Modelling self-consistently beyond general relativity

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w/Llibert Aresté Saló, Katy Clough, Phys. Rev. Lett 129 (2022) 261104, *to appear*
w/Ramiro Cayuso, Tiago França and Luis Lehner, arXiv:2303.07246

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THE ROYAL SOCIETY

Motivation

- Era of gravitational wave astronomy:

➡ New tests of the strong field regime of gravity

➡ Understanding of fundamental nature of gravity

- Current GWs data indicates that deviations from GR are small

Motivation

- Some issues in carrying out this program:
  - Predictions from (the strong field regime of) alternative theories of gravity are needed
  - No preferred alternative theory

    ➡ So what should we be looking for?
  - Many such alternative theories are not known to be well-posed

    ➡ How do we extract their predictions?

Motivation

- Possible ways forward:
  - Linearise around GR [Okounkova et al; Witek et al;…]
  - Find a well-posed formulation of the desired theory [Barausse et al.; Kovacs; Kovacs and Reall; East, Ripley; Bezares et al.; Aresté-Saló, Clough and PF]
  - “Fix” the theory à la Müller-Israel-Stewart (MIS) [Lehner et al…; Cayuso, PF, França, Lehner]

Motivation:
understand the fundamental nature of gravity

- Consider higher derivative theories: well-motivated from microscopic theories of quantum gravity (analogous to hydrodynamics)

- Effects may be enhanced in the strong field regime

- Focus on black holes. Other gravitational objects (e.g., neutron stars,...) typically require other (non-gravitational) physics

Gravity as an EFT

- Add all possible terms (in a derivative expansion) to the Einstein-Hilbert Lagrangian, consistent with the symmetries

$$I = \frac{1}{16\pi G}\int d^4x\sqrt{|g|}\left[R + \frac{1}{\Lambda_{UV}^2}(\alpha_1 R^2 + \alpha_2 R_{\mu\nu}R^{\mu\nu}) + \frac{\beta}{\Lambda_{UV}^3}\textbf{Riem}^3\ldots\right]$$

- Some terms can be removed by field redefinitions and using the lower order eoms

- The coefficients in the expansion are determined by the microscopic theory

- The EFT is only reliable at distances $L \gg \Lambda_{UV}^{-1}$

Outline

- Well-posedness of the initial value problem in gravity
- 4 derivative scalar-tensor theory (4∂ST)
- Müller-Israel-Stewart (MIS) for gravity: 8 derivative theory
- Conclusions

Well-posedness of the initial value problem

Well-posedness

- Given suitable initial data, the solution exists, is unique and it depends continuously on the initial data

  ➡ Predictive power

  ➡ Control of the “size” of the solution from the initial data (for small times)

⇒ Essential to hope to solve the equation(s) numerically

- In GR, establishing well-posedness depends on finding a suitable gauge and on the initial data

Well-posedness: GR

- Generalised harmonic coordinates: $C^\mu = \Box_g x^\mu - H^\mu = 0$ [Choquet-Bruhat]

$$\frac{1}{2} g^{\alpha\beta} g_{\mu\nu,\alpha\beta} + g^{\alpha\beta}{}_{,(\mu} g_{\nu)\alpha,\beta} + H_{(\mu,\nu)} - H_\alpha \Gamma^\alpha{}_{\mu\nu} + \Gamma^\alpha{}_{\mu\beta} \Gamma^\beta{}_{\nu\alpha} = 0$$

➡ Manifest wave-like nature of the Einstein equations

➡ Requires excision of singularities

➡ All modes propagate at the speed of light

Well-posedness: GR

- ADM-like formulations (BSSN/CCZ4) in singularity avoiding coordinates [Baumgarte, Shapiro, Shibata, Nakamura; Baker et al., Campanelli et al.]:

- Decompose the spacetime metric into space and time:

$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$

- Evolve the induced metric γ_{ij} and its "velocity" $\partial_t \gamma_{ij} \sim K_{ij}$ (i.e., extrinsic curvature)

- Coordinate freedom: choice of α and β^i → equivalent to choosing H^μ

- But it is a bit more complicated: rescale γ_{ij}, K_{ij}, use the constraints in the eoms, find "good" evolution equations for α and β^i

$$\partial_t \tilde{\gamma}_{ij} = \dots$$

$$\partial_t \chi = \dots$$

$$\partial_t K = \dots$$

$$\partial_t \tilde{A}_{ij} = \dots$$

$$\partial_t \tilde{\Gamma}^i = \dots$$

$$\partial_t \alpha = \dots$$

$$\partial_t \beta^i = \dots$$

➡ Wave-like nature of the Einstein equations obscured

➡ No singularities in the computational domain

➡ Not all modes propagate at the speed of light (issues with constraint preserving BCs)

Well-posedness: beyond GR

- Horndeski and Lovelock theories are not well posed in harmonic gauge due to degeneracies [Papallo and Reall]

- Solution: break the degeneracies → modified harmonic gauge [Kovacs and Reall]

➡ Introduce auxiliary metrics so that different modes propagate on the light cone of a different metric

