Black holes and wormholes in semiclassical gravity

Debajyoti Sarkar

IIT Indore

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Based on 1712.09914, 2112.03855, 2212.13208
and ongoing work with Sergey Solodukhin and Yohan Potaux

Broad motivation

- Almost every aspect of a black hole (BH) is fascinating!
- In classical general relativity (GR), Horizons are a key feature and provides a playground for deep theoretical questions.
- These classical solutions can be modified in various ways:
  - Backreactions from quantized matter fields (semiclassical regime)
  - Effects from quantizing geometry itself (quantum gravity)
- We will try to understand the non-perturbative fate of BH horizons within semiclassical GR.

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  - Validity of semiclassical physics near and outside horizon, (locality)
  - Black hole being a quantum system with discrete spectrum and
  - Validity of equivalence principle near the horizon. (no firewall)

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- A recent series of works also deal with derivation of Page curve by considering quantum corrections within semiclassical gravity.

Damour-Solodukhin wormholes

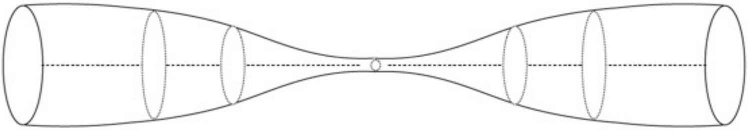
Replace black holes by wormholes via

$$-g_{tt} \to -g_{tt} + \epsilon^2 \qquad \text{with} \qquad \epsilon^2 \ll 1$$

$$ds^2_{wh} = -(g(r) + \epsilon^2)dt^2 + g(r)^{-1}dr^2 + r^2 d\Omega^2_{d-2}$$

Has the features of reproducing late-time quasi-periodic oscillations in two-point functions, instead of an exponential decay indicative of information loss.

Damour-Solodukhin 2005



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- Conclusions

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- Static, spherically symmetric and asymptotically flat BHs in 4 spacetime dimensions. Later on, black holes in 2D RST model (will be elaborated on the talk by Yohan Potaux in the afternoon).

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- Non-perturbative handle is due to the study of conformal anomaly. However, the analysis is near-horizon.
- In 4D, the effective action of semiclassical GR is fixed up to a conformal factor. We will fix this factor by casting the spacetime in a conformal form.

  Fradkin-Tseytlin '84, Dowker-Schofield '90, Mazur-Motola '01

Horizons in classical GR

- Start with a general ansatz for static, spherically symmetric metric

$$ds^2 = \Omega^2(z)\, g_{\mu\nu} dx^\mu dx^\nu = e^{2\sigma(z)} \left(dt^2 + N^2(z) dz^2 + R^2(z)(d\theta^2 + \sin^2\theta\, d\phi^2) \right) .$$

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- In the gauge $N(z) = 1/\Omega(z)$, the two independent Einstein equations then take the form $(r(z) = R(z)\Omega(z))$

$$\begin{aligned} 2rr'' + r'^2 - 1 &= 0\,, \\ \Omega(r'^2 - 1) + 2rr'\Omega' &= 0\,. \end{aligned}$$

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Aspect A: If at $\rho = z = \rho_h$, the corresponding 2-sphere is a minimal area surface, i.e. $r' = 0$, then it is necessarily a horizon. $\Omega = 0$.

Cruściel; Morris-Thorne '88

Horizons in classical GR

- In $N(z) = 1$ gauge, assuming that there exists a horizon at Schwarzschild radial coordinate $r = r_h$ with a finite temperature $T = 1/\beta$, one finds the generic near-horizon behavior as (horizon located at $z \to \infty$)

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- We will test the validity of these two aspects (starting with aspect B) within semiclassical physics where a non-perturbative result can be expected.

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- In semiclassical gravity, the classical Einstein-Hilbert action $W_{EH}[G]$ is supplemented by a quantum effective action $\Gamma[G]$ obtained by integrating out the quantum matter fields.

