On the fate of the Light Ring instability

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Journées Relativistes de Tours,
Institut Denis Poisson, Tours, France
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Mathématiques & Physique Théorique

Based on
2207.13713 with P. Cunha, E. Radu, N. Sanchis-Gual
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FCT Fundação para a Ciência e a Tecnologia
Gr@v The Gravity group @ Aveiro University, Portugal
CIDMA
REA

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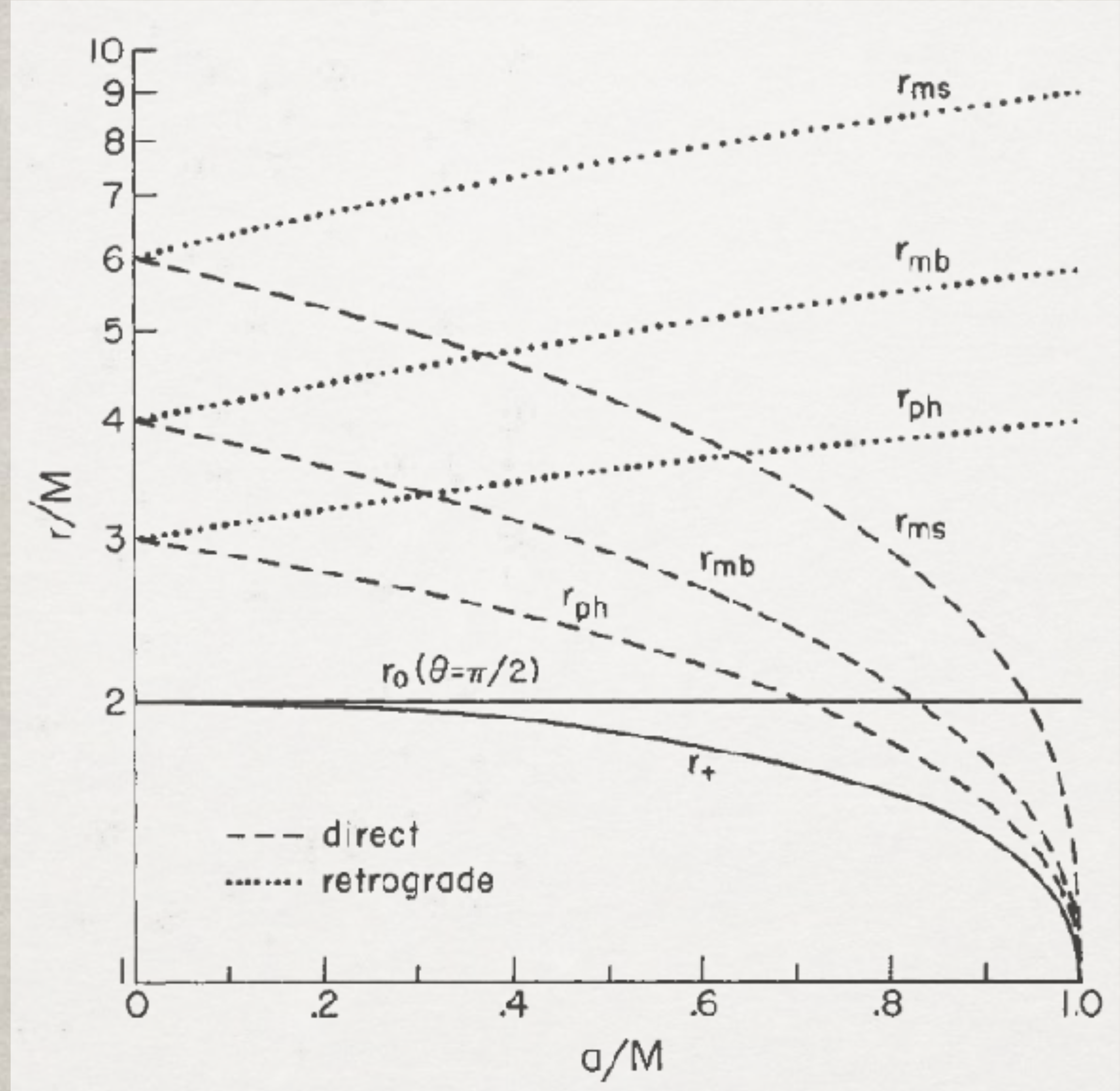

Bardeen, Press, Teukolsky , Ap. J. 178 (1972) 347

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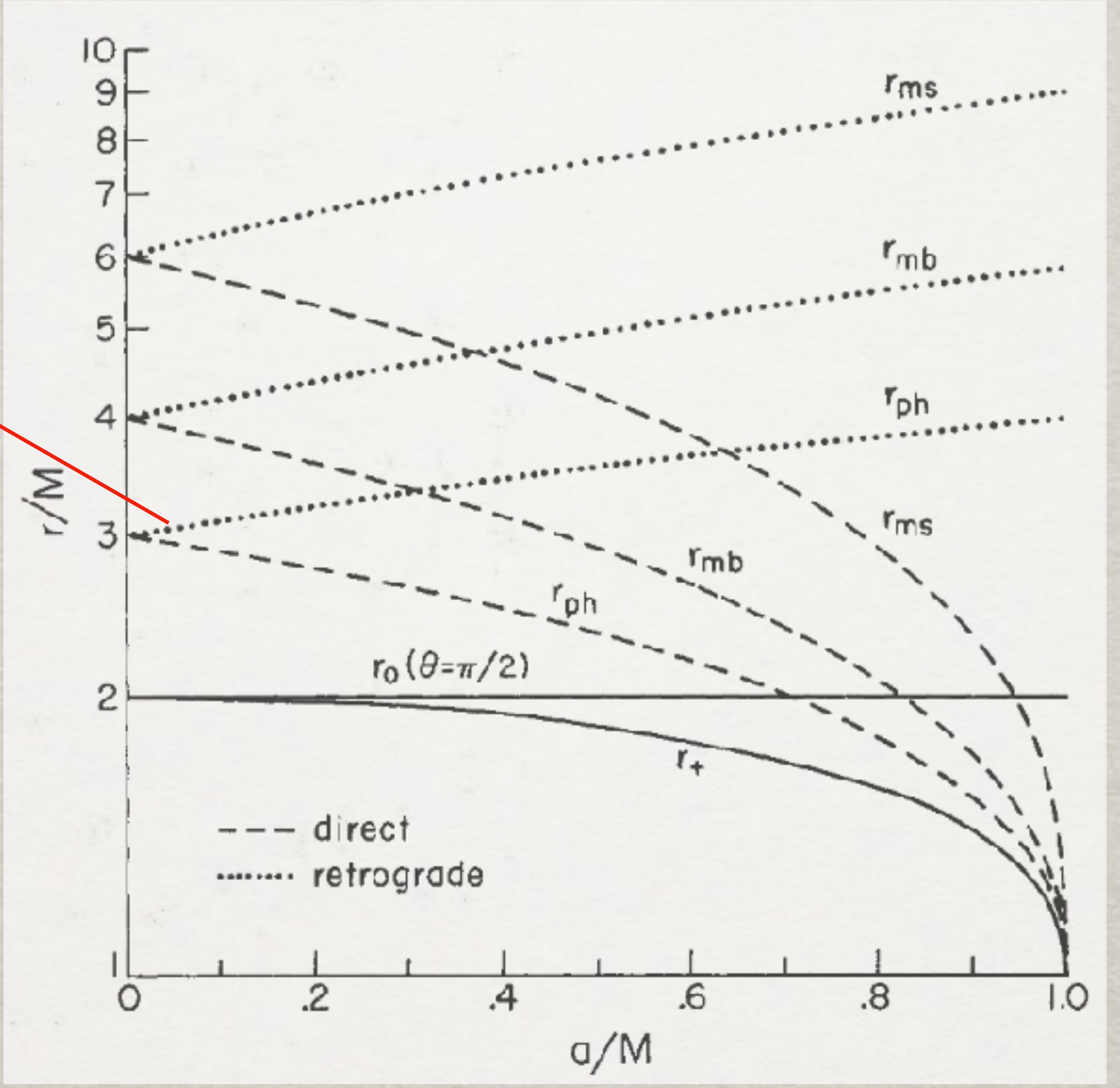

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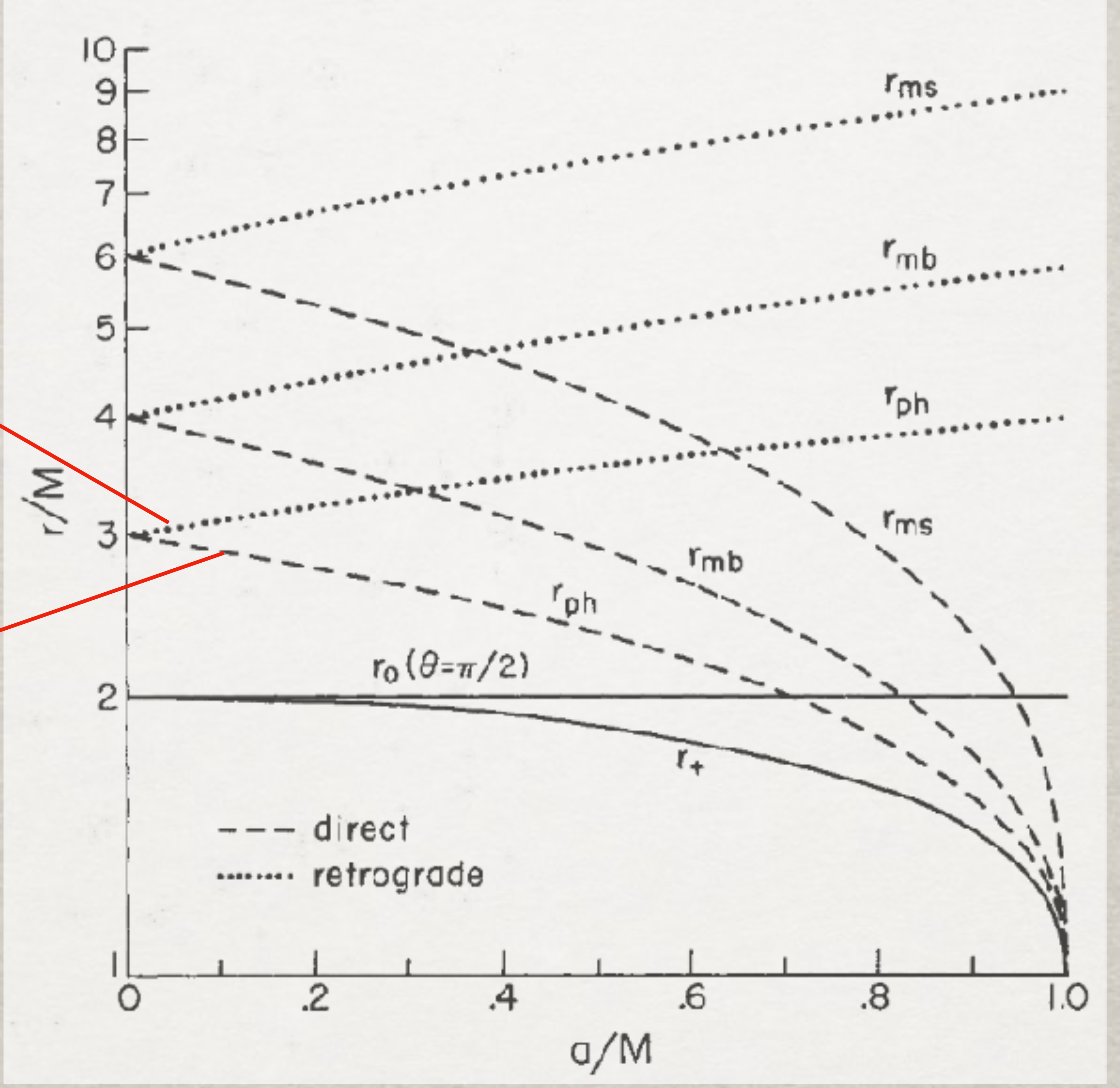

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PHYSICAL REVIEW LETTERS **124,** 181101 (2020)

Editors' Suggestion

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Central idea:
LRs are *critical points* of a potential

- determined by the Hamiltonian $\mathcal{H} = \frac{1}{2} g^{\mu\nu} p_\mu \, p_\nu = 0$.
- $2\mathcal{H} = (g^{ij} p_i \, p_j) + (g^{ab} p_a \, p_b), \qquad i \in \{r, \theta\}, \;\; a \in \{t, \varphi\}.$
  $= \quad K \quad + \quad U(r, \theta).$

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- Can be factorized as $U = (L^2 g^{tt})(\sigma - H_+)(\sigma - H_-), \qquad \sigma \equiv E/L.$

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At a LR: $\Longrightarrow$ $\boxed{\nabla H_\pm = 0}$ (critical point of $H_\pm(r,\theta)$)

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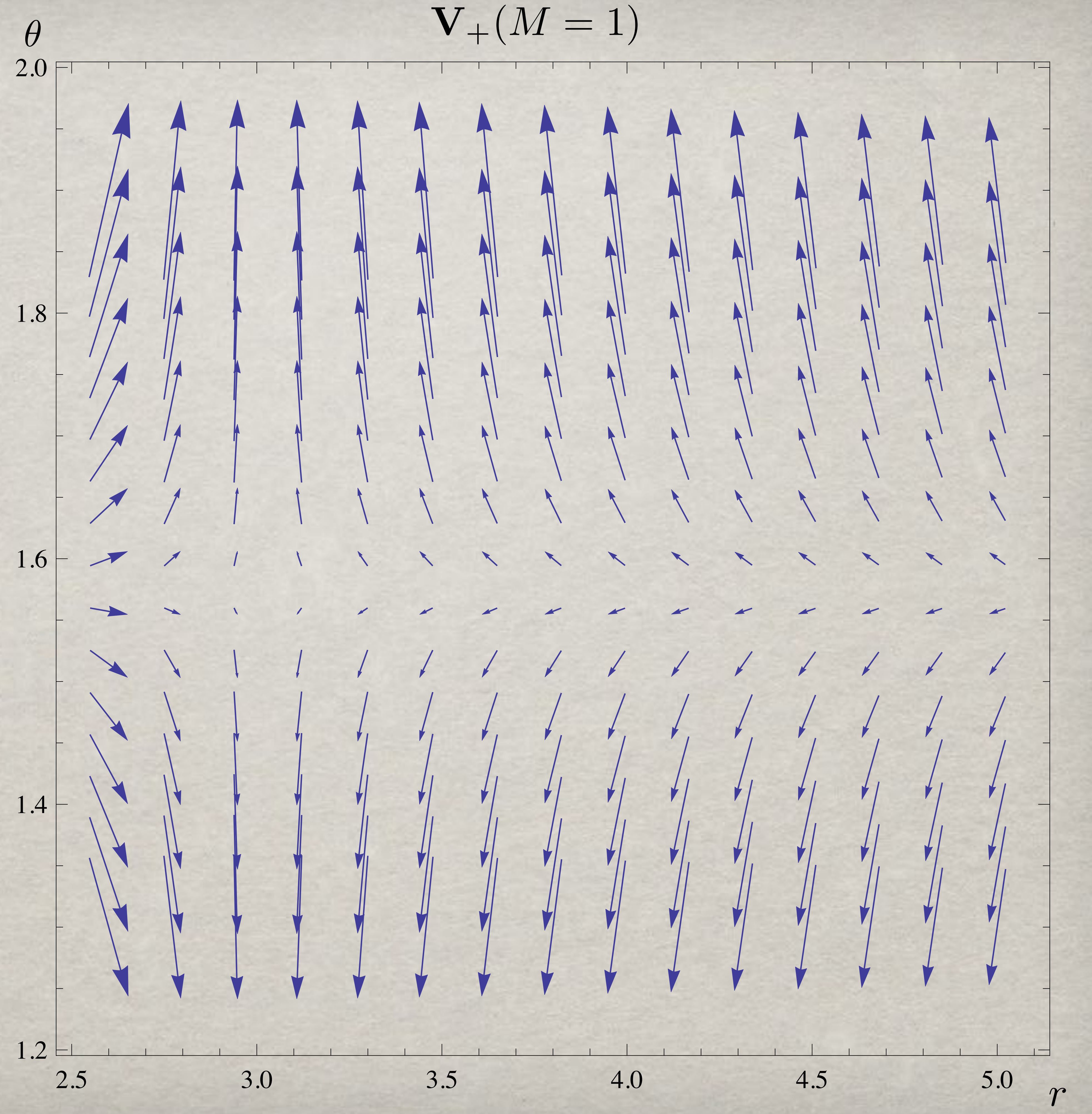

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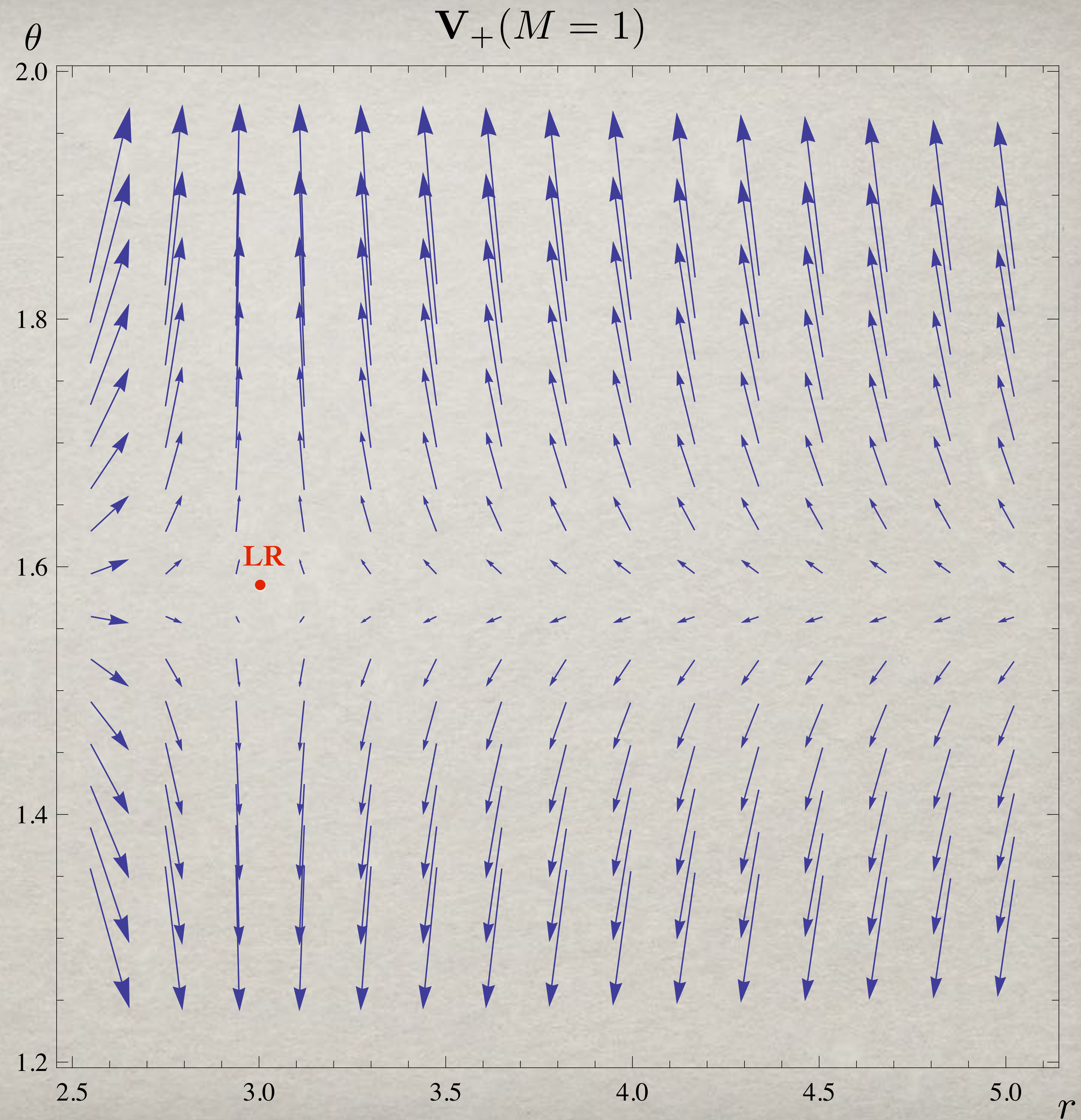

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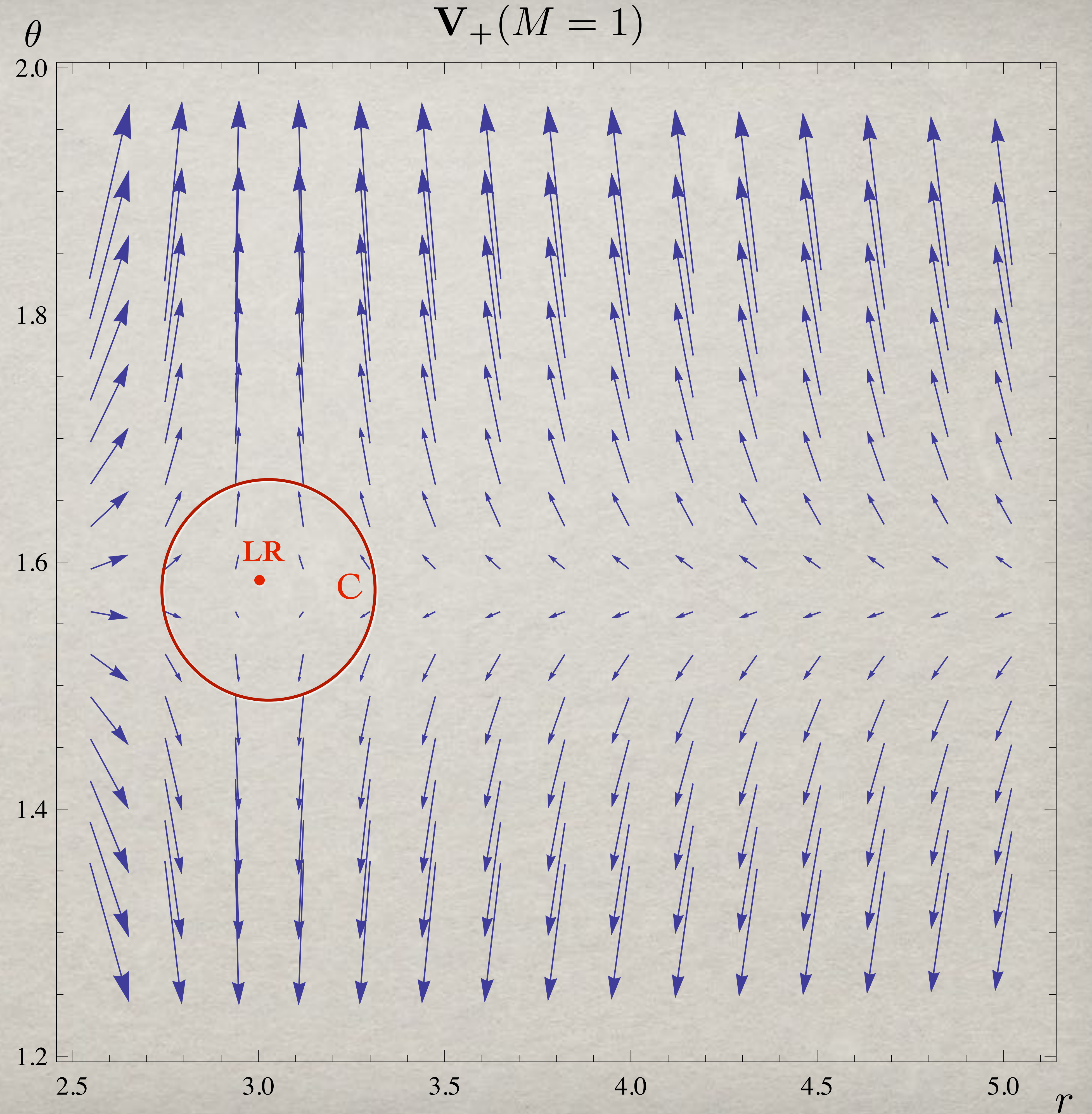

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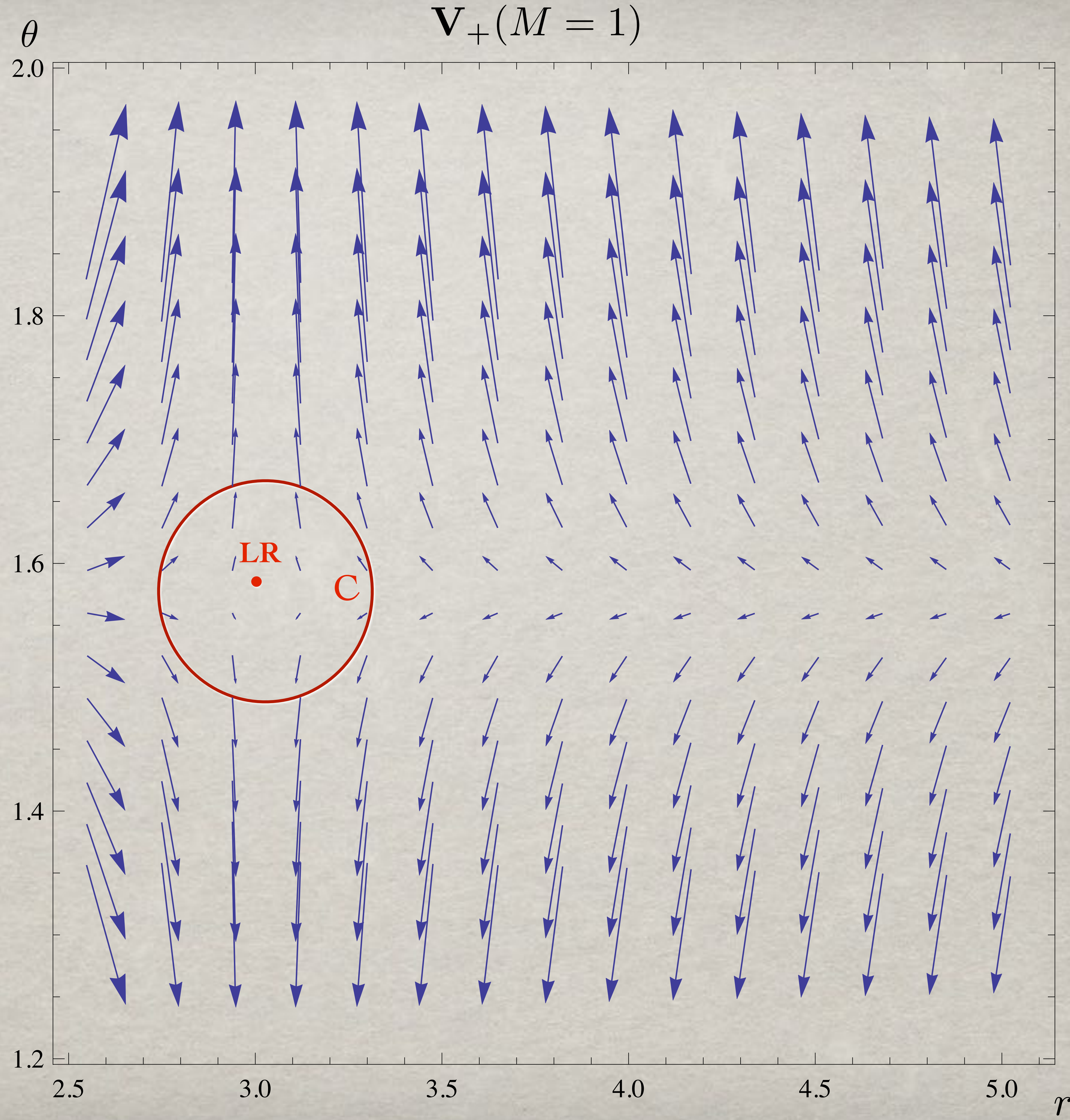

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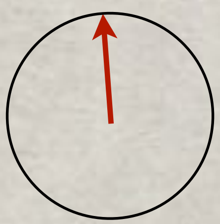

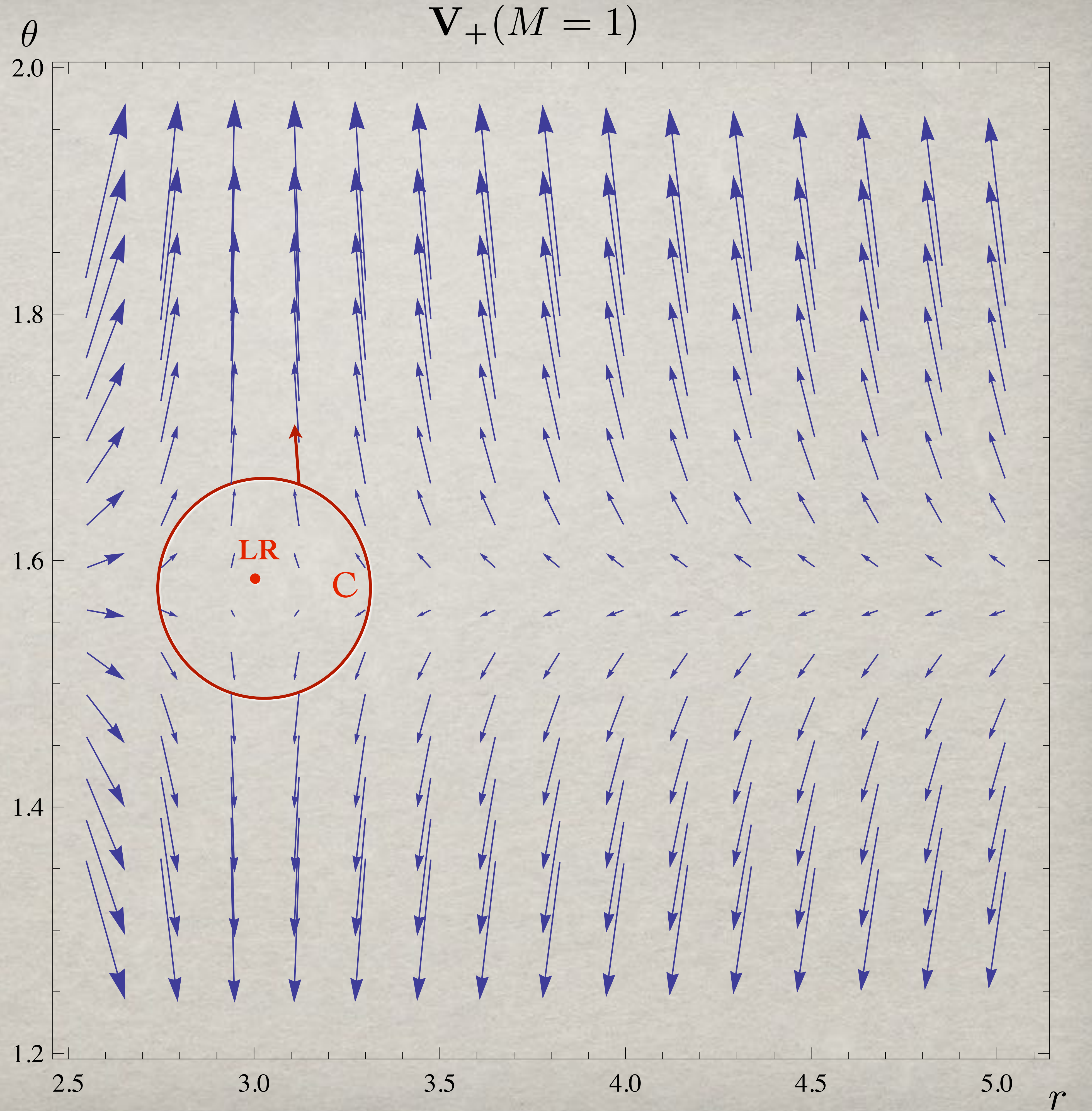

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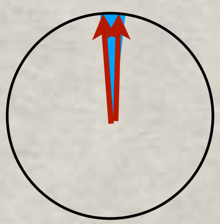

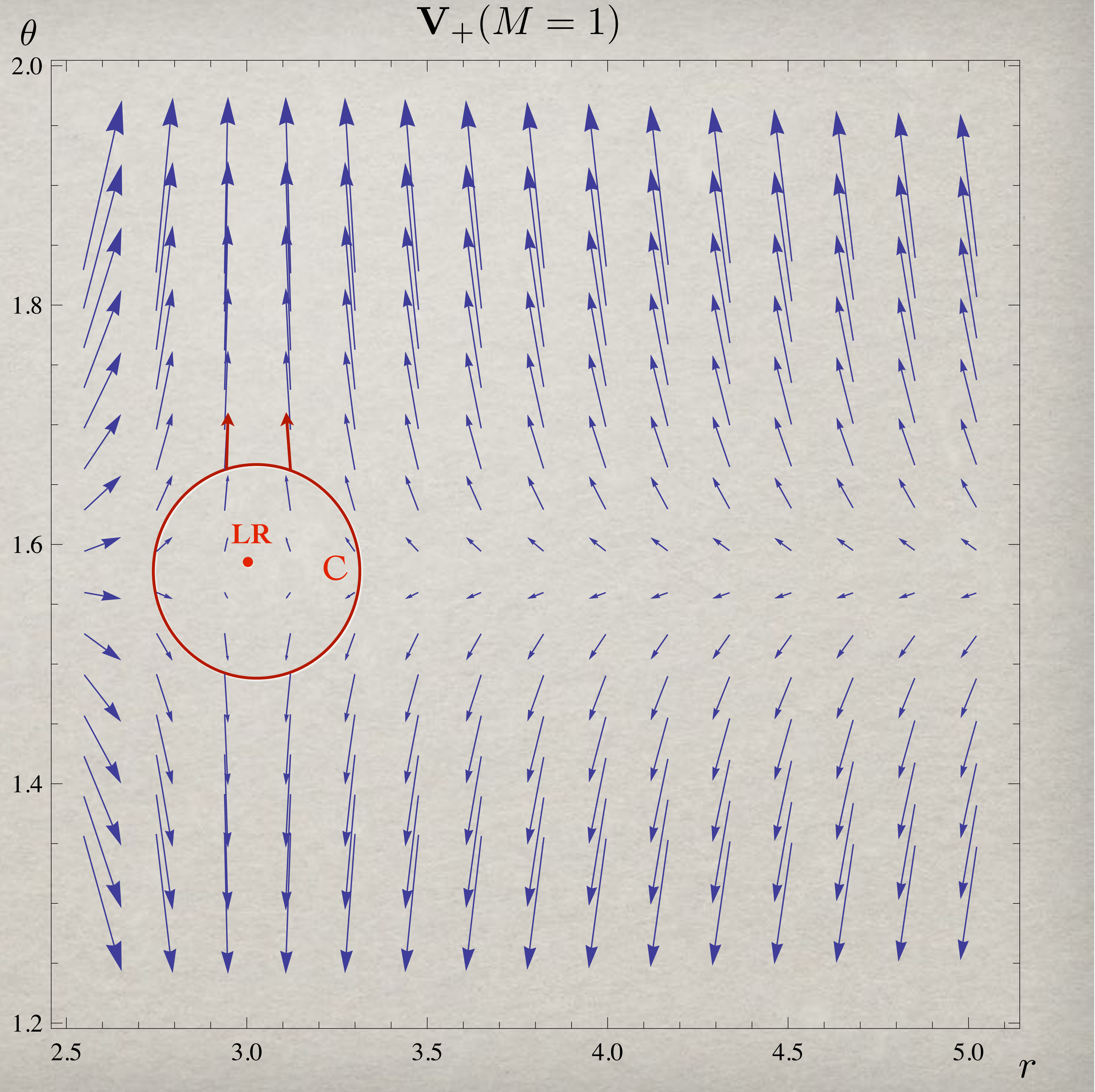

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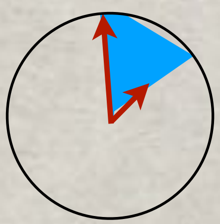

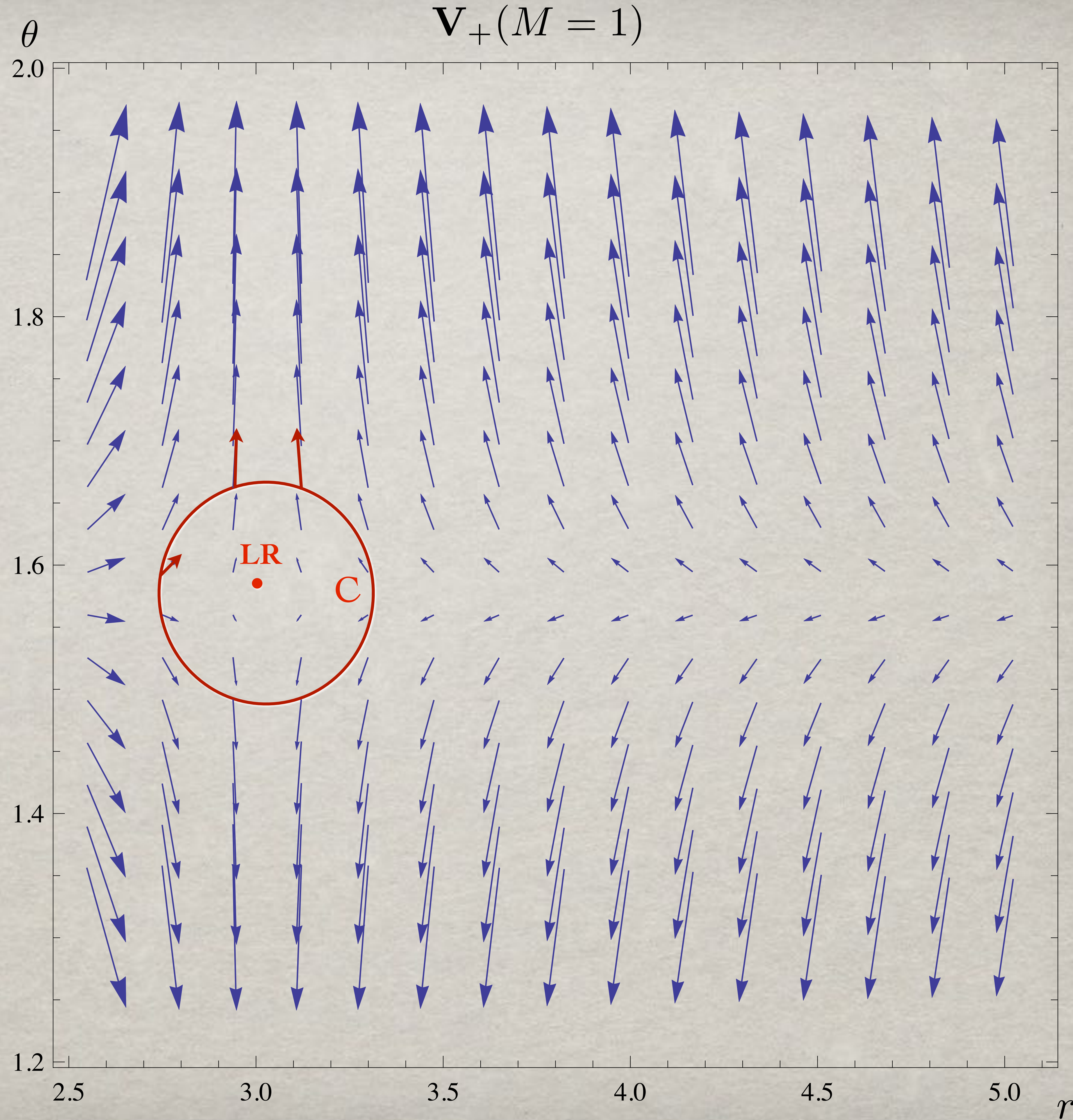

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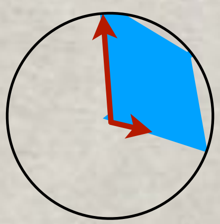

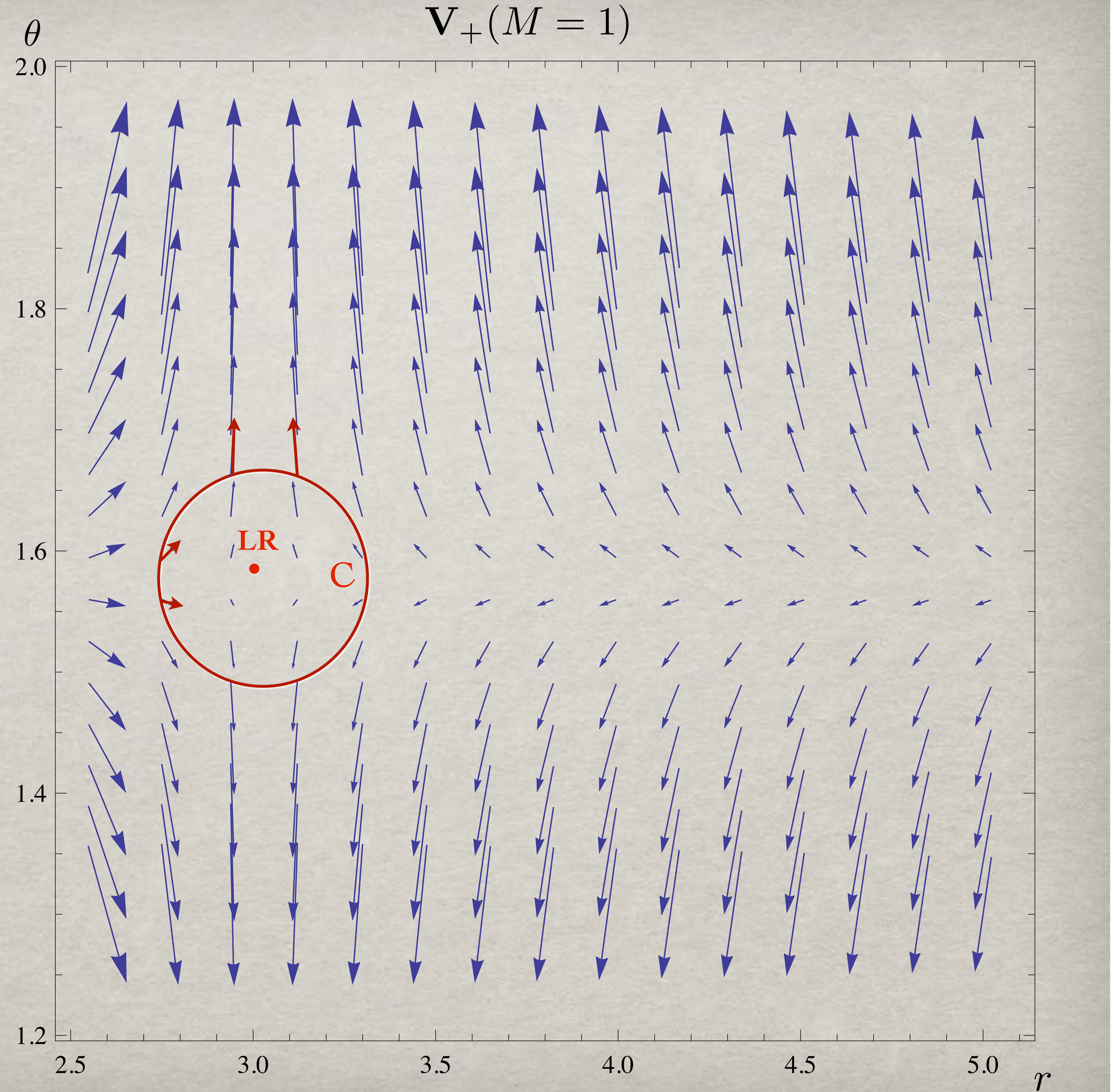

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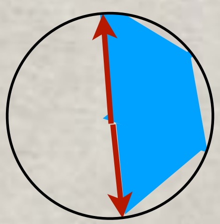

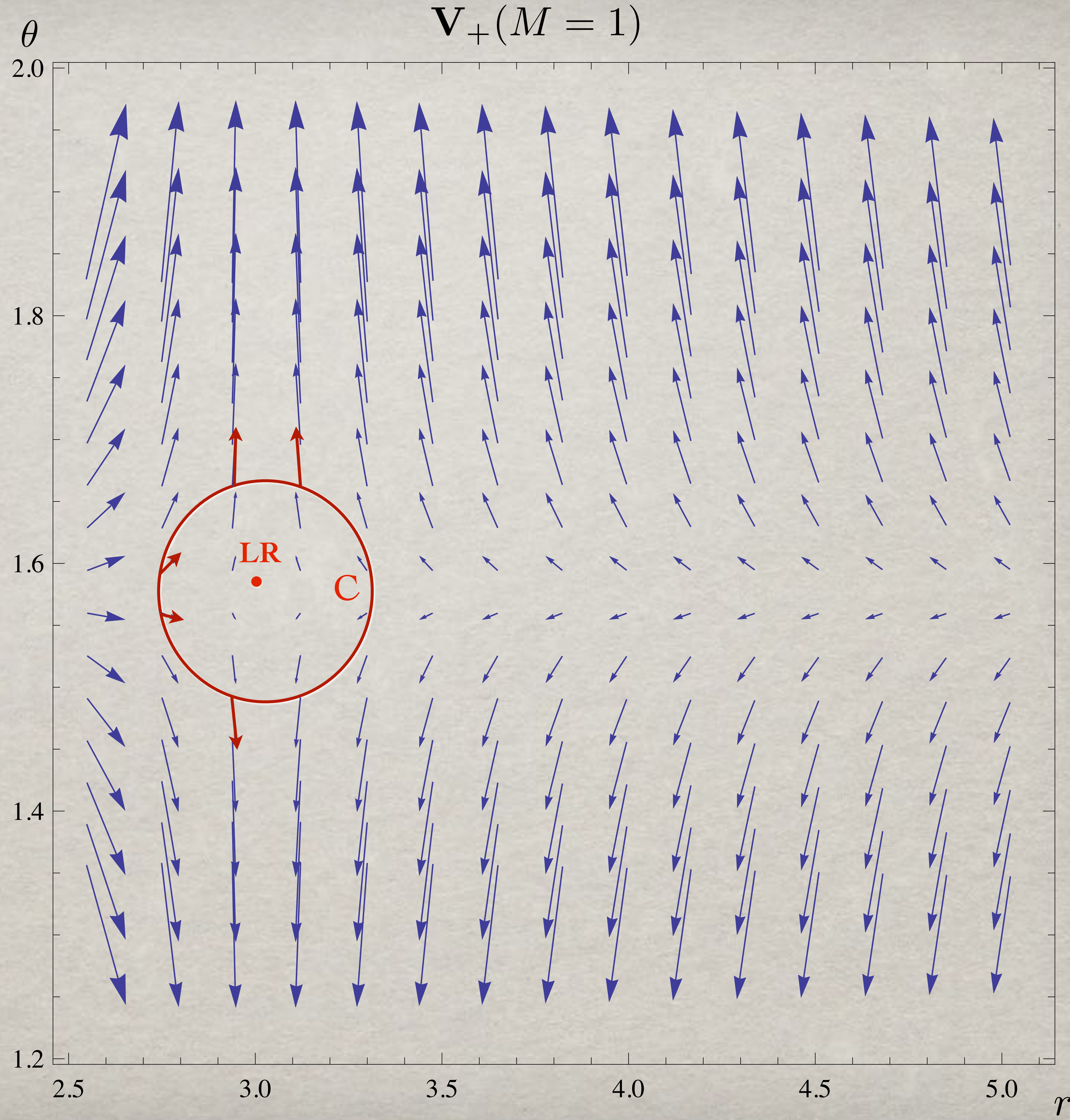

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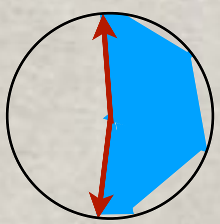

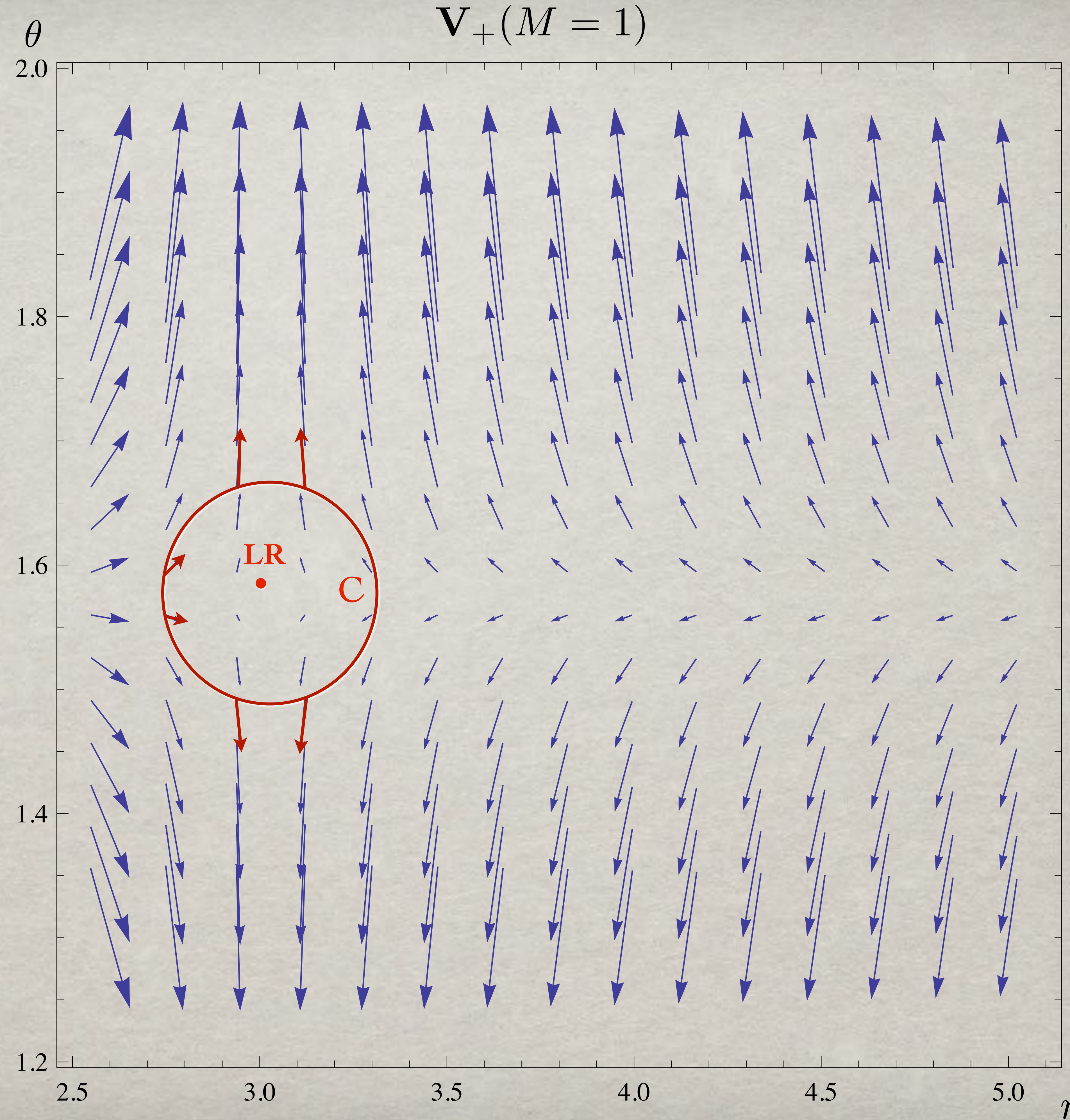

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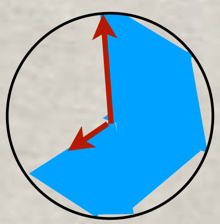

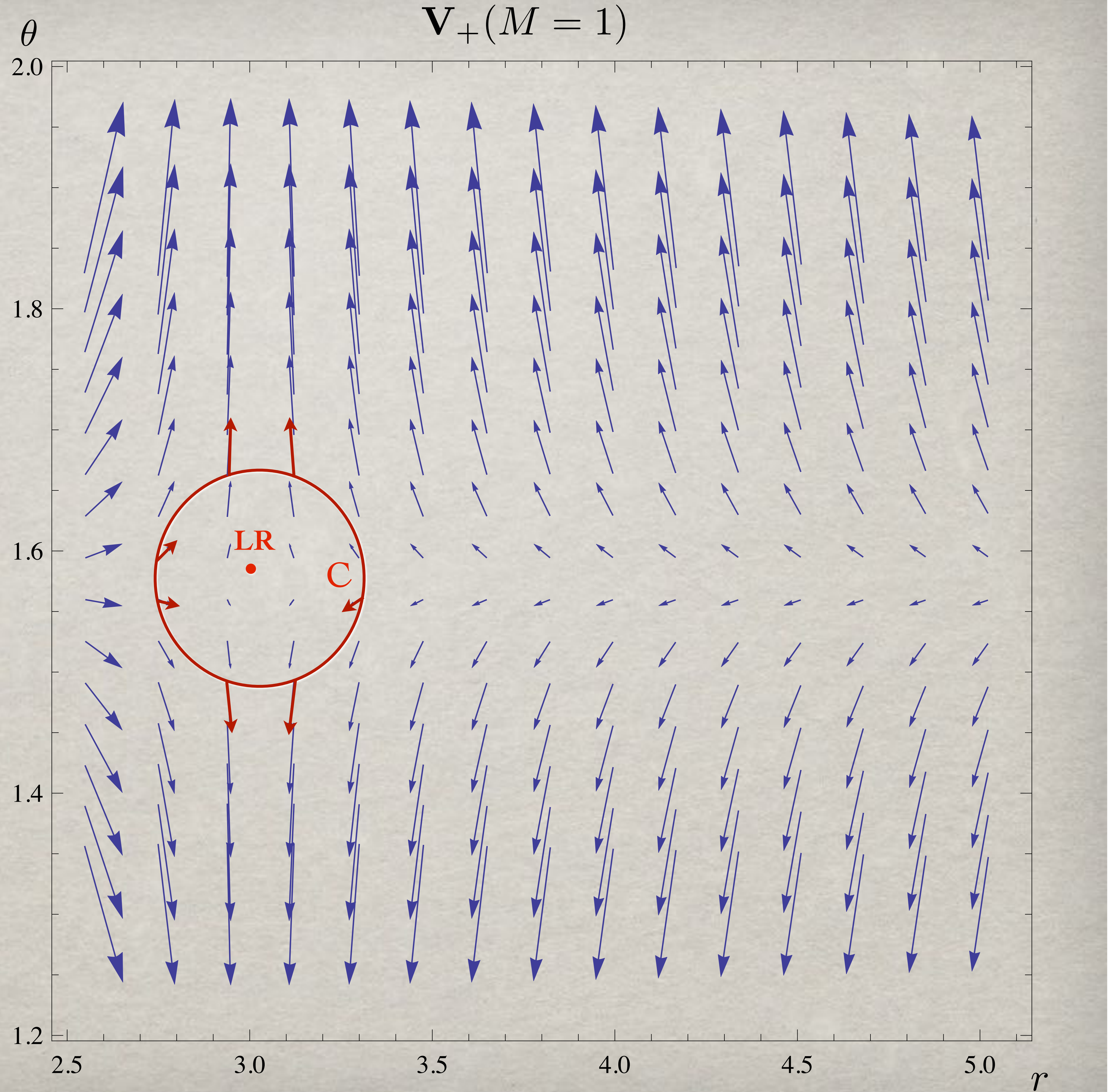

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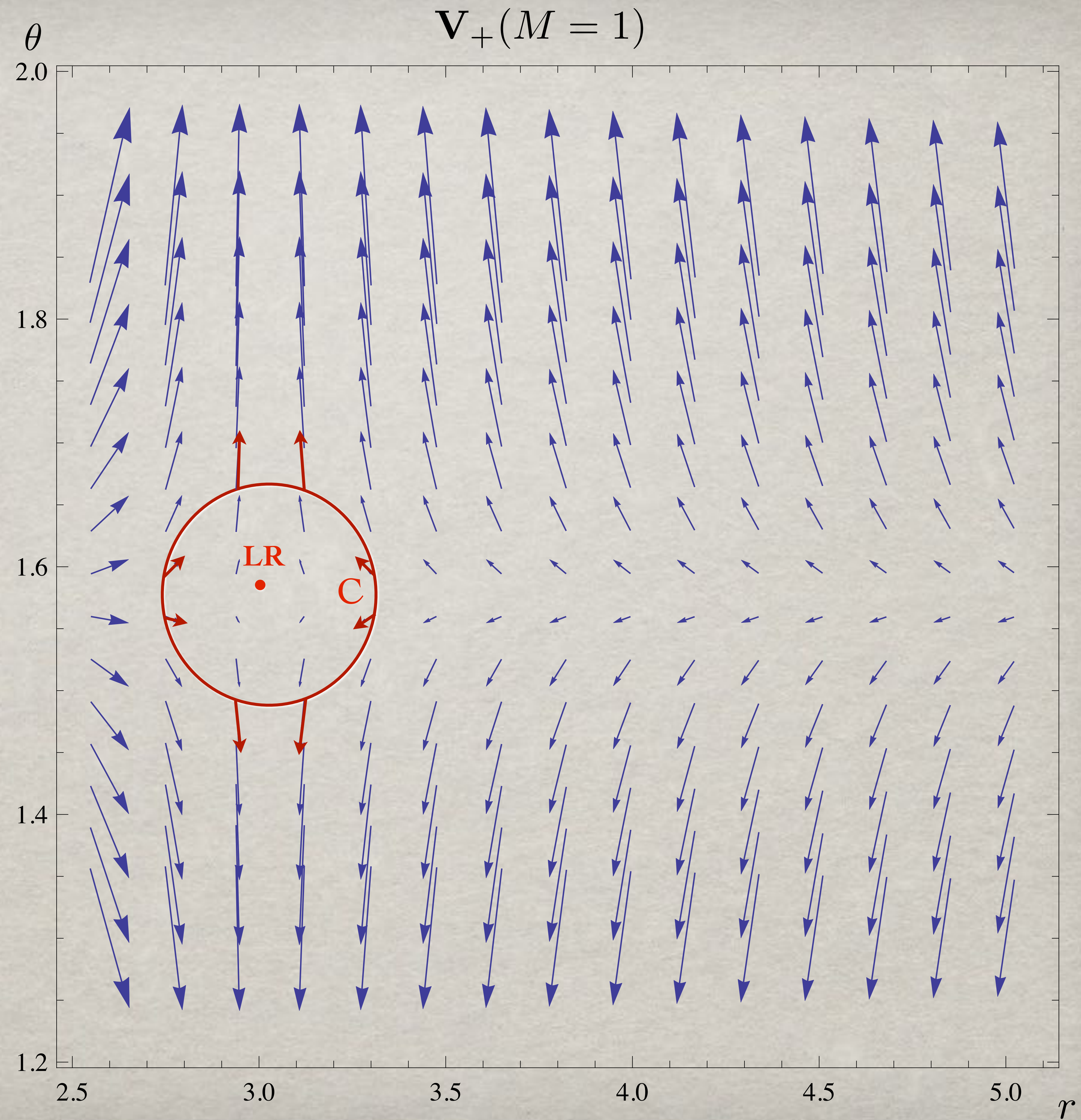

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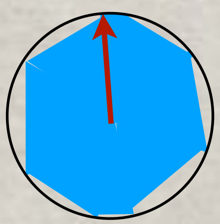