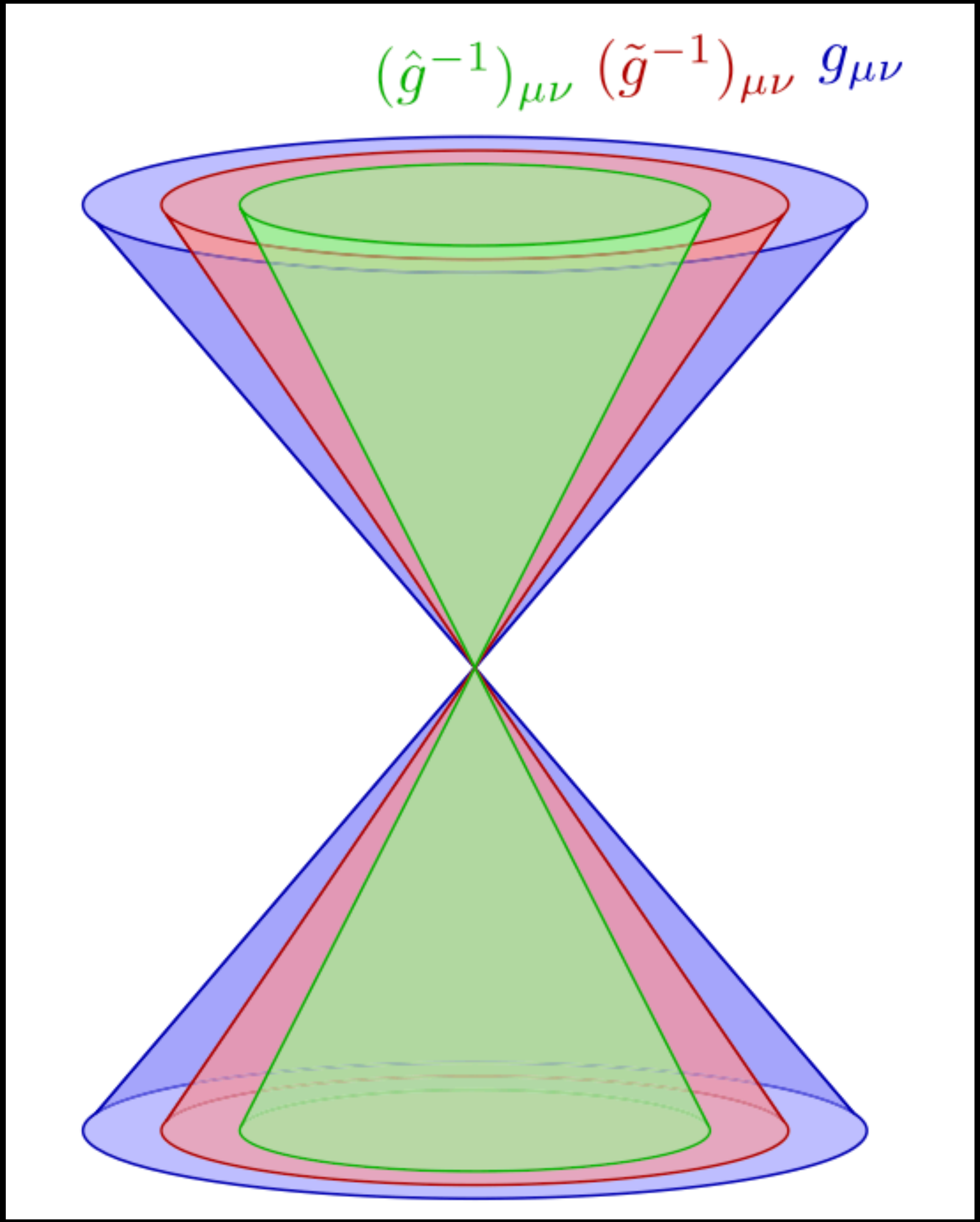


image borrowed from [Kovacs and Reall]

$$R^{\mu\nu} - \frac{1}{2} R g^{\mu\nu} - \hat{P}_{\alpha}{}^{\beta\mu\nu} \nabla_{\beta} C^{\alpha} = 0$$

$$C^{\mu} = H^{\mu} + \tilde{g}^{\rho\sigma} \Gamma^{\mu}_{\rho\sigma}$$

$$\hat{P}_{\alpha}{}^{\beta\mu\nu} = \delta_{\alpha}^{(\mu} \hat{g}^{\nu)\beta} - \frac{1}{2} \delta_{\alpha}^{\beta} \hat{g}^{\mu\nu}$$

- Modified harmonic gauge: $\tilde{g}^{\alpha\beta}\Gamma^{\mu}_{\alpha\beta} = H^{\mu}$

- Modified BSSN/CCZ4: find suitable H^{μ} that generalise the usual evolution equations for the lapse and the shift (1+log slicing and Gamma driver) [Aresté-Saló, Clough, PF]

- 4∂ST, Einstein-Gauss-Bonet, etc., are well-posed in the modified CCZ4 formulation

Most general 4-derivative scalar-tensor (4∂ST) theory of gravity

w/ Llibert Aresté Saló, Katy Clough

Most general scalar-tensor theory of gravity up to 4 derivatives

- Most general general scalar-tensor theory of gravity up to 4 derivatives [Weinberg]:

$$I = \frac{1}{16\pi}\int d^4x\sqrt{-g}\,\left[R + X - V(\phi) + g_2(\phi)\,X^2 + \lambda(\phi)\mathscr{L}_{GB}\right]$$

$$X = -\tfrac{1}{2}(\nabla_\mu\phi)(\nabla^\mu\phi) \qquad \mathscr{L}_{GB} = R^2 - 4\,R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$$