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$$W_{grav}[G] = W_{EH}[G] + \Gamma[G],$$

whereas conformal anomaly fixes

$$W_{EH}[G] + \Gamma[G] - \Gamma_0[g]$$

Conformal anomaly and effective action

A careful calculation gives:

$$
\begin{aligned}
&(4\pi\beta)^{-1} W_{grav} = \\
&\frac{1}{\kappa}\int dz\, \frac{e^{2\sigma}}{N^2}\Big(R'^2 N + 6RR'N\sigma' + 3R^2N(\sigma'^2+\sigma'') + 2RR''N - N^3 - 2RR'N' - 3R^2N'\sigma' \\
&- \frac{4a}{3(4\pi)^2}\int dz\, \frac{\sigma}{R^2N^5}\Big(N^3 + RR''N - RR'N' - R'^2N\Big)^2 \\
&- \frac{4b}{(4\pi)^2}\int dz\left[\frac{1}{N}\left(\frac{R'^2}{N^2}-1\right)\sigma'^2 + \frac{R^2\sigma'^2}{N^3}\left(\sigma'' + 2\frac{R'}{R}\sigma' - \frac{N'}{N}\sigma'\right) + \frac{R^2\sigma'^4}{2N^3}\right] \\
&+ (4\pi\beta)^{-1}\Gamma_0[g_{\mu\nu}]\,,
\end{aligned}
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$$-\frac{4a}{3(4\pi)^2}\int dz\, \frac{\sigma}{R^2N^5}\Big(N^3 + RR''N - RR'N' - R'^2N\Big)^2$$

$$-\frac{4b}{(4\pi)^2}\int dz\left[\frac{1}{N}\left(\frac{R'^2}{N^2}-1\right)\sigma'^2 + \frac{R^2\sigma'^2}{N^3}\left(\sigma'' + 2\frac{R'}{R}\sigma' - \frac{N'}{N}\sigma'\right) + \frac{R^2\sigma'^4}{2N^3}\right]$$

$$+ (4\pi\beta)^{-1}\Gamma_0[g_{\mu\nu}]\,,$$

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Here,

$$a = \frac{n_0}{120} + \frac{n_{1/2}}{20} + \frac{n_1}{10}\,,$$

$$b = \frac{n_0}{360} + \frac{11 n_{1/2}}{360} + \frac{31 n_1}{180}\,,$$

$\kappa = 8\pi G_N$ and n_s is number of fields of spin s.

Γ_0 on $S^1 \times \mathbb{M}_3$ to $S^1 \times \mathbb{H}_3$

However, in order to study the above-mentioned aspects, we need to know $\Gamma_0[g]$ to be computed in

$$ds^2(g) = dt^2 + ds^2(\gamma)\,, \quad ds^2(\gamma) = N^2(z)dz^2 + R^2(z)(d\theta^2 + \sin^2\theta d\phi^2)\,,$$

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$$\begin{aligned}
\Gamma_0[S_1^\beta \times \mathbb{M}_3] &= -\frac{\pi^2}{90\beta^3}\left(n_0 + \frac{7}{2}n_{1/2} + 2n_1\right)\int_{M_3} 1 + \frac{1}{144\beta}\left(n_{1/2} + 4n_1\right)\int_{M_3} \mathcal{R}_{\mathbb{M}} + \ldots \\
&= -\frac{2\pi^3}{45}\frac{c_H}{\beta^3}\int dz N(z)R^2(z) + \frac{\pi}{18}\frac{\lambda_H}{\beta}\int dz(N(z) + R'^2 N^{-1})|_{\text{near horizon}}
\end{aligned}$$

where

$$\mathcal{R}_{\mathbb{M}} = -\frac{2}{R^2 N^3}\left(2RNR'' - 2RR'N' - N^3 + NR'^2\right),$$

$$c_H = n_0 + \frac{7}{2}n_{1/2} + 2n_1,\ \lambda_H = n_{1/2} + 4n_1$$

Gusev-Zelnikov ’98, Hung-Myers-Smolkin ’14

Fate of aspect B in semiclassical theory

For the near-horizon ($z \to \infty$) ansatz

$$\Omega(z) = e^{-2\pi z/\beta} + \dots\,, \quad R(z) = r_h e^{2\pi z/\beta} + \dots\,,$$

the semiclassical equations give

$$\begin{aligned}
\delta_\sigma W_{grav}: \quad 0 &= \mathcal{O}\left(e^{-4\pi z/\beta}\right), \\
\delta_N W_{grav}: \quad 0 &= (360b - 2c_H - 10\lambda_H)\,\frac{\pi^2}{180\beta^4} R^2(z) \\
&= -\left(n_0 + 6n_{1/2} - 18n_1\right)\frac{\pi^2}{180\beta^4} R^2(z)\,.
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- Curiously, the equations are satisfied for $\mathcal{N} = 4$ SYM theory.
- The above statement remains true even at subleading orders.