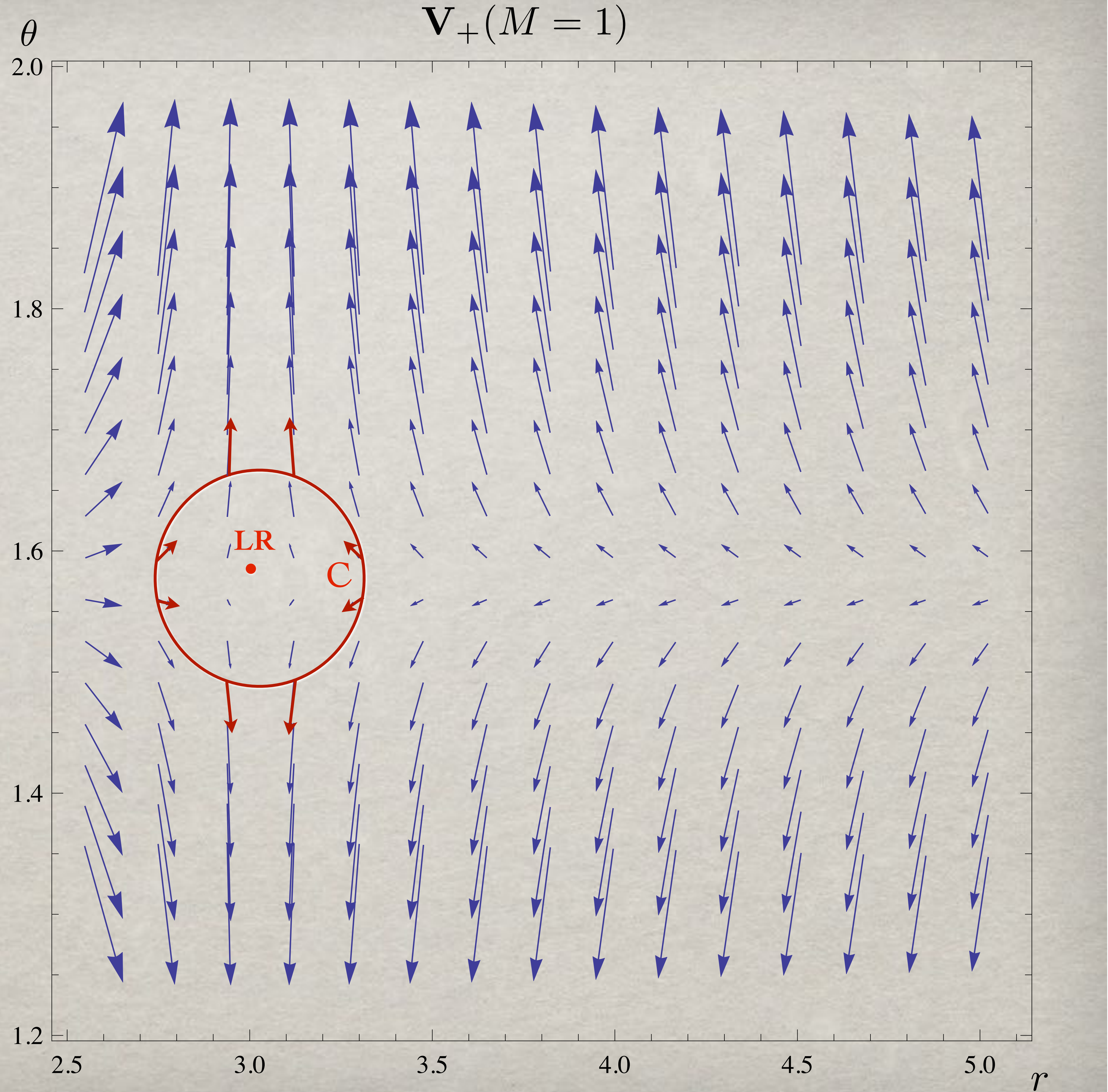

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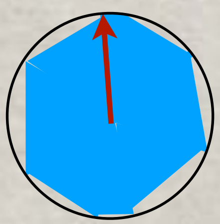
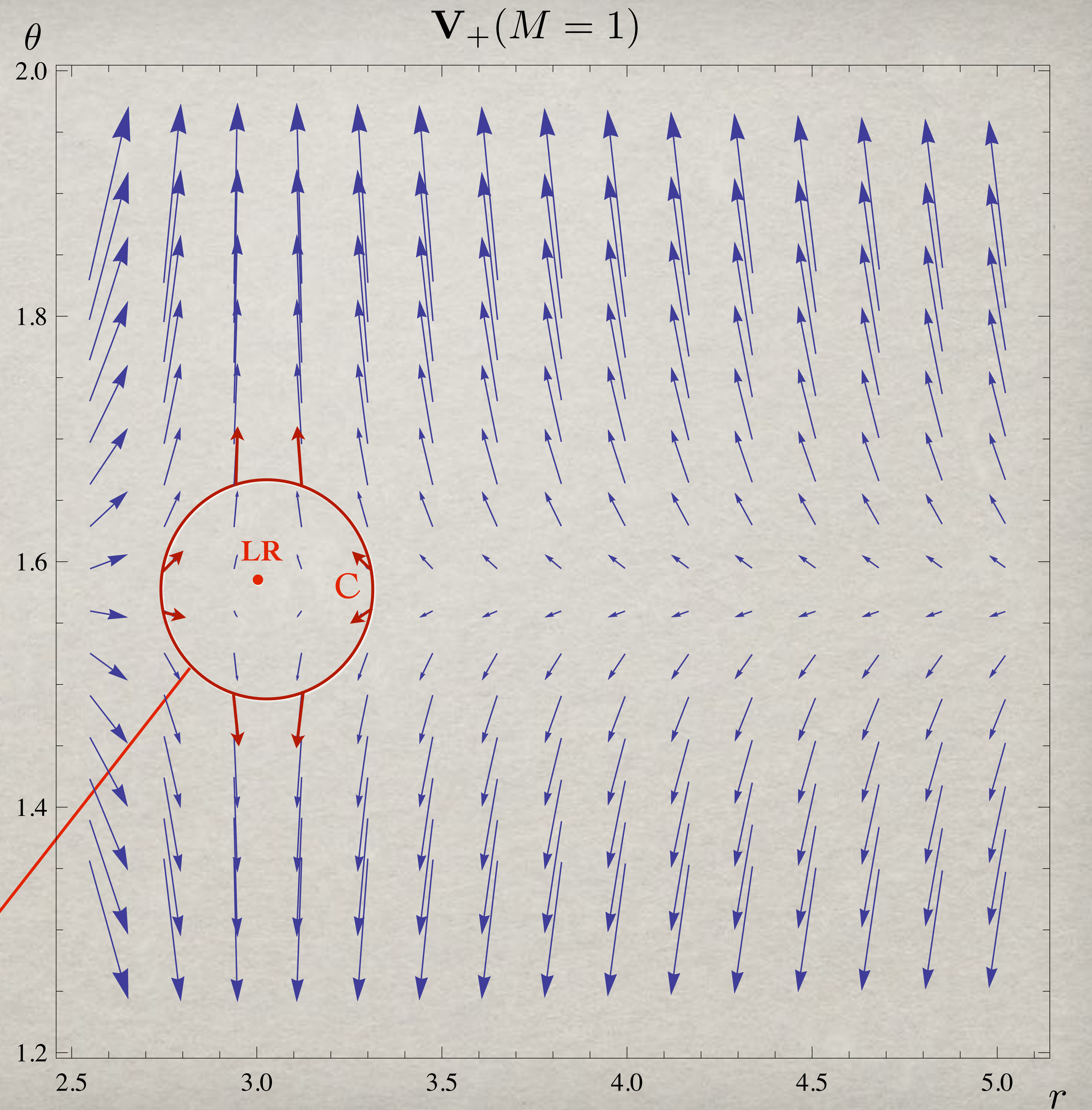


Anti-clockwise circulation of V around C gives a non-trivial clockwise winding (w=-1)



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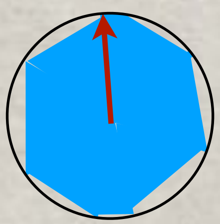

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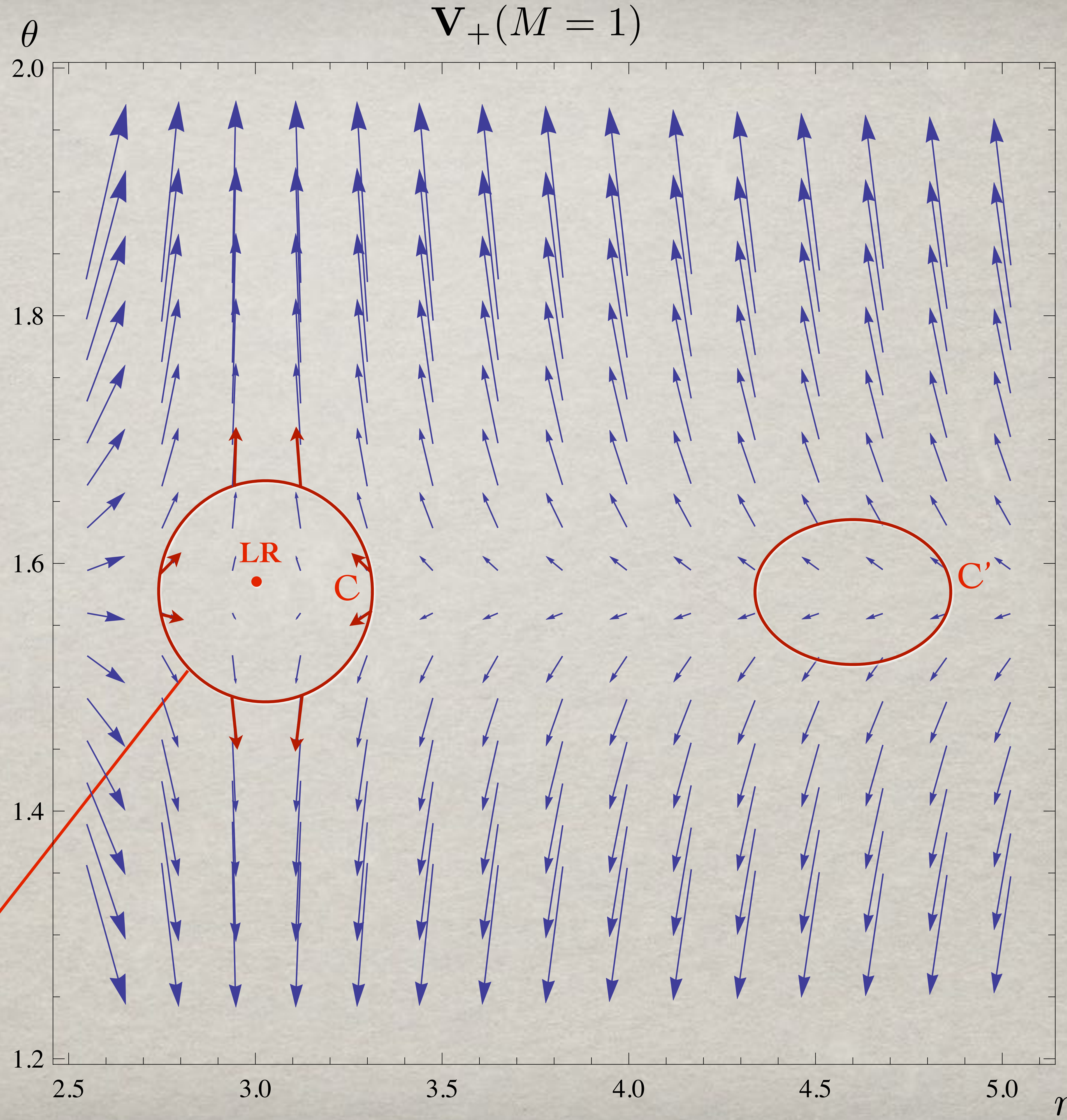

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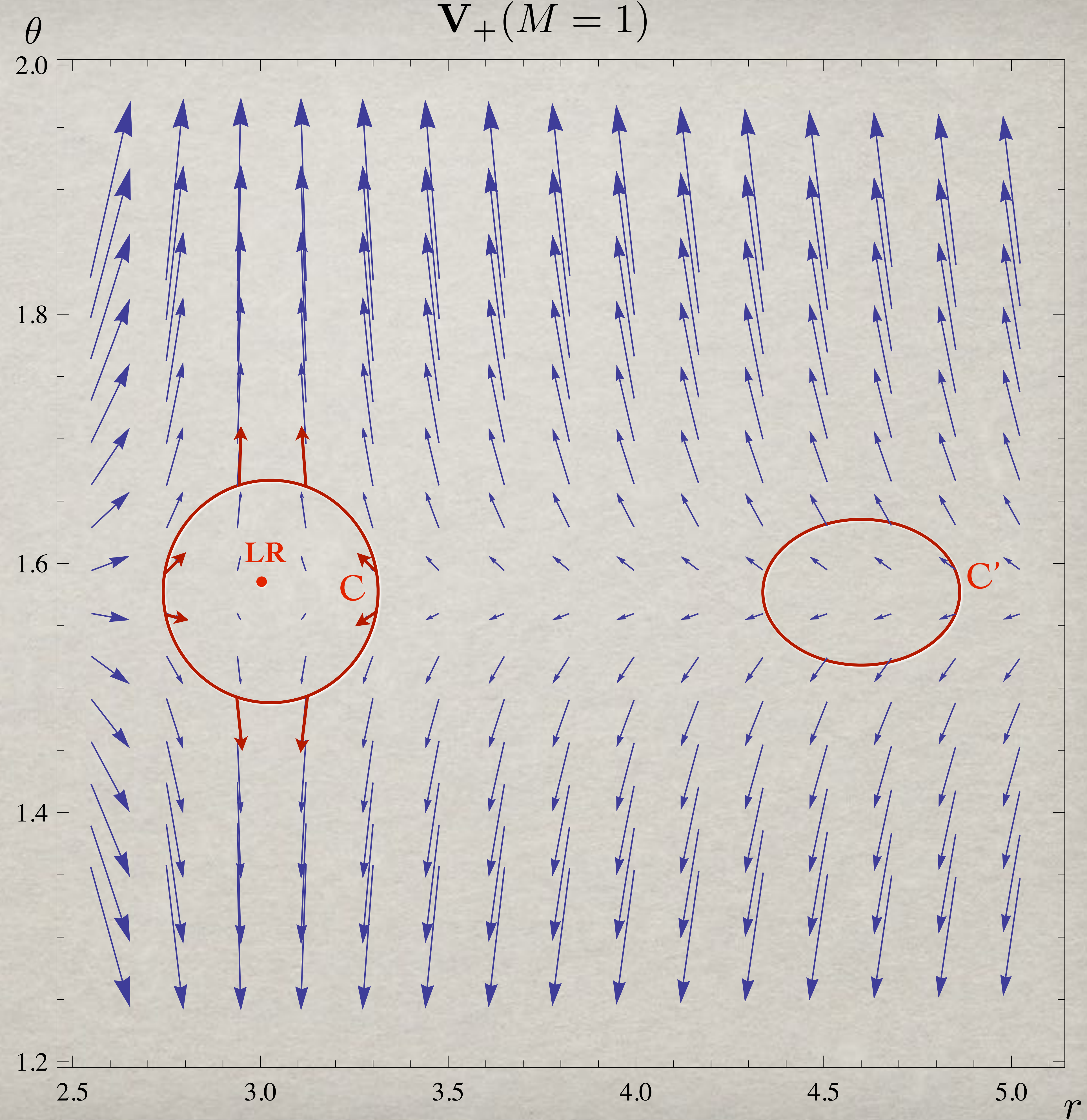

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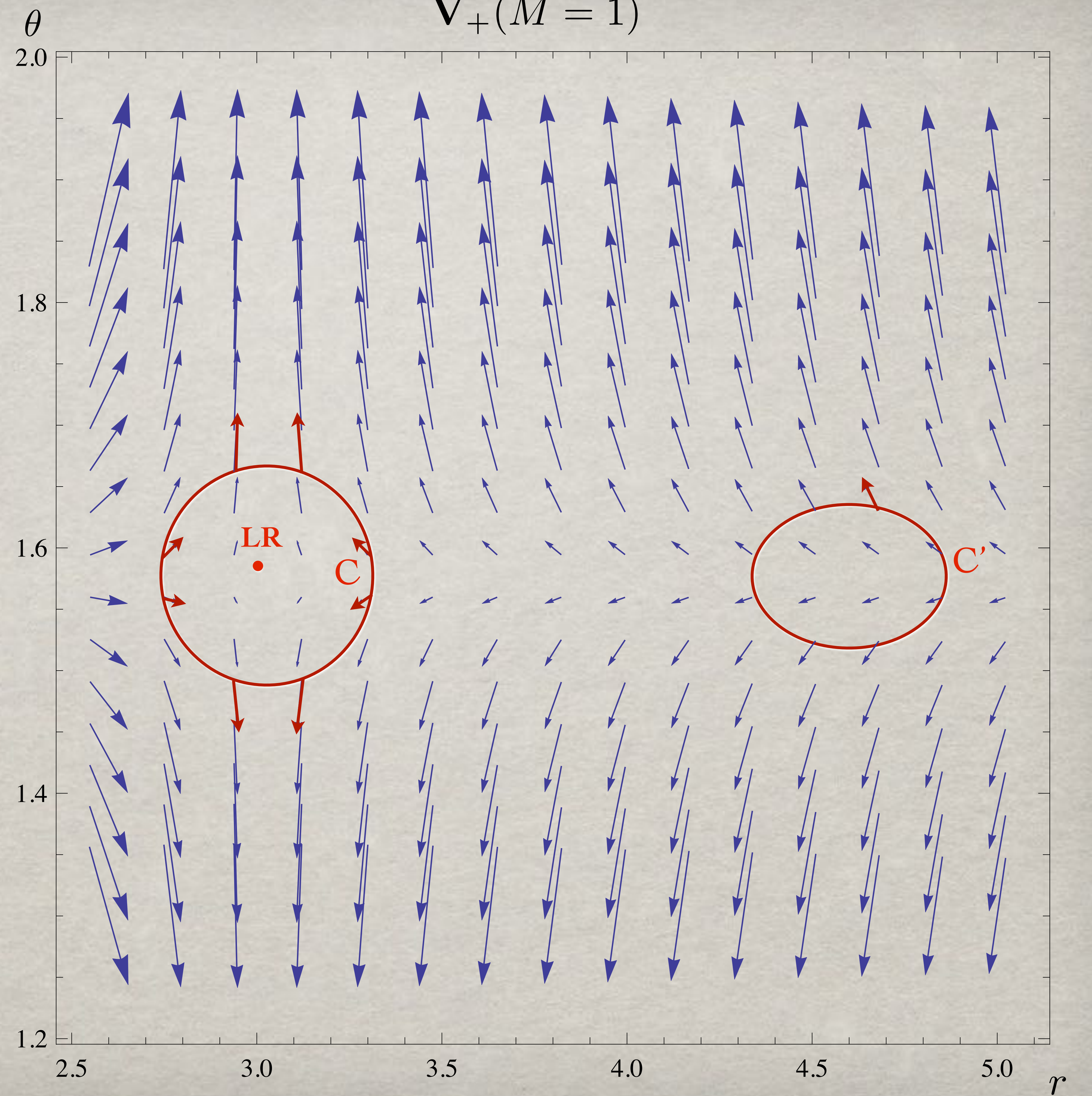

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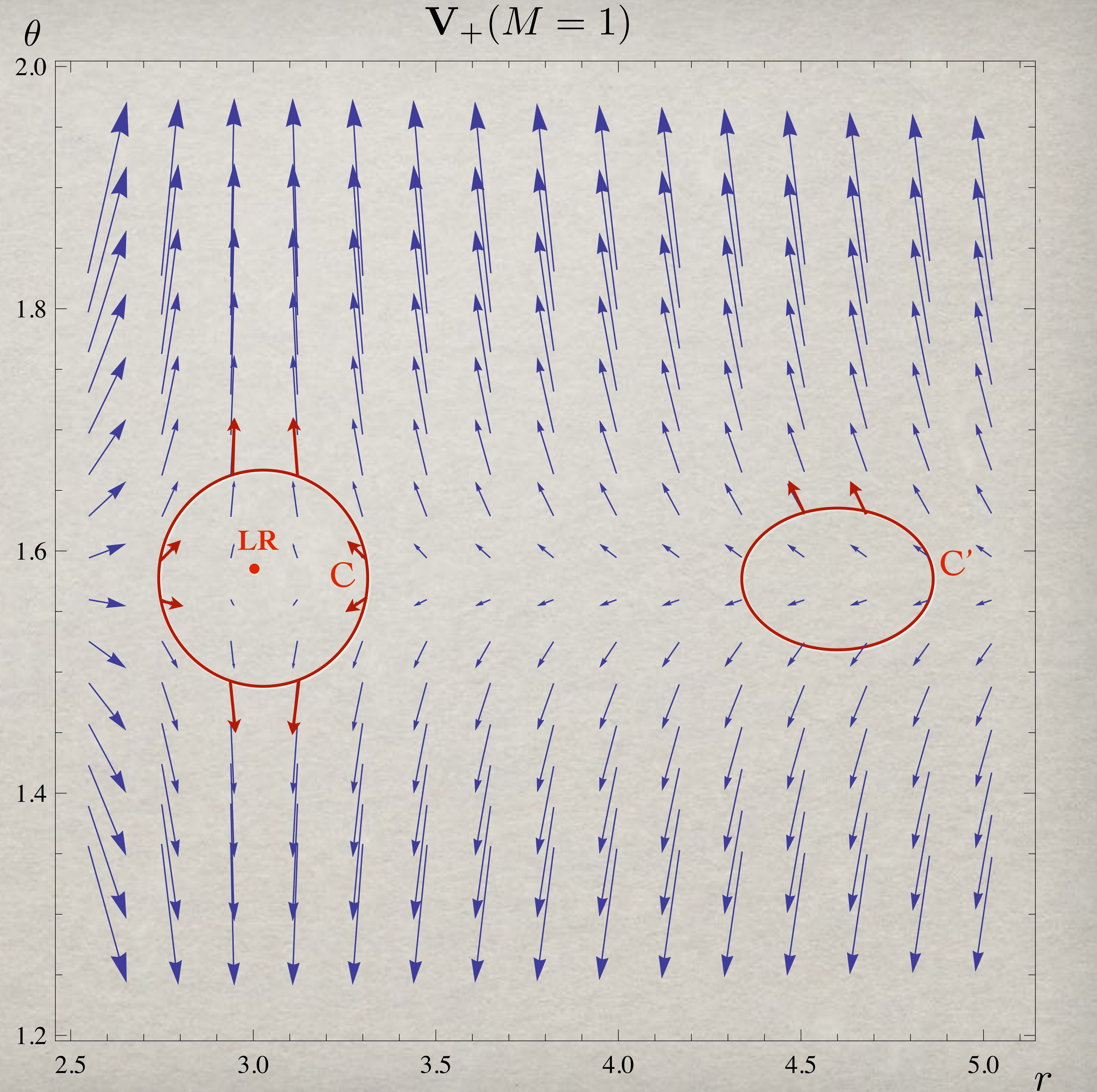
$$\mathbf{V}_\pm = \nabla H_\pm$$

Winding of the vector field



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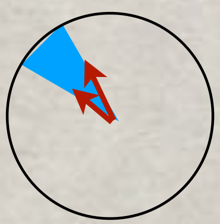
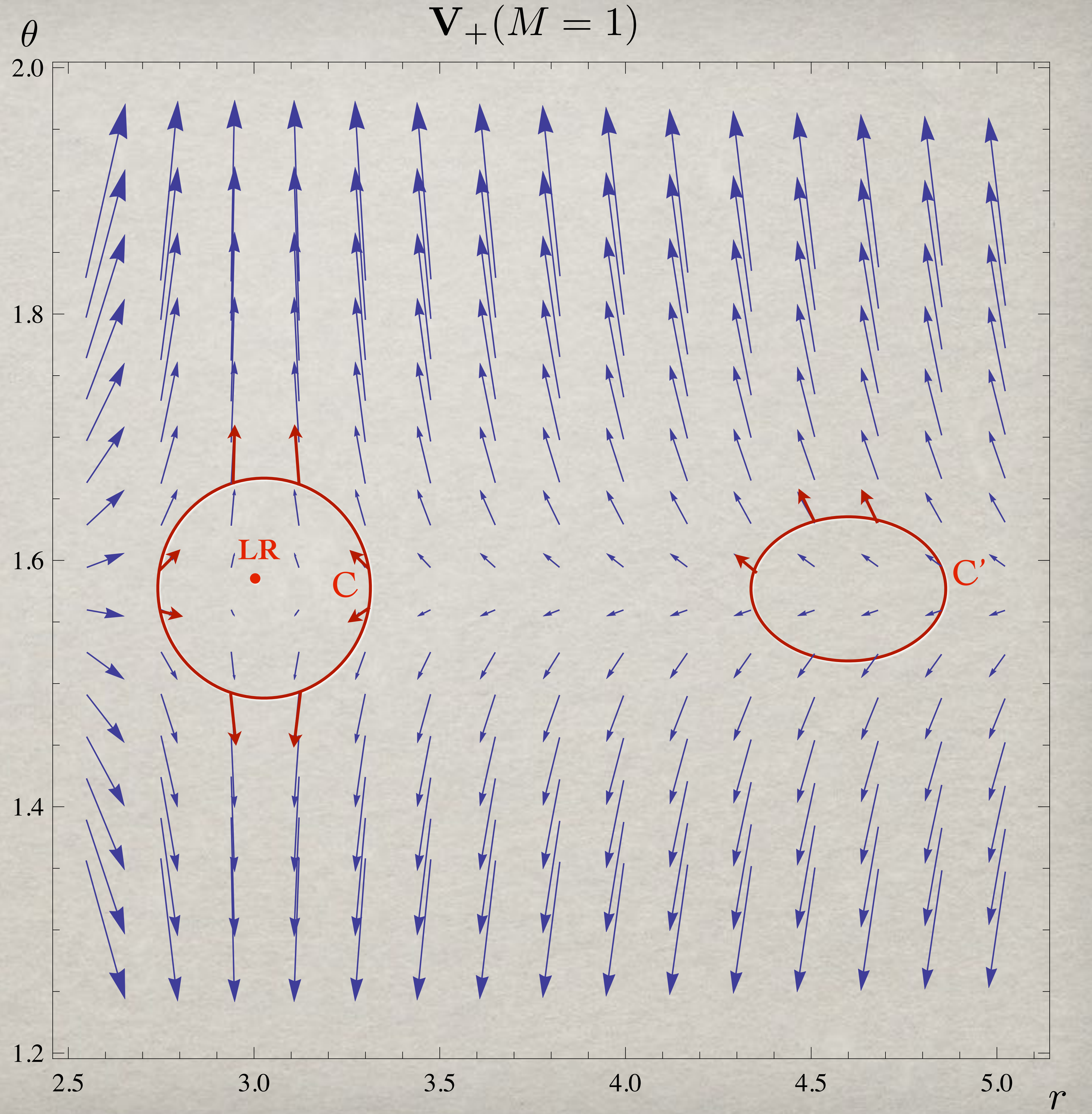
the vector field





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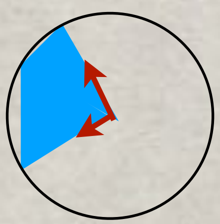

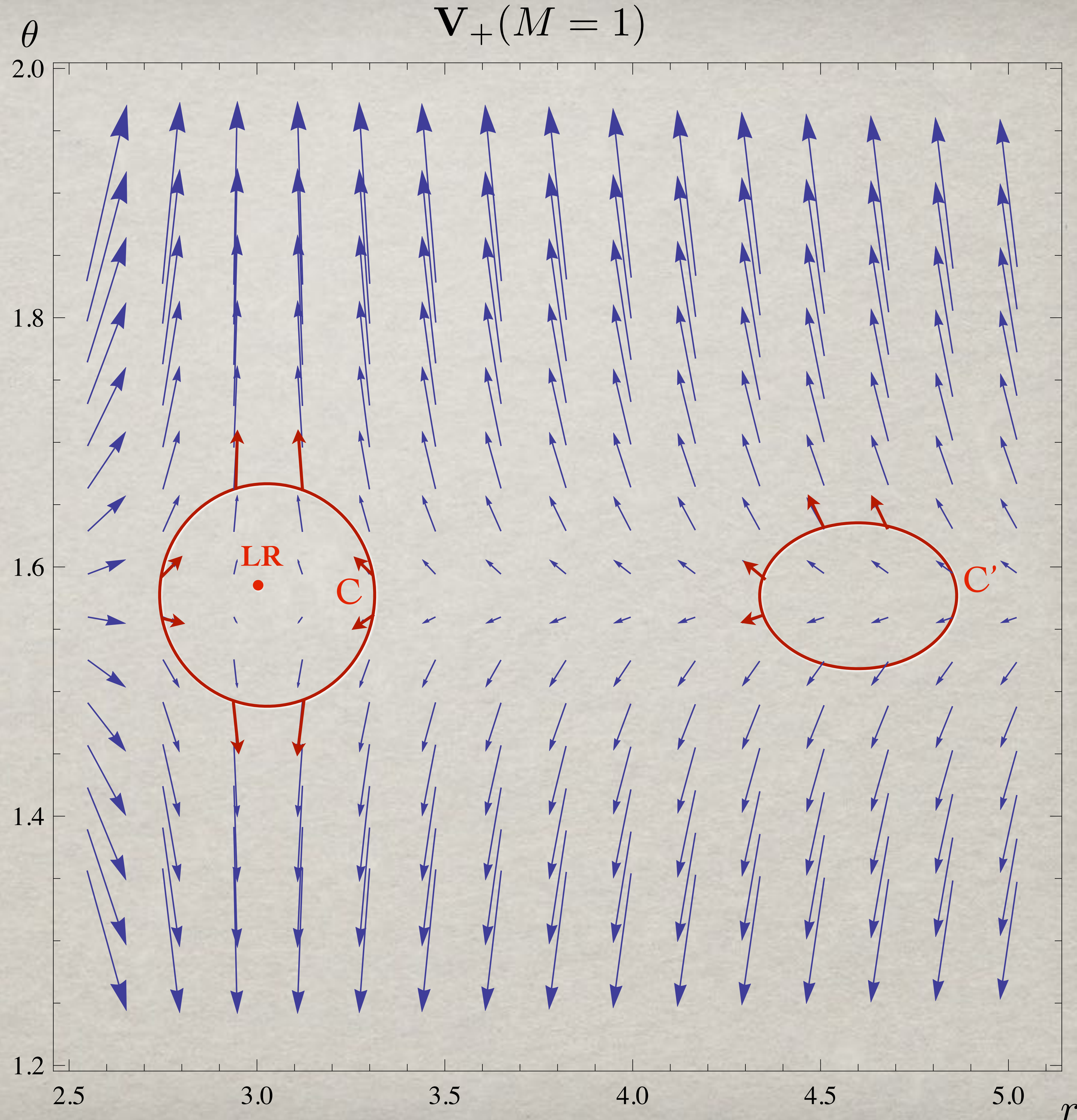

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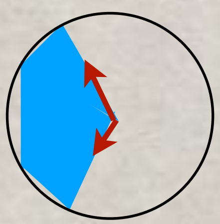

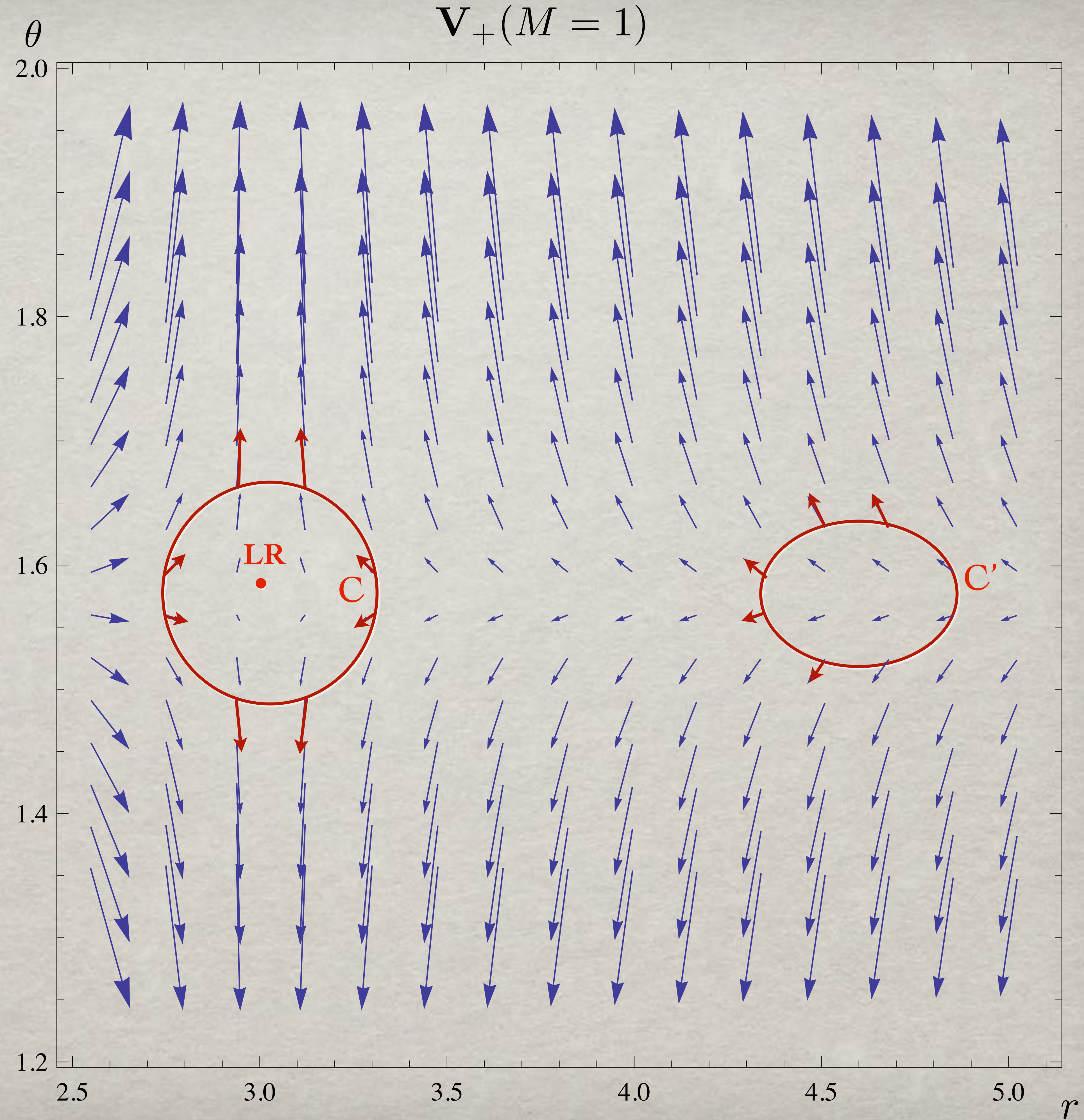

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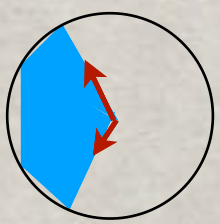

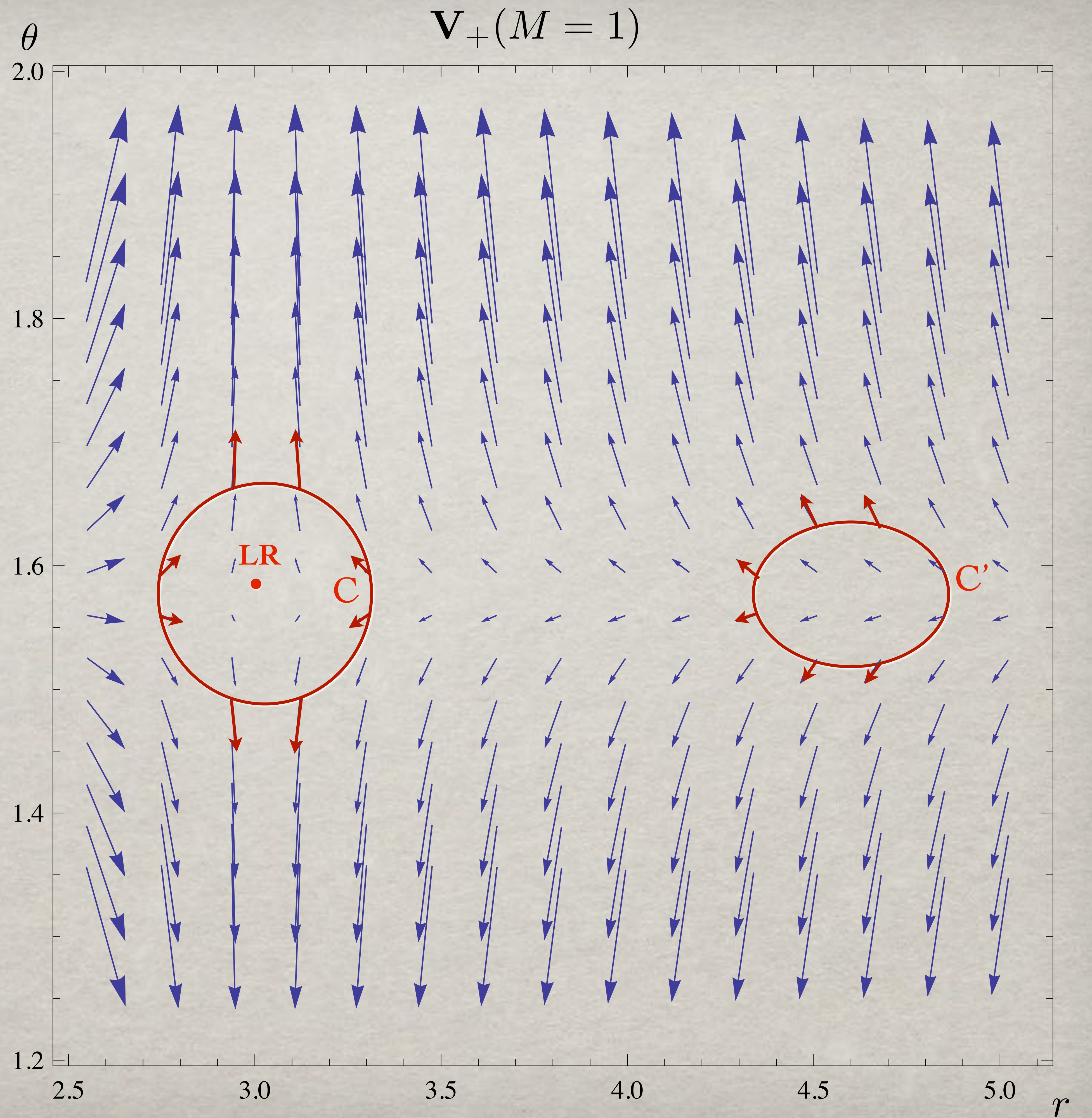

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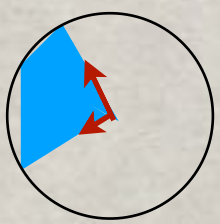

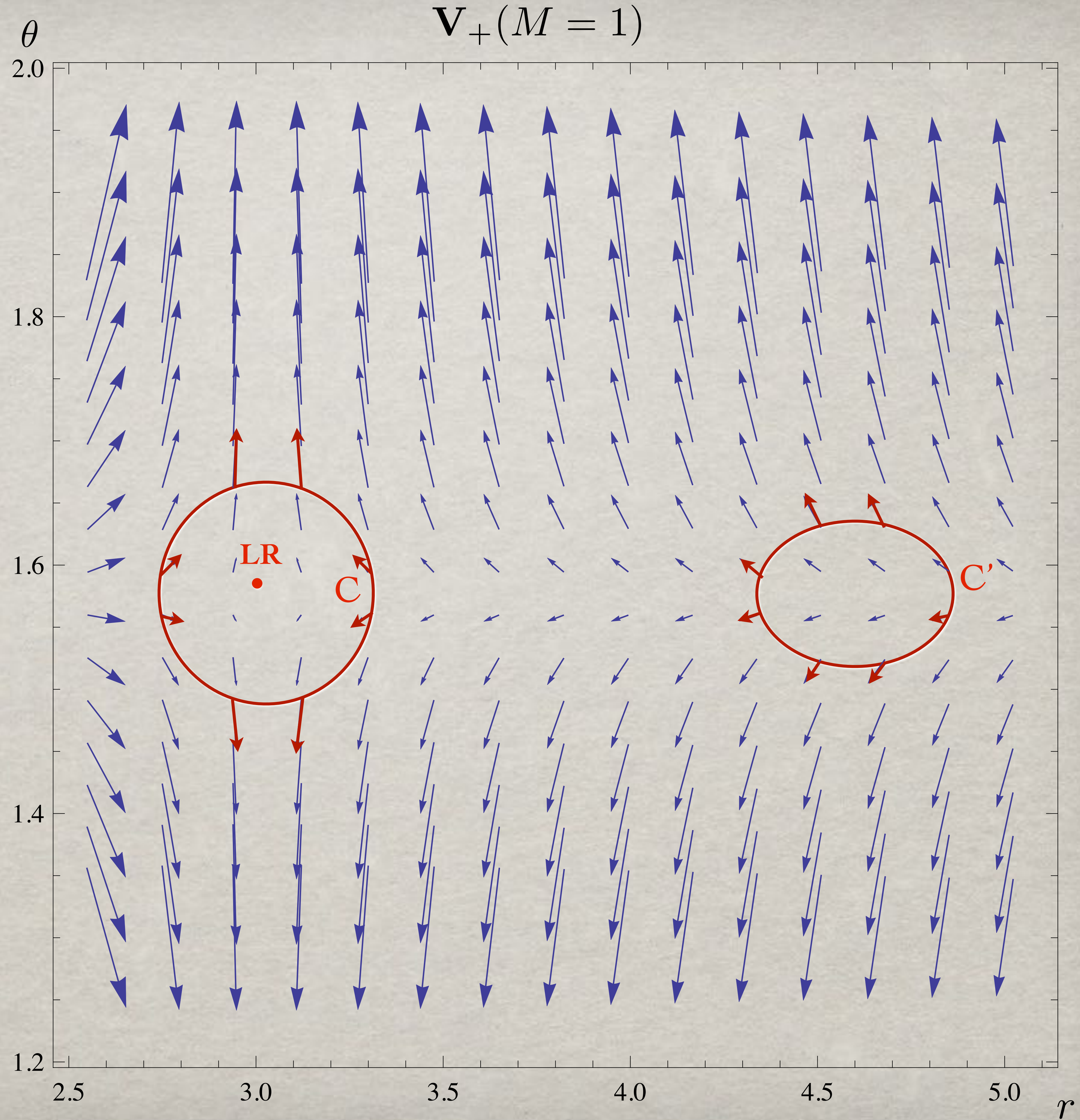

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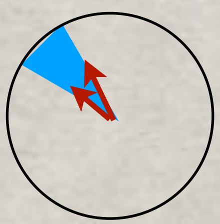

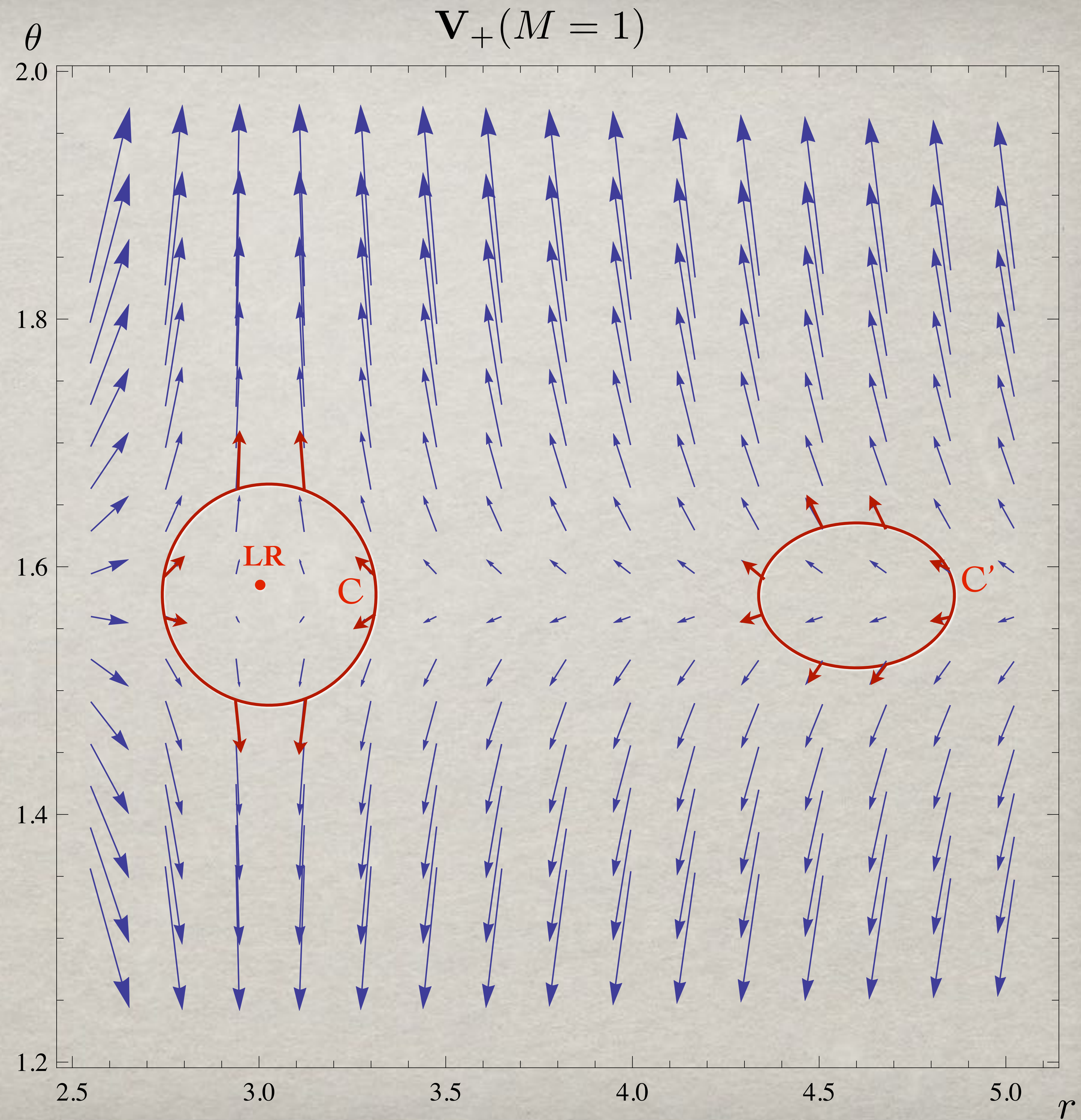

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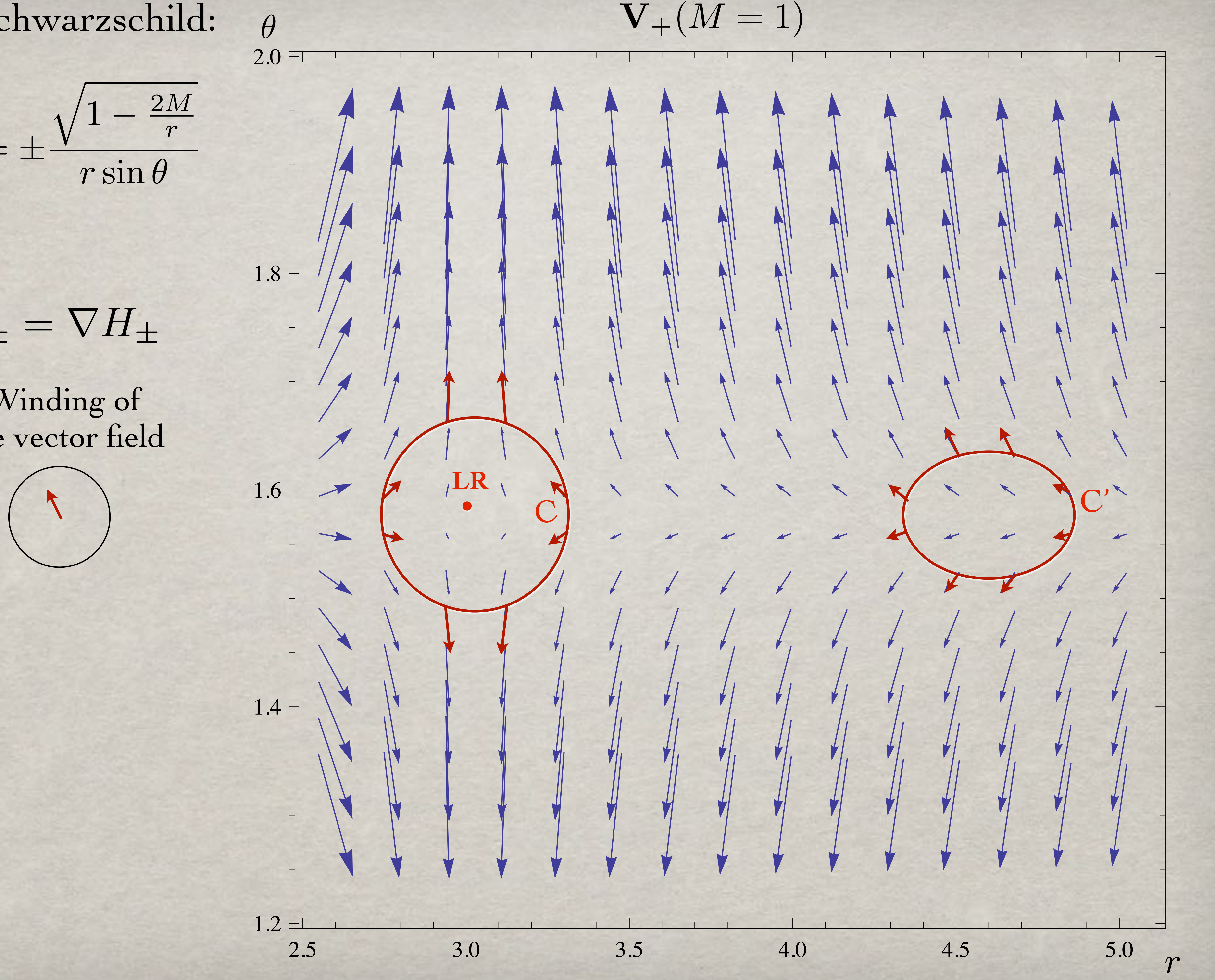

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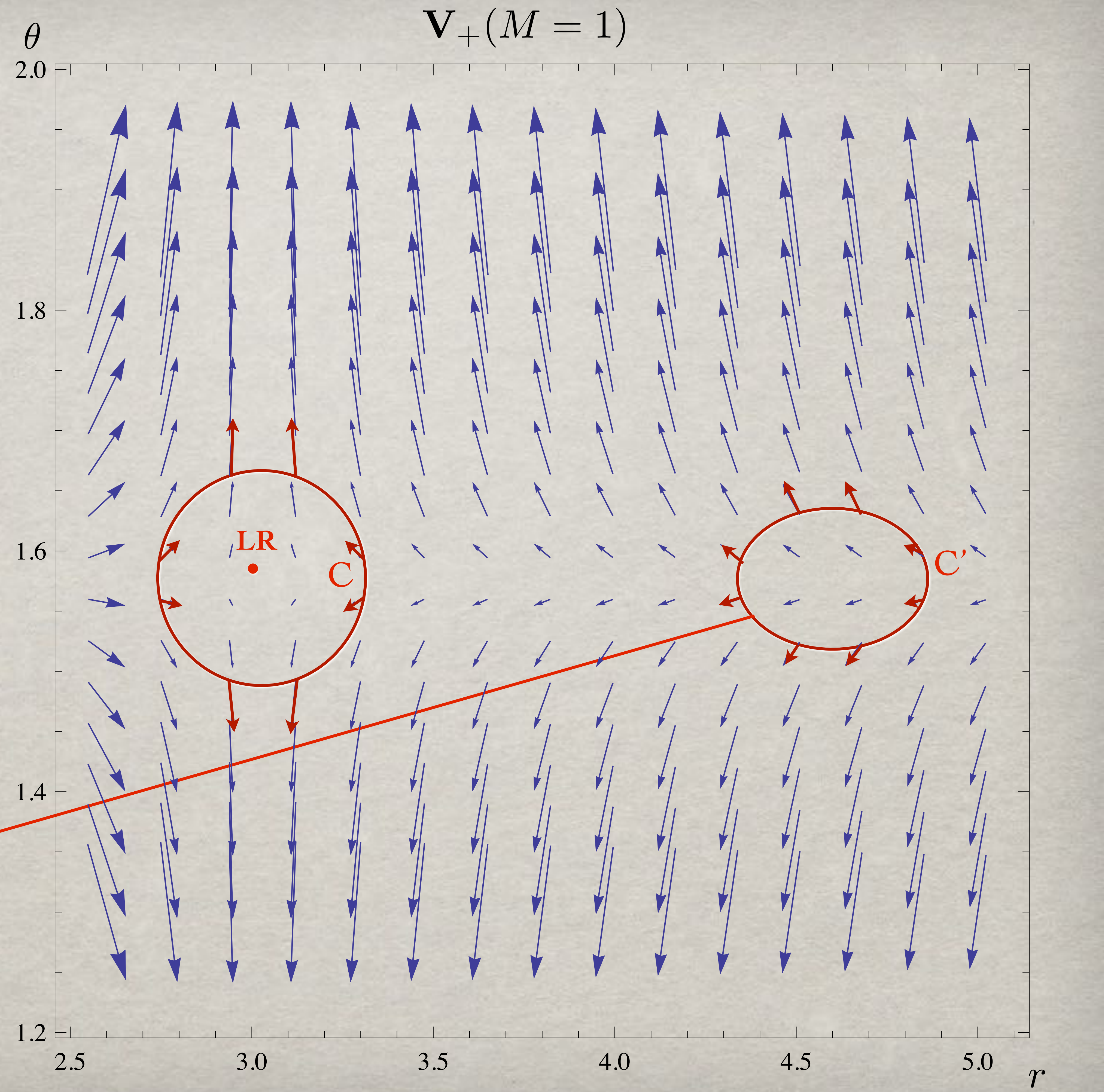
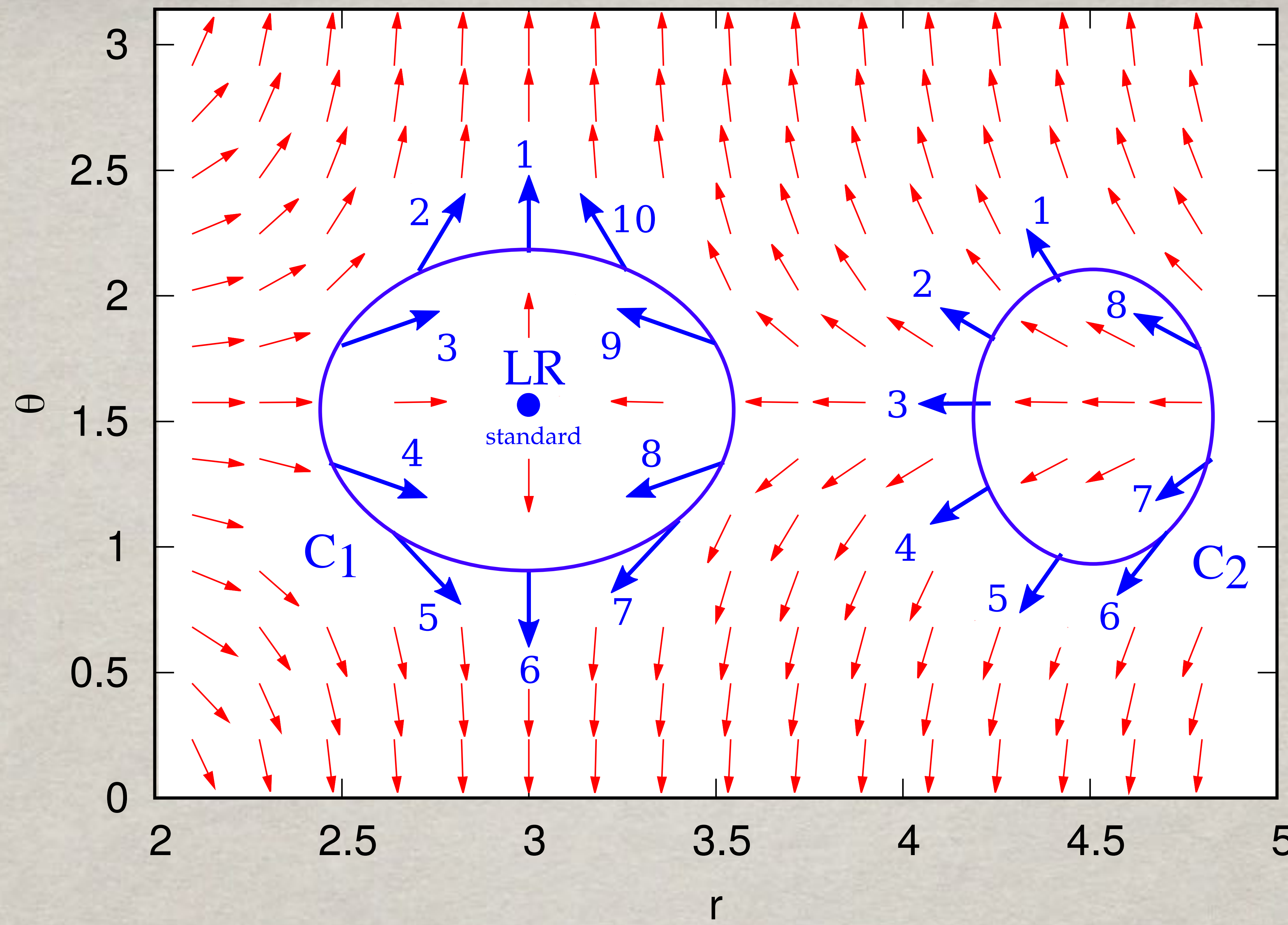
$$\mathbf{V}_{\pm} = \nabla H_{\pm}$$

Winding of
the vector field

The winding # of V
circulating around is
w=0





3
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r
LR
standard
C1
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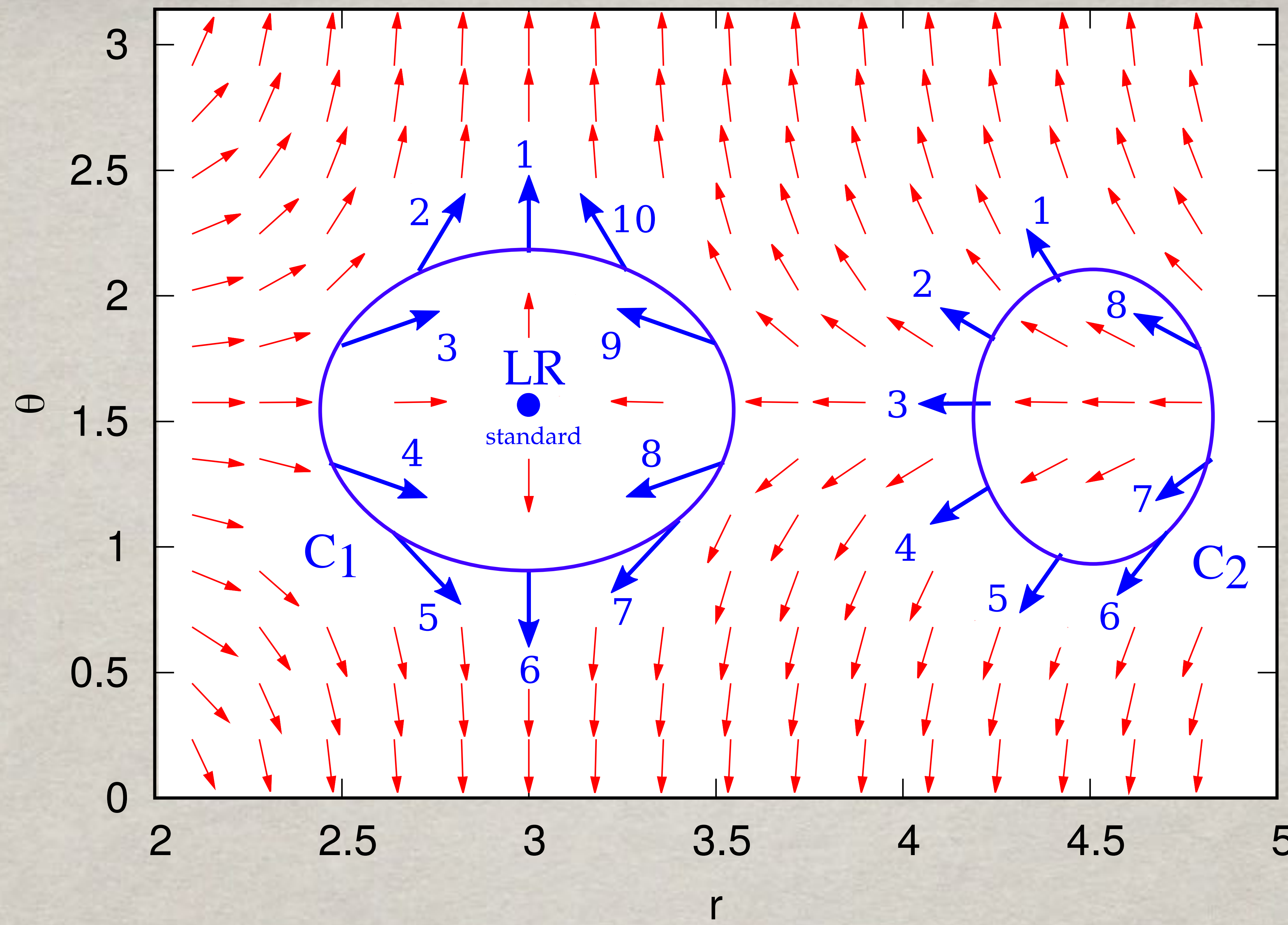
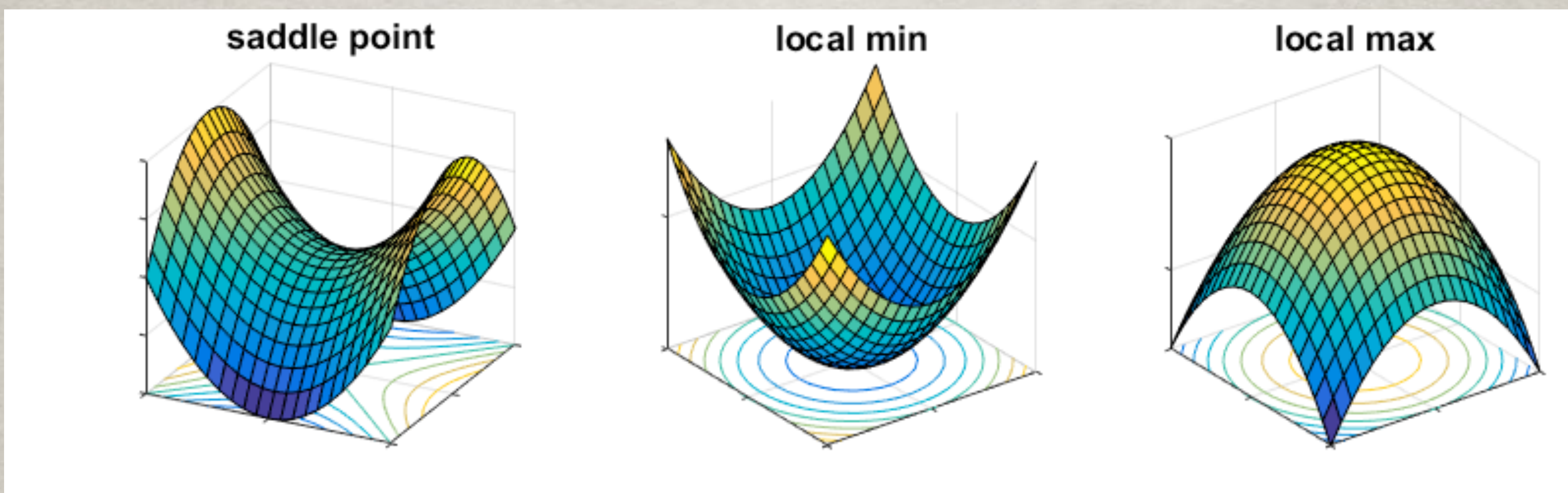


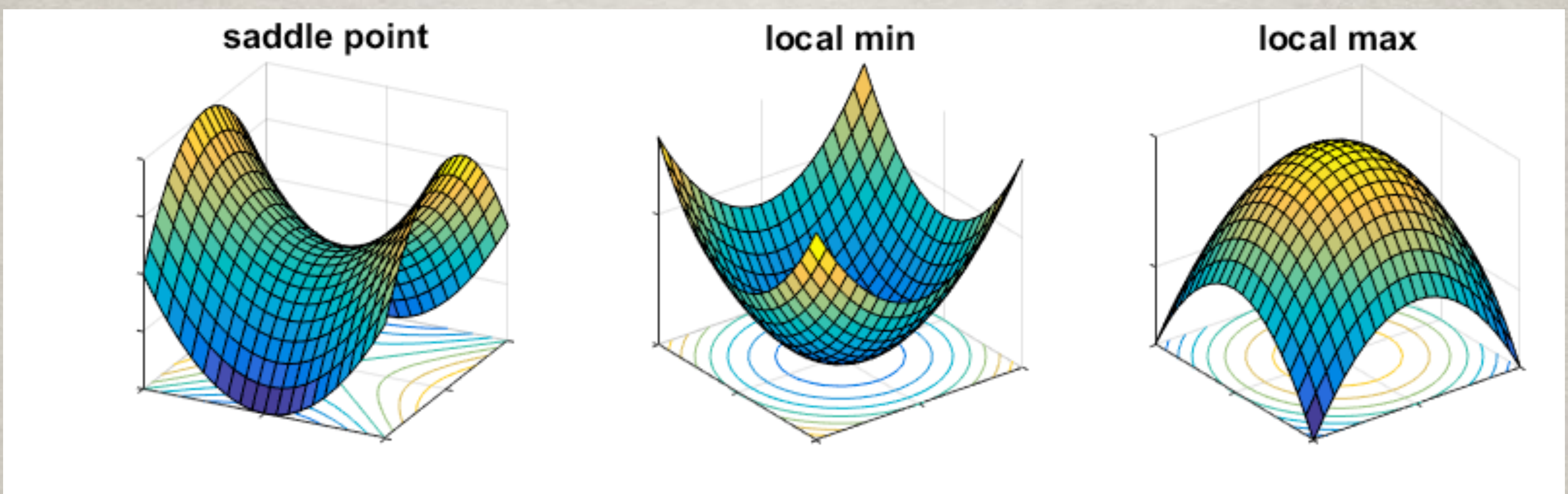
$$\oint_C d\Omega = 2\pi w \ , \qquad w \in \mathbb{Z} \ .$$

