Spontaneous scalarization of Gauss-Bonnet black holes - stationary solutions and dynamics

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Plan of the talk

- Extended scalar-tensor theories of gravity
- Basic ideas behind the spontaneous scalarization
- Spontaneous scalarization of Schwazschild and Kerr black hole in scalar-Gauss-Bonnet gravity
- Dynamics of the spontaneous scalarization in scalar-Gauss-Bonnet gravity
- Core collapse in scalar-Gauss-Bonnet gravity
- Beyond the spontaneous scalarization
- Conclusions

Scalar-tensor theories of gravity

- In scalar-tensor theories of gravity the gravitational interaction is propagated not only by the spacetime metric $g_{\mu\nu}$ but also by an additional (dynamical) scalar field φ.

$$gravitational\ field = (g_{\mu\nu}, \varphi)$$

- The scalar-tensor extensions (modifications) of GR are naturally predicted by the unifying theories (Kaluza-Klein theories) , quantum gravity and effective field theories.

- The most natural modifications of this class are the extended scalar-tensor theories where the usual Einstein-Hilbert action is supplemented with all possible algebraic curvature invariants of second order with a dynamical scalar field non-minimally coupled to these invariants (S. Weinberg, 0804.4291 [hep-th]).

Extended scalar-tensor theories of gravity

- The general form of the Einstein frame action for the ESTT is

$$S = \frac{1}{16\pi}\int \sqrt{-g}d^4x\Big[R - 2\nabla_\mu\varphi\nabla^\mu\varphi - V(\varphi) + f_1(\varphi)R^2 + f_2(\varphi)R_{\mu\nu}R^{\mu\nu} + f_3(\varphi)R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + f_4(\varphi)\,{}^*RR\Big] + S_{\text{matter}}\left[\chi, \gamma(\varphi)g_{\mu\nu}\right]$$

- The classical scalar-tensor theories are recovered for and $\gamma(\varphi) \neq 0$ *and* $f_1(\varphi) = f_2(\varphi) = f_3(\varphi) = f_4(\varphi) = 0$

- There are two very interesting and very popular sectors of ESTT, namely the dynamical Chern-Simons gravity and the scalar-Gauss-Bonnet gravity.

- The dynamical Chern-Simons gravity is defined by the action

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left[R - 2\nabla_\mu\varphi\nabla^\mu\varphi + 8\lambda^2 f(\varphi)^{\star}\mathcal{R}\mathcal{R}\right]$$

$\star RR = \frac{1}{2}\epsilon^{\mu\nu\gamma\delta} R_{\gamma\delta}{}^{\alpha\beta} R_{\mu\nu\alpha\beta}$ is the Pontryagin invariant

- The theory is parity violating – the deviations from general relativity occur only for systems that violate parity through the presence of a preferred axis. Such systems are for example the isolated rotating black holes.

- The field equations of the dynamical Chern-Simons gravity are of third order.

- The equations of the ESTT in their most general form are of order higher than two. This in general can lead to the Ostrogradski instability and to the appearance of ghosts.

- **The second special case is the Gauss-Bonnet sector (or scalar - Gauss-Bonnet theory) which is free from the mentioned problems.**
- In scalar-Gauss-Bonnet theory (sGB) the scalar field is coupled exactly to the Gauss-Bonnet invariant.
- The field equations are of second order as in general relativity and the theory is free from ghosts.

The general action of sGB theory

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left[R - 2\nabla_\mu\varphi\nabla^\mu\varphi + \lambda^2 f(\varphi)\mathcal{R}^2_{GB}\right]$$

$$\mathcal{R}^2_{GB} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}$$

Field equations

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Gamma_{\mu\nu} = 2\nabla_\mu\varphi\nabla_\nu\varphi - g_{\mu\nu}\nabla_\alpha\varphi\nabla^\alpha\varphi,$$

$$\nabla_\alpha\nabla^\alpha\varphi = -\frac{\lambda^2}{4}\frac{df(\varphi)}{d\varphi}\mathcal{R}^2_{GB},$$

where

$$\Gamma_{\mu\nu} = -R(\nabla_\mu\Psi_\nu + \nabla_\nu\Psi_\mu) - 4\nabla^\alpha\Psi_\alpha\left(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}\right) + 4R_{\mu\alpha}\nabla^\alpha\Psi_\nu + 4R_{\nu\alpha}\nabla^\alpha\Psi_\mu$$
$$-4g_{\mu\nu}R^{\alpha\beta}\nabla_\alpha\Psi_\beta + 4R^\beta_{\mu\alpha\nu}\nabla^\alpha\Psi_\beta$$

with

$$\Psi_\mu = \lambda^2\frac{df(\varphi)}{d\varphi}\nabla_\mu\varphi$$

Special class of sGB theories

- We consider coupling functions which are analytic in φ

$$f(\varphi) = f_0 + f_1\,\varphi + \frac{1}{2}\,f_2\,\varphi^2 + \ \ldots$$

- Since $R^2_{\{GB\}}$ is a topological invariant f_0 plays no role and we choose $f_0 = 0$.
- Without loss of generality we can also put $f_2 = \epsilon = \pm 1$ (by redefining the coupling constant $\lambda^2 \to |f_2|\,\lambda^2$)
- From now on we will focus on sGB theories with $f_1 = 0 <=> \frac{df}{d\varphi}(0) = 0$!

Two subclasses: $\epsilon = 1$ and $\epsilon = -1$.

- This class of sGB theories is indistinguishable from GR in the weak field regime and can differ from GR in strong curvature regime only.

Observational constraints

- Obsevational constraints on the Gauss-Bonnet coupling constant λ can be impose by using real gravitational wave events, for example GW190814 and GW151226. (L. Wong, C. Herdeiro and E. Radu, PRD (2022))

- More stringent constraints on λ can be imposed by using observations of pulsars in close binary systems (V. Danchev, D. Doneva and S.Y., PRD (2022)).

- The maximum mass of a static scalarized black hole is roughly $18\, M_{\odot}$ ($\lambda < 18\, M_{\odot}$)

Black Hole Spontaneous Scalarization

- The phenomenon of spontaneous scalarization was discovered by Damour and Esposito-Farese almost 20 years ago. In a seminal paper (T. Damour and G. Esposito-Farese, PRL (1993)) they discovered the spontaneous scalarization of neutron stars in classical scalar-tensor theories.

Spontaneous scalarization in simple words

- Beyond certain threshold in the mass of objects (in the case of the classical scalar-tensor theories) or in the spacetime curvature (in the case of ESTT), GR solutions become unstable within the bigger scalar-tensor theory. The instability leads to the development of a nontrivial scalar field and the compact objects acquire scalar hair - the compact objects get spontaneously scalarized.

Spontaneous scalarization – basic idea

- Why the spontaneous scalarization is so interesting and important? The spontaneous scalarization is the only known dynamical mechanism for endowing black holes (and other compact objects) with scalar hair without altering the predictions in the weak field limit.

- New development, July 2021 : There exists new fully nonlinear mechanism for generating black hole scalar hair different from the spontaneous scalarization! (D. Doneva and S. Y. (2021))

- In order to get insight into the theory of the spontaneous scalarization, we shall demonstrate the theoretical machinery of the black hole spontaneous scalarization on the case of static and spherically symmetric black holes in sGB gravity.
(D. Doneva and S. Y. , PRL (2018); P. Kanti et. al, PRL (2018); H. Silva et. al., PRL (2018))

Static and spherically symmetric field equations

$$ds^2 = -e^{2\Phi(r)}dt^2 + e^{2\Lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

$$\frac{2}{r}\left[1+\frac{2}{r}(1-3e^{-2\Lambda})\Psi_r\right]\frac{d\Lambda}{dr} + \frac{(e^{2\Lambda}-1)}{r^2} - \frac{4}{r^2}(1-e^{-2\Lambda})\frac{d\Psi_r}{dr} - \left(\frac{d\varphi}{dr}\right)^2 = 0,$$

$$\frac{2}{r}\left[1+\frac{2}{r}(1-3e^{-2\Lambda})\Psi_r\right]\frac{d\Phi}{dr} - \frac{(e^{2\Lambda}-1)}{r^2} - \left(\frac{d\varphi}{dr}\right)^2 = 0,$$

$$\frac{d^2\Phi}{dr^2} + \left(\frac{d\Phi}{dr}+\frac{1}{r}\right)\left(\frac{d\Phi}{dr}-\frac{d\Lambda}{dr}\right) + \frac{4e^{-2\Lambda}}{r}\left[3\frac{d\Phi}{dr}\frac{d\Lambda}{dr} - \frac{d^2\Phi}{dr^2} - \left(\frac{d\Phi}{dr}\right)^2\right]\Psi_r$$
$$-\frac{4e^{-2\Lambda}}{r}\frac{d\Phi}{dr}\frac{d\Psi_r}{dr} + \left(\frac{d\varphi}{dr}\right)^2 = 0,$$

$$\frac{d^2\varphi}{dr^2} + \left(\frac{d\Phi}{dr} - \frac{d\Lambda}{dr} + \frac{2}{r}\right)\frac{d\varphi}{dr}$$
$$-\frac{2\lambda^2}{r^2}\frac{df(\varphi)}{d\phi}\left\{(1-e^{-2\Lambda})\left[\frac{d^2\Phi}{dr^2} + \frac{d\Phi}{dr}\left(\frac{d\Phi}{dr}-\frac{d\Lambda}{dr}\right)\right] + 2e^{-2\Lambda}\frac{d\Phi}{dr}\frac{d\Lambda}{dr}\right\} = 0,$$

Spontaneous scalarization – basic idea

- Recall that we consider sGB theories which are indistinguishable from GR in the weak field limit, i.e. $\frac{df}{d\varphi}(0) = 0$ (and with the normalization $\frac{d^2 f}{d\varphi^2}(0) = \epsilon = \pm 1$. We shall first focus on the case $\epsilon = 1$.

- From the dimensionally reduced field equations it is clear that the usual Schwarzschlild black hole solution is also a black hole solution to the sGB gravity with a trivial scalar field $\varphi = 0$.

- We shall however show that the Schwarzschild solution within the certain range of the mass is unstable in the framework of our class of sGB theories.

- For this purpose we consider the perturbations of the Schwarzschild solution (with mass **M)** within the framework of the described class of sGB theories.

Spontaneous scalarization – basic idea

- In the considered class of sGB theories the equations governing the perturbations of the metric are decoupled from the equation governing the perturbation of the scalar field. The equations for metric perturbations are in fact the same as those in the pure Einstein.

- The equation governing the scalar perturbations is

$$\Box_{(0)}\delta\varphi + \frac{1}{4}\lambda^2\mathcal{R}^2_{GB(0)}\delta\varphi = 0, \qquad => \qquad \Box_{(0)}\delta\varphi = \mu^2_{eff}\delta\varphi$$

Tachyonic instability $\mu^2_{eff} = -\frac{1}{4}\lambda^2 R^2_{GB(0)} < 0,$ $\quad R^2_{GB(0)} = \frac{48M^2}{r^6}$

- The equation can be cast in Schrodinger form $\delta\varphi = \frac{u(r)}{r}e^{-i\omega t}Y_{lm}(\theta,\phi),$

$$\frac{d^2u}{dr_*^2} + [\omega^2 - U(r)]u = 0$$

with an effective potential

$$U(r) = \left(1 - \frac{2M}{r}\right)\left[\frac{2M}{r^3} + \frac{l(l+1)}{r^2} - \lambda^2\frac{12M^2}{r^6}\right]$$

Spontaneous scalarization – basic idea

- A sufficient condition for the existence of a unstable mode is

$$\int_{-\infty}^{+\infty} U(r_*)dr_* = \int_{2M}^{\infty} \frac{U(r)}{1-\frac{2M}{r}} dr < 0.$$

- We can conclude that the Schwarzshild black holes with mass satisfying $\boldsymbol{M^2 < 0.3\,\lambda^2}$ are unstable within the framework of the sGB gravity. The Schwarzshild black holes become unstable when the curvature of the horizon exceeds a certain critical value $K_H > 8.3/\lambda^4$.

