of lensing



$$s(z, m_{\text{lim}}) \equiv \left. \frac{\partial \log_{10} \bar{N}(z, L > L_{\text{lim}})}{\partial m} \right|_{m_{\text{lim}}}$$

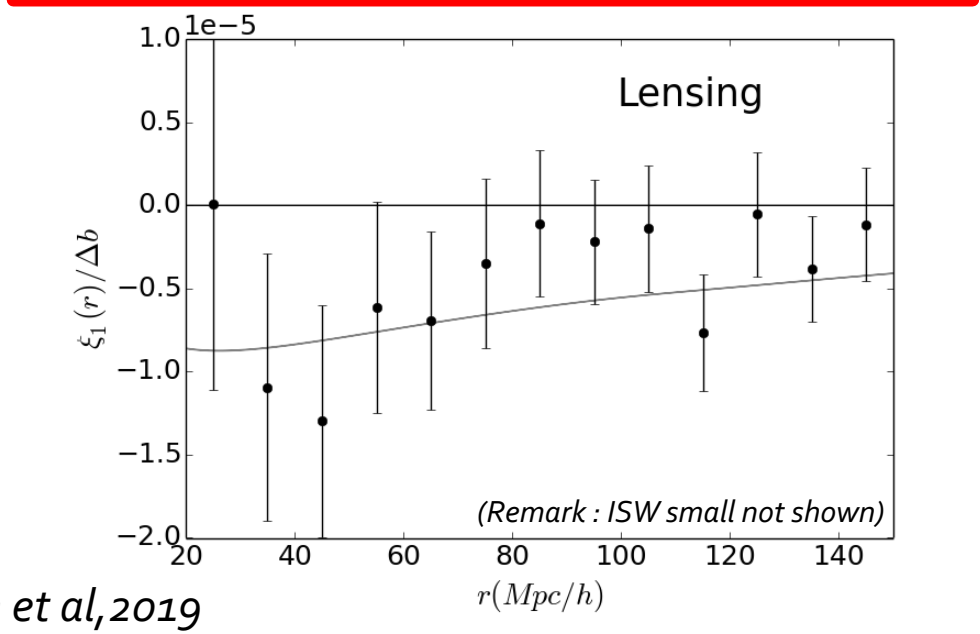
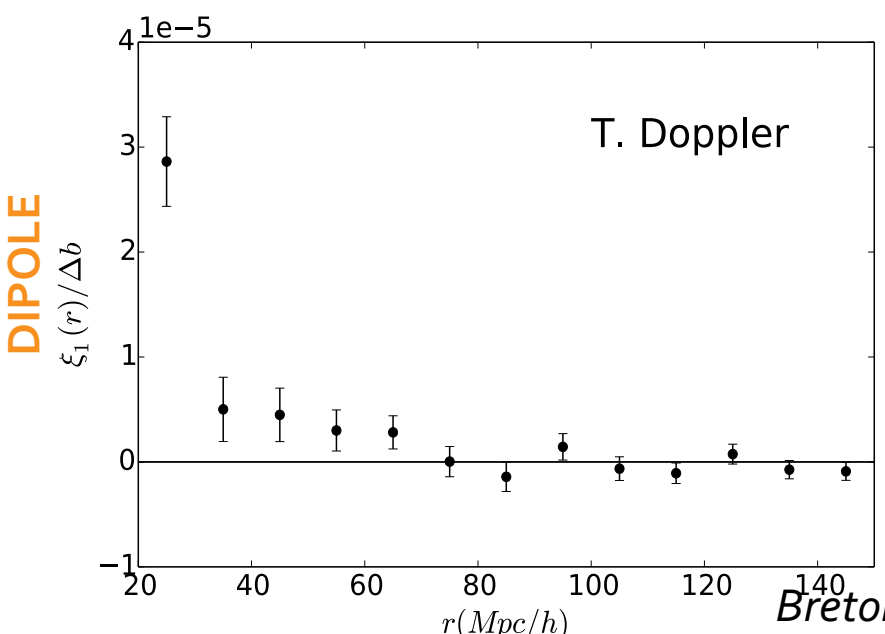
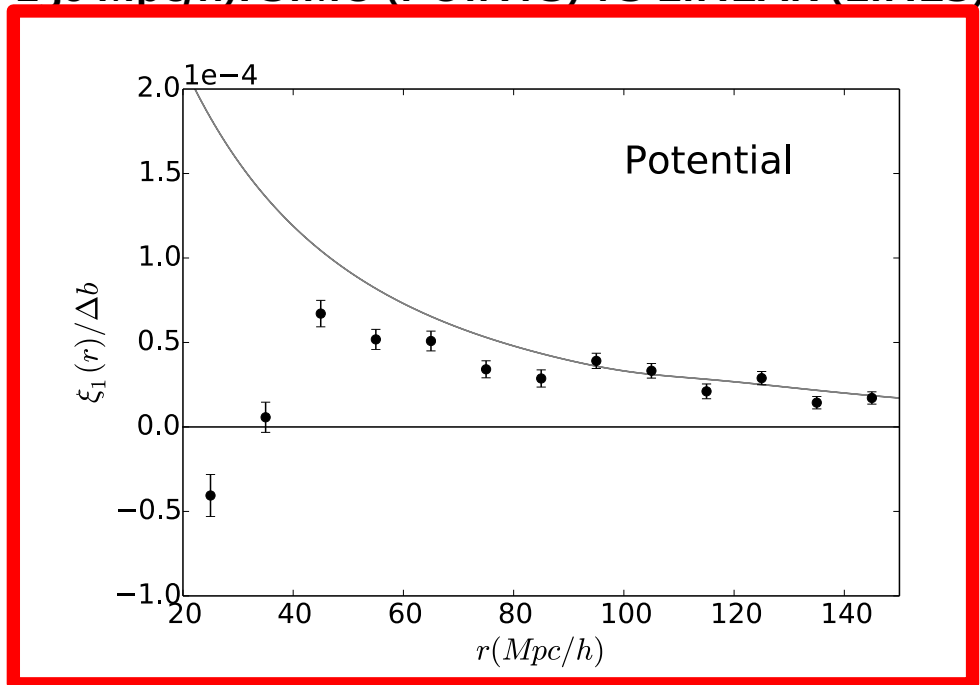
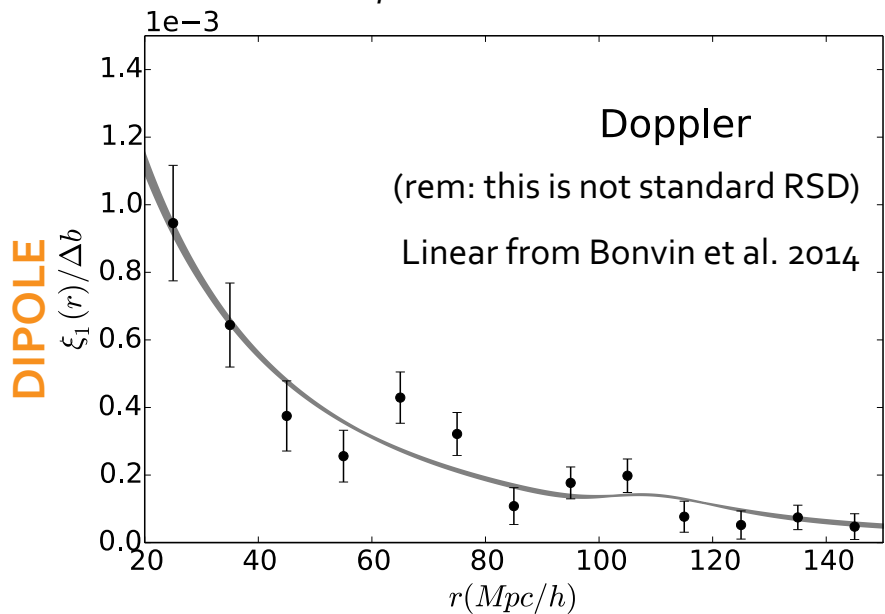
$s \sim 1.1$  for Euclid spectro survey

- Left: Effect of lensing on RSD (linear theory)
- Right: Growth rate inference at  $z=1.8$  ( $b=1$ ,  $s=1.2$ ) for a Euclid-like survey built from RayGal data



# Relativistic RSD: DIPOLE LARGE SCALES (20-150 Mpc/h): SIMU (POINTS) VS LINEAR (LINES)

MW-size halo-Group size halo cross-correlation



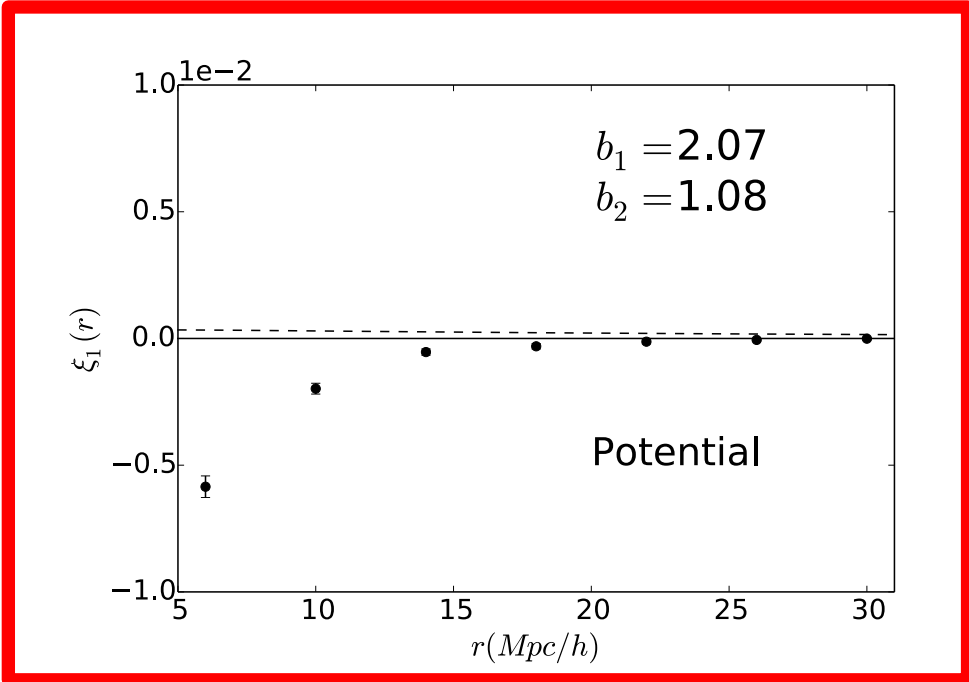
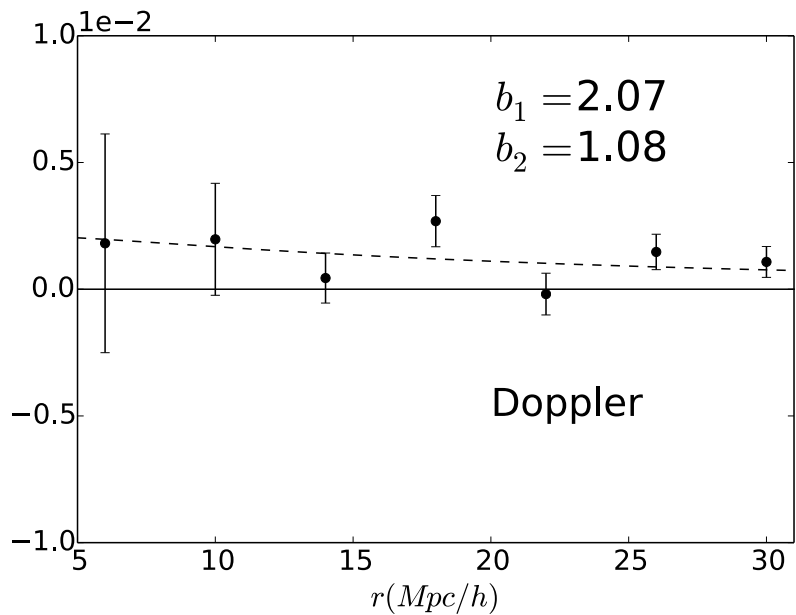
**RADIUS**

