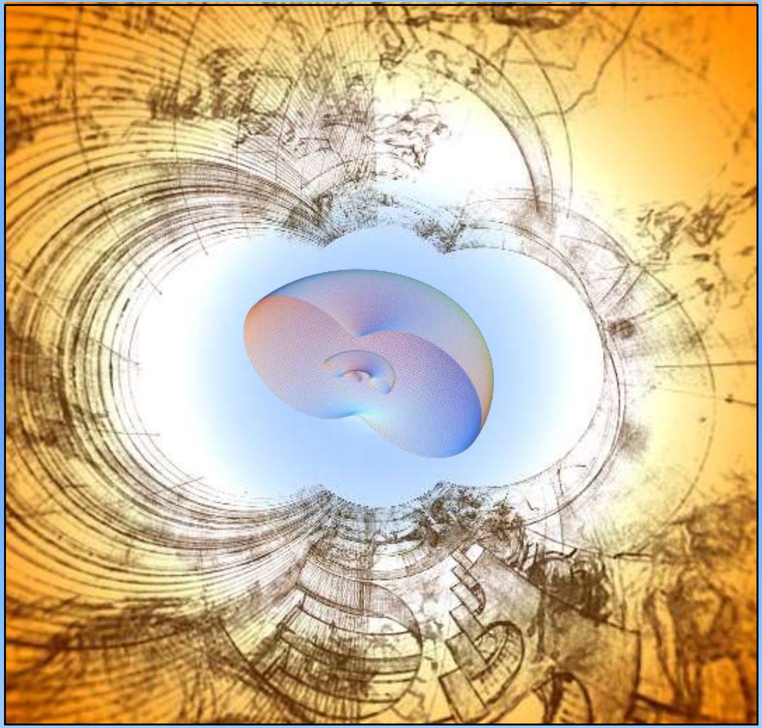
Journées Relativistes de Tours, 2.06.2023

Asymptotically flat scalar hairy black holes and solitons



Eugen Radu

Universidade de Aveiro, Portugal

based on work done (mainly) with

C. Herdeiro

the plan:

Einstein-Maxwell
(electro-vacuum)

$$\mathcal{L} = \frac{R}{16\pi G} - \frac{1}{4}F_{\alpha\beta}F^{\alpha\beta}$$

spin-one

four dimensions asymptotically flat

- fully solvable
- no solitons
- Kerr-Newman black hole
- uniqueness

Einstein-Klein-Gordon
(massive complex scalar field)

the plan:

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(electro-vacuum)

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Einstein-Klein-Gordon
(massive complex scalar field)

$$\mathcal{L} = \frac{R}{16\pi G} - \frac{1}{2} g^{\alpha\beta} \left(\partial_\alpha \Phi^* \partial_\beta \Phi + \partial_\beta \Phi^* \partial_\alpha \Phi \right) - \mu^2 \Phi^* \Phi$$

spin-zero

- no exact solutitons
- solitons: (multi) Boson Stars
- spinning black holes with scalar hair *(single)*
- non-uniqueness
- two balanced black holes

very different picture

(i) the solitons in EKG theory

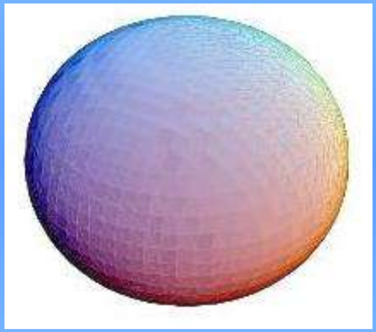
BOSON STARS

$$\mathcal{L} = \frac{R}{16\pi G} - \frac{1}{2} g^{\alpha\beta} \left(\partial_\alpha \Phi^* \partial_\beta \Phi + \partial_\beta \Phi^* \partial_\alpha \Phi \right) - \mu^2 \Phi^* \Phi$$

simplest solutions:

- static, spherically symmetric Boson Stars

  (+ their rotating generalizations)
- *"macroscopic quantum states"*
- *a subject of constant interest*



Kaup;Bonazolla-Rufinni.

single object

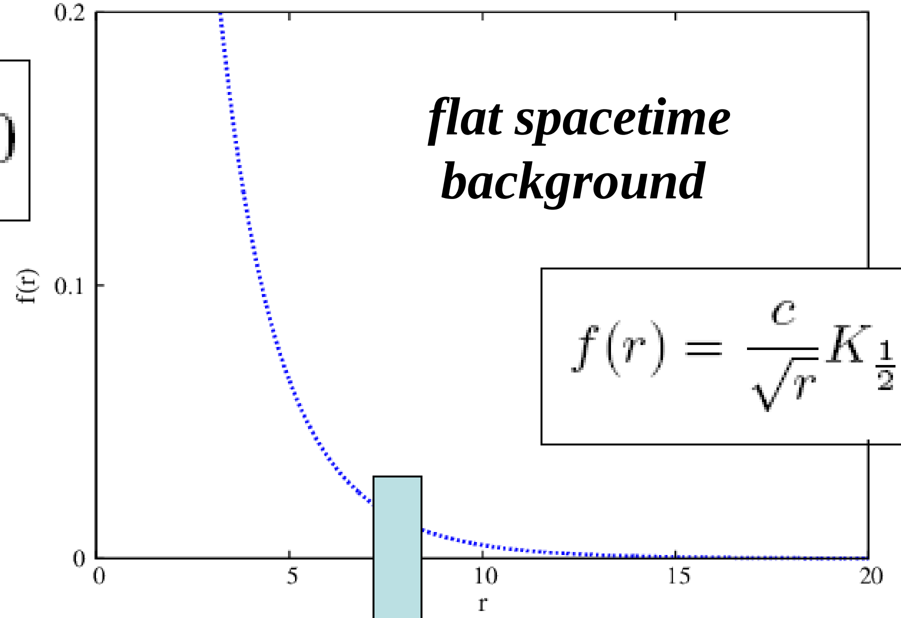
heuristic construction:

gravitational desingularization mechanism

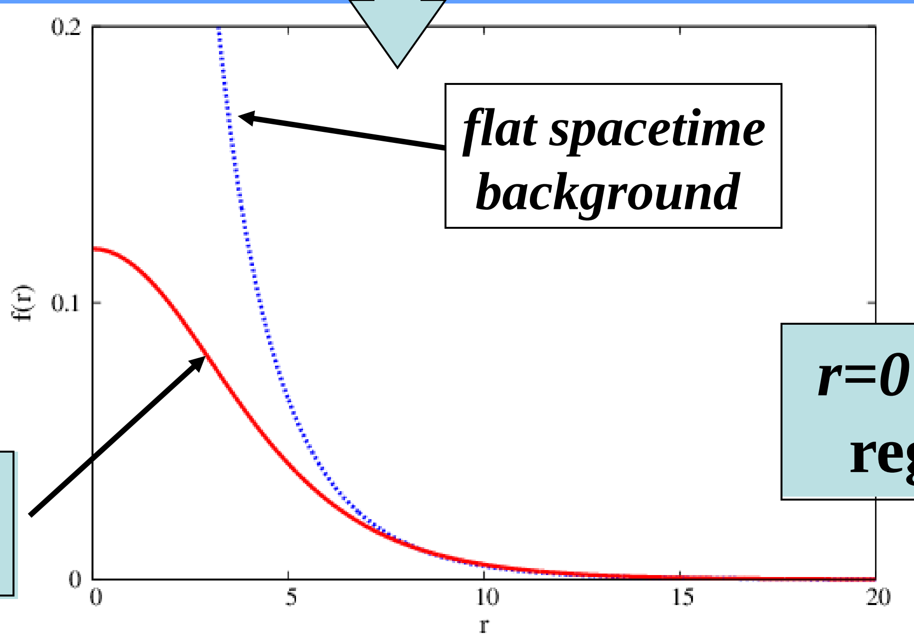
$$(\nabla^2 - \mu^2)\Phi = 0$$

spherical symmetry

$$\Phi = f(r)\, e^{-i\omega t}$$



$$f(r) = \frac{c}{\sqrt{r}} K_{\frac{1}{2}}\left(r\sqrt{\mu^2 - \omega^2}\right)$$



r=0 singularity:
regularized

(with CH, J. Kunz and Y. Shnir)

arXiv: 2008.10608 :

same mechanism works for the general solution (no isometries)

$$(\nabla^2 - \mu^2)\Phi = 0$$

general ansatz:

$$\Phi = f(r,\theta,\varphi)\, e^{-i\omega t}$$

***real** amplitude*

flat spaceime, single mode :

$$f = R_\ell(r) Y_{\ell m}(\theta,\varphi)$$

***real** spherical harmonics*

$$R_\ell(r) = \frac{c}{\sqrt{r}} K_{\frac{1}{2}+\ell}(r\sqrt{\mu^2-\omega^2})$$

asymptotically (only)

including gravity:

$$f = \sum_{\ell,m} R_\ell(r) Y_{\ell m}(\theta,\varphi)$$

no r=0 singularity

gravitational desingularization

numerics:

- solving numerically the full Einstein—Klein-Gordon equations

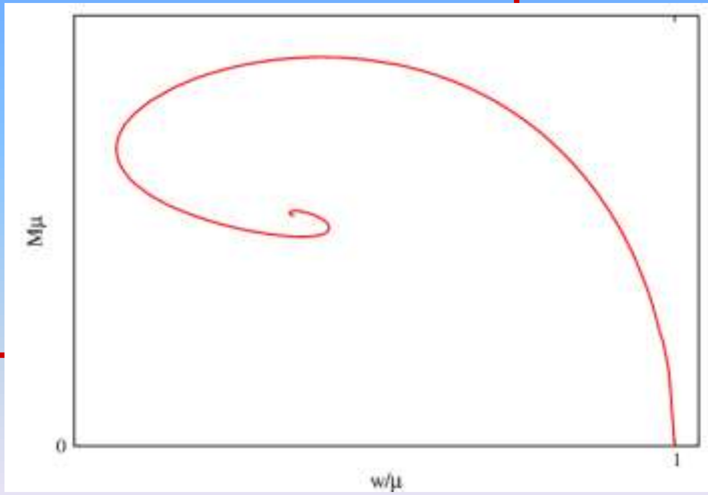
(non-perturbative approach, de Turck method**)**

$$\Phi = \underbrace{f(r,\theta,\varphi)}\, e^{-i\omega t}$$

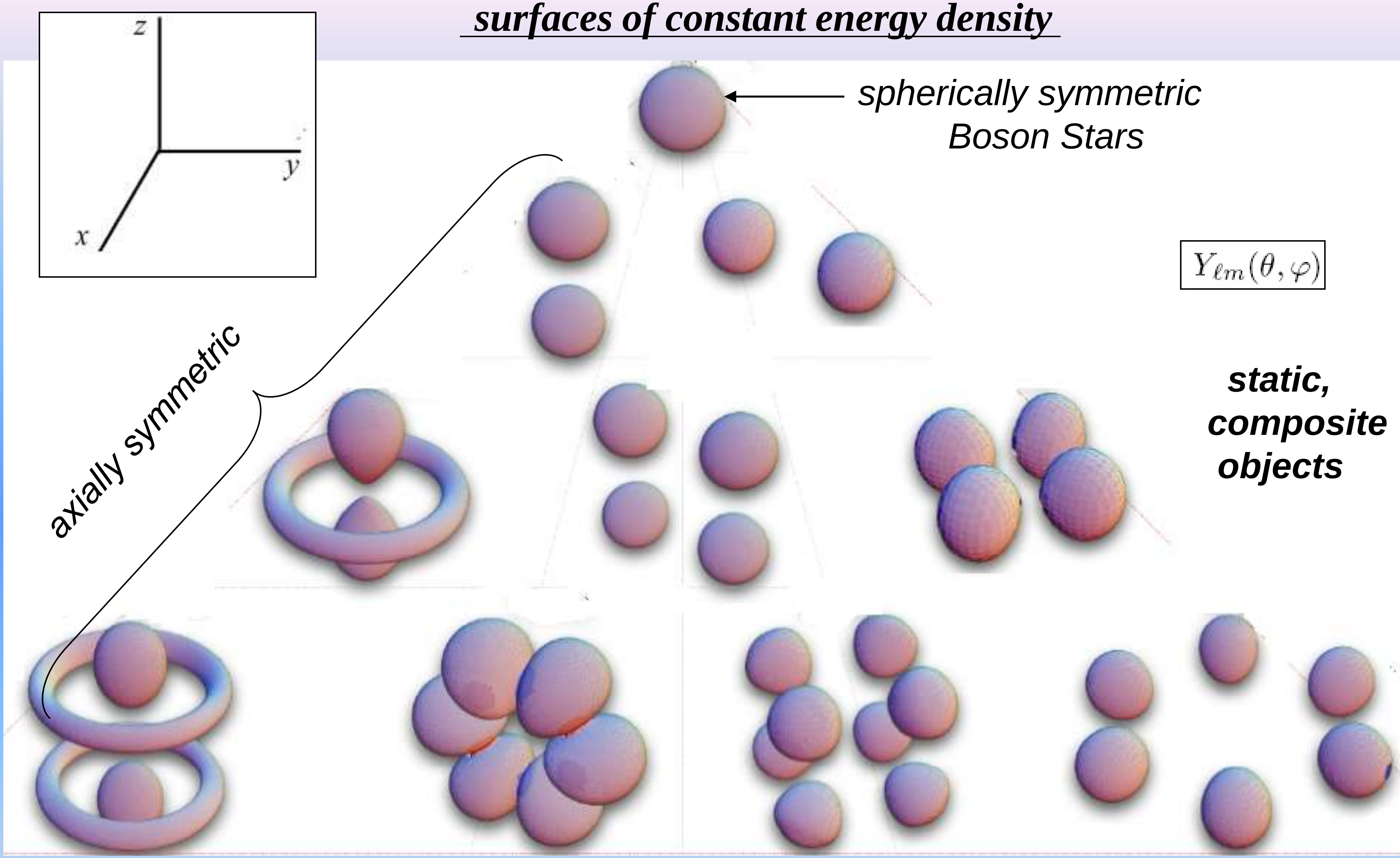
real *amplitude*

main results:

- ***new families of <u>static</u> Boson Stars***
- ***general case: <u>no isometries</u>,*** **one Killing vector** $\partial/\partial t$
- ***the solutions are regular everywhere***
- ***basic properties: as with spherical BSs***
- ***<u>multicenter, composite solutions</u>***



surfaces of constant energy density

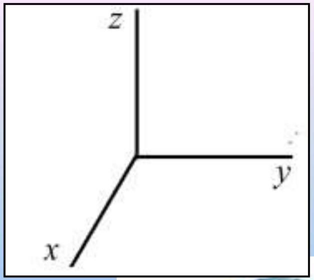


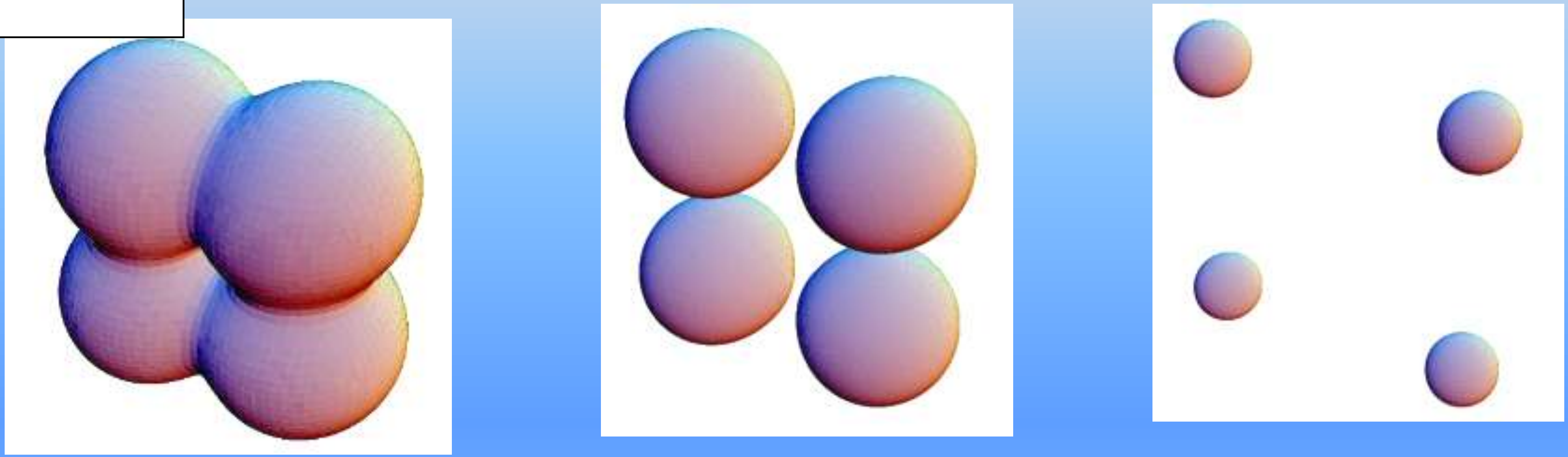
analogy with the atomic orbitals

(arXiv: 2008.10608)

(with CH, J. Kunz and Y. Shnir)