Light ring types

Light ring types



Light ring types

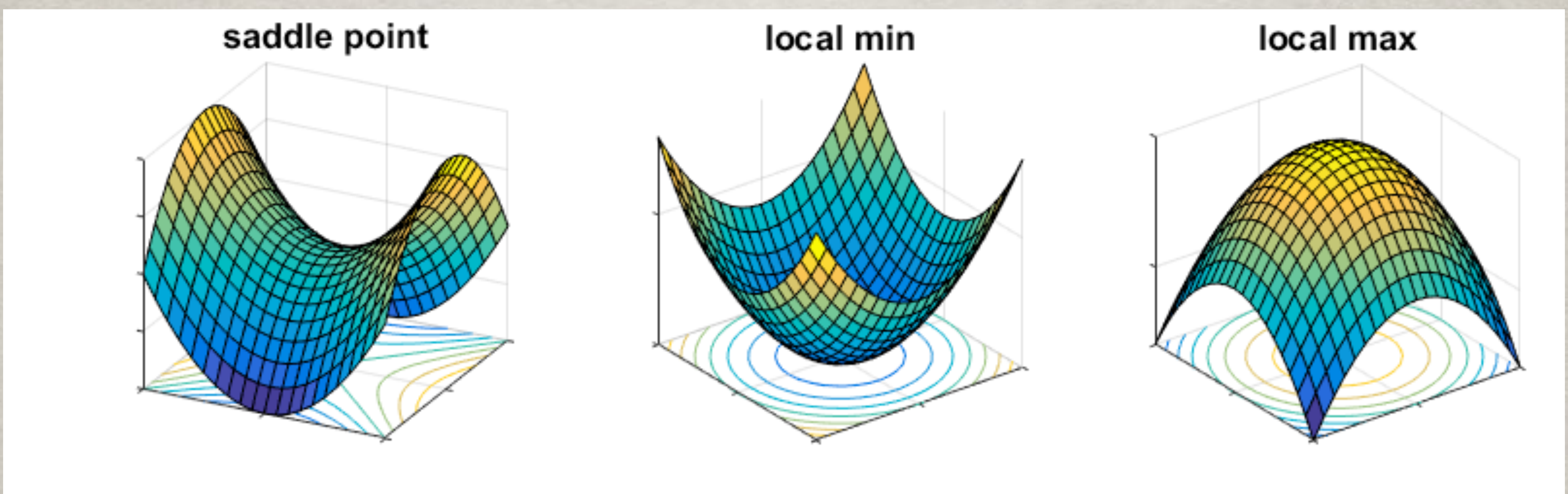


$w = -1$

Standard LRs

(Schwarzschild/Kerr like)

Light ring types

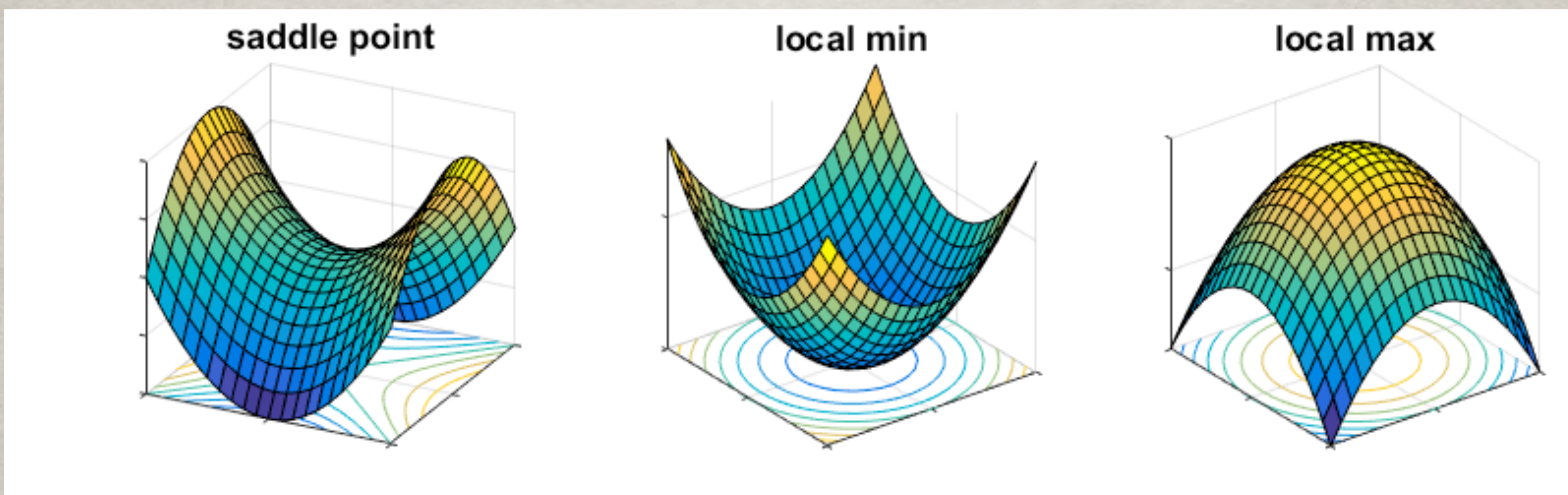


$w = -1$ $\qquad$ $w = +1$ $\qquad$ $w = +1$

Standard LRs
(Schwarzschild/Kerr like)

Exotic LRs
(Schwarzschild/Kerr unlike)

Light ring types

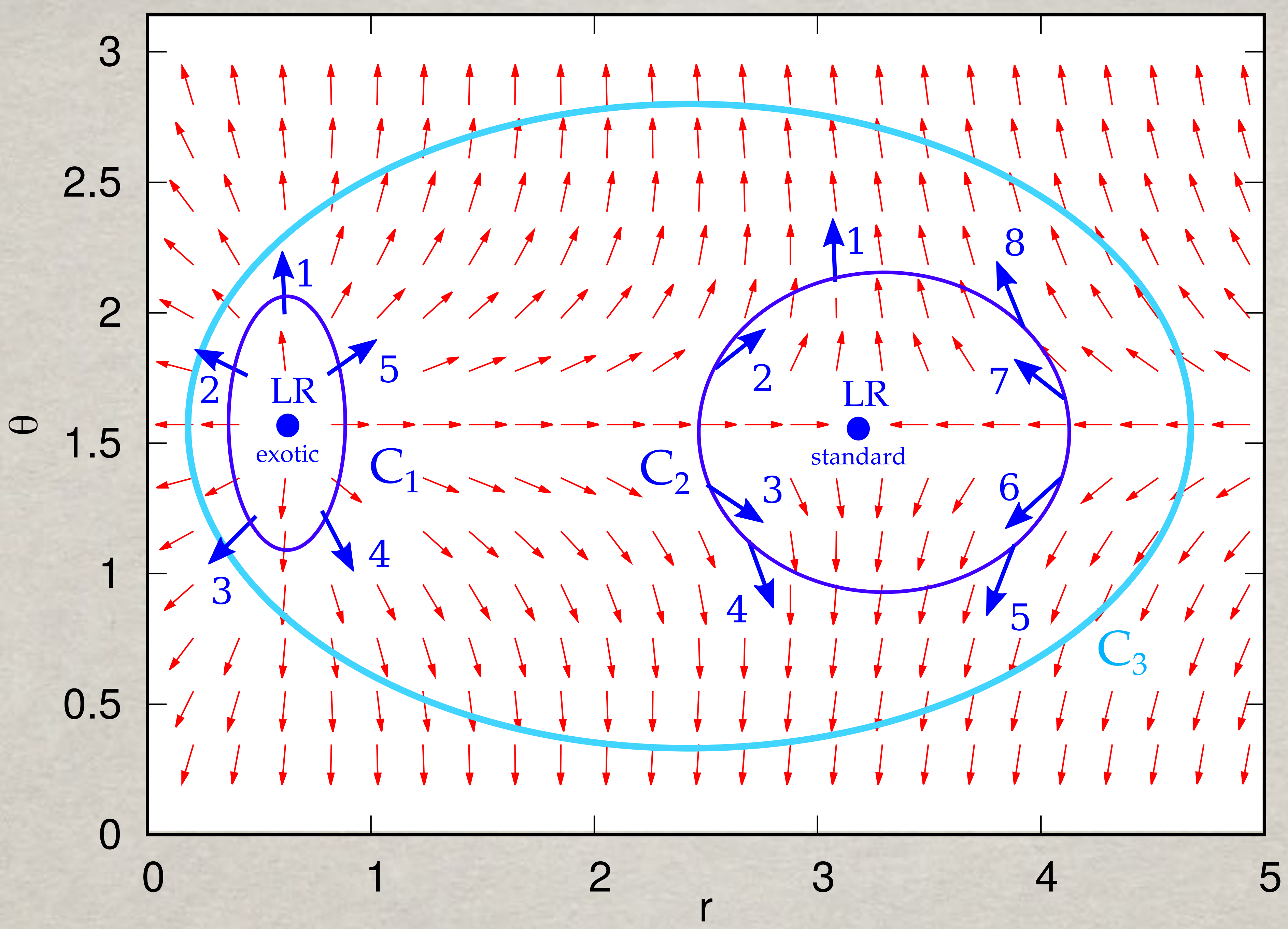


$w = -1$ $w = +1$ $w = +1$

Standard LRs
(Schwarzschild/Kerr like)

Exotic LRs
(Schwarzschild/Kerr unlike)

$$\oint_C d\Omega = 2\pi \sum_i w_i \ , \qquad w_i = -1, 1 \ .$$



3
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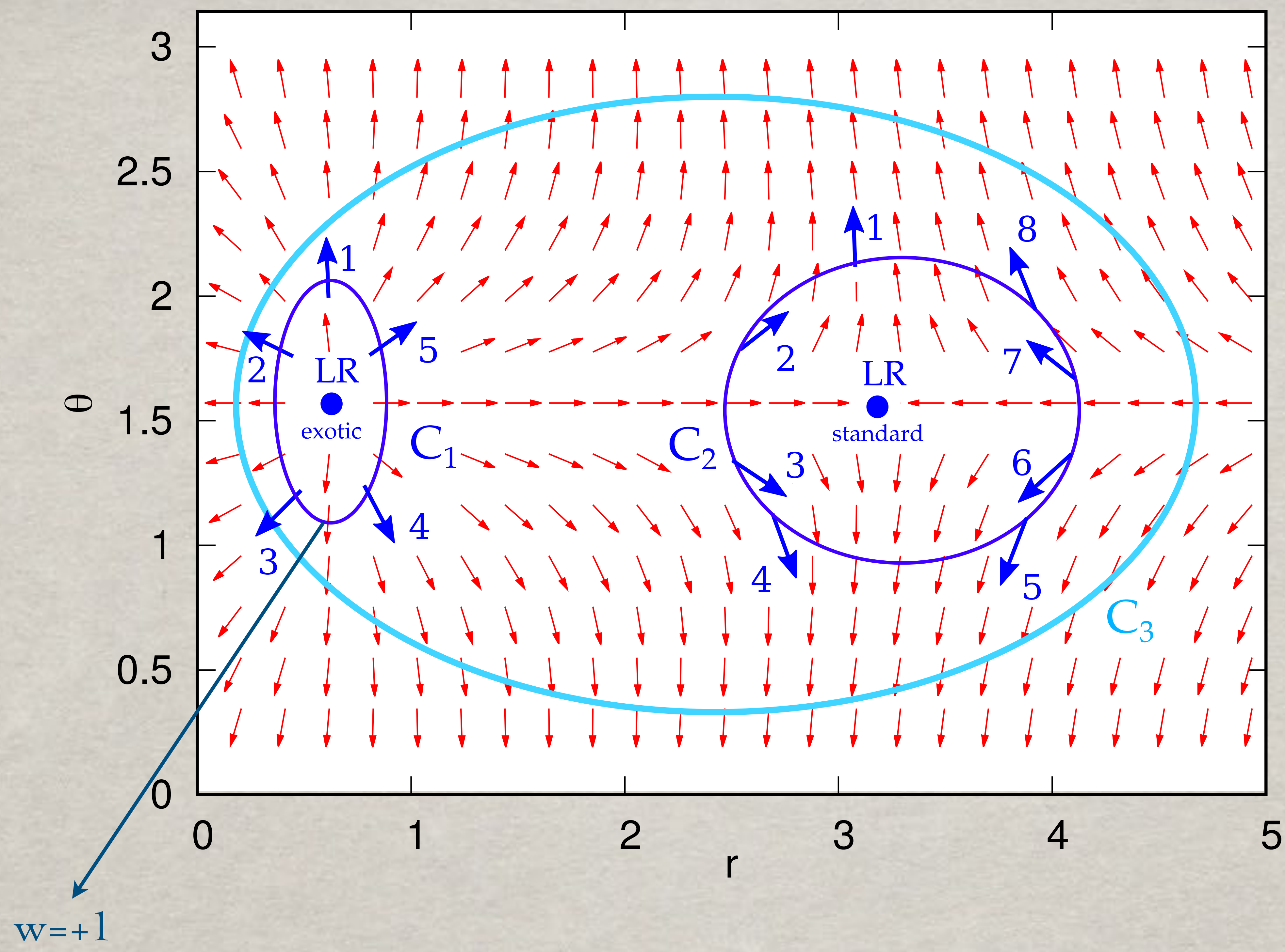
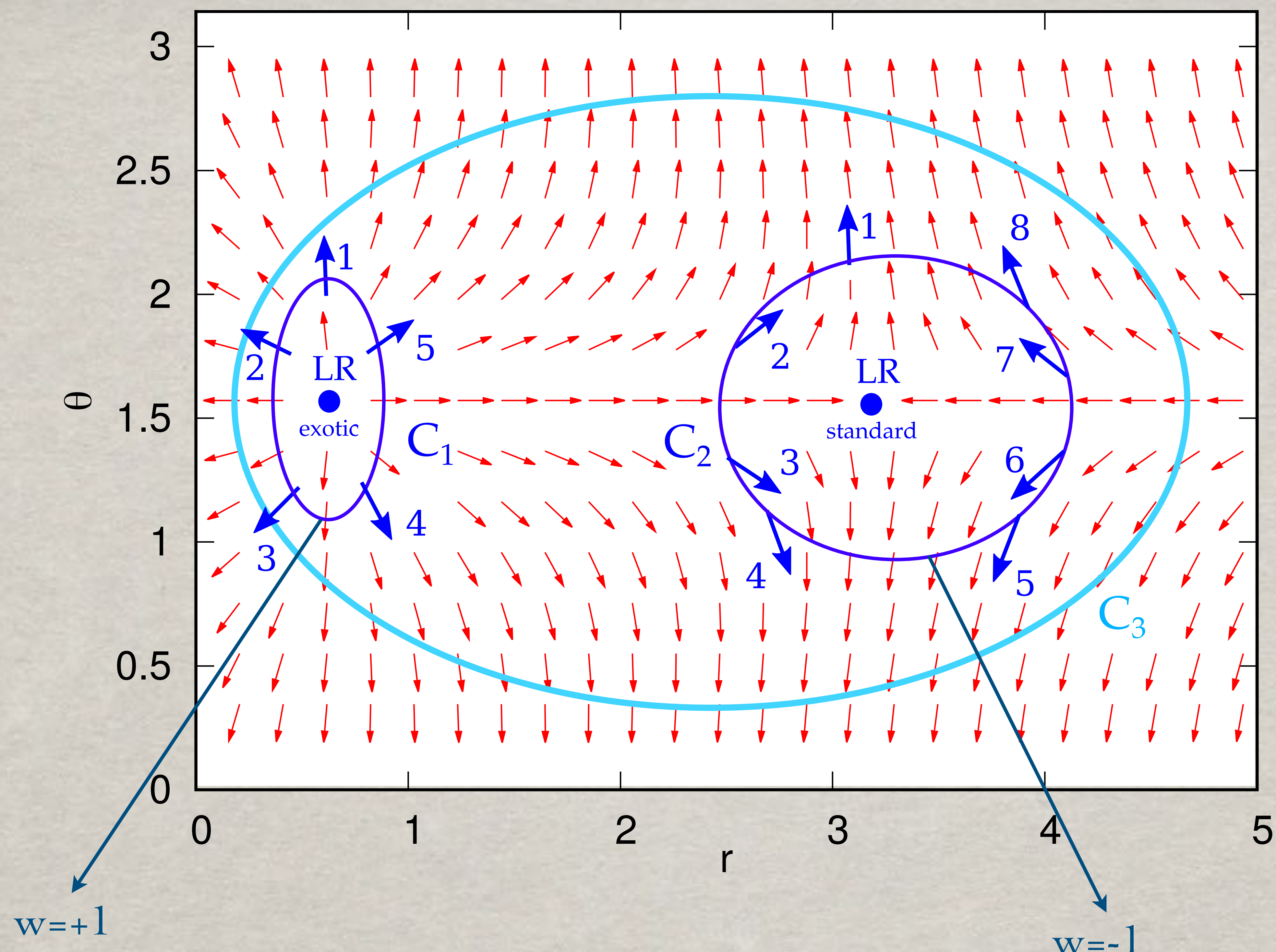


θ
r
LR
exotic
LR
standard
C1
C2
C3
w=+1



LR
exotic
LR
standard
C1
C2
C3
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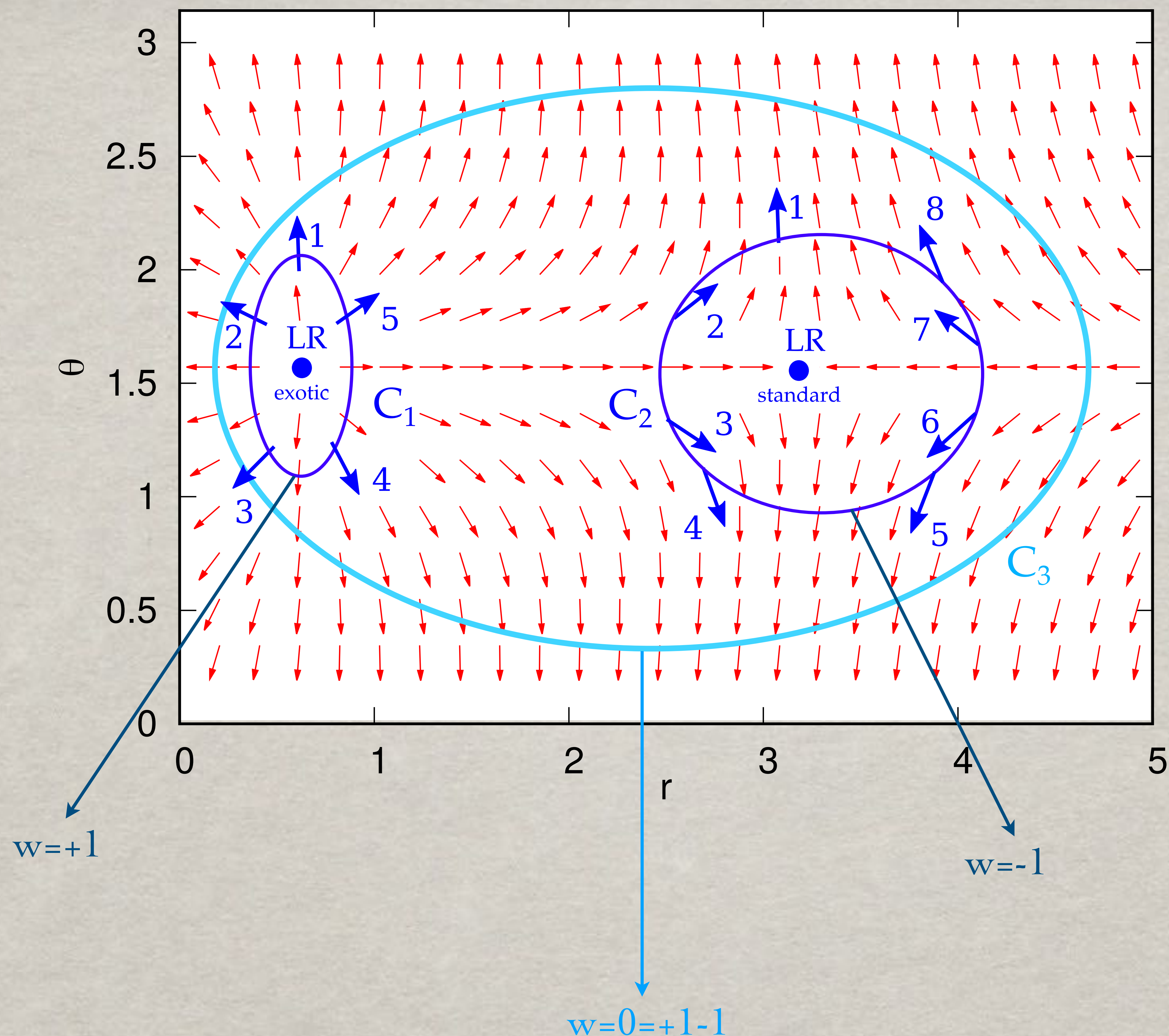
θ
r
w=+1
w=-1



3
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LR
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LR
standard
C_1
C_2
C_3
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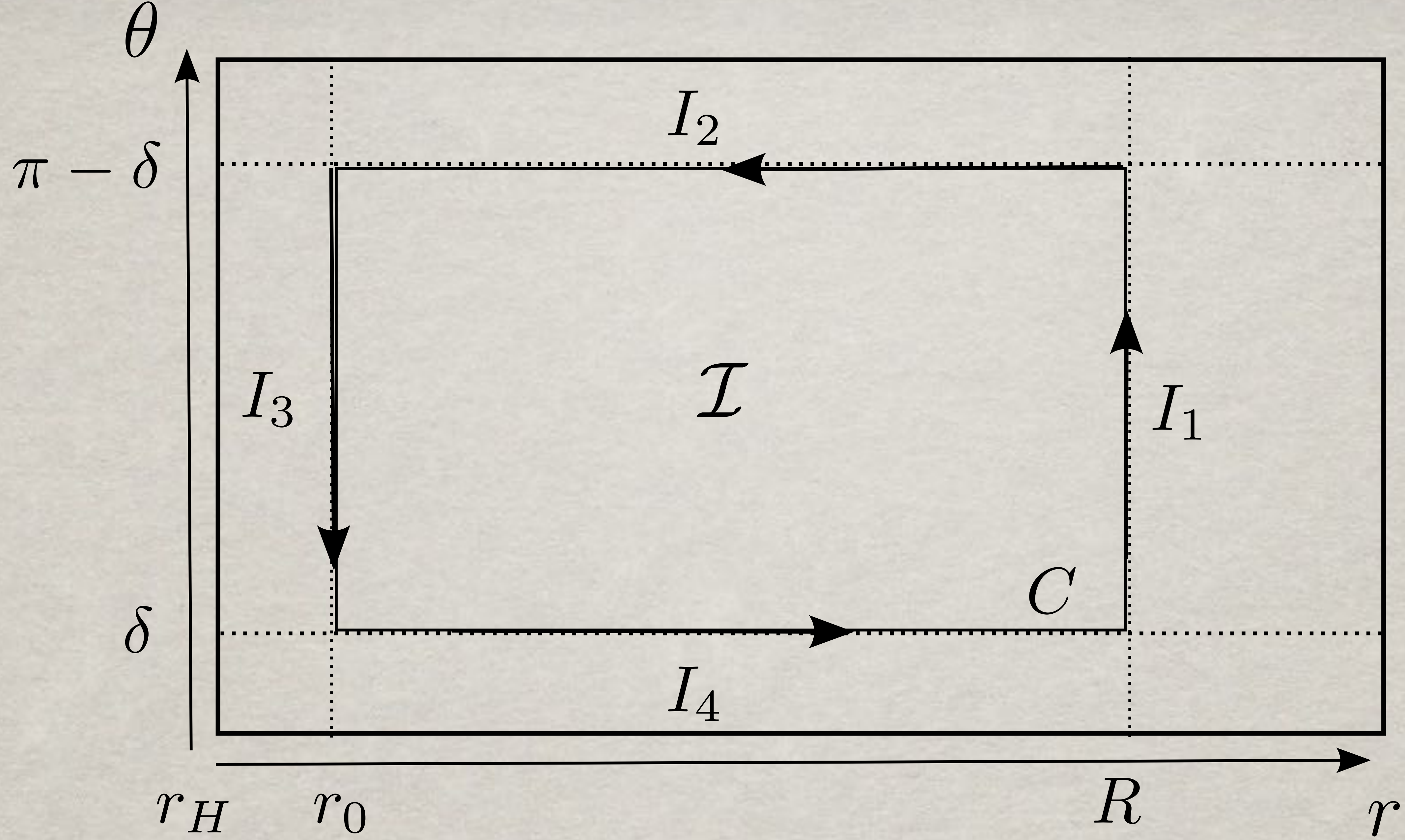
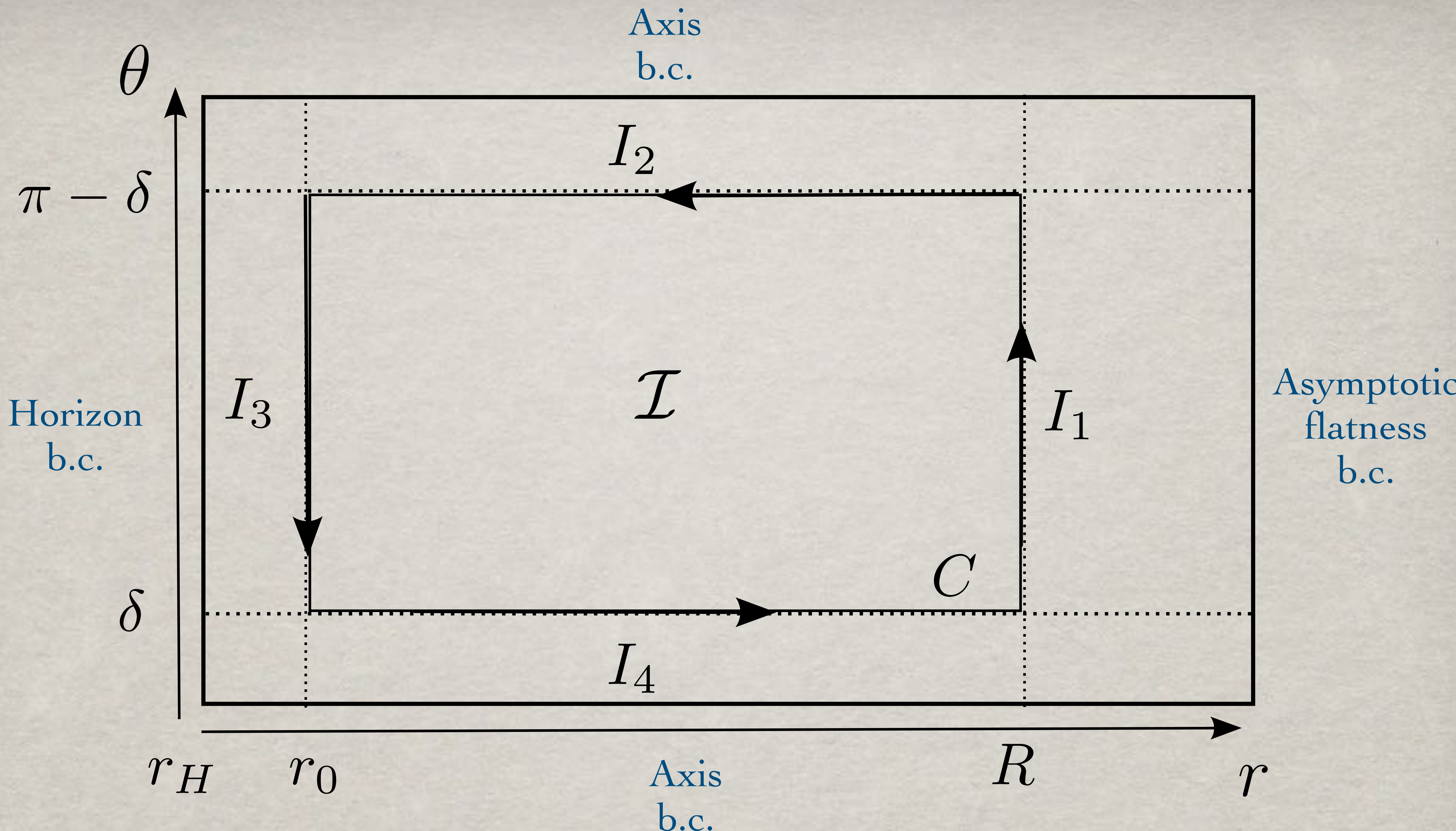
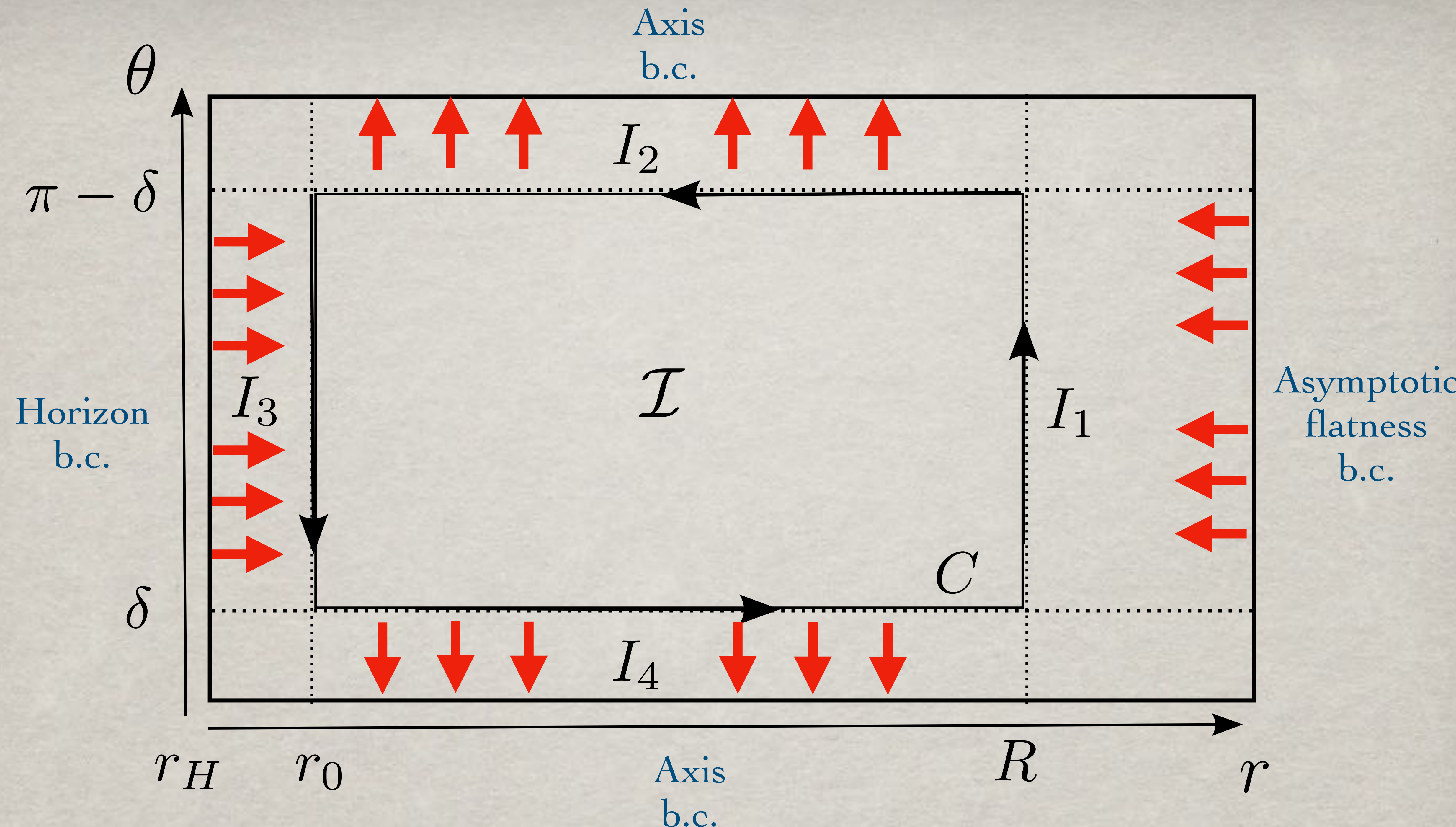
w=+1
w=0=+1-1
w=-1

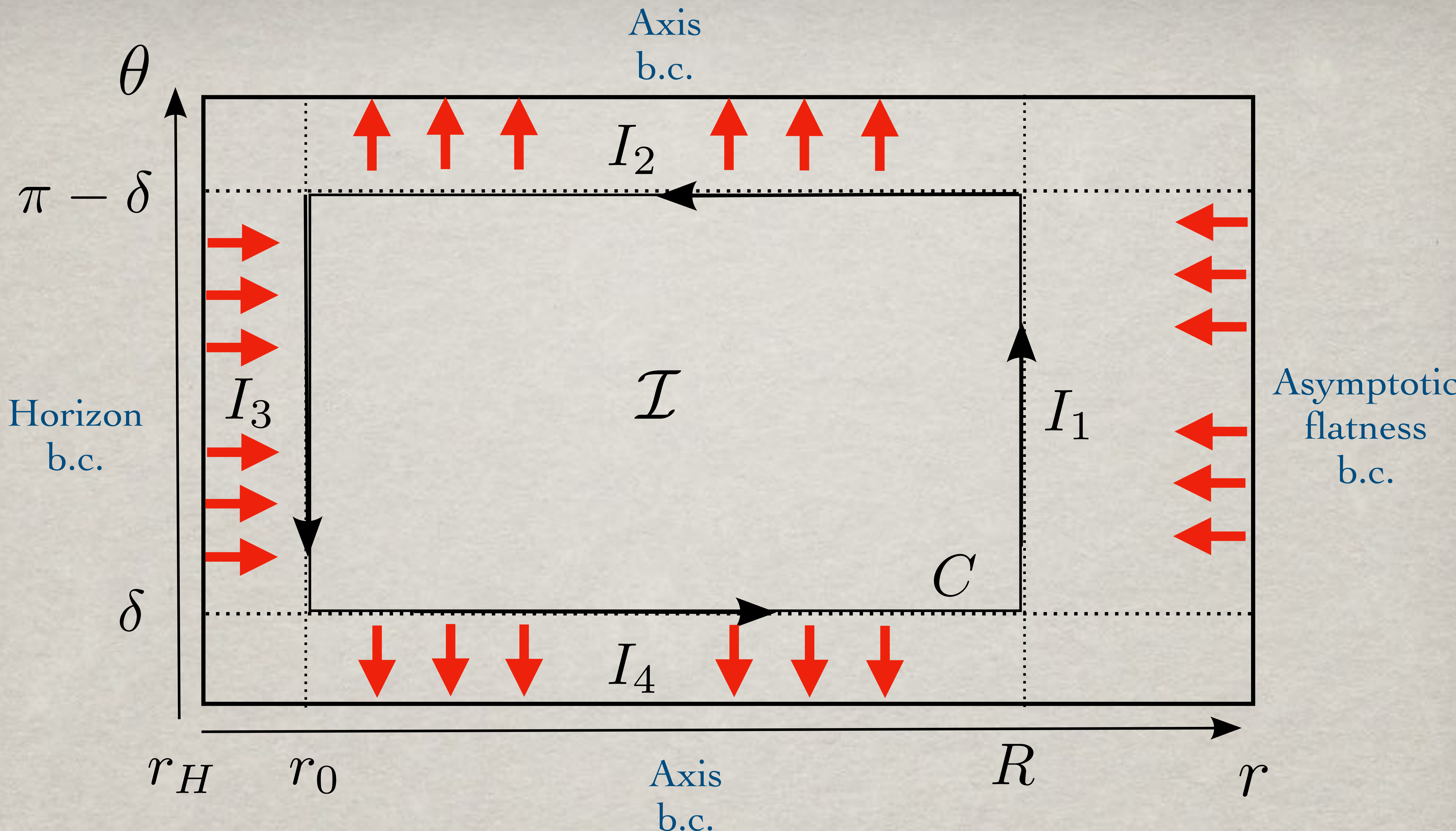


θ
$\pi - \delta$
δ
I_2
I_3
$\mathcal{I}$
I_1
C
I_4
r_H
r_0
R
r


Axis
b.c.
θ
$\pi - \delta$
I_2
Horizon
b.c.
I_3
$\mathcal{I}$
I_1
Asymptotic
flatness
b.c.
C
δ
I_4
r_H
r_0
Axis
b.c.
R
r



$$w = \lim_{R \to +\infty} \lim_{r_0 \to r_H} \left(\lim_{\delta \to 0} \oint_C d\Omega \right) = -1$$



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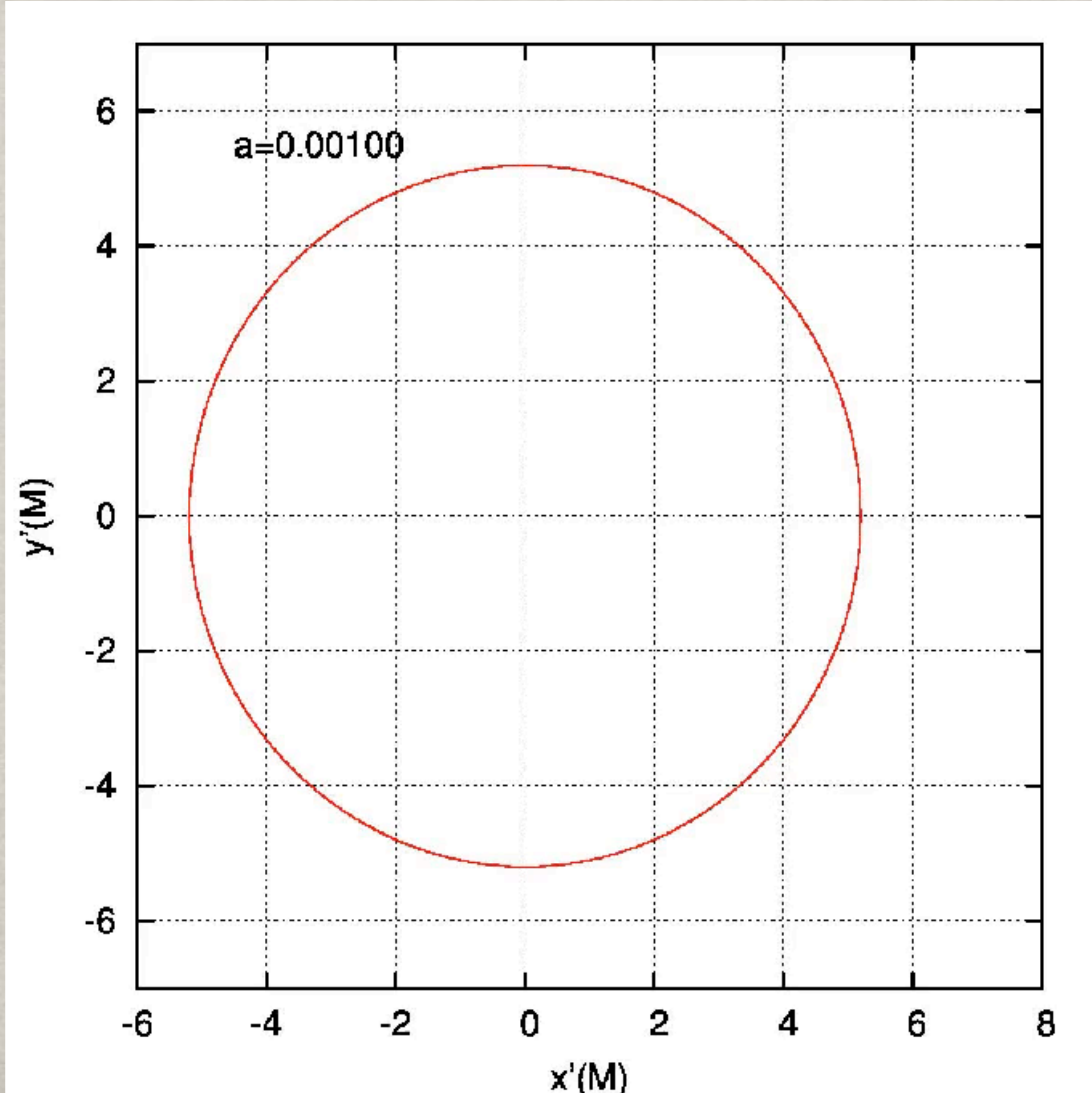
A black hole without LRs: Júnior, Cunha, CH, Crispino, Phys. Rev. D 104 (2021) 044018

Shadow edge of a Kerr BH (equatorial observation)



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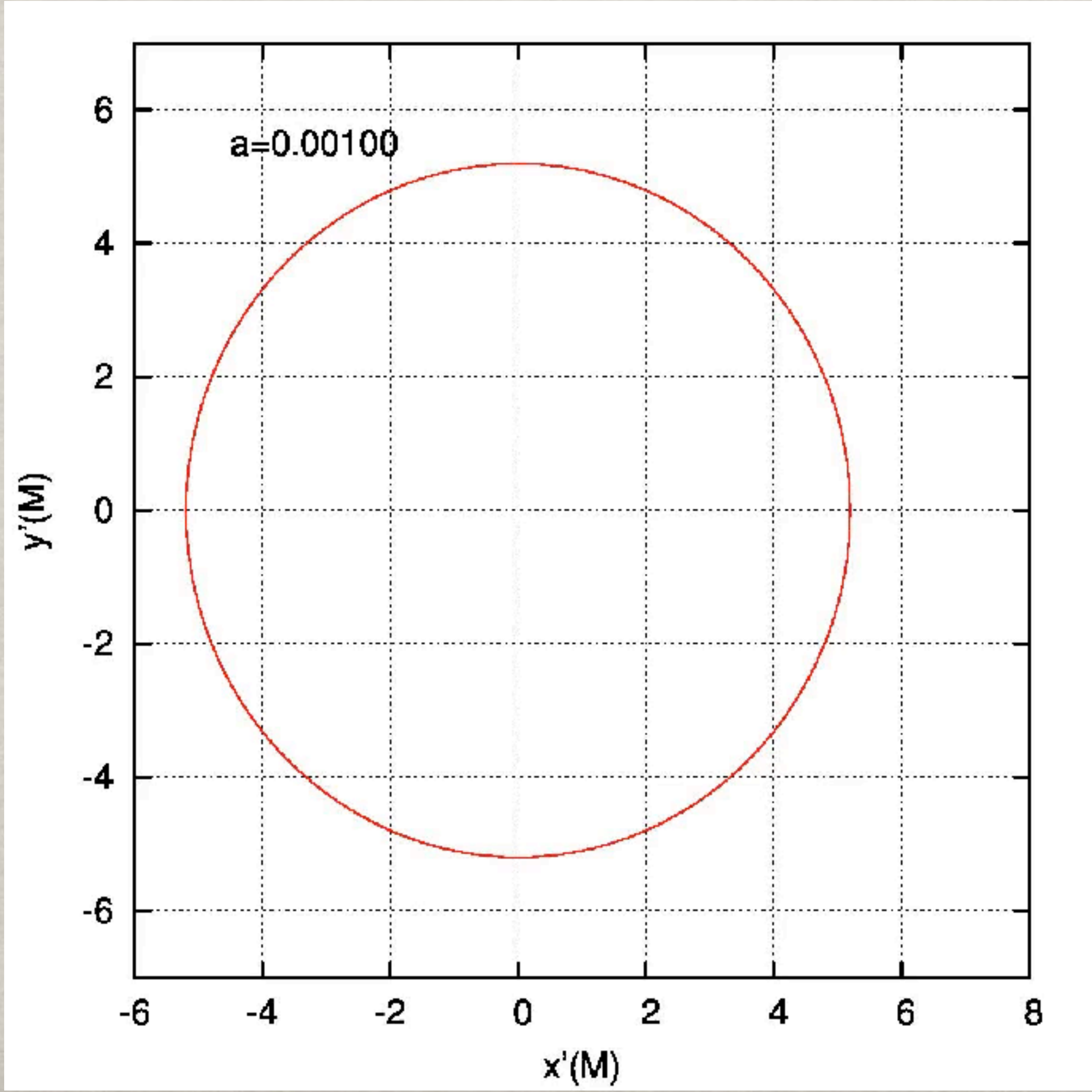

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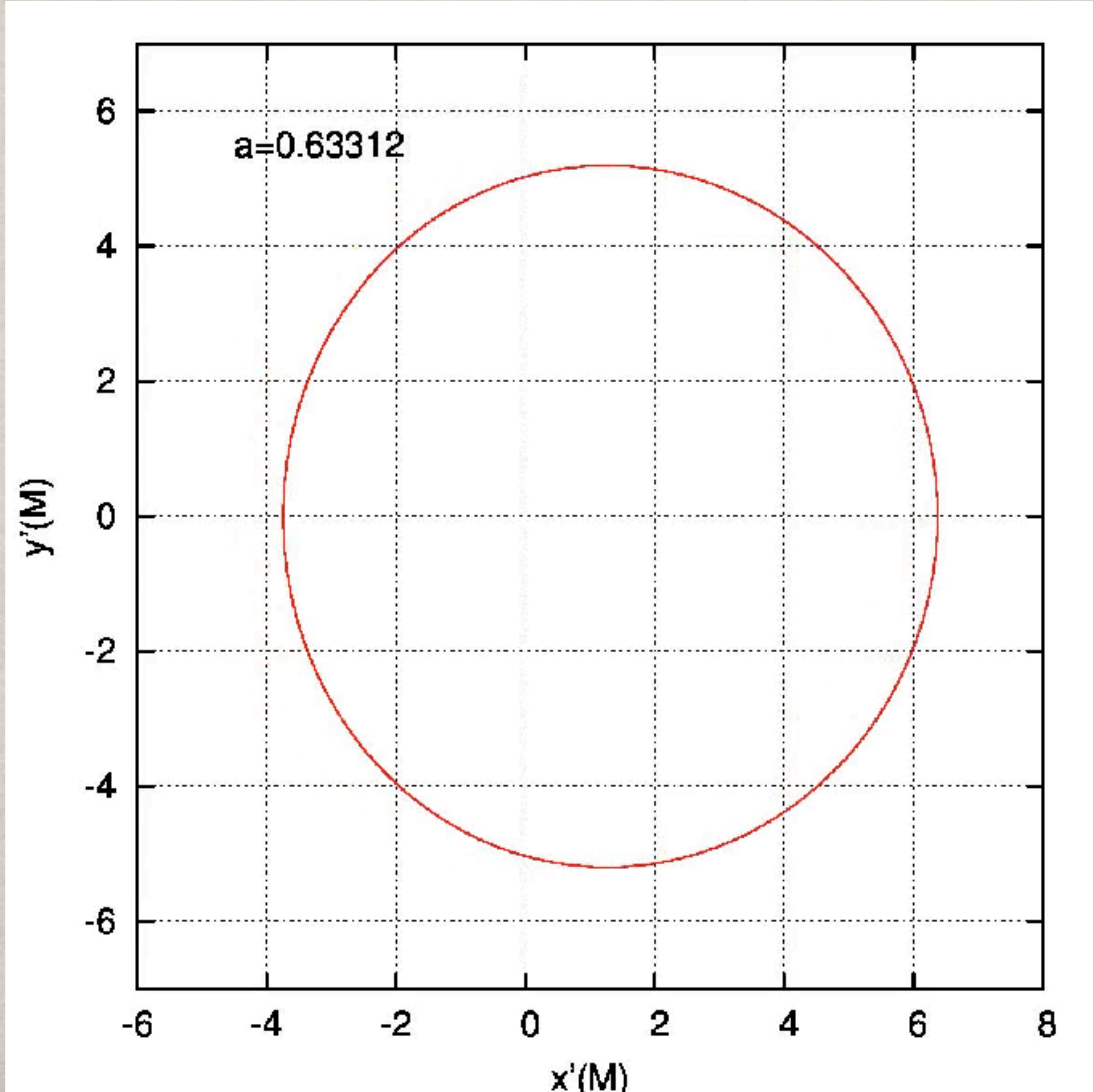

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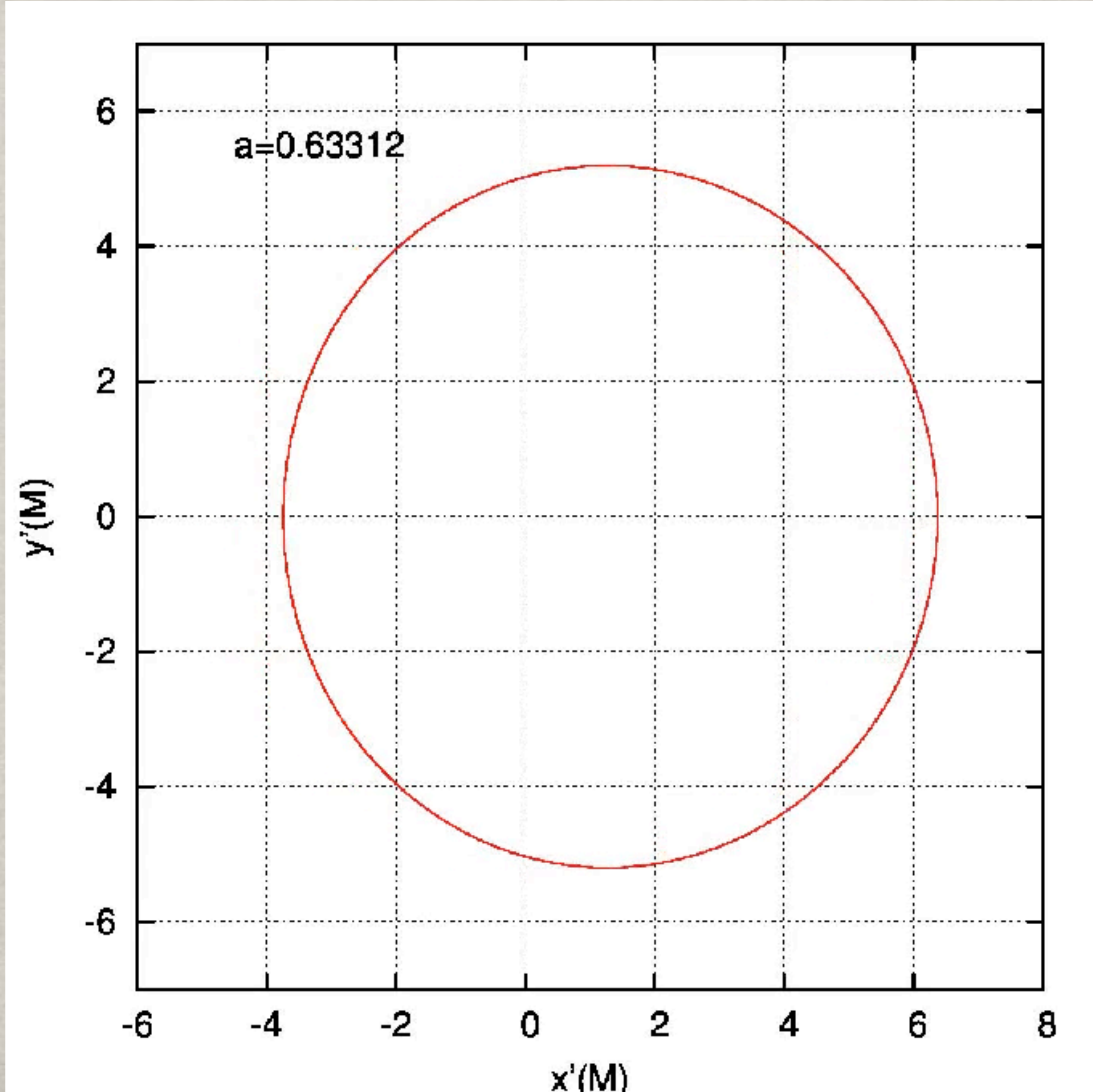

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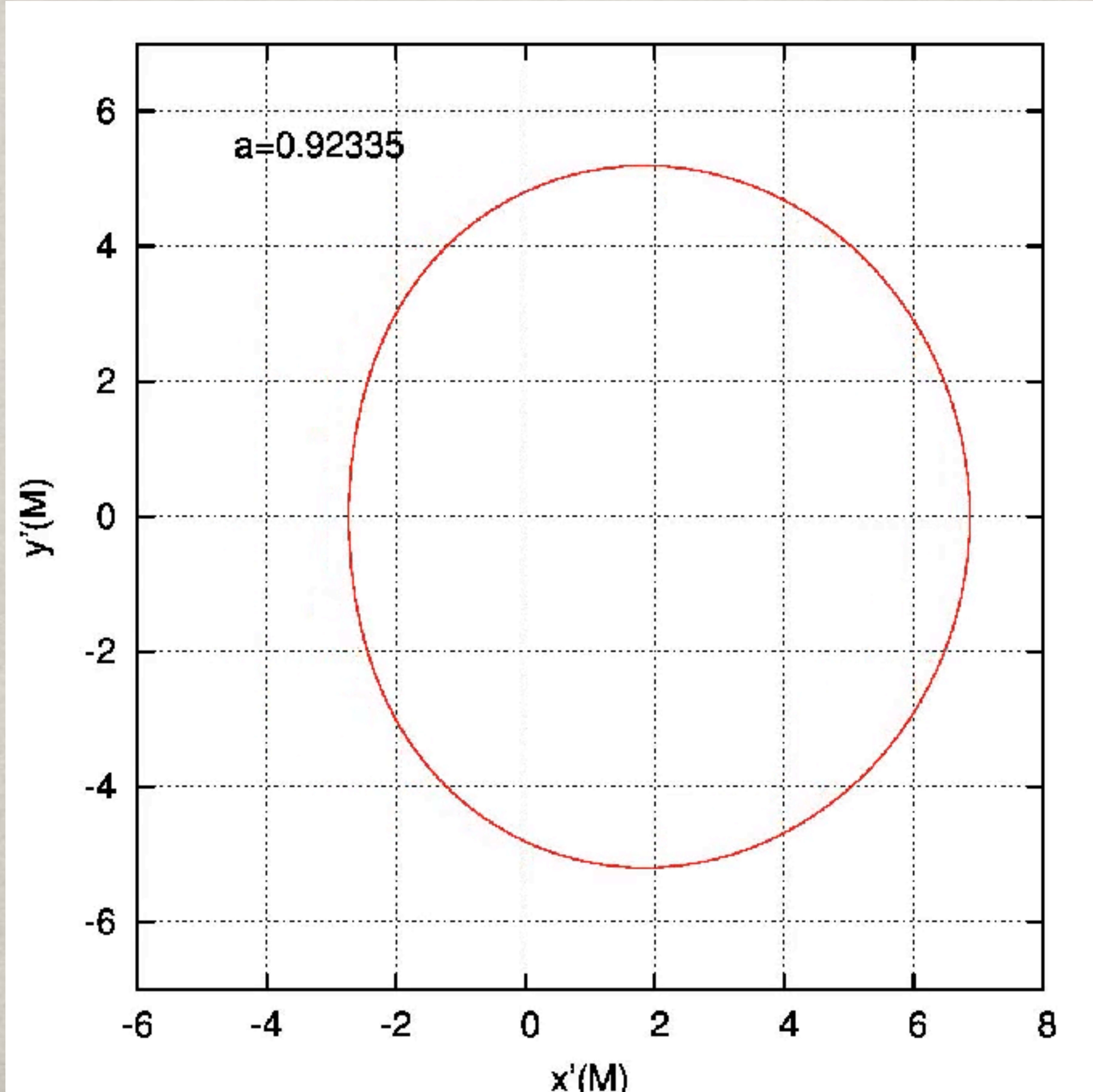

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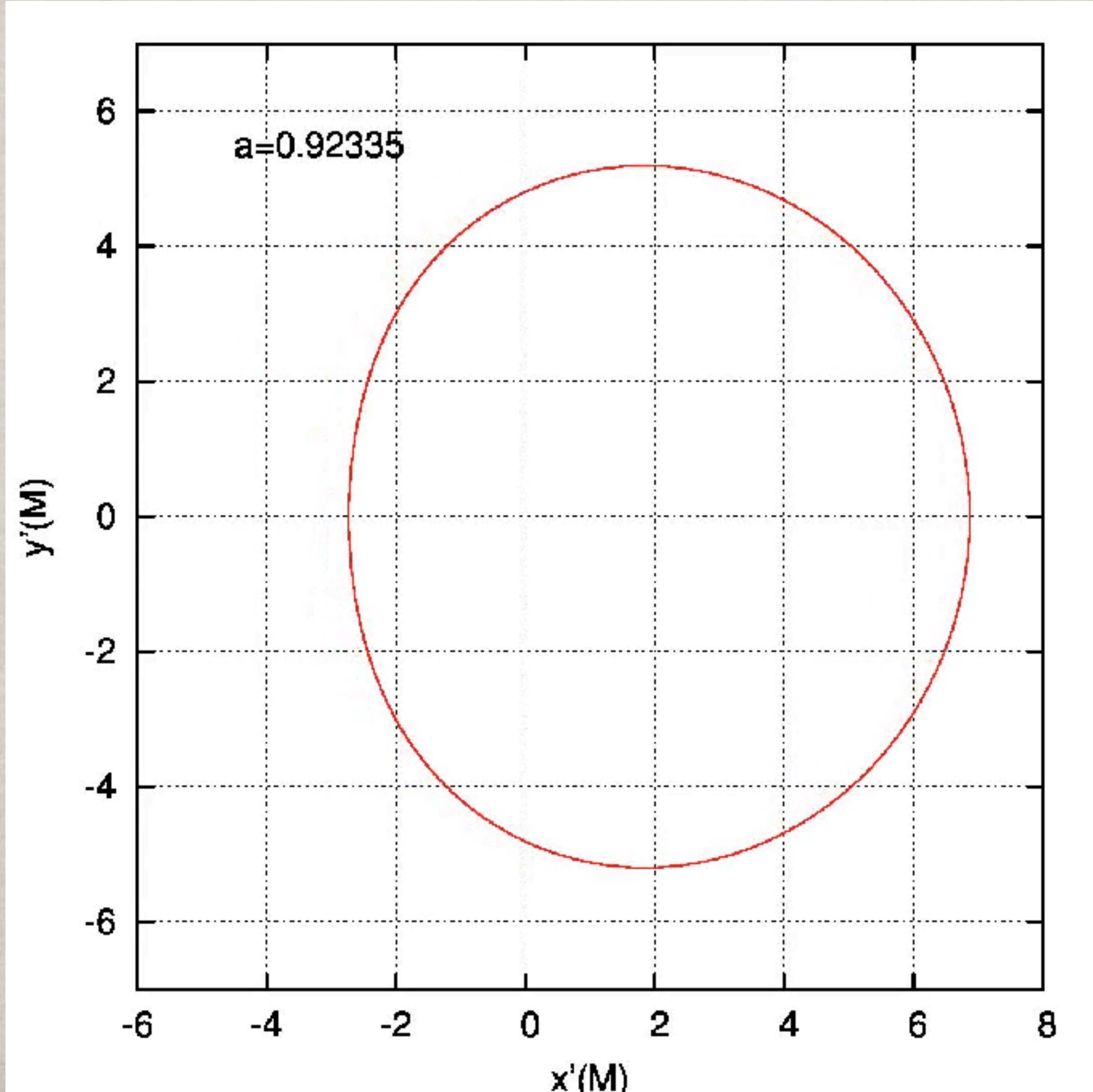

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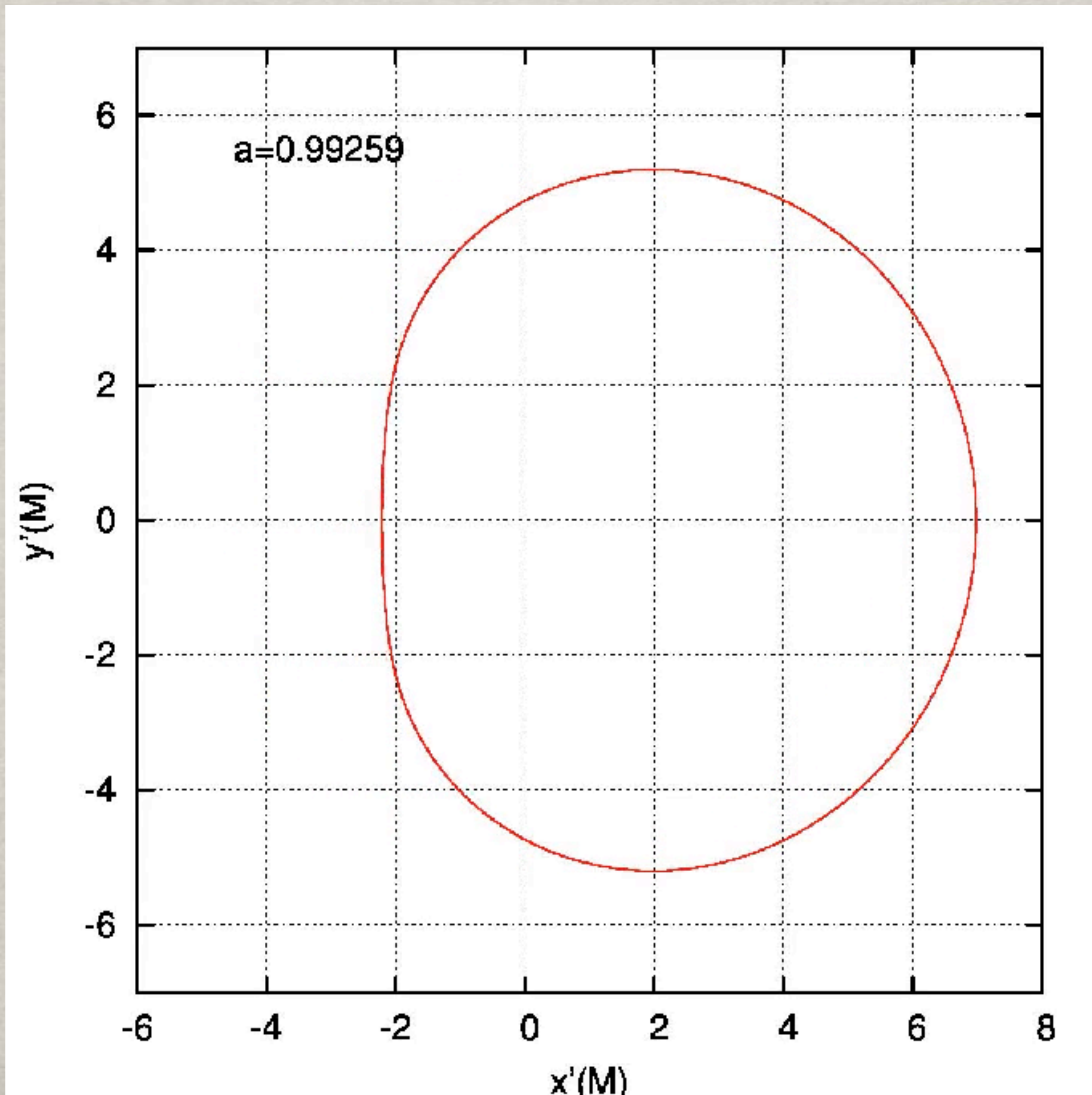