Breton et al, 2019  
ADE 2023

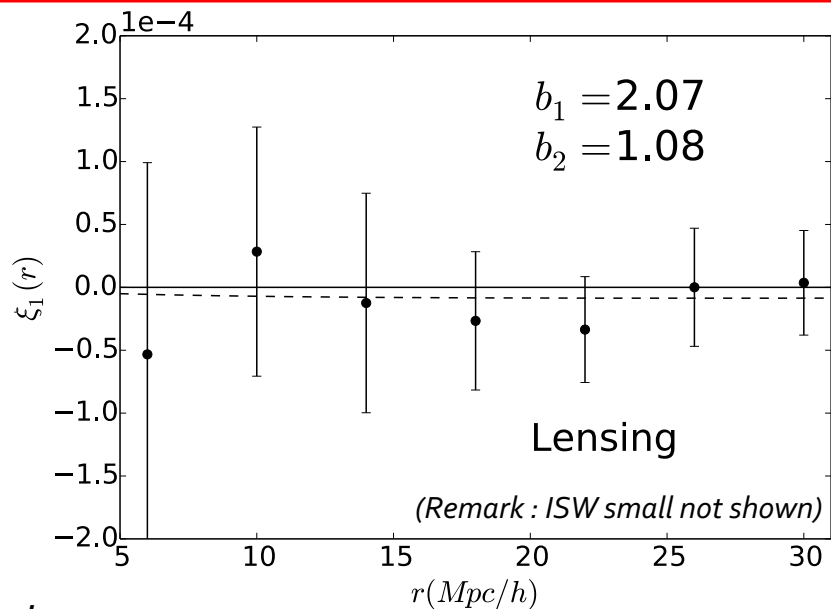
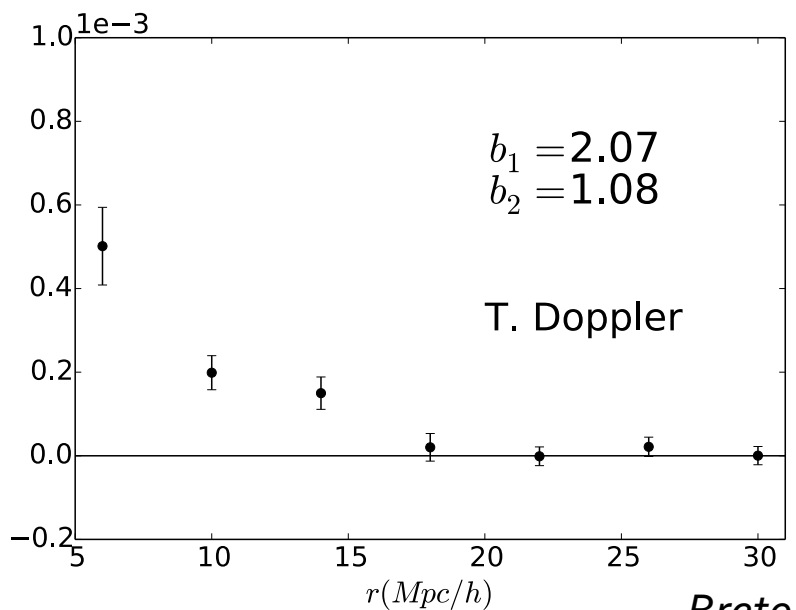
**RADIUS**

# SMALL SCALES (5-30 Mpc/h): SIMU (POINTS) VS LINEAR (DASHED LINES)

DIPOLE



DIPOLE



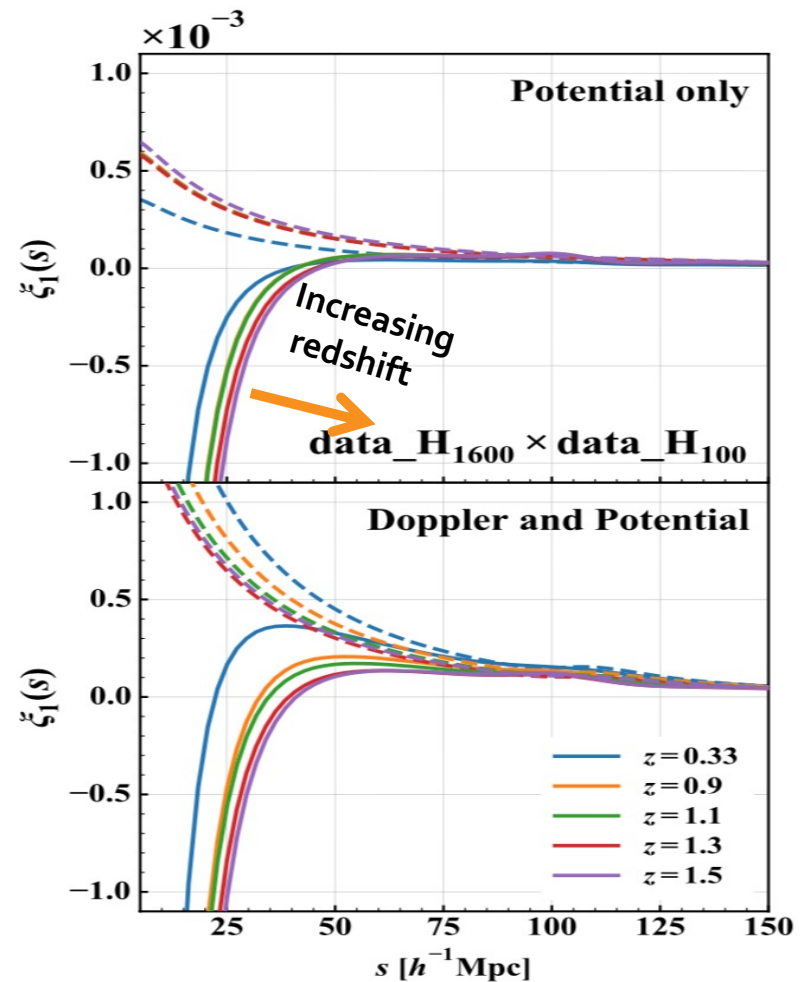
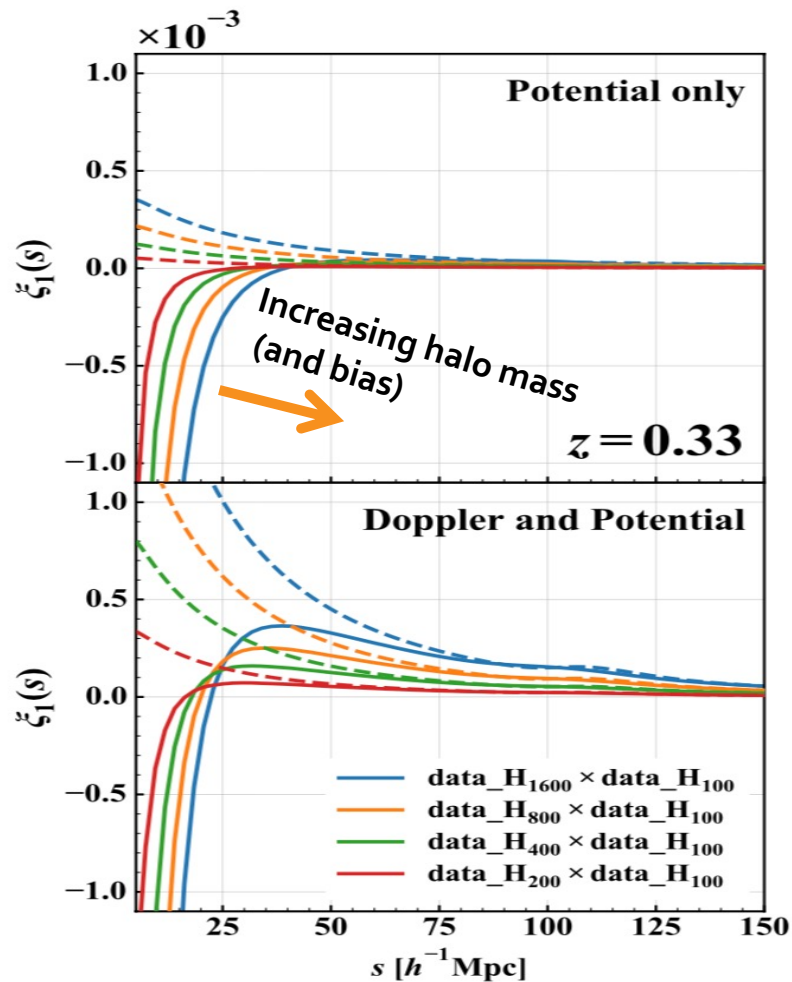
RADIUS

Breton et al, 2019  
ADE 2023

RADIUS

# Non-linear Analytical predictions

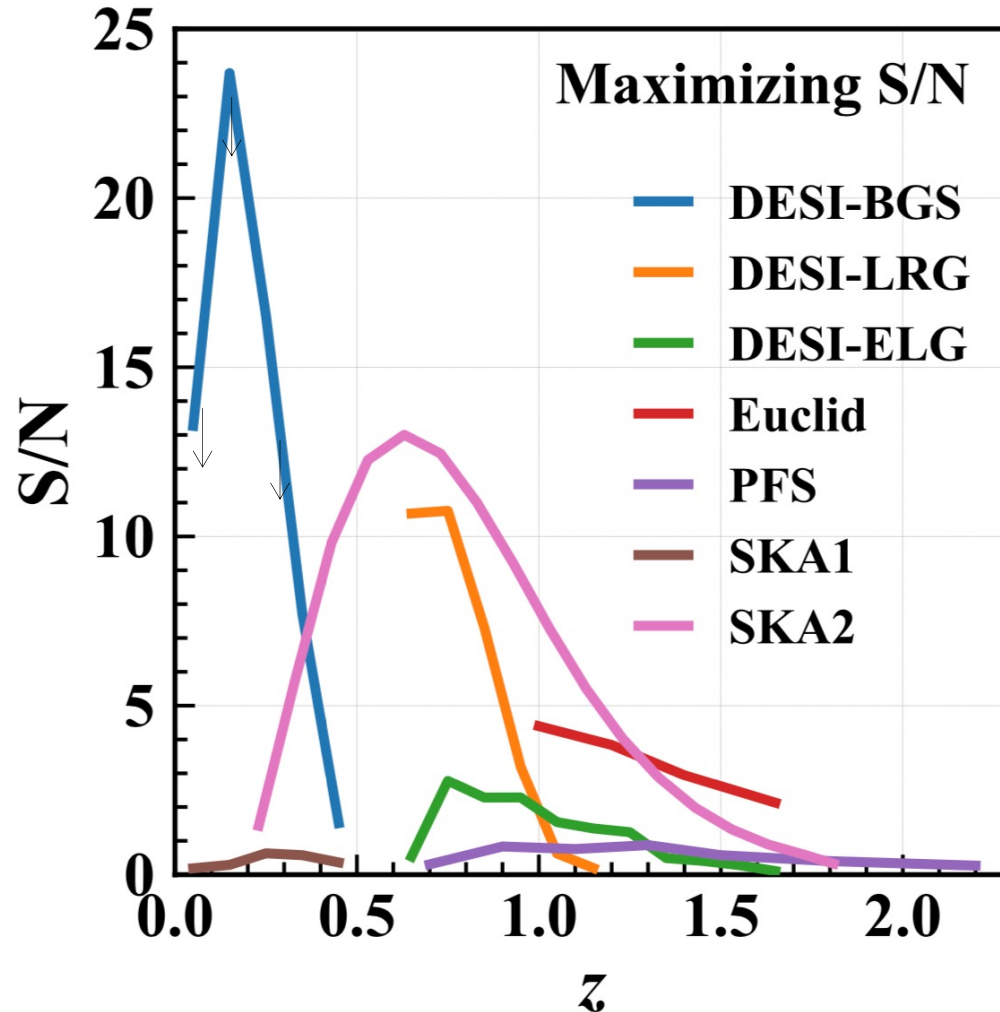
Taruya et al, 2019  
Saga et al, 2020



- Zeldovich prediction+NL halo term (e.g. NFW potential) => good prediction below 50 Mpc/h (unlike linear one)
- Increasing halo mass or redshift=> increase sign flip scale

# Detectability

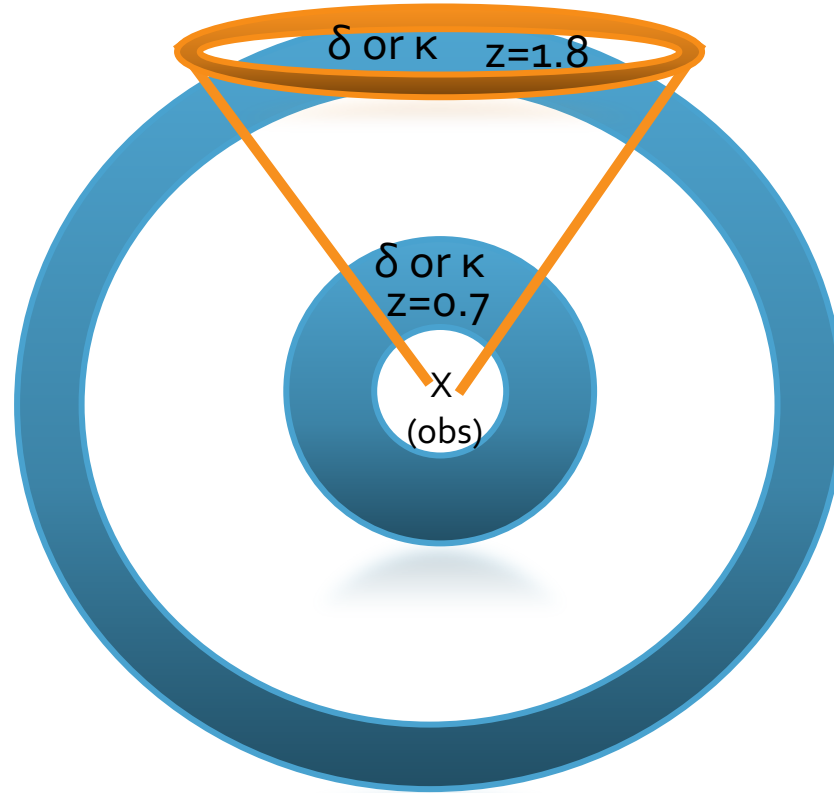
Saga et al., 2022



- *Caution: DESI-BGS is only an upper bound, you can skip this curve (accurate HOD modeling is required)*
- **S/N > 10 for DESI and SKA.**
- **New probe of the gravitational potential at cosmic scales**
- **New test of the equivalence principle: Saga et al. 2023**

# **Application 2: Relativistic Lensing Matter Clustering**

# GEOMETRY AND QUANTITIES



- Quantities: Observed matter overdensity  $\delta$  and apparent weak-lensing convergence  $\kappa$

- Statistics:  $C_l$   
 $\langle \delta \delta \rangle$  : clustering  
 $\langle \kappa \kappa \rangle$  : weak-lensing  
 $\langle \delta \kappa \rangle$  : galaxy-galaxy lensing

- Geometry: centered on observer, 2500 deg<sup>2</sup> light-cone : shells at  $z=0.7 \pm 0.2$  and  $z=1.8 \pm 0.1$

- DEFINITION:

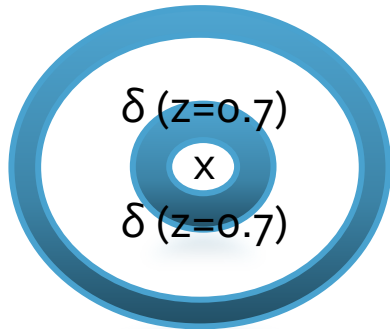
**NON-TRIVIAL RELATIVISTIC EFFECTS= DEVIATION FROM COMOVING MATTER OVERDENSITY AND BORN CONVERGENCE** (ie. mostly magnification bias MB and RSD)

- THEORY (for comparison): CLASS

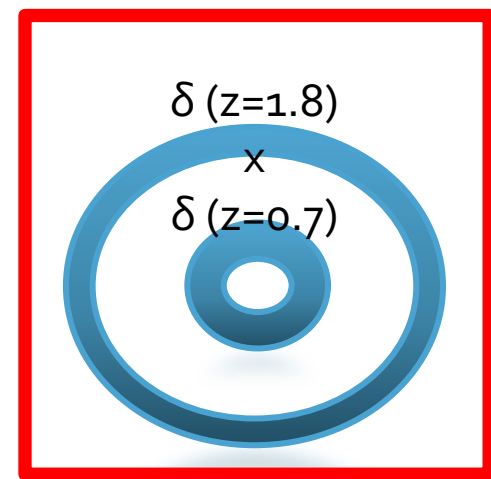
-> Linear with all relativistic effects

+ Non-linear prescription= halofit, linear mapping, RSD no Finger of God, Born lensing, etc.