static, composite configurations





surfaces of constant energy density **– same solution without isometries**

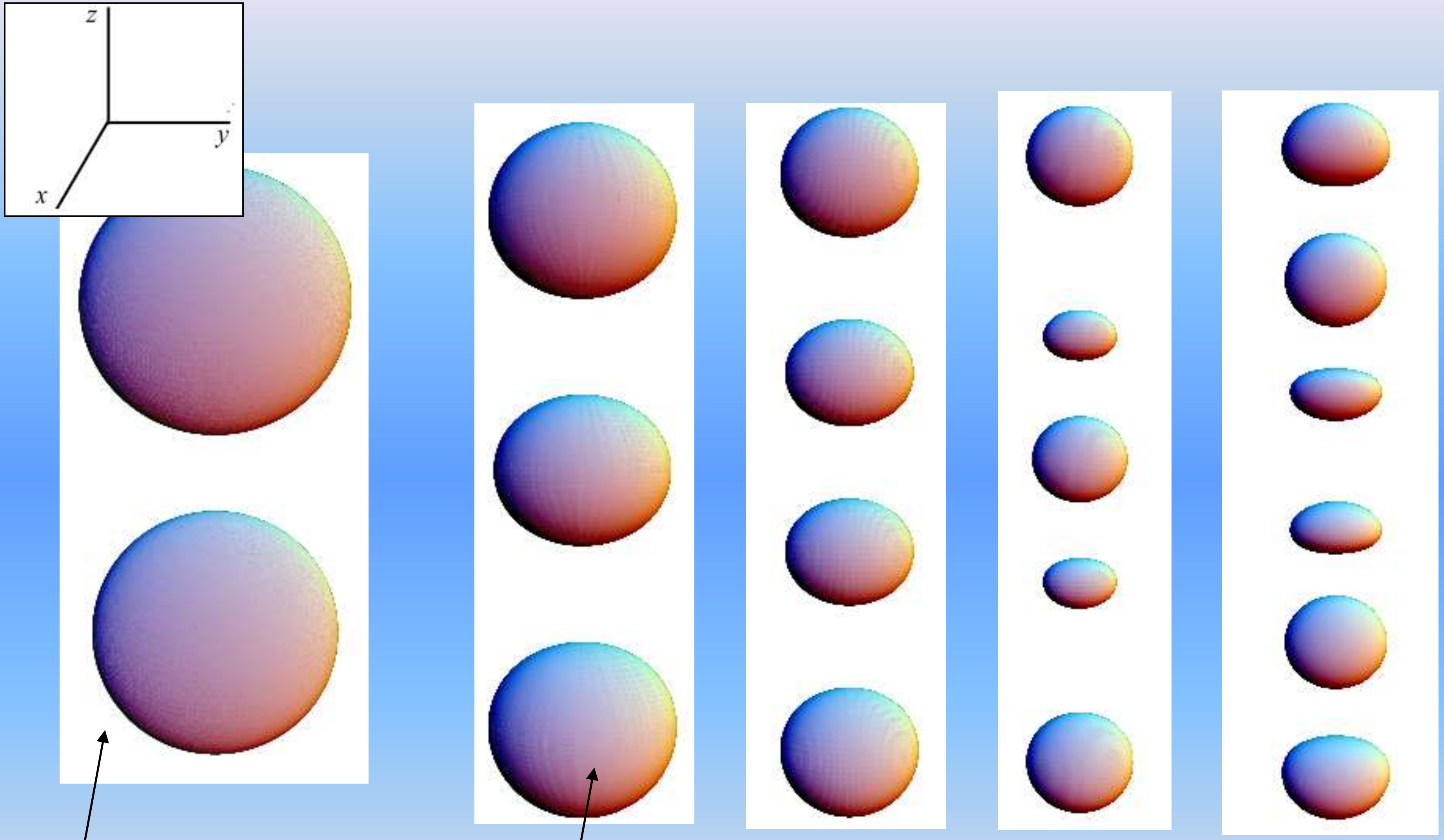
Non-trivial numerical problem:

one solves a set of 8 elliptic PDEs with dependence of (r,θ,φ)

$$ds^2 = g_{ij}dx^i dx^j + g_{tt}dt^2$$

(r,θ,φ)

Special solutions: **Boson Star Chains** (*static*)



dipole Boson Stars

triplet Boson Stars

…

to summarize (i):

self gravitating, massive, complex scalar field

$$\mathcal{L} = \frac{R}{16\pi G} - \frac{1}{2} g^{\alpha\beta} \left(\partial_\alpha \Phi^* \partial_\beta \Phi + \partial_\beta \Phi^* \partial_\alpha \Phi \right) - \mu^2 \Phi^* \Phi$$

STATIC SECTOR:

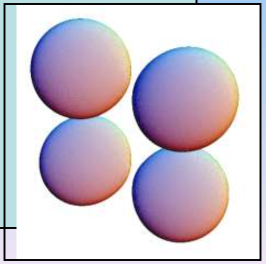
the main case considered so far..

- ***spherically symmetric Boson Stars***

+

new:

- ***Boson Stars with no isometries*** **(single Killing vector)**
- ***multicenter, composite configurations***
- ***(**also**) Boson star chains***– axially symmetry



the scalar no-hair theorems have a loophole

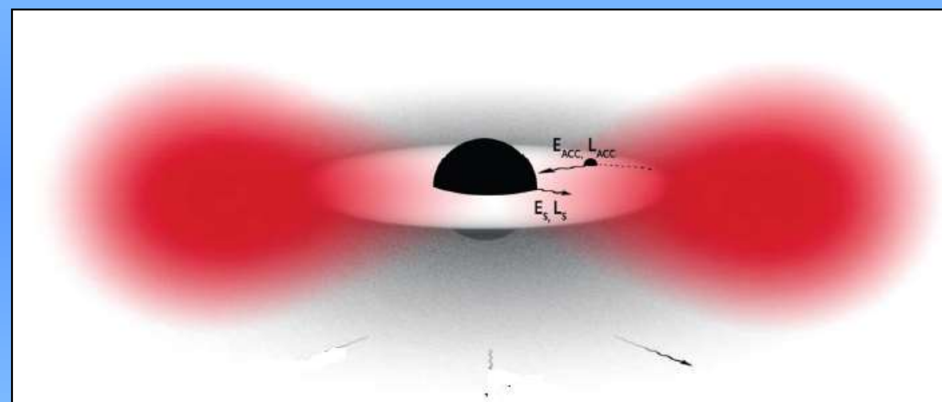
(ii) (single) ***Black Holes in EKG theory***

$$S=\int d^4x\sqrt{-g}\left[\frac{1}{16\pi G}R-\Phi^*_{,a}\Phi^{,a}-\mu^2\Phi^*\Phi\right]$$

numerics

existence proof:

Chodosh&Shlapentokh-Rothman



vacuum Kerr is a solution

regular, stationary, asymptotically flat black holes
with scalar hair

simplest example of hairy black holes

spin-one

naively, such solutions should be simpler than Kerr-Newman:

however:

richer, different pattern from Kerr(-Newman) !

spin-one

naively, such solutions should be simpler than Kerr-Newman:

however:

different pattern from Kerr(-Newman) !

synchronization condition

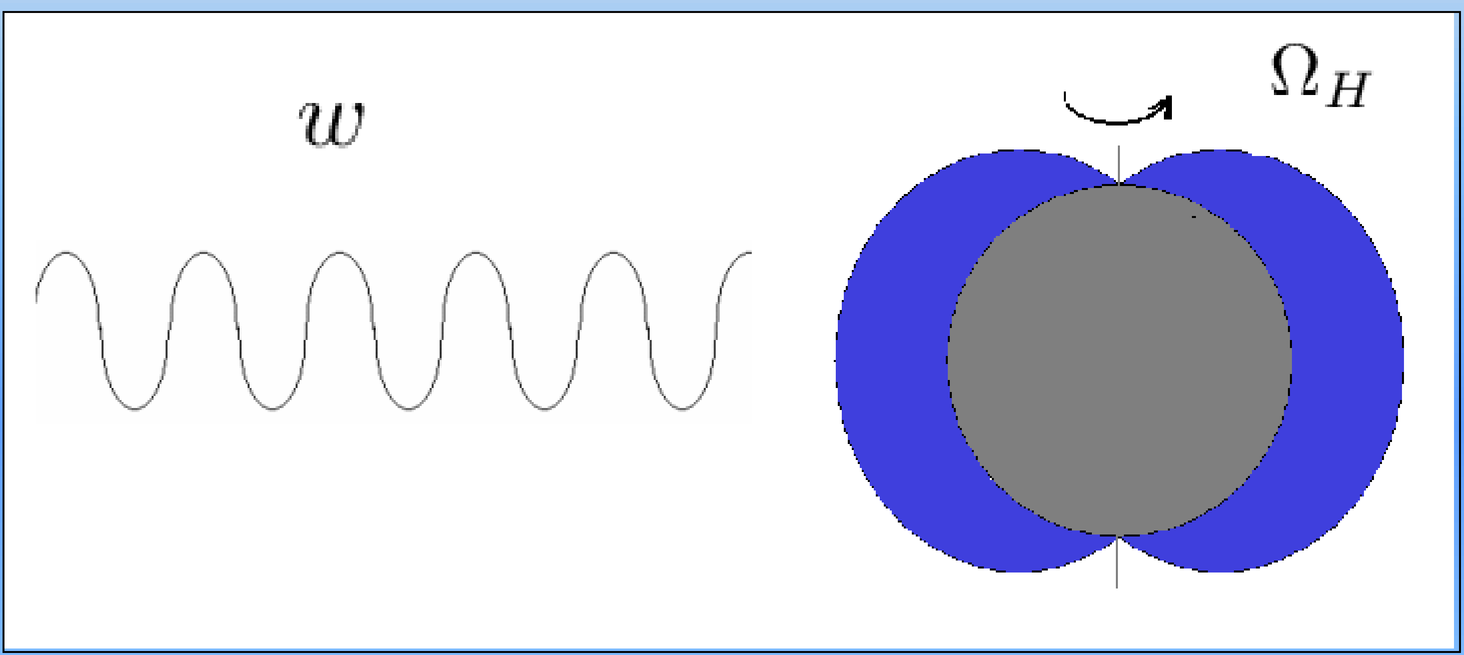
: (circumvent no-hair theorems)

- **no static limit** $w = m\Omega_H$

with $\Phi \sim e^{i(m\varphi - wt)}$

spinning black holes only!

$$\Phi \sim e^{i(m\varphi - wt)}$$



synchronization condition:

$$w = m\Omega_H$$

(zero flux)

general properties:

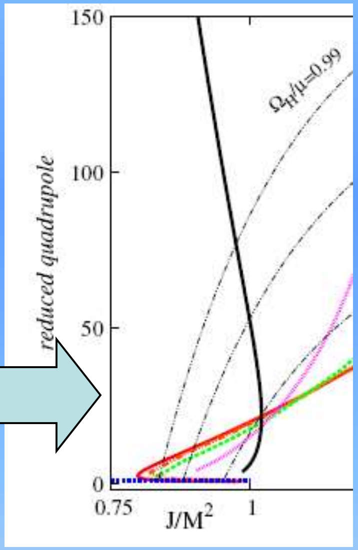
different pattern from Kerr

- **no static limit**
- **can violate Kerr bound** $J/M^2 > 1$

general properties:

different pattern from Kerr

- **no static limit**
- **violate Kerr bound**
- **different quadrupole**



general properties:

different pattern from Kerr

- **no static limit**
- **violate Kerr bound**
- **different quadrupole**
- **distinct ISCOs**

general properties:

different pattern from Kerr

- **no static limit**
- **violate Kerr bound**
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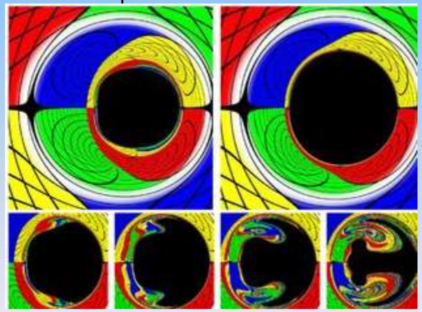
- **distinct ISCOs**
- **ergo-Saturns**

general properties:

different pattern from Kerr

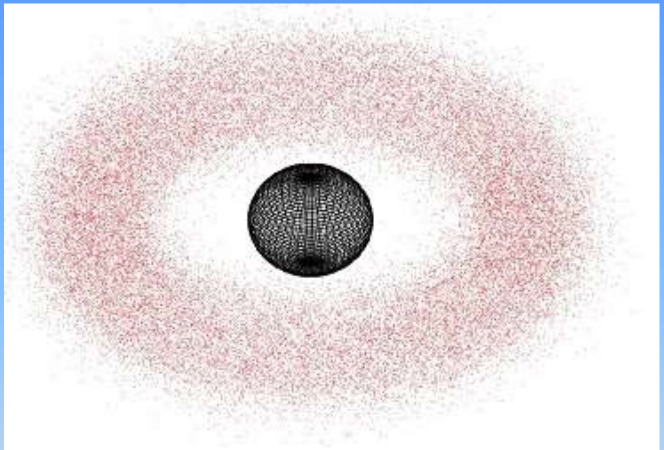
- **no static limit**
- **violate Kerr bound**
- **different quadrupole**
- **distinct ISCOs**
- **ergo-Saturns**
- **different shadows**



why is so different?

two (complementary) viewpoints:

- **<u>Boson stars</u>**: one can add a BH for spinning configurations
- **<u>Kerr black holes</u>**: branching towards a new family of solutions due to superradiant instability



hairy black hole*:* bound state *soliton*+ *Kerr horizon*

conjecture: (Herdeiro&Radu):

"a (hairless) BH which is afflicted by the superradiant instability of a given field must allow a hairy generalization with that field"

another example: **Einstein-(abelian) Proca theory:**

$$\mathcal{S} = \int d^4x \sqrt{-g} \left(\frac{R}{16\pi G} - \frac{1}{4}\mathcal{F}_{\alpha\beta}\bar{\mathcal{F}}^{\alpha\beta} - \frac{1}{2}\mu^2 \mathcal{A}_\alpha \bar{\mathcal{A}}^\alpha \right)$$

$$G_{\alpha\beta} = 8\pi G T_{\alpha\beta}, \quad \nabla_\alpha \mathcal{F}^{\alpha\beta} = \mu^2 \mathcal{A}^\beta,$$

Einstein-Proca hairy black holes: very similar properties
Herdeiro, Radu and Runarsson -*Class.Quant.Grav. 33 (2016) 154001*

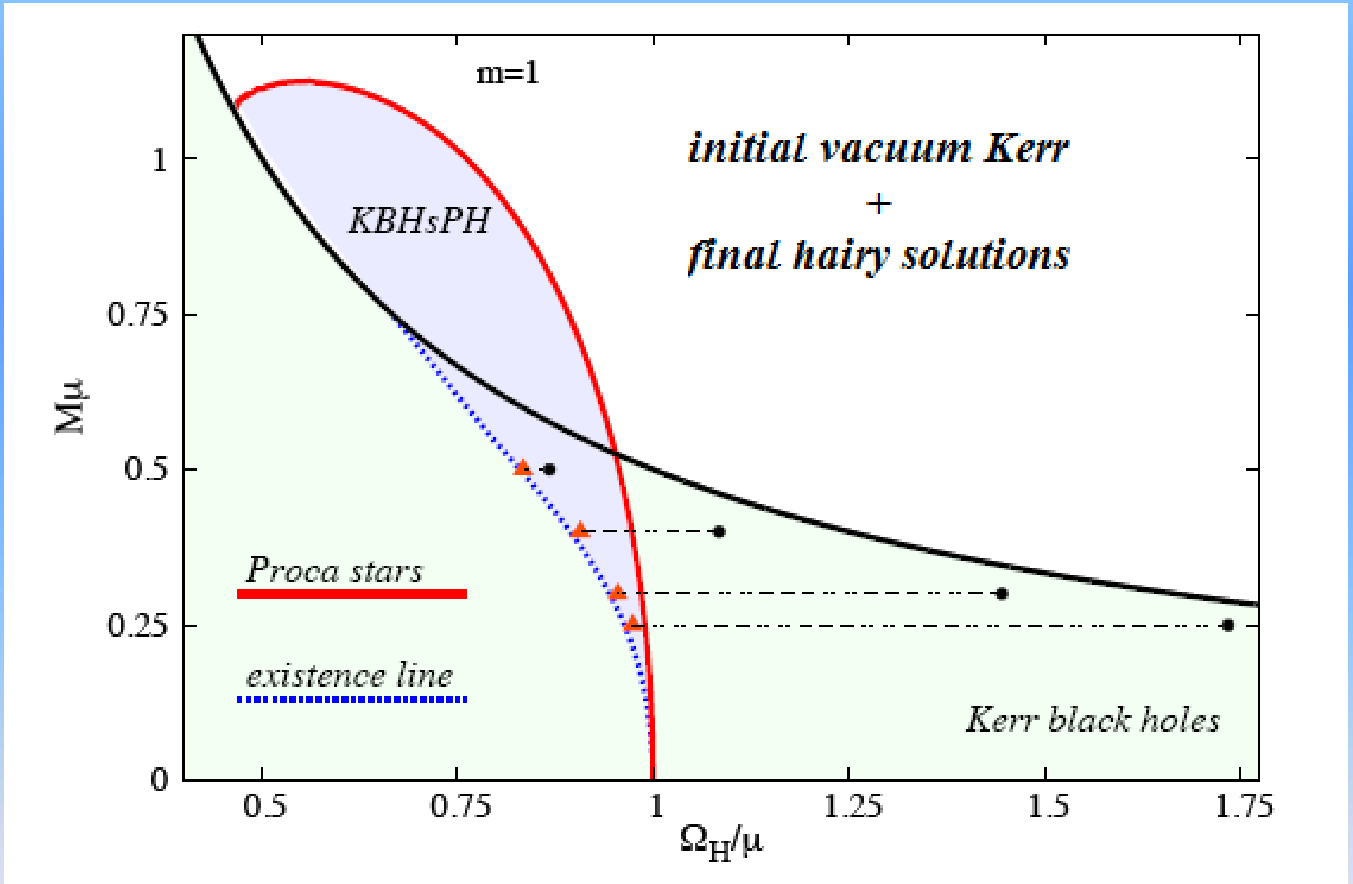
<u>these Black Holes form dynamically!</u>

PRL 119, 041101 (2017)
Selected for a Viewpoint in *Physics*
PHYSICAL REVIEW LETTERS
week ending 28 JULY 2017

Superradiant Instability and Backreaction of Massive Vector Fields around Kerr Black Holes

William E. East[1] and Frans Pretorius[2]

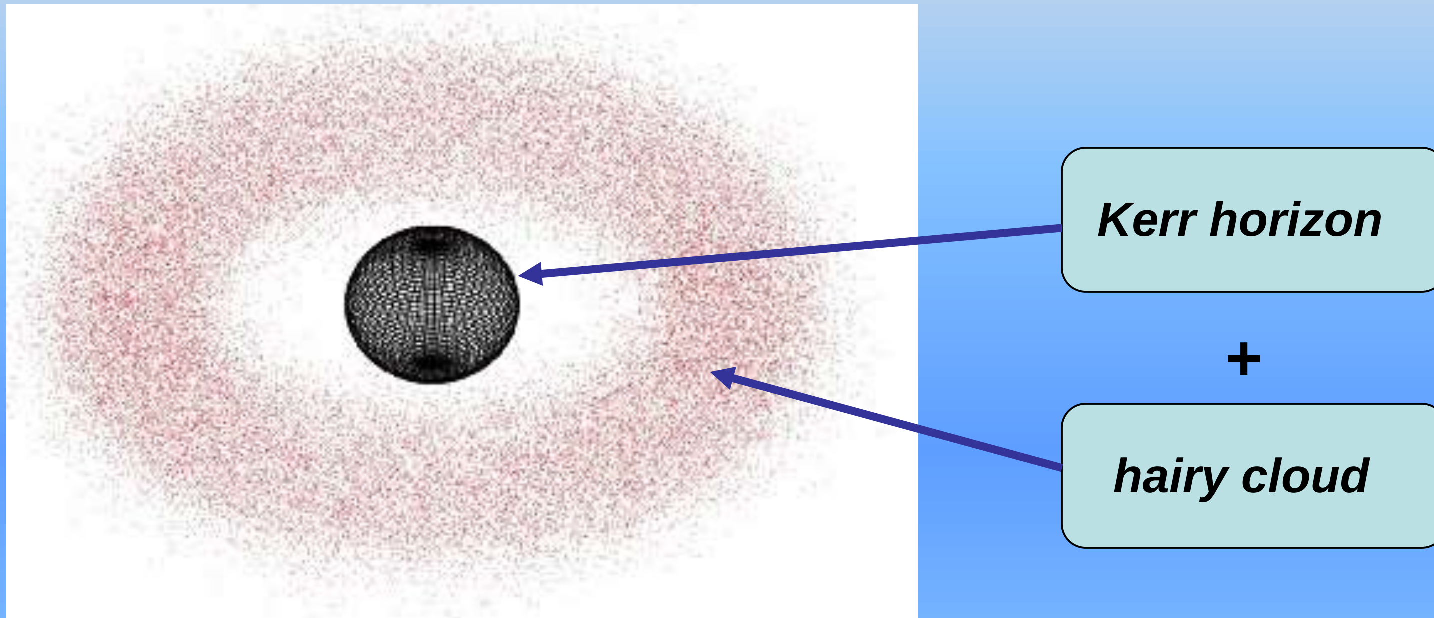
first dynamical counter-example to the no-hair conjecture



the endpoints of evolution matches the known hairy black holes

an analytic model (arXiv:1706.06597)

hairy black hole: bound state ***soliton***+ ***Kerr horizon***



horizon quantities

(A_H, T_H, Ω_H)