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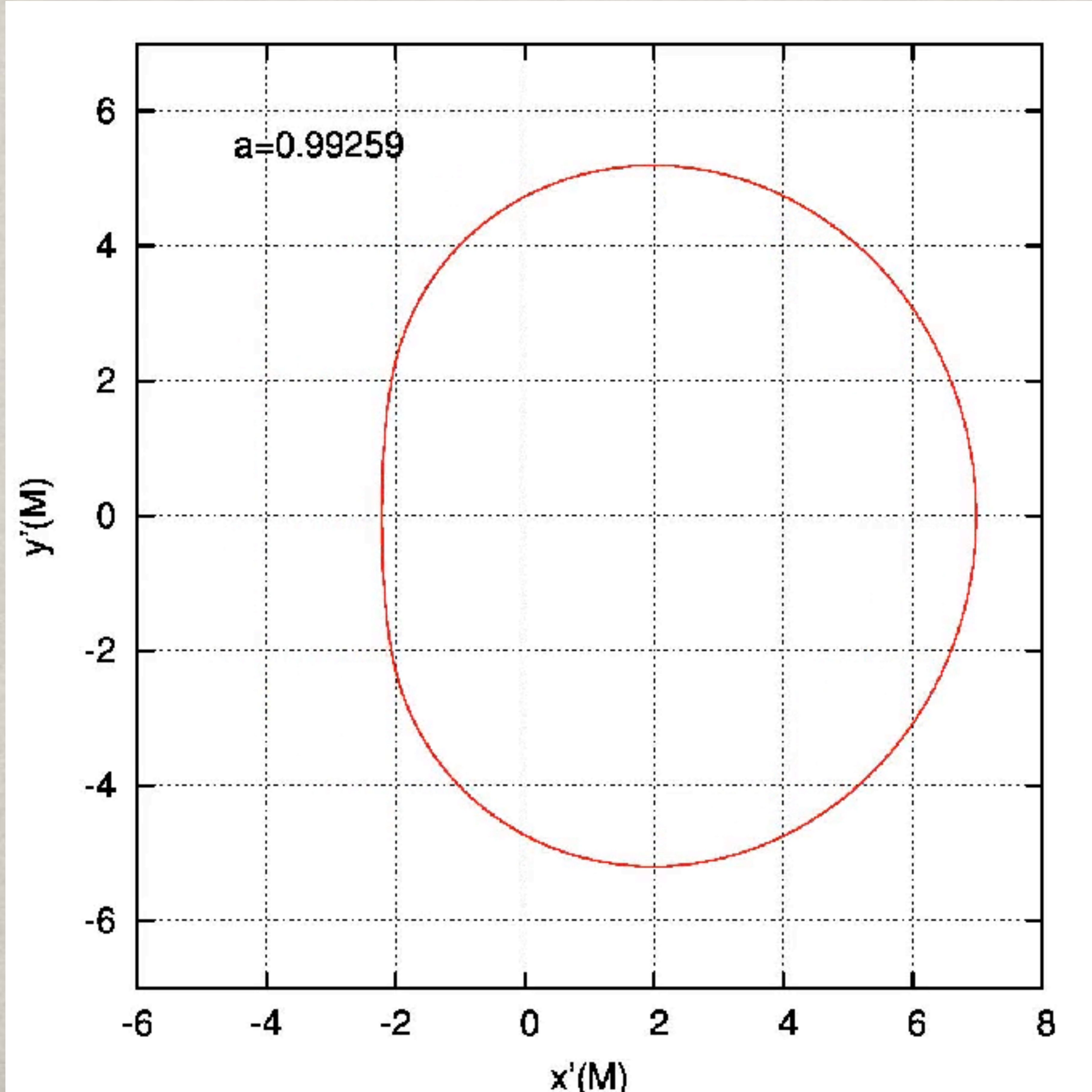

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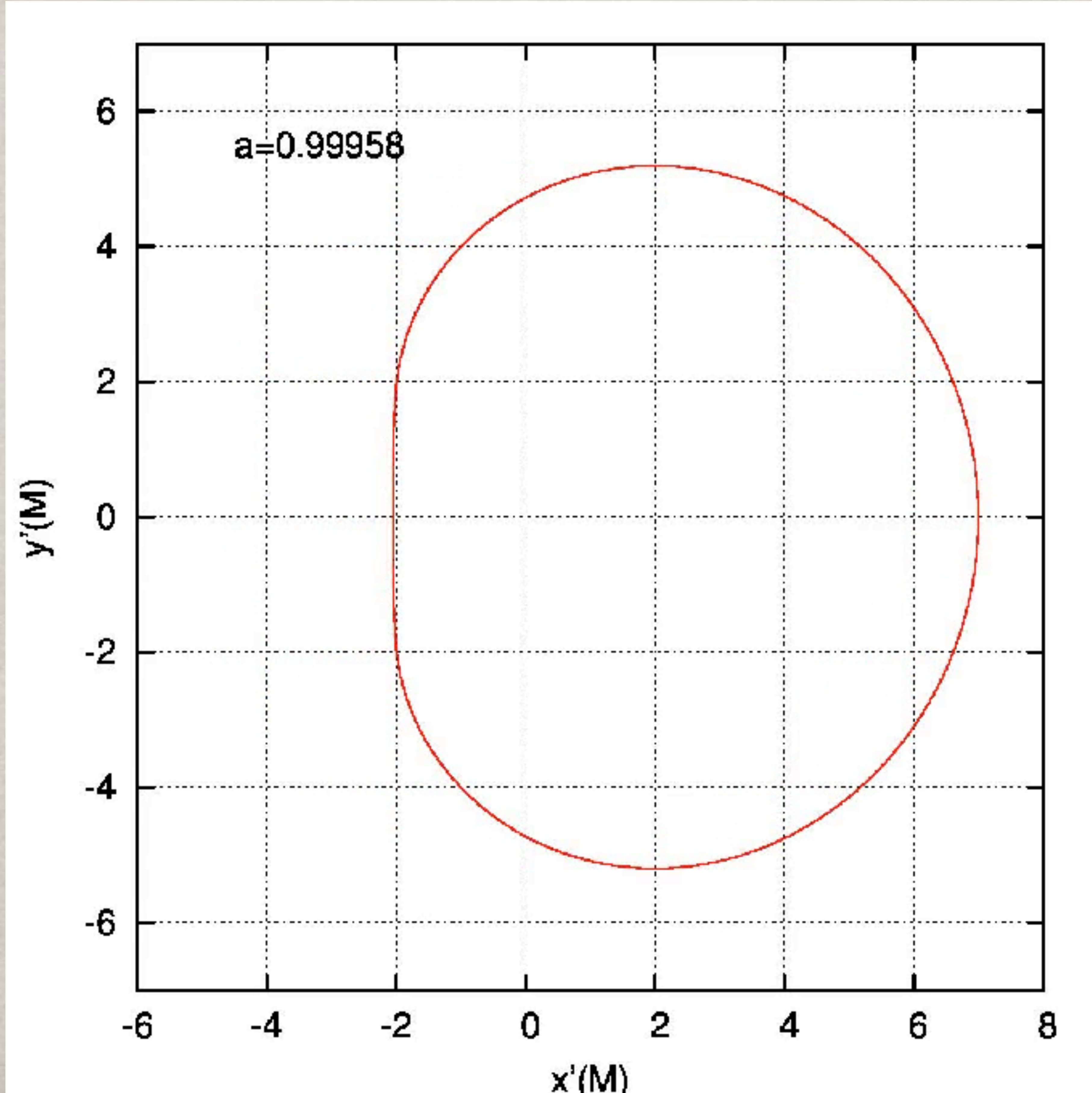

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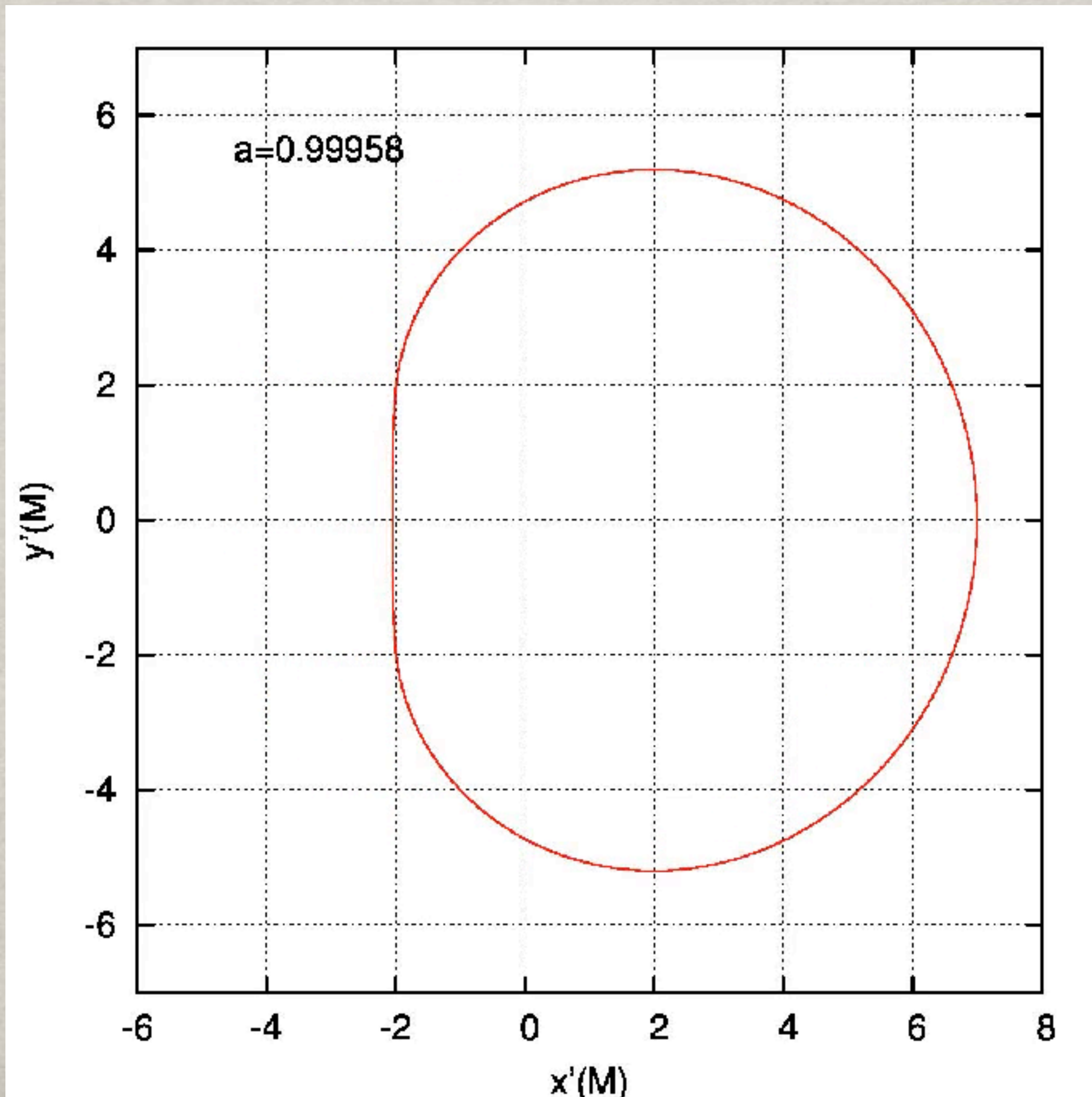

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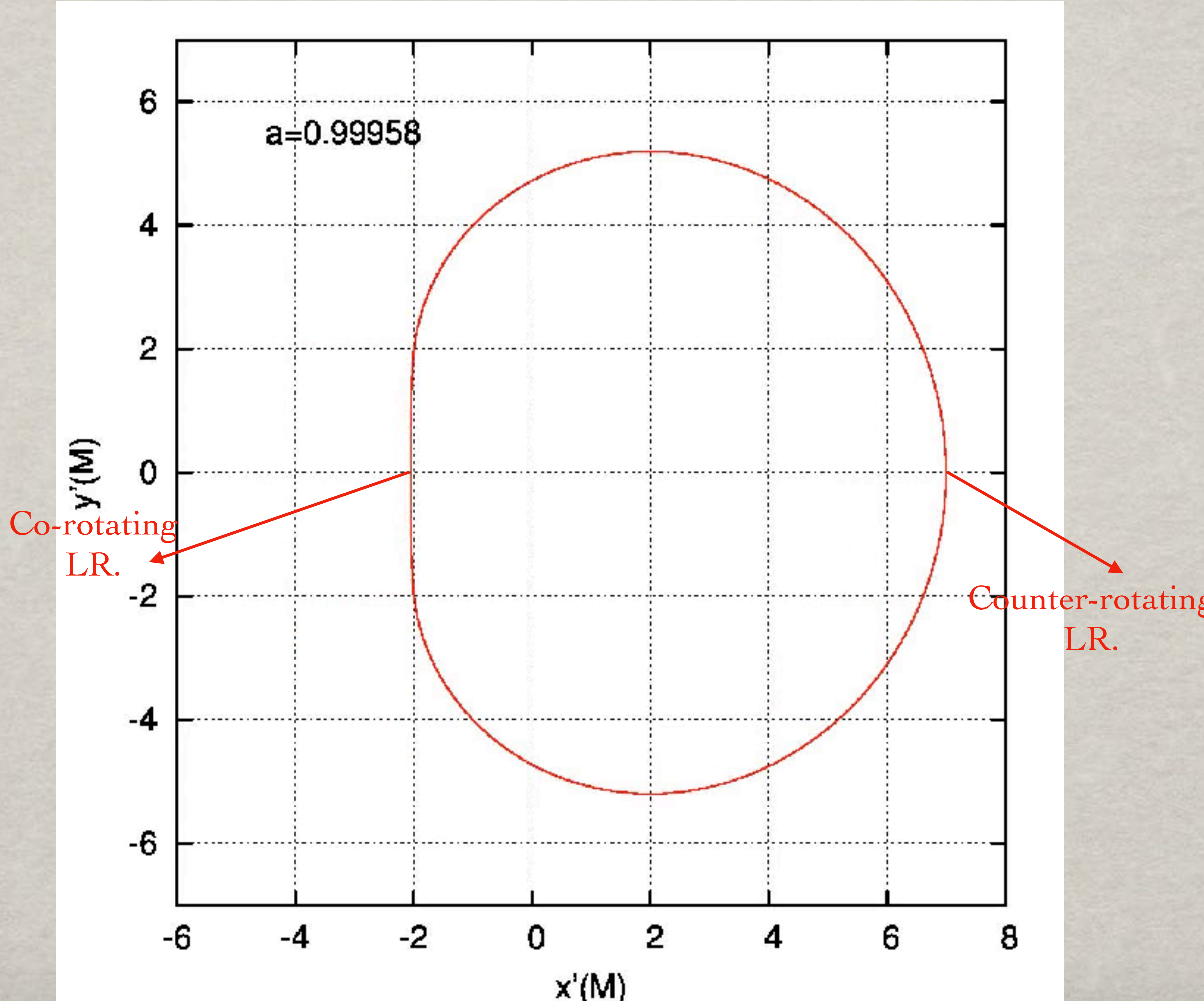

The ringdown is associated to the light ring:

THE ASTROPHYSICAL JOURNAL, **172**:L95–L96, 1972 March 15

COMMENTS ON THE "VIBRATIONS" OF A BLACK HOLE

C. J. GOEBEL
University of Wisconsin, Physics Department, Madison
Received 1972 January 4; revised 1972 January 27

ABSTRACT

It is shown that the "vibrations of a black hole" of Press are gravitational waves in spiral orbits close to the well-known unstable circular orbit at $r = 3M$. The corresponding "vibrations" of a spinning black hole are discussed. It is emphasized that these "vibrations" provide, not a source, but only a temporary storage, of high-frequency gravitational radiation.

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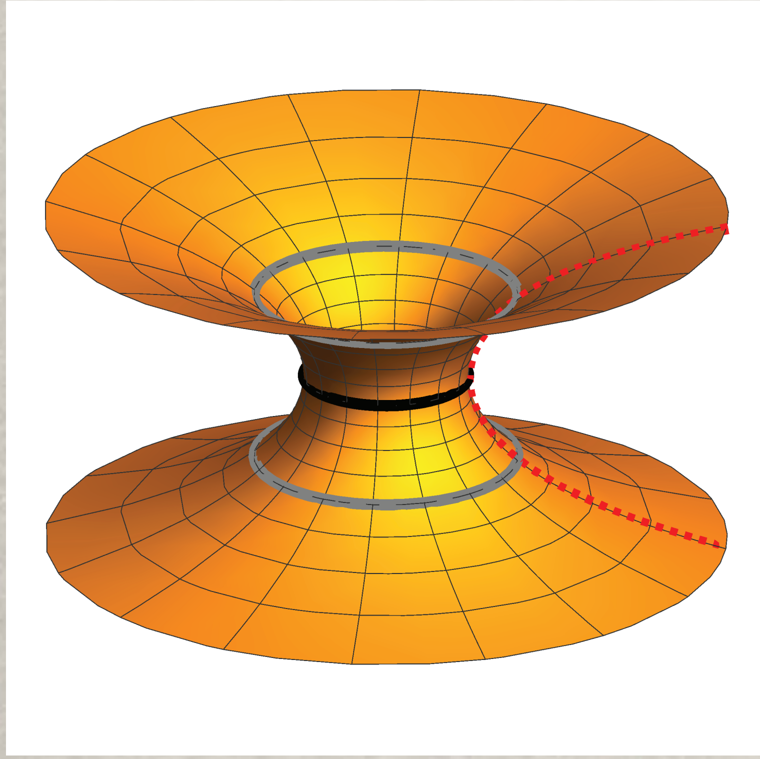
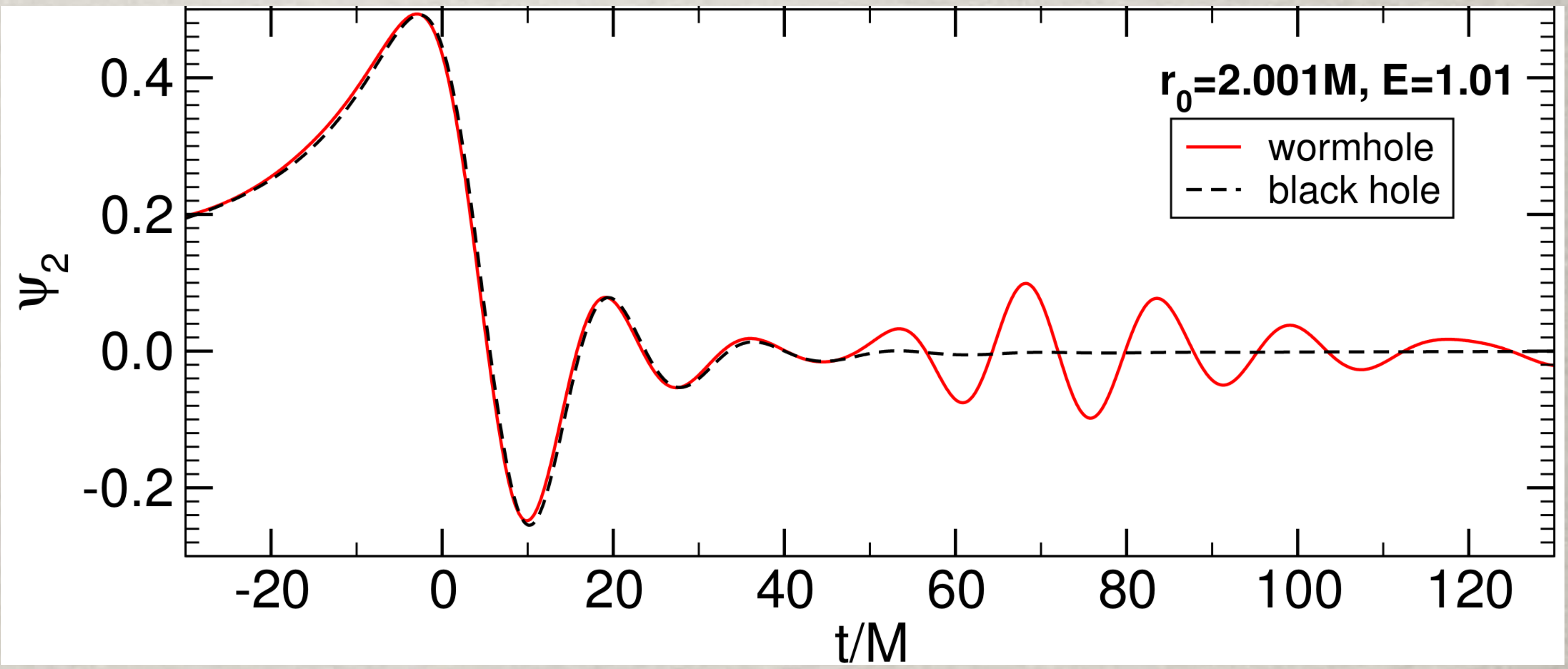
Cardoso, Miranda, Berti, Witek and Zanchin, Phys. Rev. D 79 (2009) 064016

$$\omega_{QNM} = \Omega_c \ell - i \left(n + \frac{1}{2} \right) |\lambda|$$

So, a hypothetical horizonless ultracompact ECO (i.e. with a similar light ring) could vibrate similarly, initially…





Cardoso, Franzin, Pani, PRL 117 (2016) 089902

Are there black hole mimickers with similar light rings
to black holes but no “event horizon”
they could mimic the black hole phenomenology.

Can there be such speculative ultra-compact ECOs?

On the fate of the Light Ring instability

Plan:

1) The "*black hole (BH) hypothesis*": BHs and light rings (LRs)

2) The "*exotic compact object (ECO) hypothesis*": ECOs and LRs

3) The LR instability and an explicit test of its fate

4) Discussion and final thoughts

The Exotic Compact Object (ECO) hypothesis

Many models of horizonless compact objects (black hole mimickers) without horizons or singularities have been proposed.

a) "geons", realized by Boson stars (Schunck, Mielke, CQG 20 (2003) R301; Jetzer, Phys. Rept. 220 (1992) 163) and Proca stars (Brito, Cardoso, CH, Radu, PLB 752 (2016) 291); can form dynamically (Seidel, Suen, PRL 72 (1994) 2516); Perturbatively stable Gleiser and Watkins, NPB 319 (1989) 733; Lee and Pang, NPB 315 (1989) 477; Can be studied dynamically in binaries (Liebling and Palenzuela LRR 20 (2017) 5)

b) wormholes (Morris and Thorne, Am. J. Phys. 56 (1988) 395-412)

c) gravastars (Mazur and Mottola, gr-qc/0109035)

d) fuzzballs (Mathur, Fortsch. Phys. 53 (2005) 793)

e) … See e.g. Pani and Cardoso, Nature Astron. 1 (2017) 9, 586

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Can they explain the observations?
Are they good theoretically?

A theorem for ultracompact ECOs that form from incomplete gravitational collapse

PRL **119,** 251102 (2017) PHYSICAL REVIEW LETTERS week ending 22 DECEMBER 2017

Light-Ring Stability for Ultracompact Objects

Pedro V. P. Cunha,[1,2] Emanuele Berti,[3,2] and Carlos A. R. Herdeiro[1]
[1]*Departamento de Física da Universidade de Aveiro and CIDMA, Campus de Santiago, 3810-183 Aveiro, Portugal*
[2]*CENTRA, Departamento de Física, Instituto Superior Técnico, Universidade de Lisboa, Avenida Rovisco Pais 1, 1049 Lisboa, Portugal*
[3]*Department of Physics and Astronomy, The University of Mississippi, University, Mississippi 38677, USA*
(Received 3 August 2017; revised manuscript received 18 October 2017; published 18 December 2017)

We prove the following theorem: axisymmetric, stationary solutions of the Einstein field equations formed from classical gravitational collapse of matter obeying the null energy condition, that are everywhere smooth and ultracompact (i.e., they have a light ring) must have at least *two* light rings, and one of them is *stable*. It has been argued that stable light rings generally lead to nonlinear spacetime instabilities. Our result implies that smooth, physically and dynamically reasonable ultracompact objects are not viable as observational alternatives to black holes whenever these instabilities occur on astrophysically short time scales. The proof of the theorem has two parts: (i) We show that light rings always come in pairs, one being a saddle point and the other a local extremum of an effective potential. This result follows from a topological argument based on the Brouwer degree of a continuous map, with no assumptions on the spacetime dynamics, and, hence, it is applicable to any metric gravity theory where photons follow null geodesics. (ii) Assuming Einstein's equations, we show that the extremum is a local minimum of the potential (i.e., a stable light ring) if the energy-momentum tensor satisfies the null energy condition.

DOI: 10.1103/PhysRevLett.119.251102

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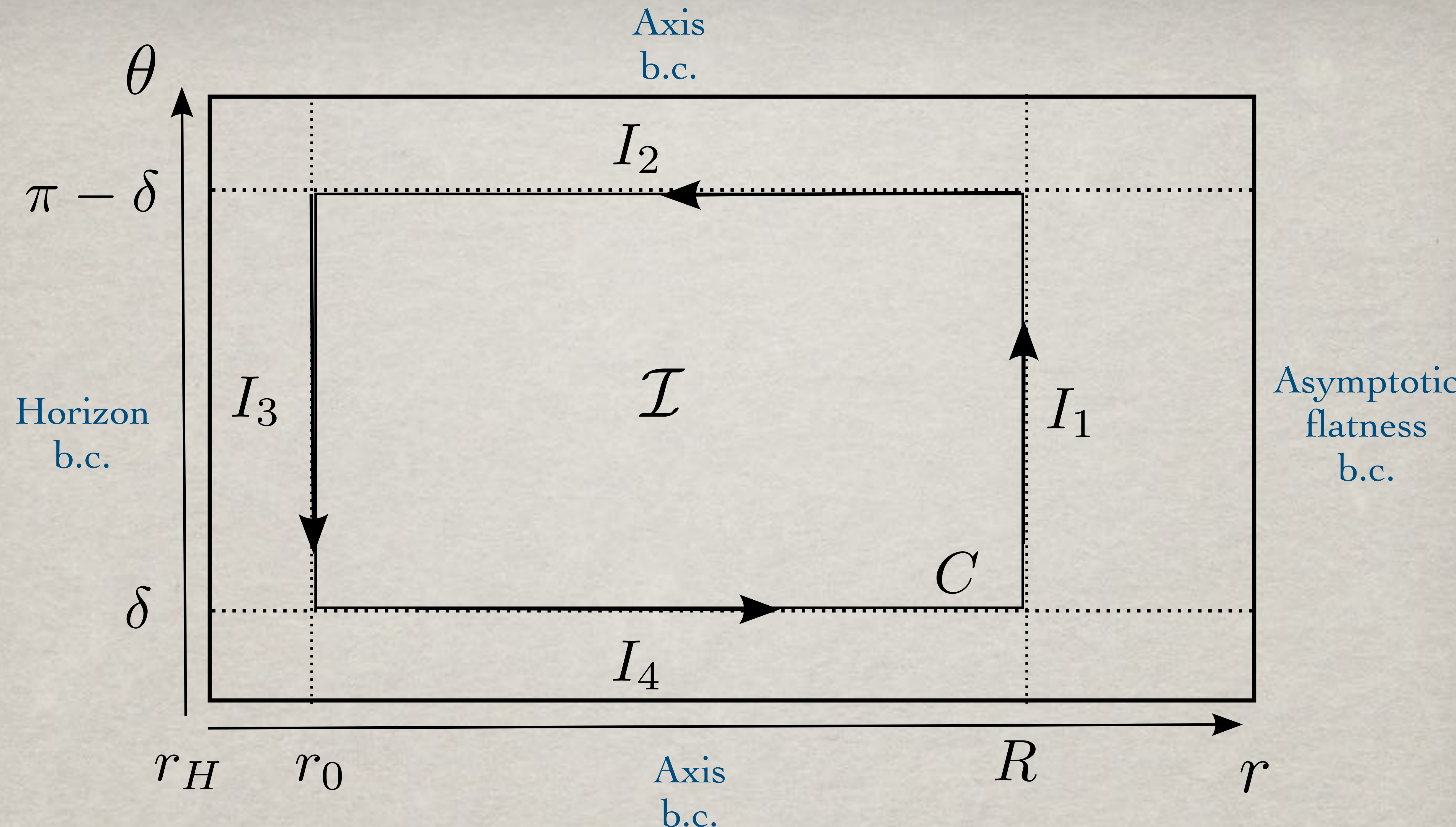
[3]*Department of Physics and Astronomy, The University of Mississippi, University, Mississippi 38677, USA*

(Received 3 August 2017; revised manuscript received 18 October 2017; published 18 December 2017)

[redacted] axisymmetric, stationary solutions of the Einstein field equations formed from classical gravitational collapse of matter obeying the null energy condition, that are everywhere smooth and ultracompact (i.e., they have a light ring) must have at least *two* light rings, and one of them is *stable*. [redacted] implies that smooth, physically and dynamically reasonable ultracompact objects are not viable as observational alternatives to black holes whenever these instabilities occur on astrophysically short time scales. The proof of the theorem has two parts: (i) We show that light rings always come in pairs, one being a saddle point and the other a local extremum of an effective potential. This result follows from a topological argument based on the Brouwer degree of a continuous map, with no assumptions on the spacetime dynamics, and, hence, it is applicable to any metric gravity theory where photons follow null geodesics. (ii) Assuming Einstein's equations, we show that the extremum is a local minimum of the potential (i.e., a stable light ring) if the energy-momentum tensor satisfies the null energy condition.

DOI: 10.1103/PhysRevLett.119.251102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$\pi - \delta$
Horizon
b.c.
I_3
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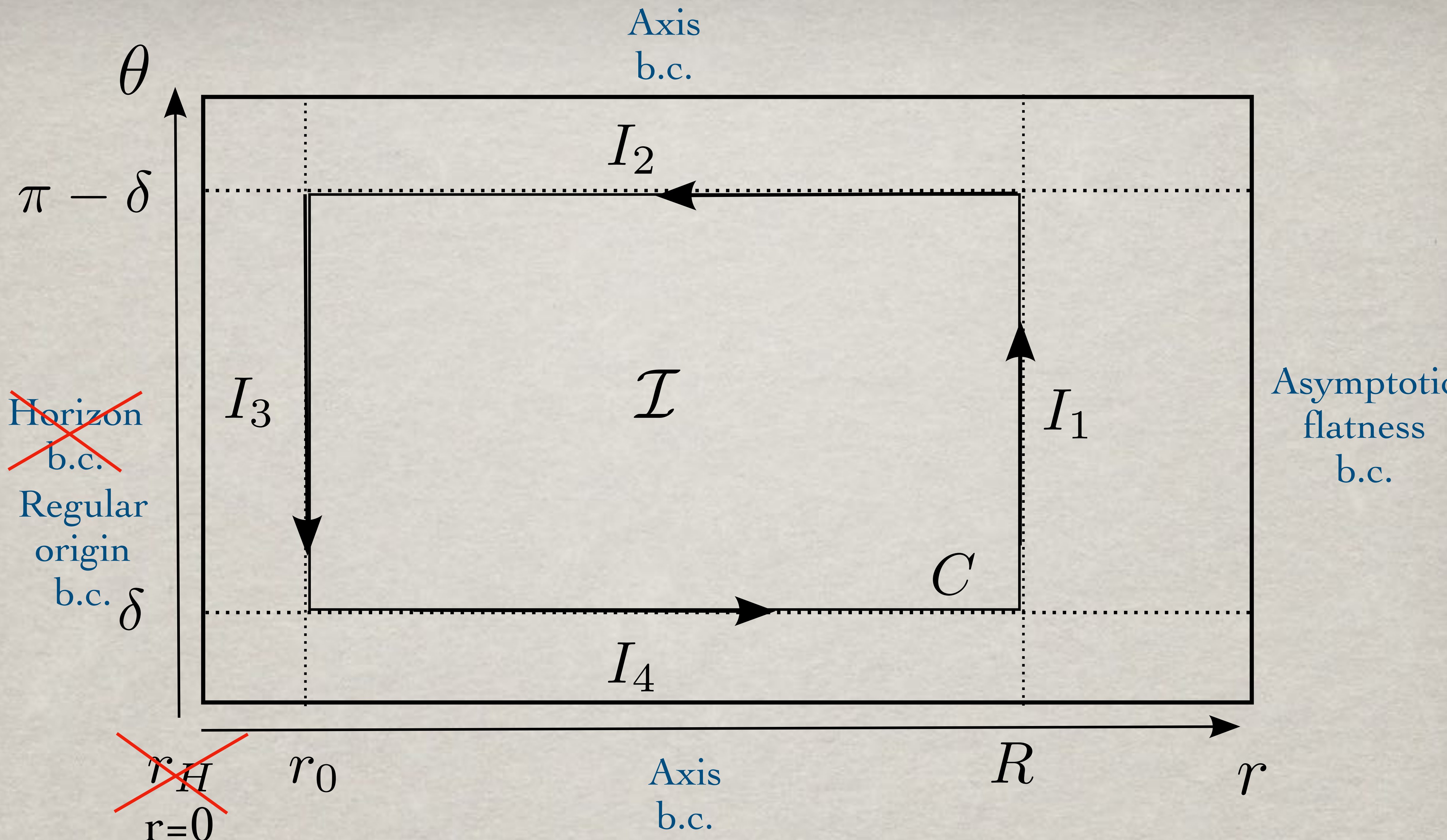
I_1
Asymptotic
flatness
b.c.
C
δ
I_4
r_H
r_0
R
r
Axis
b.c.



Axis
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θ
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$\pi - \delta$
Horizon
b.c.
Regular
origin
b.c.
δ
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C
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flatness
b.c.
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r_H
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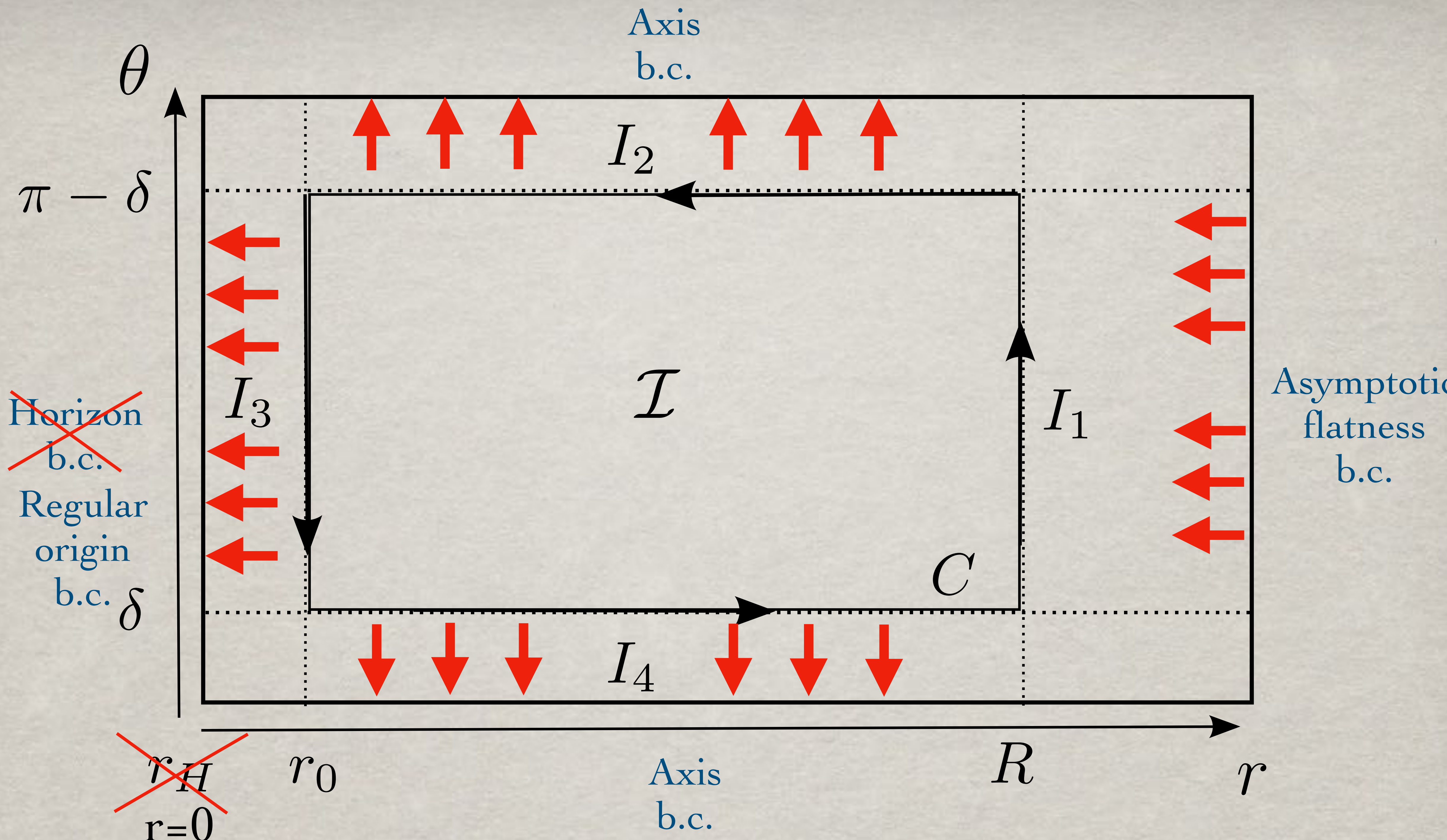
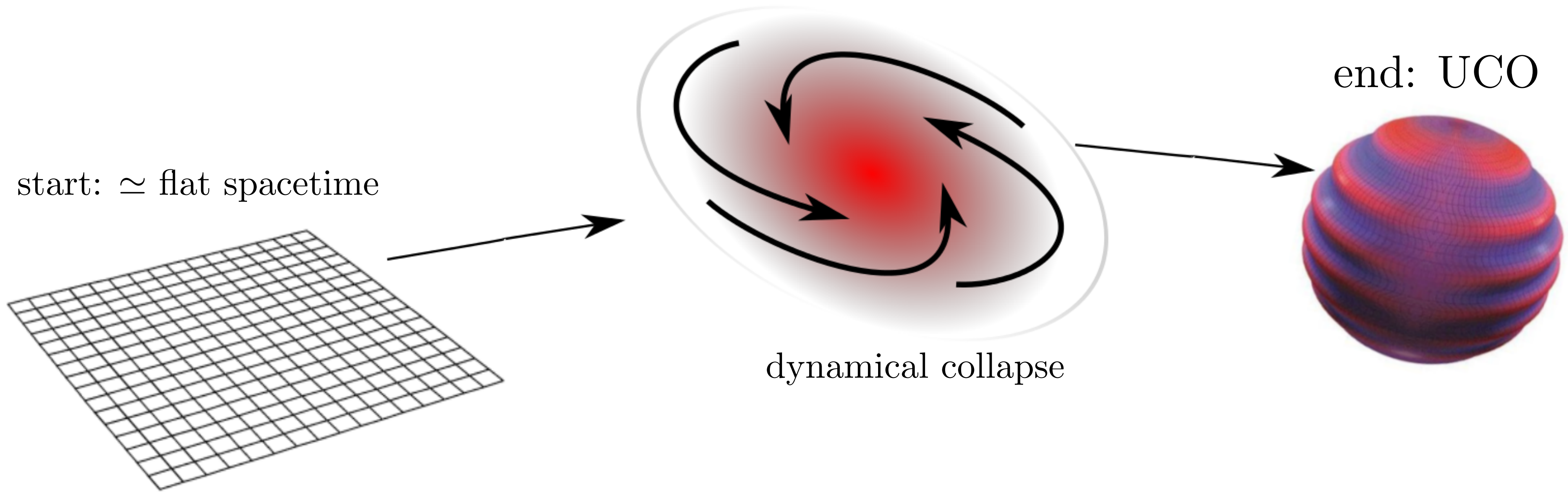
r_0
Axis
b.c.
R
r

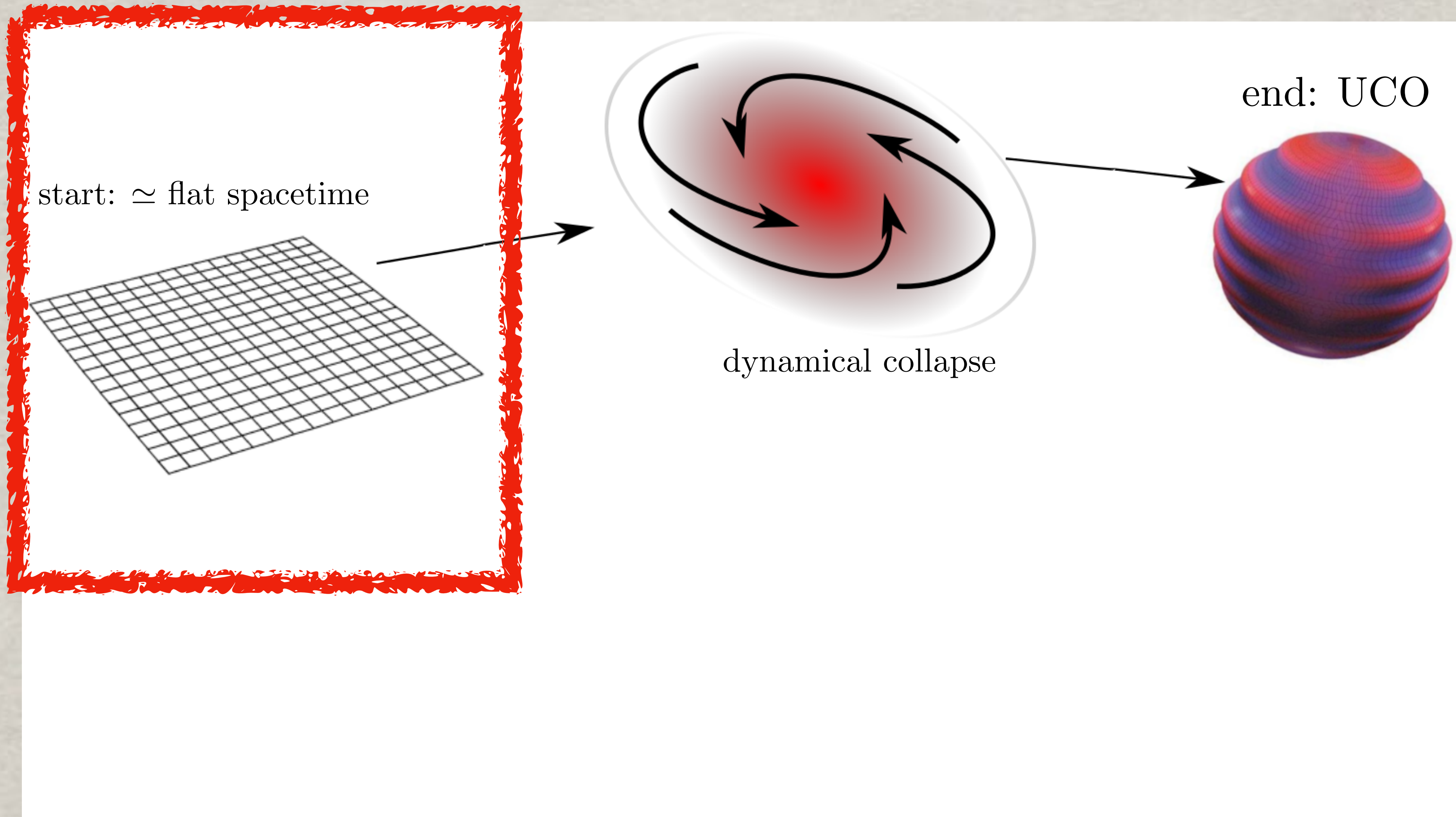


$$w = \lim_{R \to +\infty} \lim_{r_0 \to 0} \left(\lim_{\delta \to 0} \oint_C d\Omega \right) = 0$$

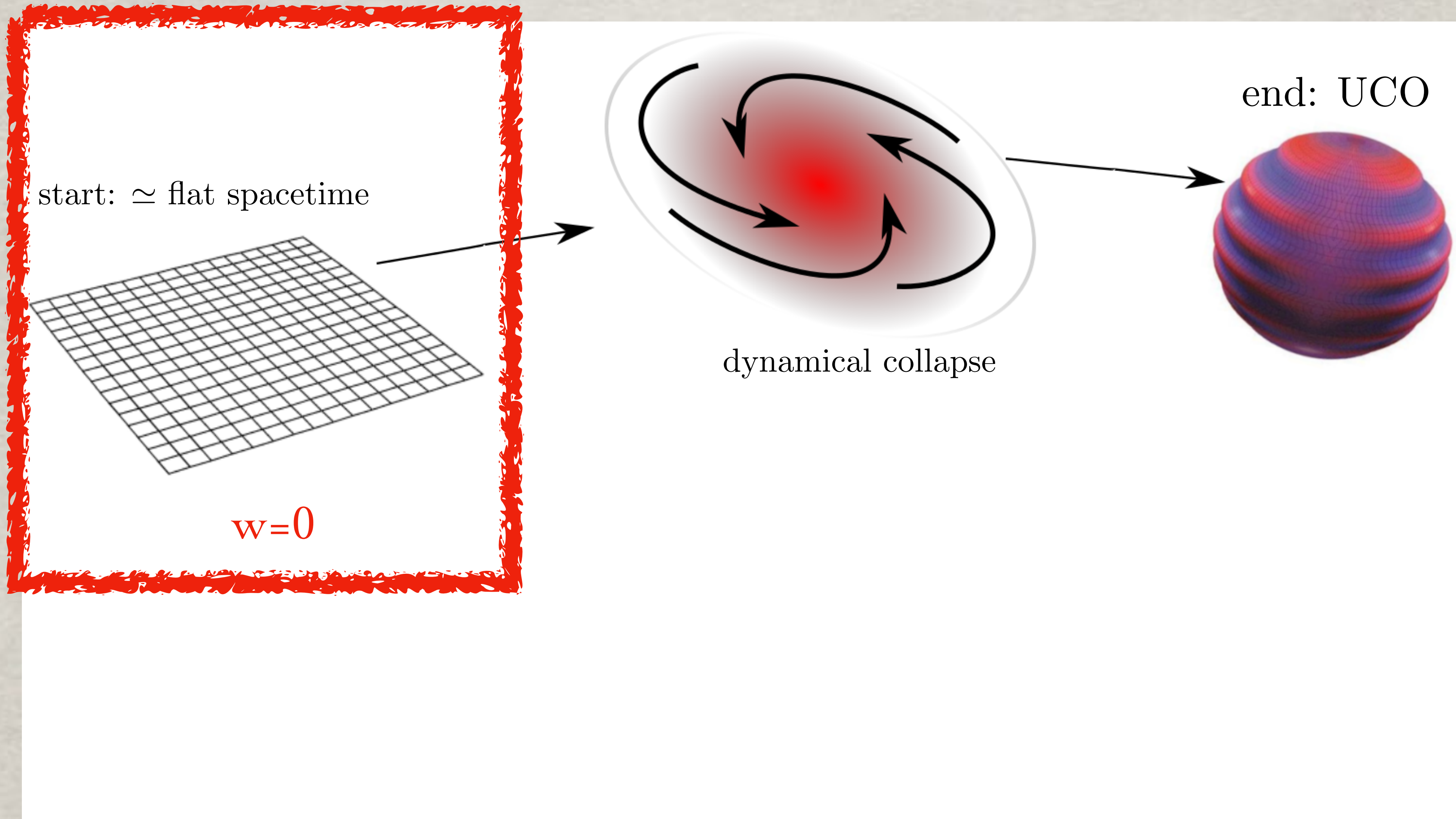
A generic dynamical picture



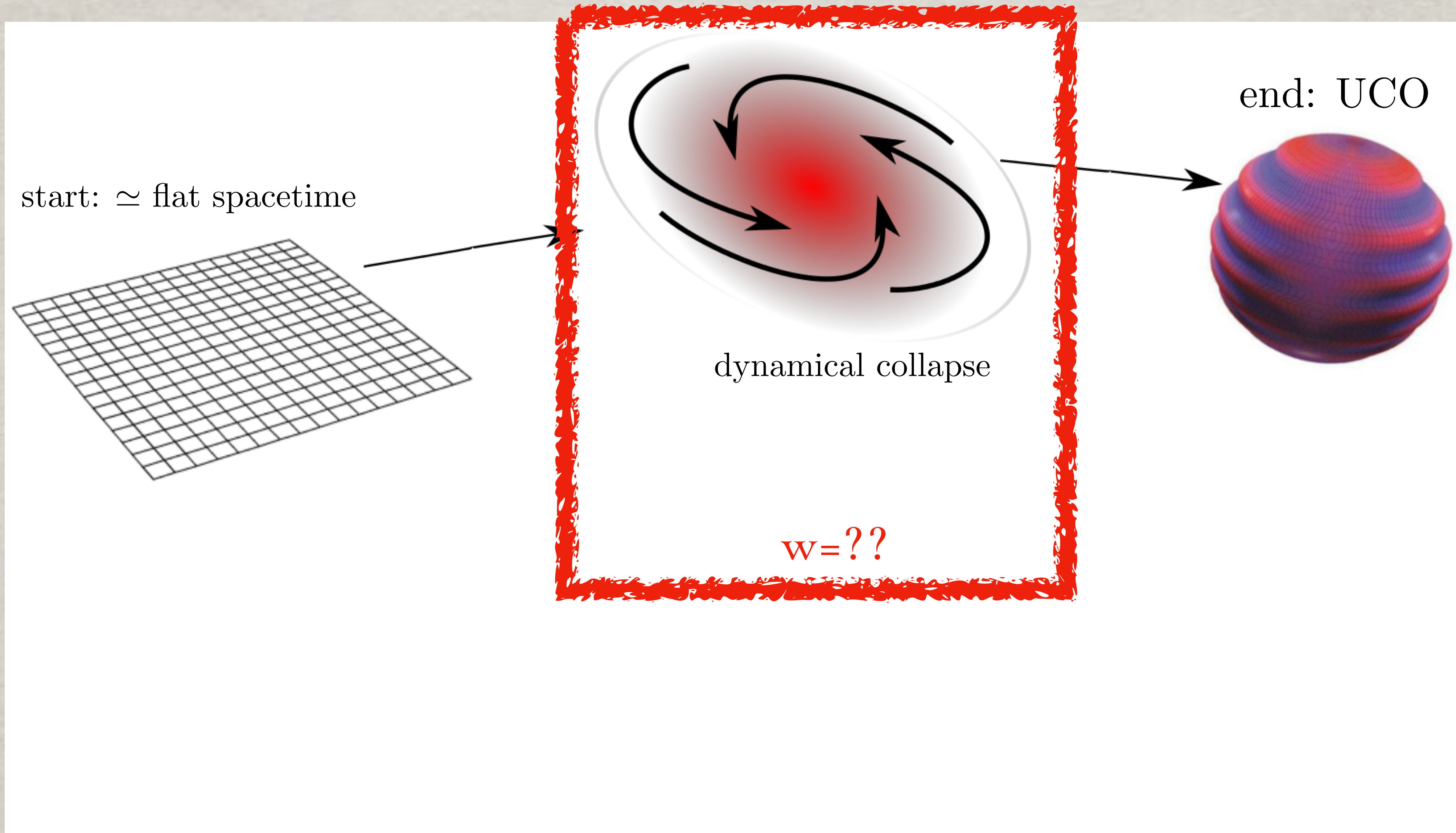
A generic dynamical picture



A generic dynamical picture



A generic dynamical picture



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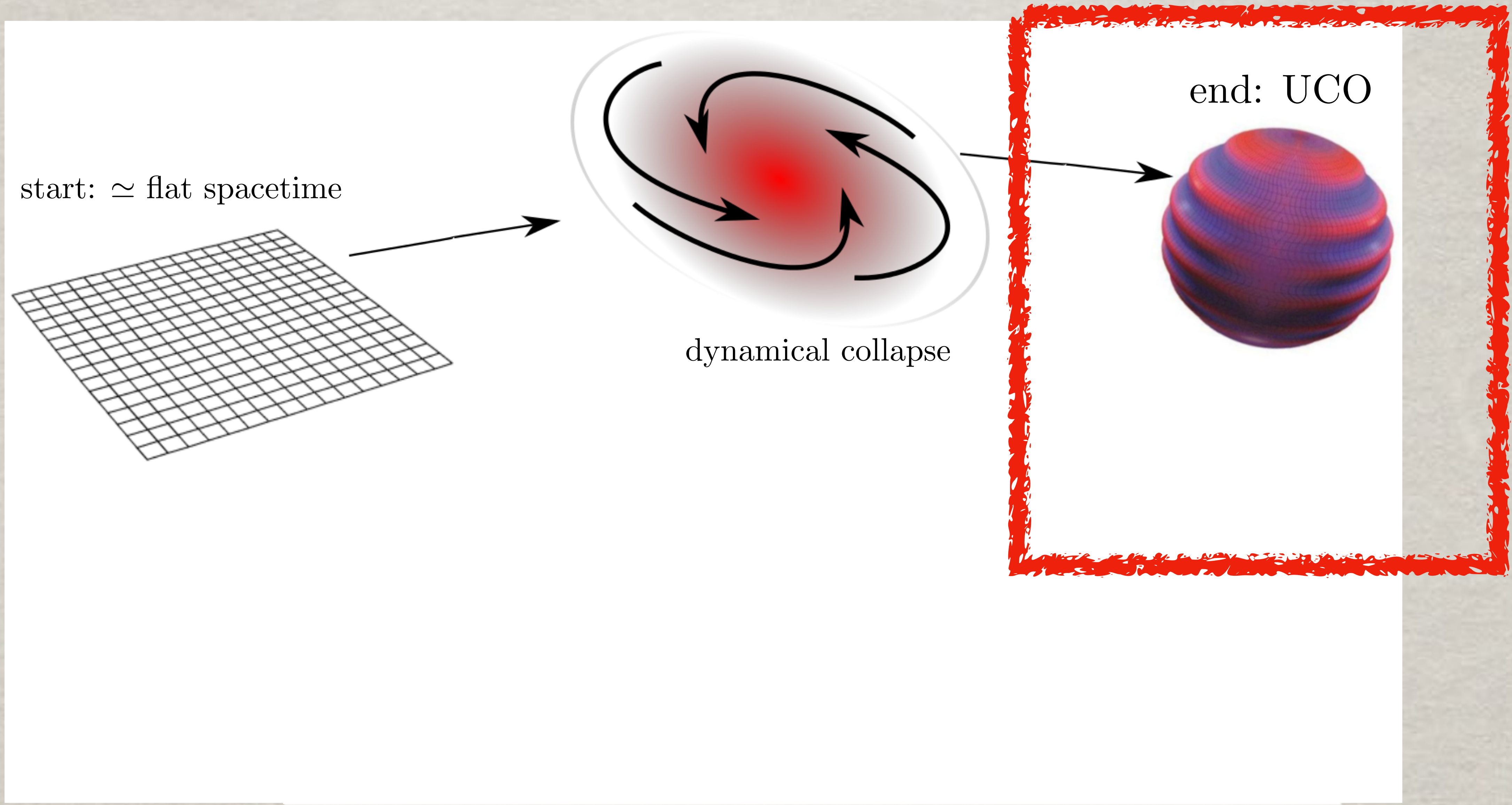

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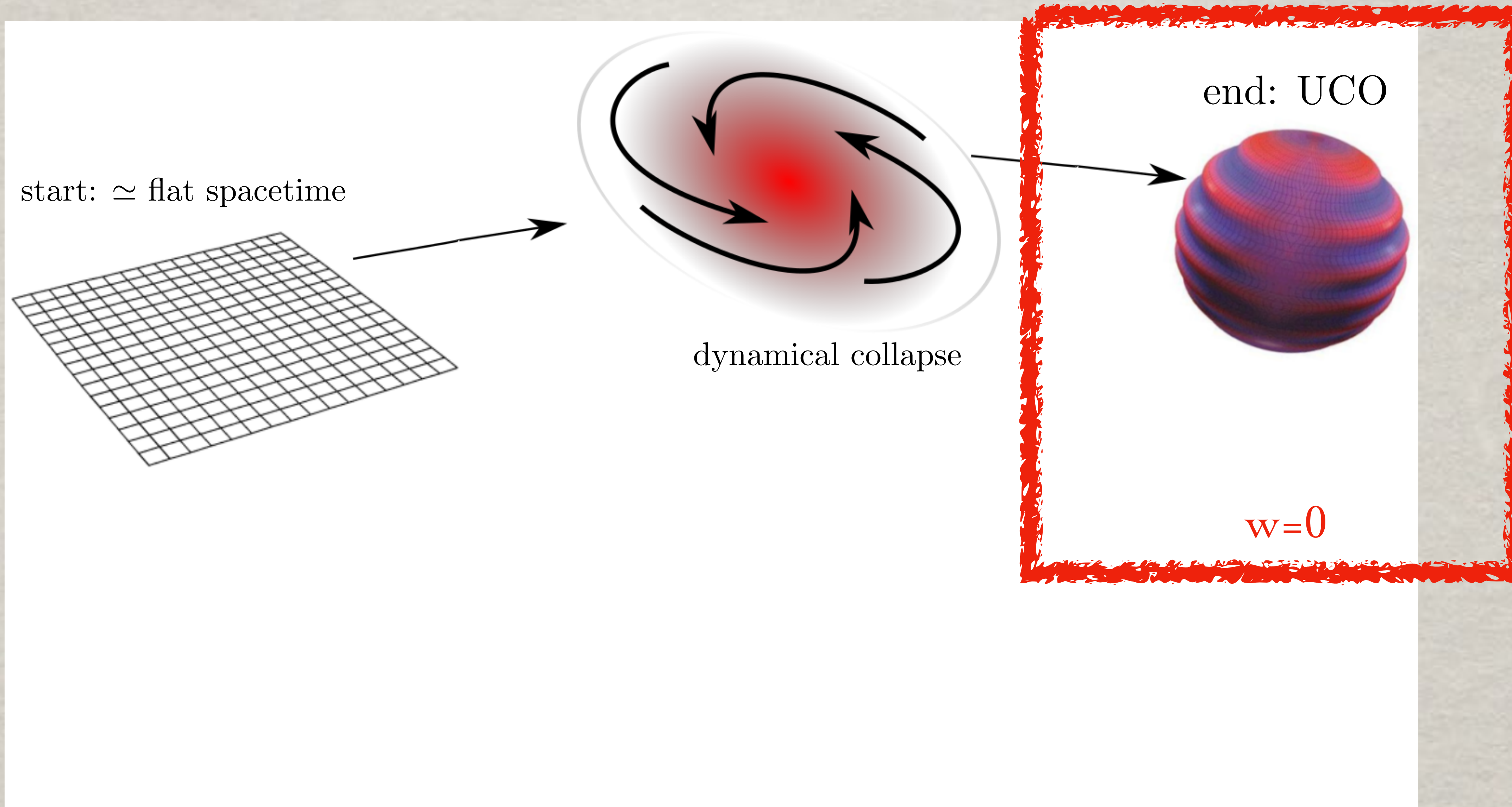