- In other words, the scalar field can be excited only in the strong curvature regime when the spacetime curvature exceeds the critical value $K_H > 8.3/\lambda^4$.

- This result naturally leads us to the conjecture that, in our class of GB theories and in the interval where the Schwarzschild is unstable, there exist black hole solutions with nontrivial scalar field.

Black holes with curvature induced scalarization in GB gravity

- In order to obtain the black hole solutions with a nontrivial scalar field we solve numerically the system of reduced field equations with the following boundary conditions:

$$\Phi|_{r\to\infty} \to 0, \quad \Lambda|_{r\to\infty} \to 0, \quad \varphi|_{r\to\infty} \to 0$$

$$e^{2\Phi}|_{r\to r_H} \to 0, \quad e^{-2\Lambda}|_{r\to r_H} \to 0.$$

- The regularity of the scalar field and its first and second derivatives on the black hole horizon gives one more condition. Black hole solutions exist only when

$$r_H^4 > 24\lambda^4 \left(\frac{df}{d\varphi}(\varphi_H)\right)^2$$

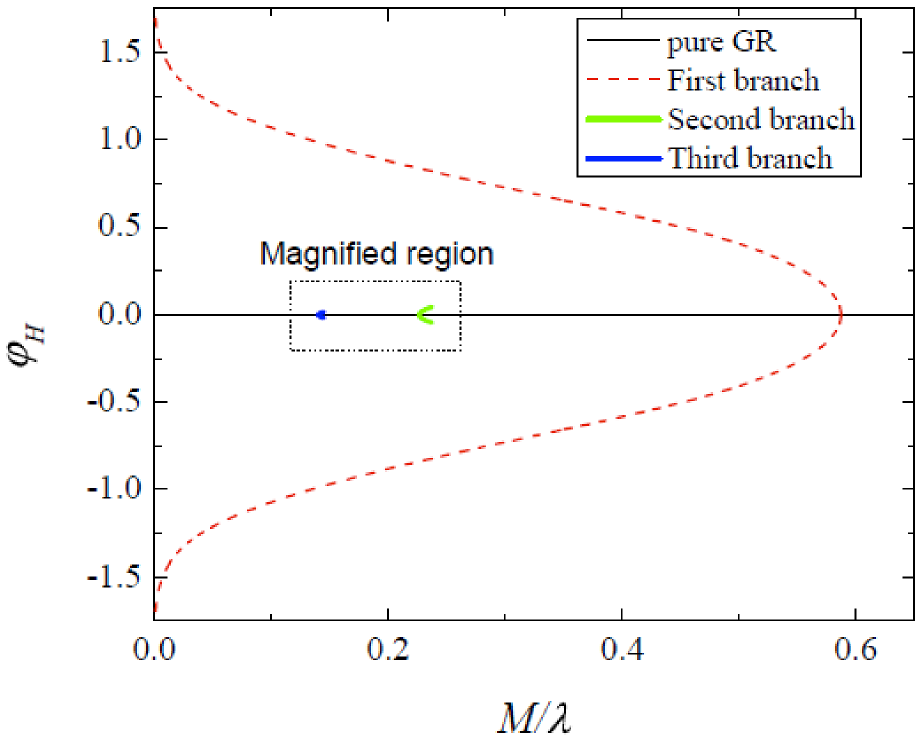
- Solving the fully nonlinear problem we found that, in addition to the trivial (Schwarzschild) solution, there exist several nontrivial branches of black hole solutions.

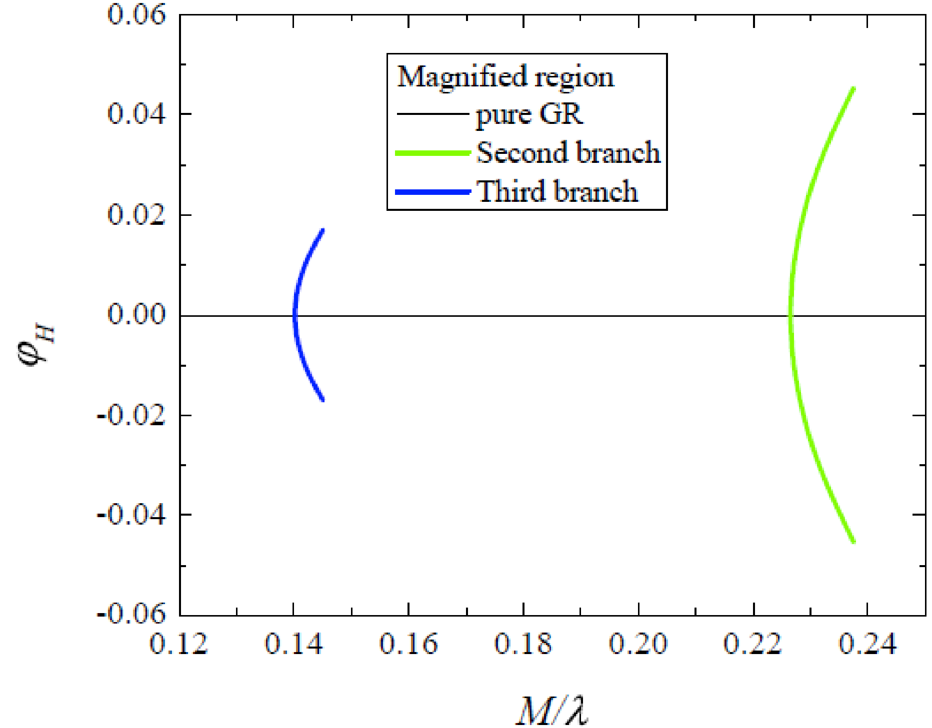
- The different nontrivial branches of solutions are characterized by different number of zeros of the scalar field. For the first branch (the red dashed line in Fig. 1) there are no zeros of $\boldsymbol{\varphi}$ as one can see in the left panel of Fig. 2, the next one (green line) has one zero while the third one (blue line) has two zeros as one can see in Fig. 3.

- All the nontrivial branches start from a bifurcation point at the trivial branch and they span either to $\boldsymbol{r_H = 0}$ (the first nontrivial branch) or they are terminated at some nonzero $\boldsymbol{r_H}$ (all the other nontrivial branches). The reason for termination of the branch at nonzero $\boldsymbol{r_H}$ is that beyond this point the black hole existence condition is violated.

Scalarized Schwarzschild black hole

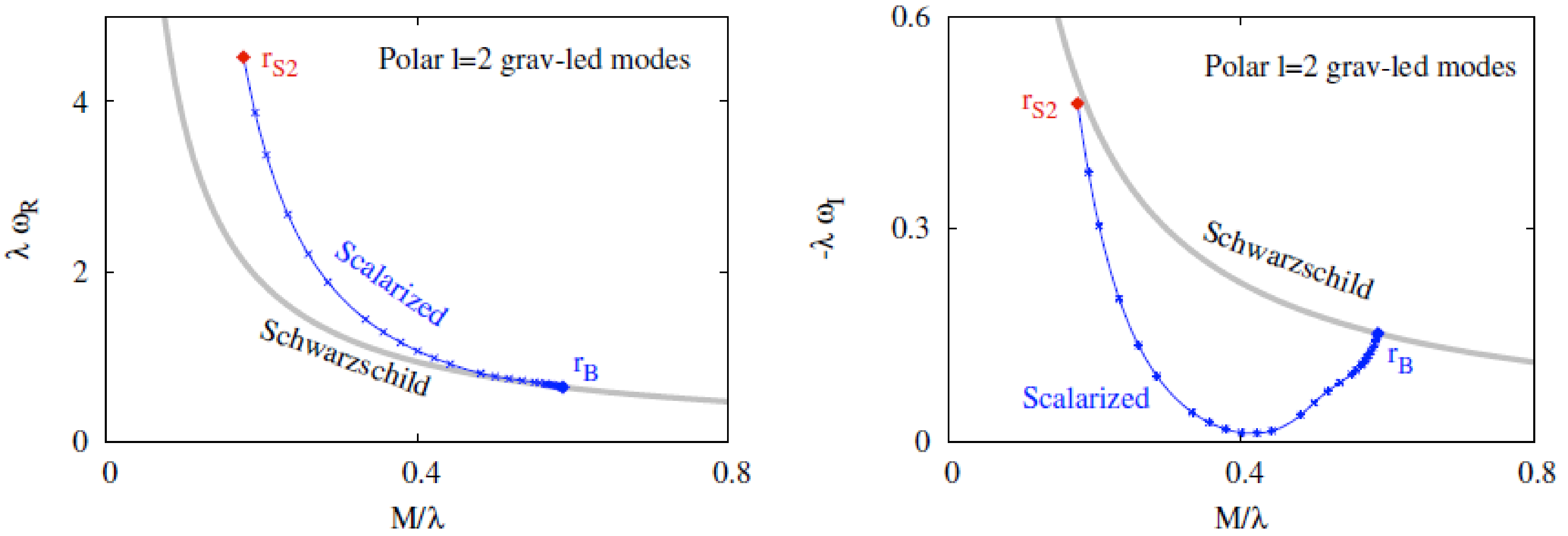
Results for coupling function $f(\varphi) = \frac{1}{12}\left[1 - \exp(-6\varphi^2)\right]$





Stability of the scalarized solutions

- We investigated the axial and polar quasinormal modes of the scalarized solutions. On this base we showed that the fundamental branch of the scalarizied solutions is stable while the other branches are unstable. (Blazquez-Salcedo et. al.)



Spin-induced scalarization of Kerr black holes

- The spin induced scalarization of Kerr black hole is in fact curvature induced scalarization, however with $\frac{d^2 f}{d\varphi^2}(0) = \epsilon = -1$. In this case the Schwarzschild black hole can not scalarize (it is stable against scalar perturbations) and only the rotating Kerr black hole can scalarize - hence the name “spin-induced”.

(A. Dima, E. Barausse, N. Franchini, T. Sotiriou, PRL (2020);
D. Doneva, C. Kruger, L. Collodel and S. Y. , PRD (2020))

- In order to study the stability of the Kerr black hole within the sGB gravity we follow the known method. Tensor and scalar perturbations decouple and only the scalar ones can trigger an instability.

$$\Box_{(0)}\delta\varphi + \frac{\epsilon}{4}\lambda^2\mathcal{R}^2_{GB(0)}\delta\varphi = 0$$

- The Gauss-Bonnet invariant for the Kerr solution is given by

$$\mathcal{R}^2_{GB(0)} = \frac{48M^2}{\Sigma^6}(r^2 - a^2\cos^2\theta)(r^4 - 14a^2r^2\cos^2\theta + a^4\cos^4\theta)$$

where $\Sigma = r^2 + a^2\cos^2\theta$.