(M,J) → **(M_H, J_H)**

→

universality relations which fit well the numerical results *hairiness parameters*

$$M_{(\psi)} = M - M_H$$

$$J_{(\psi)} = J - J_H$$

$$p \equiv \frac{M_{(\psi)}}{M} \qquad q \equiv \frac{J_{(\psi)}}{J}$$

standard normalization:

$$j \equiv \frac{J}{M^2}, \quad a_H \equiv \frac{A_H}{16\pi M^2}, \quad w_H \equiv \Omega_H M, \quad t_H \equiv 8\pi M T_H$$

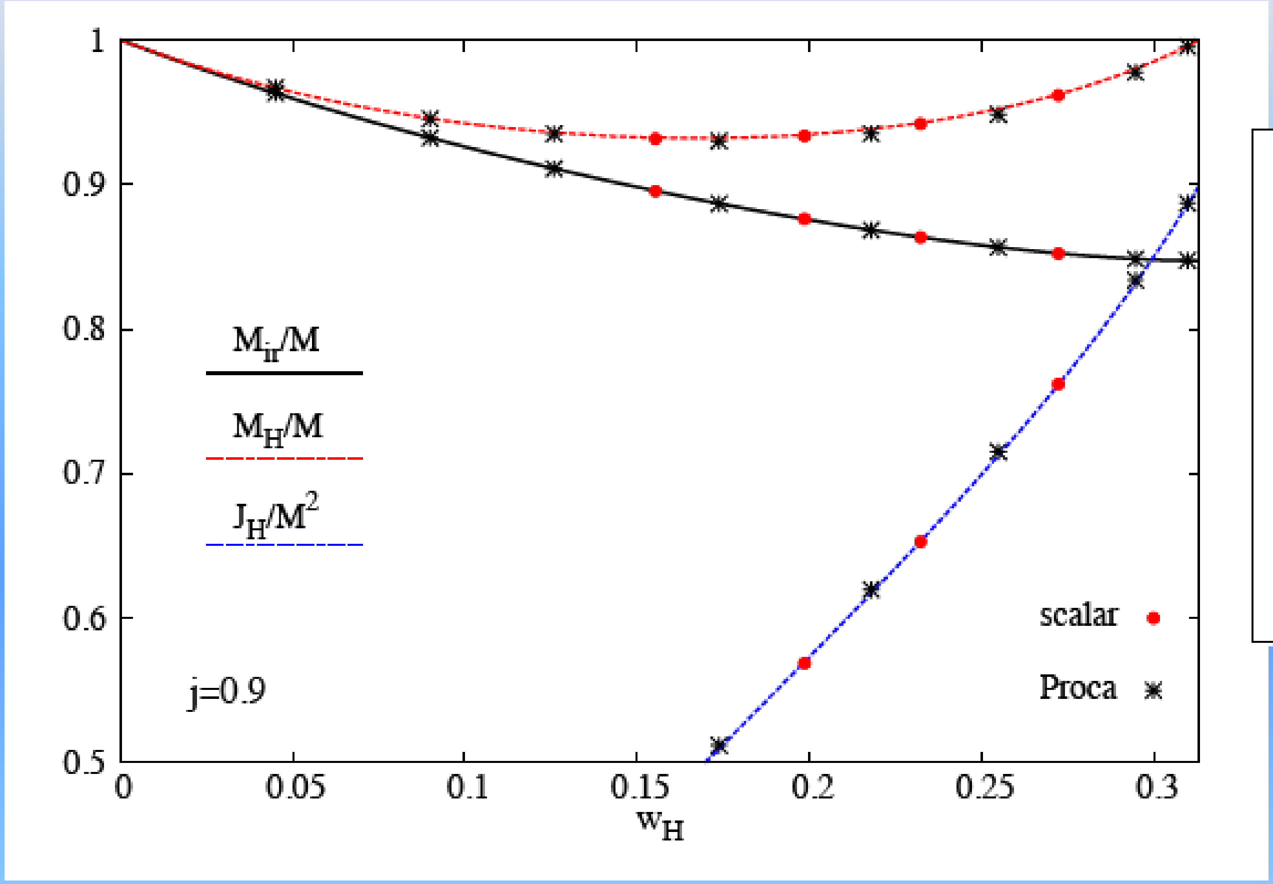
(M,J) → (M_H, J_H) + **1st law**

$$q = p\frac{1+4(1-p)^2 w_H^2}{p+4(1-p)^2 w_H^2} \qquad j = \frac{p+4(1-p)^2 w_H^2}{w_H(1+4(1-p)^2 w_H^2)}$$

$$a_H = \frac{(1-p)^2}{1+4(1-p)^2 w_H^2} \qquad t_H = \frac{1-4(1-p)^2 w_H^2}{1-p}$$

arXiv:1706.06597

universality!



$$j \equiv \frac{J}{M^2}$$

$$a_H \equiv \frac{A_H}{16\pi M^2}$$

$$p \equiv \frac{M_{(\psi)}}{M}$$

$$w_H \equiv \Omega_H M$$

online data + MATHEMATICA files

http://gravitation.web.ua.pt/index.php?q=node/716

simple application:

$$a_H^3 - 2(1-p)a_H^2 + \left[\frac{j^2}{4} + (1-p)^2\right] a_H = \frac{j^2(1-p)^2}{4}$$

the solution for small-*p*

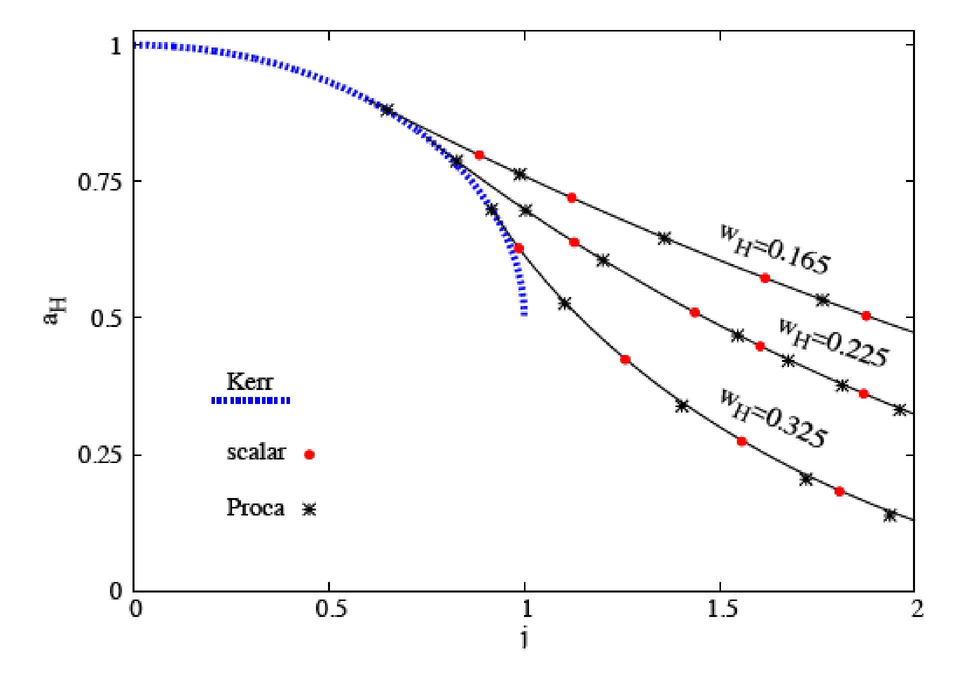
$$A_H(M,J) = A_H^{(Kerr)}(M,J) + \left[\frac{(1+\sqrt{1-j^2} - \frac{1}{2}j^2)(1+\sqrt{1-j^2})}{j^2\sqrt{1-j^2}}\right] M_{(\psi)}^2$$