## Clustering (density x density)

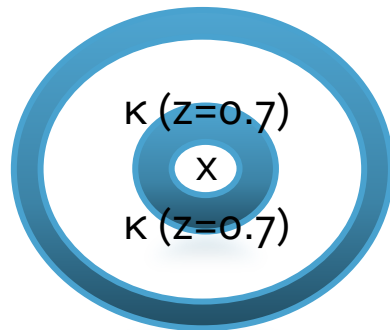


$$\delta(z=1.8) \times \delta(z=1.8)$$

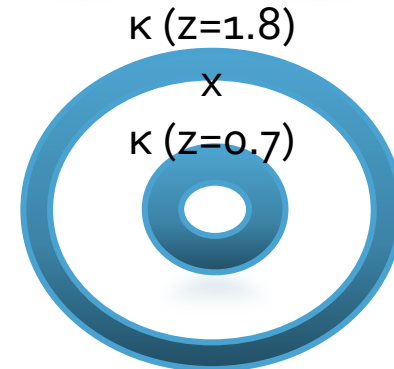


Purely  
relativistic  
often  
omitted

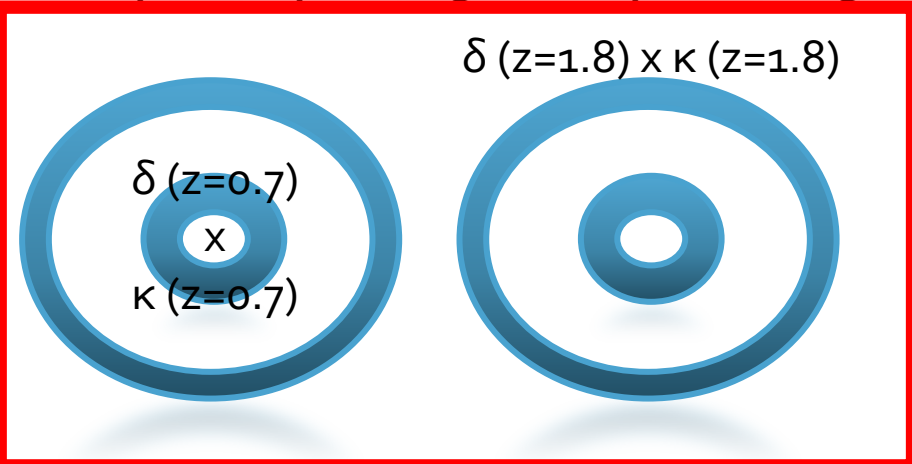
## Weak-lensing (convergence x convergence)



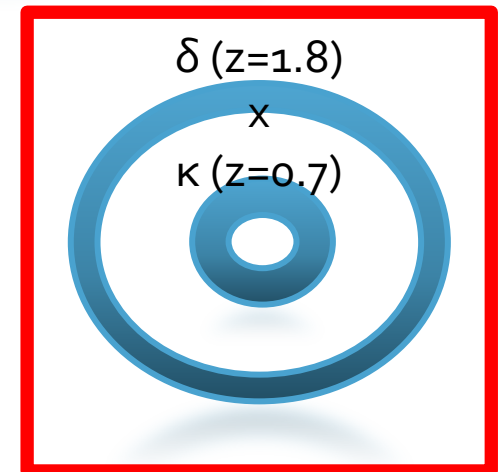
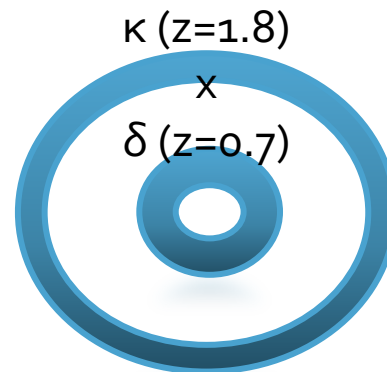
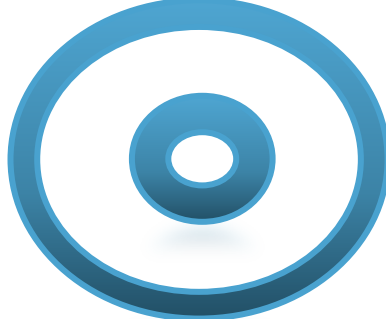
$$\kappa(z=1.8) \times \kappa(z=1.8)$$



## Galaxy-Galaxy lensing (density x convergence)



$$\delta(z=1.8) \times \kappa(z=1.8)$$

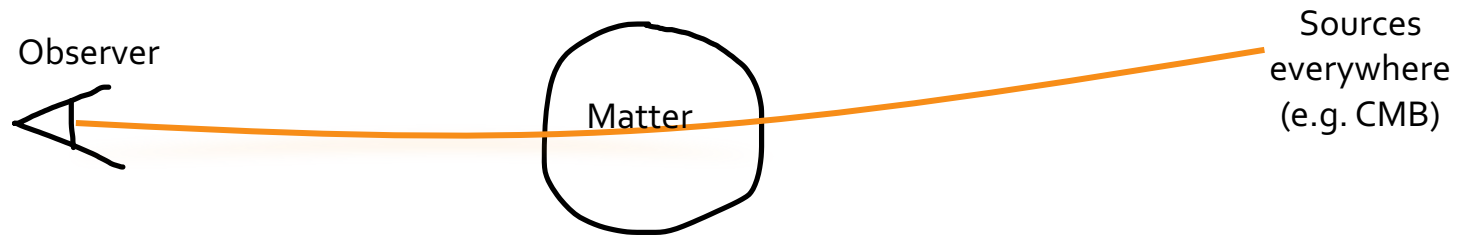




# IMPORTANT TAKE HOME MESSAGE

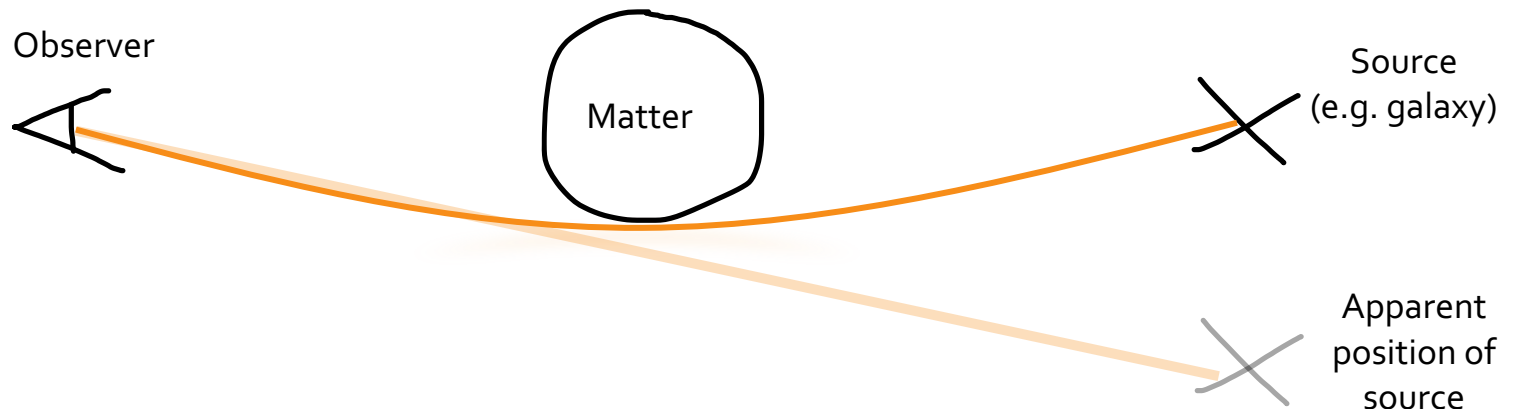
## SOURCE AVERAGING VERY DIFFERENT FROM ANGULAR AVERAGING

### ANGULAR AVERAGING:



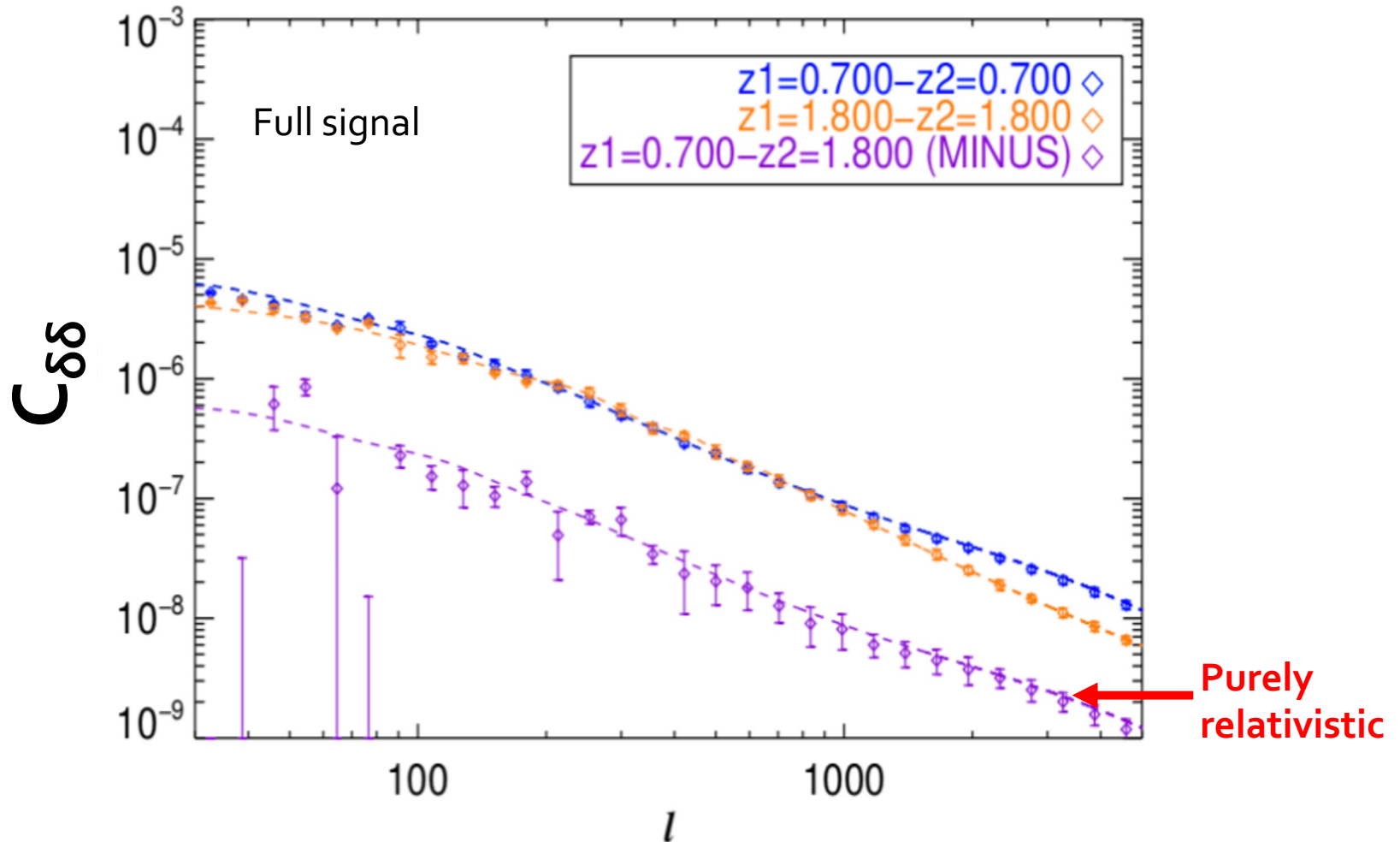
Usual approach: Born, post-Born, multiple-lens => minor correction