- Before presenting the explicit form of the perturbation equation, it is convenient to introduce the Kerr azimuthal coordinate ϕ_* and the tortoise coordinate x defined by

$$d\phi_* = d\phi + \frac{a}{\Delta}\, dr \qquad \mathrm{d}x = \frac{r^2+a^2}{\Delta}\, \mathrm{dr}$$

with $\Delta = r^2 - 2Mr + a\hat{}2$

- Working with ϕ_* allows us to get rid of some unphysical pathologies near the horizon

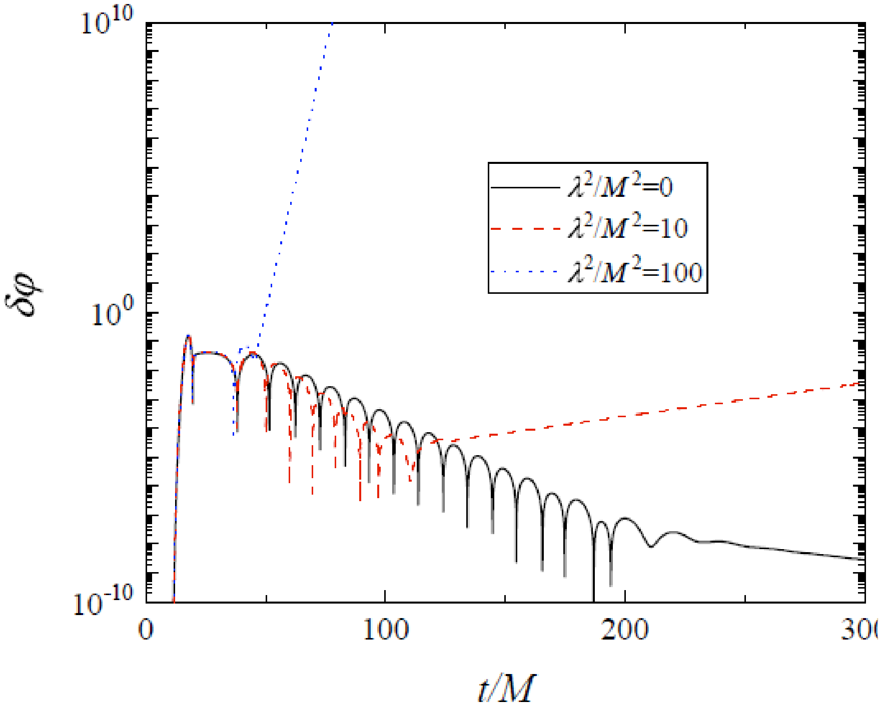
- The explicit form of the perturbation equation is

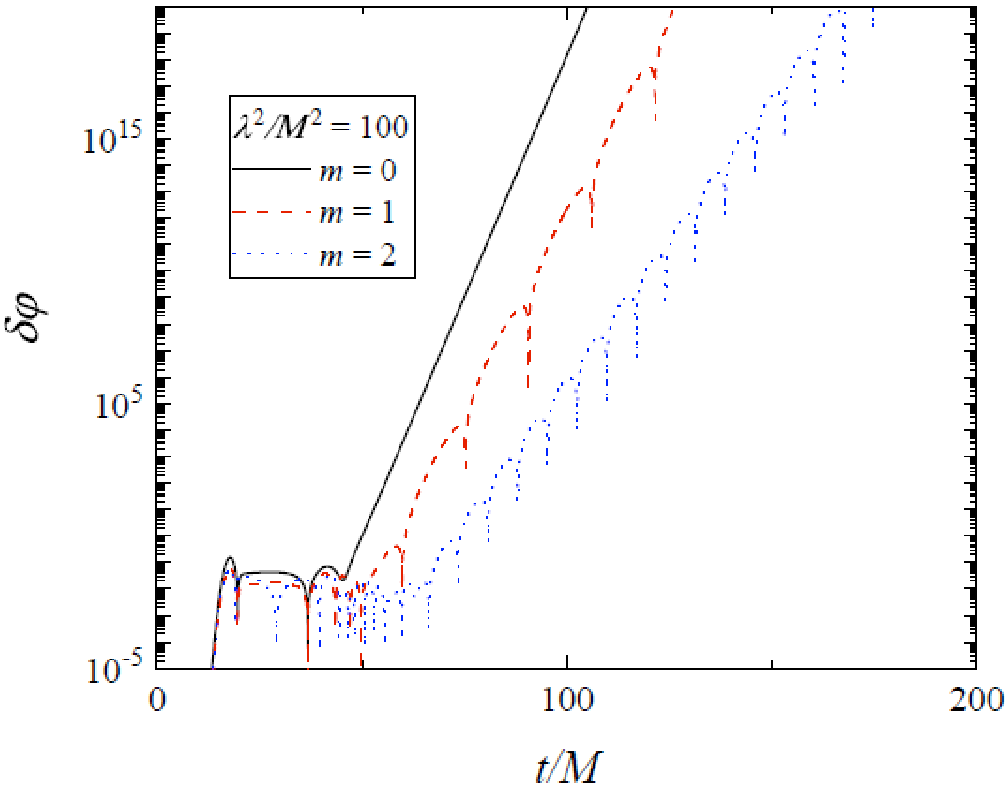
$$-\left[(r^2+a^2)^2-\Delta a^2\sin^2\theta\right]\partial_t^2\delta\varphi+(r^2+a^2)^2\partial_x^2\delta\varphi+2r\Delta\partial_x\delta\varphi-4Mar\partial_t\partial_{\phi_*}\delta\varphi$$
$$+2a(r^2+a^2)\partial_x\partial_{\phi_*}\delta\varphi+\Delta\left[\frac{1}{\sin\theta}\partial_\theta(\sin\theta\partial_\theta\delta\varphi)+\frac{1}{\sin^2\theta}\partial^2_{\phi_*}\delta\varphi\right]$$
$$=\lambda^2\frac{12M^2\Delta}{\Sigma^5}(r^2-a^2\cos^2\theta)(r^4-14a^2r^2\cos^2\theta+a^4\cos^4\theta)\delta\varphi.$$

- Since the background solution is axisymmetric we can separate the azimuthal dependence in the standard way

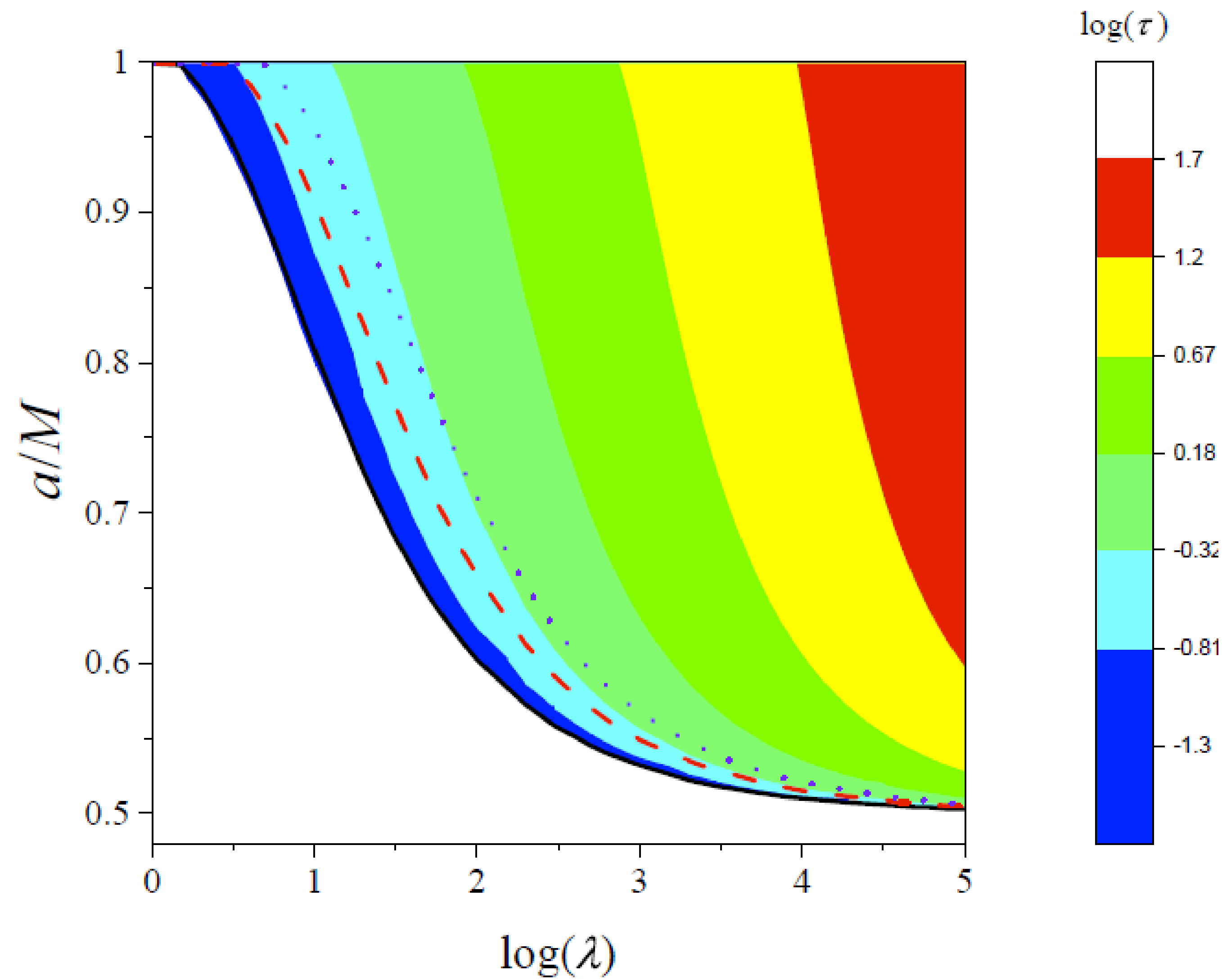
$$\delta\varphi(t,x,\theta,\phi_*)=\delta\varphi(t,x,\theta)e^{im\phi_*},$$

Scalarized Kerr black hole

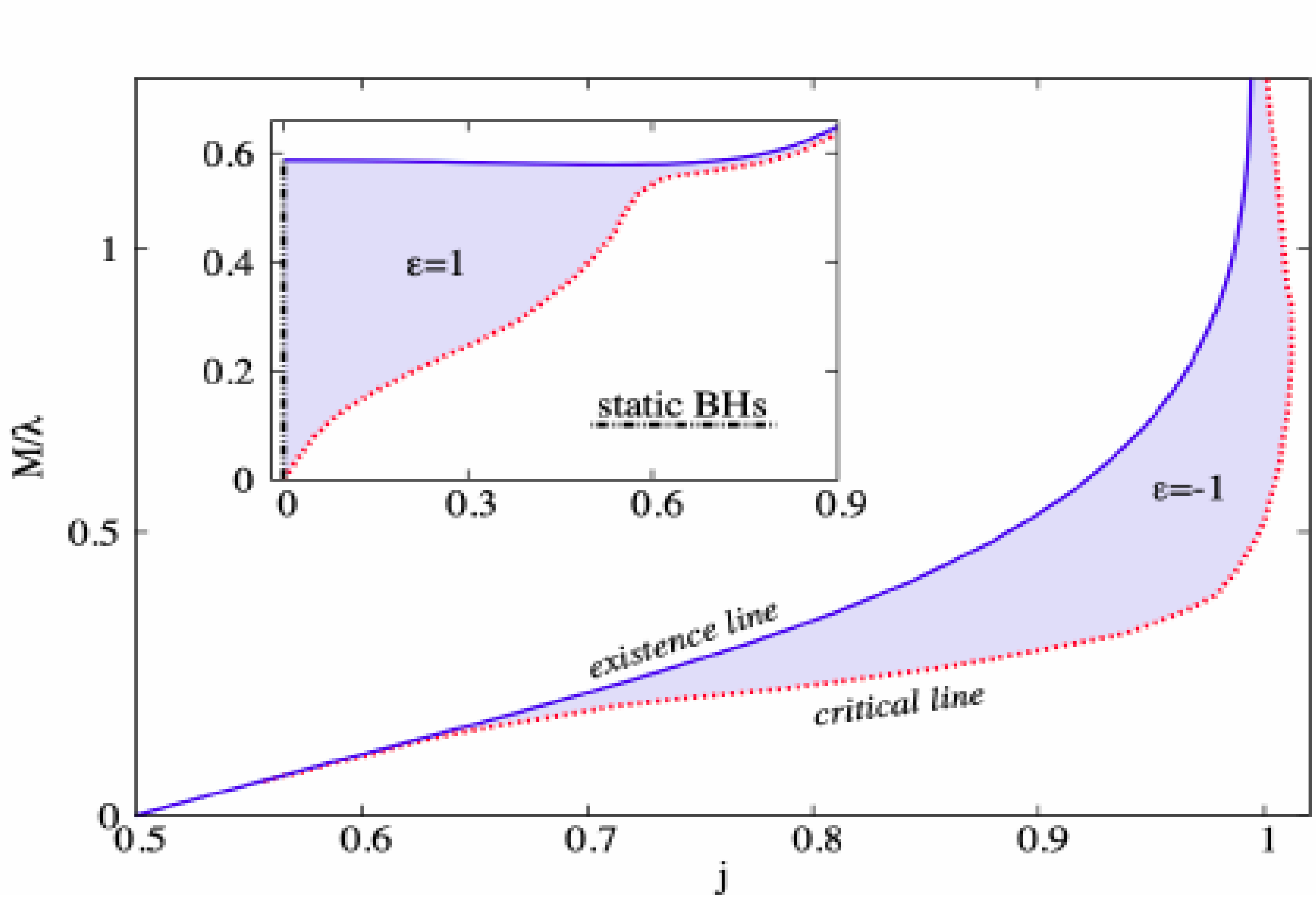




Scalarized Kerr black hole



- The stationary solutions describing scalarized Kerr black holes were constructed numerically by C. Herdeiro, E. Radu, H. Silva T. Sotiriou, and N. Yunes, PRL (2020) and by E. Berti, L. Collodel, B. Kleihaus and J. Kunz, PRL (2020)



Dynamics of the spontaneous scalarization

- The tachyonic instability that triggers the spontaneous scalarization is to a large extent well understood and extensively studied both in the static and the rotating regimes.