- EFT of inflation
- 2nd order eoms!
- Our choices:

$$V(\phi) = 0, \quad g_2(\phi) = g_2 \quad \lambda(\phi) = \tfrac{\lambda^{GB}}{4}\,\phi \quad \textbf{or} \quad \lambda(\phi) = \frac{\lambda^{GB}}{\gamma}(1 - e^{-\gamma\phi^2})$$

Weak vs Strong coupling

- We take an EFT approach:

  ➡ We consider the full theory but in a regime where the higher derivative terms in the eoms are small at all times

  ➡ Compatible with non-linearities being important and consistent to neglect higher derivative terms in the action

  ➡ Well-posedness holds

- In practice we monitor that the weak coupling condition is satisfied

$$|g_2 L^{-2}| \ll 1\,, \quad |\lambda'(\phi)\, L^{-2}| \ll 1\,, \quad L^{-1} = \textbf{sup}\{\,|R_{\mu\nu\rho\sigma}|^{\frac{1}{2}}, |\nabla_\mu \phi|\,, |\nabla_\mu \nabla_\nu \phi|^{\frac{1}{2}}\}$$

Weak vs Strong coupling

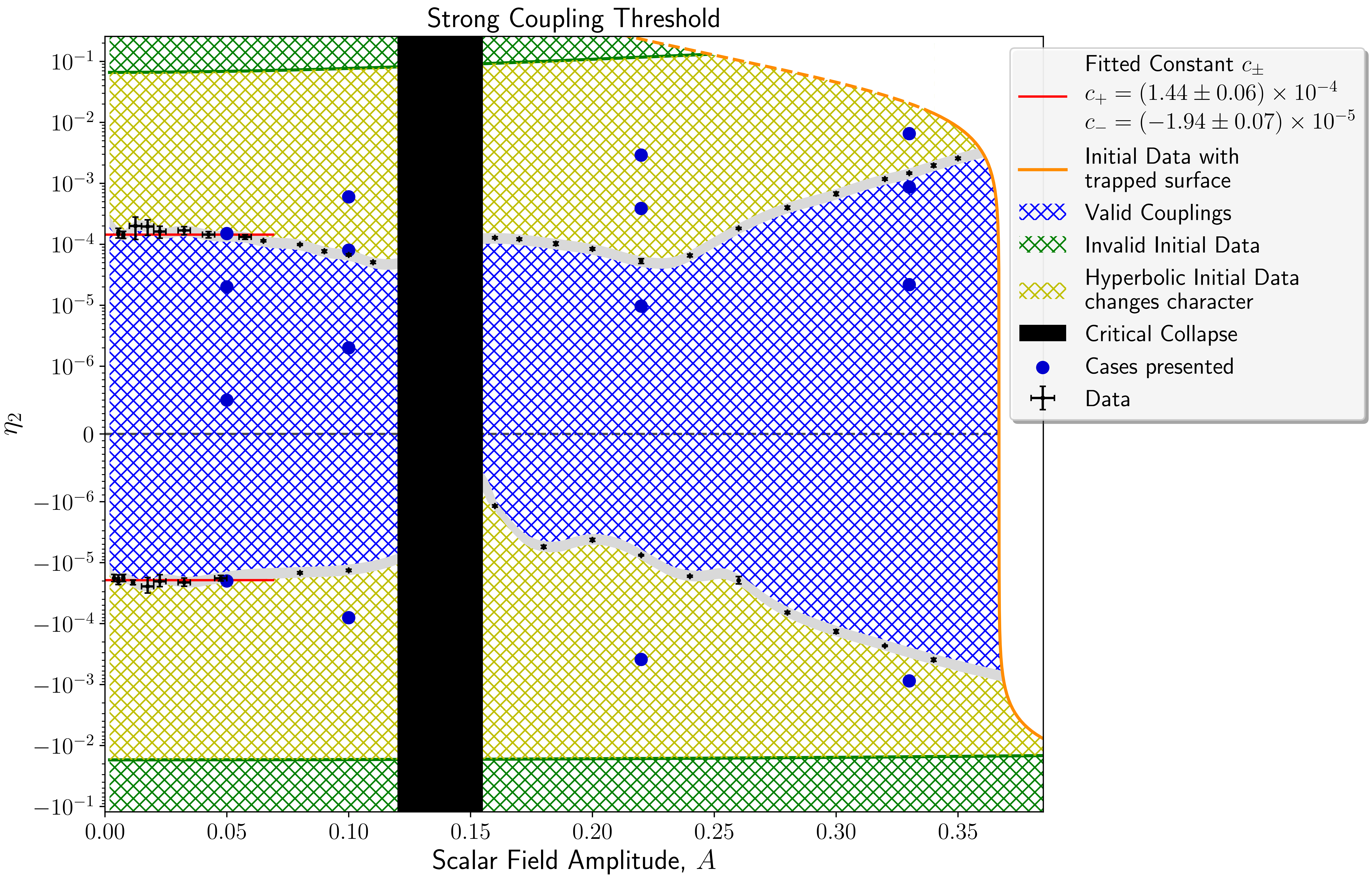
- Cases: $h^{\mu\nu}\nabla_\mu\nabla_\nu\phi = \lambda'(\phi)\mathscr{L}_{GB}$
  - $\lambda(\phi) = \frac{\lambda^{GB}}{4}\phi$: shift-symmetric case → Kerr is not a solution, only hairy black holes
  - $\lambda(\phi) = \lambda^{GB}\gamma^{-1}(1 - e^{-\gamma\phi^2})$: Kerr and hairy black holes are solutions
- The evolution of the scalar field is controlled by an effective metric:

$$h^{\mu\nu} = g^{\mu\nu}(1 + g_2 X) - 2g_2(\nabla^\mu\phi)(\nabla^\nu\phi)$$

➡ Hyperbolicity can break down in the strongly coupled regime

➡ Shocks can form from smooth initial data

[Ripley and Pretorius; Bernard et al; PF and França; Bezares et al.]