### SOURCE AVERAGING:



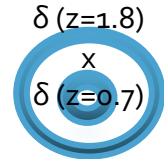
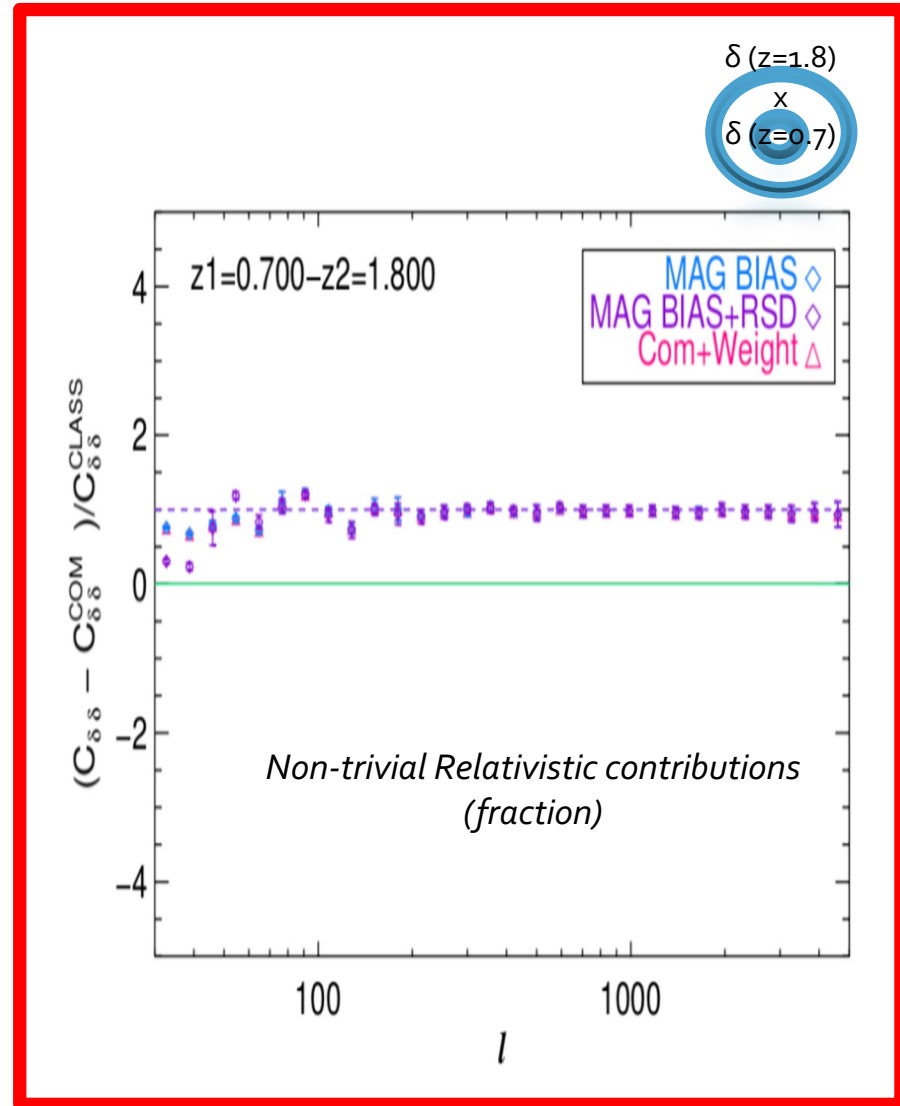
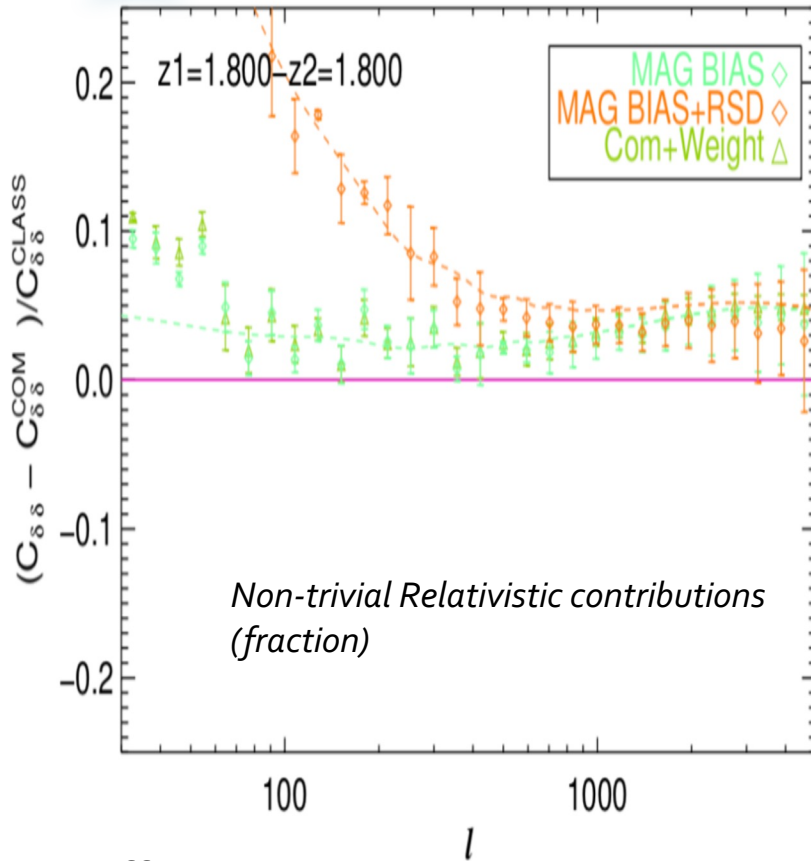
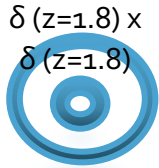
In Magrathea, geodesics finders => light crosses exactly the source and the observer  
**strong effects (magnification bias etc), not only a post-born correction**

# Matter angular (cross-)power spectra with relativistic contributions



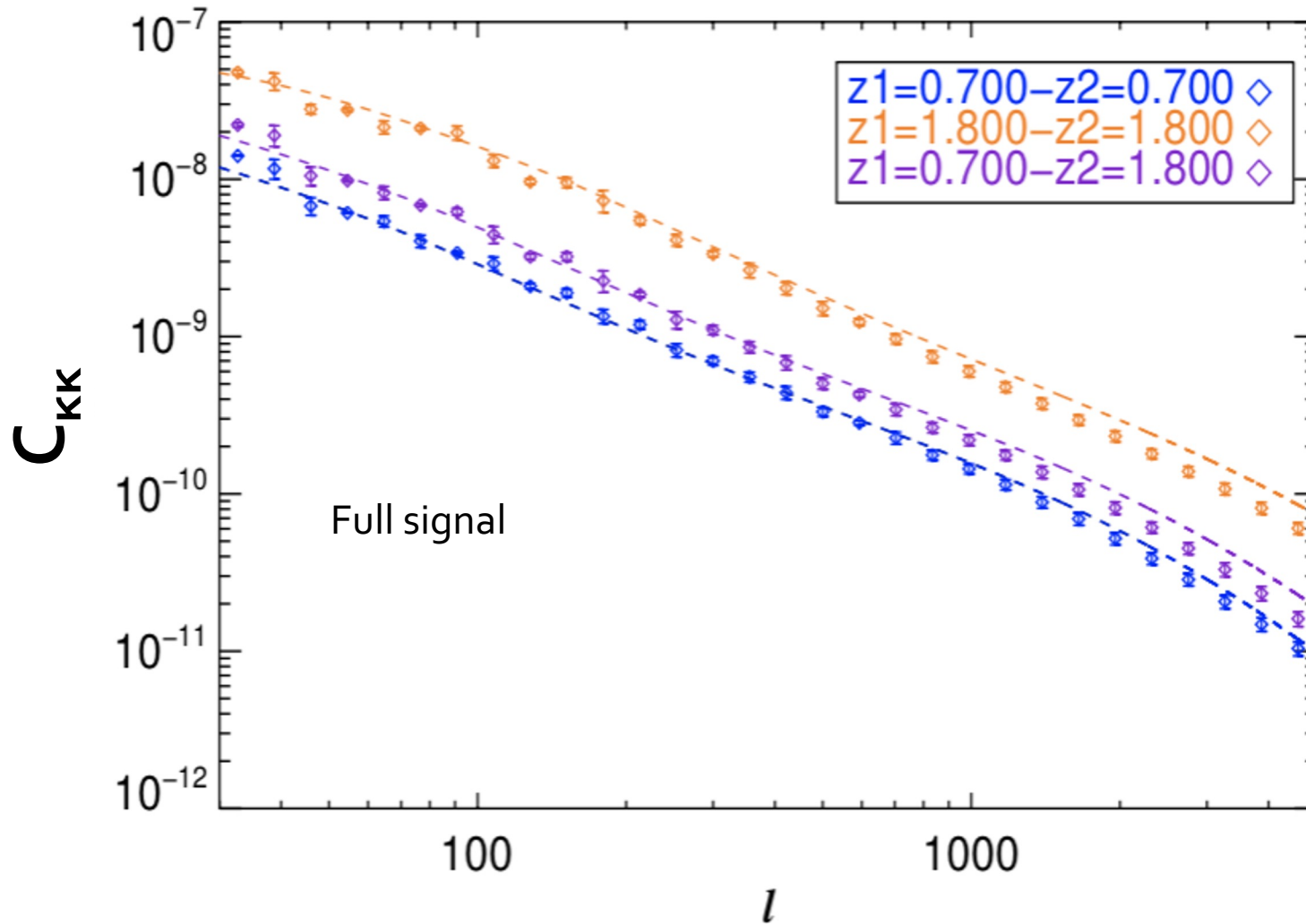
- **Good agreement with Class** (dashed lines)
- 3D matter  $P(k)$  calibrated on RayGal (otherwise halofit errors induce  $\sim 5\%$  errors)

# Matter angular (cross-)power spectra: magnification bias and RSD effect



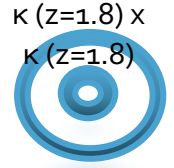
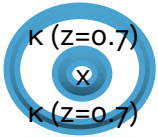
- RSD effect at large scale
- Magnification Bias (MB)  $\Rightarrow \delta_{\text{obs}} \approx \delta_{\text{com}} - 2\kappa_{\text{Born}}$  (for flux-limited survey  $-2\kappa \rightarrow (5s-2)\kappa$ )
- MB effect at every scale (+ dominate for distance shell)
- Not shown here at low redshift **Class doesn't capture Fingers-of-god effect**

# Convergence angular (cross-)power spectra spectra with relativistic contributions

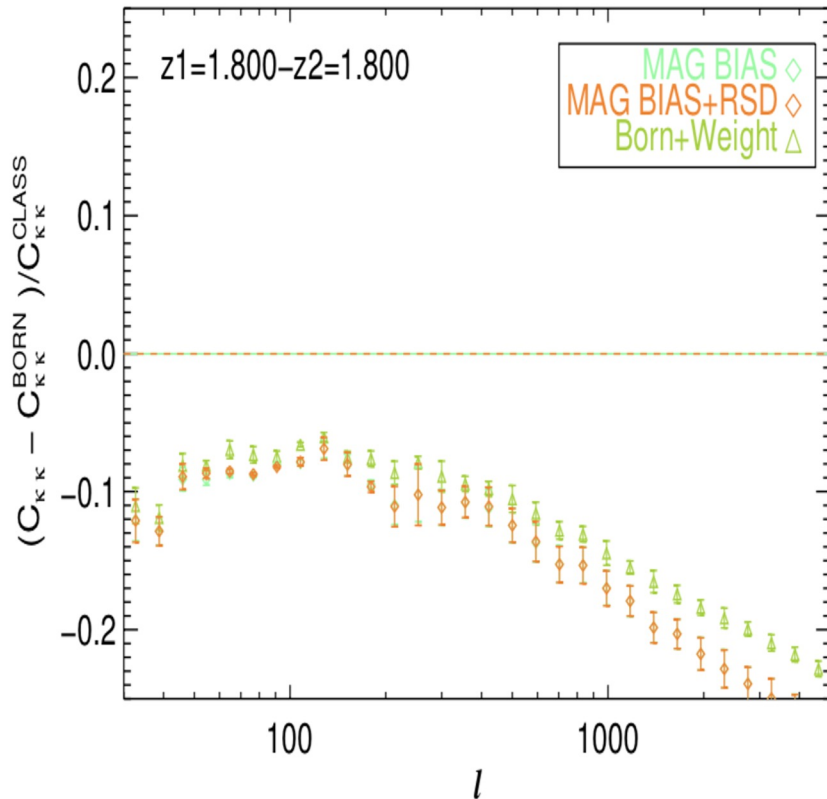


- Reasonable agreement with class at large scales

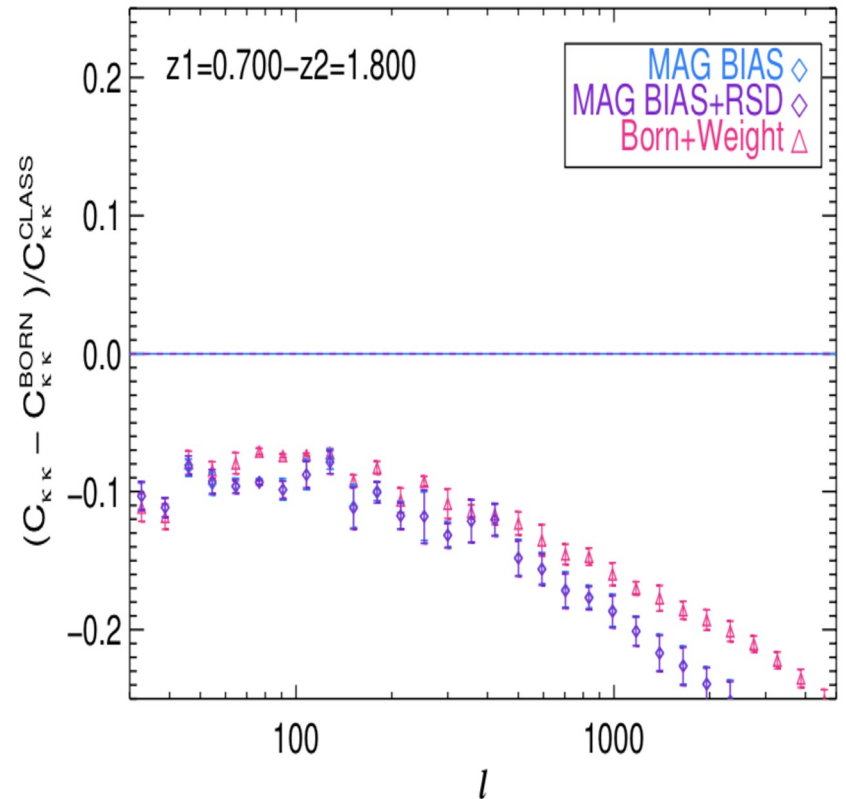
# Convergence angular (cross-)power spectra: magnification bias and RSD effect



