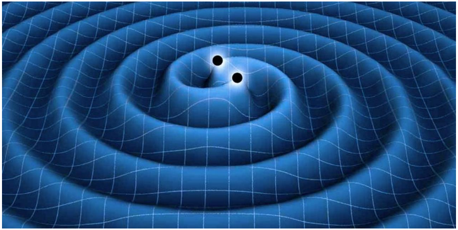
Quasinormal modes of higher-derivative Kerr black holes

Pablo A. Cano

based on JHEP 05 (2019) w/ Alejandro Ruipérez
Phys. Rev. D 102 (2020), 044047 w/ Kwinten Fransen, Thomas Hertog
Phys. Rev. D 105 (2022) 2, 024064 w/ KF, TH and Simon Maenaut
arXiv: 2304.02663 and 23XX.XXXXX w/ KF, TH and SM

Journèes Relativistes de Tours, 2 June 2023

Gravitational waves → probes of gravity

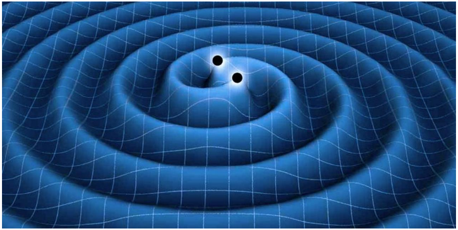


$$R_{\mu\nu} = 0?$$

$$g_{\mu\nu}^{\text{BH}} = g_{\mu\nu}^{\text{Kerr}}(M, a)?$$

Introduction

Gravitational waves $\longrightarrow$ probes of gravity



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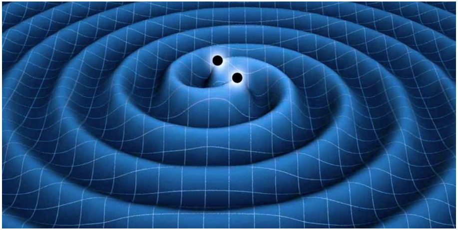

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Look for specific deviations from GR $\longrightarrow$ **higher-derivative corrections**

$$\mathcal{L} = R + \ell^2\mathcal{R}^2 + \ell^4\mathcal{R}^3 + \ldots$$

Natural EFT extension of GR. Scale ℓ essentially unconstrained

Can we really see higher-derivative corrections? Depends on the scale ℓ

The first corrections are $\sim \ell^4 Riem^3$. The relative deviation Δ with respect to GR is of order

$$\Delta \sim \frac{\ell^4 (GM)^2}{r^6}$$

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$$\Delta_{Surf.Sun} \sim \left(\frac{\ell}{5\times 10^8\text{km}}\right)^4 , \quad \Delta_{Surf.Earth} \sim \left(\frac{\ell}{2\times 10^8\text{km}}\right)^4 , \quad \Delta_{BH}(10M_{\odot}) \sim \left(\frac{\ell}{40\text{km}}\right)^4$$

Improve constraints by a factor $\sim 10^{30}$

Where to look for higher-derivative corrections?

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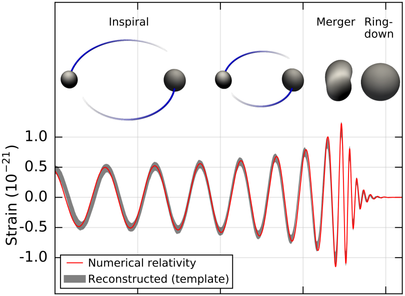

Ringdown: most powerful test of GR

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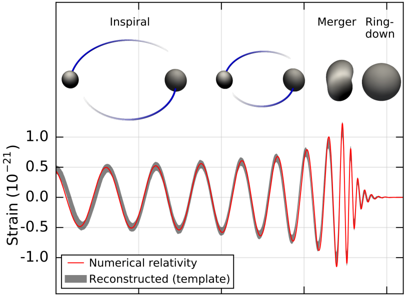