- The same applies to the scalarized solutions produced by the spontaneous scalarization, both in the static and the rotating cases.

- However, the very dynamics of the spontaneous scalarization, i.e. the process of forming scalarized black holes from non-scalarized ones, is much less studied and understood.

- The dynamics of spontaneous scalarization of the Kerr solution is studied in D. Doneva and S. Y., PRD (2021).

- Solving the scalarization dynamics in its full generality and nonlinearity for rotating black holes is a formidable task and that is why, a first step, we consider a simplified but nonlinear dynamical model.

- We study the spontaneous scalarization dynamics in the “decoupling limit” – we numerically evolve the nonlinear scalar field equation on the fixed geometric background of a Kerr black hole (or a Schwarzschild black hole as a particular case). In other words, we neglect the back reaction of the scalar field dynamics on the spacetime geometry.

- Although simplified our model captures the basic qualitative and even quantitative features of the full nonlinear dynamics. In any case, considering the decoupling limit is a very good approximation of the full nonlinear picture in the vicinity of the bifurcation point where the back reaction of the scalar field on the geometry is small.

- Following our approach we consider the nonlinear wave equation for the scalar field on the Kerr background

$$\nabla_\alpha \nabla^\alpha \varphi = -\frac{\lambda^2}{4}\frac{df(\varphi)}{d\varphi}\mathcal{R}^2_{GB}$$

- In explicit form the nonlinear wave equations takes the form

$$- \left[(r^2+a^2)^2 - \Delta a^2 \sin^2\theta\right] \partial_t^2 \varphi + (r^2+a^2)^2 \partial_x^2 \varphi + 2r\Delta \partial_x \delta\varphi - 4Mar\partial_t \partial_{\phi_*} \varphi$$
$$+2a(r^2+a^2)\partial_x \partial_{\phi_*}\varphi + \Delta\left[\frac{1}{\sin\theta}\partial_\theta(\sin\theta\partial_\theta\varphi) + \frac{1}{\sin^2\theta}\partial^2_{\phi_*}\varphi\right]$$
$$= -\lambda^2 \frac{12M^2\Delta}{\Sigma^5}(r^2 - a^2\cos^2\theta)(r^4 - 14a^2r^2\cos^2\theta + a^4\cos^4\theta)\frac{df(\varphi)}{d\varphi}.$$

with $\Delta = r^2 - 2Mr + a\hat{}2$ $\quad d\phi_* = d\phi + \frac{a}{\Delta}\, dr \quad dx = \frac{r^2+a^2}{\Delta}\, \mathrm{dr}$

- We consider two coupling functions:

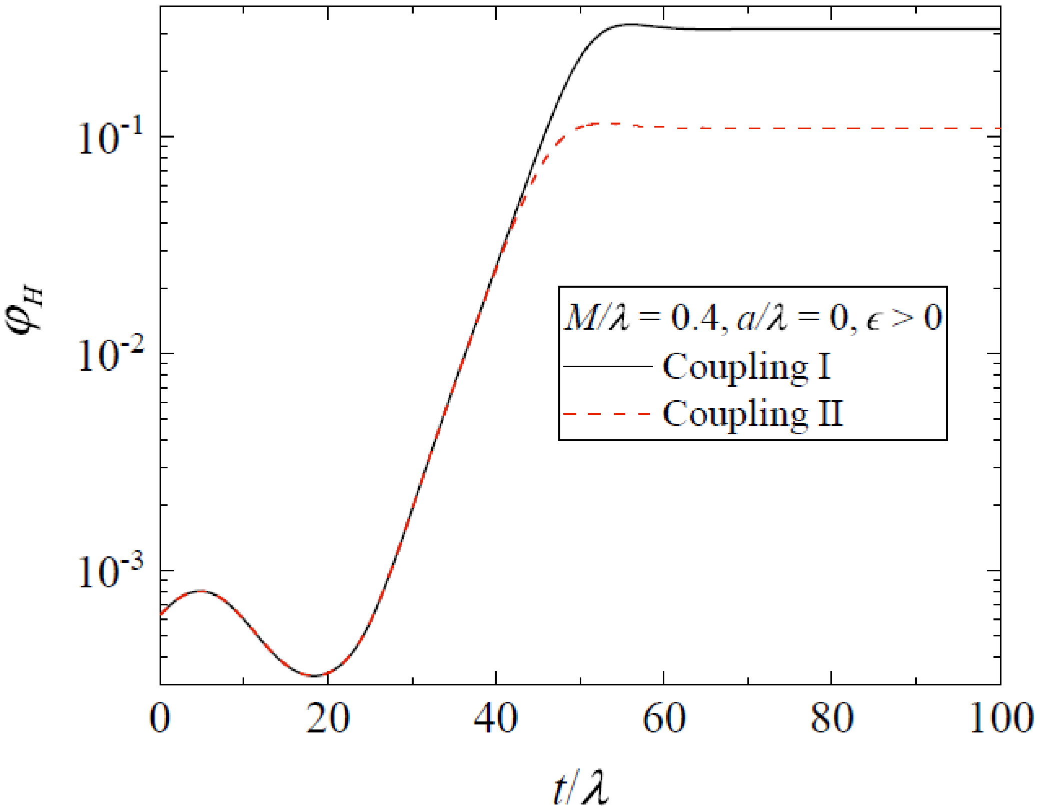
$$f(\varphi)_{\mathrm{CaseI}} = \frac{\epsilon}{2\beta}\left(1 - e^{-\beta\varphi^2}\right),$$
$$f(\varphi)_{\mathrm{CaseII}} = \frac{\epsilon}{\beta^2}\left(1 - \frac{1}{\cosh(\beta\varphi)}\right),$$

where β is a parameter and $\epsilon = \pm 1$. For $\epsilon = 1$ both rotating and nonrotating black holes can scalarized while the case of $\epsilon = -1$ is responsible for the spin-induced scalarization where black holes can scalarize only if they are rotating fast enough.

- In the numerical calculations we have chosen $\beta = 12$.

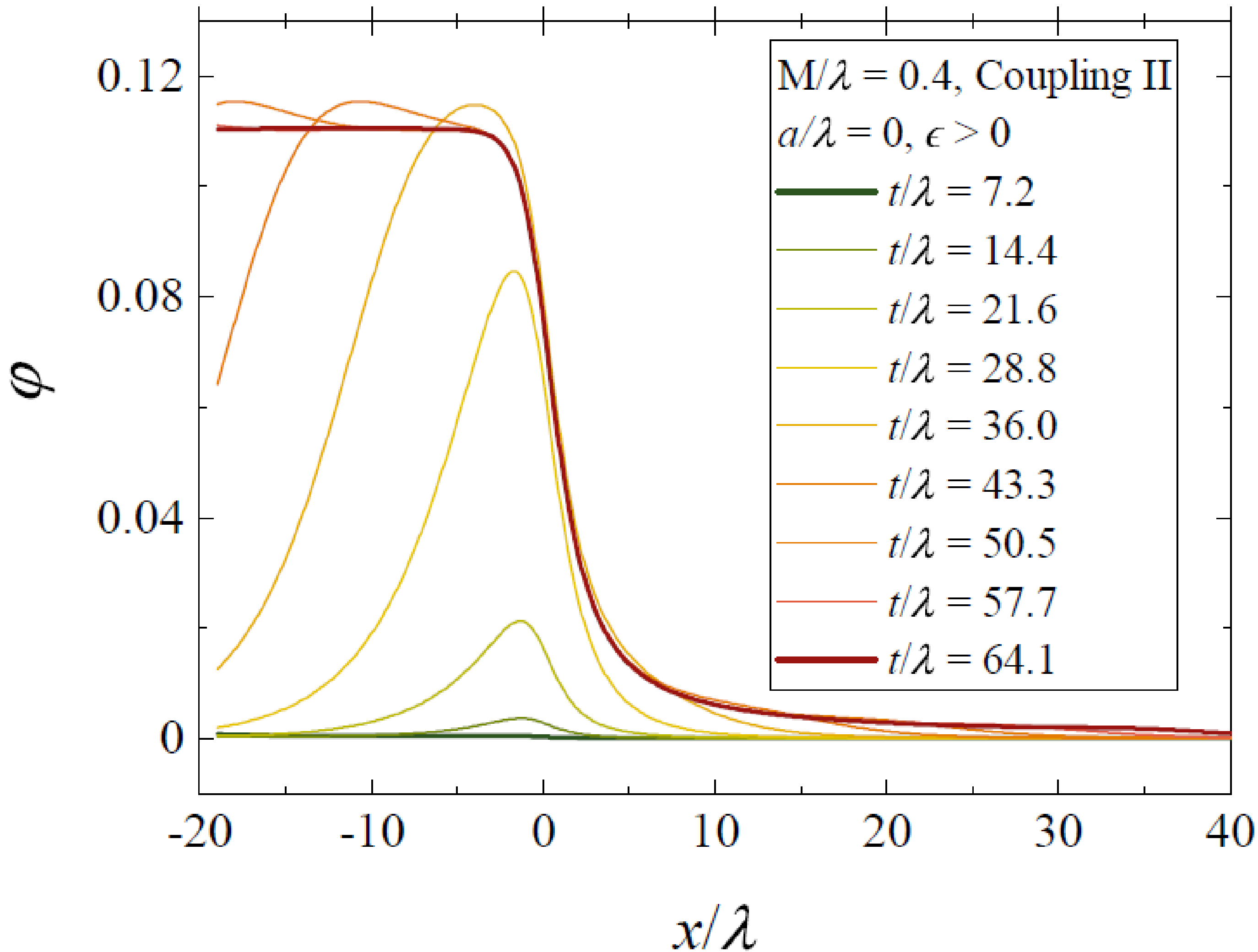
- The boundary conditions we impose when evolving our equation is that the scalar field has the form of an outgoing wave at infinity and an ingoing wave at the black hole horizon. Thus we start the simulations with a Gauss pulse (in radial direction) located far outside the black hole.