Non-trivial Relativistic contributions (fraction)

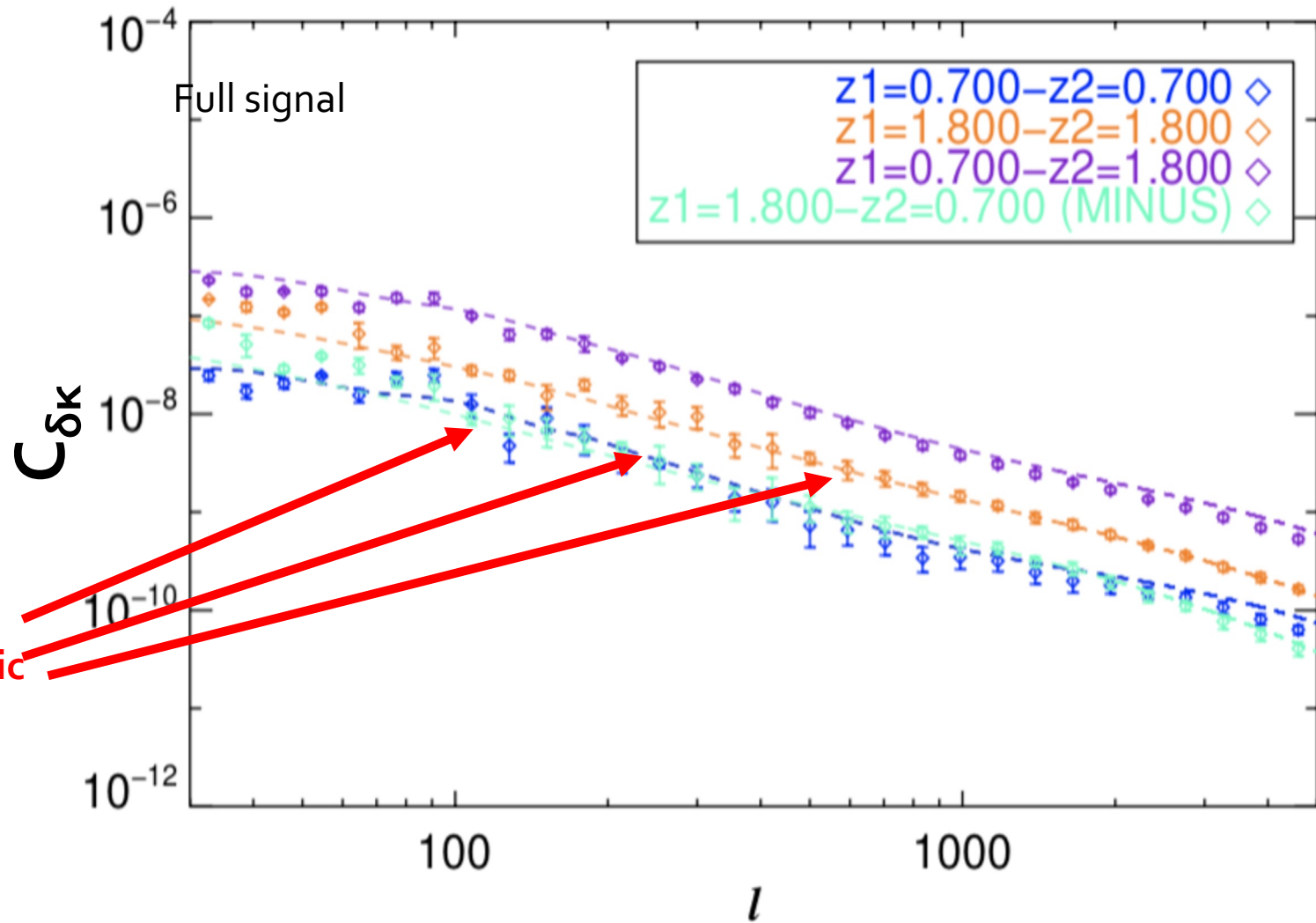


Non-trivial Relativistic contributions (fraction)



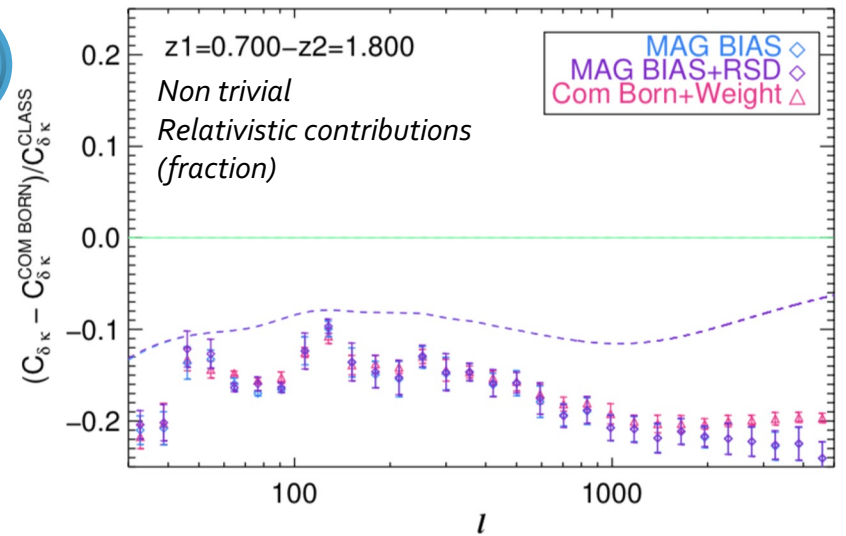
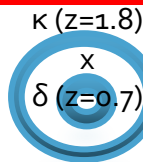
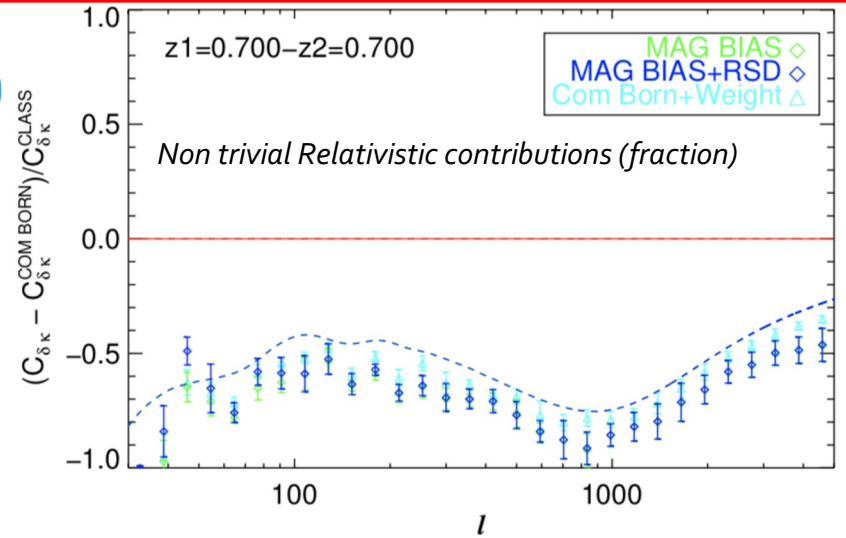
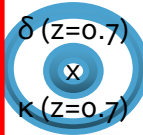
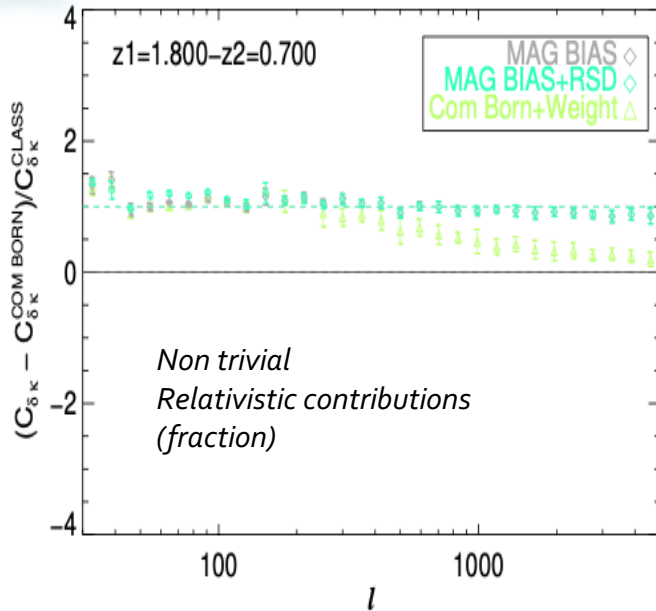
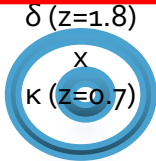
- **Cannot compute the effect of magnification bias on the convergence with Class (as it is related to the bispectrum)**
- $\kappa_{\text{obs}} \approx \kappa_{\text{Born}} (1 - 2 \kappa_{\text{Born}})$
- MB effect on convergence Cl means that **shear and convergence power spectra differ!**

# Density-convergence (cross-)power spectra with relativistic contributions



- Good agreement with Class

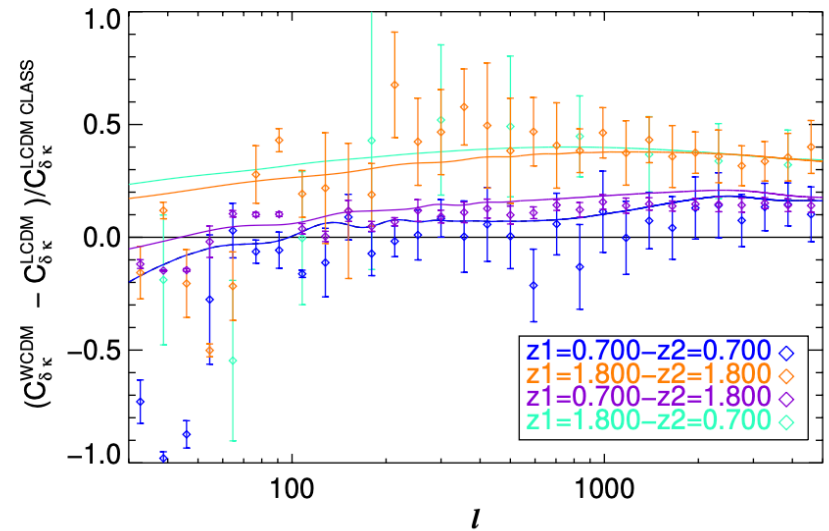
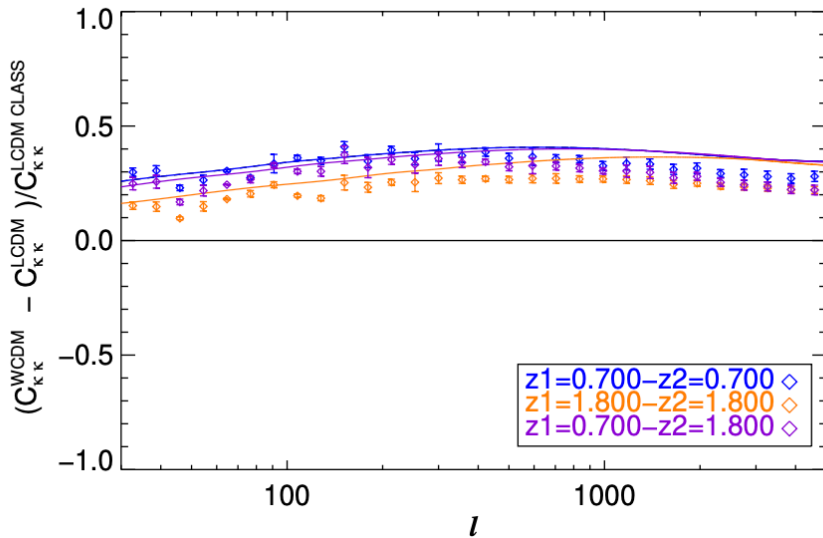
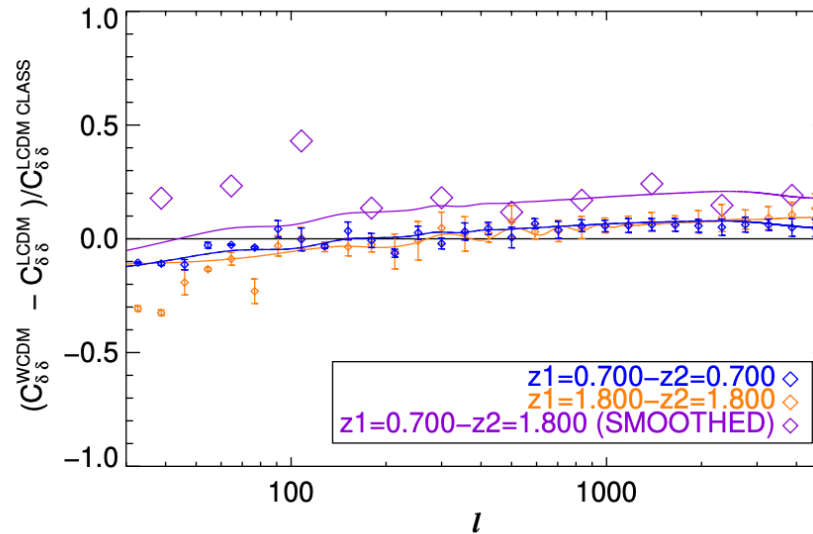
# Density-convergence (cross-)power spectra: magnification bias and RSD effect