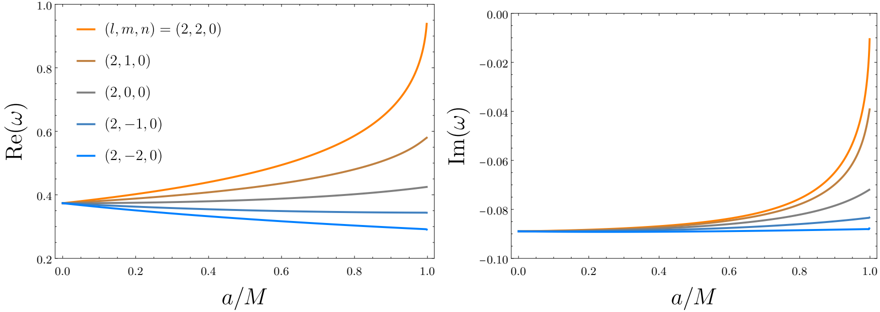
Ringdown: most powerful test of GR

Controlled by the **quasinormal modes**

$$h = \sum_{n=0} c_n e^{-i\omega_n t} \,, \qquad \mathrm{Im}(\omega_n) < 0$$

- QNMs only depend on the final black hole $\longrightarrow$ mass and angular momentum
- Studied through perturbation theory
- Depend on the photon-sphere physics $\longrightarrow$ small length scale $\longrightarrow$ sensitive to short-distance modifications of GR

QNM frequencies of Kerr black holes: $\omega_{l,m,n}$



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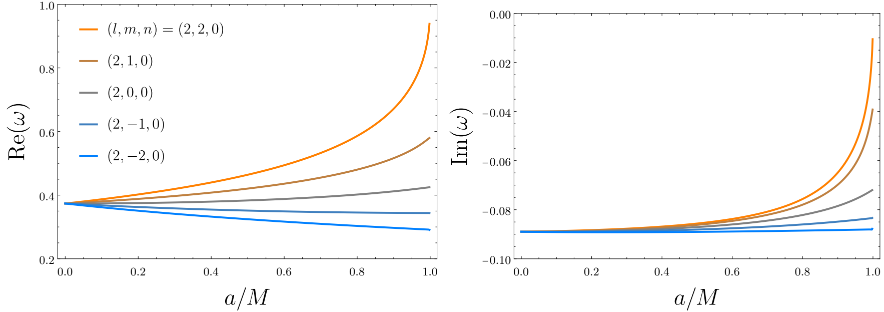

What about **modifications of GR?**

$$\mathcal{L} = R + \text{corrections} \quad \Rightarrow \quad \omega = \omega^{\text{Kerr}} + \delta\omega$$

Computation of $\delta\omega$: **crucial** to test these theories, but **remarkably challenging**

[Gualtieri, Pierini '21, '22], [Wagle, Yunes, Silva '21], [Srivastava, Chen, Shankaranarayanan '21], [Li, Wagle, Chen, Yunes '22], [Hussain, Zimmerman '22]

In this talk: **general EFT extension of GR**

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{|g|}\Big\{R + \ell^4\left(\lambda_{\rm ev}R_{\mu\nu}{}^{\rho\sigma}R_{\rho\sigma}{}^{\delta\gamma}R_{\delta\gamma}{}^{\mu\nu} + \lambda_{\rm odd}R_{\mu\nu}{}^{\rho\sigma}R_{\rho\sigma}{}^{\delta\gamma}\tilde{R}_{\delta\gamma}{}^{\mu\nu}\right)$$
$$+\ell^6\left(\epsilon_1\mathcal{C}^2 + \epsilon_2\tilde{\mathcal{C}}^2 + \epsilon_3\mathcal{C}\tilde{\mathcal{C}}\right) + \mathcal{O}(\ell^8)\Big\}$$

$$\mathcal{C} = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\,,\quad \tilde{\mathcal{C}} = R_{\mu\nu\rho\sigma}\tilde{R}^{\mu\nu\rho\sigma}\,,\quad \tilde{R}_{\mu\nu\rho\sigma} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}R^{\alpha\beta}{}_{\rho\sigma}$$