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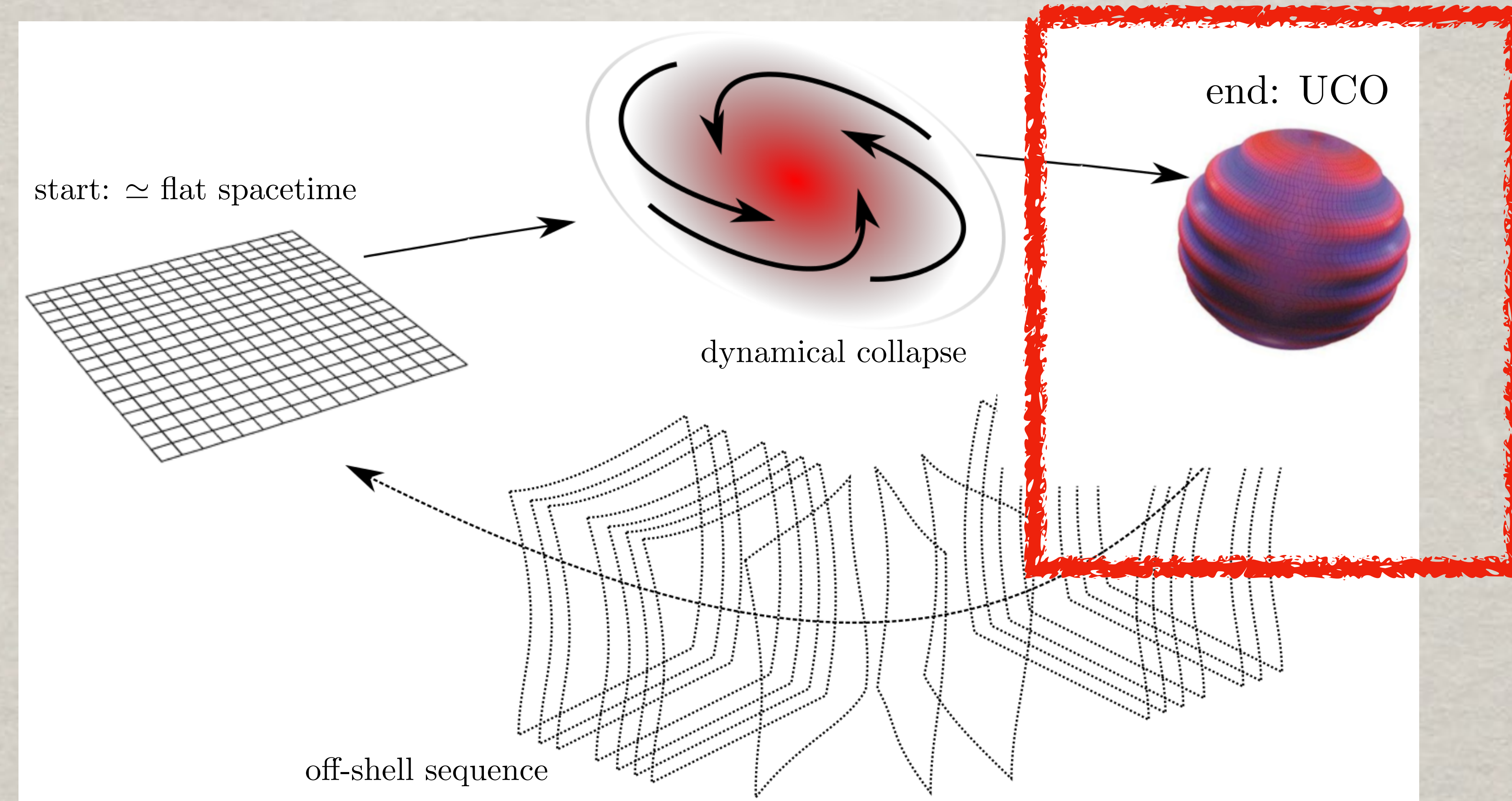
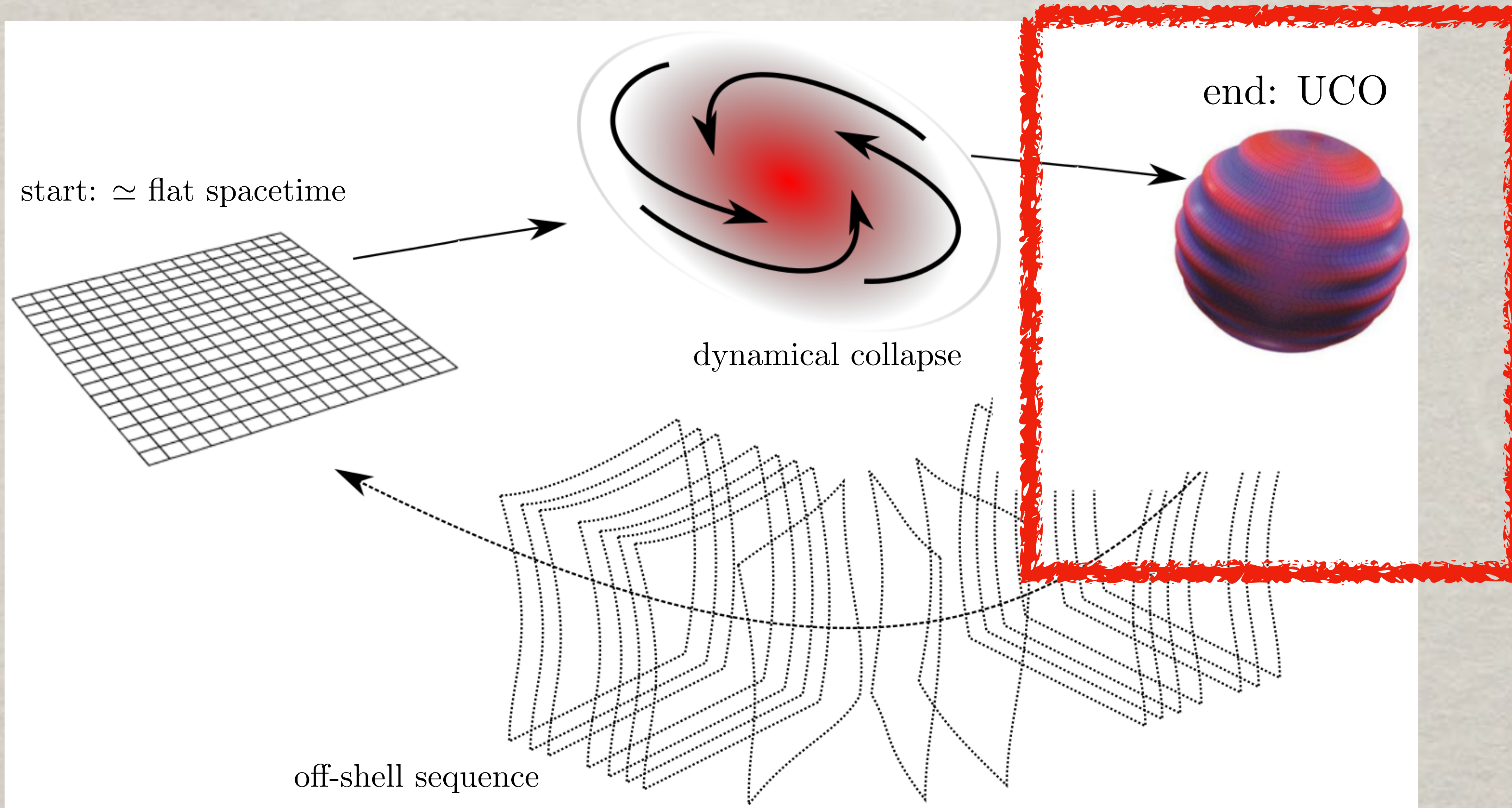
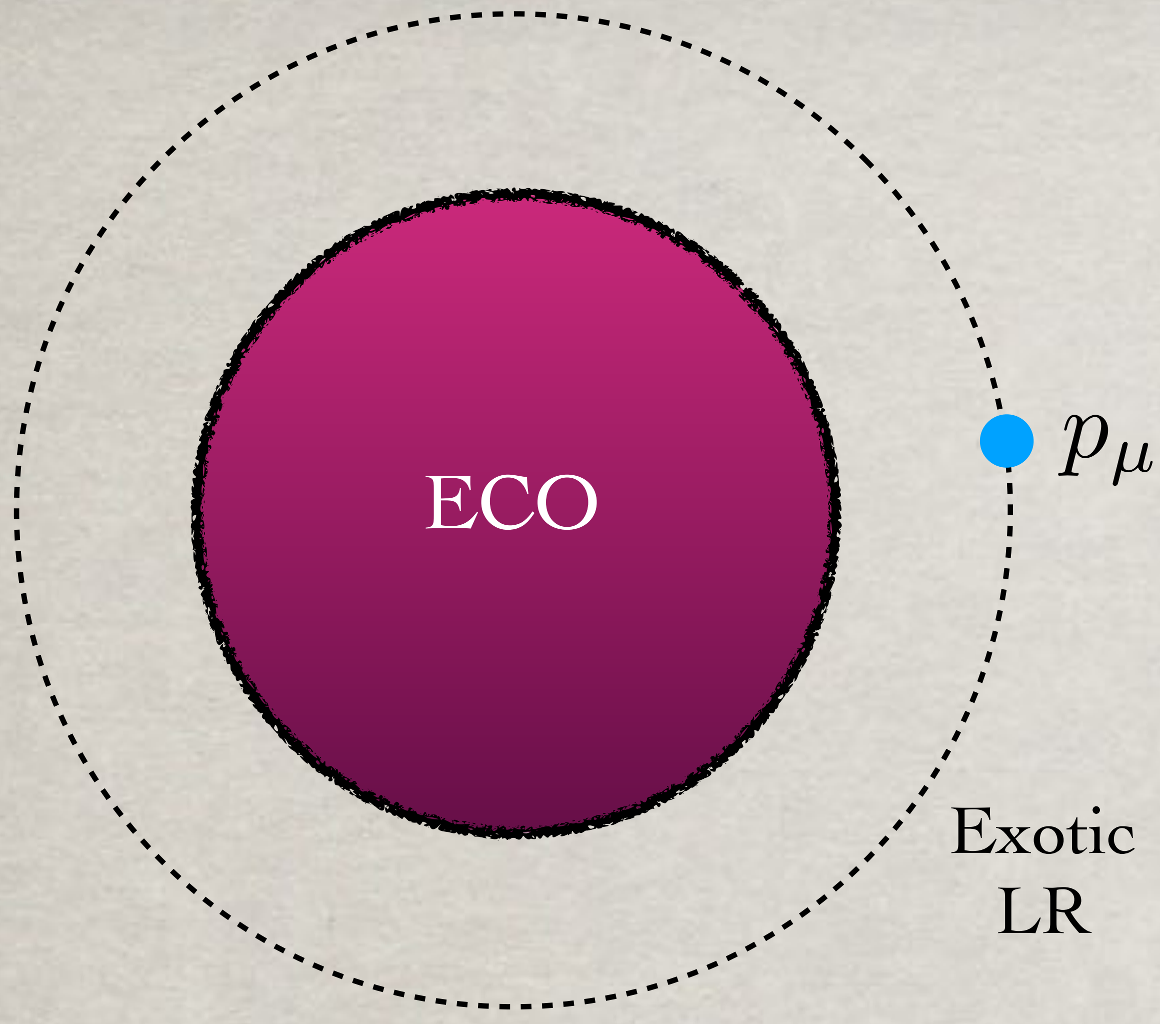


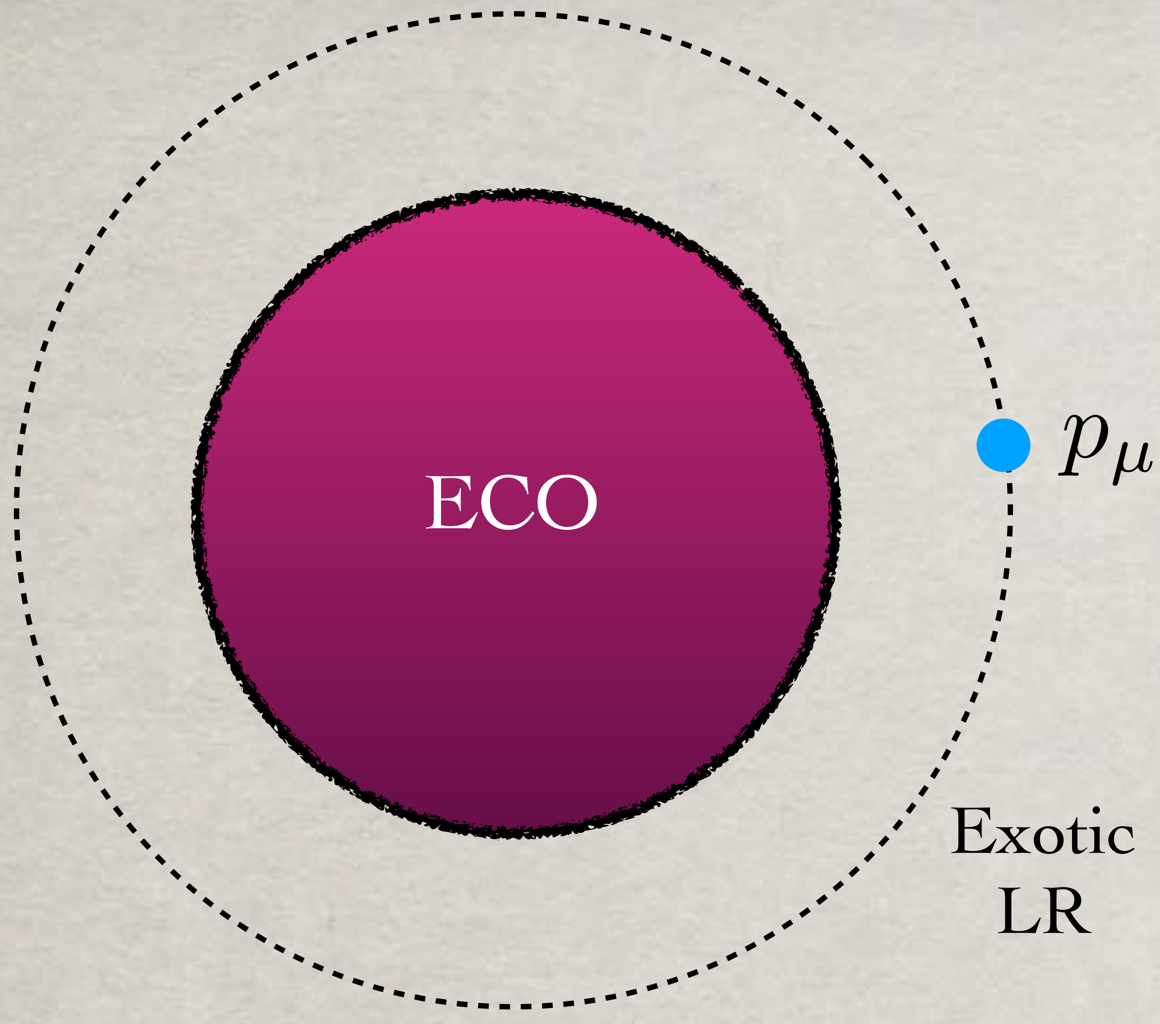
A generic dynamical picture



Punch line:

any (stationary, axi-symmetric, circular, topologically trivial) ECO that forms from an incomplete gravitational collapse which has a standard LR, must have an exotic one as well.


ECO
p_μ
Exotic
LR


ECO
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Exotic
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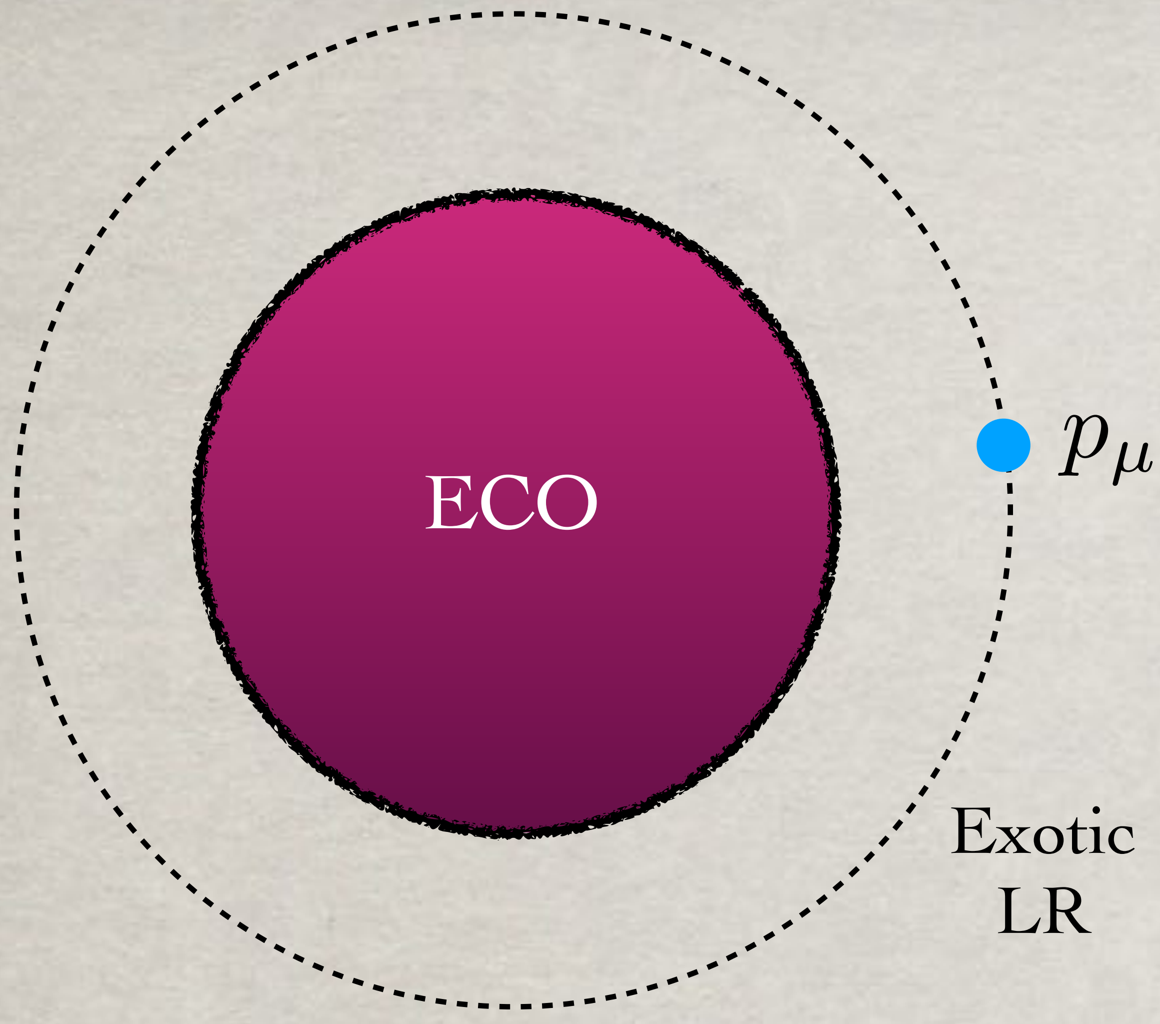
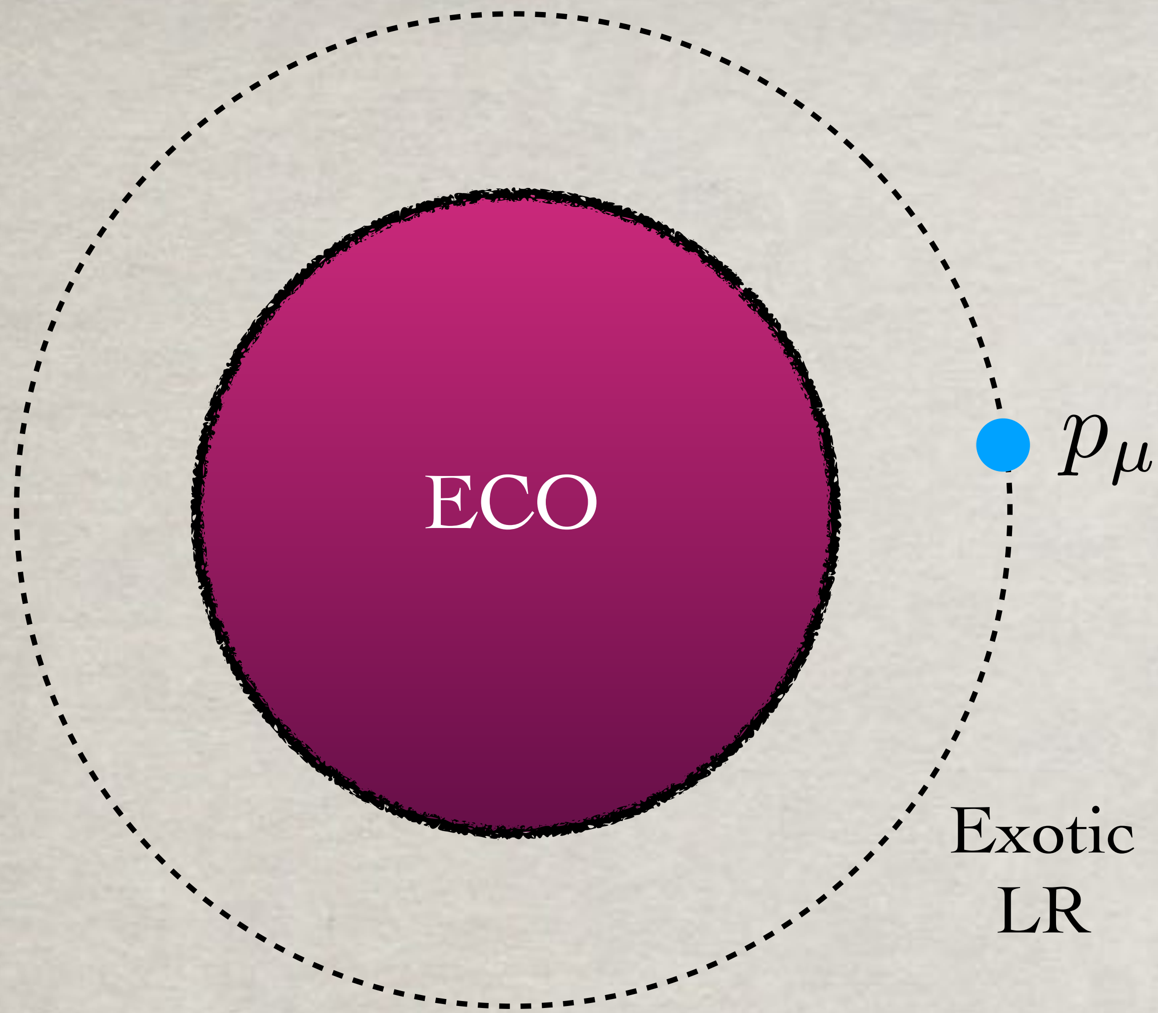


At the LR:

$$G^{\mu\nu}p_\mu p_\nu = \frac{1}{2}\partial_i\partial^i U$$

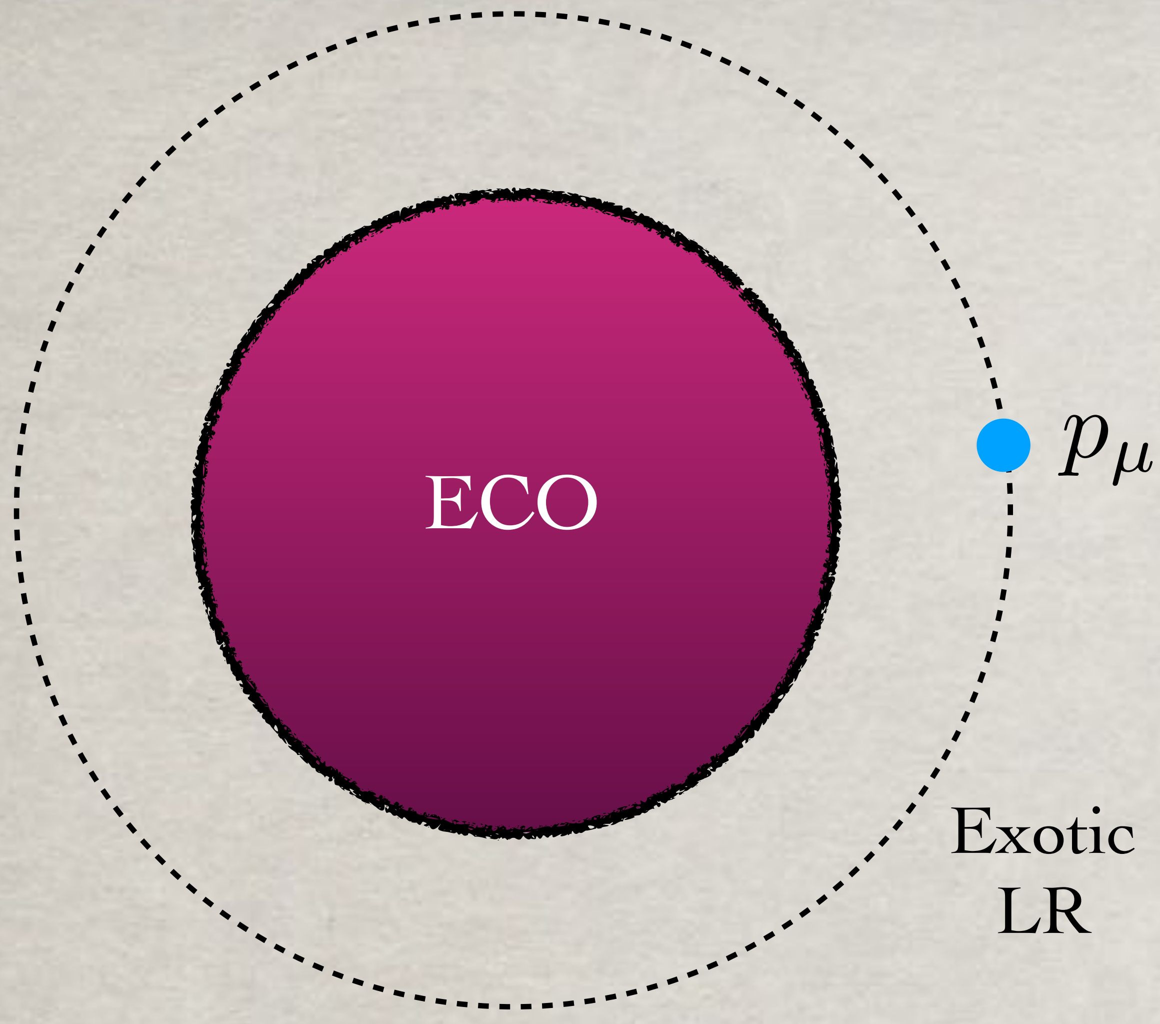


At the LR:

$$G^{\mu\nu}p_\mu p_\nu = \frac{1}{2}\partial_i\partial^i U$$

Assuming GR:

$$T^{\mu\nu}p_\mu p_\nu = \frac{1}{16\pi}\partial_i\partial^i U$$



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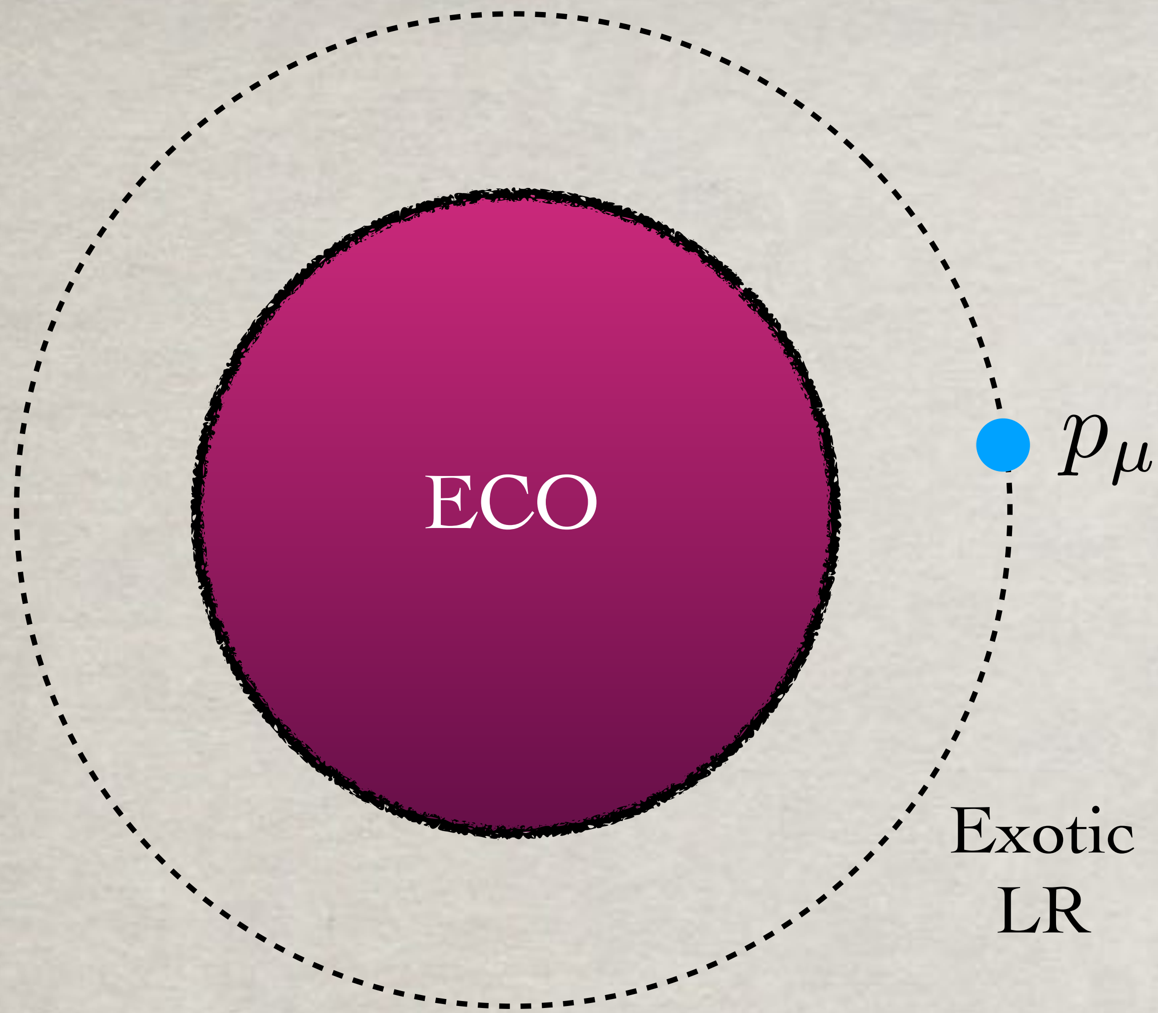
If the LR is a local maximum of U, then the NEC is violated:

$$\partial_i\partial^i U < 0 \implies T^{\mu\nu}p_\mu p_\nu < 0$$



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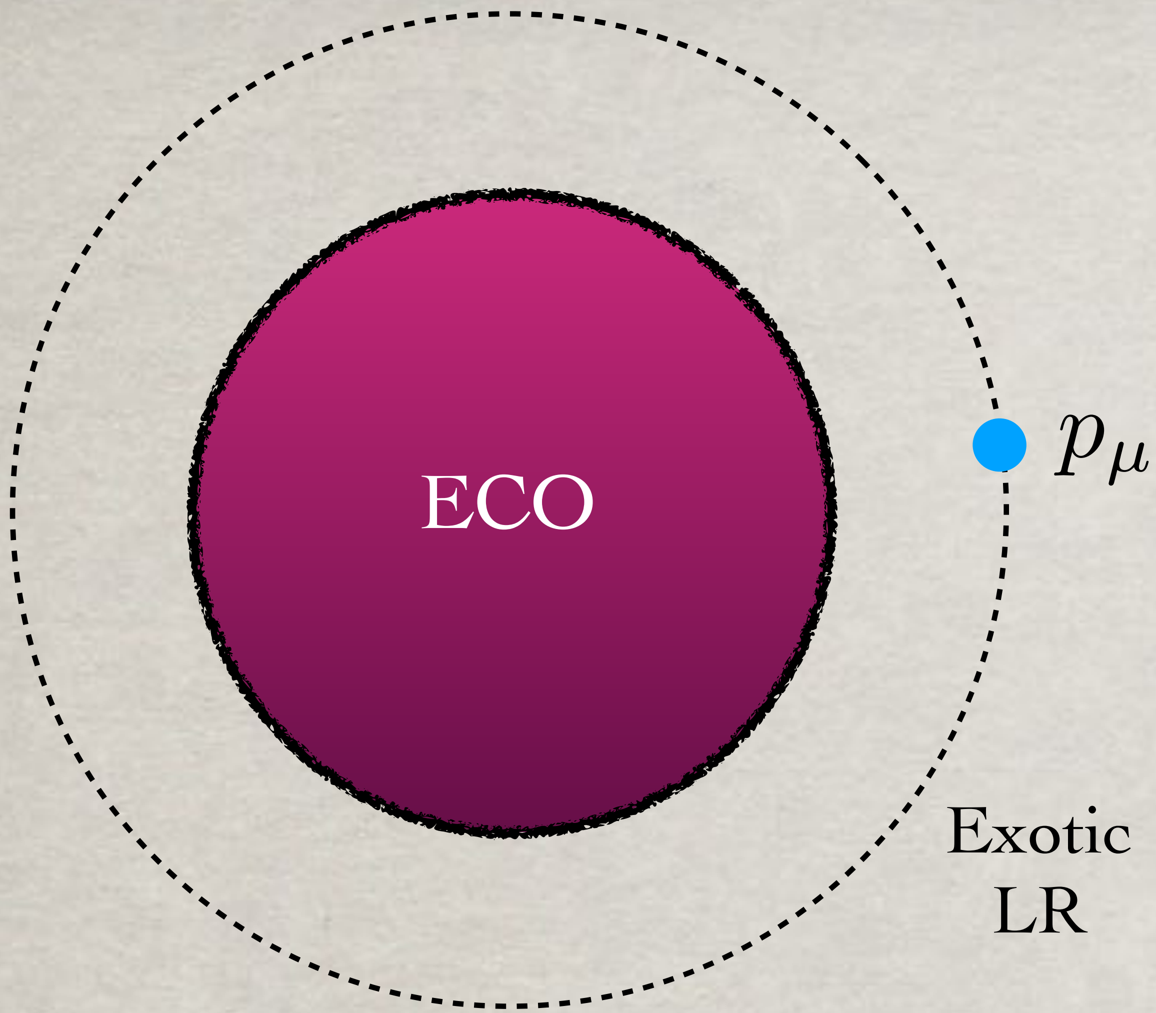
If one imposes the null energy condition, the exotic one must be fully stable.



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If one imposes the null energy condition, the exotic one must be fully stable.

In modified gravity one may proceed similarly in terms of an effective energy-momentum tensor.

On the fate of the Light Ring instability

Plan:

1) The "*black hole (BH) hypothesis*": BHs and light rings (LRs)

2) The "*exotic compact object (ECO) hypothesis*": ECOs and LRs

3) The LR instability and an explicit test of its fate

4) Discussion and final thoughts

"There is a crack in everything,
that is how the light gets in"

L. Cohen

"There is a crack in everything,
that is how the light gets in"

L. Cohen

It has been suggested: J. Keir, Class.Quant.Grav. 33 (2016) no.13, 135009; Benomio, arXiv:1809.07795

- Treating scalar linear waves as a model for nonlinear perturbations.
- Considering spherically symmetric spacetimes exhibiting stable Light Rings.
- Showing that linear waves cannot (uniformly) decay faster than logarithmically.
- Such slow decay is highly suggestive of a *nonlinear instability*.

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- Such slow decay is highly suggestive of a *nonlinear instability*.

The existence of a stable light ring is a (potentially) generic obstruction for any ultracompact ECO that can form from classical GR dynamics.

A good testing ground is an ECO model that:

- Is free of other instabilities (perturbative, ergoregion, …)

- Is dynamically robust (prone to non-linear evolutions)

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Bosonic stars

An artists impression of the collision of two bosonic stars
Credits: Nicolás Sanchis-Gual y Rocío García Souto

Bosonic stars

Bosonic stars

Check list:

1) Appear in a well motivated and consistent physical model;

Boson stars: General Relativity minimally coupled to massive bosonic fields

2) Have a dynamical formation mechanism;

Gravitational cooling

3) Be (sufficiently) stable.

Some solutions are stable

Bosonic stars

Check list:

1) Appear in a well motivated and consistent physical model;

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3) Be (sufficiently) stable.

Some solutions are stable

Subtleties:

- scalar rotating stars are unstable in the simplest model;

Sanchis-Gual, Di Giovanni, Zilhão, CH, P. Cerda-Duran, Font and Radu, Phys. Rev. Lett. 123 (2019) 221101

- The initial value problem may become ill-defined for self-interacting Proca fields.

Clough et al. ArXiv:2204.10868; Coates and Ramazanoglu, ArXiv:2205.07784; You and Zhang ArXiv:2204.11324

A testing ground for the LR (trapping) instability

PHYSICAL REVIEW LETTERS **130**, 061401 (2023)

Editors' Suggestion   Featured in Physics

Exotic Compact Objects and the Fate of the Light-Ring Instability

Pedro V. P. Cunha,[1] Carlos Herdeiro,[1] Eugen Radu,[1] and Nicolas Sanchis-Gual[2,1]

[1]*Departamento de Matemática da Universidade de Aveiro and Centre for Research and Development in Mathematics and Applications (CIDMA), Campus de Santiago, 3810-183 Aveiro, Portugal*

[2]*Departamento de Astronomía y Astrofísica, Universitat de València, Dr. Moliner 50, 46100 Burjassot (València), Spain*

(Received 12 August 2022; accepted 6 December 2022; published 7 February 2023)

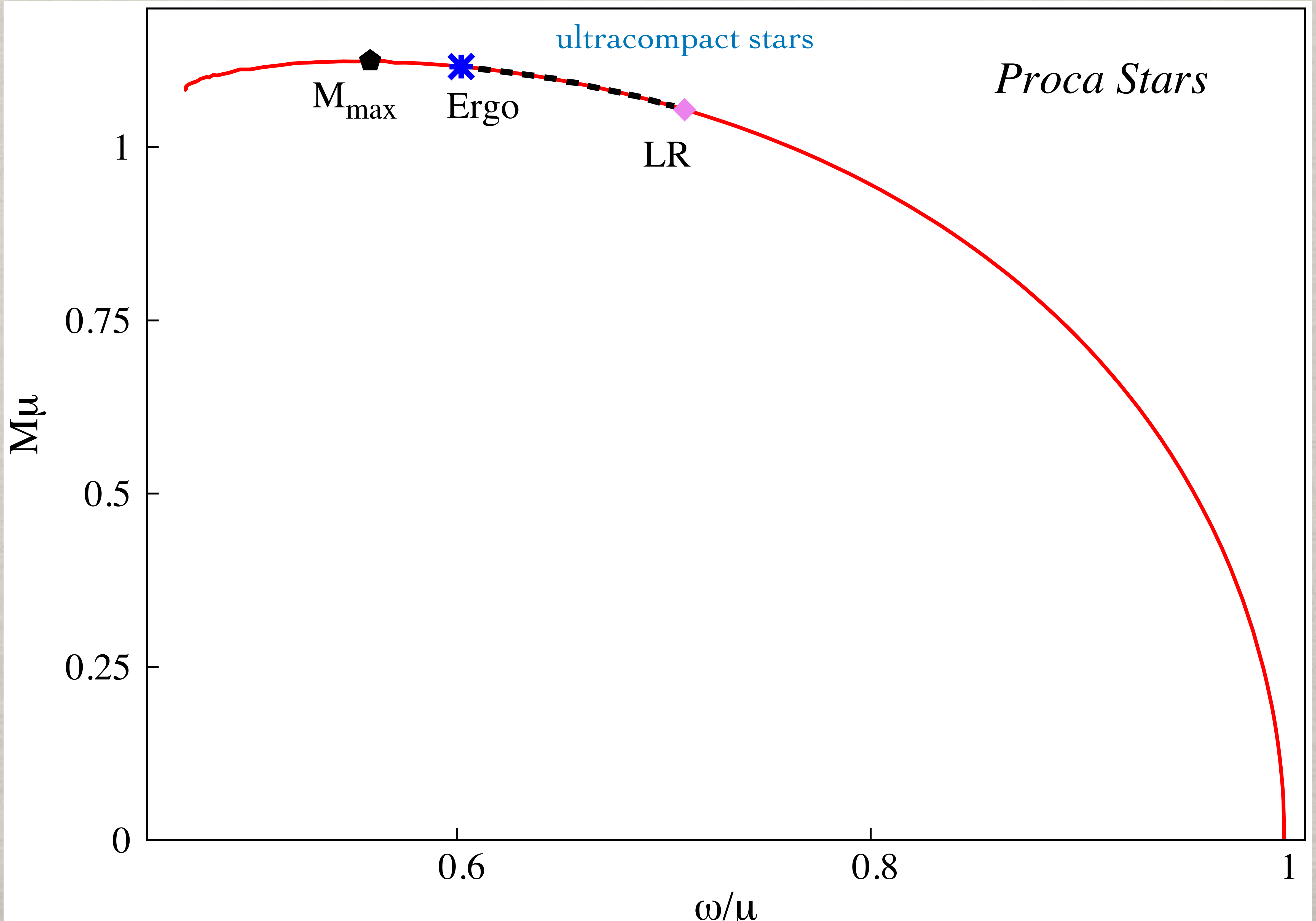
Ultracompact objects with light rings (LRs) but without an event horizon could mimic black holes (BHs) in their strong gravity phenomenology. But are such objects dynamically viable? Stationary and axisymmetric ultracompact objects that can form from smooth, quasi-Minkowski initial data must have at least one *stable* LR, which has been argued to trigger a spacetime *instability*; but its development and fate have been unknown. Using fully nonlinear numerical evolutions of ultracompact bosonic stars free of any other known instabilities and introducing a novel adiabatic effective potential technique, we confirm the LRs triggered instability, identifying two possible fates: migration to nonultracompact configurations or collapse to BHs. In concrete examples we show that typical migration (collapse) timescales are not larger than $\sim 10^3$ light-crossing times, unless the stable LR potential well is very shallow. Our results show that the LR instability is effective in destroying horizonless ultracompact objects that could be plausible BH imitators.

DOI: 10.1103/PhysRevLett.130.061401

Spinning (mini)-Proca stars

$$\mathcal{L}_m = -\frac{1}{4}\mathcal{F}_{\alpha\beta}\bar{\mathcal{F}}^{\alpha\beta} - \frac{1}{2}\mu^2\mathcal{A}_\alpha\bar{\mathcal{A}}^\alpha$$

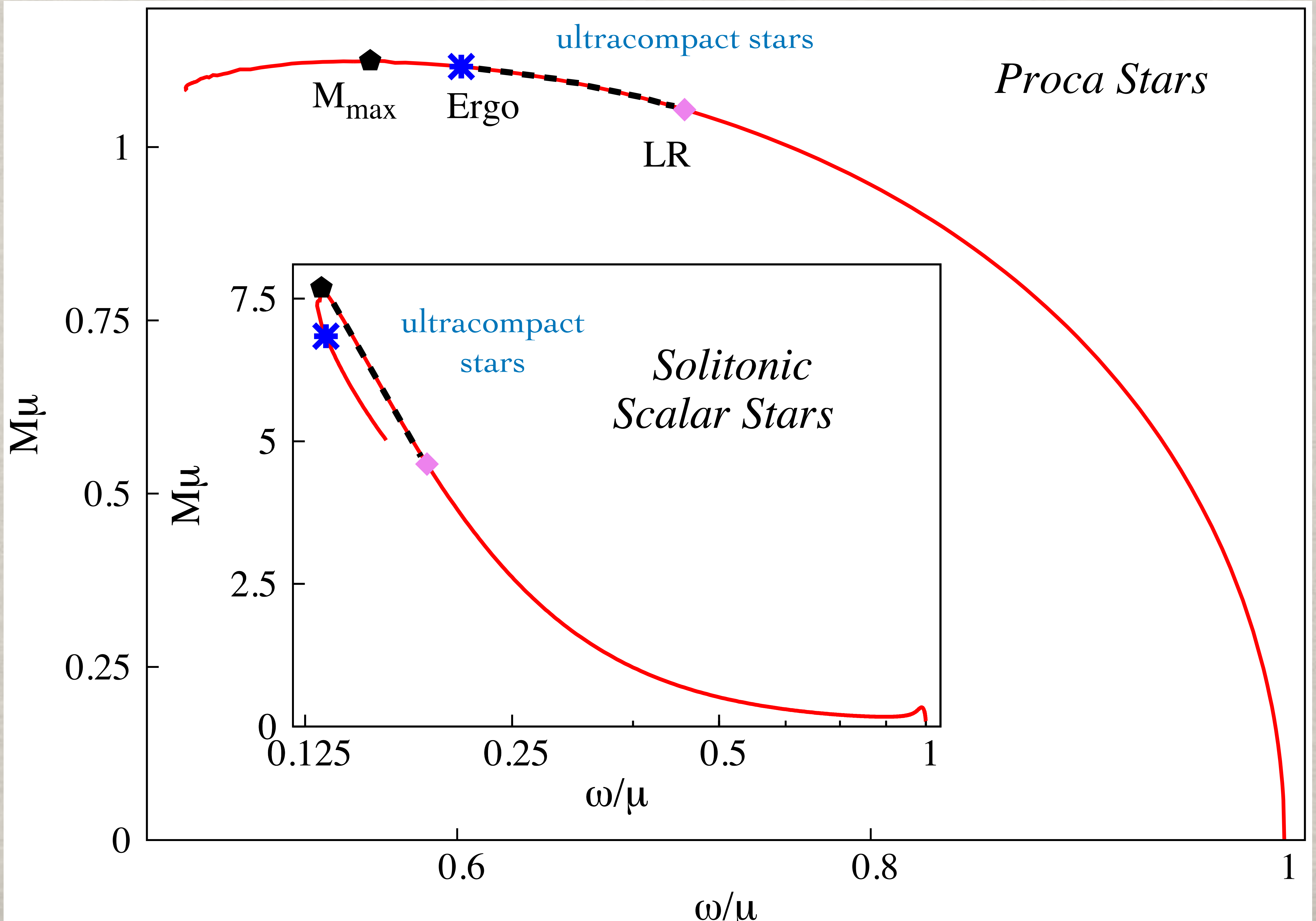
$$\mathcal{L} = \frac{R}{16\pi G} + \mathcal{L}_m$$



$$\mathcal{L} = \frac{R}{16\pi G} + \mathcal{L}_m$$

$$\mathcal{L}_m = -\frac{1}{4}\mathcal{F}_{\alpha\beta}\bar{\mathcal{F}}^{\alpha\beta} - \frac{1}{2}\mu^2\mathcal{A}_\alpha\bar{\mathcal{A}}^\alpha$$ Spinning (mini)-Proca stars

$$\mathcal{L}_m = -\partial_\alpha\Phi\partial^\alpha\bar{\Phi} - \mu^2|\Phi|^2\left[1 - \frac{2|\Phi|^2}{\sigma_0^2}\right]^2$$ Spinning solitonic scalar stars



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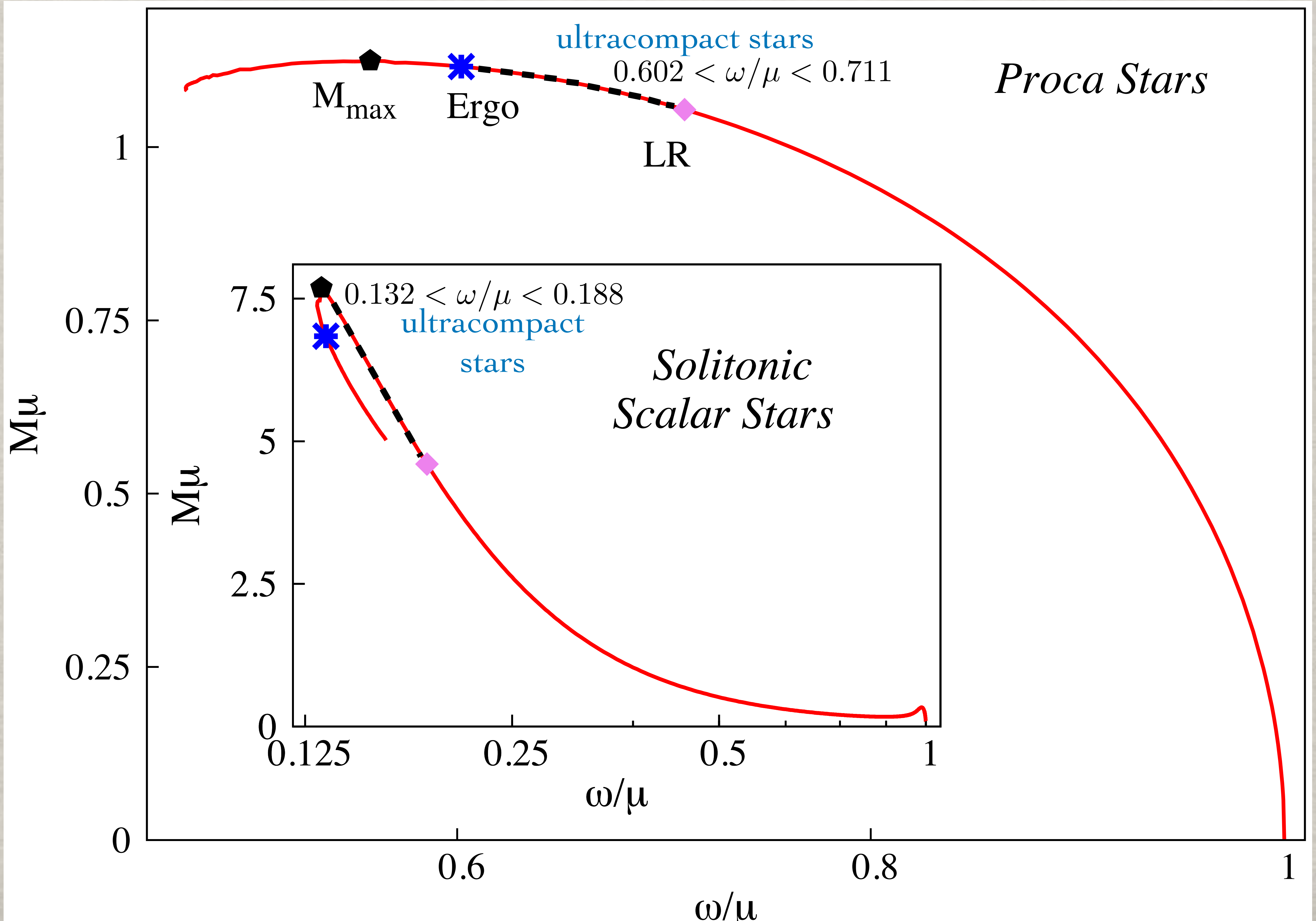

Now we perform fully non-linear numerical relativity evolutions

Sanchis-Gual, Di Giovanni, Zilhão, CH, P. Cerda-Duran, Font and Radu, Phys. Rev. Lett. 123 (2019) 221101

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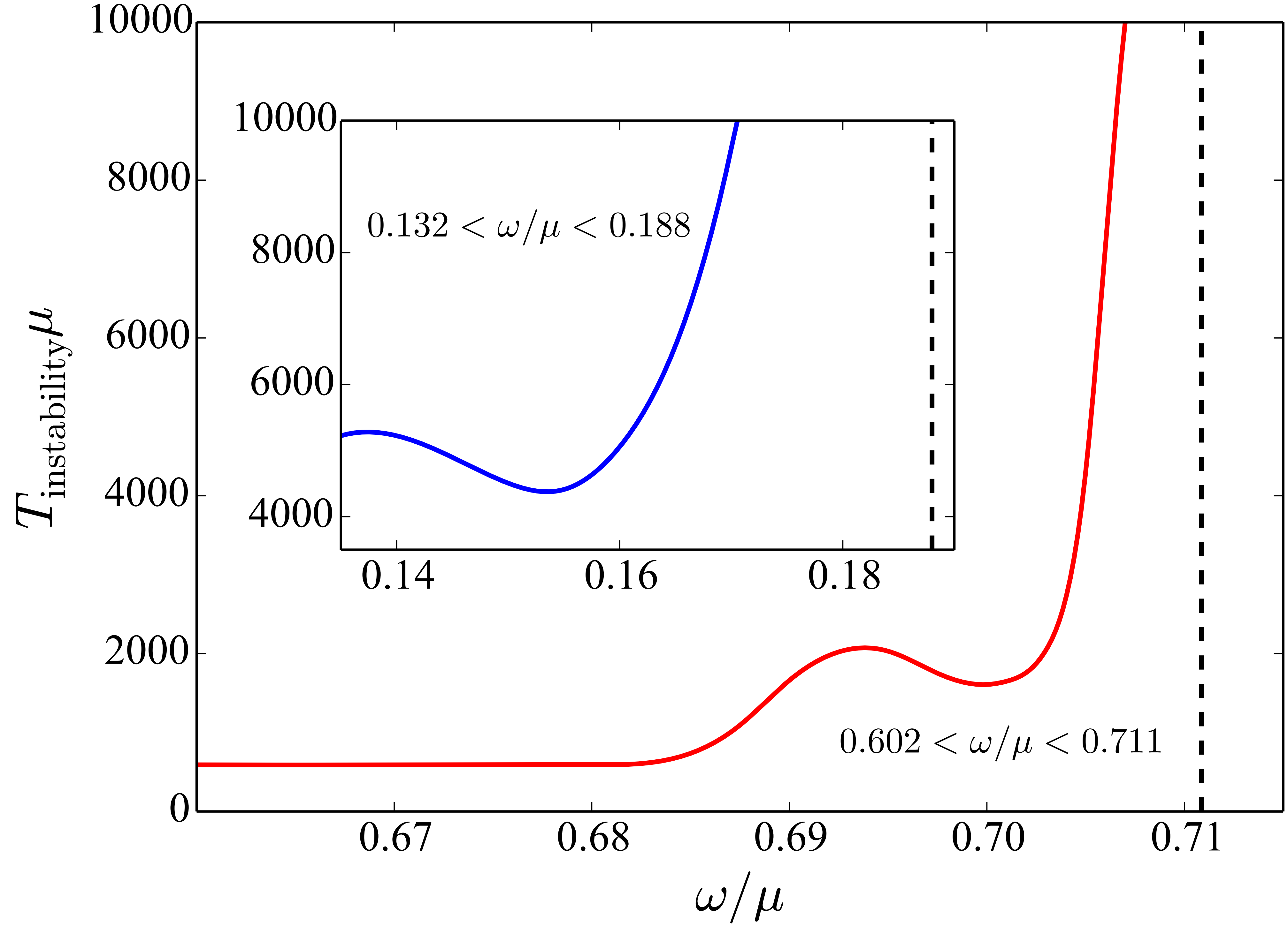

Cunha, CH, Radu and Sanchis-Gual, 2207.13713

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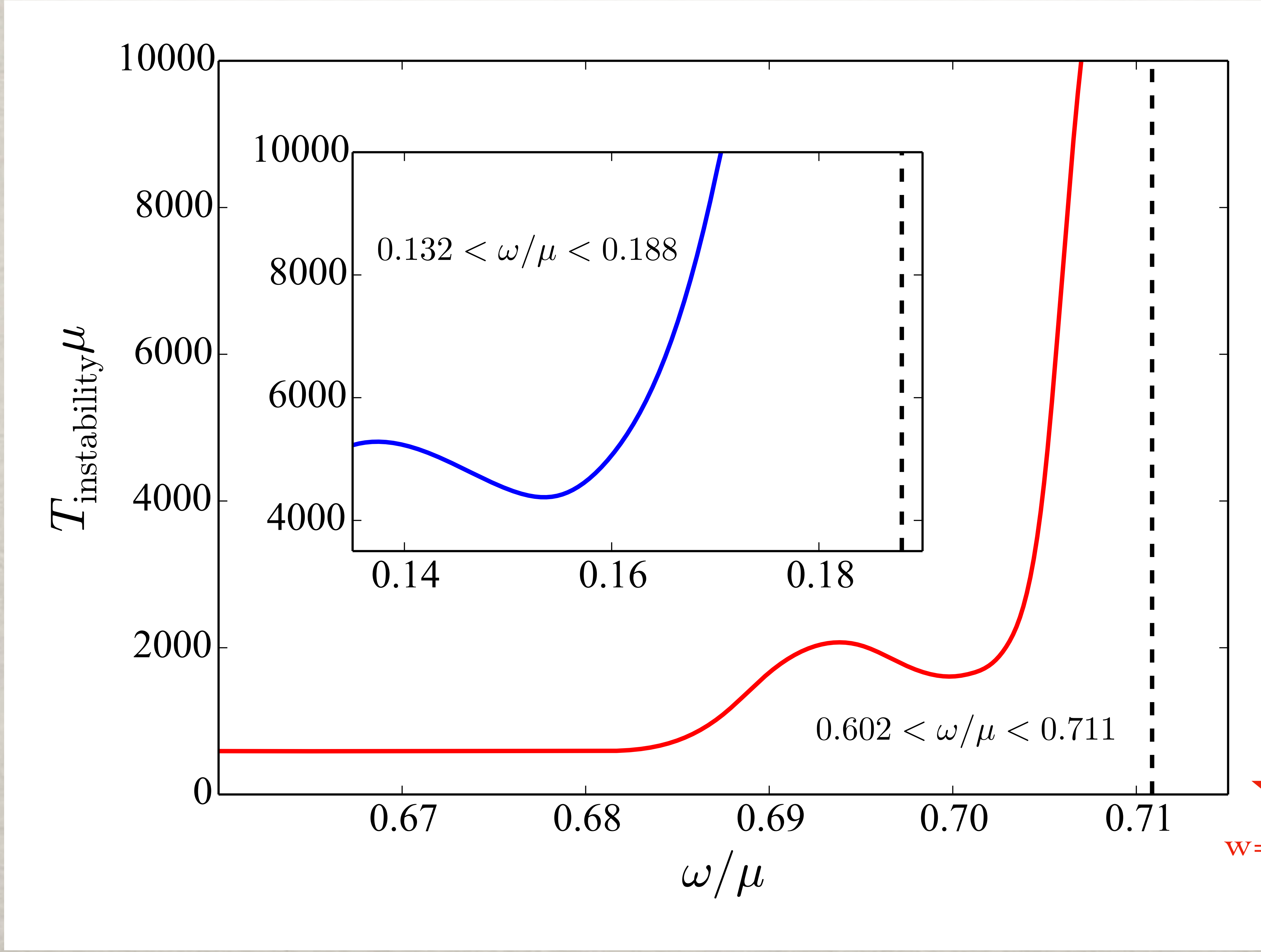

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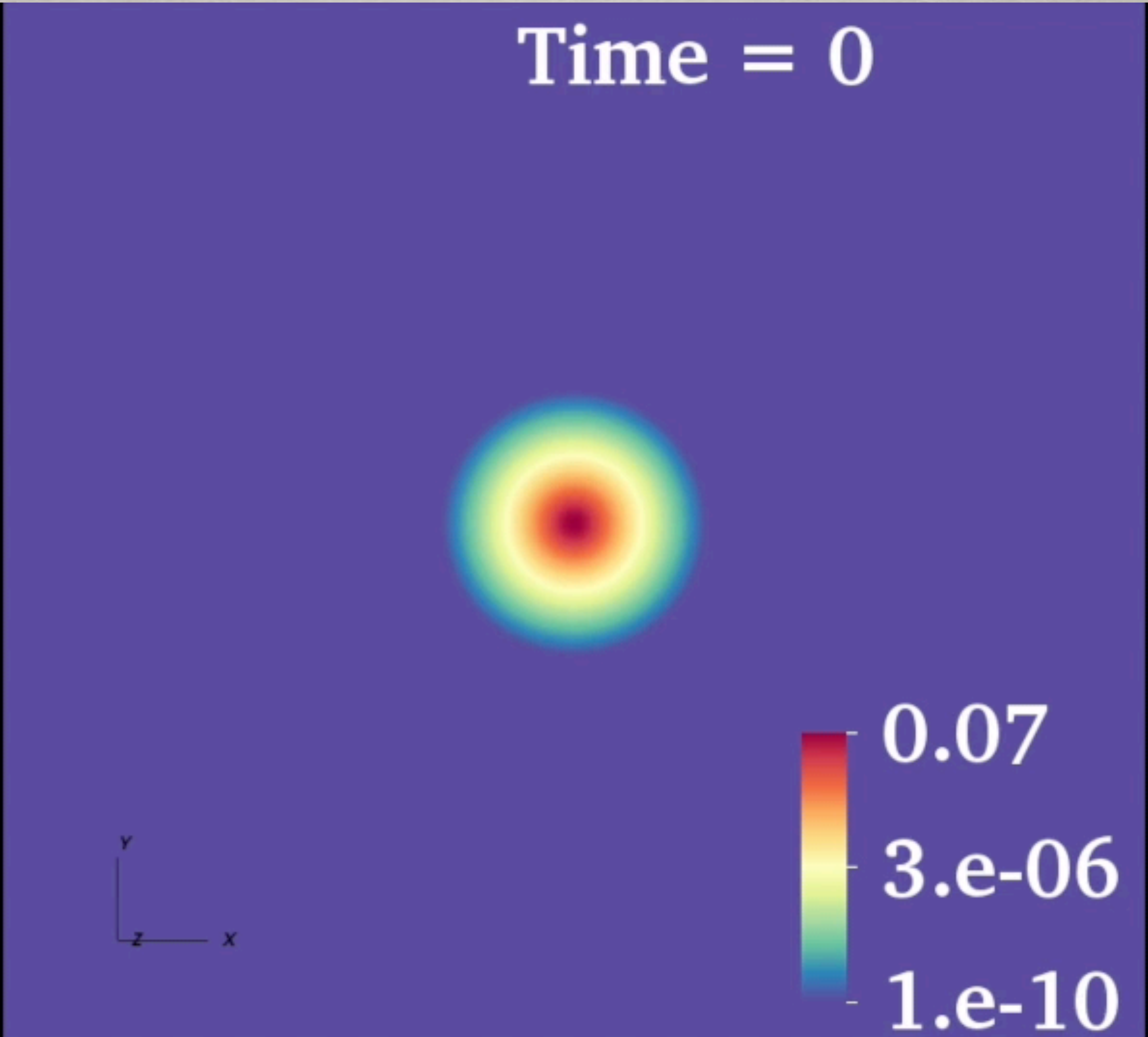
Cunha, CH, Radu and Sanchis-Gual, 2207.13713