Dynamics in spherical symmetry

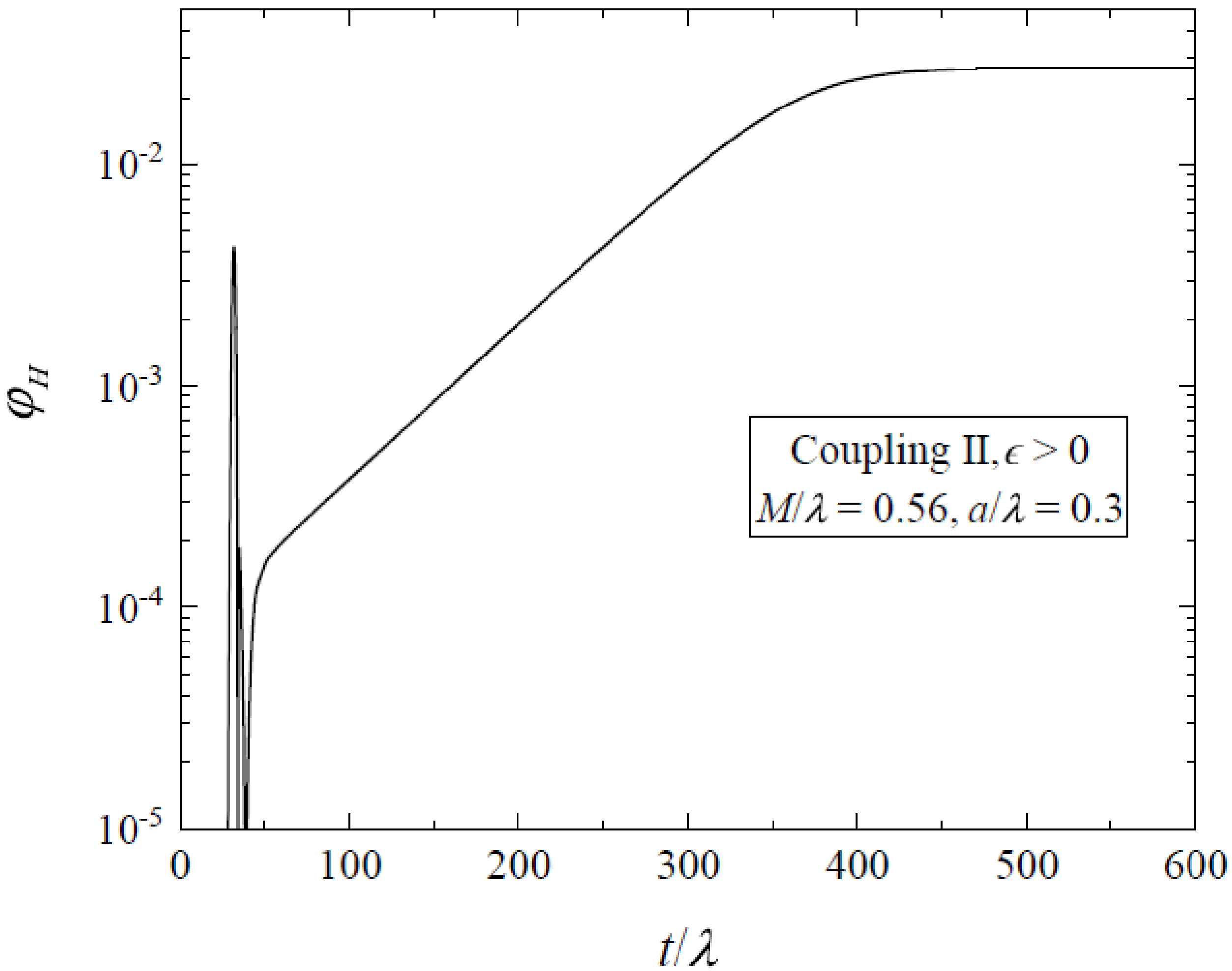


- The simulations confirm that while the path to reach the equilibrium black hole might be strongly dependent on the initial perturbation, the end state is unaltered.

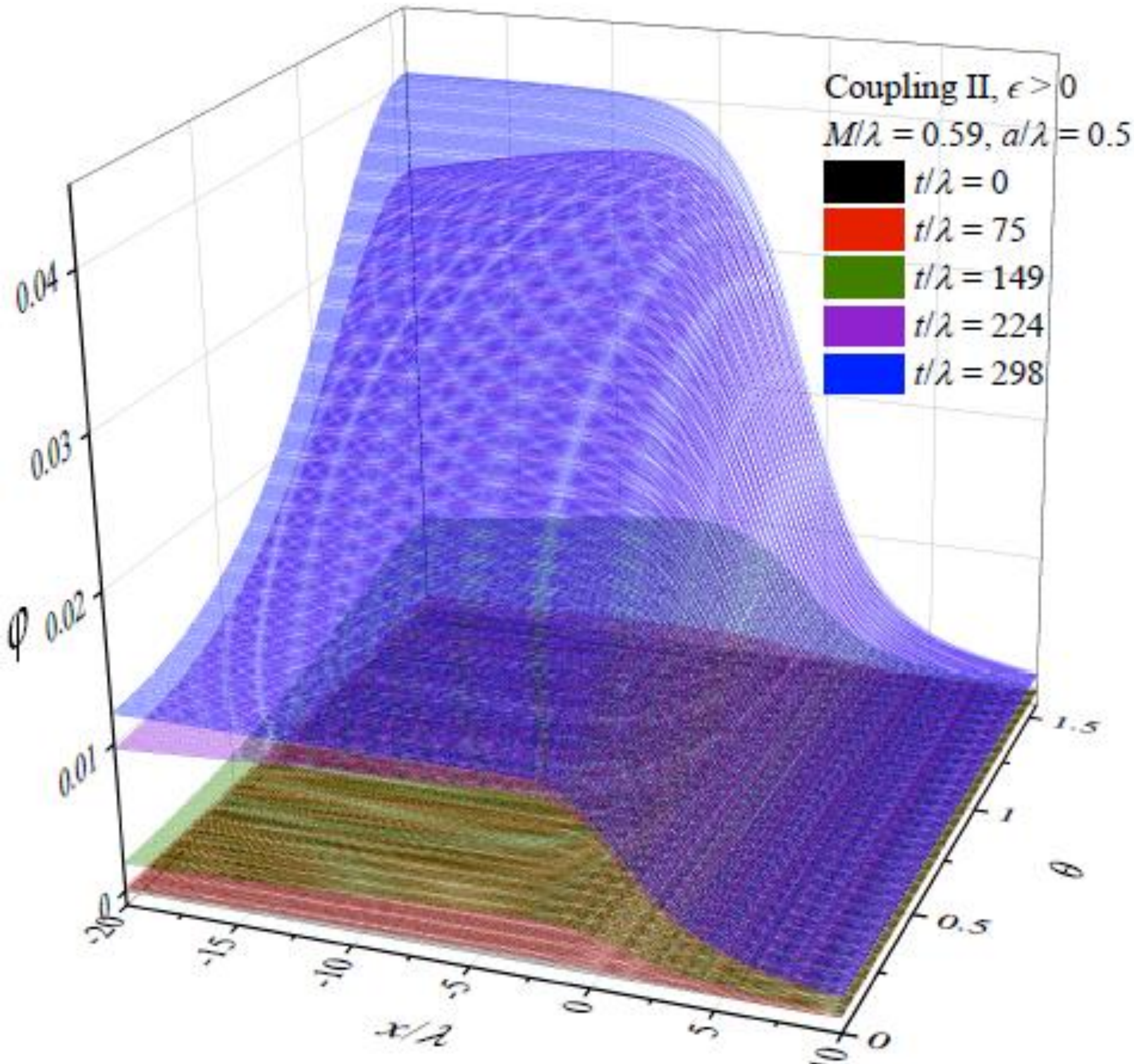
Dynamics in spherical symmetry



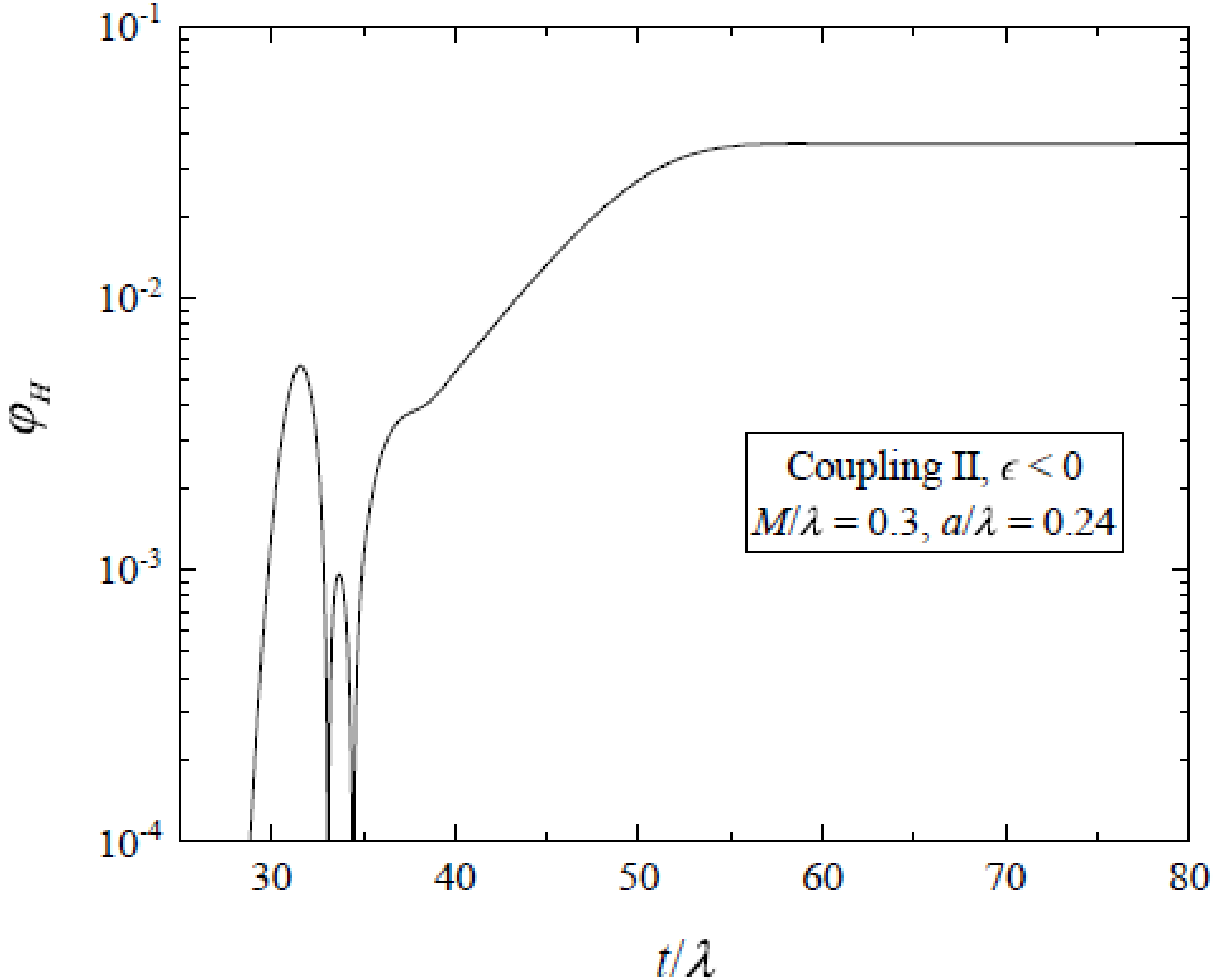
Dynamics of the Kerr black hole scalarization