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Goal: obtain $\delta\omega$ at first order in the couplings

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- Introduce null tetrad $\{l_\mu, n_\mu, m_\mu, \bar{m}_\mu\}\,, \quad l^\mu n_\mu = -1\,, \quad m^\mu \bar{m}_\mu = 1$

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- Key property: the Kerr metric **algebraically special** (Petrov type D):

$$\Psi_0^{(0)} = \Psi_1^{(0)} = \Psi_3^{(0)} = \Psi_4^{(0)} = 0$$

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- For perturbations around Kerr, $\delta\Psi_0$ and $\delta\Psi_4$ satisfy decoupled equations

$$\boxed{\mathcal{D}_{+2}\delta\Psi_0 = 0\,,\quad \mathcal{D}_{-2}\delta\Psi_4 = 0}$$

These are the **Teukolsky equations**. Arise as a combination of Bianchi identities $\nabla_{[\alpha}R_{\mu\nu]\rho\sigma} = 0$

Perturbations of Kerr black holes

Teukolsky equations are decoupled, homogeneous second-order equations

Furthermore, they are **separable**

$$\delta\Psi_0 = e^{-i\omega t + im\phi} R_2(r) S_2(x)\,, \quad \delta\Psi_4 = e^{-i\omega t + im\phi}(r - iax)^{-4} R_{-2}(r) S_{-2}(x)$$

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$$\frac{d}{dx}\left[(1-x^2)\frac{dS_s}{dx}\right]+\left[(a\omega)^2x^2-2sa\omega x+B_{lm}-\frac{(m+sx)^2}{1-x^2}\right]S_s=0$$

$$\Delta^{-s+1}\frac{d}{dr}\left[\Delta^{s+1}\frac{dR_s}{dr}\right]+VR_s=0$$

where $x=\cos\theta$, $\Delta=r^2-2Mr+a^2$

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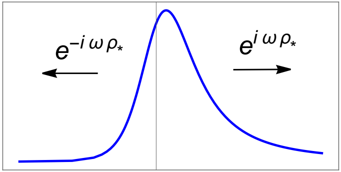
$\longrightarrow$ **1-dim. Schrödinger equation**

$$\frac{d^2\varphi}{d\rho_*^2} + \left(\omega^2 - V(\rho_*, \omega)\right)\varphi = 0$$

$$\varphi \propto R_s\,, \quad \rho_* = \rho_*(r)$$



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- Non-separabiity
- EOM of higher order in derivatives

Universal Teukolsky equation Kwinten Fransen; Li, Wagle, Chen, Yunes; Hussain, Zimmerman

$$\mathcal{E}_{\pm 2}\left(\Psi_i, \Phi\right) = 0\,, \qquad \text{where} \qquad \Phi = \{e^a_\mu, \gamma_{abc}, R_{ab}\}$$

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- For Ricci-flat Petrov type D it reduces to Teukolsky

$$\mathcal{D}_{\pm 2}(\delta\Psi_{0,4}) = 0$$

Perturbations of rotating black holes beyond GR

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There are still outstanding challenges

How to decouple this? How to obtain radial equations?

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1. Evaluate the Universal Teukolsky equations $\longrightarrow$ needs background geometry
2. Express the equations only in terms of $\delta\Psi_{0,4} \longrightarrow$ Metric reconstruction $h_{\mu\nu}(\delta\Psi_{0,4})$
3. Effective separation of the equation

$$\text{Assume } \delta\Psi_0 = e^{-i\omega t+im\phi} R_s^{lm}(r) S_s^{lm}(x) \quad \longrightarrow \quad \text{project on } S_s^{lm}(x)$$