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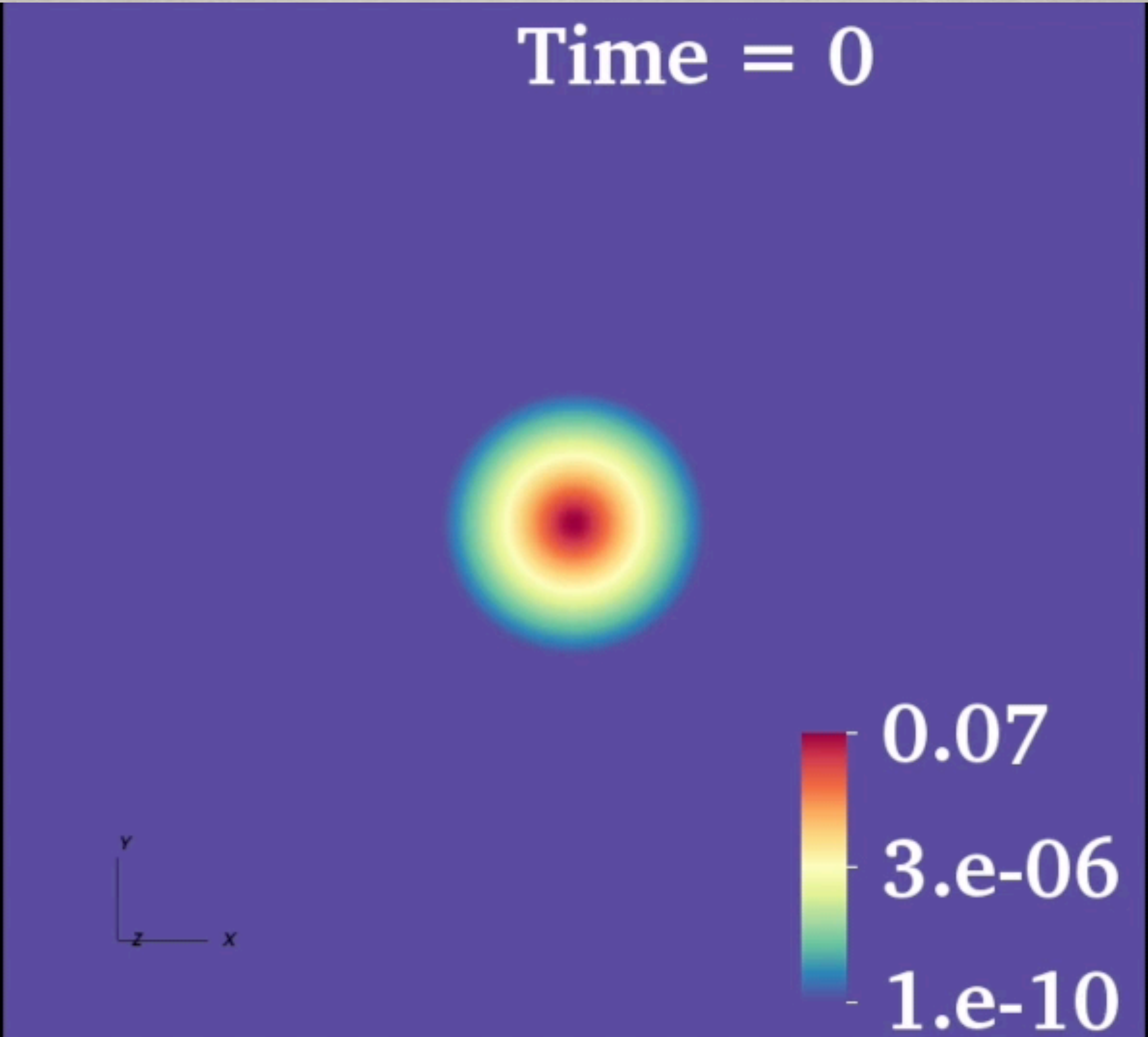
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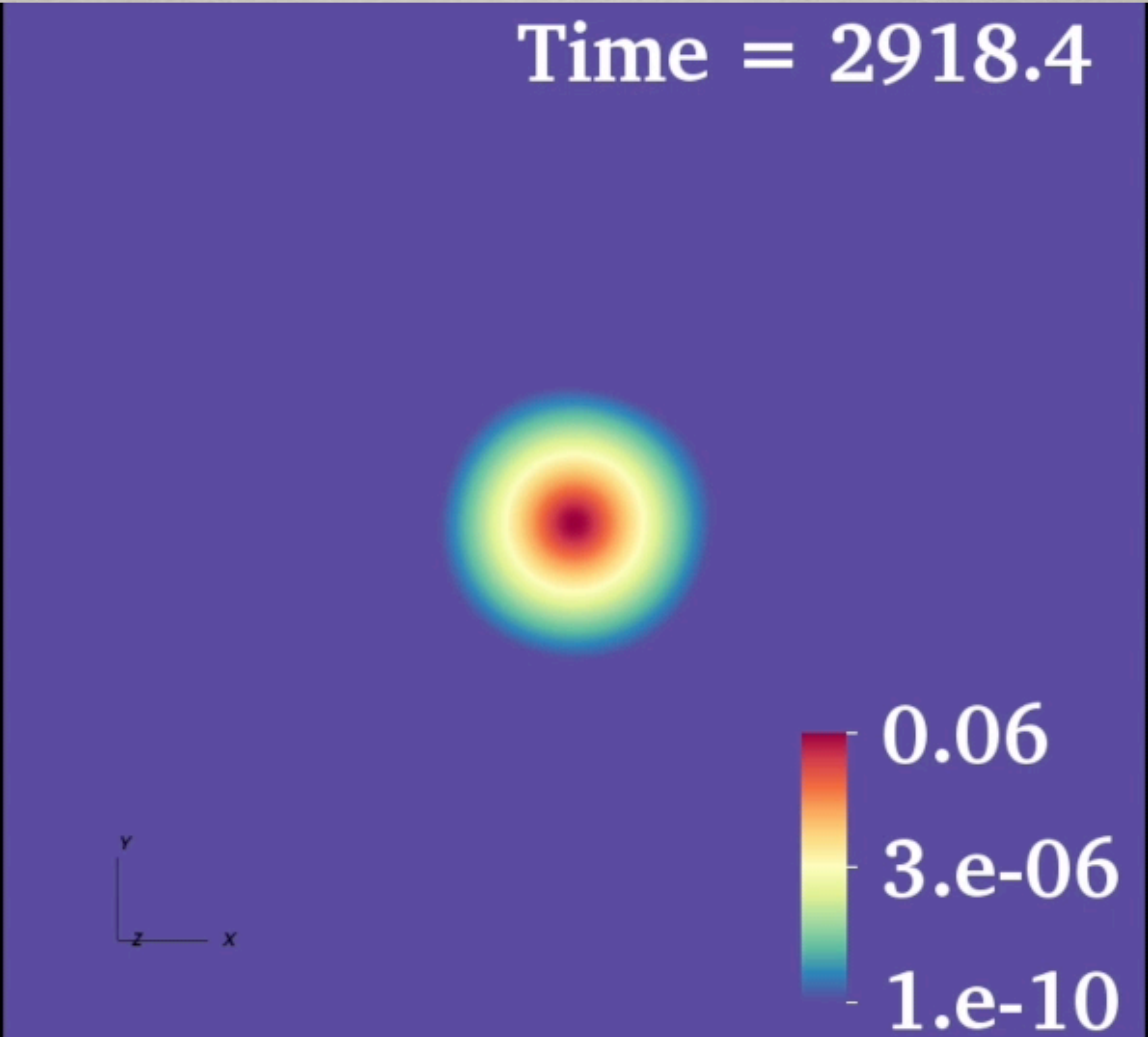
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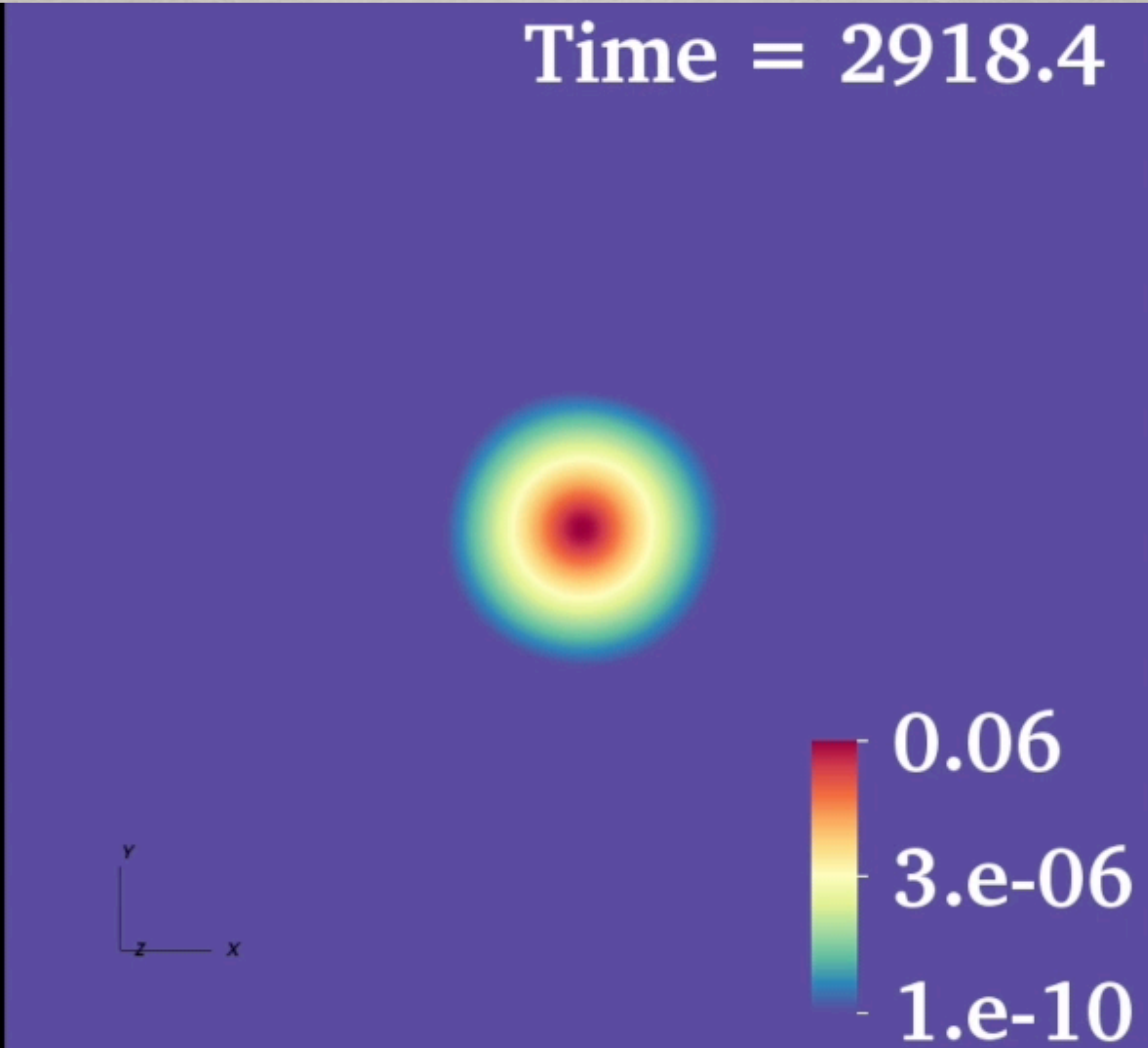
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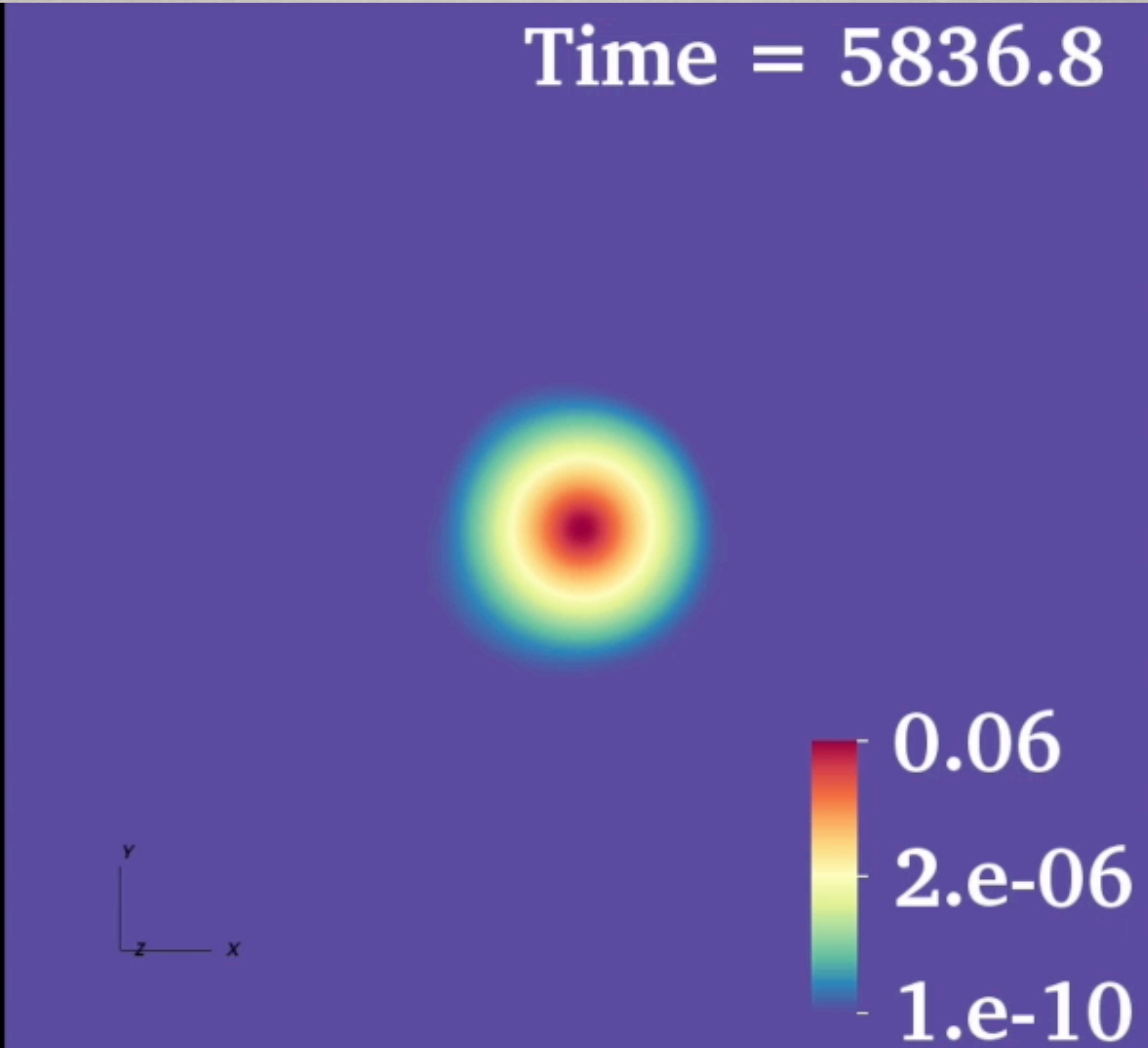
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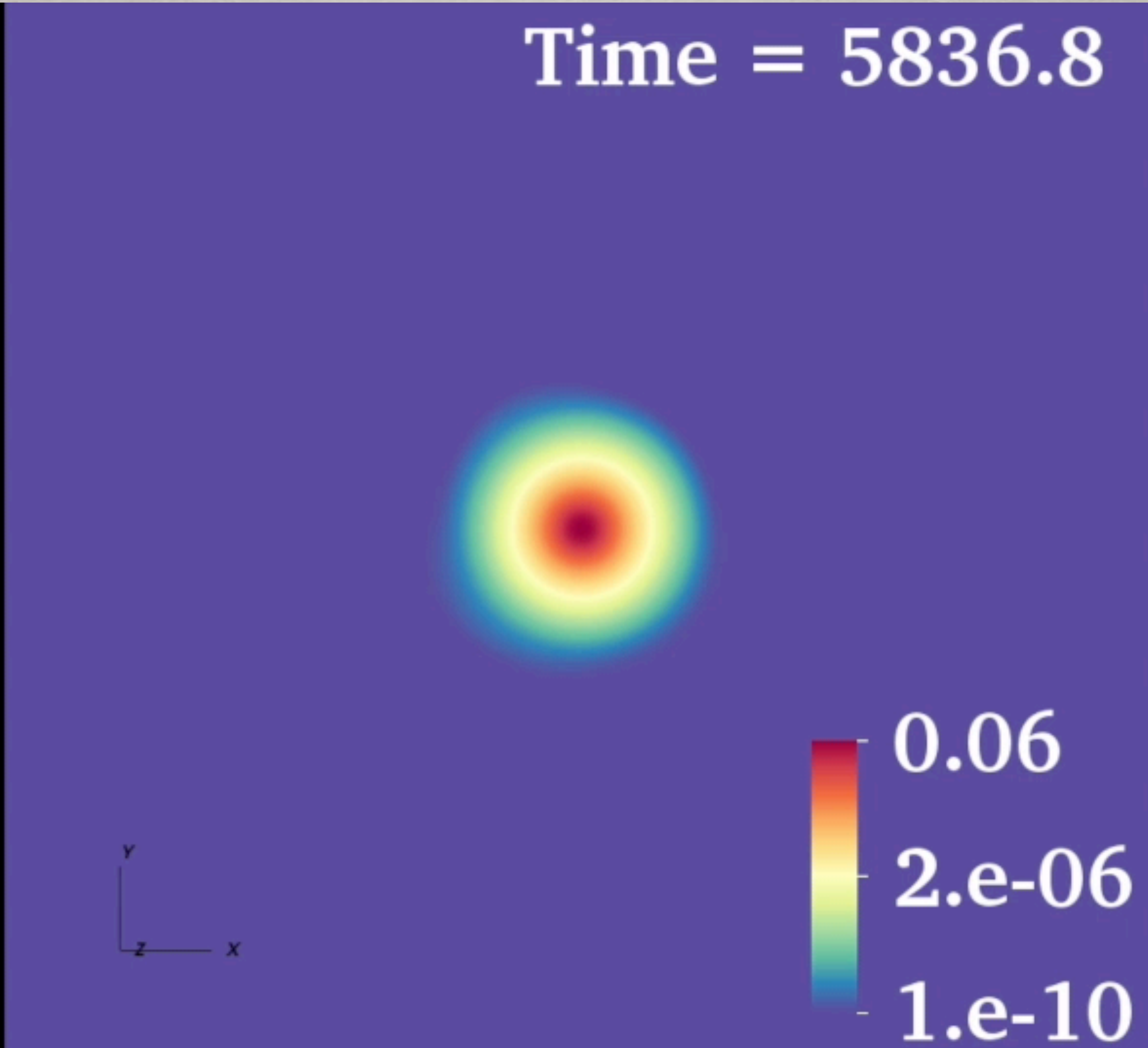
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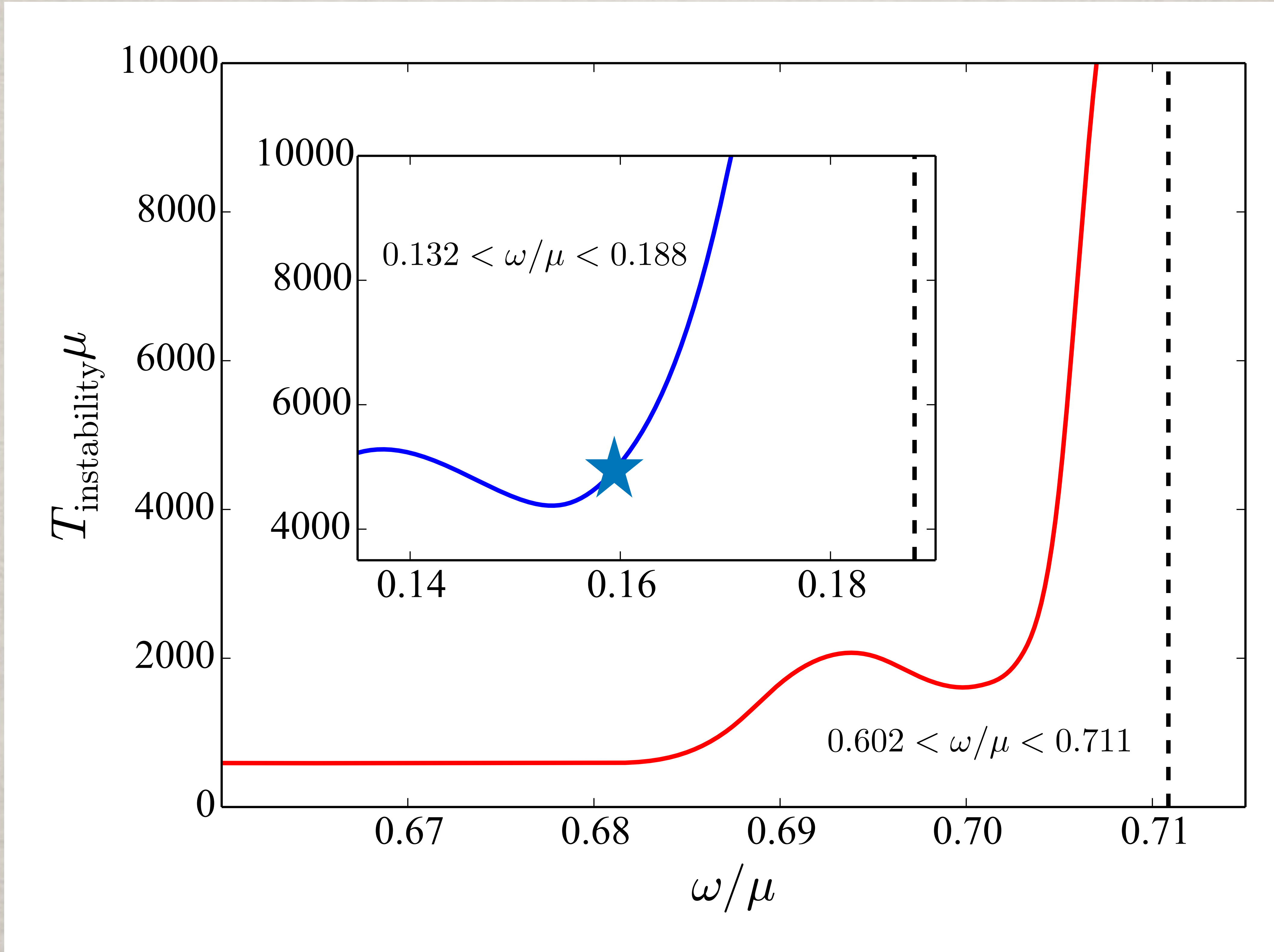
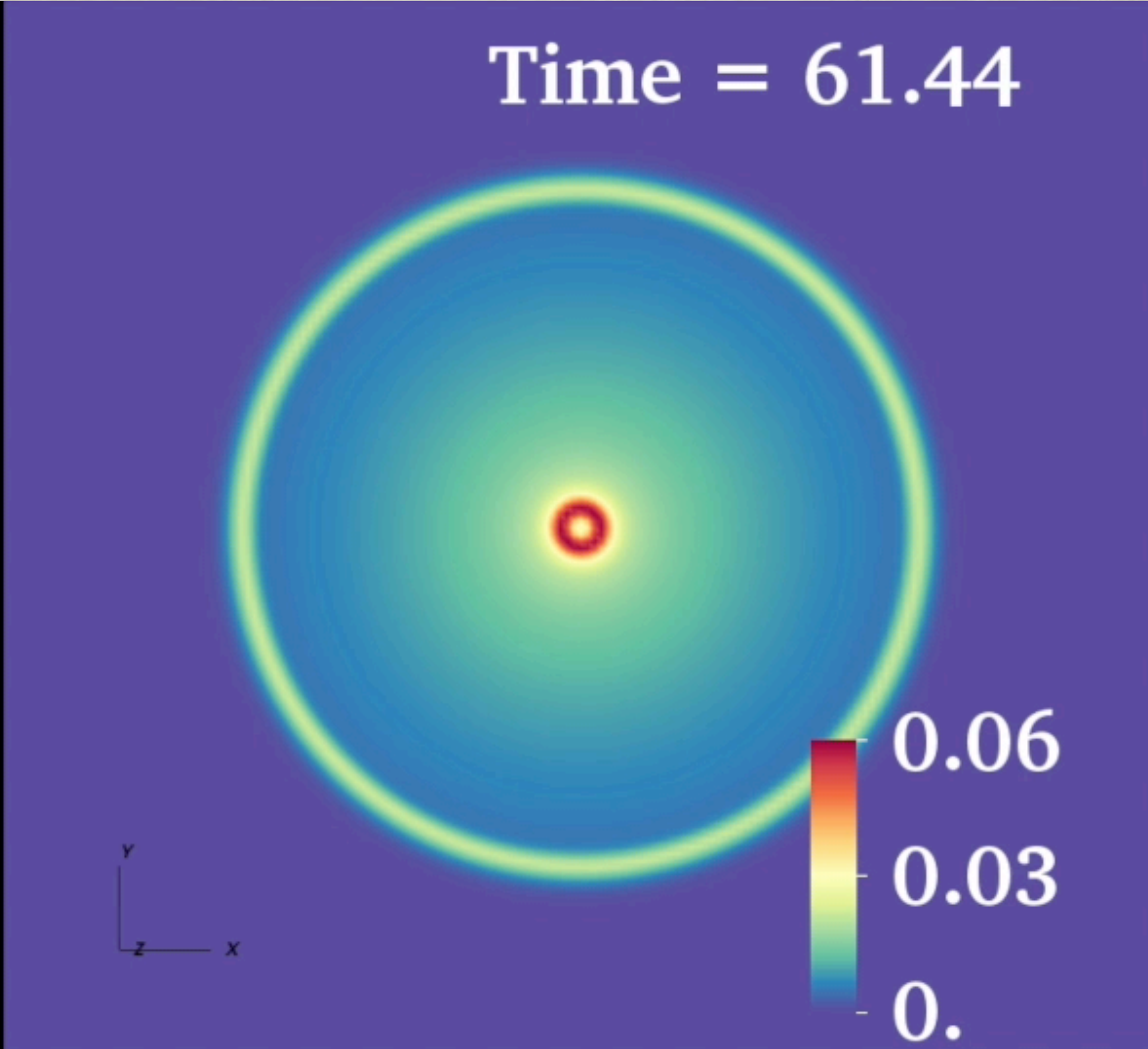


Cunha, CH, Radu and Sanchis-Gual, 2207.13713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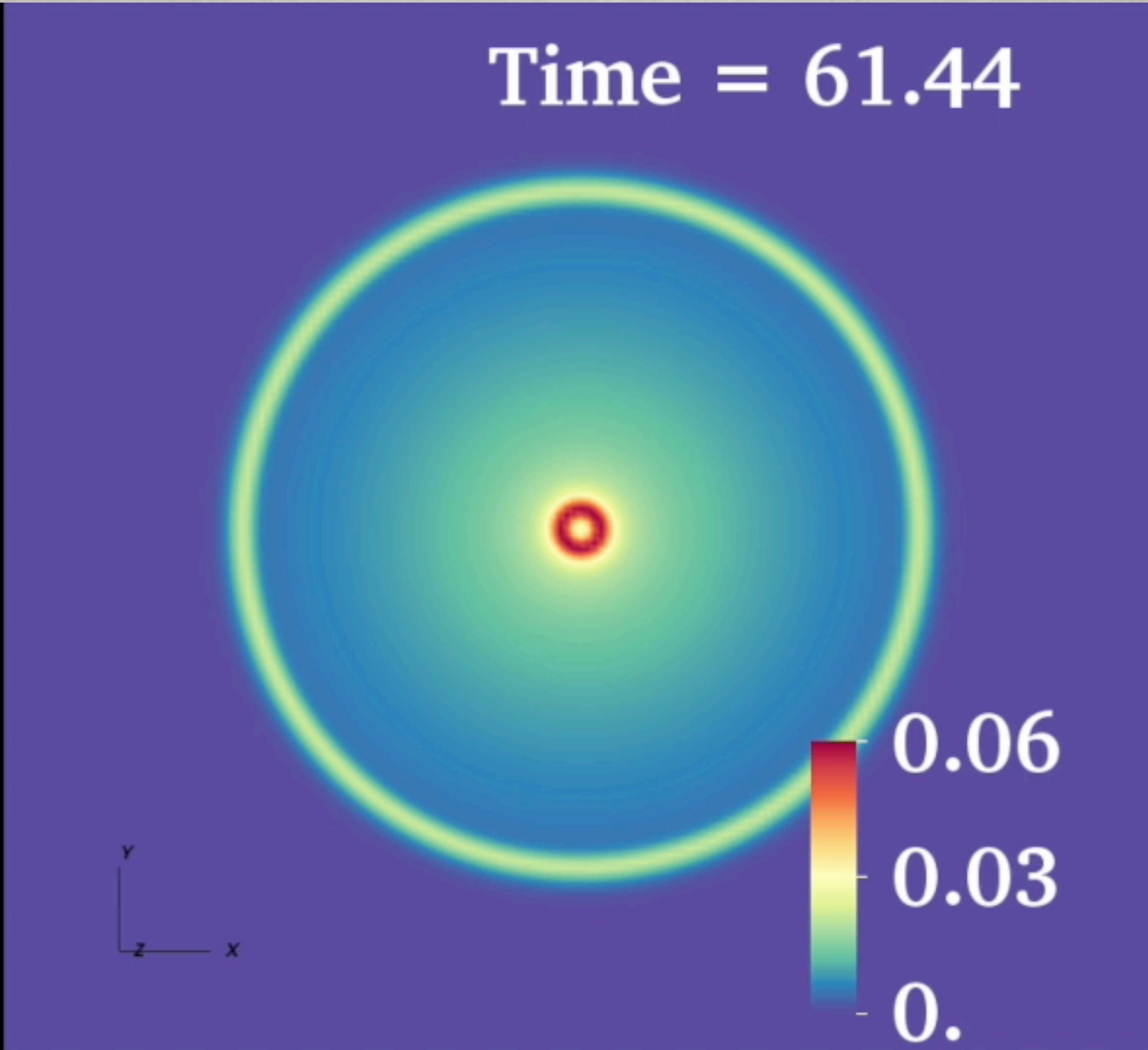
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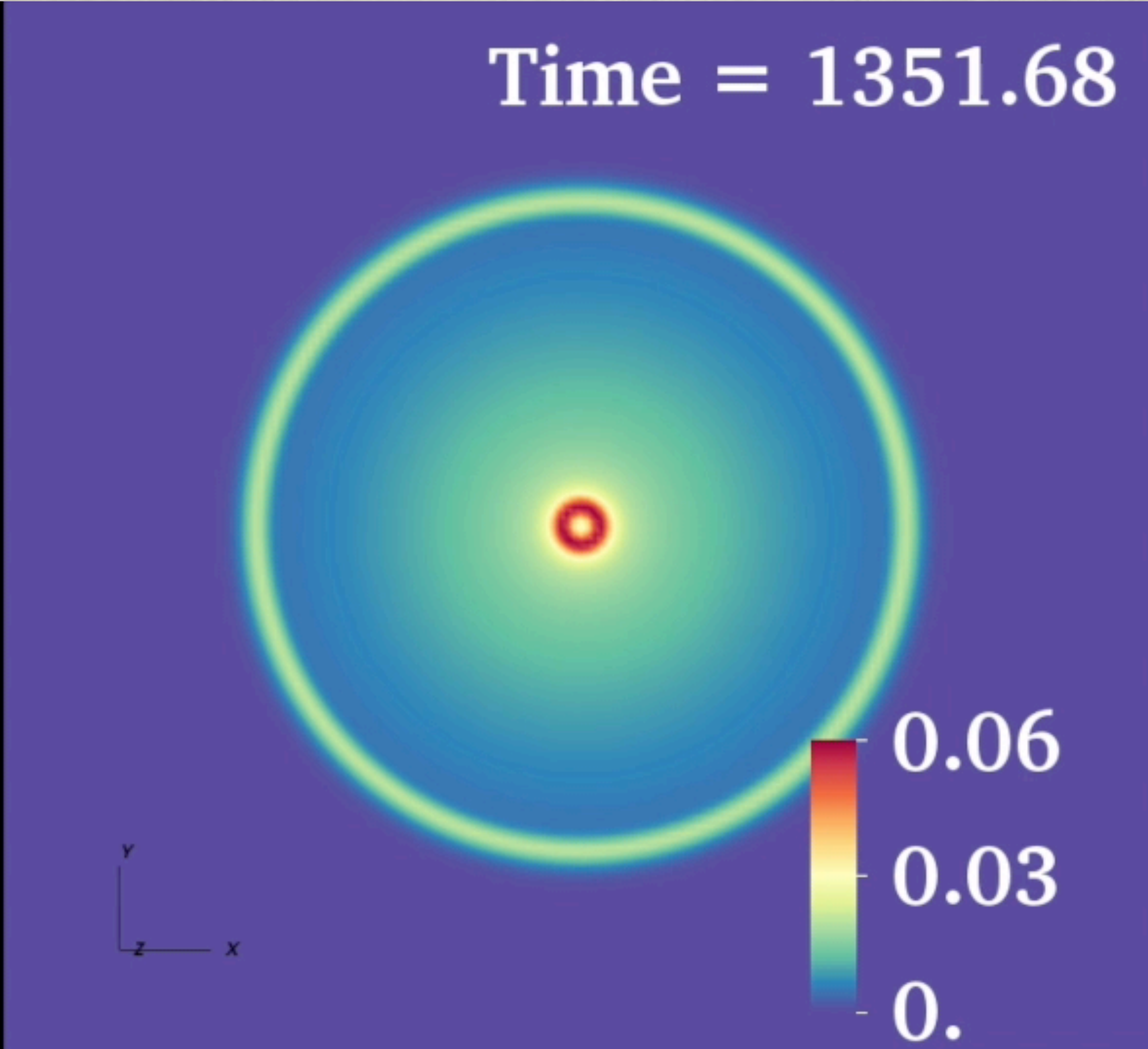
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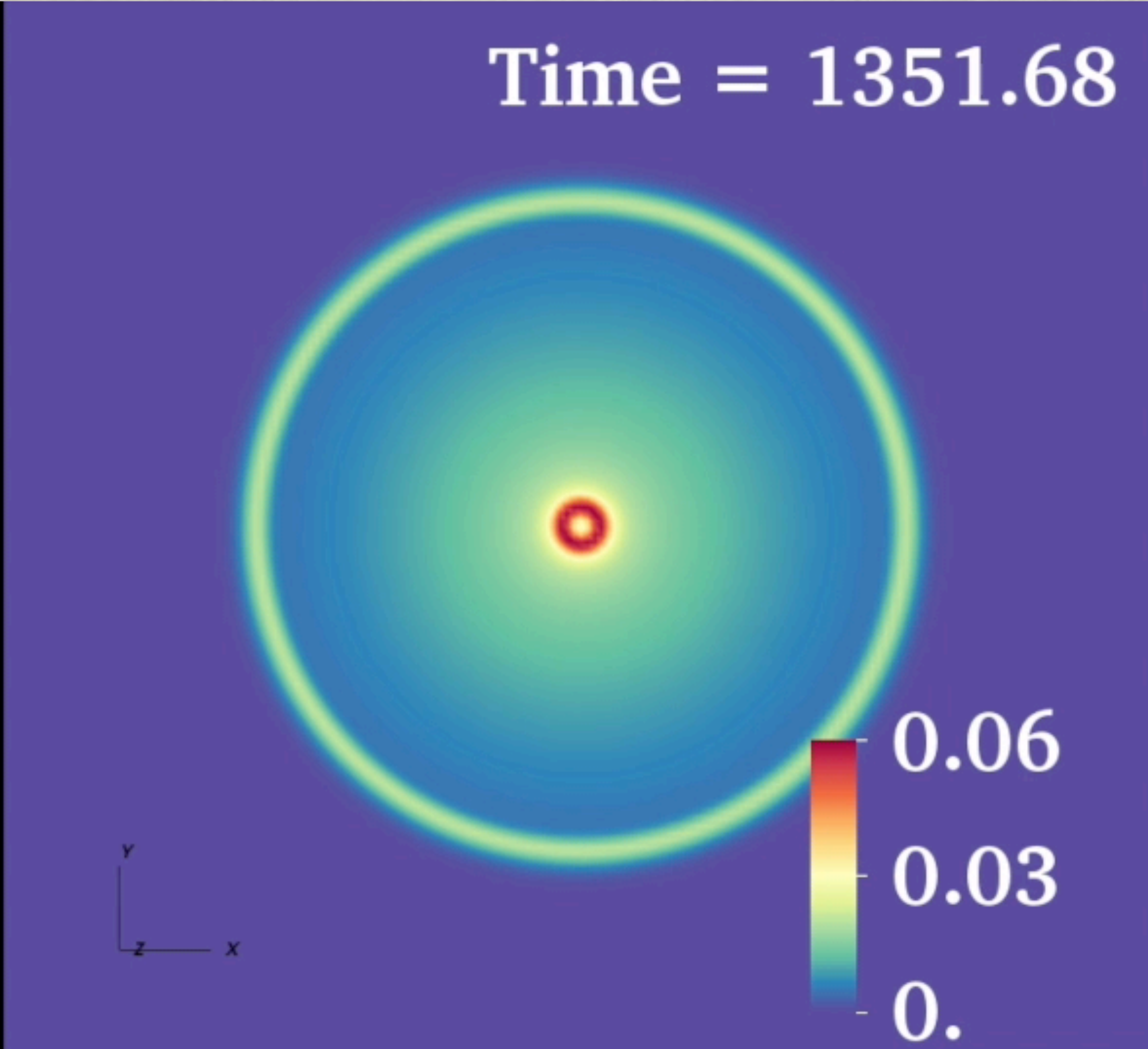

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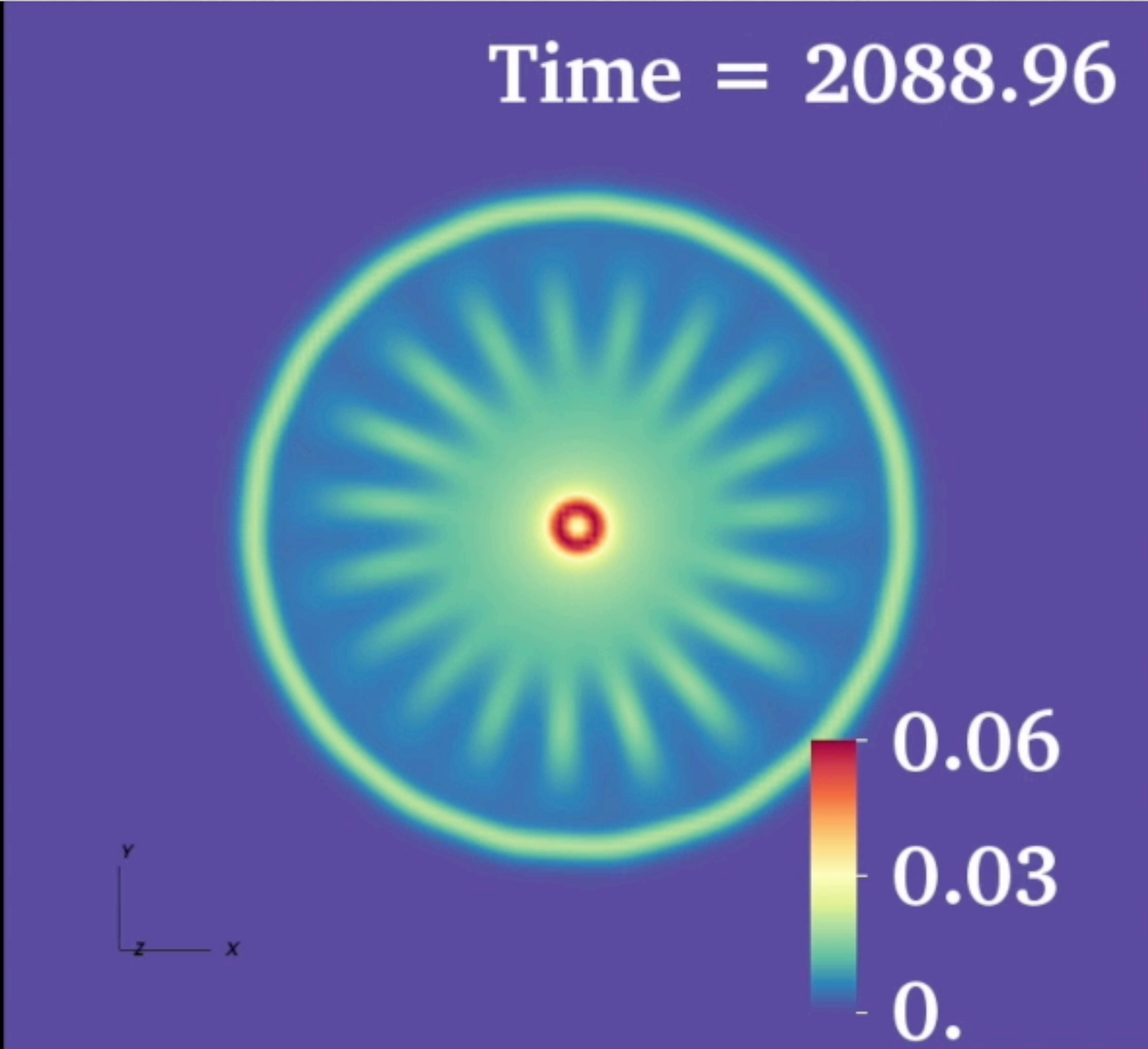

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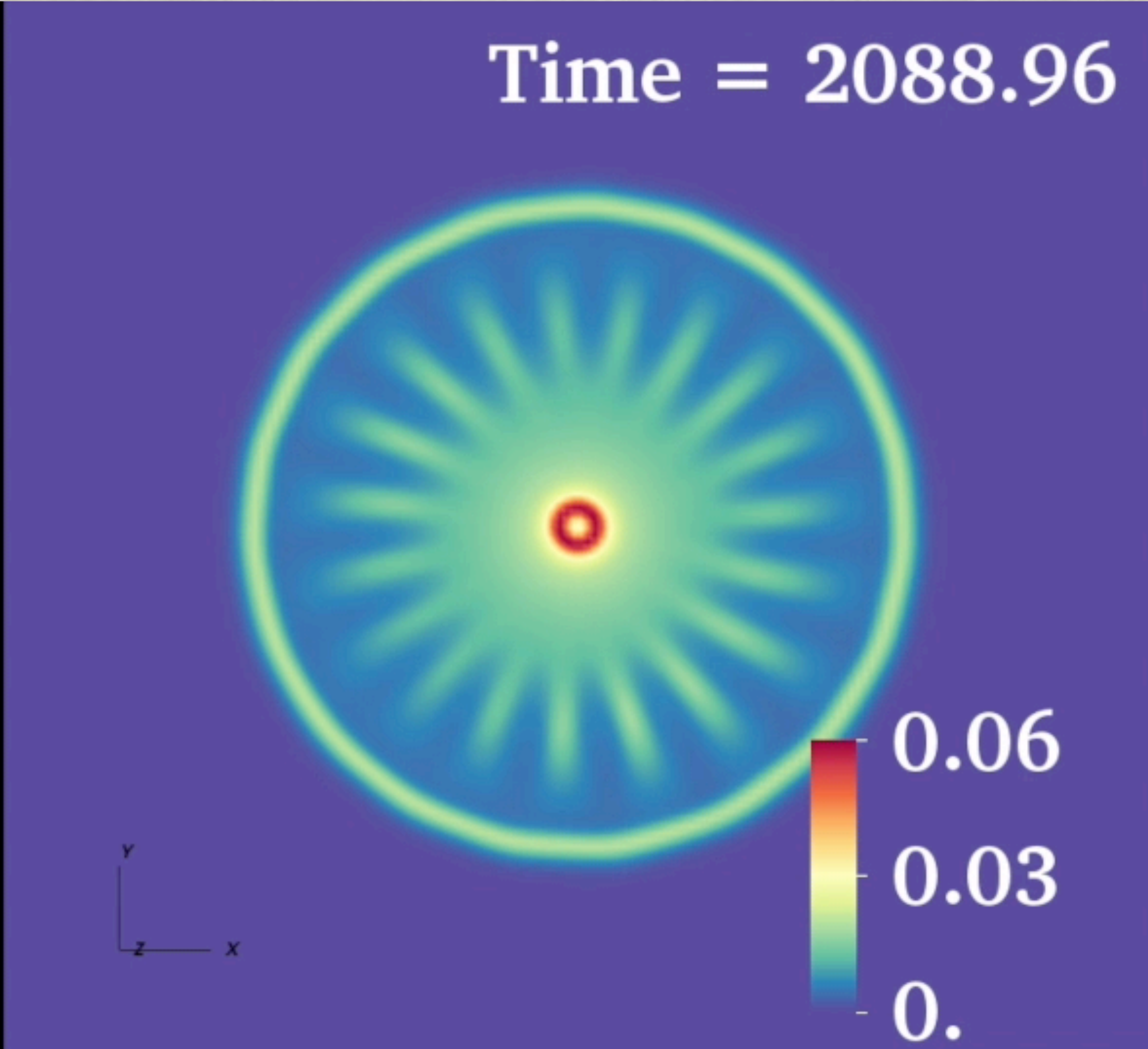
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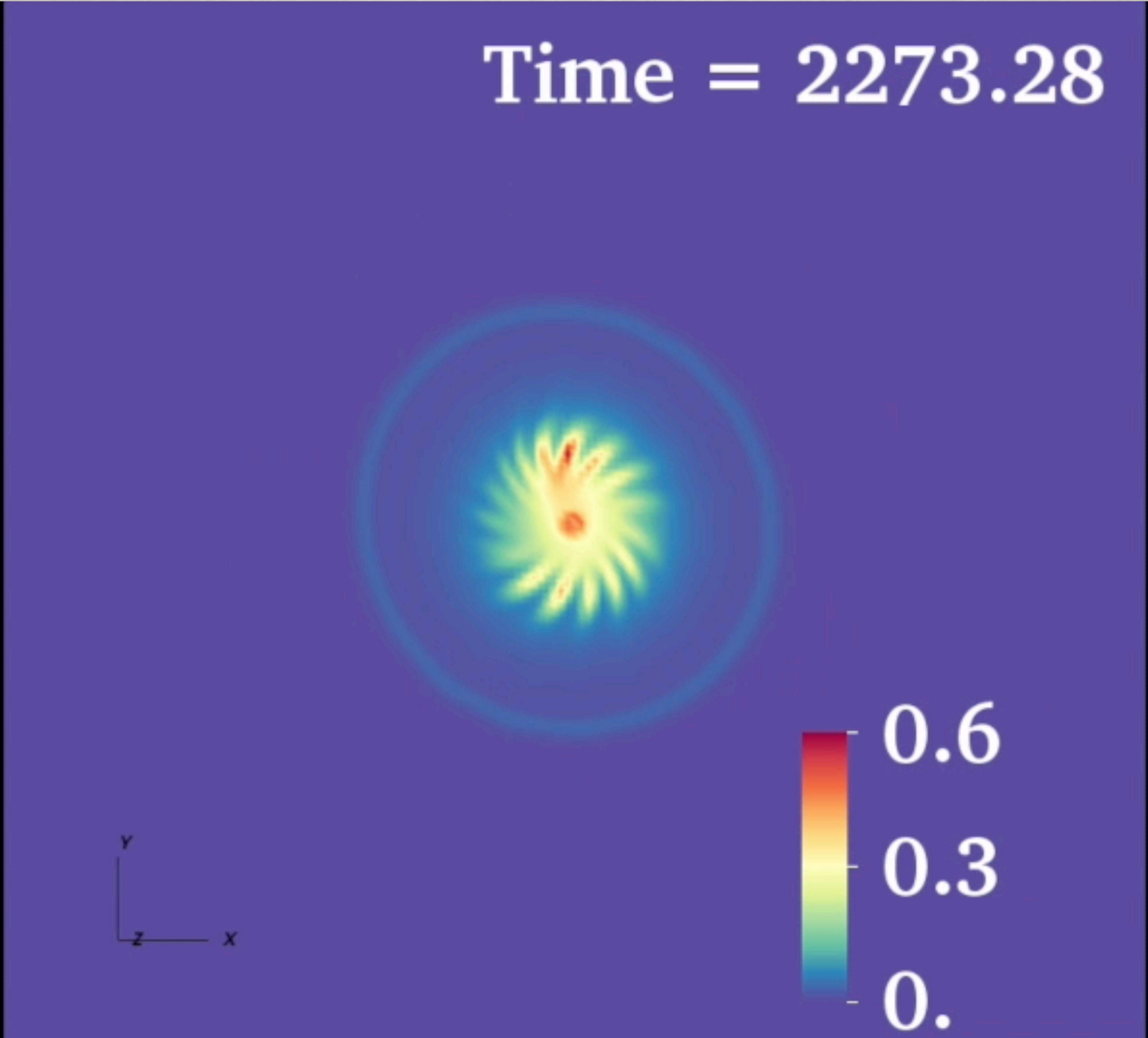

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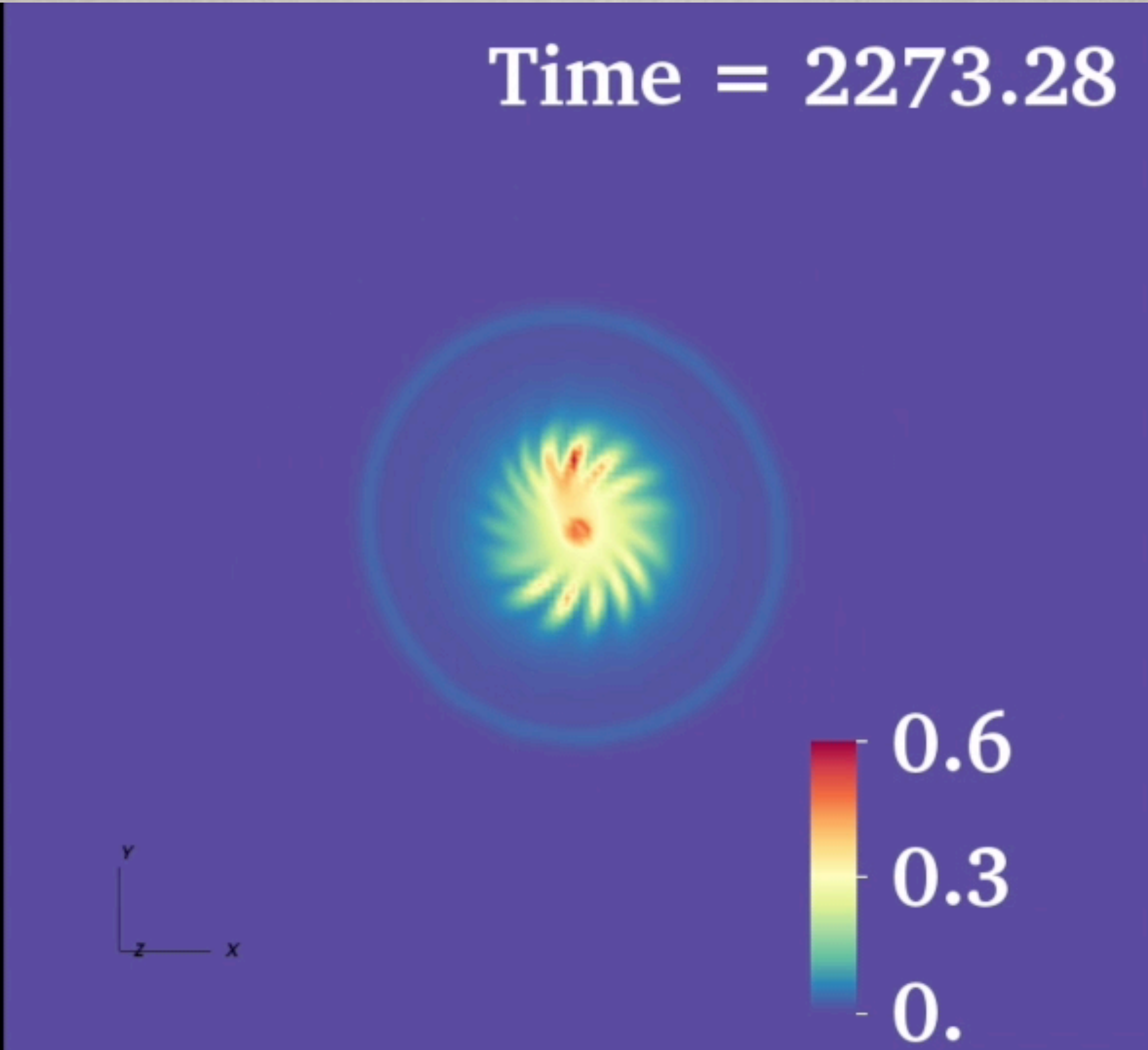
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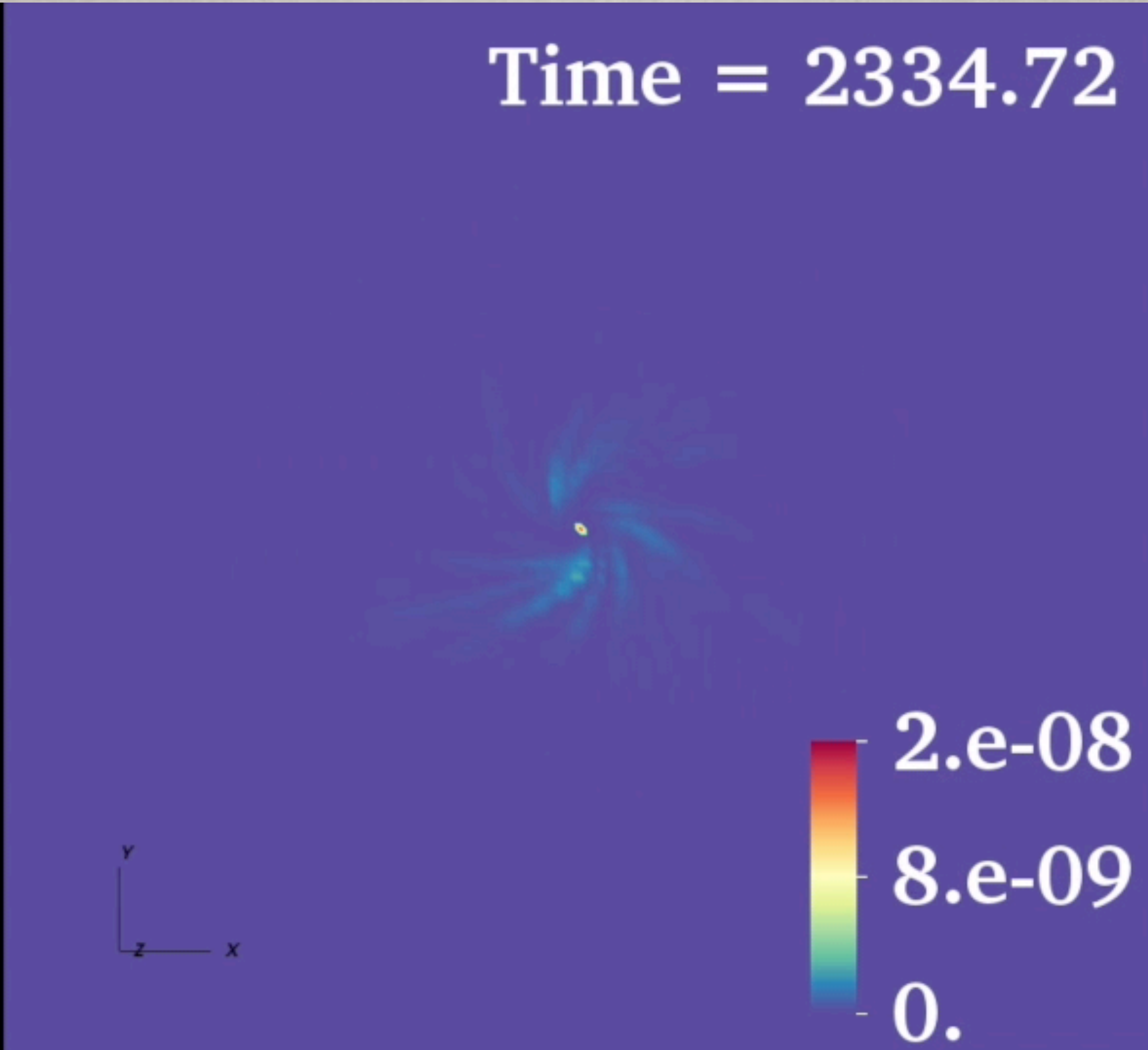
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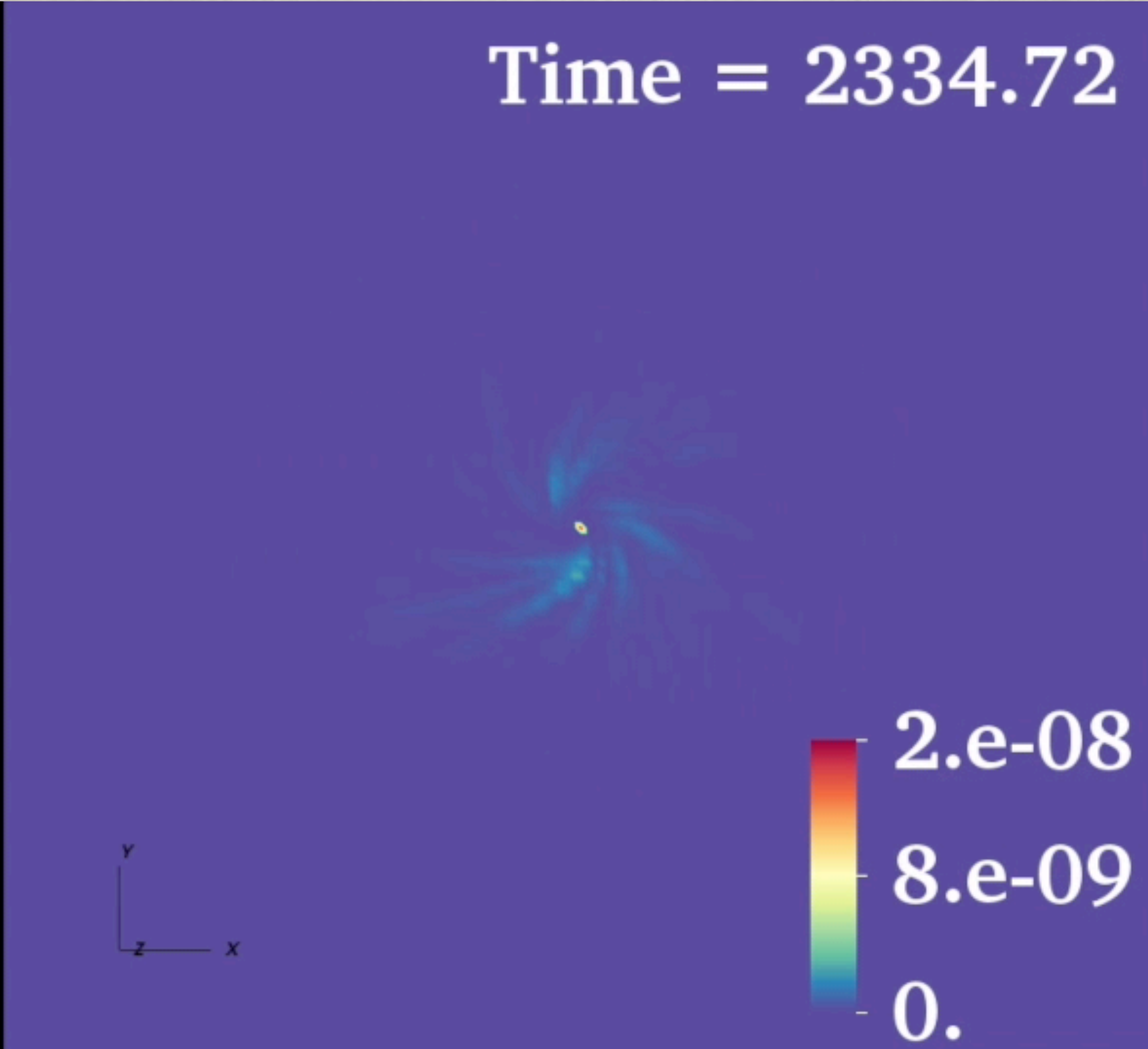

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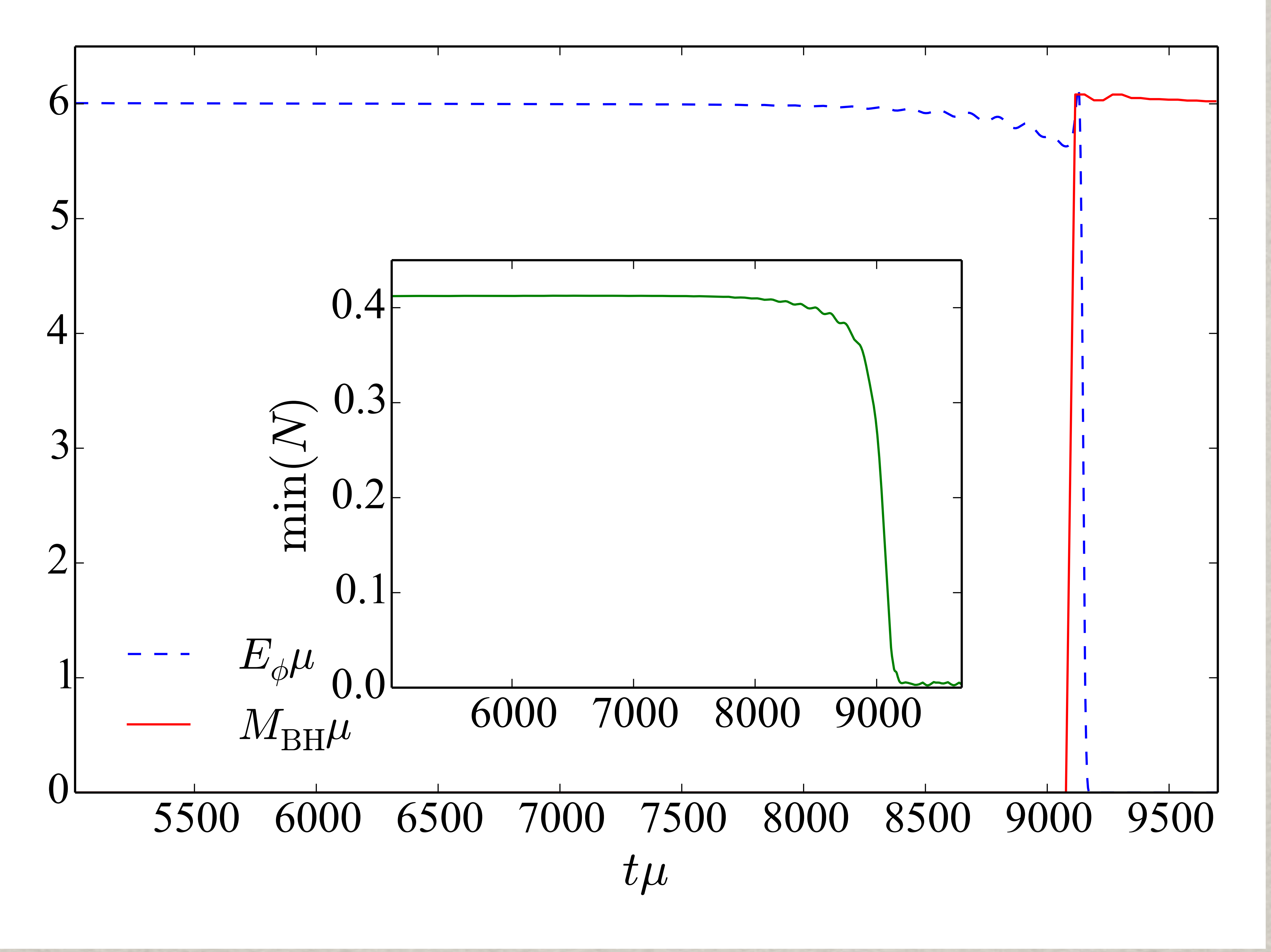

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2
1
0
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6000
6500
7000
7500
8000
8500
9000
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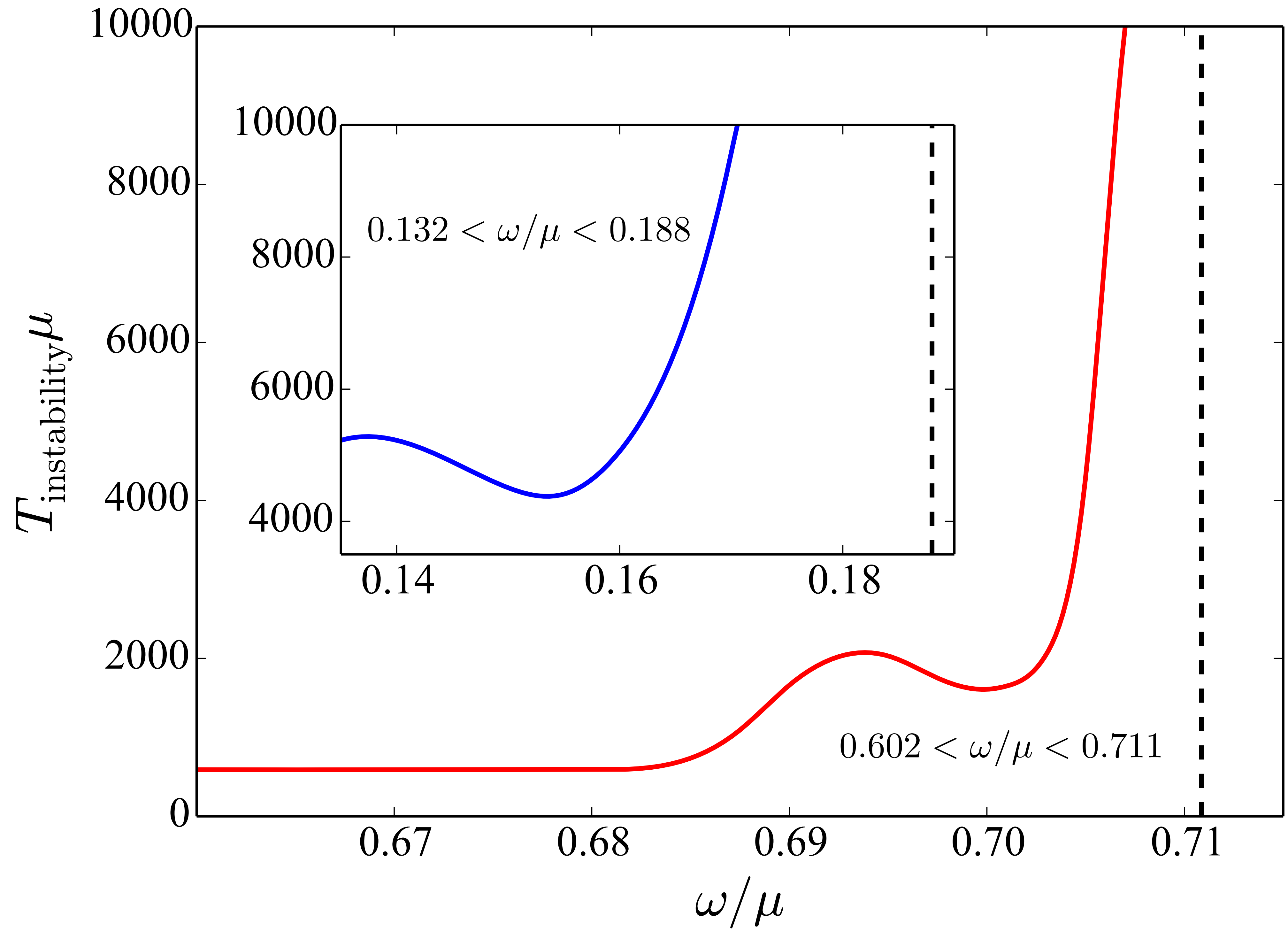
$t\mu$
$E_\phi \mu$
$M_{\rm BH}\mu$
min(N)
0.4
0.3
0.2
0.1
0.0
6000
7000
8000
9000