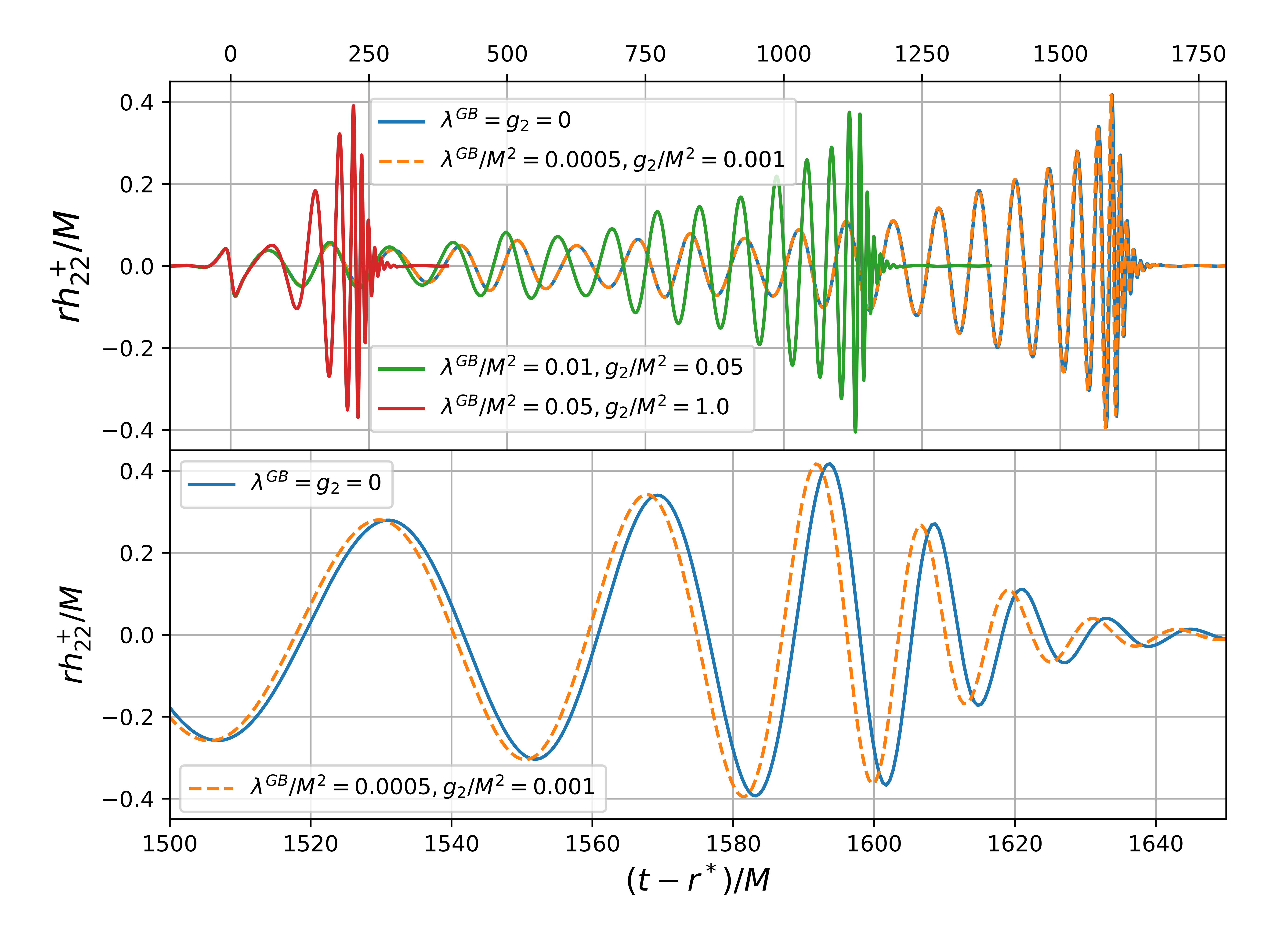


[PF and França]