(1) Rotating black hole solutions PAC, Ruipérez '19:

$$ds^2 = -\left(1 - \frac{2Mr}{\Sigma} - H_1\right) dt^2 - (1 + H_2)\,\frac{4Mar(1-x^2)}{\Sigma} dt d\phi$$
$$+ (1 + H_3)\,\Sigma \left(\frac{dr^2}{\Delta} + \frac{dx^2}{1-x^2}\right) + (1 + H_4)\left(r^2 + a^2 + \frac{2Mra^2(1-x^2)}{\Sigma}\right)(1-x^2)d\phi^2$$

where Σ and Δ are given by

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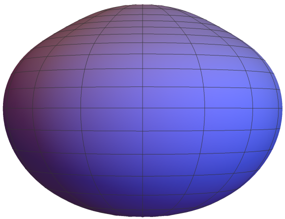
Power series in χ: **analytic solution**

$$H_i = \sum_{n=0}^{\infty} \chi^n \sum_{p=0}^{n} \sum_{k=0}^{k_{\max}} H_i^{(n,p,k)} \frac{x^p}{r^k}$$

$n = 14$ accurate for $\chi \sim 0.7$



(2) Metric reconstruction Dolan, Kavanagh, Wardell '21

$$h_{\mu\nu} = -\frac{i}{3M\omega}\nabla_\beta\left[\zeta^4\nabla_\alpha C_{(\mu\ \ \nu)}^{\ \ \ \alpha\ \ \beta}\right] - \frac{i}{3M\omega}\nabla_\beta\left[(\zeta^*)^4\nabla_\alpha \bar{C}_{(\mu\ \ \nu)}^{\ \ \ \alpha\ \ \beta}\right]$$

$$C_{\mu\alpha\nu\beta} = 4\left(\psi_0 n_{[\mu}\bar{m}_{\alpha]}n_{[\nu}\bar{m}_{\beta]} + \psi_4 l_{[\mu}m_{\alpha]}l_{[\nu}m_{\beta]}\right)$$

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The radial functions can be related by

$$R_{+2}^* = q_{+2}R_{+2} \qquad R_{-2} = C_{+2}\Delta^2\left(\mathcal{D}_0\right)^4\left(\Delta^2 R_{+2}\right)$$
$$R_{-2}^* = q_{-2}R_{-2} \qquad R_{+2} = C_{-2}(\mathcal{D}_0^\dagger)^4 R_{-2}$$

$q_{\pm2} \longrightarrow$ polarization, $C_{\pm2} \longrightarrow$ Starobinsky-Teukolsky constants, $C_{+2}C_{-2} = \mathcal{K}^{-2}$

(2) Metric reconstruction

The Teukolsky variables are proportional to the metric variables

$$\delta\Psi_s = P_s \psi_s\,, \quad \delta\Psi_s^* = P_s^* \psi_s^*\,.$$

The constants P_s, P_s^* depend on the polarization parameters q_s and ST constants C_s and are given by

$$P_{\pm 2} = \frac{1}{2} \pm \frac{iD_2 q_{\pm 2}}{24M\omega} \mp \frac{iC_{\pm 2} q_{\mp 2} \mathcal{K}^2}{6M\omega}\,,$$
$$P^*_{\pm 2} = \frac{1}{2} \pm \frac{iD_2}{24M\omega q_{\pm 2}} \mp \frac{iC_{\pm 2} \mathcal{K}^2}{6M\omega q_{\pm 2}}$$

(3) Decoupling and separation of the equation

1. Start with

$$\delta\Psi_s = e^{-i\omega t + im\phi}\zeta^{s-2}\sum_{l} R_s^{lm}(r) S_s^{lm}(x)$$

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3. Project the equation onto the spheroidal harmonics

$$\mathfrak{D}_s^2 R_s^{lm} = \lambda\sum_{l',s'}\mathcal{D}_{s'}^{ll'}R_{s'}^{l'm} + c.c.$$

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5. Decouple $s=\pm 2$ and conjugates by using $q_{\pm 2}$ and ST identities

Result: **corrected radial Teukolsky equations**

$$\Delta^{-s+1}\frac{d}{dr}\left[\Delta^{s+1}\frac{dR_s}{dr}\right]+\left(V+\lambda\delta V\right)R_s=0\,,\qquad \delta V=\sum_{n=-2}^{4}A_n r^n$$