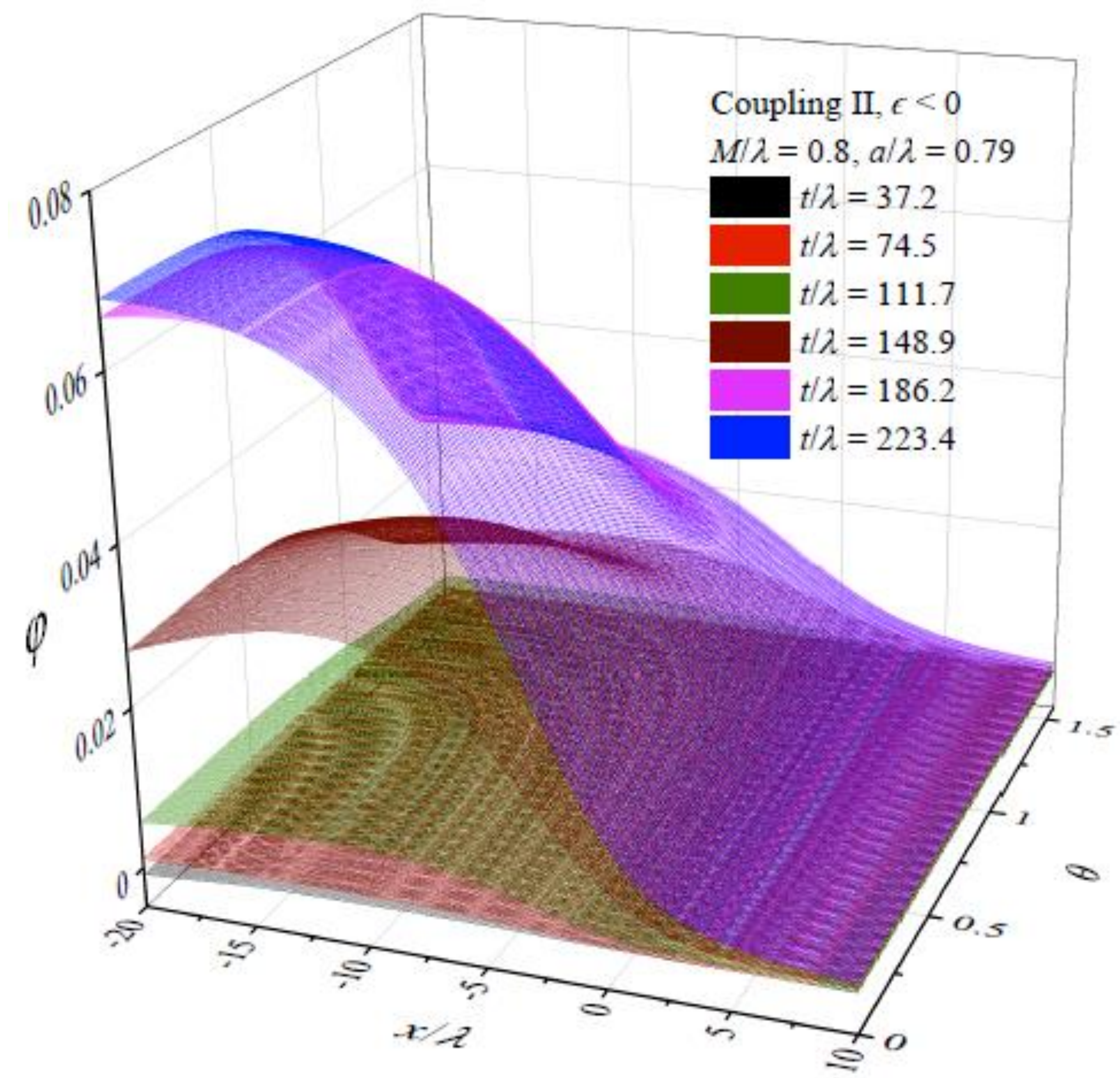
Dynamics of the spontaneous scalarization



Dynamics of the spin-induced scalarization



Dynamics of the spontaneous scalarization



Realistic physical mechanism for the formation of isolated scalarized BHs and NSs

- The gravitational collapse can produce scalarized black holes and scalarized neutron stars starting with initial state with no scalar field present
  (H.-J. Kuan, D. Doneva and S. Y., PRL (2021))

- We have studied the spherically-symmetric core-collapse in Gauss-Bonnet gravity in its full nonlinearity.

- Depending on the progenitors, the coupling constants β and λ, and the sign of ϵ, the final outcome of the core collapse and the path to reach it can vary significantly

$$ds^2 = -e^{2\Phi(t,r)}dt^2 + e^{2\Lambda(t,r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

$$m(r,t) = \frac{r}{2}(1 - e^{-2\Lambda})$$

- We will model the matter as a perfect fluid with energy-momentum tensor $T_{\mu\nu} = \rho h u_\mu u_\nu + p g_{\mu\nu}$, $h = 1 + e + \frac{p}{\rho}$; ρ – rest–mass density, p – fluid pressure e – specific internal fluid energy, h – specific enthalpy

$$u^\mu = \frac{1}{\sqrt{1 - v^2}}(e^{-\Phi}, ve^{-\Lambda}, 0, 0)$$

- The high-resolution shock-capturing numerical schemes use the conserved fluid variable instead of the primitive ones ρ, v and p.

$$D = \frac{\rho e^\Lambda}{\sqrt{1 - v^2}}, \; S^r = \frac{\rho h v}{1 - v^2}, \; \tau = \frac{\rho h}{1 - v^2} - p - D$$

- Equations for the metric functions in terms of the conserved variables

$$\partial_r m = 4\pi r^2(\tau + D),$$
$$\partial_t m = -4\pi r^2 S^r e^{\Phi-\Lambda},$$
$$\partial_r \Phi = \frac{e^{2\Lambda}}{r^2}\left[m + 4\pi r^3(S^r v + p)\right]$$

- Hydrodynamical equations in conservative form

$$\partial_t \mathbf{U} + \frac{1}{r^2}\partial_r\left(r^2 e^{\Phi-\Lambda}\mathbf{f}(\mathbf{U})\right) = \mathbf{s}(\mathbf{U})$$

- The state vector $\mathbf{U}$ of the conserved variables is give $\mathbf{U} = [D, S^r, \tau]$

- The flux vector $\boldsymbol{f}(\boldsymbol{U})$ and the source vector $\boldsymbol{s}(\boldsymbol{U})$ are defined by

$$\mathbf{f}(\mathbf{U}) = [Dv, S^r v + p, S^r - Dv],$$
$$\mathbf{s}(\mathbf{U}) = [0, (S^r v - \tau - D)e^{\Phi+\Lambda}(8\pi r p + \frac{m}{r^2})$$
$$+ e^{\Phi+\Lambda} p \frac{m}{r^2} + 2e^{\Phi-\Lambda}\frac{p}{r}, 0].$$

- The pressure consists of a cold and a thermal part, $p = p_c + p_{th}$ and $e = e_c + e_{th}$ where the cold parts of the and the internal energy are given by

$$p_c = K_1 \rho^{\Gamma_1}, \quad e_c = \frac{K_1}{\Gamma_1 - 1}\rho^{\Gamma_1 - 1}, \quad \text{for} \quad \rho \le \rho_{\text{nucl}},$$
$$p_c = K_2 \rho^{\Gamma_2}, \quad e_c = \frac{K_2}{\Gamma_2 - 1}\rho^{\Gamma_2 - 1}, \quad \text{for} \quad \rho > \rho_{\text{nucl}},$$

and for the thermal part we assume $p_{th} = (\Gamma_{th} - 1)(e - e_{th})$ with $\Gamma_1 = 1.3$, $\Gamma_2 = 2.5$, $\Gamma_{th} = 1{,}35$ and $\rho_{nucl} = 2\,.10^{14}\ g\ cm^{-3}$

Scalarization through core-collapse

- The equation for the scalar field

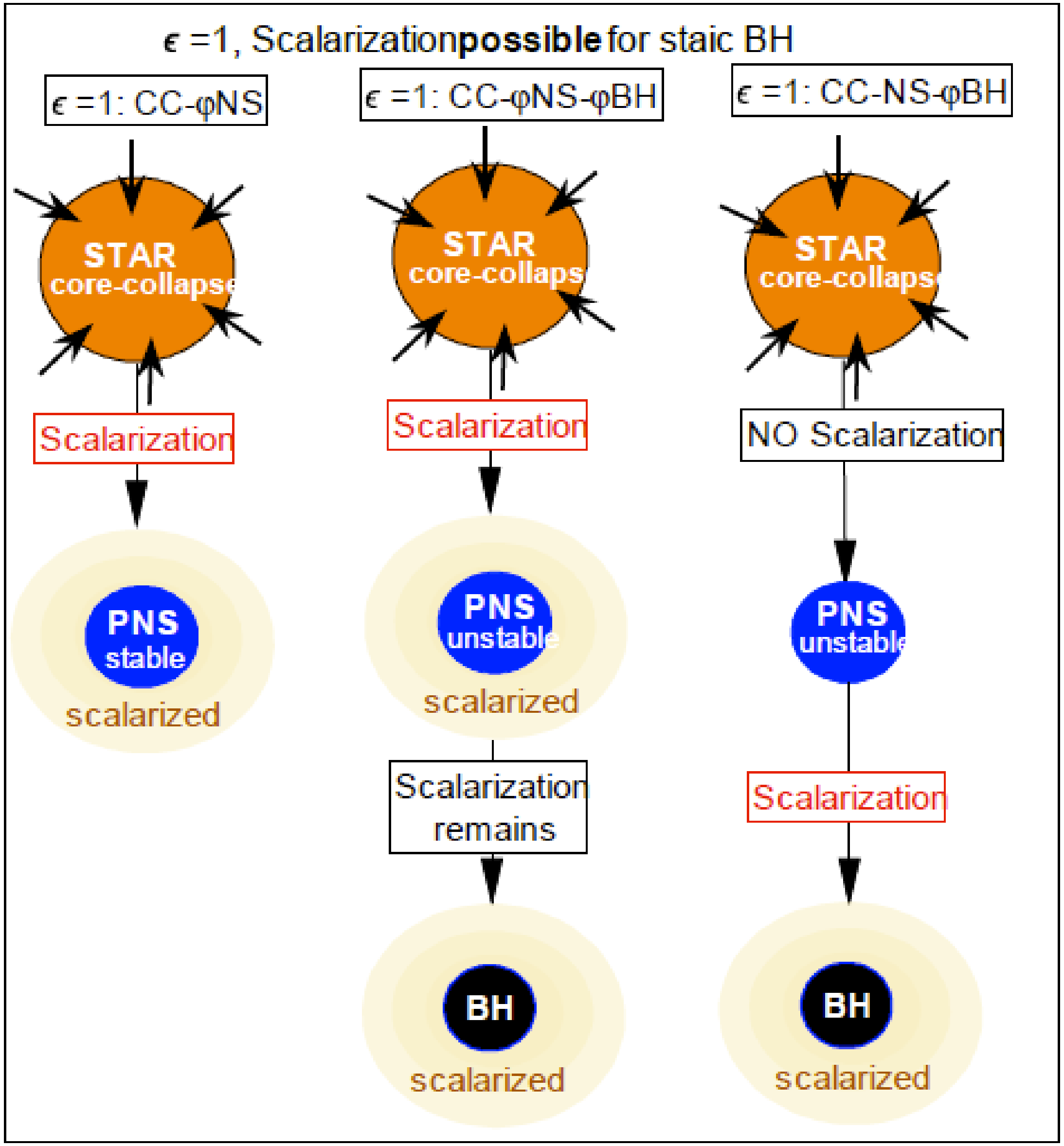
$$-\partial_t^2\varphi + (\partial_t\Phi - \partial_t\Lambda)\partial_t\varphi + e^{2\Phi-2\Lambda}\partial_r^2\varphi + e^{2\Phi-2\Lambda}(\partial_r\Phi - \partial_r\Lambda + \frac{2}{r})\partial_r\varphi =$$
$$-2\frac{df(\varphi)}{d\varphi}\frac{\lambda^2}{r^2}e^{2\Phi}\left\{(1-e^{-2\Lambda})\left[\frac{1}{r}(\partial_r\Phi - \partial_r\Lambda)e^{-2\Lambda} - 8\pi p\right] - 2\partial_r\Phi\partial_r\Lambda e^{-4\Lambda} + 2(\partial_t\Lambda)^2 e^{-2\Phi-2\Lambda}\right\}$$