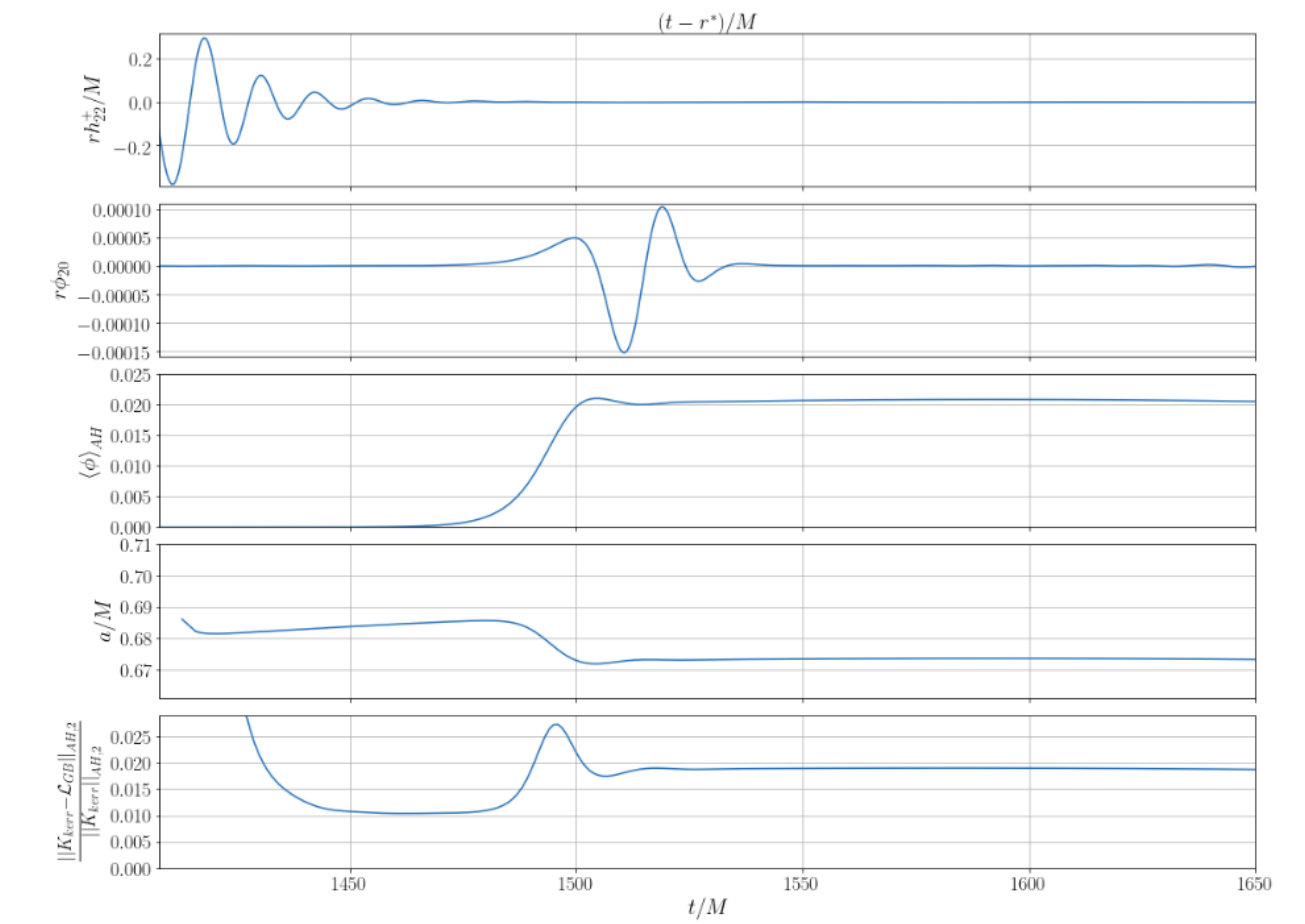
Black hole binaries

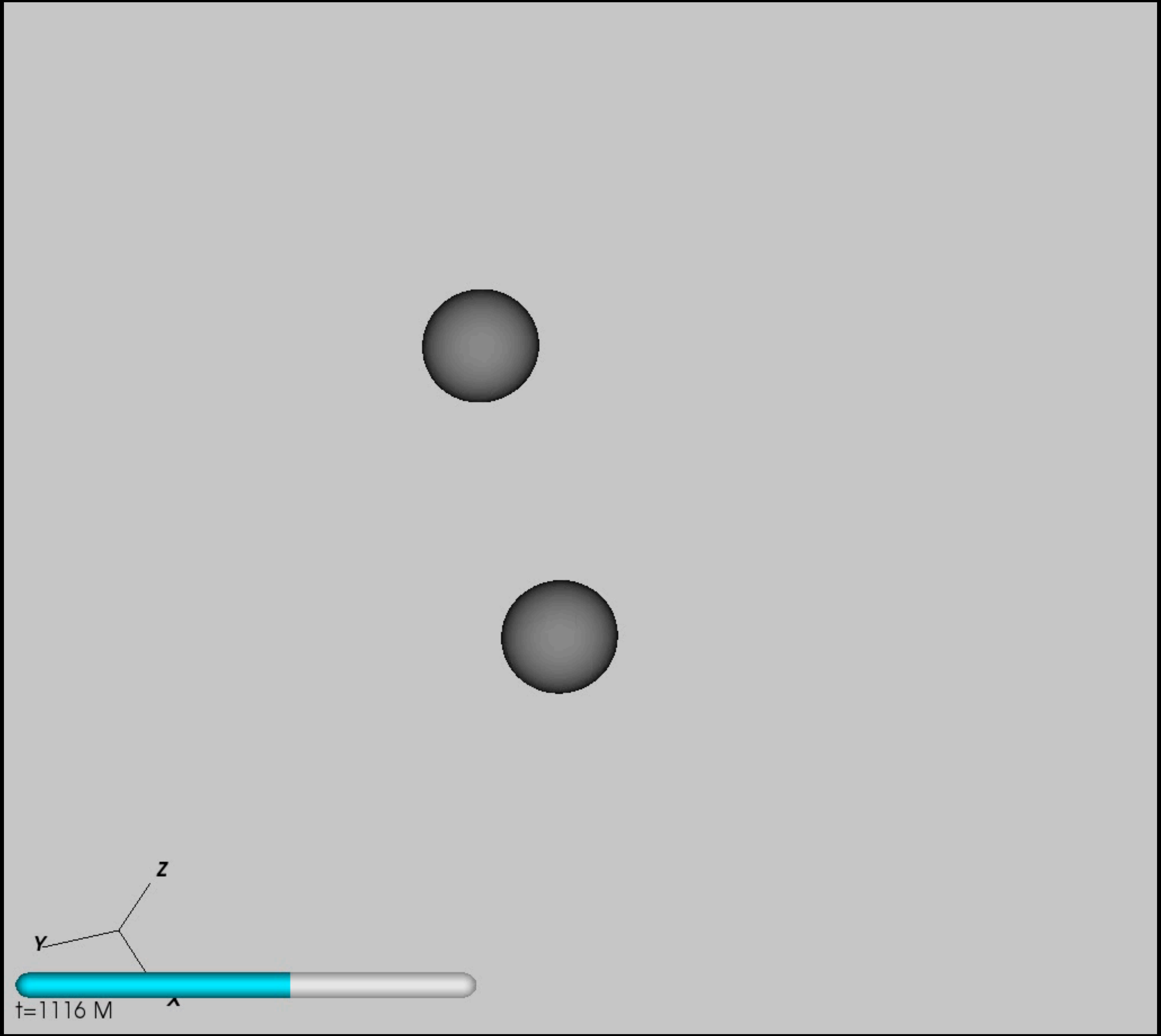
- Initial data corresponding to two superposed non-spinning GR black holes → small initial constraint violations
- Initial configuration is in the weakly coupled regime
- ~11 quasi-circular orbits
- Monitor the weak coupling condition
- “Excise” a portion of the interior of the AH