- MB  $\Rightarrow \delta_{\text{obs}} \approx \delta_{\text{com}} - 2\kappa_{\text{Born}}$  &  $\kappa_{\text{obs}} \approx \kappa_{\text{Born}} (1 - 2\kappa_{\text{Born}})$ .
- MB effect in Class is included but only for the density not the convergence  $\Rightarrow$  deviations
- Interesting non-trivial configurations : including some with the convergence at lower or equal redshift than the density shell  $\Rightarrow$  the cosmological signal is not negligible



# Cosmological dependance (relative difference between $\Lambda$ CDM ( $w=-1.2$ ) & LCDM )



**=> RELATIVISTIC LENSING-MATTER CLUSTERING IS A POWERFUL COSMOLOGICAL PROBE**

# CONCLUSION

- **Goal:** Understand the **connection from the “real universe” to the “apparent universe”** to find new probes of DE=> need to model **all weak-field relativistic effect** (i.e. like for CMB but in non-linear regime)

- New **PUBLIC DATA**

- Don't hesitate to download the **RAYGALGROUPSIMS** (or in short **RayGal**) relativistic halo catalogues and maps to make your own test (traditional snapshot data are also available)

- Very simple files with angular position, redshift and distortion matrix
- Magrathea geodesics-finder => SOURCE AVERAGING available

- **Relativistic effects in RSD**

- For the 1<sup>st</sup> time **all the (kinematical) DIPOLE effects are modeled accurately in weak field from lin. to deep NL scales**
- The most important contribution after wide-angle RSD is **the GRAVITATIONAL POTENTIAL** at low redshift

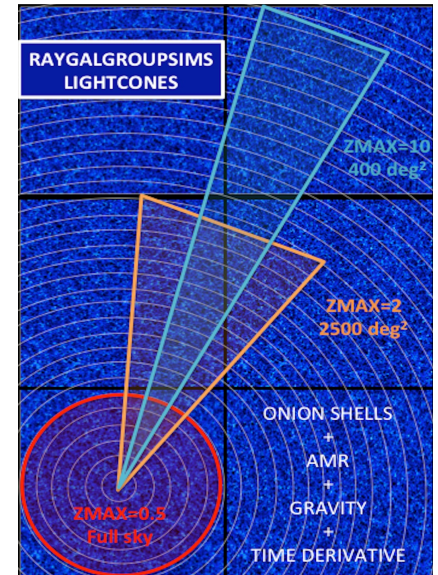
- Detectability of grav. Pot. in DESI, SKA and possibly Euclid (Saga et al, 2022).

- **Relativistic effects and weak-lensing (3x2pts):**

- good agreement with CLASS at quasi-linear scales
- **subtle effects in NL regime** (Finger-of-gods effect in angular correlation, **MAGNIFICATION BIAS** on the convergence power spectra, **non-trivial configuration in GGL**)=> powerful cosmo probe

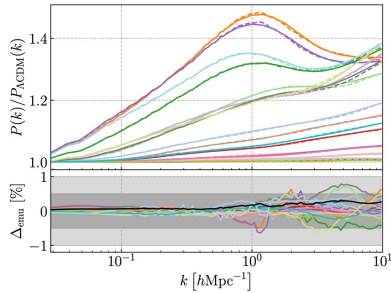
- **Very general approach, many extensions:**

- Many Other possible applications (theory/simulation/observation) : doppler lensing, ISW, fluctuations of cosmic distances, cluster studies (WL, RSD, gravitational redshift), etc...



# ONGOING WORK

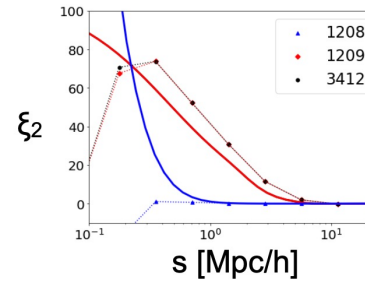
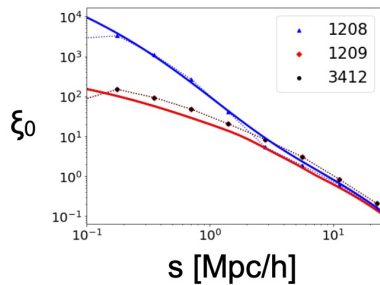
- **OTHER MODELS: e-Mantis emulator** for the 3D matter power-spectrum in **F(R) GRAVITY** by I.SAEZ-CASARES **Saez-Casares et al. 2023** <https://pypi.org/project/emantis/> (+application to Euclid forecast on going)



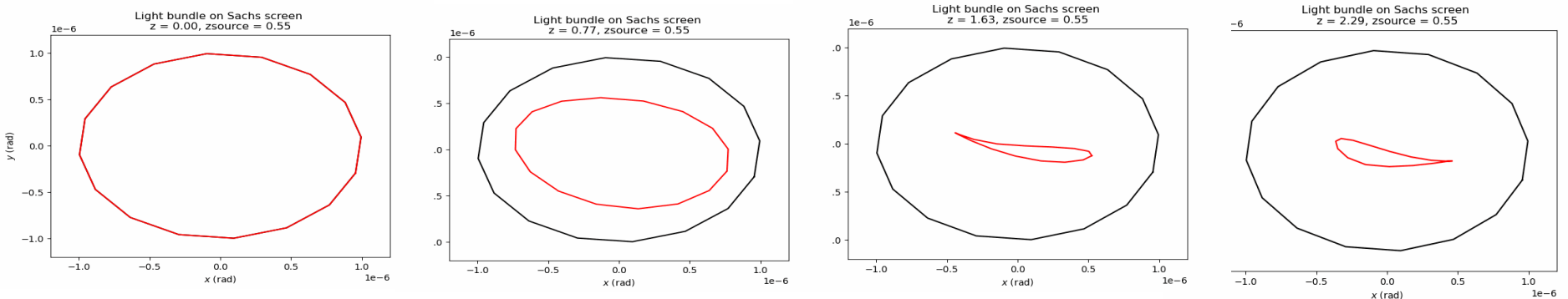
pypi v1.0.4 DOI 10.5281/zenodo.7738362

**e-MANTIS: Emulator for Multiple observable ANalysis in extended cosmological TheorieS**

- **OTHER PROBES: relativistic effect in the halo-matter cross-correlation (S. SAGA, preliminary results)**



- **TOWARD STRONG LENSING: Strong-lensing weak-lensing connexion (M-A BRETON: New simulation 220 Mpc/h 2048<sup>3</sup> particles + narrow light-cone).** Preliminary results: evolution of a light bundle.



- **ProGraceRay ANR project (2024-2028): the suite of all of this! Ad: we are looking for a postdoc about MODIFIED GRAVITY & N-body simulations** (see inspirehep or jobregister if interested)

# BACKUP SLIDES

Philosophy exam for everybody in the room

**What is the link between**

**THE « REAL » UNIVERSE**

**and**

*THE OBSERVED UNIVERSE*

?

# PROBLEMS

- **Redshift perturbations:** modification of the apparent redshift (i.e. inferred distance) of structures
- **Weak-lensing:** modification of the apparent angular position, shapes, luminosities of structures

=> **The cosmological signal is blurred**

## BUT

**Redshift perturbations**-> information about velocity fields (and more) at source location

**Weak Lensing** -> information about potentials along the line-of-sight

⇒ **NEW COSMOLOGICAL INFORMATIONS FROM WEAK LENSING (WL) AND RELATIVISTIC REDSHIFT SPACE DISTORTIONS (RSD)**

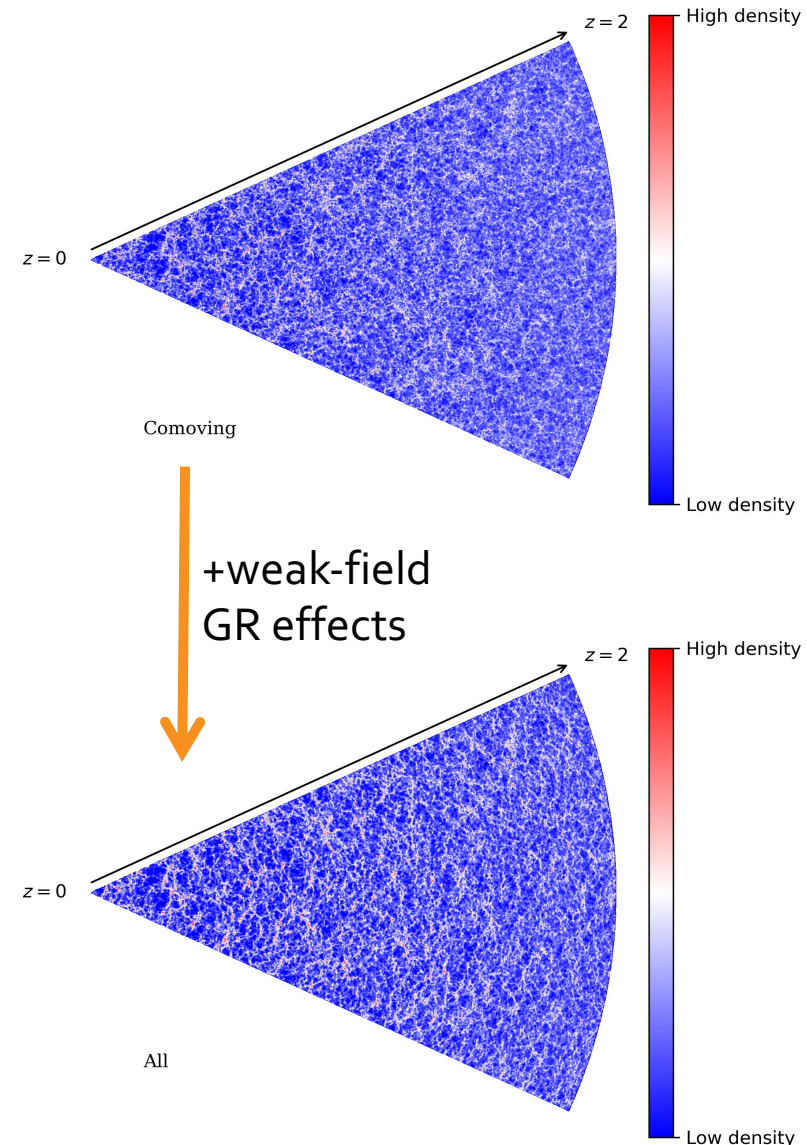


# RayGal simulation suite with General Relativistic Ray-Tracing

Breton et al. 2019;  
Rasera et al. 2022

*Weak-field GR approach from linear to non-linear scales...*

- Large and well resolved HPC N-body simulations ( $4096^3$  part.  $L=2.625$  Gpc/h)
- Standard cosmology ( $w=-1$ ) + **alternative dark energy model** ( $w=-1.2$ )
- **Ray-tracing** including all **general relativistic effects** in the weak field regime at high-resolution
- **Billion light-rays** launched
- For the first time, **identification of light rays going exactly from the source to the observer.**
- Unique **halos catalogues** including **beyond state-of-the-art weak-lensing and redshift space distortions** (Doppler effect, gravitational redshift, weak-lensing, ISW).