strictly positive

$$j \equiv \frac{J}{M^2}$$

$$a_H \equiv \frac{A_H}{16\pi M^2}$$

$$p \equiv \frac{M_{(\psi)}}{M}$$



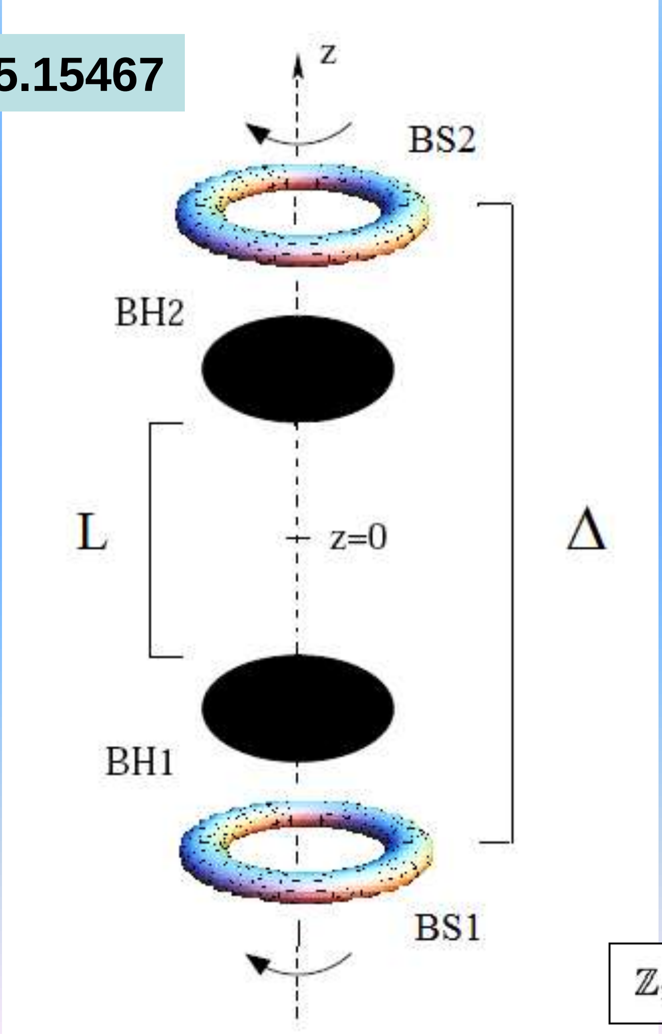
the hairy solutions and ***entropically favoured*** over Kerr black holes

(iii) the double Black Hole system in EKG theory

$$S=\int d^4x\sqrt{-g}\left[\frac{1}{16\pi G}R-\Phi^*_{,a}\Phi^{,a}-\mu^2\Phi^*\Phi\right]$$

not possible in (electro-)vacuum

arXiv: 2305.15467



- no singularities (also conical)
- crucial ingredient: *the dipolar BSs limit*
- likely holds for other systems with two solitons

synchronization condition

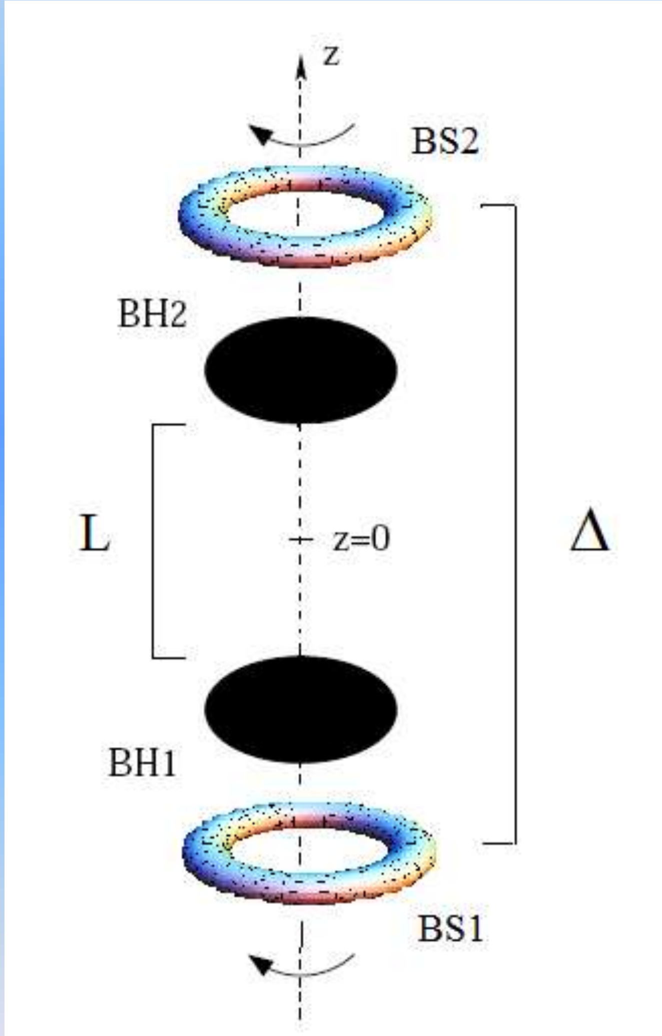
$$w=m\Omega_H$$

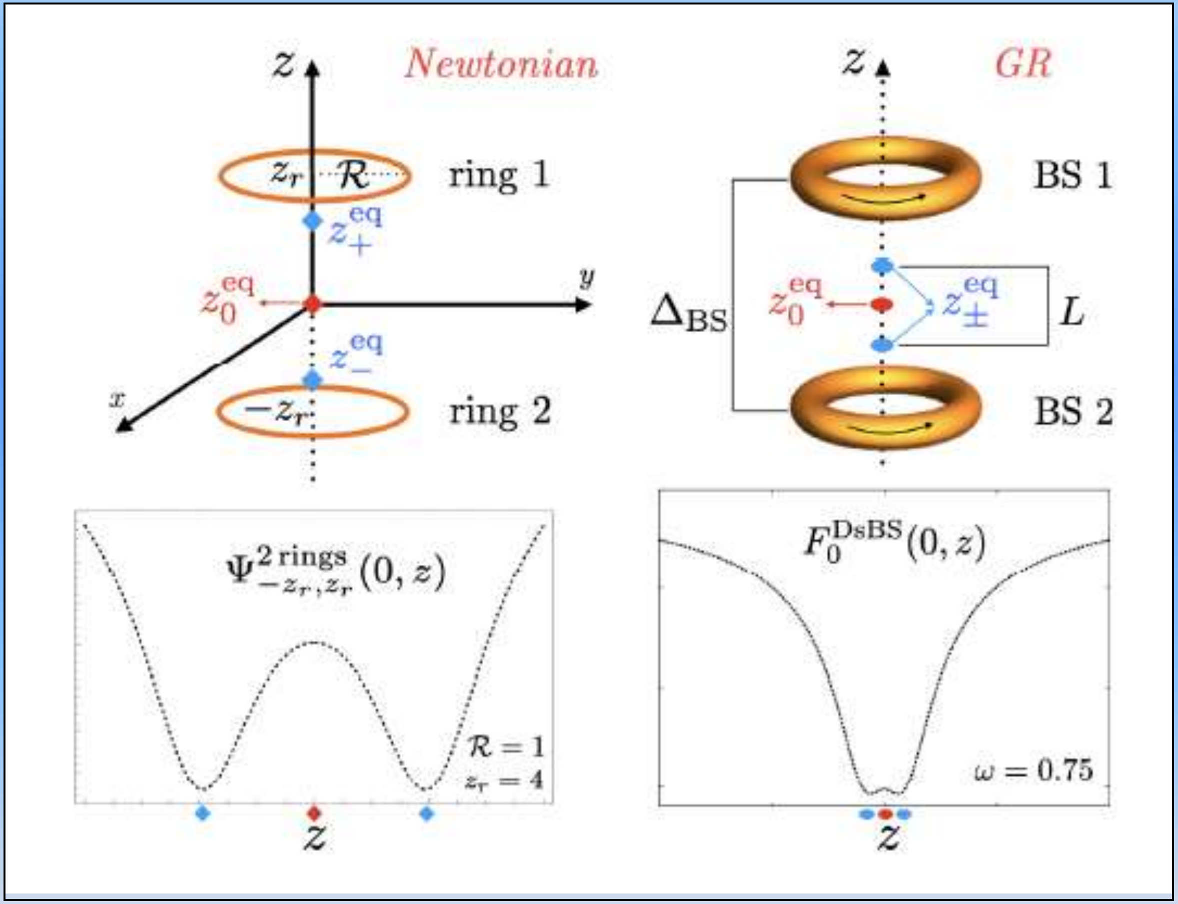
$\mathbb{Z}_2$-*symmetric solutions*

crucial ingredient:

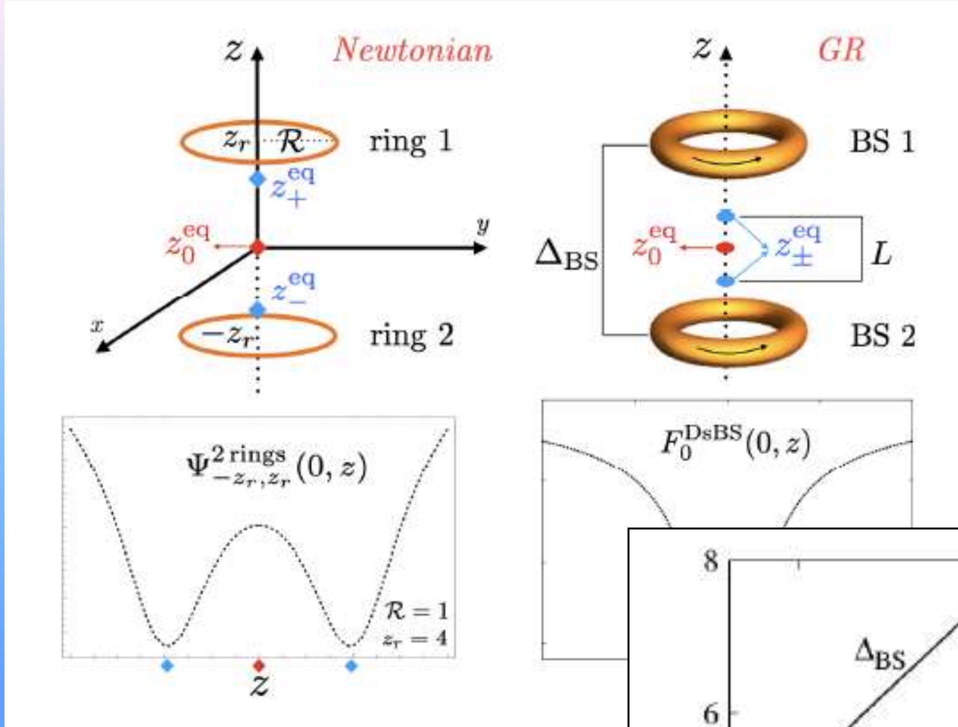
the dipolar BSs limit

- the existence of (timelike) equilibrium points at finite distance on the *z*-axis