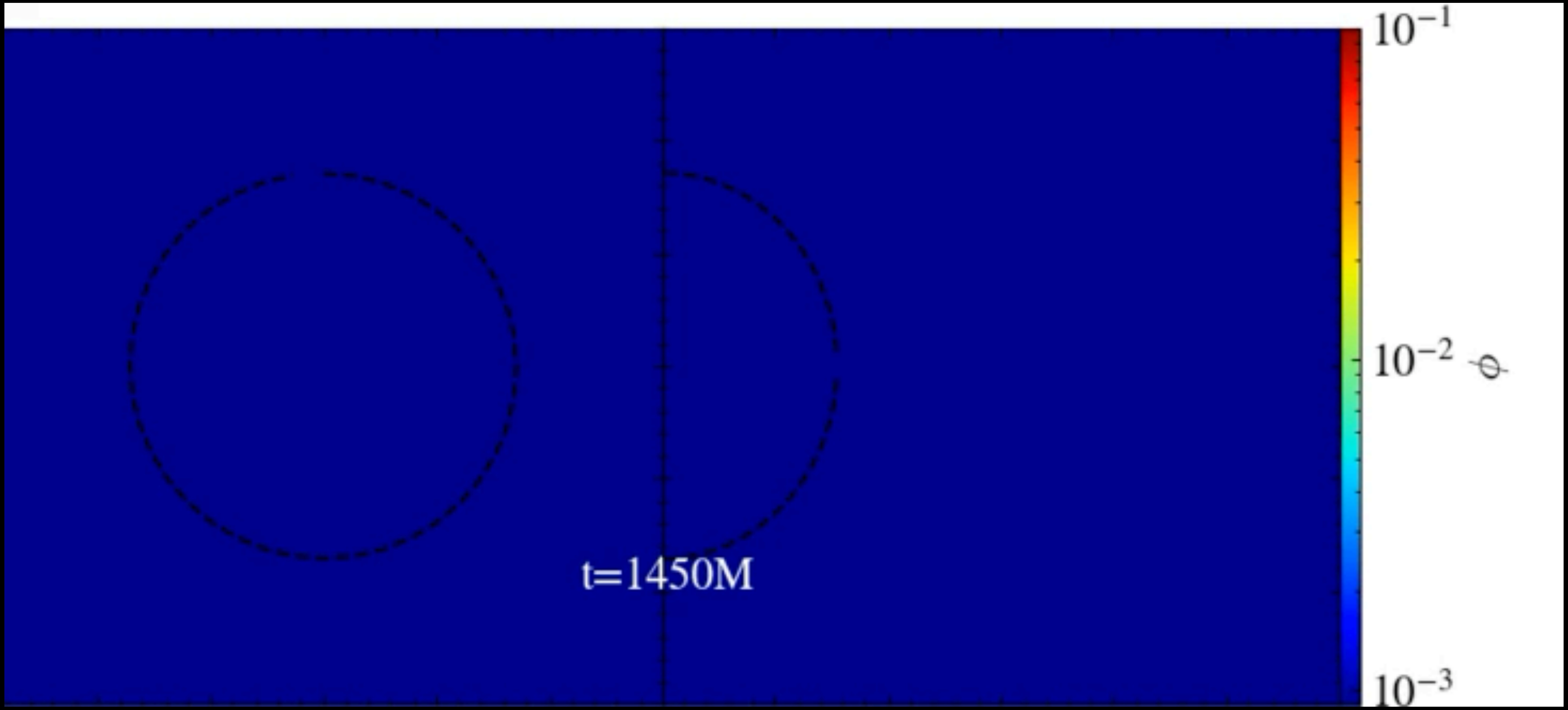
$\lambda(\phi) = \frac{\lambda^{GB}}{4}\phi$ theory