Coefficients A_n analytic power series in χ: $A_n=\sum_{k=0}^{\infty}\chi^k A_{n,k}$

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Coefficients A_n analytic power series in χ: $A_n=\sum_{k=0}^{\infty}\chi^k A_{n,k}$

In total **4 equations** ($s=\pm 2$ and the conjugates)

For each harmonic numbers (l,m) these equations depend on

- Frequency ω
- Polarization $q_{\pm 2}$
- Starobinsky-Teukolsky constants $C_{\pm 2}$ ("gauge" freedom)

Observe: for a QNM, all the equations are solved **by the same frequency**

Parity-preserving corrections

$$\omega = \omega^{\rm Kerr} + \frac{\ell^4 \lambda_{\rm ev}}{M^5}\delta\omega(\chi)$$

- Modes of even and odd parity decouple: $q_{+2} = q_{-2} = \pm 1$
- We can compute $\delta\omega$ either from the $s = +2$ or $s = -2$ equations
- The result should also be independent of $C_{\pm 2}$

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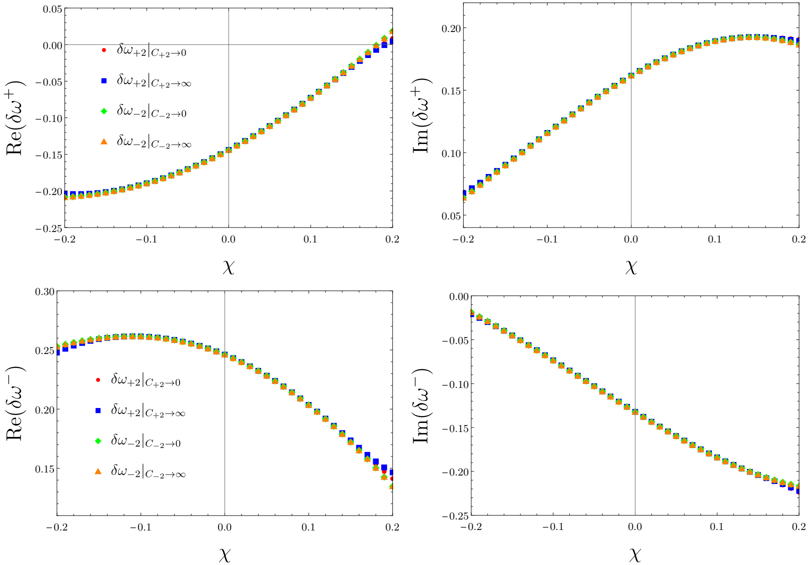
We consider four different estimations of $\delta\omega$

$$\delta\omega_{+2}\Big|_{C_{+2}=0}\,, \quad \delta\omega_{+2}\Big|_{C_{+2}=\infty}\,, \quad \delta\omega_{-2}\Big|_{C_{-2}=0}\,, \quad \delta\omega_{-2}\Big|_{C_{-2}=\infty}$$