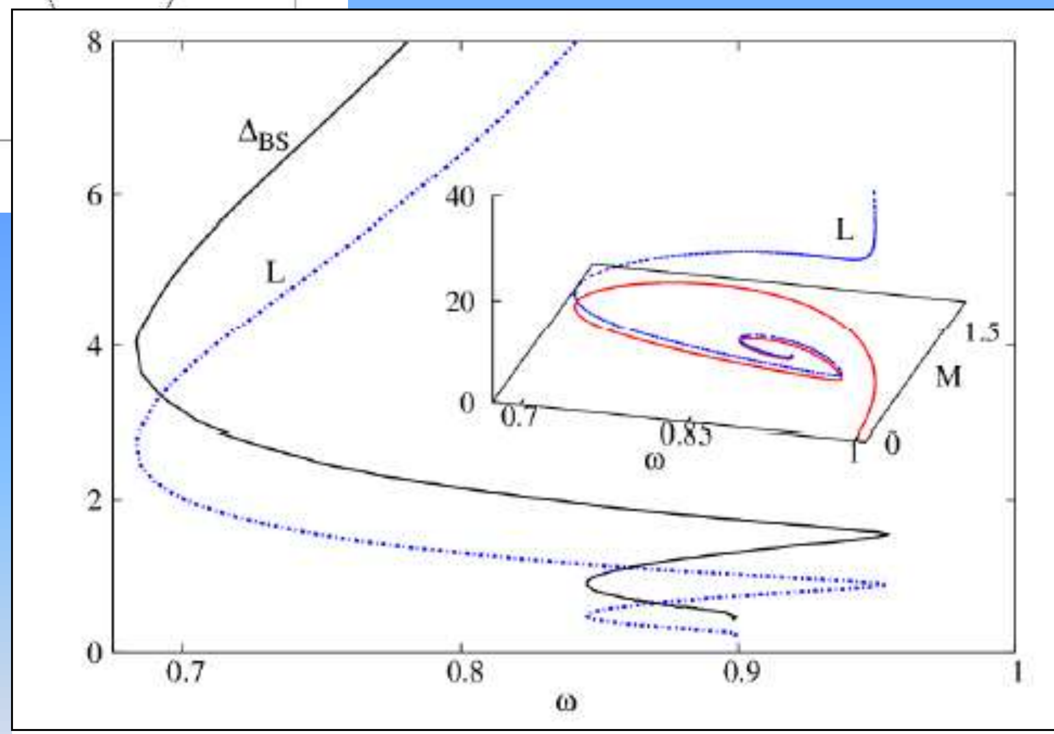


$$\ddot{x}^\alpha + \Gamma^\alpha_{\beta\gamma}\dot{x}^\beta\dot{x}^\gamma = 0$$



$$\ddot{x}^\alpha + \Gamma^\alpha_{\beta\gamma}\dot{x}^\beta\dot{x}^\gamma = 0$$

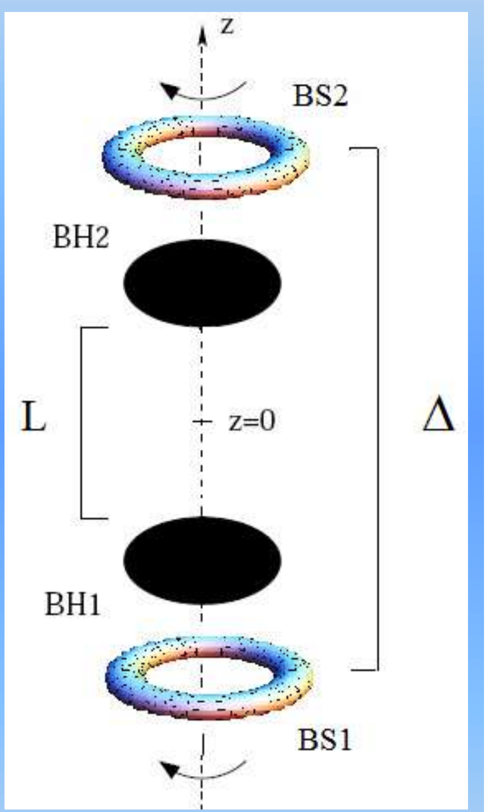
(timelike)
equilibrium points

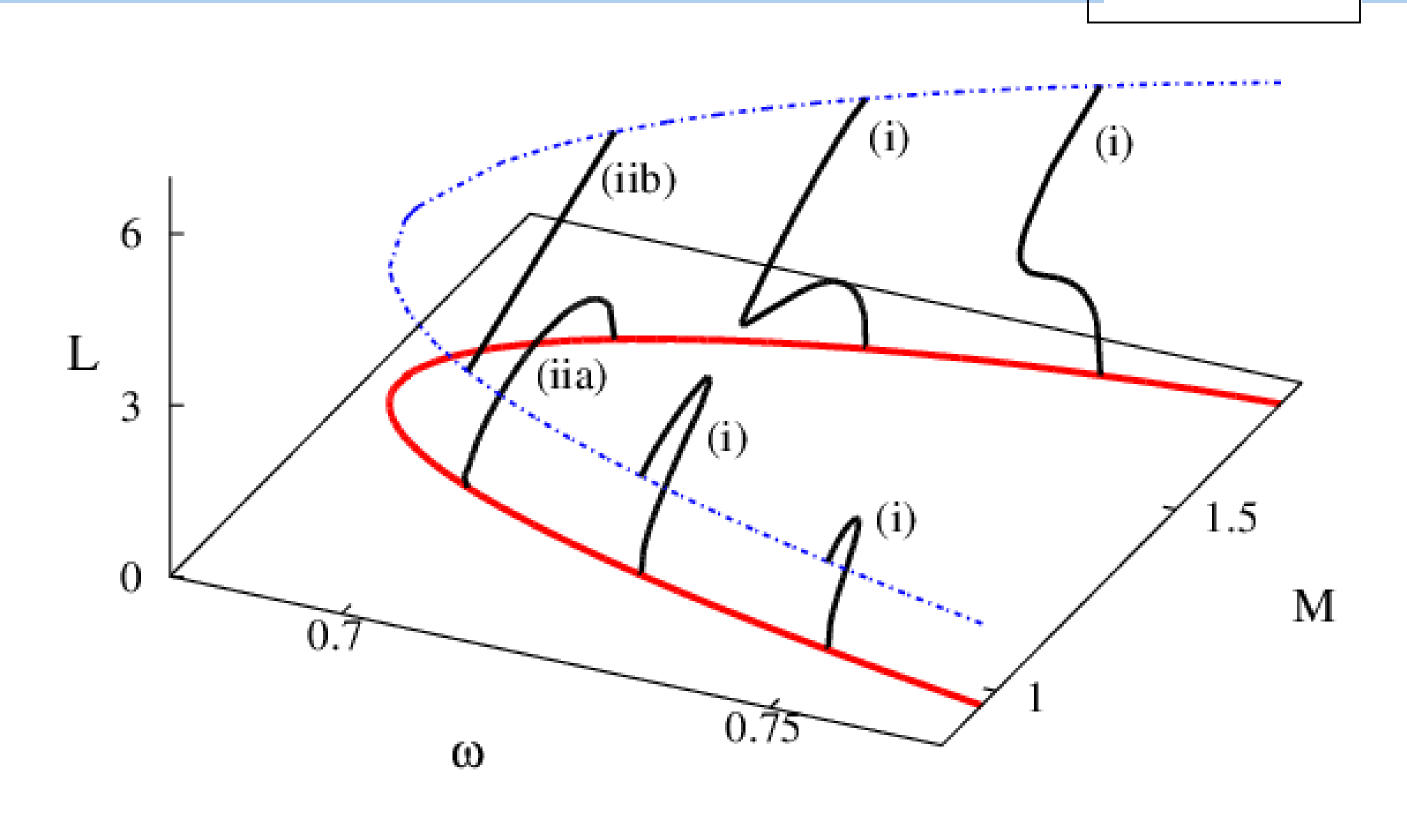


dipolar Boson Stars

the emerging picture *(families of solutions at constant frequency)*

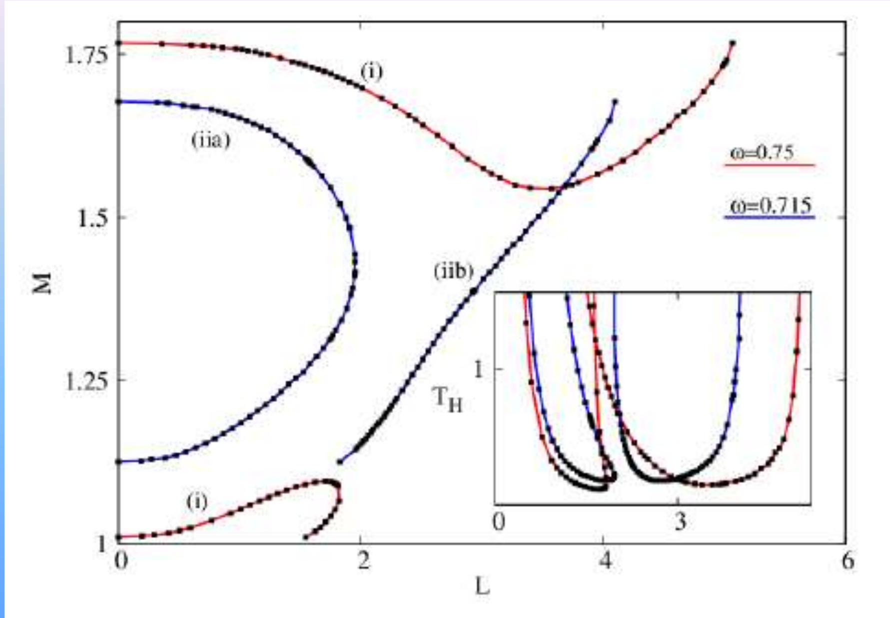
$w = m\Omega_H$

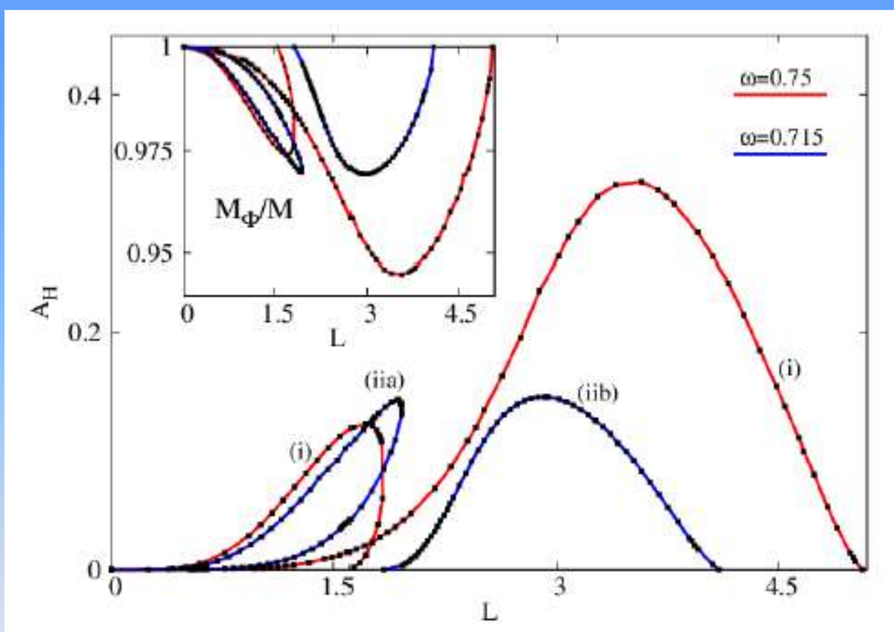


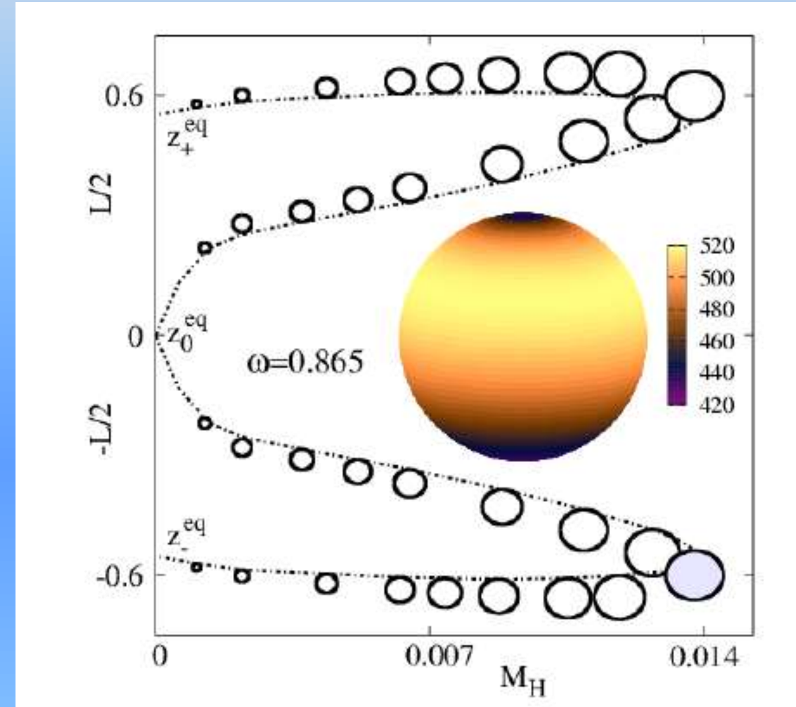


families of Black Holes

- form a sequence 'inside' the same dipolar Boson Stars (*i*)
- interpolate between two different dipolar Boson Stars (*iia*, *iib*)


M
L
1.75
1.5
1.25
1
0
2
4
6
(i)
(iia)
(iib)
ω=0.75
ω=0.715
T_H
3


A_H
L
0.4
0.2
0
1.5
3
4.5
(i)
(iia)
(iib)
ω=0.75
ω=0.715
M_Φ/M
1
0.975
0.95


L/2
-L/2
0.6
0
-0.6
z_+^eq
z_0^eq
z_-^eq
ω=0.865
520
500
480
460
440
420
0.007
0.014
M_H

to summarize:

Einstein-Maxwell
(electro-vacuum)