Now we perform fully non-linear numerical relativity evolutions

Sanchis-Gual, Di Giovanni, Zilhão, CH, P. Cerda-Duran, Font and Radu, Phys. Rev. Lett. 123 (2019) 221101

http://gravitation.web.ua.pt/node/1740



Cunha, CH, Radu and Sanchis-Gual, 2207.13713

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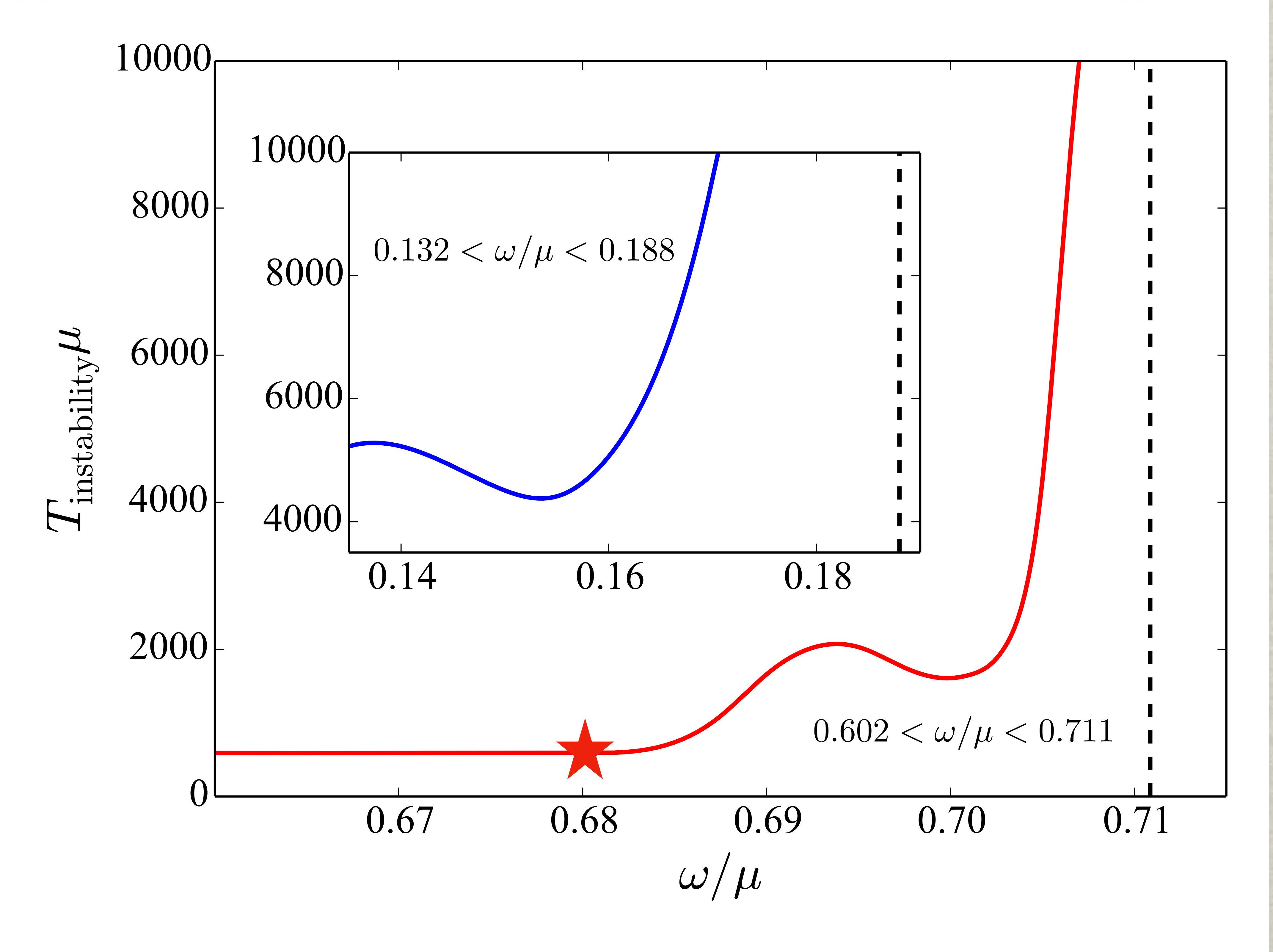

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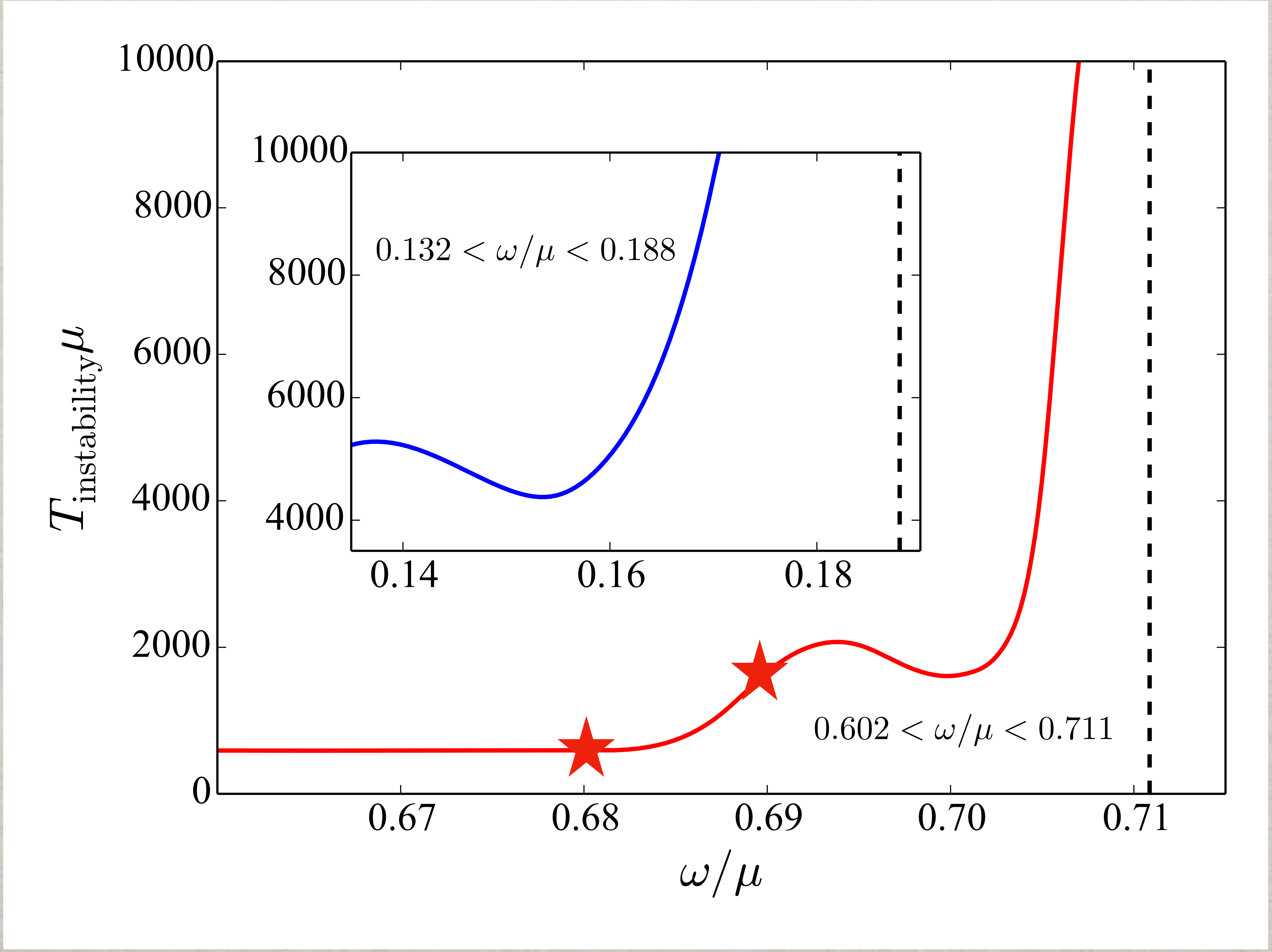

Cunha, CH, Radu and Sanchis-Gual, 2207.13713

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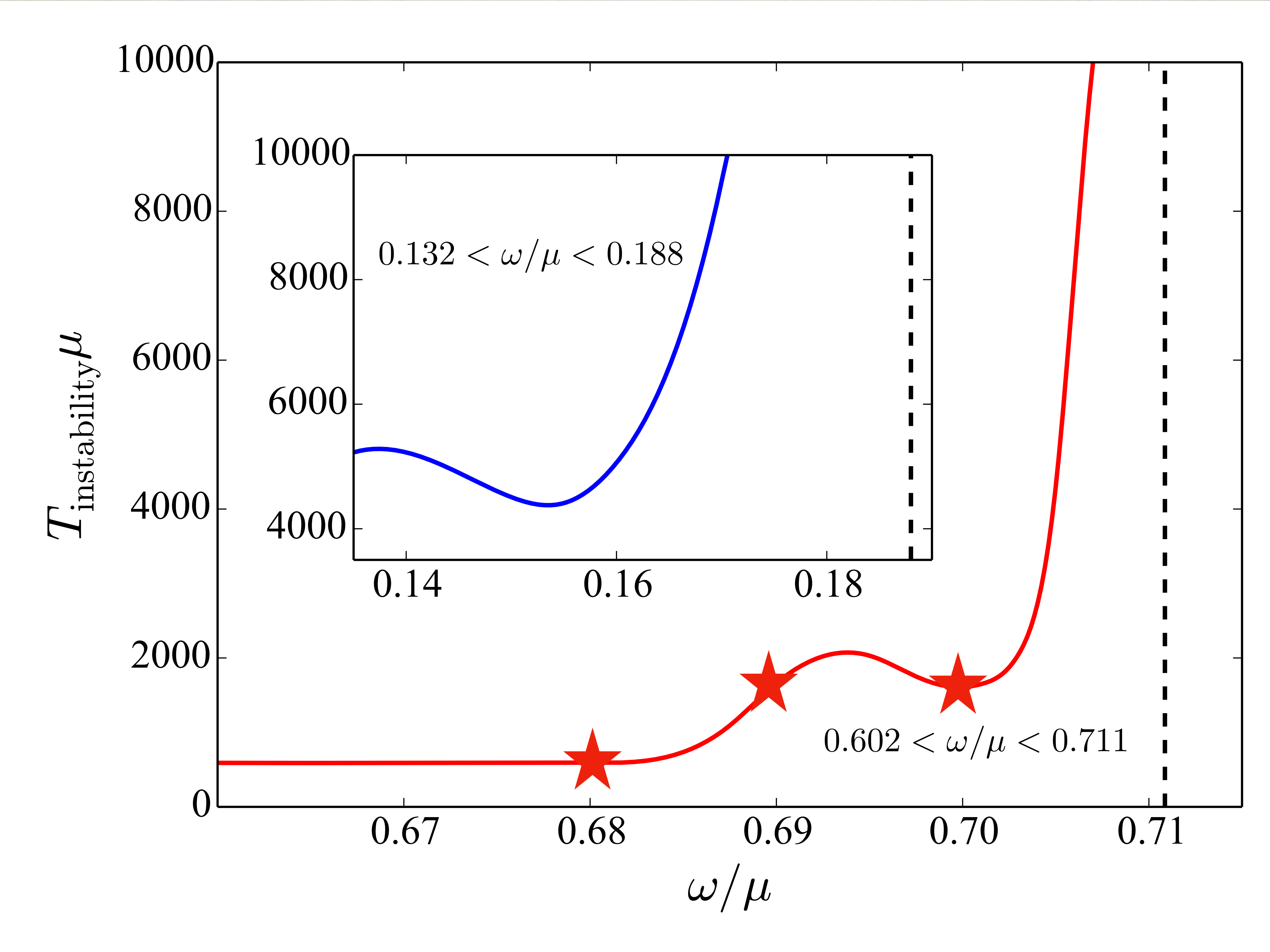
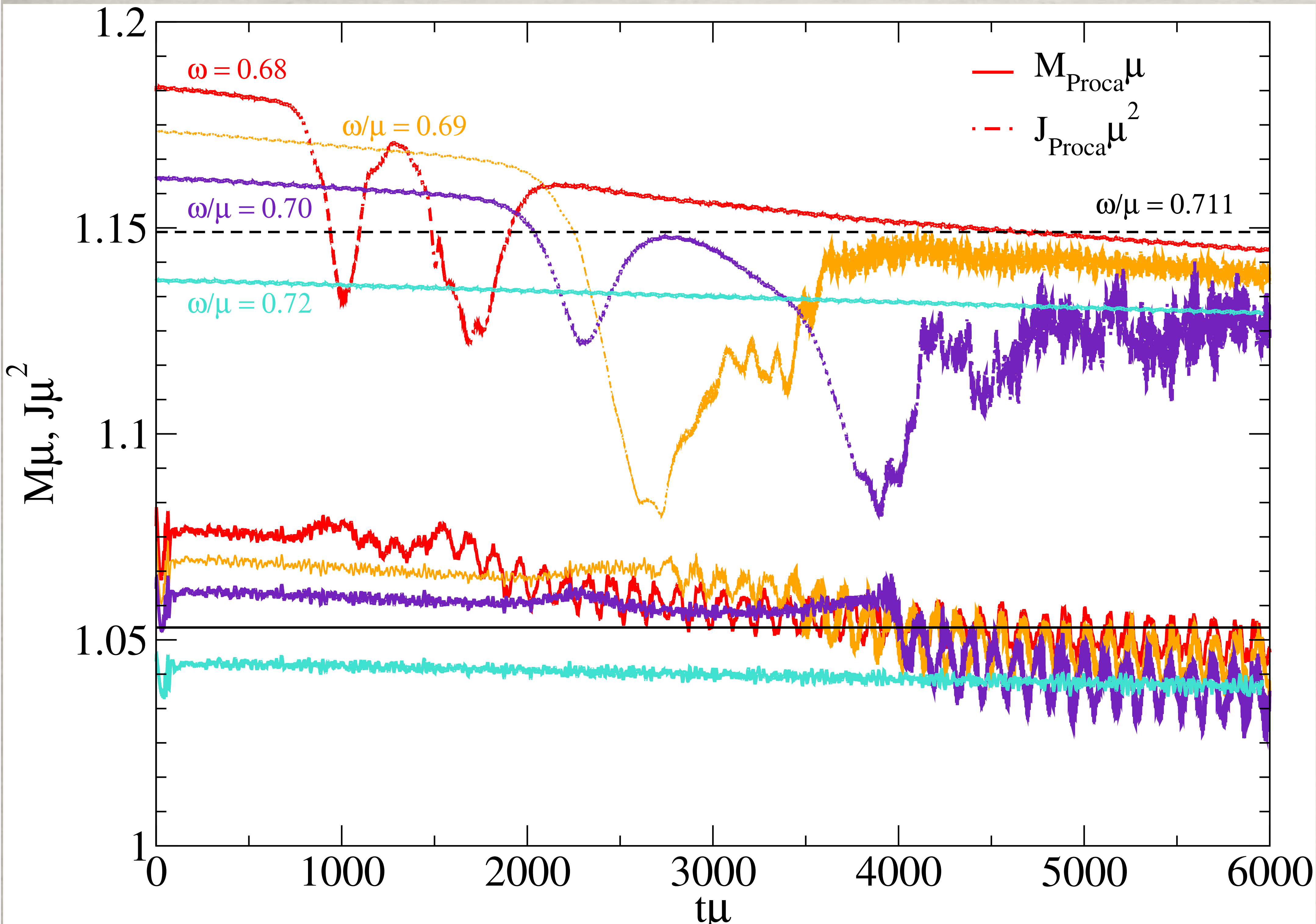
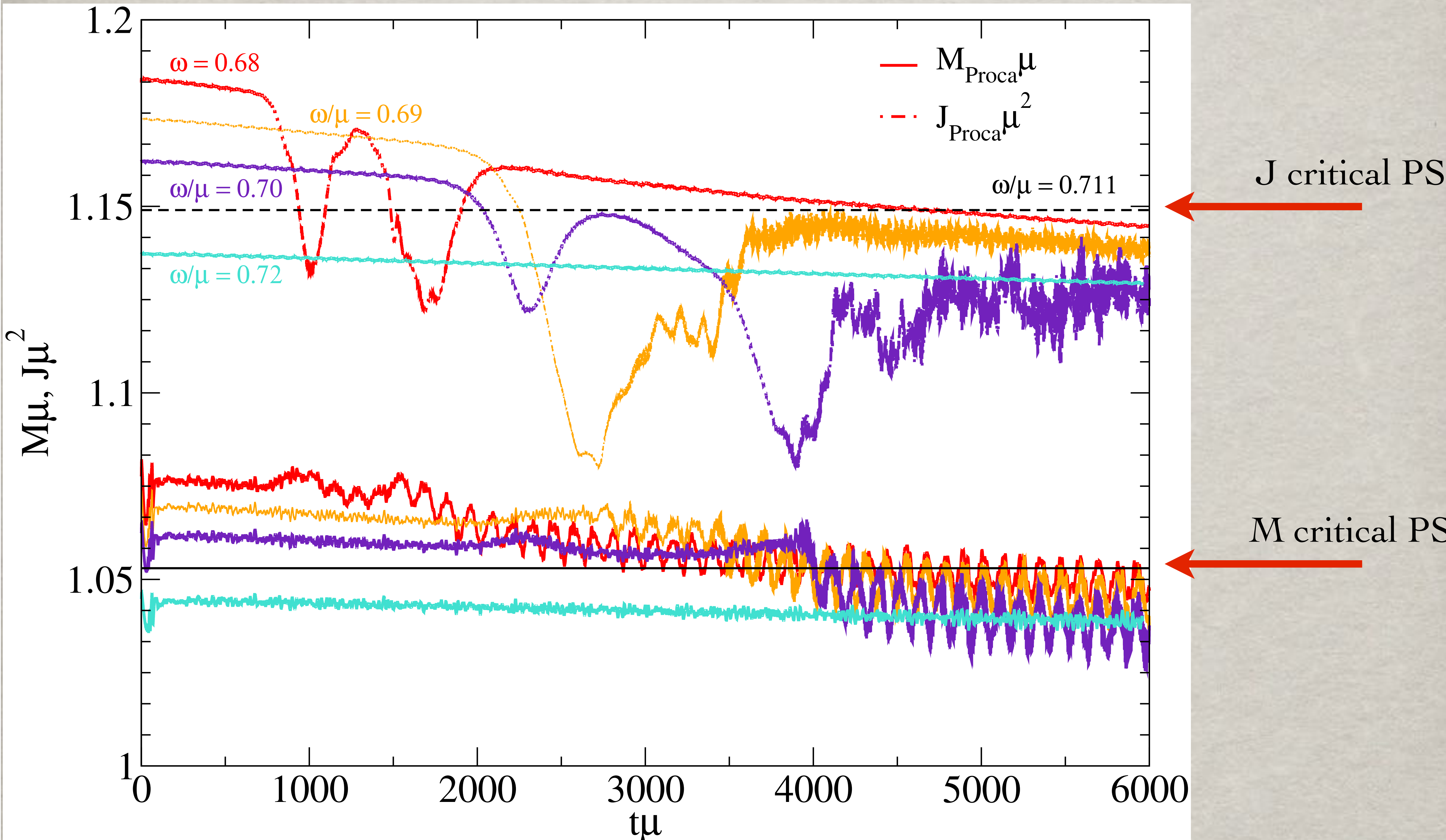


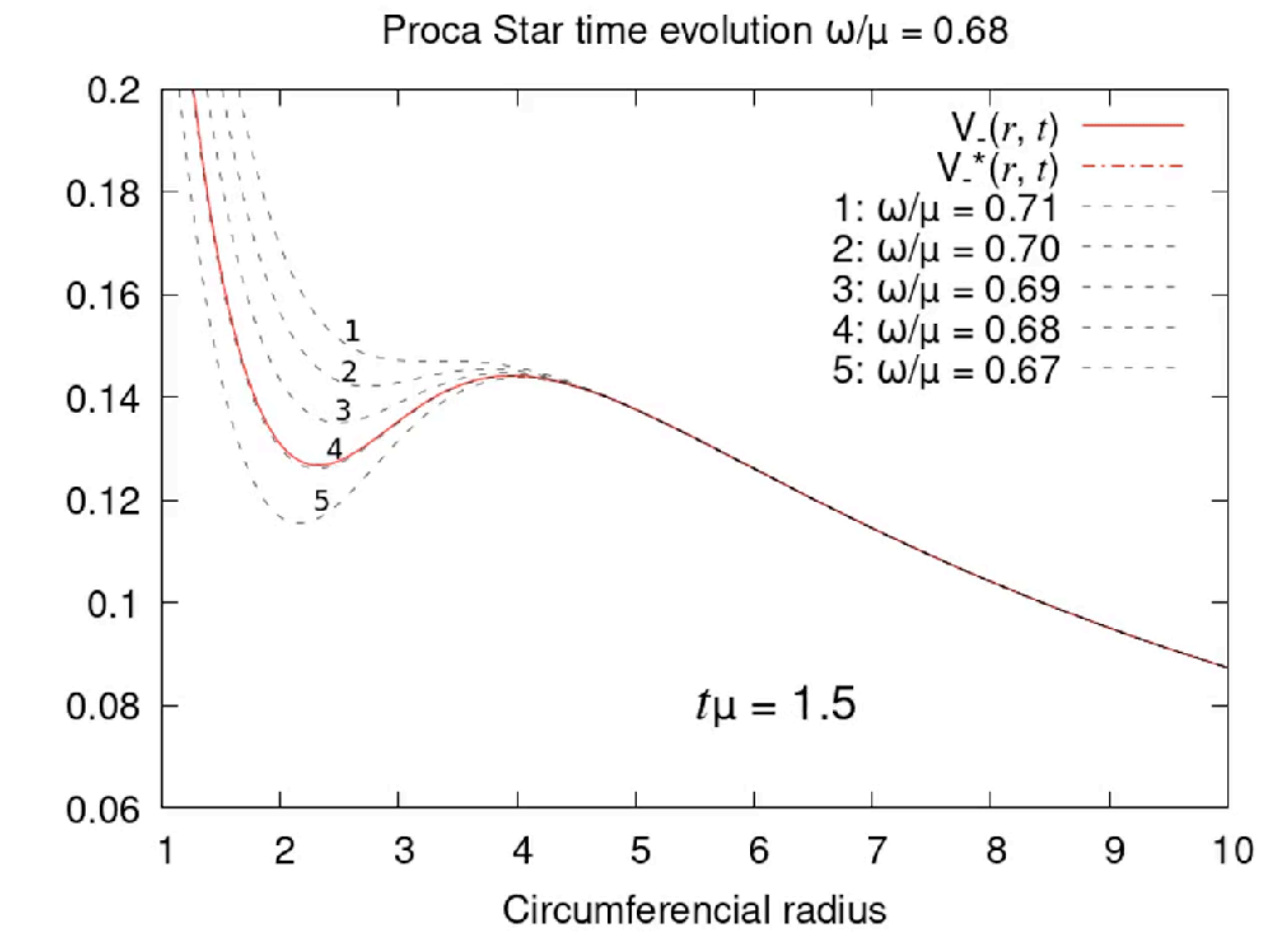
Cunha, CH, Radu and Sanchis-Gual, 2207.13713



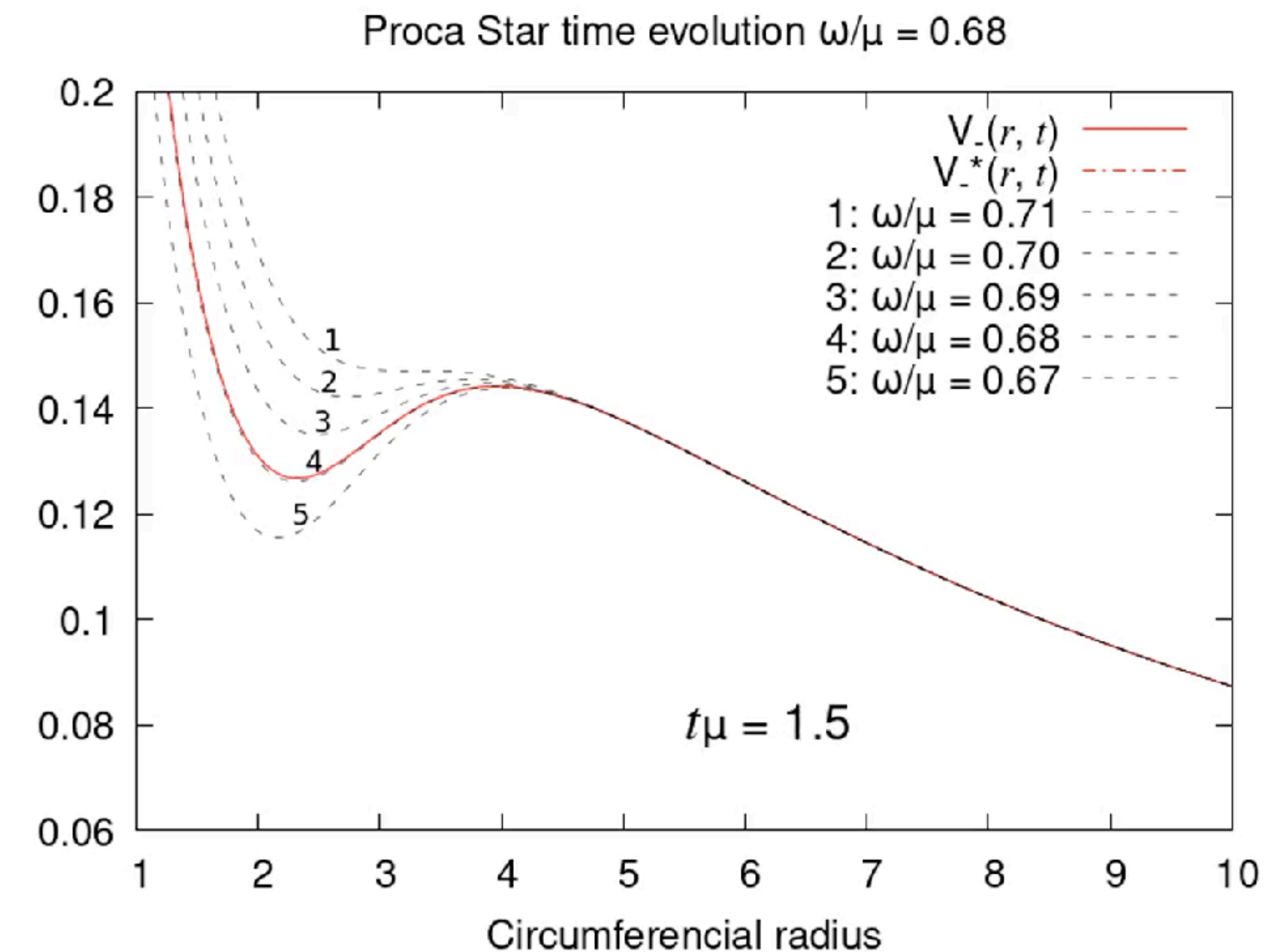
ω = 0.68
ω/μ = 0.69
ω/μ = 0.70
ω/μ = 0.72
ω/μ = 0.711
$M_{Proca}\mu$
$J_{Proca}\mu^2$
$M\mu$, $J\mu^2$
tμ
1
1.05
1.1
1.15
1.2
0
1000
2000
3000
4000
5000
6000



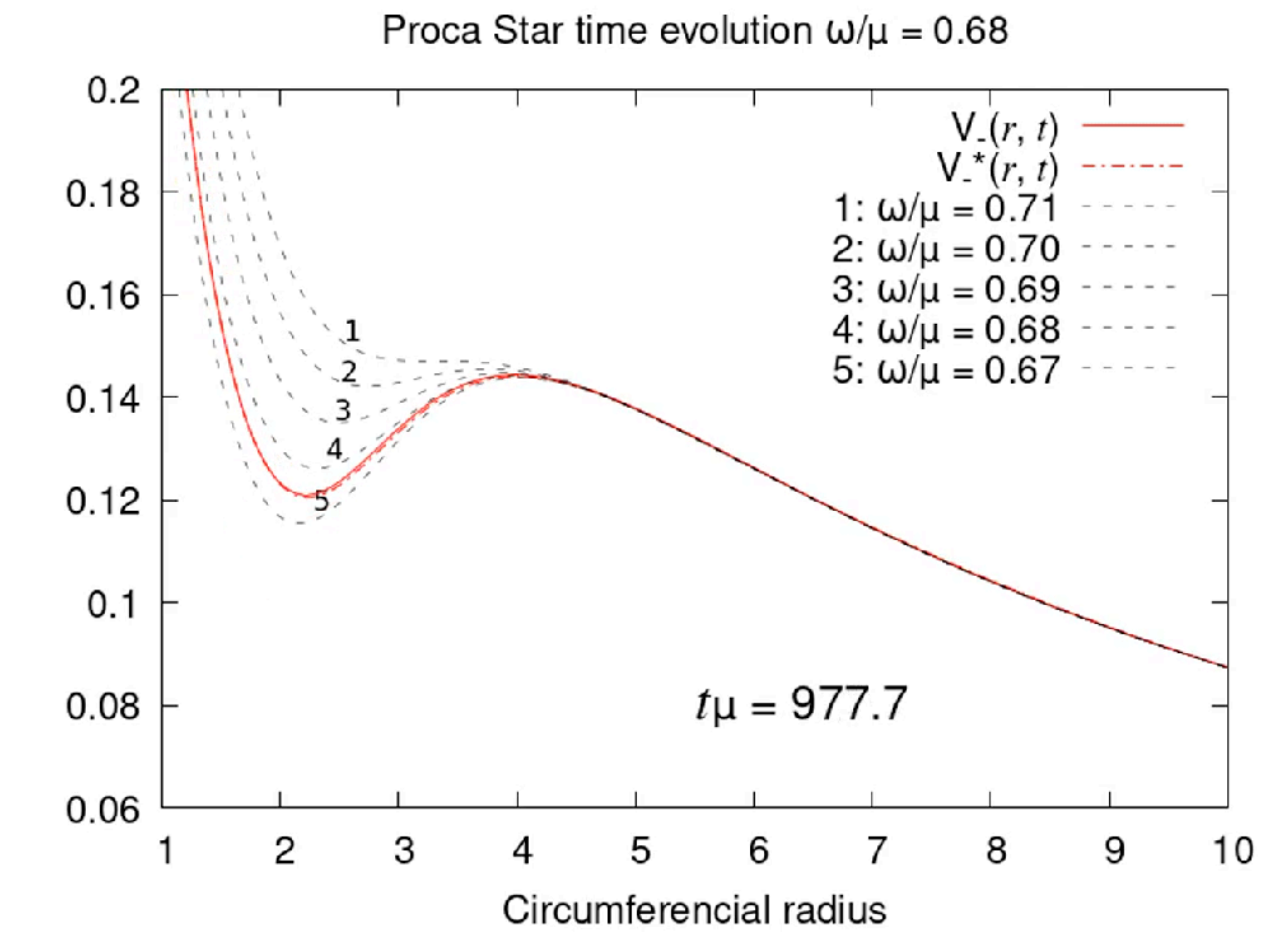
ω = 0.68
ω/μ = 0.69
ω/μ = 0.70
ω/μ = 0.72
ω/μ = 0.711
$M_{Proca}\mu$
$J_{Proca}\mu^2$
J critical PS
M critical PS
Mμ, Jμ²
tμ
1.2
1.15
1.1
1.05
1
0
1000
2000
3000
4000
5000
6000



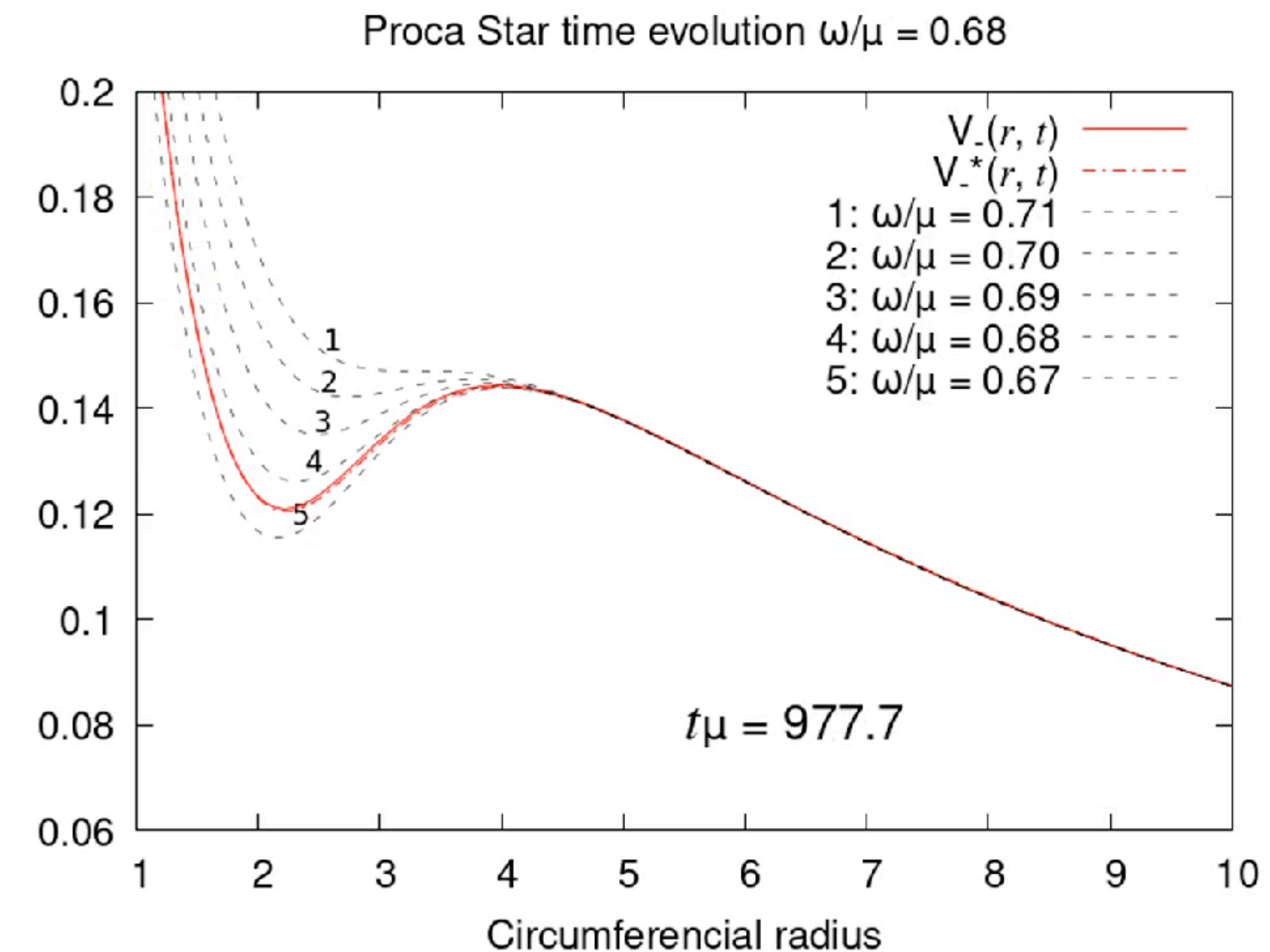
Proca Star time evolution ω/μ = 0.68
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
1
2
3
4
5
6
7
8
9
10
V-(r, t)
V-*(r, t)
1: ω/μ = 0.71
2: ω/μ = 0.70
3: ω/μ = 0.69
4: ω/μ = 0.68
5: ω/μ = 0.67
tμ = 1.5
Circumferencial radius

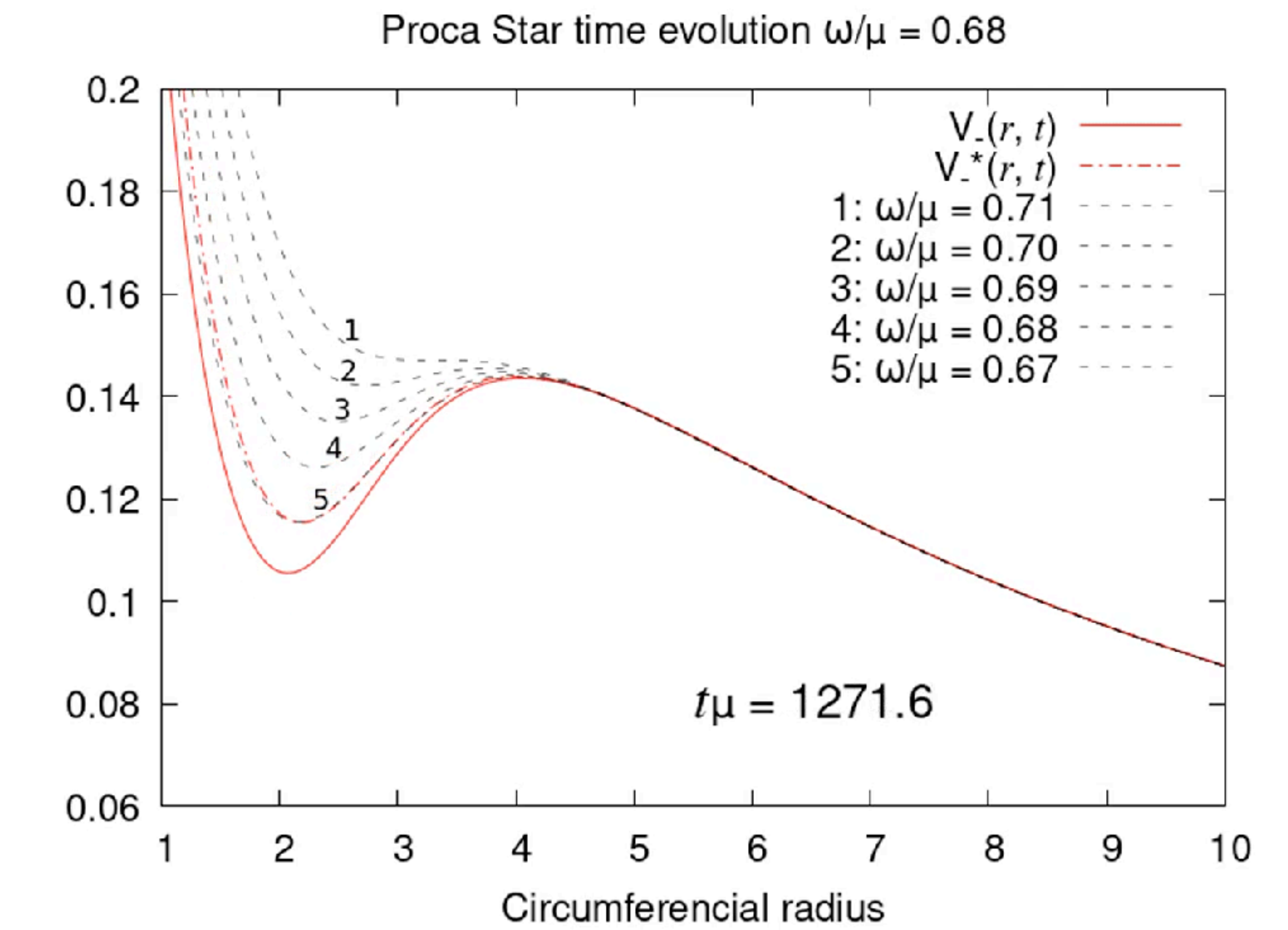


Proca Star time evolution ω/μ = 0.68
V₋(r, t)
V₋*(r, t)
1: ω/μ = 0.71
2: ω/μ = 0.70
3: ω/μ = 0.69
4: ω/μ = 0.68
5: ω/μ = 0.67
1
2
3
4
5
tμ = 1.5
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
1
2
3
4
5
6
7
8
9
10
Circumferencial radius

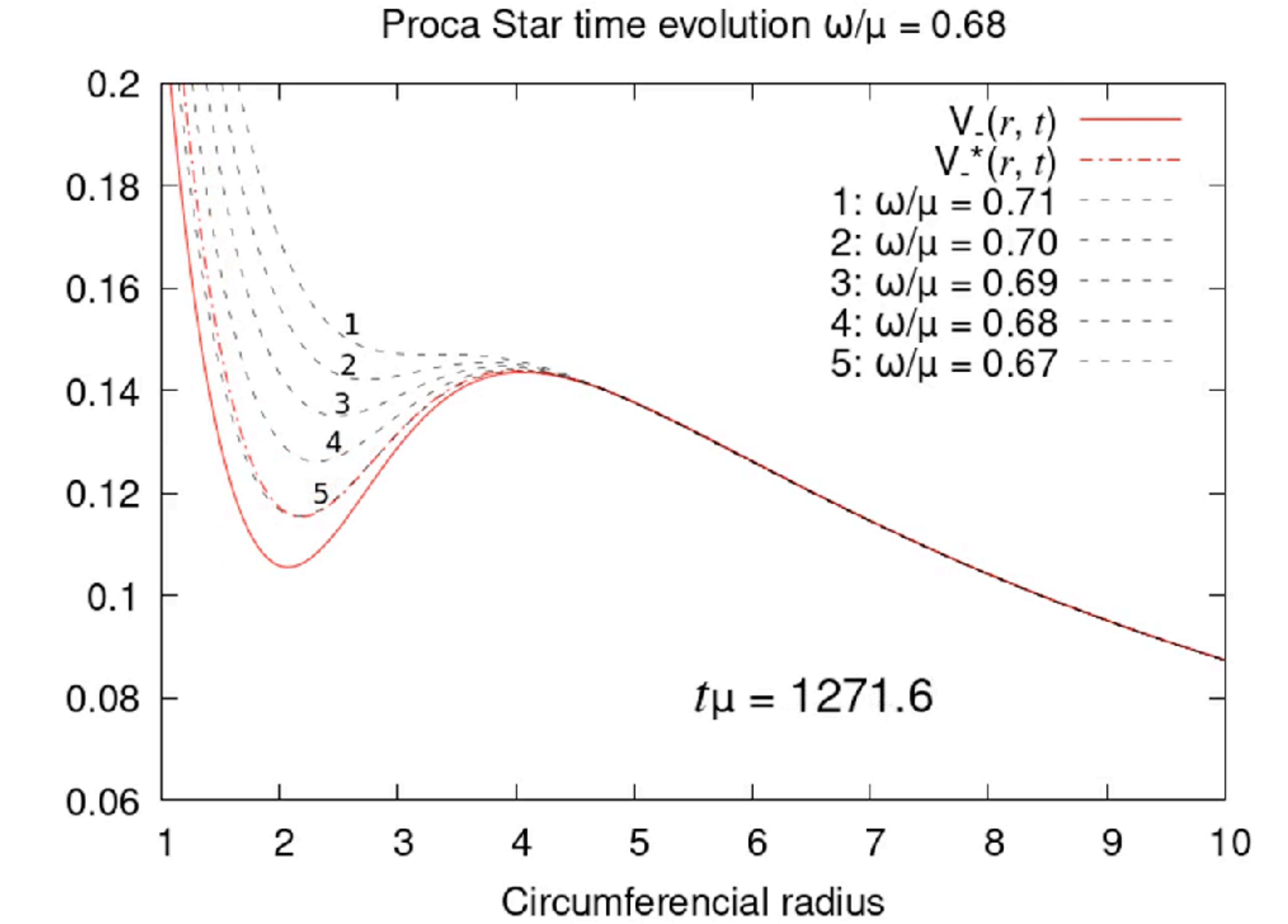


Proca Star time evolution ω/μ = 0.68
V-(r, t)
V-*(r, t)
1: ω/μ = 0.71
2: ω/μ = 0.70
3: ω/μ = 0.69
4: ω/μ = 0.68
5: ω/μ = 0.67
1
2
3
4
5
tμ = 977.7
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
1
2
3
4
5
6
7
8
9
10
Circumferencial radius

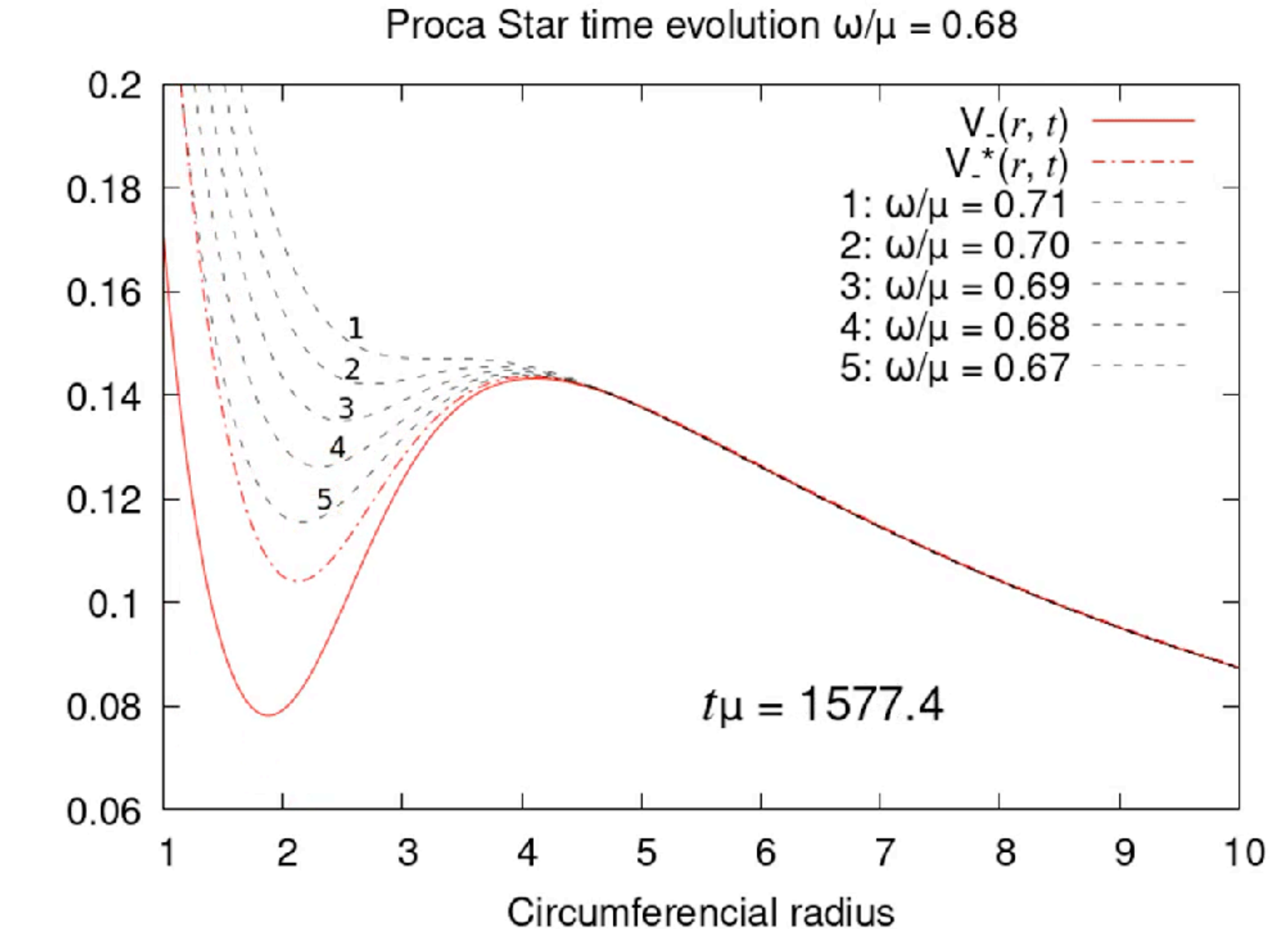

Proca Star time evolution ω/μ = 0.68
V-(r, t)
V-*(r, t)
1: ω/μ = 0.71
2: ω/μ = 0.70
3: ω/μ = 0.69
4: ω/μ = 0.68
5: ω/μ = 0.67
1
2
3
4
5
tμ = 977.7
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
1
2
3
4
5
6
7
8
9
10
Circumferencial radius



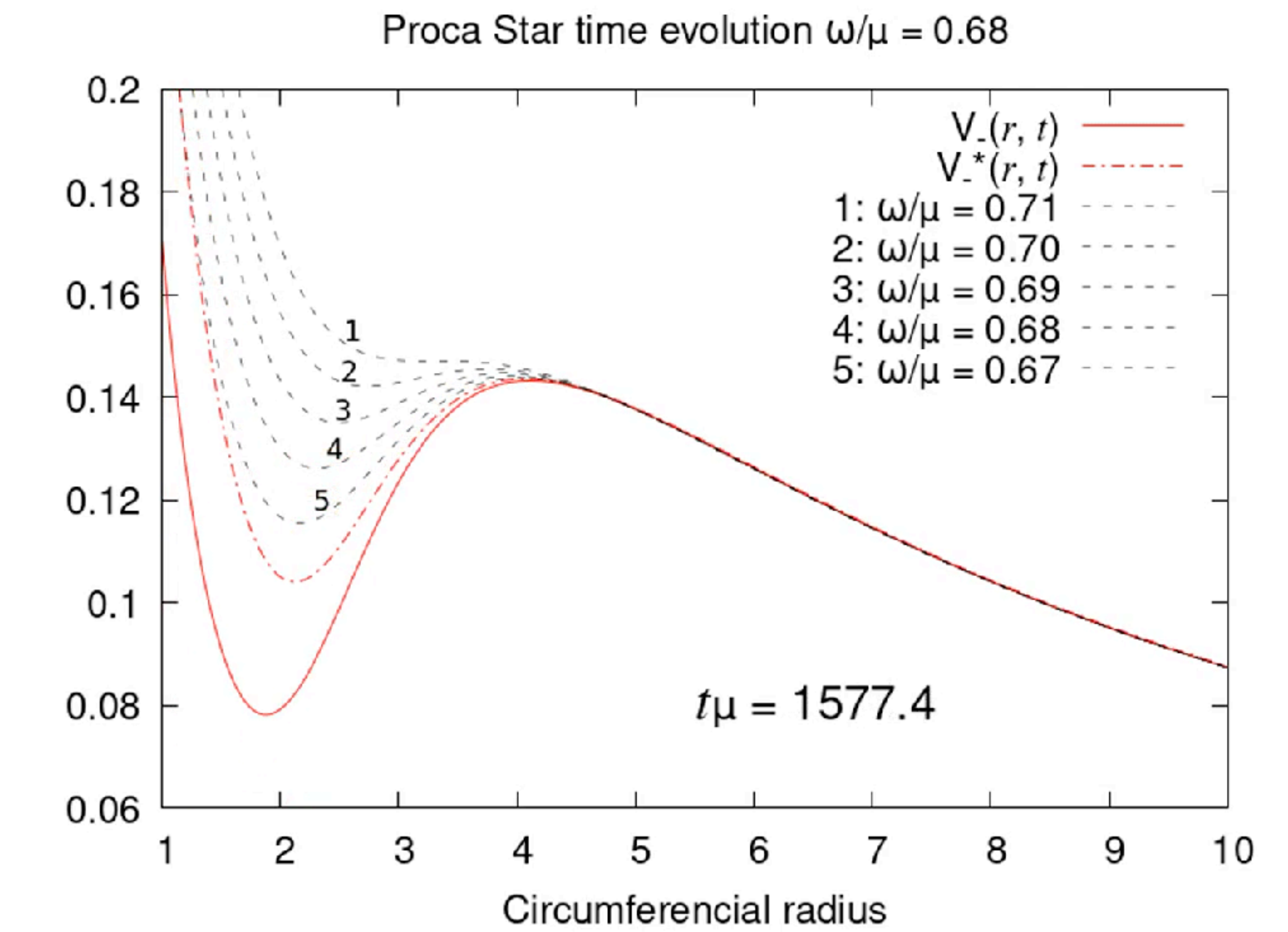
Proca Star time evolution ω/μ = 0.68
V₋(r, t)
V₋*(r, t)
1: ω/μ = 0.71
2: ω/μ = 0.70
3: ω/μ = 0.69
4: ω/μ = 0.68
5: ω/μ = 0.67
1
2
3
4
5
tμ = 1271.6
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
1
2
3
4
5
6
7
8
9
10
Circumferencial radius



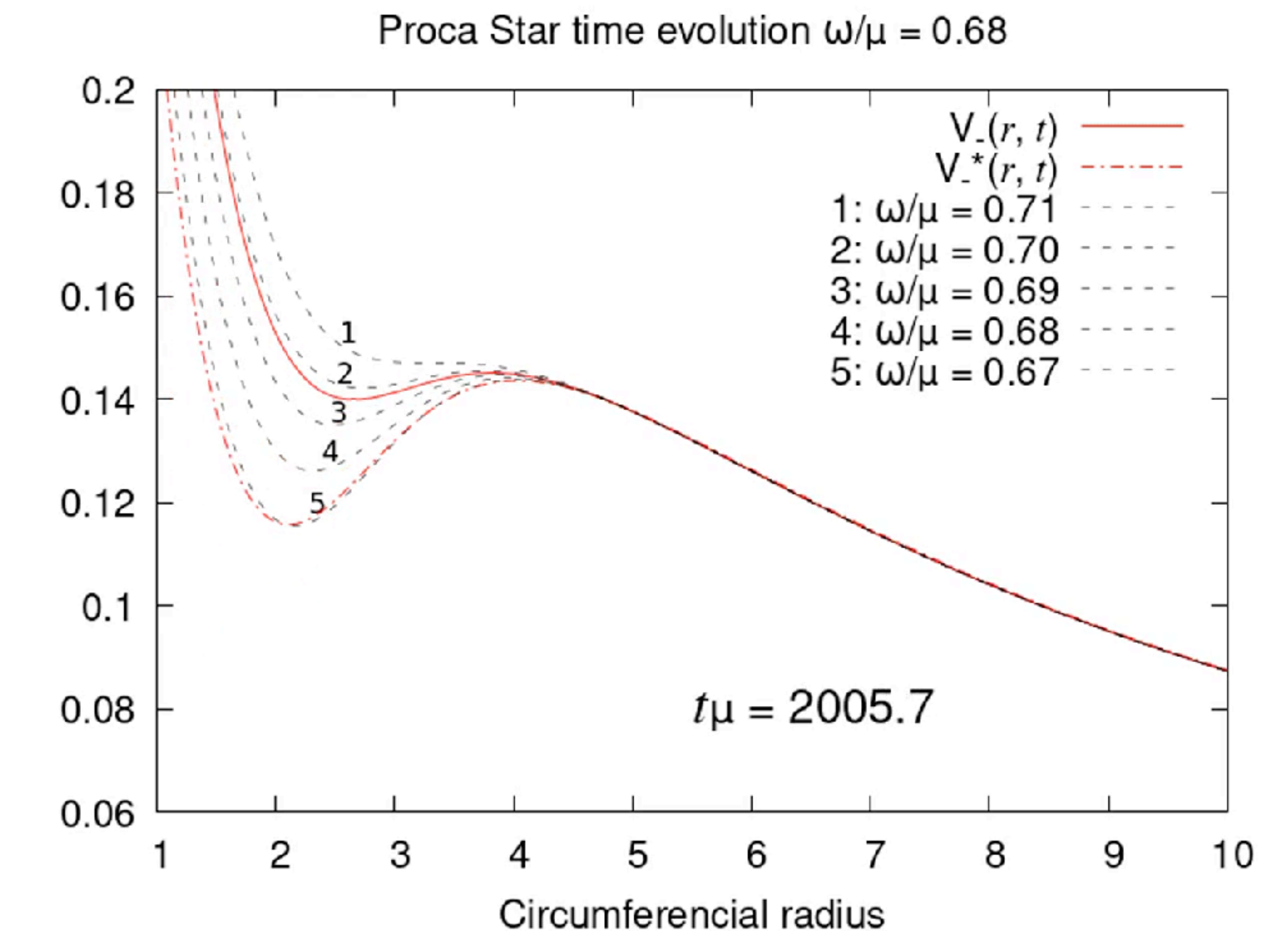
Proca Star time evolution ω/μ = 0.68
V-(r, t)
V-*(r, t)
1: ω/μ = 0.71
2: ω/μ = 0.70
3: ω/μ = 0.69
4: ω/μ = 0.68
5: ω/μ = 0.67
1
2
3
4
5
tμ = 1271.6
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
1
2
3
4
5
6
7
8
9
10
Circumferencial radius



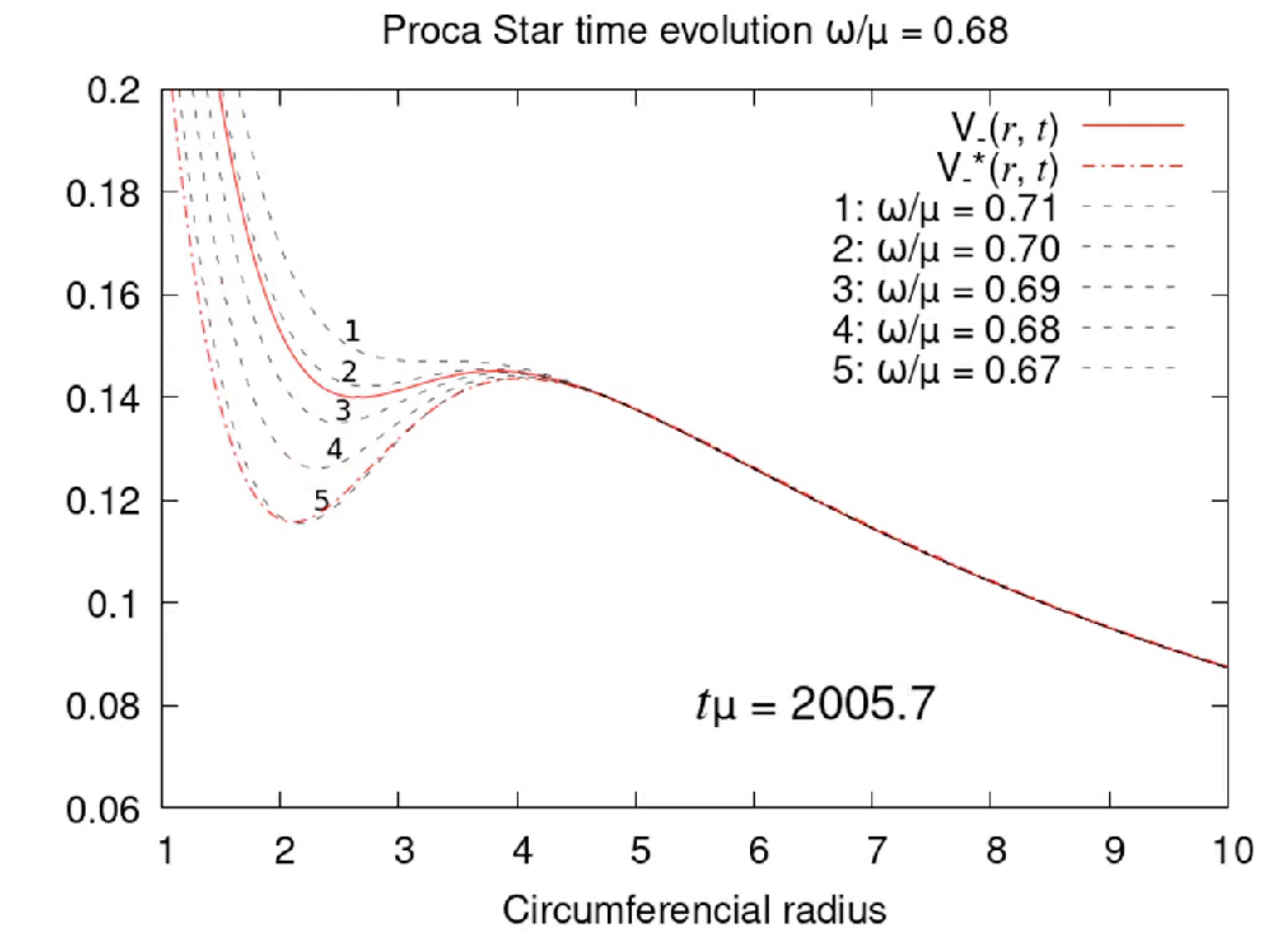
Proca Star time evolution ω/μ = 0.68
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
1
2
3
4
5
6
7
8
9
10
V-(r, t)
V-*(r, t)
1: ω/μ = 0.71
2: ω/μ = 0.70
3: ω/μ = 0.69
4: ω/μ = 0.68
5: ω/μ = 0.67
1
2
3
4
5
tμ = 1577.4
Circumferencial radius