Consistency test: they should all agree

Corrections to the QNM frequencies

Result: $(l, m) = (2, 2)$ **modes at** $\mathcal{O}(\chi^6)$



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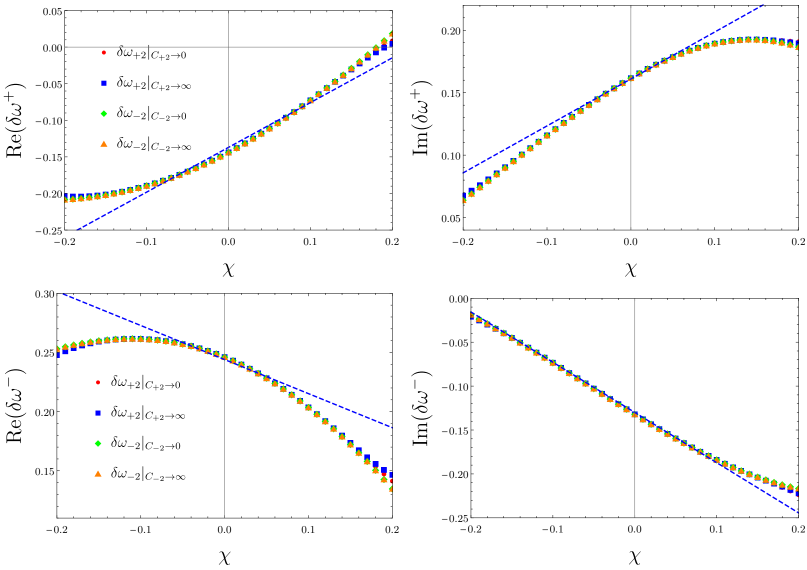

Remarks

- Consistency tests are satisfied
  1. The $s = +2$ and $s = -2$ yield the same results ✓
  2. Results independent of $C_{\pm 2}$ ✓
  3. Reproduce results at linear order in the spin ✓

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- Consistency tests are satisfied
  1. The $s = +2$ and $s = -2$ yield the same results ✓
  2. Results independent of $C_{\pm 2}$ ✓
  3. Reproduce results at linear order in the spin ✓
- Results converge for higher χ if we increase the order of the expansion
- The $s = -2$ equation converges much faster than $s = +2$

Corrections to the QNM frequencies

Larger spins: expansion $\mathcal{O}(\chi^{12})$

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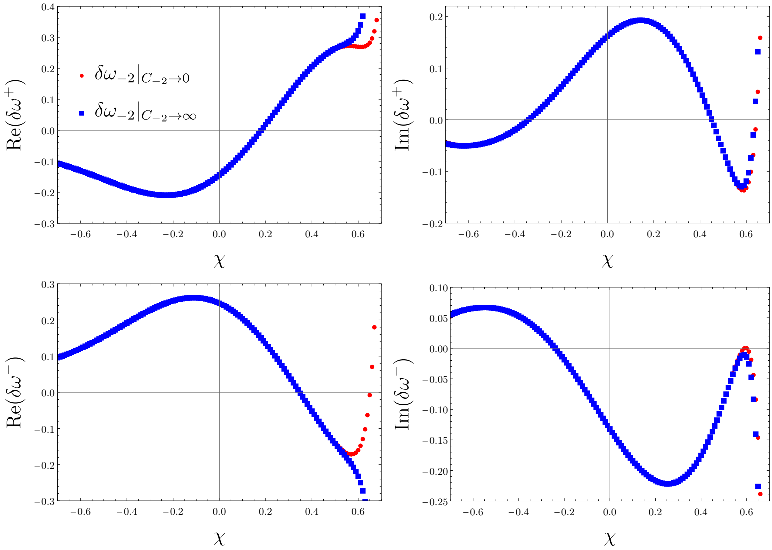

Parity-breaking corrections

$$\omega = \omega^{\text{Kerr}} + \frac{\ell^4 \lambda_{\text{odd}}}{M^5} \delta\omega(\chi)$$

- Modes of even and odd parity are coupled $\longrightarrow$ obtain $q_{\pm 2}$ together with ω by solving all the equations
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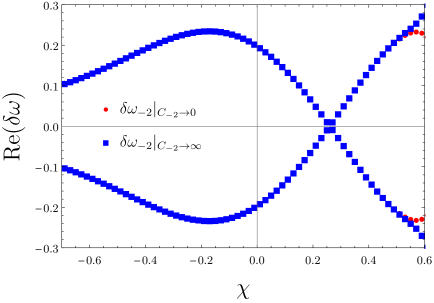
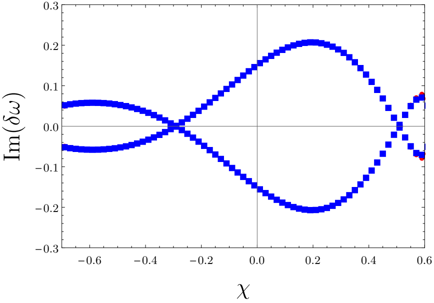
finding agreement