# Apparent distribution of sources:

## redshift space distortions with the relativistic terms

- APPARENT POSITION SOURCE: we have access to direction  $\vec{\beta}$  and redshift  $z_O$
- POSITION INTERPRETED ASSUMING HOMOGENEOUS FLRW (ex:  $dr \approx c dz/H$  if no lens)  
REDSHIFT AND ANGLE MODIFICATIONS

$$z_O = \frac{a_O}{a_S} \left( 1 + \frac{\vec{v}_S \cdot \vec{n}}{c} - \frac{\phi_S - \phi_O}{c^2} + \frac{v_S^2}{2c^2} - 2 \int_{\eta_S}^{\eta_O} \frac{\dot{\phi}}{c^2} d\eta \right) - 1$$

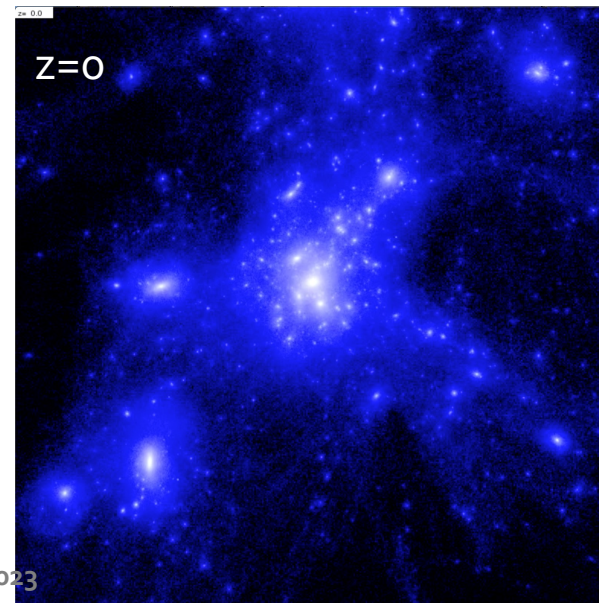
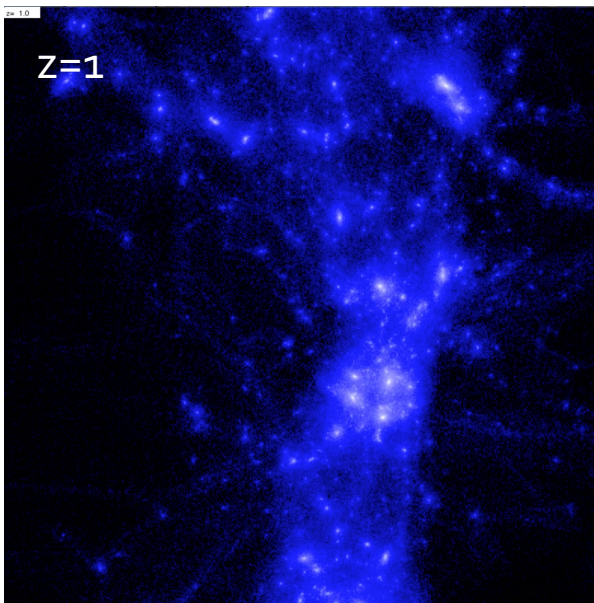
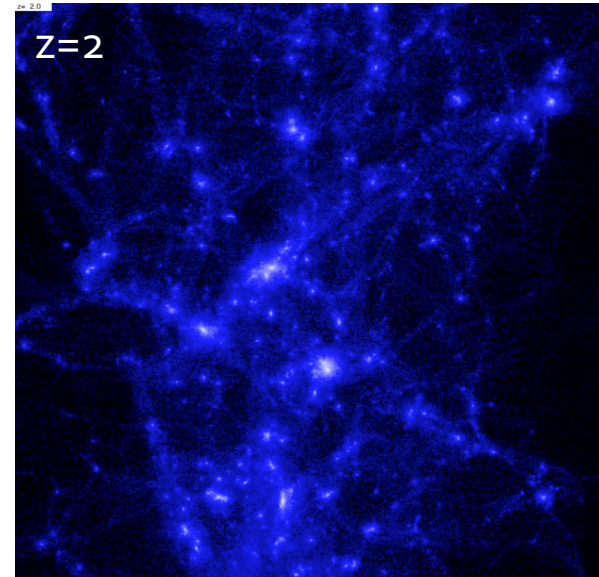
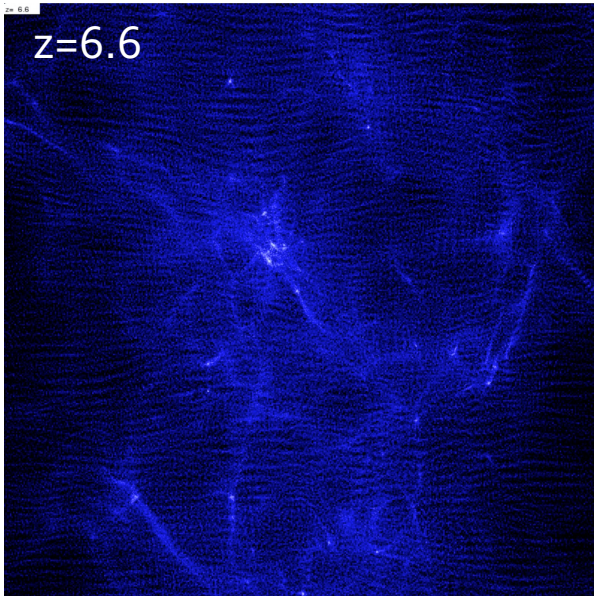
HOMOGENEOUS
DOPPLER
GRAVITATIONAL REDSHIFT
TRANSVERSE DOPPLER
ISW-RS  
 $\vec{\beta} = \vec{\theta} - \vec{\alpha}$   
LENSING

- OBSERVED DENSITY IS GIVEN BY (NON-LINEAR MAPPING)

$$(1 + \delta_{obs}) dV_{obs} = (1 + \delta_{real}) dV_{real}$$

A tentative answer with modern physics (i.e. weak field general relativity) and modern methods (i.e. simulations)

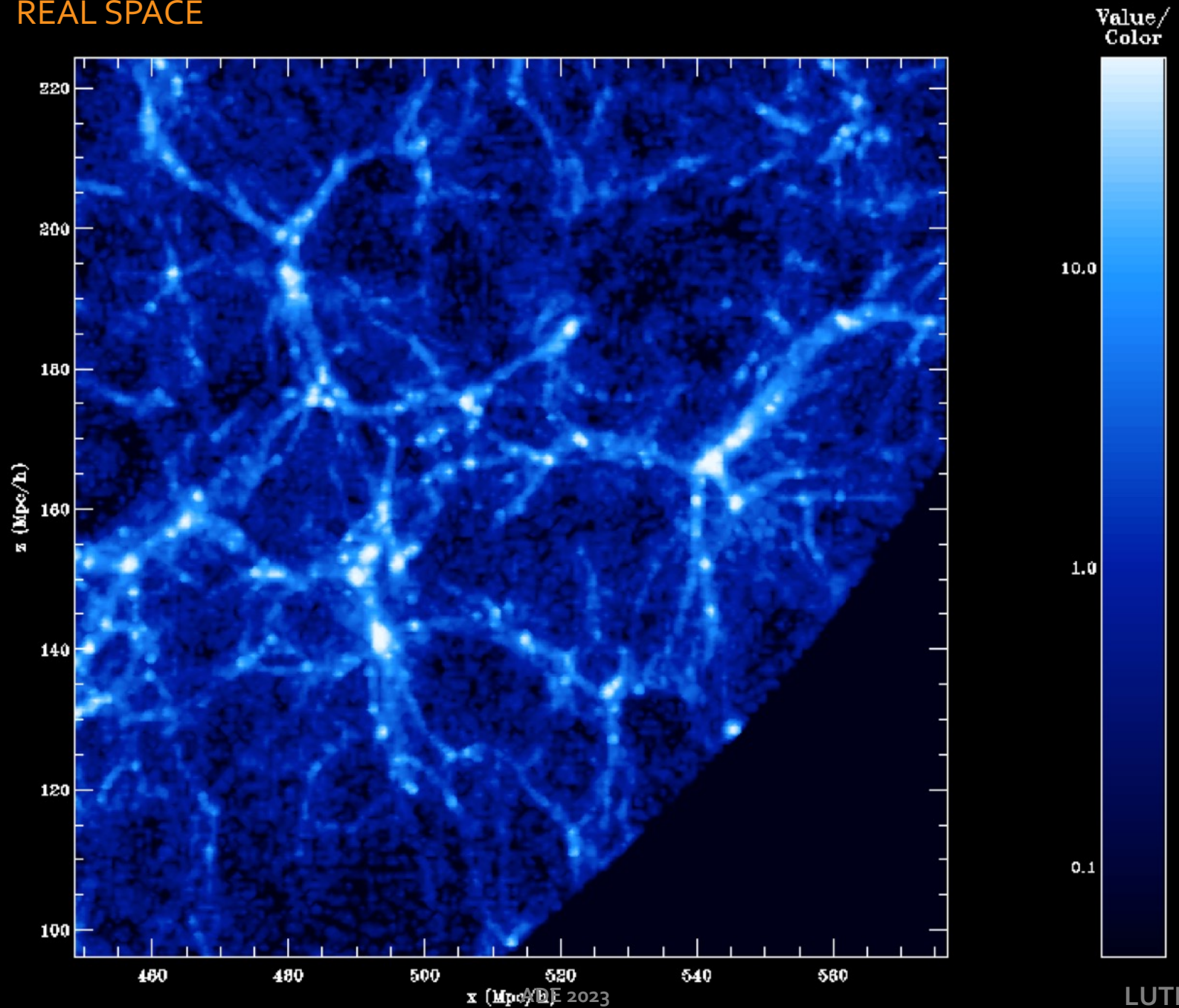
STEP 1 : Build a (virtual) « real » universe by running N-Body sims:  $4096^3$  particles,  $(2.6 \text{ Gpc}/h)^3$ ,  $\Lambda$ CDM & wCDM



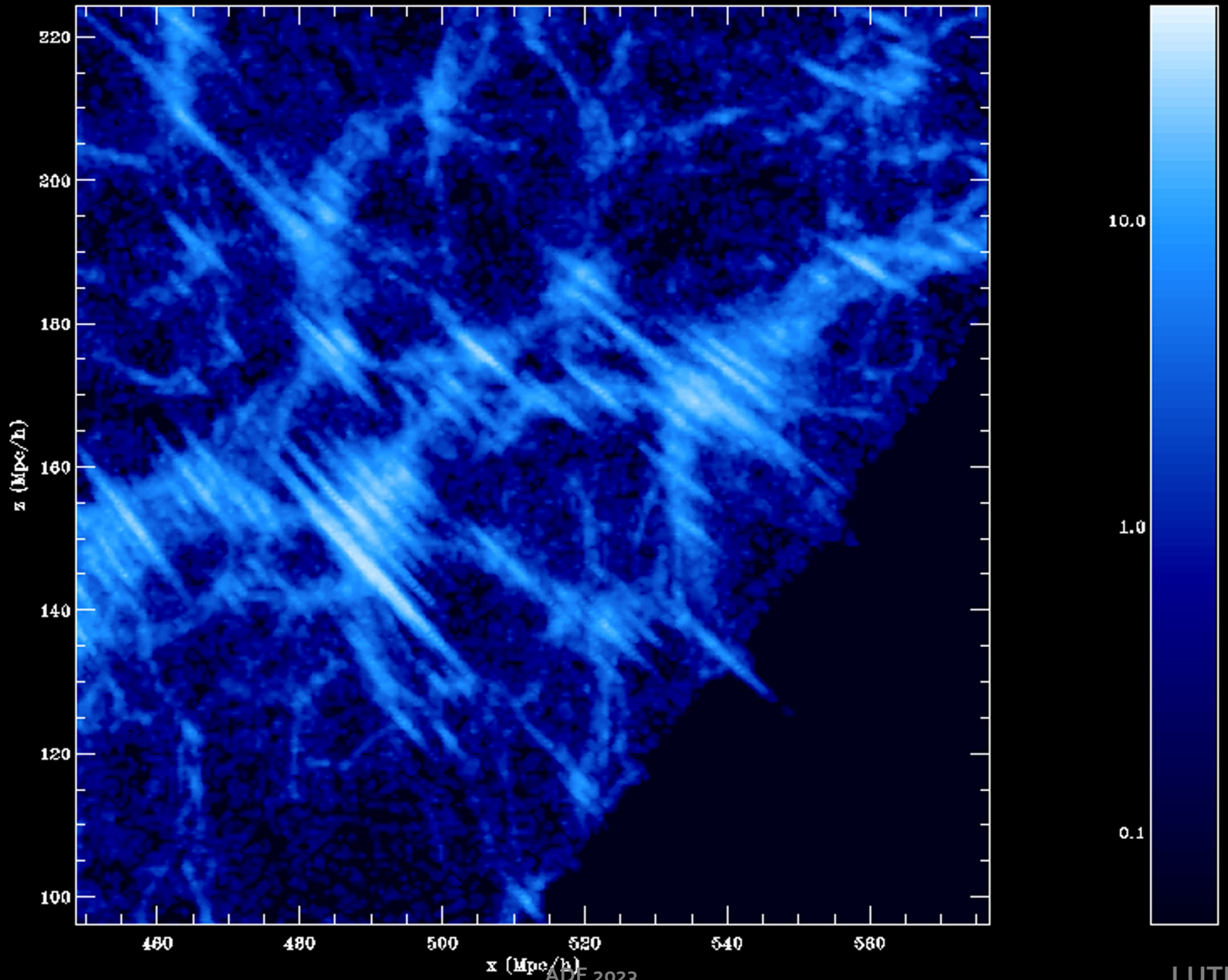
*Illustrative example of the formation of one large halo in a simulation*  
LUTH



# REAL SPACE



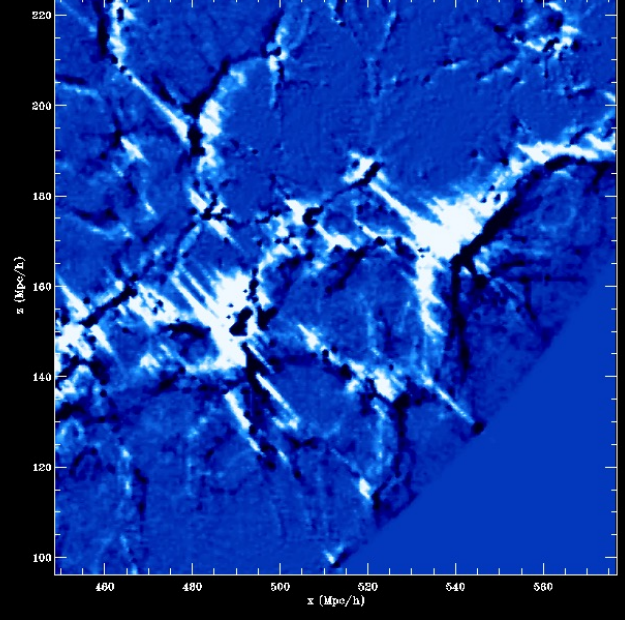
# REDSHIFT SPACE WITH ALL CONTRIBUTIONS (RSD+RELATIVISTIC)