- Coupling function

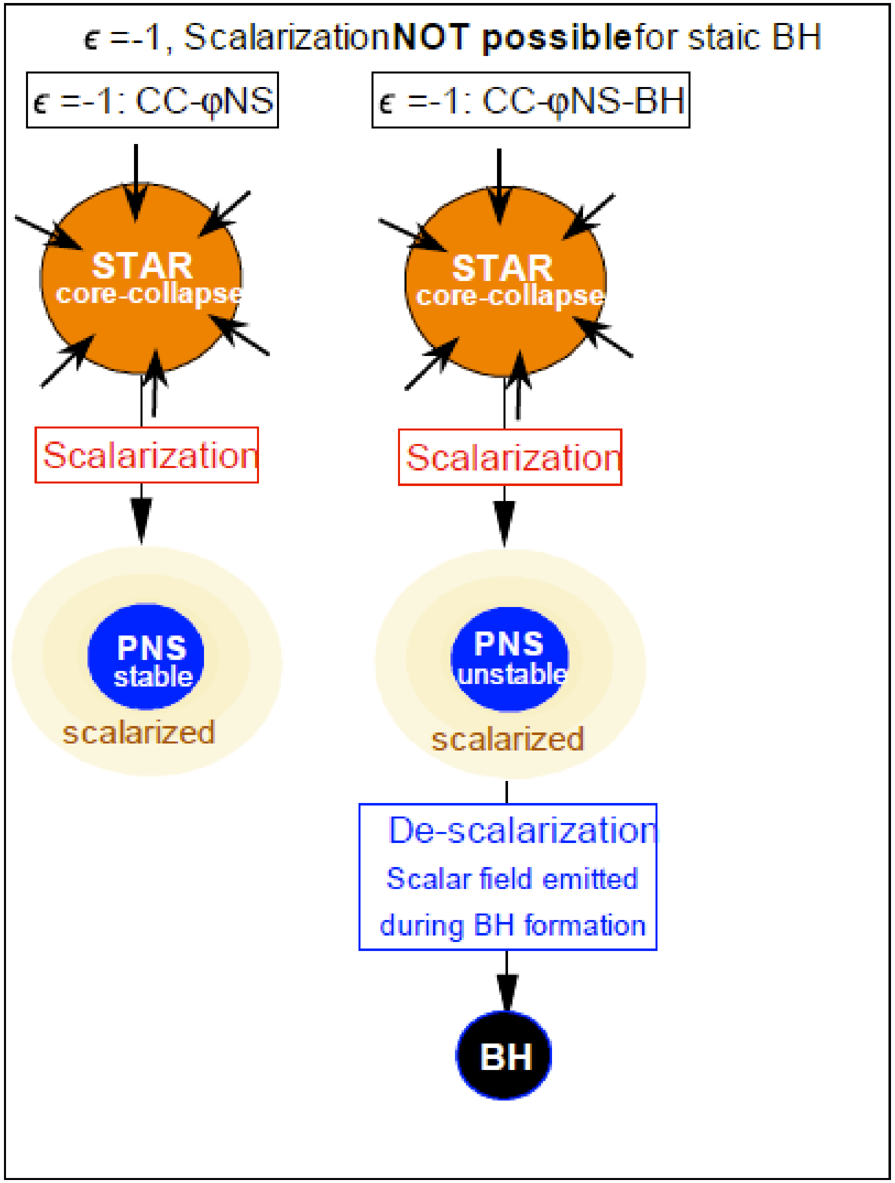
$$f(\varphi) = \frac{\epsilon}{2\beta}\left[\exp(-\beta\varphi^2) - 1\right], \quad \beta > 0, \quad \epsilon = \pm 1.$$

- Depending on the properties of the progenitor, the coupling constants λ and β, and the sign of ϵ, the final outcome of the gravitational collapse and the path to reach it can vary significantly.

Scalarization through core-collapse



Scalarization through core-collapse



Beyond spontaneous scalarization

- Let us consider sGB theories that do not allow for spontaneous scalarization!

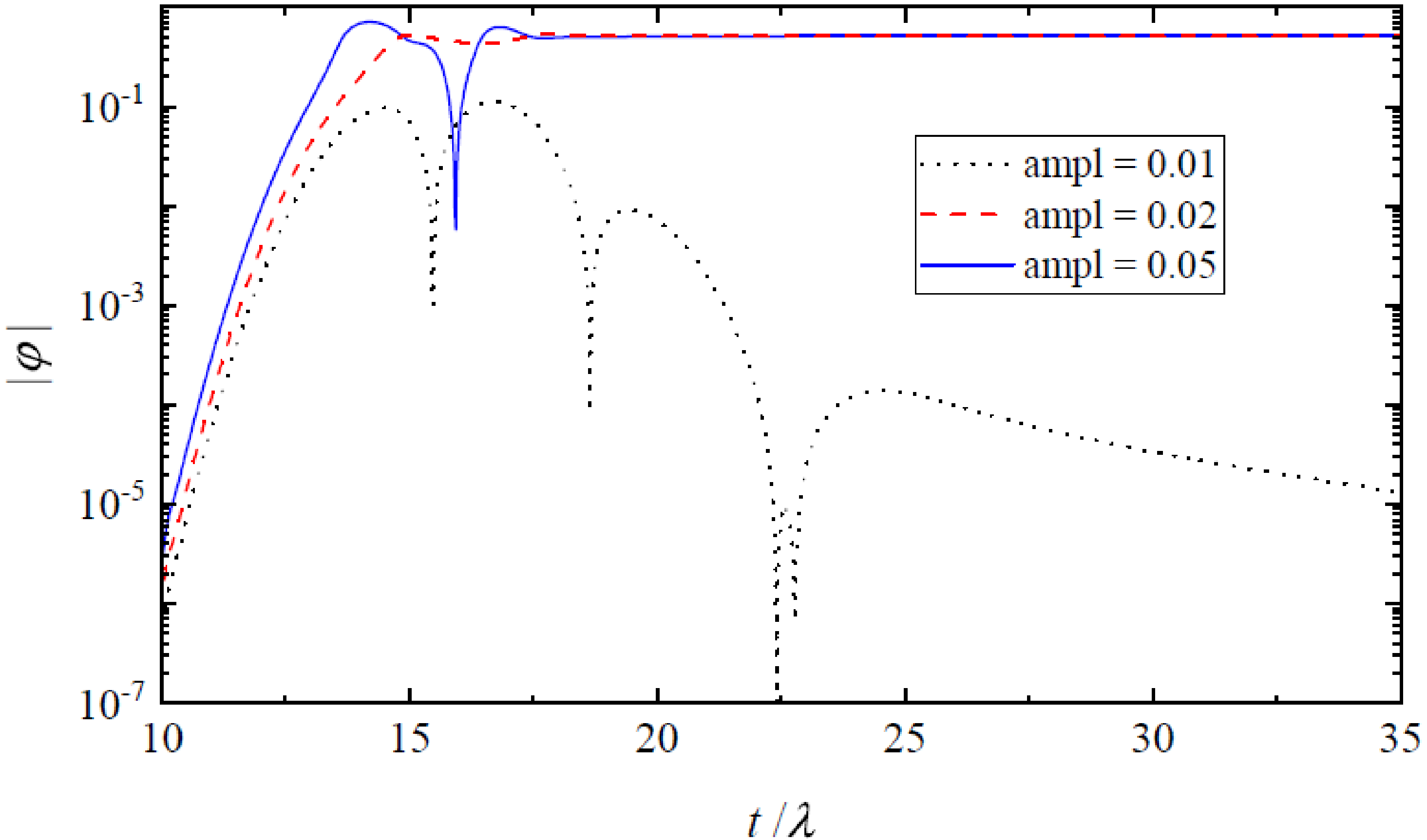
$$\frac{df}{d\varphi}(0) = 0 \;\; and \;\; \frac{d^2 f}{d\varphi^2}(0) = 0$$

- With these conditions imposed on the coupling function, **no tachyonic instability is possible, however, the sGB theories admit all the stationary solutions of GR!**

- **The Schwarzschild solution is linearly stable even within the sGB gravity!**

- **Now, we ask the following question: What happens to the Schwarzschild black hole within the sGB gravity if the black hole is acted by nonlinear perturbations?**

$$f(\varphi) = \frac{1}{2\beta}\left(1 - e^{-\beta\varphi^4}\right) \quad \beta = 50, \quad \frac{M}{\lambda} = 0.1$$



➤ Independent approach to the scalarized phases of the Schwarzschild black holes

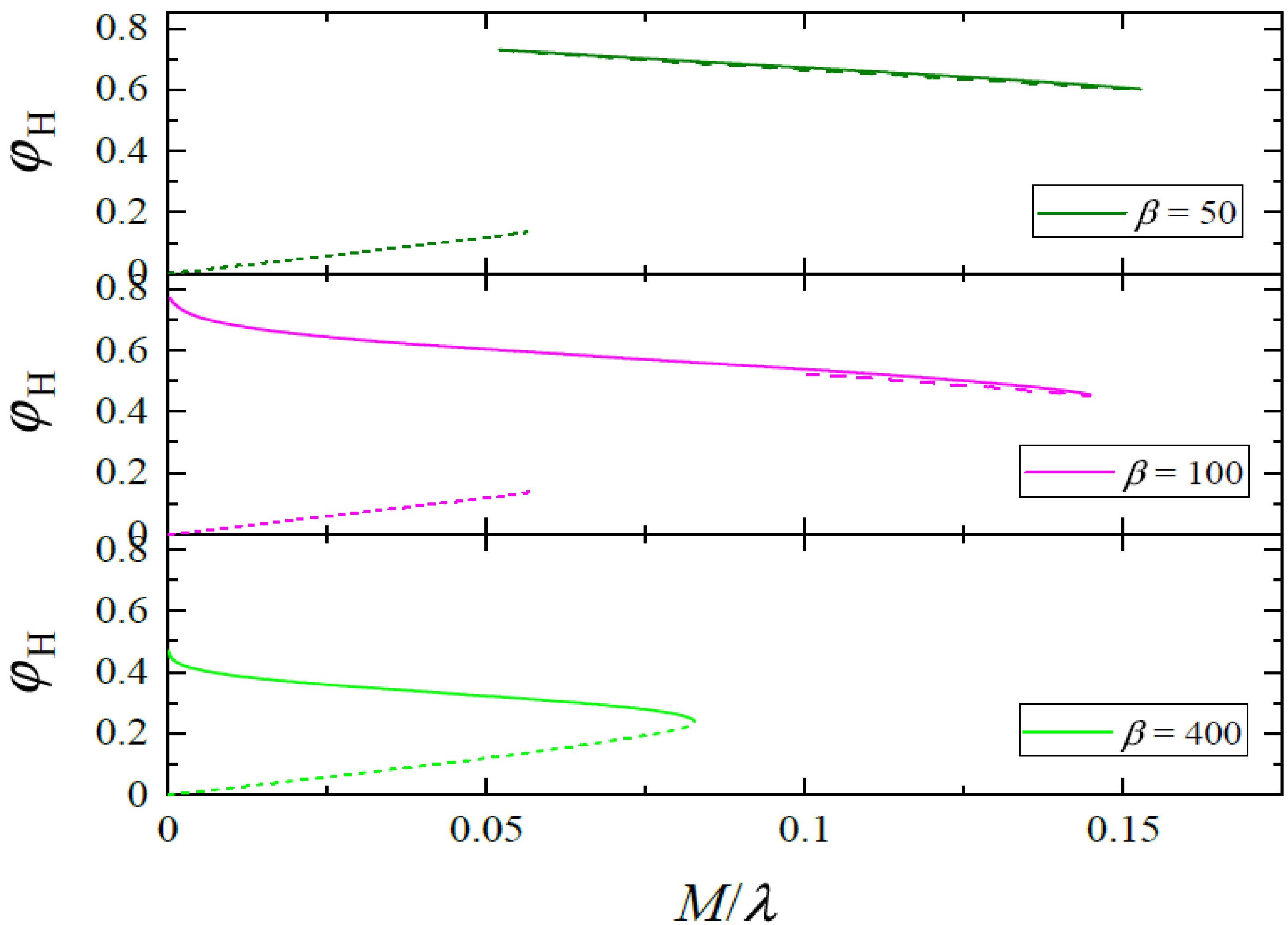
$$\frac{2}{r}\left[1+\frac{2}{r}(1-3e^{-2\Lambda})\Psi_r\right]\frac{d\Lambda}{dr}+\frac{(e^{2\Lambda}-1)}{r^2}-\frac{4}{r^2}(1-e^{-2\Lambda})\frac{d\Psi_r}{dr}-\left(\frac{d\varphi}{dr}\right)^2=0,$$

$$\frac{2}{r}\left[1+\frac{2}{r}(1-3e^{-2\Lambda})\Psi_r\right]\frac{d\Phi}{dr}-\frac{(e^{2\Lambda}-1)}{r^2}-\left(\frac{d\varphi}{dr}\right)^2=0,$$