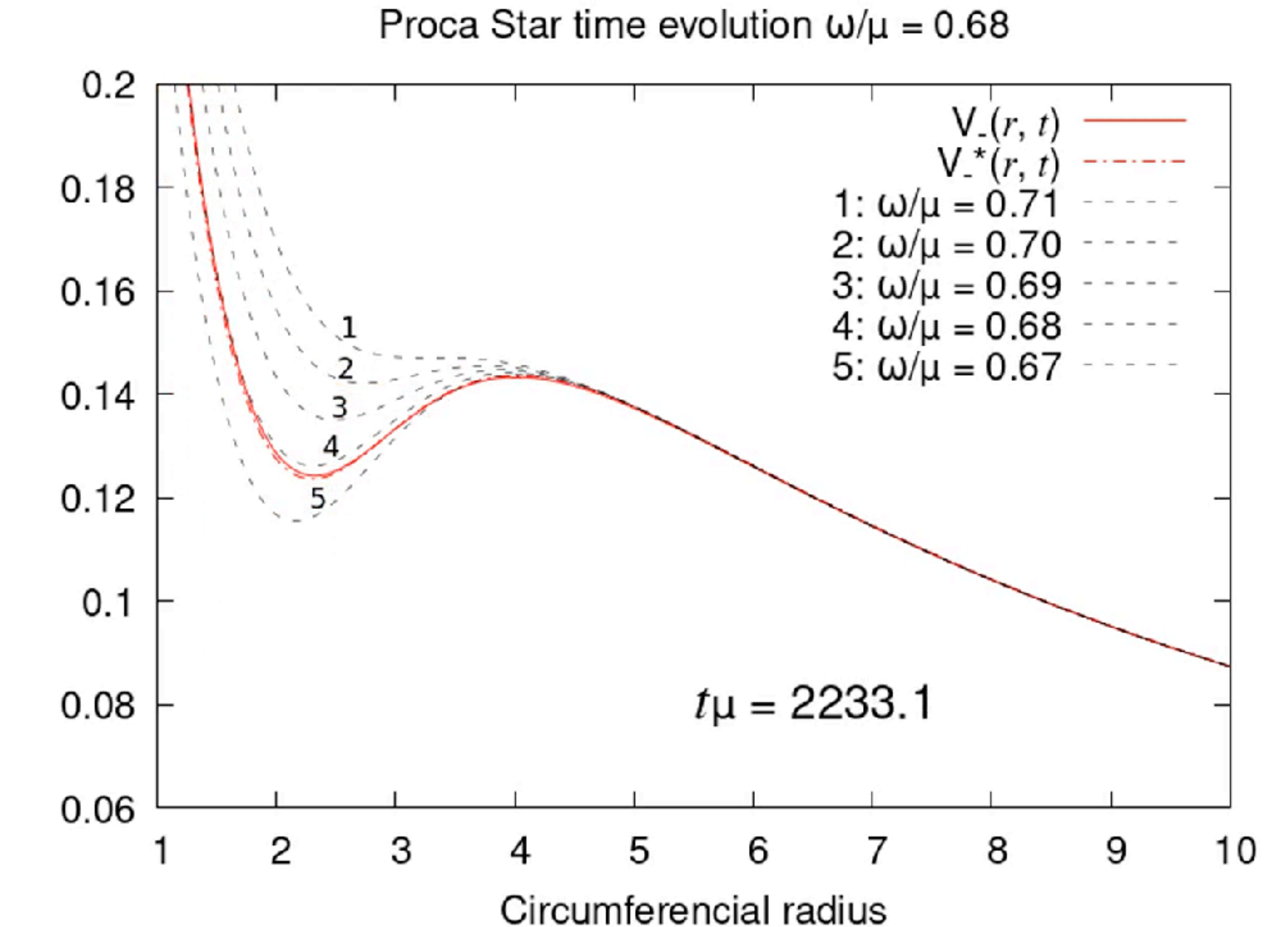
Proca Star time evolution ω/μ = 0.68
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
1
2
3
4
5
6
7
8
9
10
V-(r, t)
V-*(r, t)
1: ω/μ = 0.71
2: ω/μ = 0.70
3: ω/μ = 0.69
4: ω/μ = 0.68
5: ω/μ = 0.67
1
2
3
4
5
tμ = 1577.4
Circumferencial radius

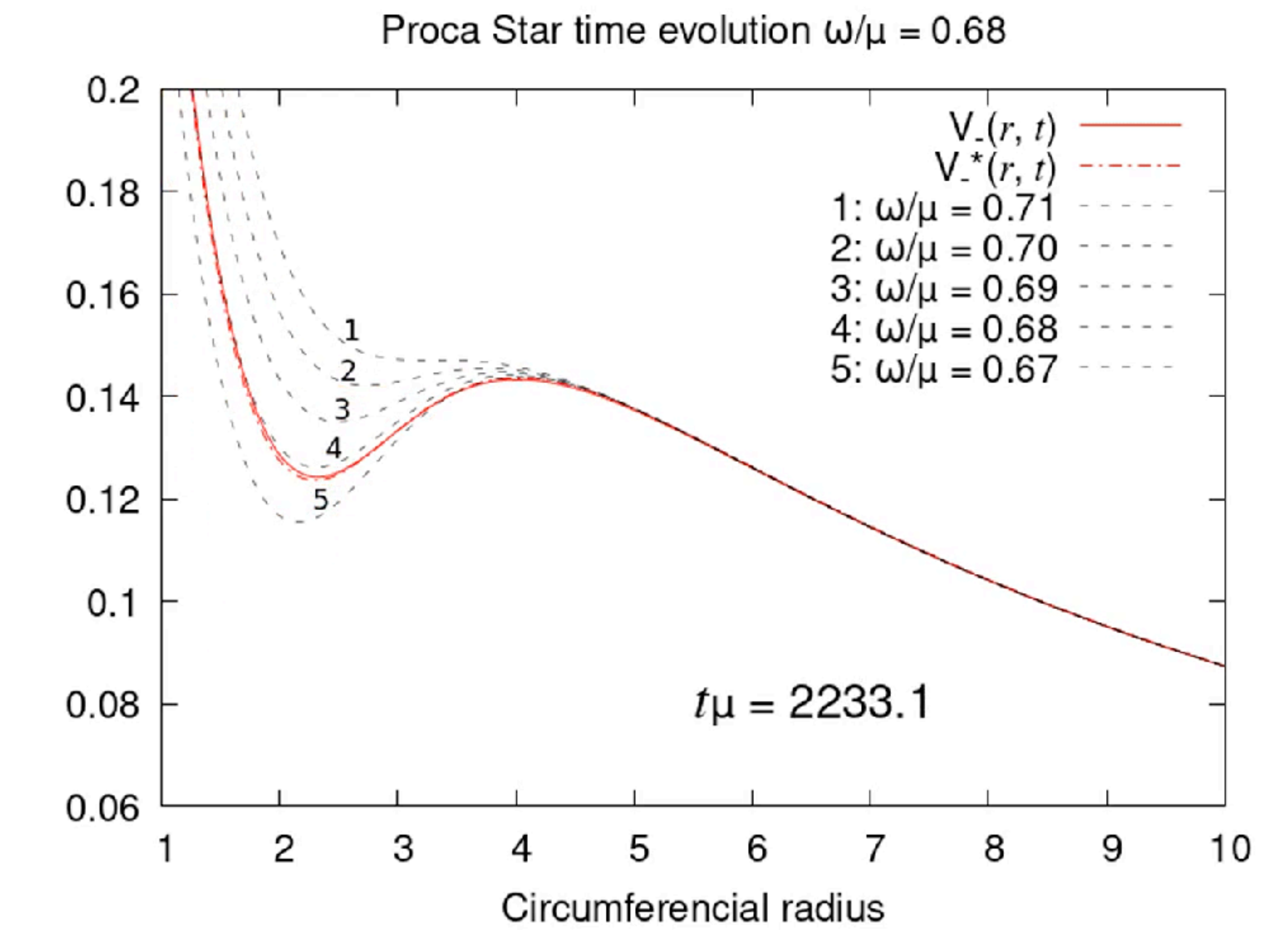


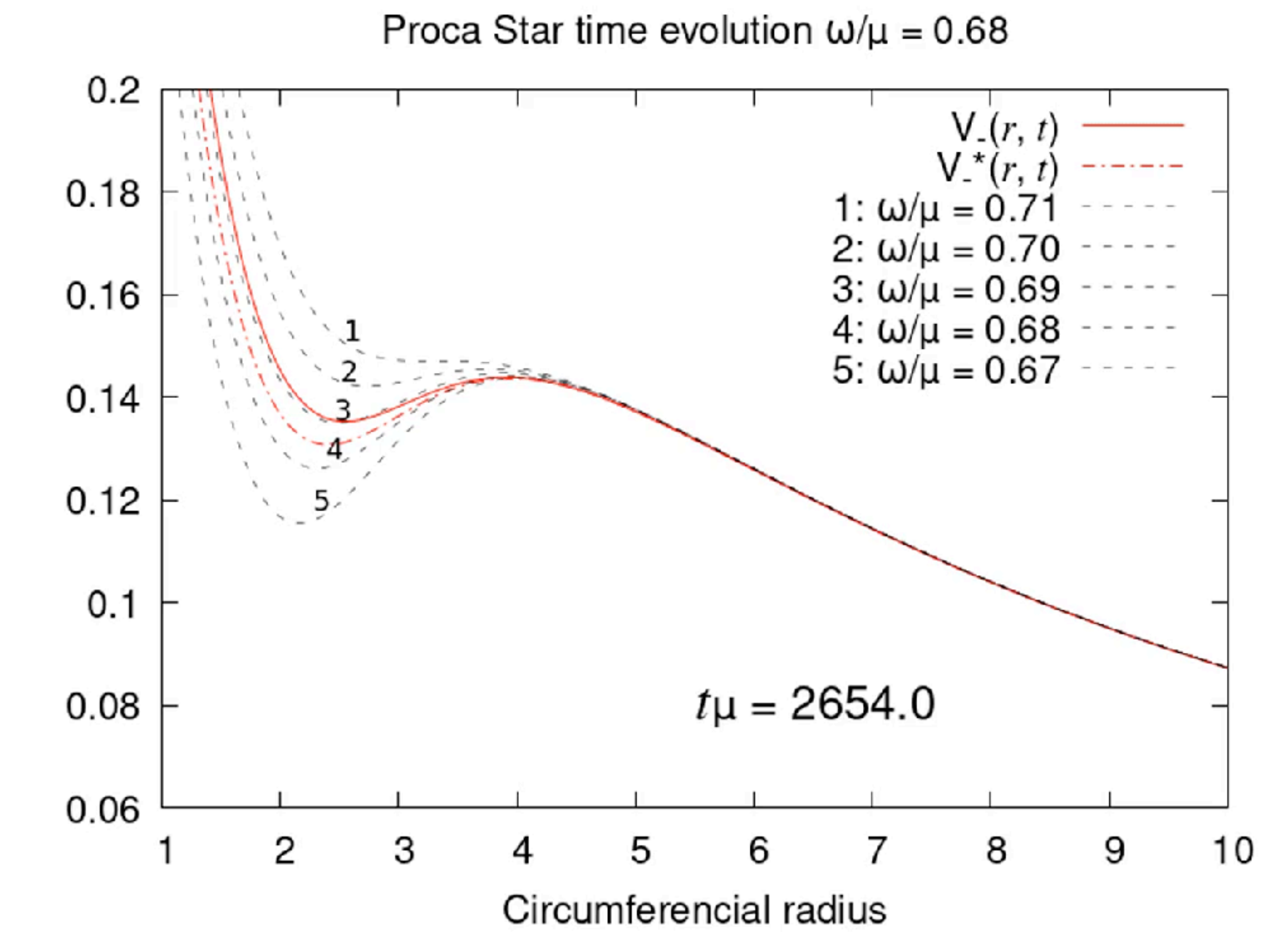
Proca Star time evolution ω/μ = 0.68
V-(r, t)
V-*(r, t)
1: ω/μ = 0.71
2: ω/μ = 0.70
3: ω/μ = 0.69
4: ω/μ = 0.68
5: ω/μ = 0.67
1
2
3
4
5
tμ = 2005.7
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
1
2
3
4
5
6
7
8
9
10
Circumferencial radius



Proca Star time evolution ω/μ = 0.68
V-(r, t)
V-*(r, t)
1: ω/μ = 0.71
2: ω/μ = 0.70
3: ω/μ = 0.69
4: ω/μ = 0.68
5: ω/μ = 0.67
tμ = 2005.7
Circumferencial radius


Proca Star time evolution ω/μ = 0.68
V-(r, t)
V-*(r, t)
1: ω/μ = 0.71
2: ω/μ = 0.70
3: ω/μ = 0.69
4: ω/μ = 0.68
5: ω/μ = 0.67
tμ = 2233.1
Circumferencial radius
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
1
2
3
4
5
6
7
8
9
10


Proca Star time evolution ω/μ = 0.68
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
1
2
3
4
5
6
7
8
9
10
V-(r, t)
V-*(r, t)
1: ω/μ = 0.71
2: ω/μ = 0.70
3: ω/μ = 0.69
4: ω/μ = 0.68
5: ω/μ = 0.67
tμ = 2233.1
Circumferencial radius



Proca Star time evolution ω/μ = 0.68
V-(r, t)
V-*(r, t)
1: ω/μ = 0.71
2: ω/μ = 0.70
3: ω/μ = 0.69
4: ω/μ = 0.68
5: ω/μ = 0.67
tμ = 2654.0
Circumferencial radius
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
1
2
3
4
5
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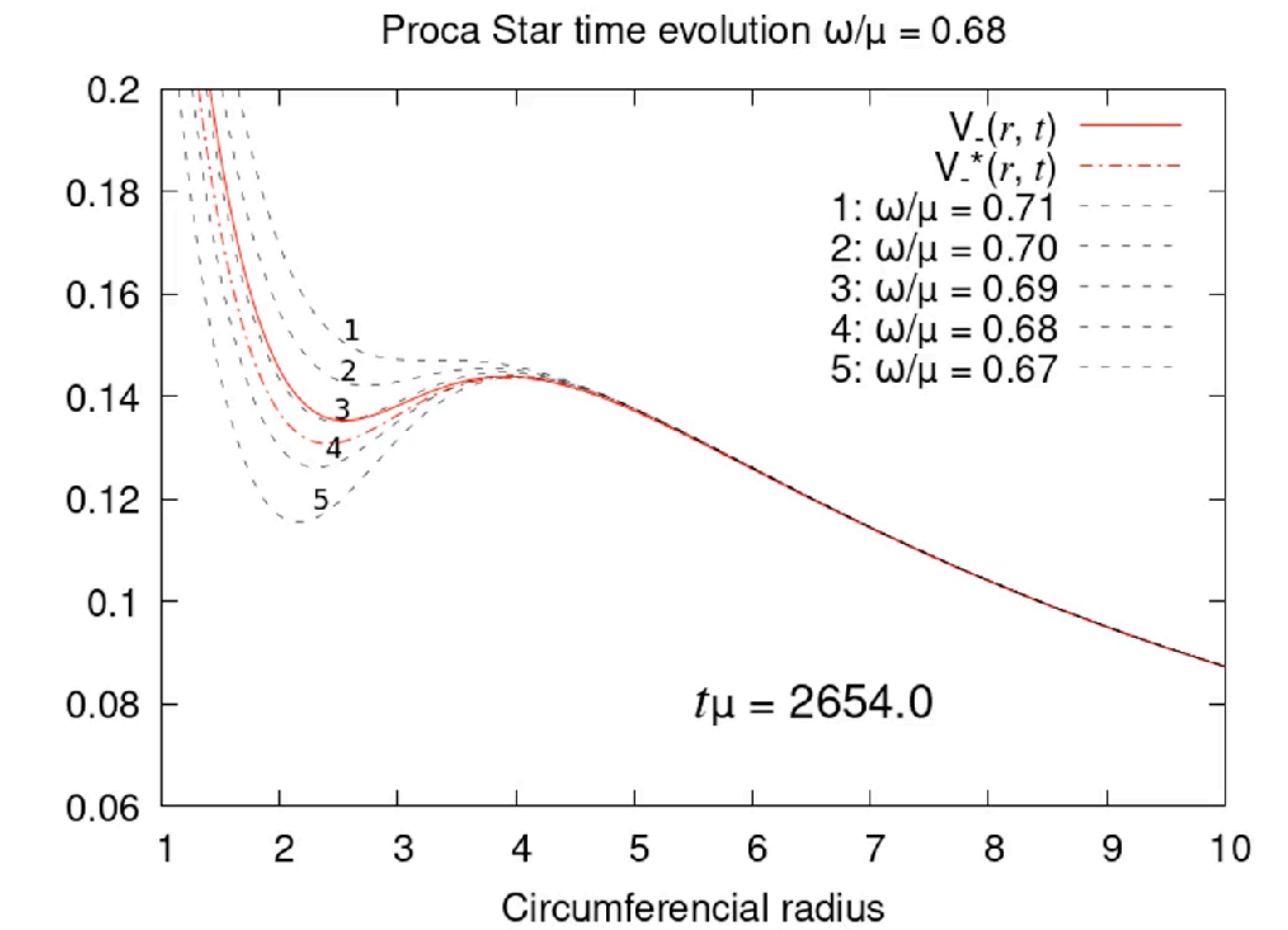
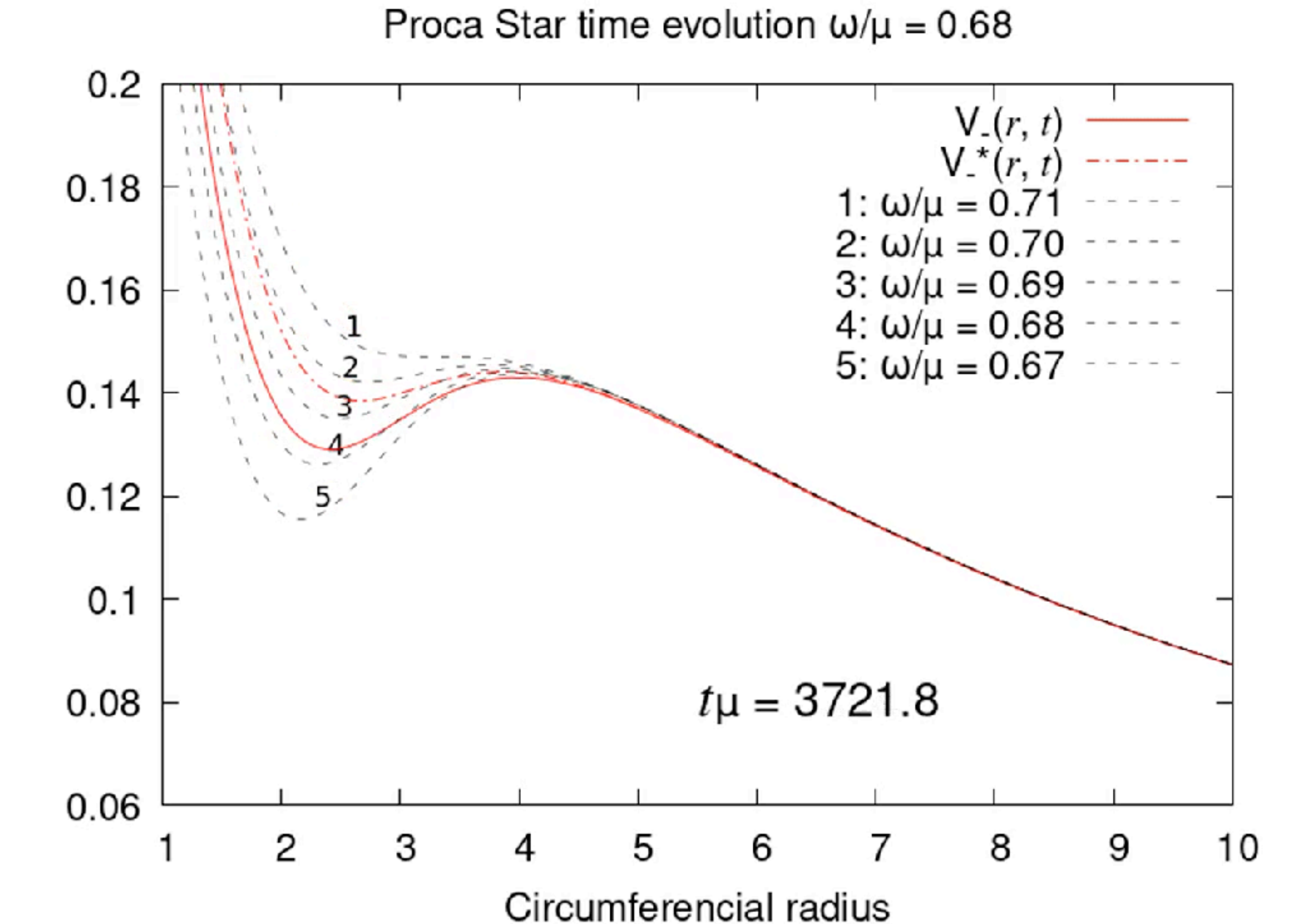
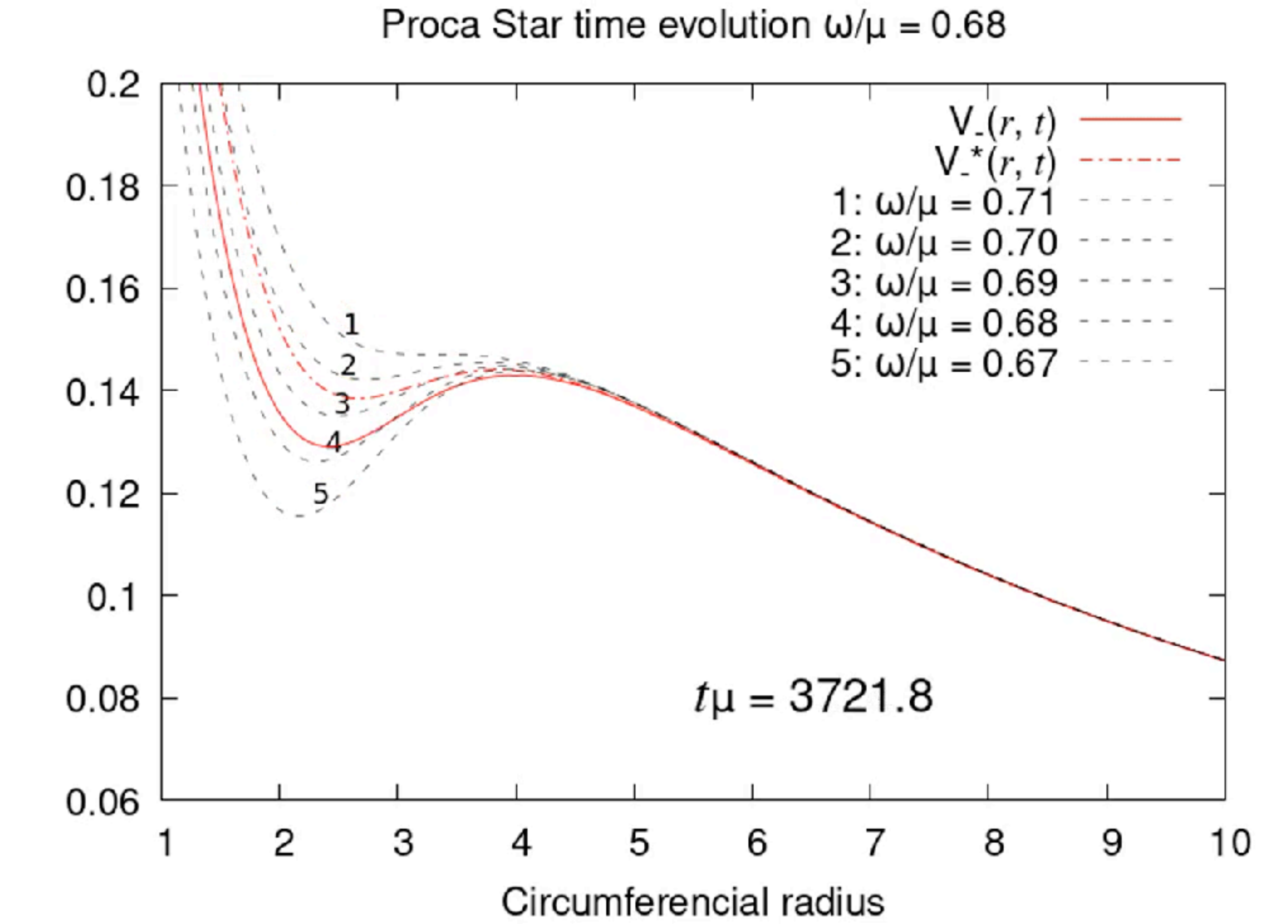
10



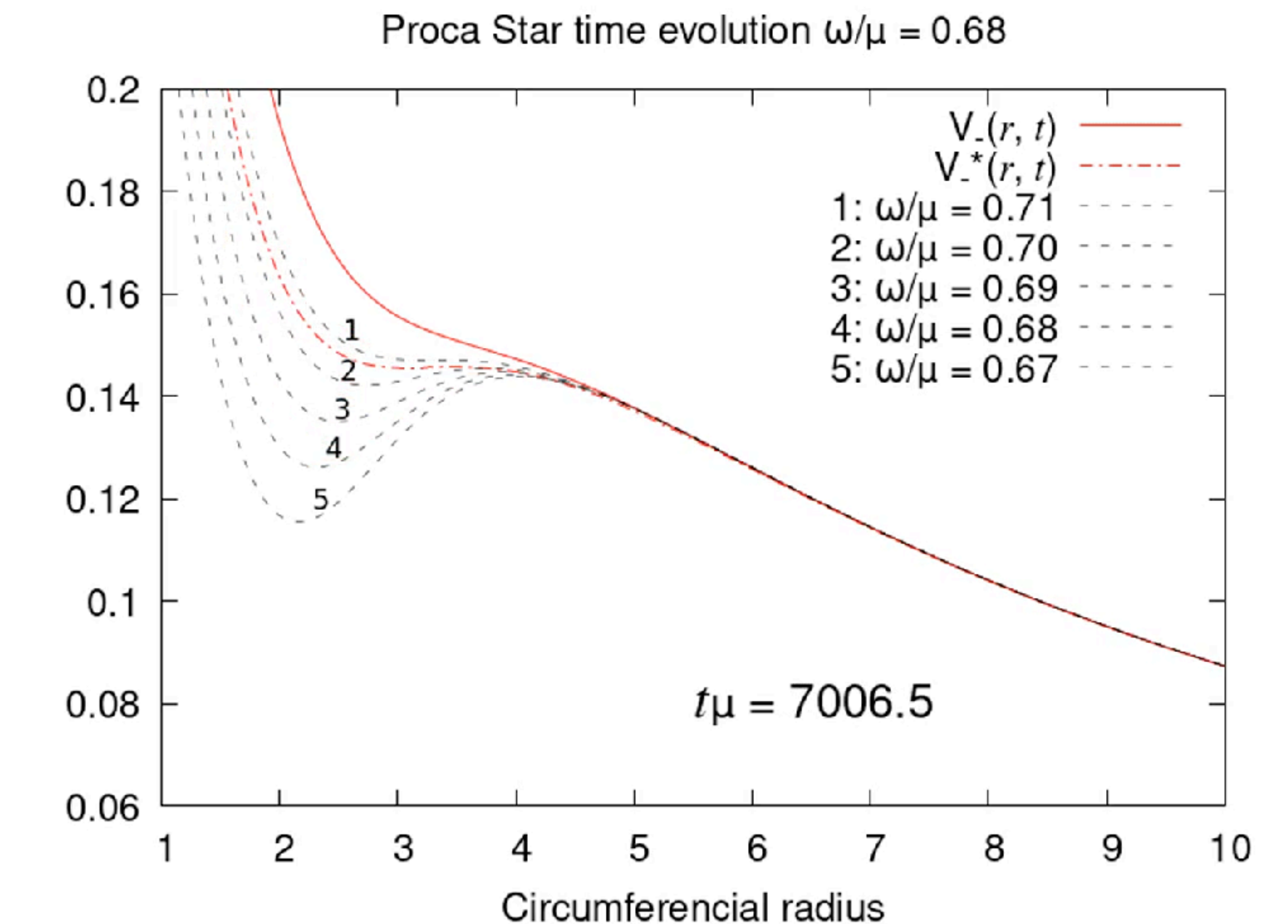
Proca Star time evolution ω/μ = 0.68
V-(r, t)
V-*(r, t)
1: ω/μ = 0.71
2: ω/μ = 0.70
3: ω/μ = 0.69
4: ω/μ = 0.68
5: ω/μ = 0.67
tμ = 2654.0
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
1
2
3
4
5
6
7
8
9
10
Circumferencial radius



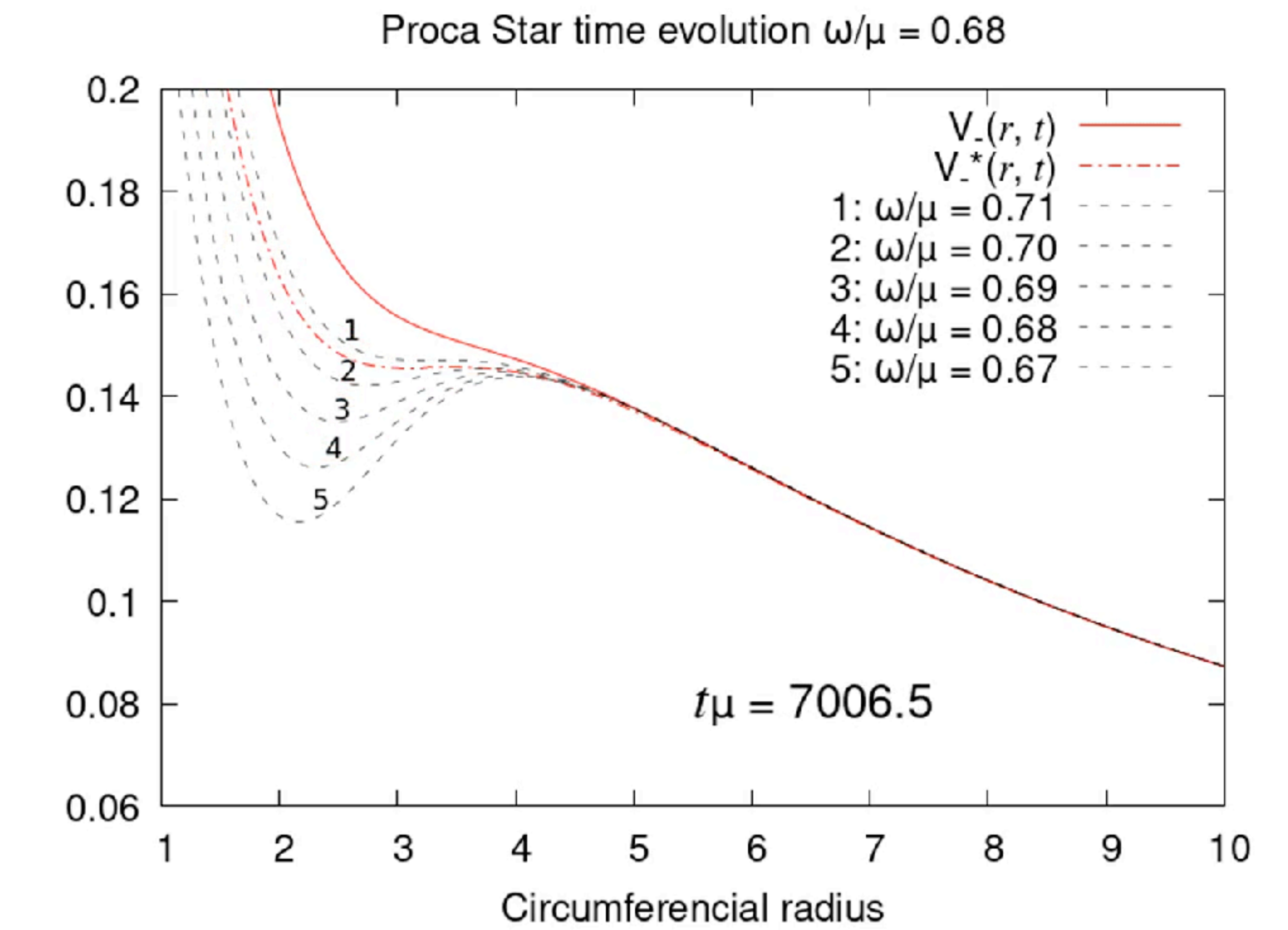
Proca Star time evolution ω/μ = 0.68
V-(r, t)
V-*(r, t)
1: ω/μ = 0.71
2: ω/μ = 0.70
3: ω/μ = 0.69
4: ω/μ = 0.68
5: ω/μ = 0.67
tμ = 3721.8
Circumferencial radius



Proca Star time evolution ω/μ = 0.68
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
1
2
3
4
5
6
7
8
9
10
V-(r, t)
V-*(r, t)
1: ω/μ = 0.71
2: ω/μ = 0.70
3: ω/μ = 0.69
4: ω/μ = 0.68
5: ω/μ = 0.67
1
2
3
4
5
tμ = 3721.8
Circumferencial radius



Proca Star time evolution ω/μ = 0.68
V₋(r, t)
V₋*(r, t)
1: ω/μ = 0.71
2: ω/μ = 0.70
3: ω/μ = 0.69
4: ω/μ = 0.68
5: ω/μ = 0.67
1
2
3
4
5
tμ = 7006.5
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
1
2
3
4
5
6
7
8
9
10
Circumferencial radius



Proca Star time evolution ω/μ = 0.68
V-(r, t)
V-*(r, t)
1: ω/μ = 0.71
2: ω/μ = 0.70
3: ω/μ = 0.69
4: ω/μ = 0.68
5: ω/μ = 0.67
1
2
3
4
5
tμ = 7006.5
0.2
0.18
0.16
0.14
0.12
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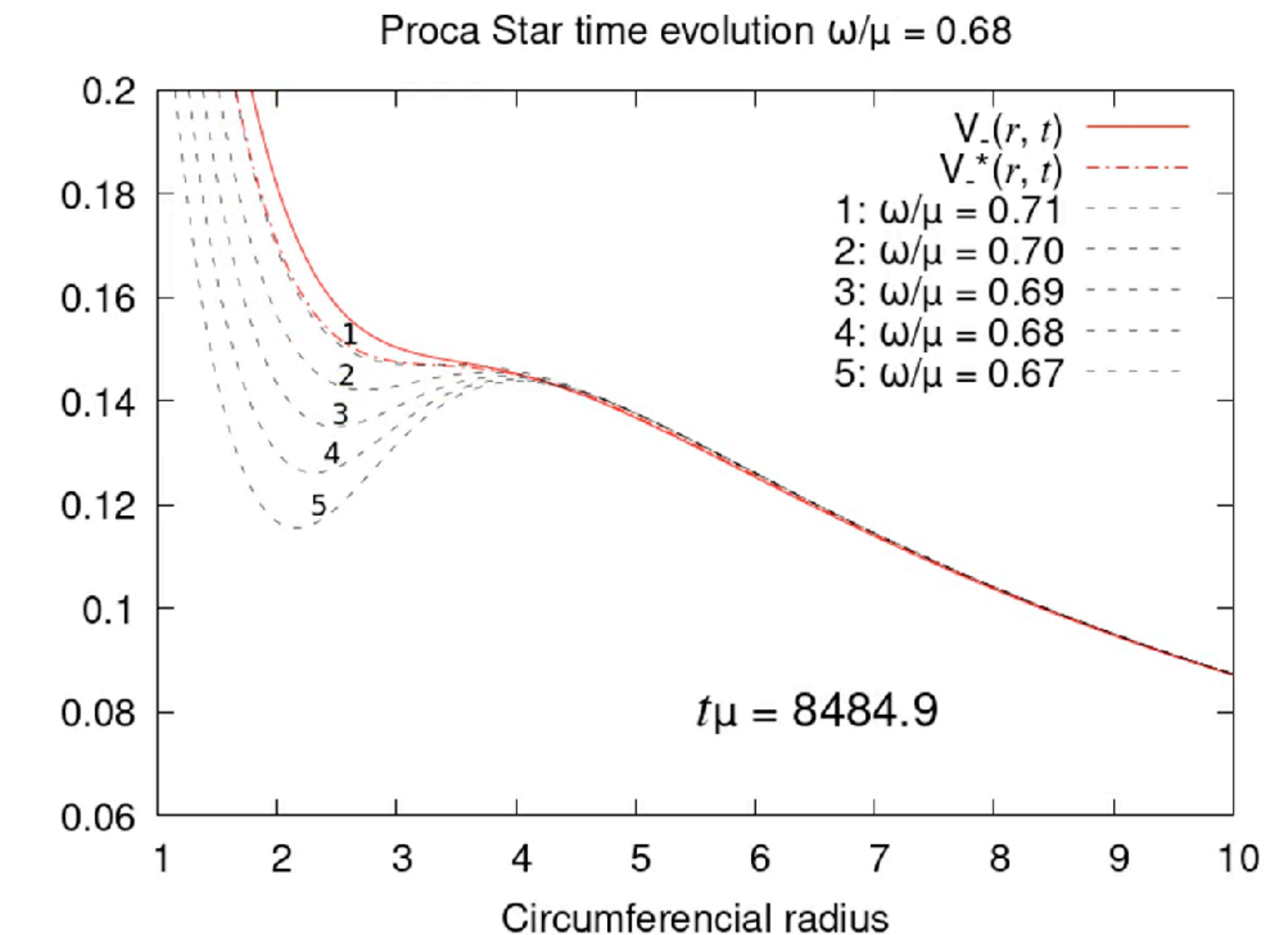
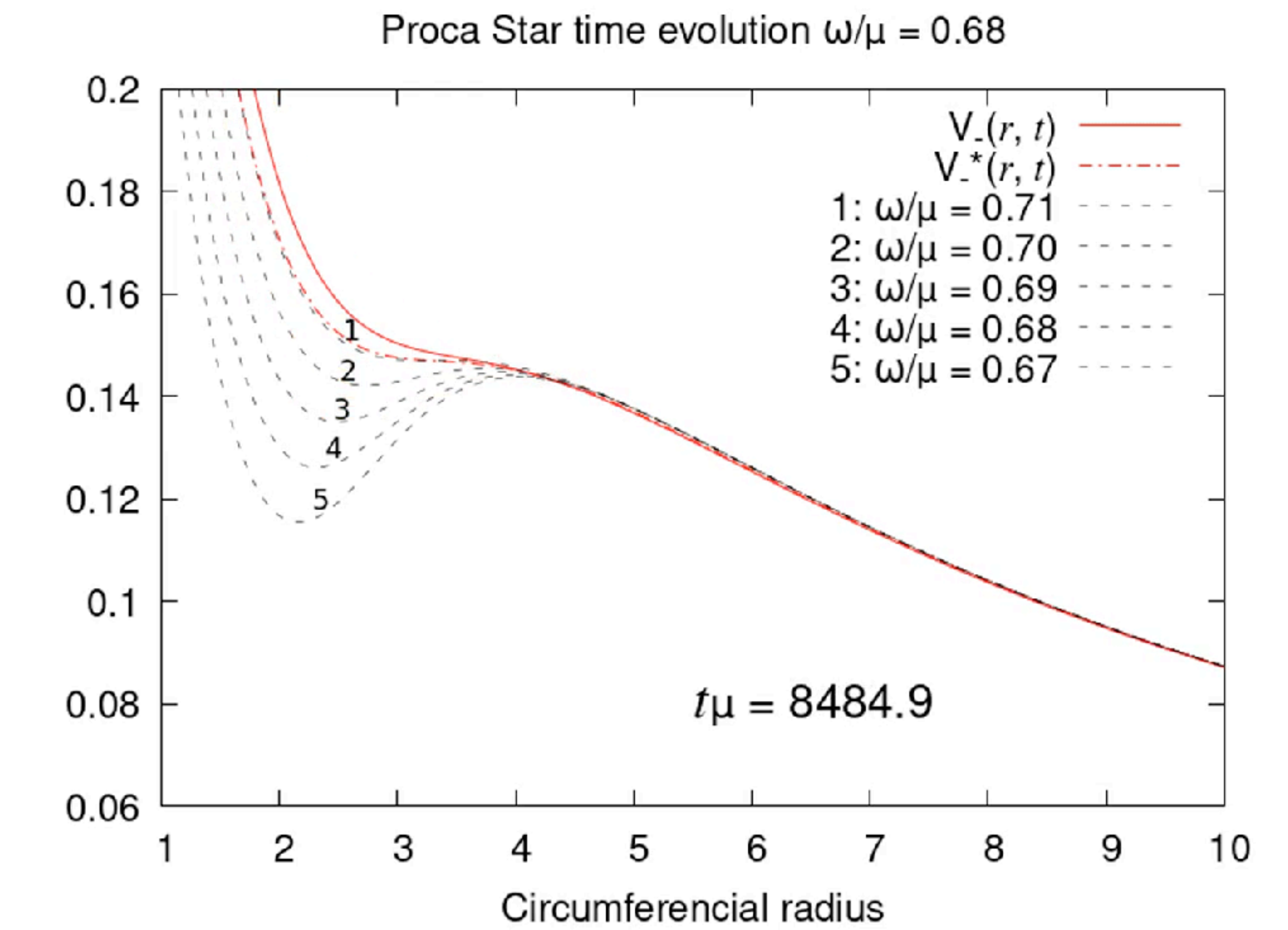
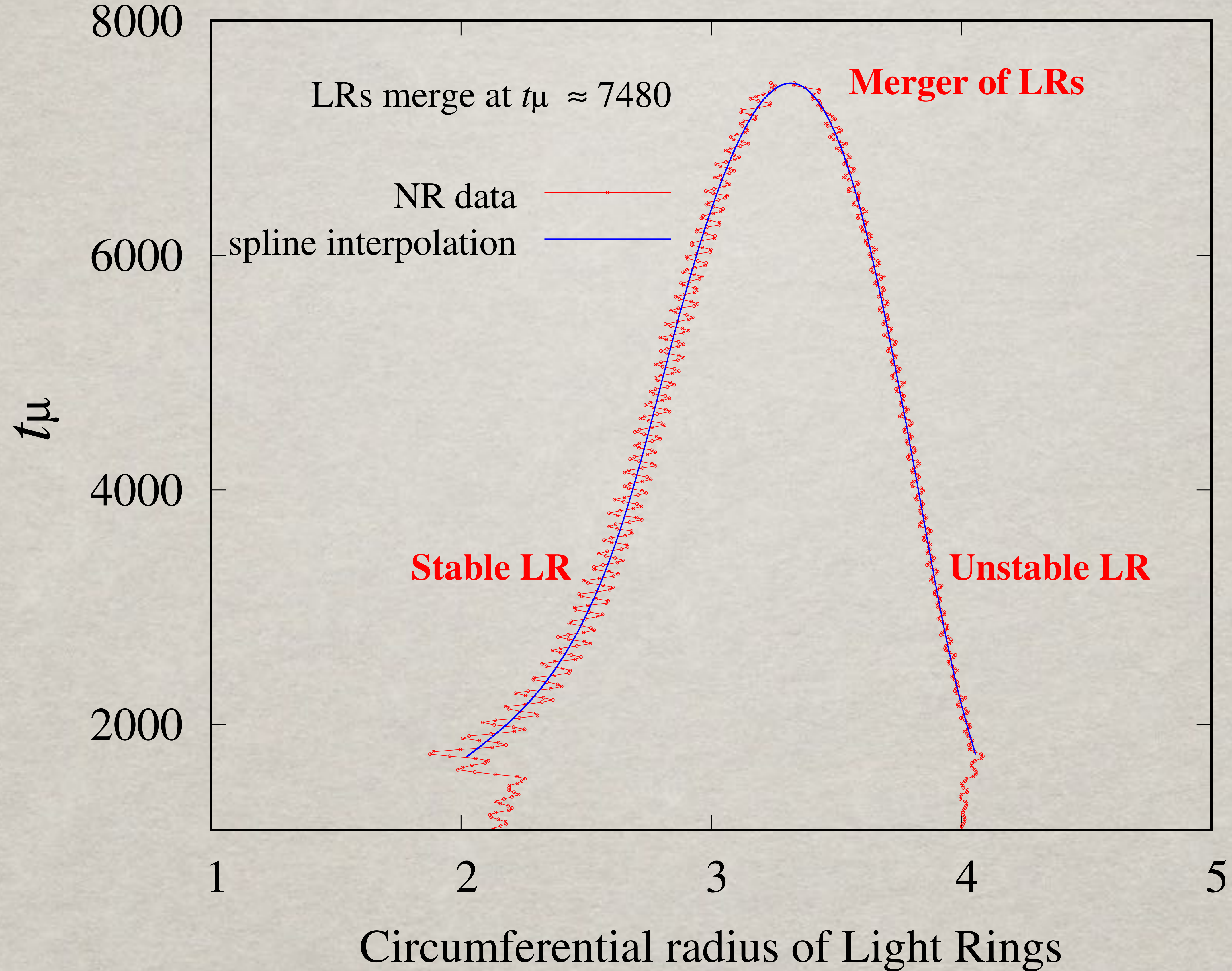
0.08
0.06
1
2
3
4
5
6
7
8
9
10
Circumferencial radius



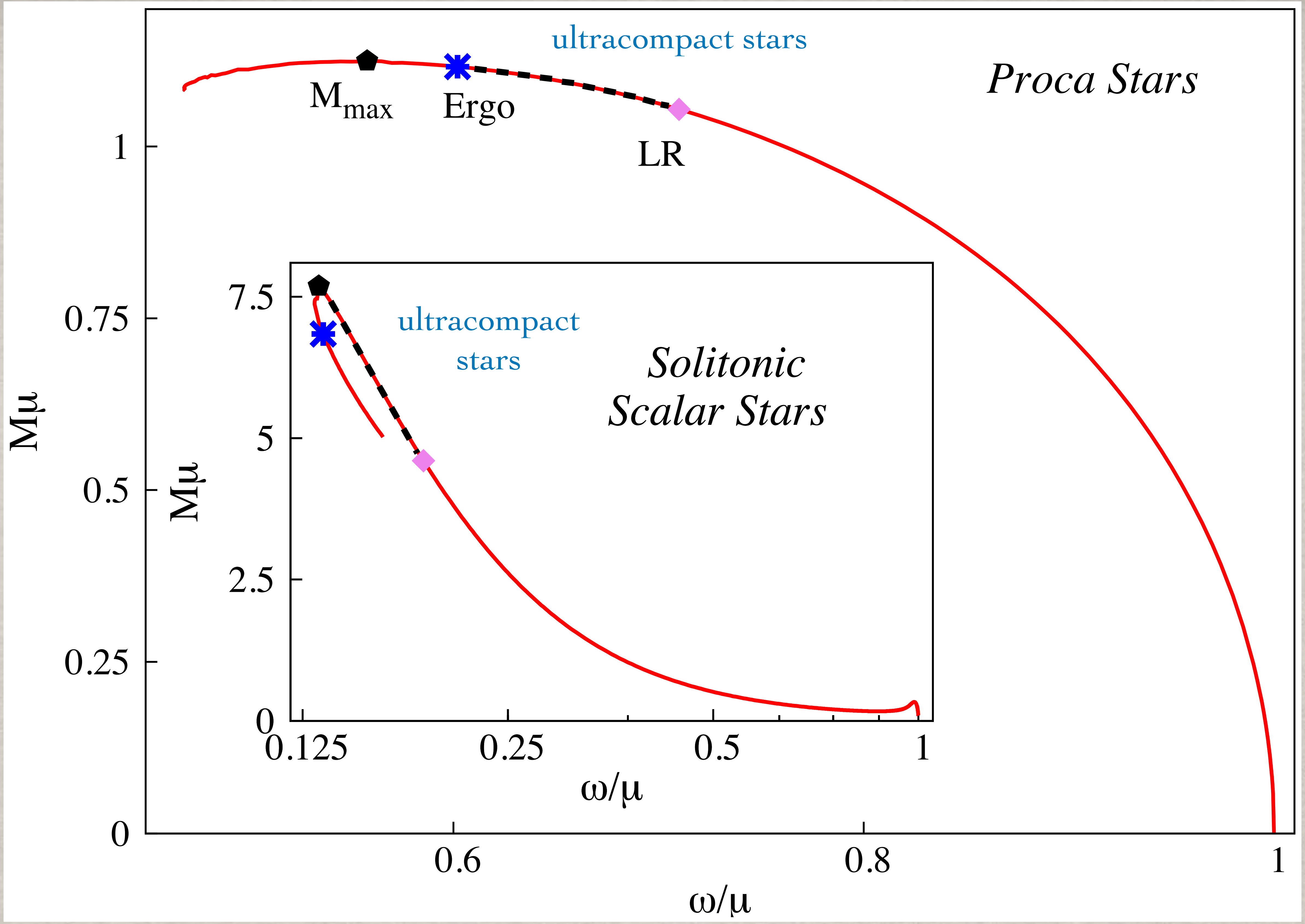
Proca Star time evolution ω/μ = 0.68
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
1
2
3
4
5
6
7
8
9
10
Circumferencial radius
V-(r, t)
V-*(r, t)
1: ω/μ = 0.71
2: ω/μ = 0.70
3: ω/μ = 0.69
4: ω/μ = 0.68
5: ω/μ = 0.67
tμ = 8484.9



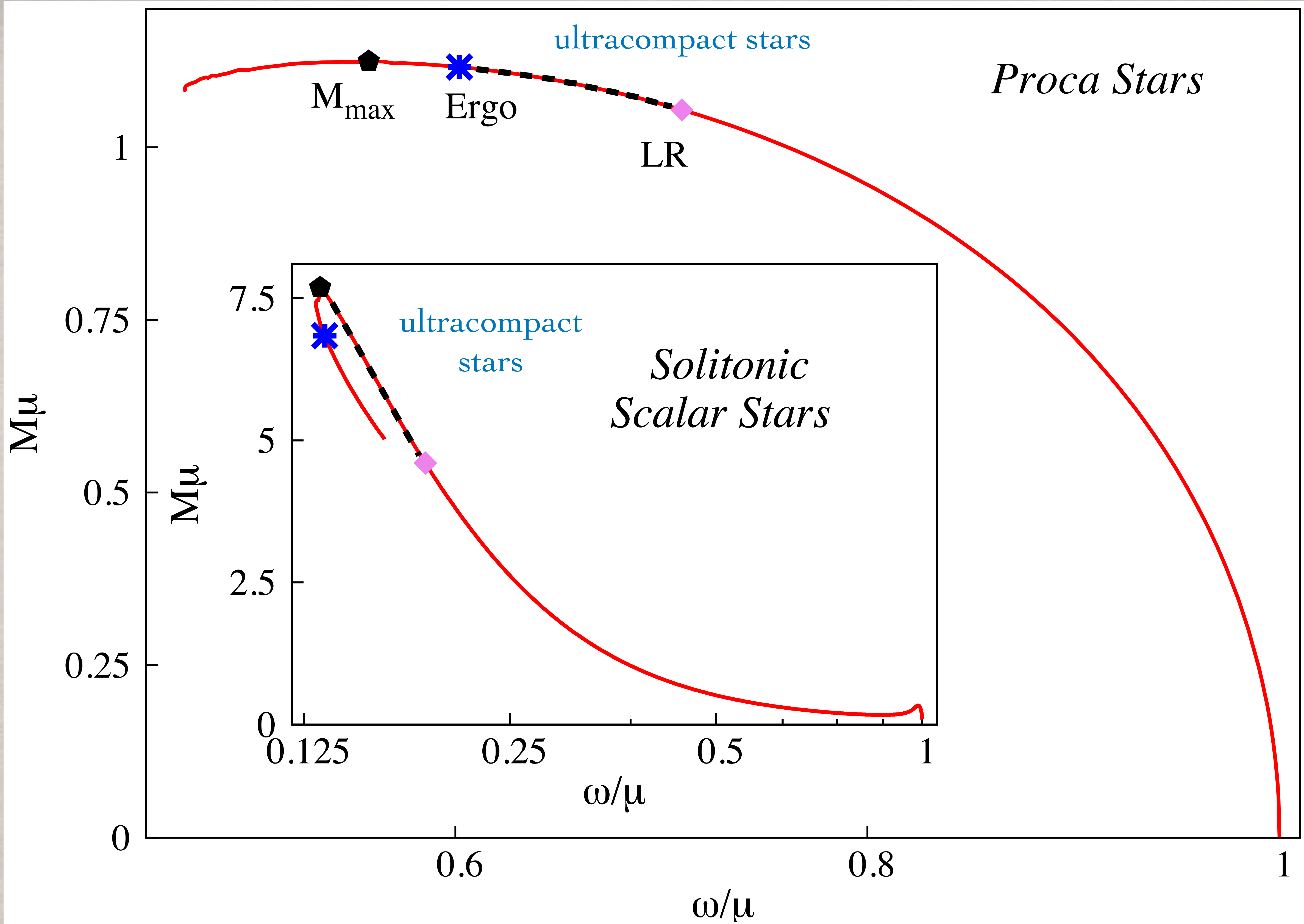
Proca Star time evolution ω/μ = 0.68
V-(r, t)
V-*(r, t)
1: ω/μ = 0.71
2: ω/μ = 0.70
3: ω/μ = 0.69
4: ω/μ = 0.68
5: ω/μ = 0.67
tμ = 8484.9
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
1
2
3
4
5
6
7
8
9
10
Circumferencial radius



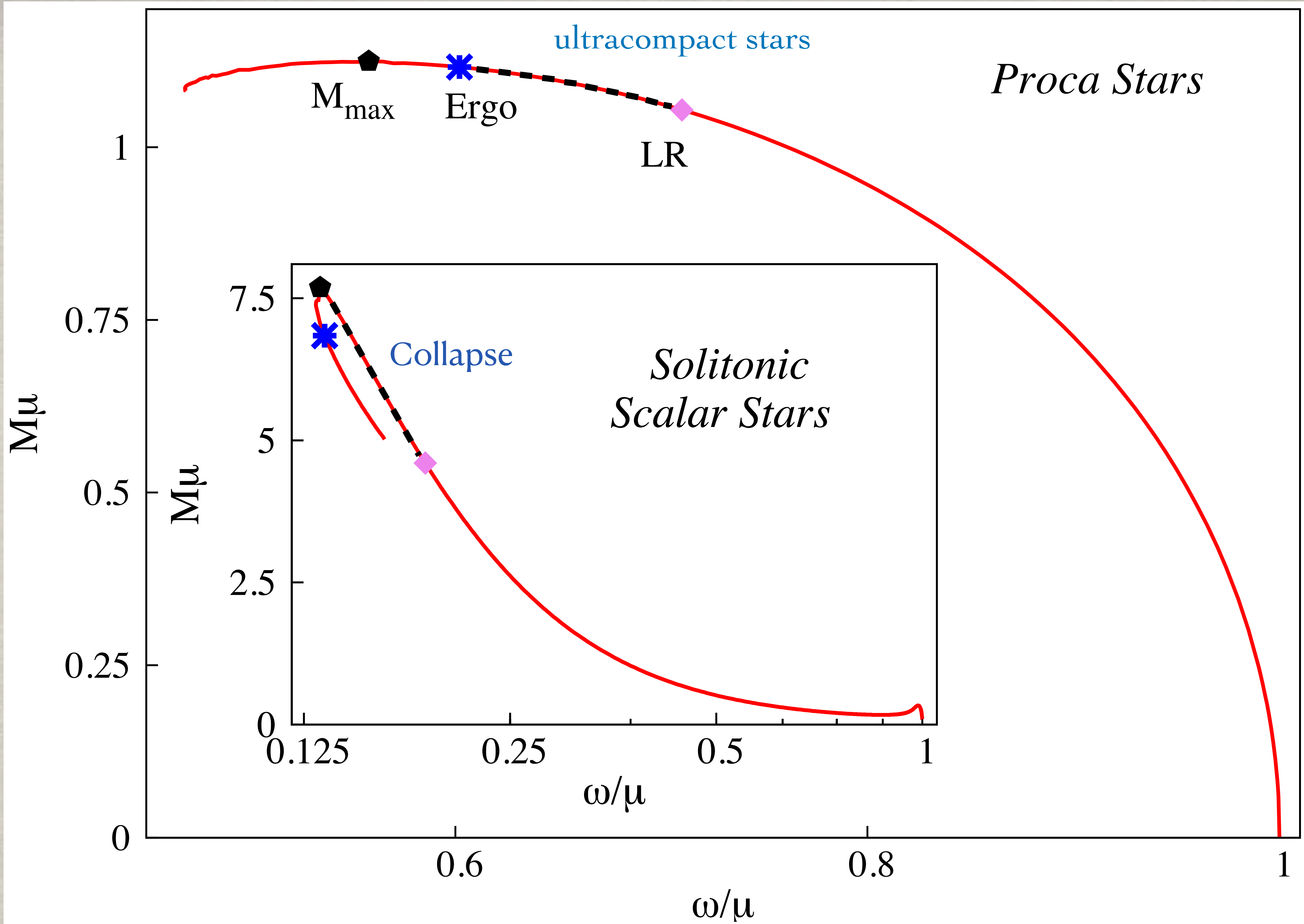
LRs merge at tμ ≈ 7480
Merger of LRs
NR data
spline interpolation
Stable LR
Unstable LR
tμ
8000
6000
4000
2000
1
2
3
4
5
Circumferential radius of Light Rings



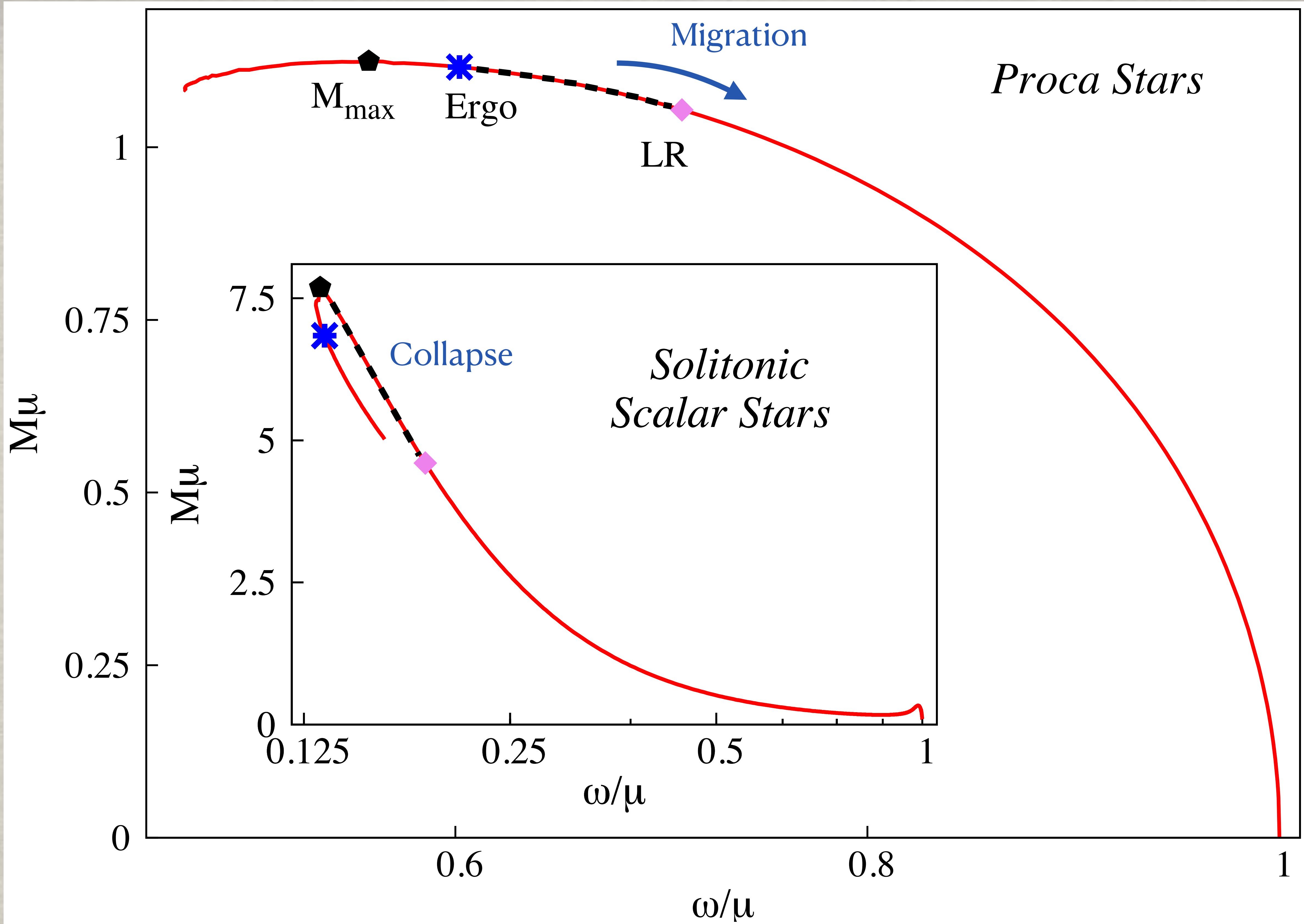
ultracompact stars
Proca Stars
M_{max}
Ergo
LR
1
0.75
0.5
0.25
0
Mμ
0.6
0.8
1
ω/μ
7.5
5
2.5
0
ultracompact
stars
Solitonic
Scalar Stars
Mμ
0.125
0.25
0.5
1
ω/μ



1) The LR instability occurs;
2) Its time scale is astrophysically short, except if close to the critical solution
3) There are two possible fates - collapse and migration



1) The LR instability occurs;
2) Its time scale is astrophysically short, except if close to the critical solution
3) There are two possible fates - collapse and migration



1) The LR instability occurs;
2) Its time scale is astrophysically short, except if close to the critical solution
3) There are two possible fates - collapse and migration

On the fate of the Light Ring instability

Plan:

1) The "*black hole (BH) hypothesis*": BHs and light rings (LRs)

2) The "*exotic compact object (ECO) hypothesis*": ECOs and LRs

3) The LR instability and an explicit test of its fate

4) Discussion and final thoughts

Can there be ultracompact ECOs?

Can there be ultracompact ECOs?

Our results provide evidence against them:

- The LR instability is real and it needs not be too long lived, except near the critical solution, leading (at least) to collapse or migration.
- This questions the plausibility of ultracompact ECOs, that have a plausible formation mechanism within GR.

Can there be ultracompact ECOs?

Our results provide evidence against them:

- The LR instability is real and it needs not be too long lived, except near the critical solution, leading (at least) to collapse or migration.
- This questions the plausibility of ultracompact ECOs, that have a plausible formation mechanism within GR.

But:

- These are just a couple of families of examples; are we missing some generality?
- There details in the analysis for which a deeper understanding would be necessary (non-monotonic instability time scale, loss of axi-symmetry, non-linear character of the instability, spatial correlation with stable LR,…).
- Are there reasonable ultracompact ECOs without a stable LR? (Drop the assumptions: circularity, axi-symmetry; topological triviality)
- Can ECOs mimic BHs even if they are not ultracompact?

On the fate of the Light Ring instability

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Departamento de Matemática e CIDMA, Universidade de Aveiro, Portugal

Journées Relativistes de Tours,
Institut Denis Poisson, Tours, France
June 1st 2023

INSTITUT DENIS POISSON
ORLÉANS • TOURS
Mathématiques & Physique Théorique

Thank you for your attention
Merci pour votre attention

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