$$\lambda(\phi) = \frac{\lambda^{GB}}{\gamma}(1 - e^{-\gamma\phi^2})$$ theory




Z
Y
X
t=1116 M



t=1450M
10^{-1}
10^{-2}
10^{-3}
ϕ

Müller-Israel-Stewart (MIS) for an 8-derivative theory of gravity

w/ Ramiro Cayuso, Tiago França and Luis Lehner

Eight derivative theory of gravity

- Most general higher derivative theory of gravity (in vacuum) up to 8 derivatives:

$$I = \int dx^4 \sqrt{-g} \left(R - \frac{1}{\Lambda^6} \mathscr{C}^2 - \frac{1}{\tilde{\Lambda}^6} \tilde{\mathscr{C}}^2 - \frac{1}{\Lambda_-^6} \mathscr{C} \tilde{\mathscr{C}} \right)$$

$$\mathscr{C} = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} \,, \quad \tilde{\mathscr{C}} = R_{\mu\nu\rho\sigma} \tilde{R}^{\mu\nu\rho\sigma} \,, \quad \tilde{R}^{\mu\nu\rho\sigma} = \frac{1}{2} \epsilon_{\mu\nu}{}^{\alpha\beta} R_{\alpha\beta\rho\sigma}$$

➡ EOMs with 4th order derivatives ($\epsilon \equiv \Lambda^{-6}$):

$$\begin{aligned} G_{\mu\nu} = 8\epsilon \Big\{ & \mathscr{C} \Big[\Box R_{\mu\nu} - \frac{1}{2} \nabla_\mu \nabla_\nu R - \frac{1}{16} \mathscr{C} g_{\mu\nu} - R_{\mu\lambda} R^\lambda{}_\nu \\ & + R^{\alpha\beta} R_{\mu\alpha\nu\beta} + \frac{1}{2} R_{\mu\sigma\rho\lambda} R_\nu{}^{\sigma\rho\lambda} \Big] \\ & + 2 (\nabla^\alpha \mathscr{C}) \Big[\nabla_\alpha R_{\mu\nu} - \nabla_{(\mu} R_{\nu)\alpha} \Big] + R_\mu{}^\alpha{}_\nu{}^\beta \nabla_\alpha \nabla_\beta \mathscr{C} \Big\} \end{aligned}$$

- No mathematical theory for general higher than 2nd order PDEs

- *How is one to approach the study of this theory and its physical predictions?*

MIS: relativistic viscous hydrodynamics

- 2nd order stress tensor of a relativistic viscous (conformal) fluid:

$$T_{\mu\nu} = \frac{\rho}{d-1}(d\, u_\mu u_\nu + \eta_{\mu\nu}) + \Pi_{\mu\nu}$$

$$\Pi_{\mu\nu} = -2\,\eta\,\sigma_{\mu\nu} + 2\,\eta\,\tau_\Pi \left(\langle u^\alpha \partial_\alpha \sigma_{\mu\nu} \rangle + \frac{1}{d-1}\,\sigma_{\mu\nu}\,\partial_\alpha u^\alpha \right) + \langle \lambda_1\,\sigma_{\mu\alpha}\sigma_\nu{}^\alpha + \lambda_2\,\sigma_{\mu\alpha}\omega_\nu{}^\alpha + \lambda_3\,\omega_{\mu\alpha}\omega_\nu{}^\alpha \rangle$$

➡ $\partial_\mu T^{\mu\nu} = 0$ are third order PDEs. How does one solve them?

- MIS formulation: promote $\Pi_{\mu\nu}$ to a new dynamical variable with eom

$$\Pi_{\mu\nu} = -\,2\,\eta\,\sigma_{\mu\nu} - \tau_\Pi \left(\langle u^\alpha \partial_\alpha \Pi_{\mu\nu} \rangle + \frac{d}{d-1}\,\Pi_{\mu\nu}\,\partial_\alpha u^\alpha \right) + \left\langle \frac{\lambda_1}{\eta^2}\,\Pi_{\mu\alpha}\Pi_\nu{}^\alpha - \frac{\lambda_2}{\eta}\,\Pi_{\mu\alpha}\omega_\nu{}^\alpha + \lambda_3\,\omega_{\mu\alpha}\omega_\nu{}^\alpha \right\rangle$$

➡ the eoms are 1st order and $\Pi_{\mu\nu} \to -\,2\,\eta\,\sigma_{\mu\nu}$ on a timescale set by τ_Π

MIS for gravity

- Order reduction: Ric $\sim \mathcal{O}(\epsilon)$ $\Rightarrow$ keep only the $\mathcal{O}(\epsilon)$ terms in the EOMs

$$G_{\mu\nu} = \epsilon \left(4\, \mathscr{C}\, W_{\mu}{}^{\alpha\beta\gamma}\, W_{\nu\alpha\beta\gamma} - \frac{1}{2}\, g_{\mu\nu}\, \mathscr{C}^2 + 8\, W_{\mu}{}^{\alpha}{}_{\nu}{}^{\beta}\, \nabla_\alpha \nabla_\beta \mathscr{C} \right) , \quad \mathscr{C} = W_{\mu\nu\rho\sigma} W^{\mu\nu\rho\sigma}$$

- Want:
  - Well-posed, 2nd order (in time) equations
  - Consistently incorporate the small corrections at long wavelengths whilst controlling to flow of energy to the UV
  - Capture non-linearities whilst remaining in the regime of validity of EFT