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- Hence we impose $r' = \Omega' = 0$ and perform a local analysis of the semiclassical equations near the turning point.
- The equations are ($N = 1/\Omega$ gauge)

$$\delta_\sigma W_{grav} : \frac{2\Omega}{\kappa}\left(1 - 2rr'' - r^2\frac{\Omega''}{\Omega}\right) + \frac{\bar{a}}{r^2\Omega}\left(\Omega + \Omega rr'' - r^2\Omega''\right)^2 + \bar{b}\Omega'' = 0\,,$$

$$\delta_N W_{grav} : -\frac{\Omega^2}{\kappa} - \frac{\bar{a}}{r^2}\ln\Omega^{-1}\left[(\Omega rr'' - r^2\Omega'')^2 - \Omega^2\right] - \frac{\gamma r^2}{\beta^4\Omega^2} + \frac{\lambda}{\beta^2} = 0\,,$$

where $\bar{a} = a/12\pi^2$, $\bar{b} = b/2\pi^2$, $\gamma = c_H\pi^2/90$ and $\lambda = \lambda_H/72$.

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- The best way to proceed is by defining a new variable y such that

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$$y^2 = 1 - \frac{r^2}{\kappa \bar{a} \ln \Omega^{-1}} \left(\frac{\gamma \kappa r^2}{\beta^4 \Omega^4} - \frac{\lambda \kappa}{\beta^2 \Omega^2} + 1 \right) .$$

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- The positivity condition $y^2 \geqslant 0$ imposes important constraints on possible values of Ω and β. One can show that this choice is consistent with other constraints.
- Since $a > 0$, $b > 0$, $\gamma > 0$, the positivity condition $y^2 > 0$ can be rewritten in the form of an inequality

$$\Omega^4 \ln\frac{\Omega_0}{\Omega} > \frac{\gamma r^4}{\bar{a}\beta^4} > 0\,, \quad \text{with} \quad \Omega_0 = e^{-\frac{r^2}{\bar{a}\kappa}}$$

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- In turn, this can be viewed as a bound on the possible 'temperature',

$$T^4 = \frac{1}{\beta^4} < \frac{\bar{a}}{\gamma r^4}\Omega^4 \ln\frac{\Omega_0}{\Omega} < \frac{1}{4}\frac{\bar{a}}{\gamma r^4}\Omega_0^4\,,$$

  where in the last inequality we used that $\Omega^4 \ln \Omega_0/\Omega \leqslant \Omega_0^4/(4e) \leqslant \Omega_0^4/4$.

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  where in the last inequality we used that $\Omega^4 \ln \Omega_0/\Omega \leqslant \Omega_0^4/(4e) \leqslant \Omega_0^4/4$.

- Temperature of the semiclassical geometry that replaces the classical black hole is much less than the Hawking temperature.

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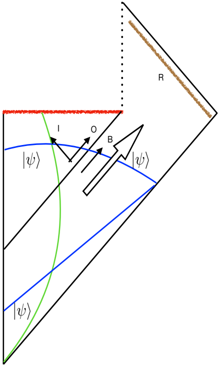
- Bound on temperature. Experimental signatures for early universe black holes. Lower temperature with longer life span.
- All the results boil down to the known black hole case in the classical limit.
- Analytic and exact results for 2D Dilaton gravity. Static cases yield no horizon scenarios for Boulware state. Dynamic case provides a framework to pose information loss problem for the hybrid state, where a Page curve can be analytically obtained.

Thank you for your attention!

Backup slides

Motivation from Information Problem

The old version of information problem due to Hawking (pure state $\Rightarrow$ mixed state) can be restated by considering BH complementarity, which demands that the early Hawking radiations are *not* independent of the sub-horizon modes. However, 'nice-slice' physics demand otherwise.



$|\psi\rangle$: State of the infalling information at various spaeclike nice slices.
I and O: Ingoing and outgoing modes inside horizon.
B: Modes just outside the horizon.
R: Modes in early Hawking radiation which are maximally entangled with O after Page time t_P.