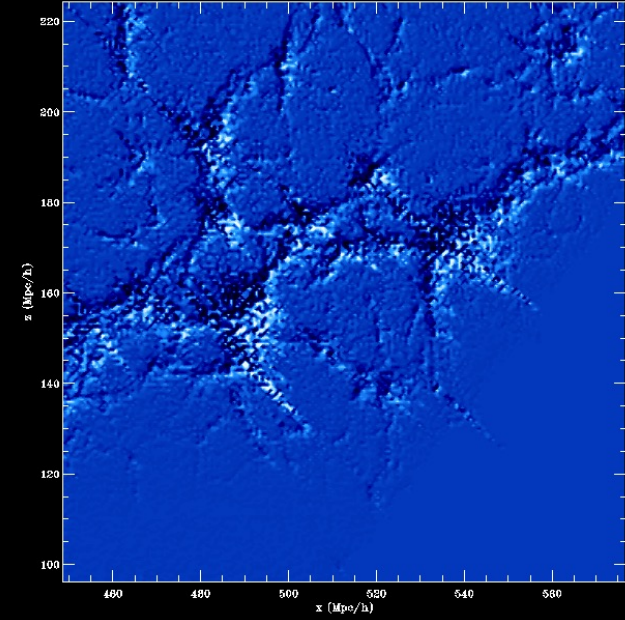
# DOPPLER

Value/Color



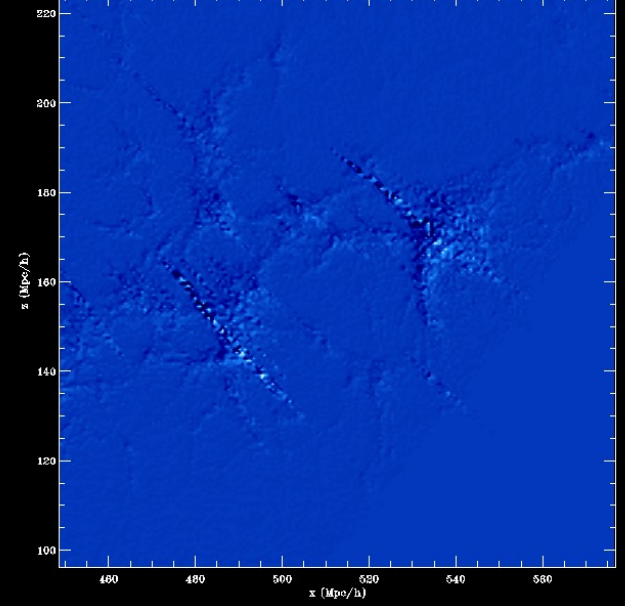
# GRAVITATIONAL REDSHIFT

Value/Color



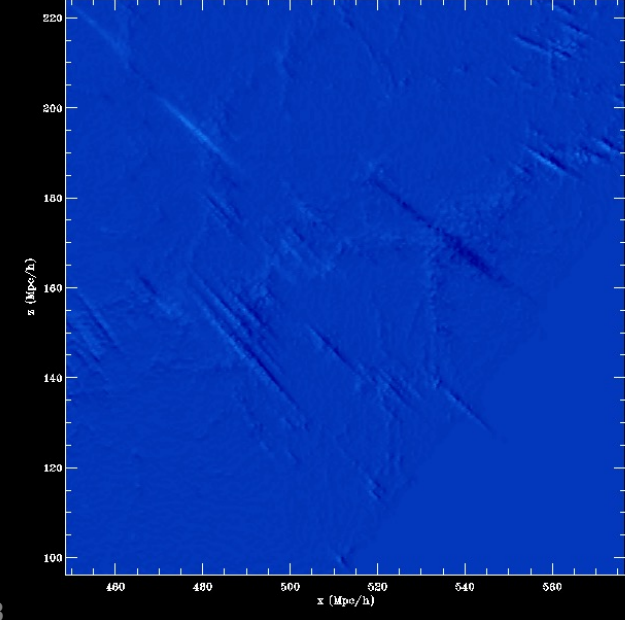
# TRANSVERSE DOPPLER

Value/Color

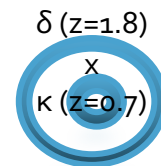
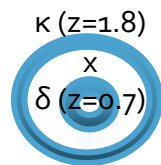
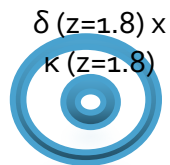
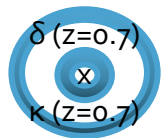
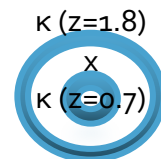
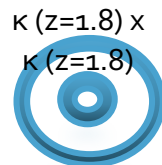
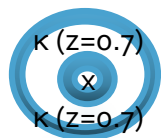
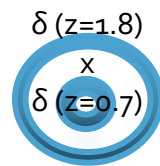
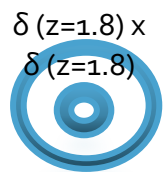
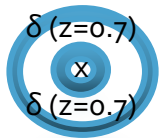


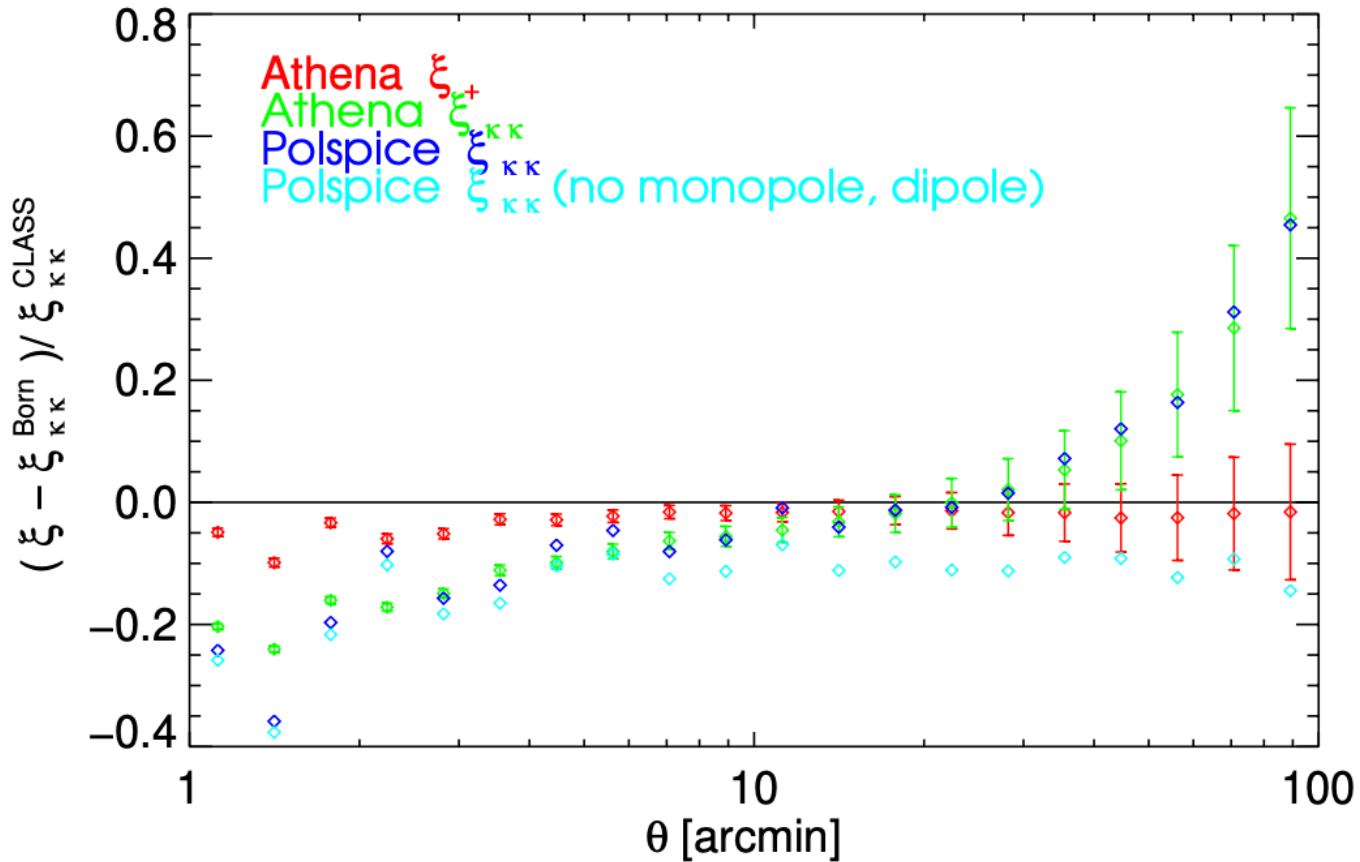
# INTEGRATED TERMS

Value/Color

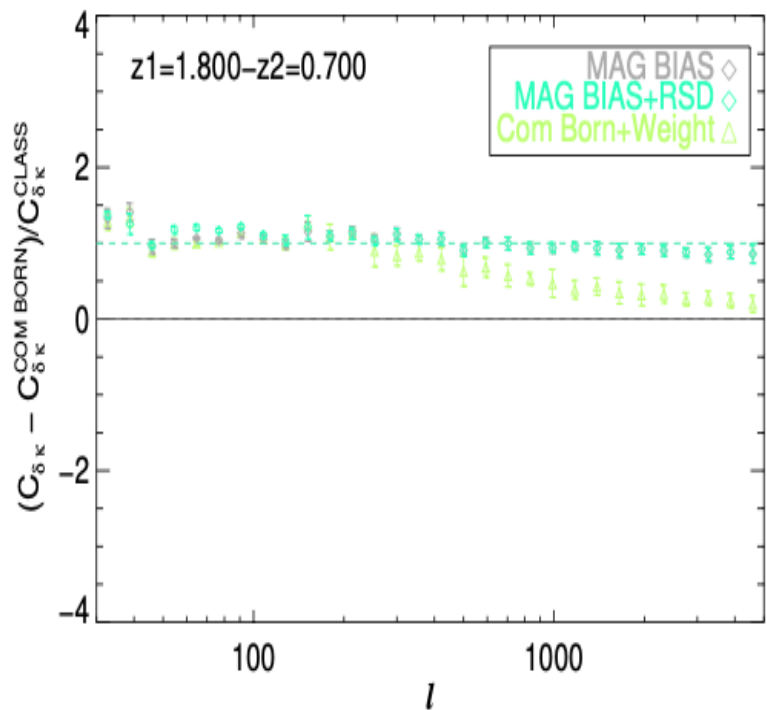




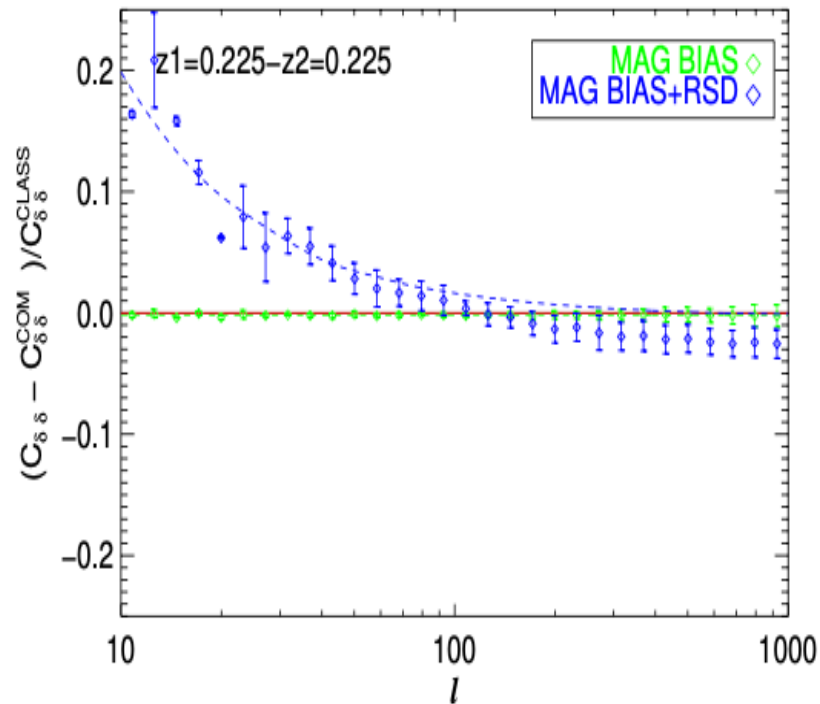




**Fig. B.1.** Relative difference between lensing angular two point correlation function on the source catalogue accounting for the dilution bias and the Born convergence angular two point correlation function. In red and green diamonds we show the measurements of cosmic shear and convergence correlation function using ATHENA, and in blue and light blue we show the results using the same methodology as in Sect. 3, keeping and removing the monopole and dipole, respectively.



**Fig. 13.** Non-trivial reverse configuration of gravitational convergence at  $z = 0.7$  and matter density at  $z = 1.8$ , similar to the bottom-right panel of Fig. 12. MB is in grey, MB+RSDs in cyan, and the  $|\mu_{\text{Born}}|^{-1}$  weight MB estimate in light green. Relativistic effects almost reach 100%, in agreement with CLASS. This configuration turns out to be a sensitive probe of the lensing convergence spectrum.



**Fig. 14.** Relativistic effects at low redshift on matter power spectrum  $C_{\delta_1\delta_2}(\ell, z_1 = 0.225, z_2 = 0.225)$  (symbols are RAYGAL measurements, and lines are CLASS predictions). The MB effect (green) and RSDs(+MB) effect (blue) are shown. The trends are similar compared to higher redshifts, but the RSD effect plays a dominant role and the MB effects are smaller. Finger-of-God effects (ignored by CLASS) are also present.