We use the $s = -2$ equation to find the results at higher χ

Corrections to the QNM frequencies

Parity-breaking corrections: $(l, m) = (2, 2)$ mode at $\mathcal{O}(\chi^{12})$





Observe: for the two different polarizations $\delta\omega^{+} = -\delta\omega^{-}$

Conclusions

- New era of experimental gravity. We can probe GR and its extensions. QNMs are a key feature to test these theories
- Calculation of QNMs of highly-rotating BHs in HDG is a highly difficult problem
- We have provided the first working approach to compute these QNMs
- Ongoing work: higher spin and higher order in EFT (quartic terms)
- Future challenges: comparison with other approaches, understanding near-extremal black holes
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Thank you for your attention!

Bonus

Universal $s = +2$ Teukolsky equation

$$\mathcal{O}_2^{(0)}(\Psi_0) + \mathcal{O}_2^{(1)}(\Psi_1) + \mathcal{O}_2^{(2)}(\Psi_0) = 8\pi\left(\mathcal{T}_2^{(0)} + \mathcal{T}_2^{(1)} + \mathcal{T}_2^{(2)}\right)$$

where

$$\begin{aligned}
\mathcal{O}_2^{(0)} &= 2\left[(\text{Þ} - 4\rho - \rho^*)(\text{Þ}' - \rho') - (\eth - 4\tau - \tau'^*)(\eth' - \tau') - 3\Psi_2\right] ,\\
\mathcal{O}_2^{(1)} &= 4\left[2\kappa\left(\text{Þ}' - \rho'^*\right) - 2\sigma\left(\eth' - \tau^*\right) + 2\left(\text{Þ}'\kappa\right) - 2\left(\eth'\sigma\right) + 5\Psi_1\right] ,\\
\mathcal{O}_2^{(2)} &= 6\left[\kappa\kappa' - \sigma\sigma'\right] ,
\end{aligned}$$

$$\begin{aligned}
\mathcal{T}_2^{(0)} &= (\eth - \tau'^* - 4\tau)[(\text{Þ} - 2\rho^*)T_{lm} - (\eth - \tau'^*)T_{ll}]\\
&+ (\text{Þ} - 4\rho - \rho^*)[(\eth - 2\tau'^*)T_{lm} - (\text{Þ} - \rho^*)T_{mm}] ,\\
\mathcal{T}_2^{(1)} &= \frac{1}{2}\left[\sigma\text{Þ} - \kappa\eth\right]T - \left[3\sigma\left(\text{Þ}' - \rho'^*\right) - \sigma'^*\left(\text{Þ} - 4\rho - \rho^*\right) - \text{Þ}\left(\sigma'^*\right)\right]T_{ll}\\
&- 2\left[\sigma\left(\eth - \tau - \tau'^*\right) + \eth\left(\sigma\right)\right]T_{l\bar{m}} + \left[3\sigma\left(\eth' - 2\tau^*\right) + 3\kappa\left(\text{Þ}' - 2\rho'^*\right)\right]T_{lm}\\
&- \left[3\kappa\left(\eth' - \tau^*\right) - \kappa^*\left(\eth - 4\tau - \tau'^*\right) - \eth\left(\kappa^*\right)\right]T_{mm}\\
&+ \left[\kappa\eth + \sigma\left(\text{Þ} - 2\rho - 2\rho^*\right) + 2\text{Þ}\left(\sigma\right) - \Psi_0\right]\left(T_{ln} + T_{m\bar{m}}\right)\\
&- 2\left[\kappa\left(\text{Þ} - \rho - \rho^*\right) + \text{Þ}\left(\kappa\right)\right]T_{nm} ,\\
\mathcal{T}_2^{(2)} &= 3\left[\kappa\kappa'^* T_{ll} + \sigma\sigma^* T_{mm}\right] .
\end{aligned}$$