Equations of motion (EOM)

$\delta_\sigma W_{grav}$:

$$\frac{2e^{2\sigma}}{\kappa}\left[\frac{2RR''}{N}+\frac{6RR'\sigma'}{N}-\frac{2RR'N'}{N^2}+\frac{R'^2}{N}+\frac{3R^2\sigma''}{N}-\frac{3R^2\sigma'N'}{N^2}+\frac{3R^2\sigma'^2}{N}-N\right]$$

$$+\frac{a}{6\pi^2}\left[-\frac{R''}{RN}-\frac{R''^2}{2N^3}-\frac{R'^3N'}{RN^4}-\frac{R'^2N'^2}{2N^5}+\frac{R'N'}{RN^2}-\frac{R'^4}{2R^2N^3}+\frac{R'^2}{R^2N}+\frac{R'R''N'}{N^4}+\frac{R'^2}{R\ldots}\right.$$

$$\left.-\frac{N}{2R^2}\right]+\frac{b}{\pi^2N^4}\left[RNR''\sigma'^2+\frac{1}{2}NR'^2\sigma''-3RR'\sigma'^2N'-\frac{3}{2}R'^2\sigma'N'+RNR'\sigma'^3+NR'^2\sigma\ldots\right.$$

$$\left.+2RNR'\sigma'\sigma''+NR'R''\sigma'-\frac{3}{2}R^2\sigma'^3N'+\frac{3}{2}R^2N\sigma'^2\sigma''-\frac{1}{2}N^3\sigma''+\frac{1}{2}N^2\sigma'N'\right]=0$$

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$$\frac{2e^{2\sigma}}{\kappa}\left[\frac{2RR''}{N}+\frac{6RR'\sigma'}{N}-\frac{2RR'N'}{N^2}+\frac{R'^2}{N}+\frac{3R^2\sigma''}{N}-\frac{3R^2\sigma'N'}{N^2}+\frac{3R^2\sigma'^2}{N}-N\right]$$

$$+\frac{a}{6\pi^2}\left[-\frac{R''}{RN}-\frac{R''^2}{2N^3}-\frac{R'^3N'}{RN^4}-\frac{R'^2N'^2}{2N^5}+\frac{R'N'}{RN^2}-\frac{R'^4}{2R^2N^3}+\frac{R'^2}{R^2N}+\frac{R'R''N'}{N^4}+\frac{R'^2}{R\ldots}\right.$$

$$\left.-\frac{N}{2R^2}\right]+\frac{b}{\pi^2N^4}\left[RNR''\sigma'^2+\frac{1}{2}NR'^2\sigma''-3RR'\sigma'^2N'-\frac{3}{2}R'^2\sigma'N'+RNR'\sigma'^3+NR'^2\sigma\ldots\right.$$

$$\left.+2RNR'\sigma'\sigma''+NR'R''\sigma'-\frac{3}{2}R^2\sigma'^3N'+\frac{3}{2}R^2N\sigma'^2\sigma''-\frac{1}{2}N^3\sigma''+\frac{1}{2}N^2\sigma'N'\right]=0$$

$\delta_N W_{grav}$:

$$0=\frac{1}{4\pi\beta}\delta_N\Gamma_0+\frac{e^{2\sigma}}{\kappa N^2}\left[\left(R'+R\sigma'\right)\left(R'+3R\sigma'\right)-N^2\right]$$

$$+\frac{b\sigma'^2}{8\pi^2N^4}\left[-2N^2+8RR'\sigma'+6R'^2+3R^2\sigma'^2\right]$$

$$+\frac{a}{12\pi^2R^2}\left[\frac{R^2\sigma R''^2}{N^4}+\frac{2R^2R'^2\sigma'N'}{N^5}+\frac{2RR'^3\sigma'}{N^4}-\frac{2RR'\sigma'}{N^2}+\frac{2R^2\sigma R'^2N''}{N^5}\right.$$

$$\left.-\frac{5R^2\sigma R'^2N'^2}{N^6}+\frac{\sigma R'^4}{N^4}-\frac{2R^2R'''\sigma R'}{N^4}-\frac{2R^2R'R''\sigma'}{N^4}+\frac{4R^2\sigma R'R''N'}{N^5}-\sigma\right]$$

Aspect A for $\lambda \neq 0$

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The temperature bound becomes

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- For large BHs, i.e. for $\kappa/r^2 << 1$,

$$\Omega'' = \frac{\Omega}{r^2\left(3 - \frac{\bar{b}\kappa}{2r^2}\right)}\left(1 - 2y + \frac{\bar{a}\kappa}{2r^2}(1+y)^2\right),$$

$$rr'' = \frac{1}{\left(3 - \frac{\bar{b}\kappa}{2r^2}\right)}\left[\left(1 + \frac{\bar{a}\kappa}{2r^2}\right) + \left(1 - \frac{\left(-\bar{a} + \frac{\bar{b}}{2}\right)\kappa}{r^2}\right)y + \frac{\bar{a}\kappa}{2r^2}y^2\right].$$

can be shown to be positive. This fixes regimes of validity of the variable y.