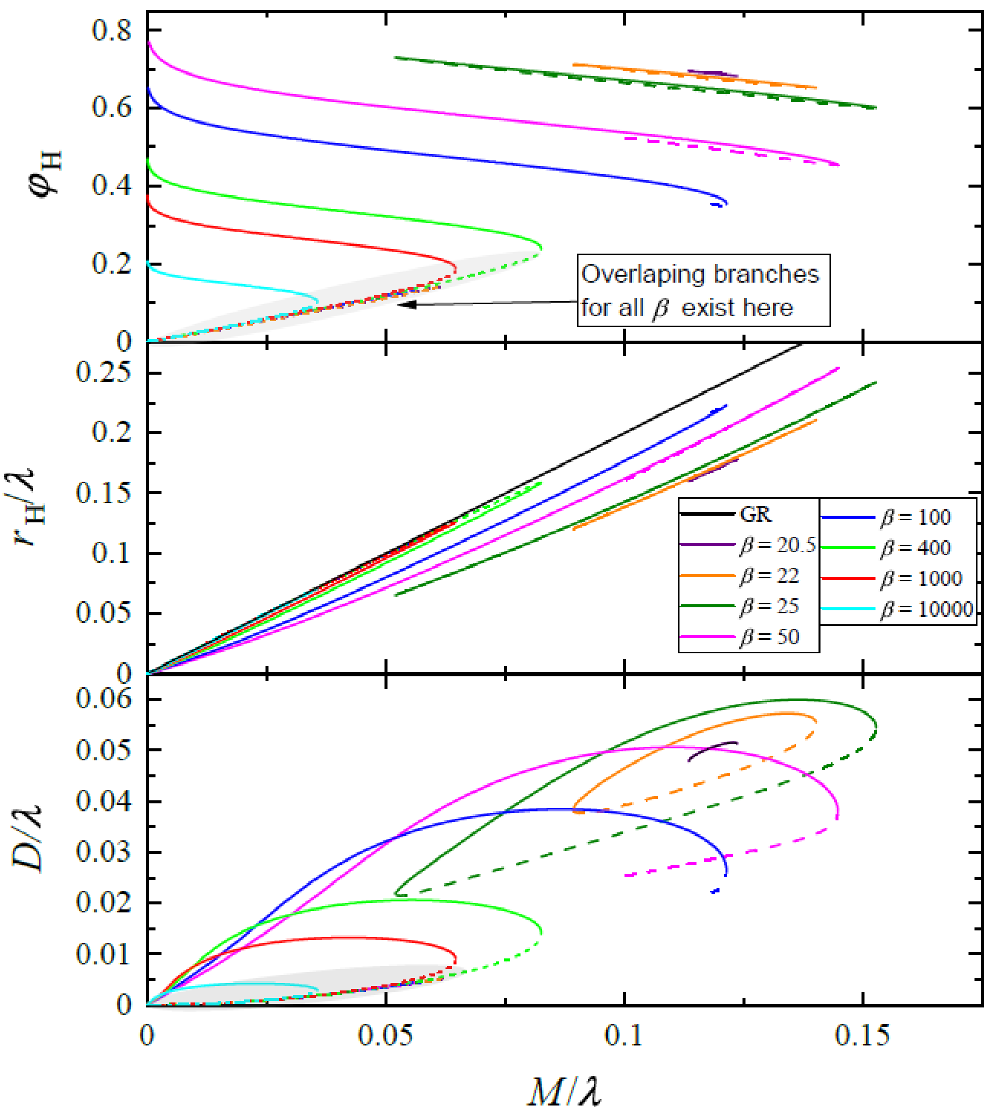
$$\frac{d^2\Phi}{dr^2}+\left(\frac{d\Phi}{dr}+\frac{1}{r}\right)\left(\frac{d\Phi}{dr}-\frac{d\Lambda}{dr}\right)+\frac{4e^{-2\Lambda}}{r}\left[3\frac{d\Phi}{dr}\frac{d\Lambda}{dr}-\frac{d^2\Phi}{dr^2}-\left(\frac{d\Phi}{dr}\right)^2\right]\Psi_r$$
$$-\frac{4e^{-2\Lambda}}{r}\frac{d\Phi}{dr}\frac{d\Psi_r}{dr}+\left(\frac{d\varphi}{dr}\right)^2=0,$$

$$\frac{d^2\varphi}{dr^2}+\left(\frac{d\Phi}{dr}-\frac{d\Lambda}{dr}+\frac{2}{r}\right)\frac{d\varphi}{dr}$$
$$-\frac{2\lambda^2}{r^2}\frac{df(\varphi)}{d\phi}\left\{(1-e^{-2\Lambda})\left[\frac{d^2\Phi}{dr^2}+\frac{d\Phi}{dr}\left(\frac{d\Phi}{dr}-\frac{d\Lambda}{dr}\right)\right]+2e^{-2\Lambda}\frac{d\Phi}{dr}\frac{d\Lambda}{dr}\right\}=0,$$

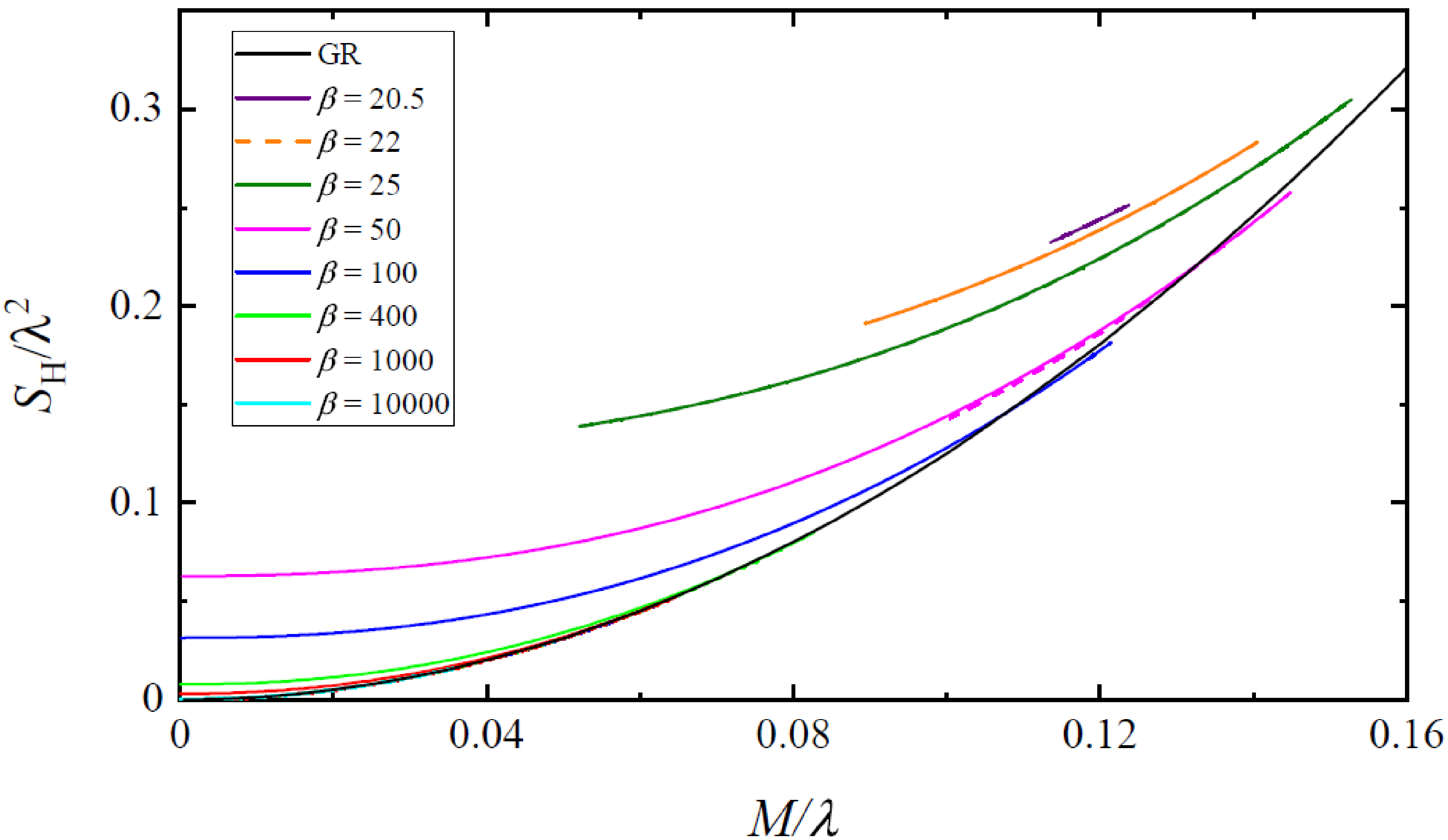
$\Phi|_{r\to\infty}\to 0,\ \ \Lambda|_{r\to\infty}\to 0,\ \ \varphi|_{r\to\infty}\to 0$ $\quad e^{2\Phi}|_{r\to r_H}\to 0,\ \ e^{-2\Lambda}|_{r\to r_H}\to 0.$ $\quad r_H^4>24\lambda^4\left(\frac{df}{d\varphi}(\varphi_H)\right)^2$


0.8
0.6
0.4
0.2
0
φ_H
$\beta = 50$
$\beta = 100$
$\beta = 400$
0
0.05
0.1
0.15
M/λ

Beyond spontaneous scalarization



Beyond spontaneous scalarization



Beyond spontaneous scalarization

- Up to now it was thought that the spontaneous scalarization is the only dynamical mechanism for endowing black holes (and other compact objects) with a scalar hair without altering the weak field limit of the theory. We have shown that there is another mechanisim, different from the spontaneous scalarization, which can generate scalar hair without altering the weak field limit.

- The new mechanism is fully nonlinear and the main ingredient of this mechanism is the nonlinear instability of the general relativistic solutions – contrary to the spontaneous scalarization which is characterized with a linear instability of the GR solutions. The new mechanism operates where the spontaneous scalarization (tachyonic instability) is impossible.

- The nonlinear mechanism works also for rotating black holes and neutron stars.

Conclusions and future perspectives

- Spontaneous scalarization is a very interesting nonlinear effect allowing for large deviations from GR while keeping the weak field regime unaltered.
- Can be sourced by the curvature of the spacetime itself, matter, exotic fields, nonlinear electrodynamics, introduction of different nonminimal couplings, etc.
- Interesting observational signatures are expected that can help us further constrain the strong field regime of gravity.

Remains to be done:

❖ Dynamics of scalarized BH and NS should be further studied.

❖ Construction of scalarized BH in other classes of alternative theories.

❖ Further understanding of the problems appearing for certain theories allowing scalarization and determining whether they are viable and how to overcome the problems.

THANK YOU!