$$\mathcal{L} = \frac{R}{16\pi G} - \frac{1}{4} F_{\alpha\beta} F^{\alpha\beta}$$

spin-one

- fully solvable
- no solitons
- Kerr-Newman black hole
- uniqueness

four dimensions asymptotically flat

Einstein-Klein-Gordon
(massive complex scalar field)

$$\mathcal{L} = \frac{R}{16\pi G} - \frac{1}{2} g^{\alpha\beta} \left(\partial_\alpha \Phi^* \partial_\beta \Phi + \partial_\beta \Phi^* \partial_\alpha \Phi \right) - \mu^2 \Phi^* \Phi$$

spin-zero

- no exact solutitons
- solitons: (multi) Boson Stars
- spinning black hole with scalar hair
- non-uniqueness
- two balanced black holes

very different picture

to summarize:

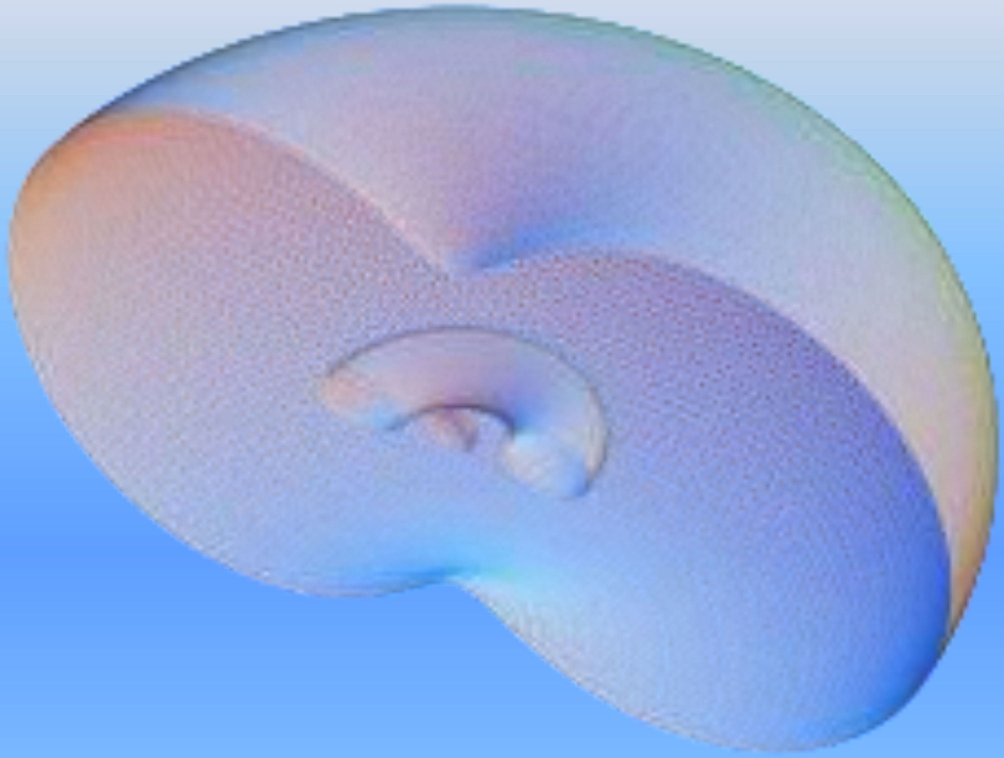
Einstein-Klein-Gordon system possesses *solitons* and hairy *black holes* (including balanced binaries)

….still a lot of work to be done

reconsider everything what is known for (electro-)vacuum GR

- most important: ***stability?***

Q: ***are there any scalar fields in Nature apart from Higgs?***



many thanks for your attention!