MIS for gravity

- Model problem: $\Box\,\phi = -\,\epsilon\,\partial_t^4\phi$
  - Option A: $\partial_t^4\phi = \partial_t^2 C(\phi)$ with $C(\phi) = \partial_x^2\phi$
  - Option B: $\partial_t^4\phi = \partial_x^4\phi$

➡ Both options lead to blow up

- Solution for the model problem: $\Box\,\phi = -\,\epsilon\,\partial_t^2\hat{C}$

$$\tau\partial_0\hat{C} + \sigma(\partial_t^2 - 2\beta^i\partial_{ti} + \beta^i\beta^j\partial_{ij})\hat{C} = C(\phi) - \hat{C}$$

  - $\hat{C} \to C(\phi)$ on a timescale σ/τ
  - σ and τ can be chosen to minimise the difference and dependence of the solution on them while preserving numerical stability

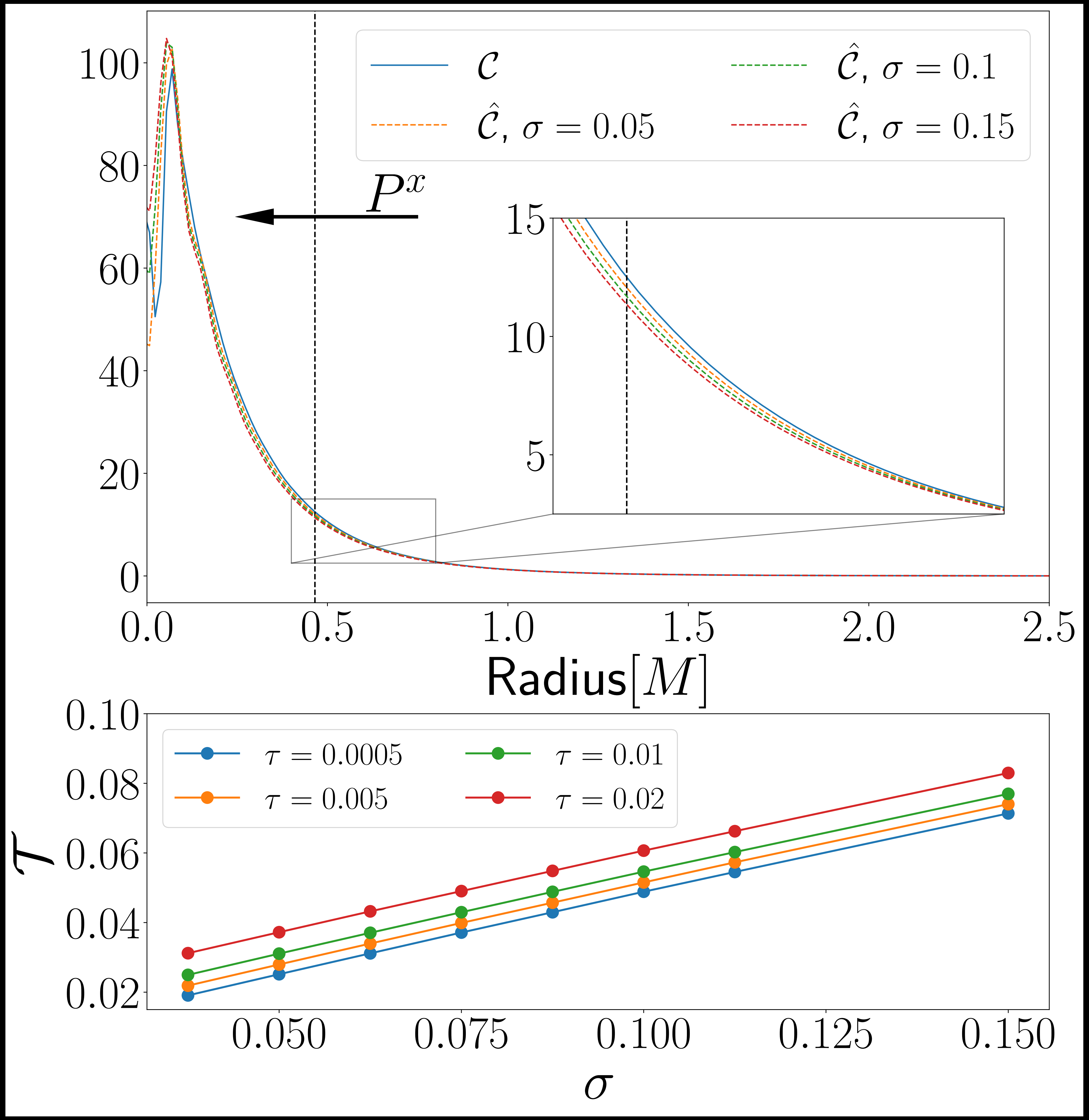
MIS for gravity

- Our solution: $$G_{\mu\nu} = \epsilon\left(4\,\hat{\mathscr{C}}\,W_{\mu}{}^{\alpha\beta\gamma}\,W_{\nu\alpha\beta\gamma} - \frac{1}{2}\,g_{\mu\nu}\,\hat{\mathscr{C}}^2 + 8\,W_{\mu}{}^{\alpha}{}_{\nu}{}^{\beta}\,\nabla_{\alpha}\nabla_{\beta}\hat{\mathscr{C}}\right)$$
$$(\partial_t^2 - 2\beta^i\partial_{ti} + \beta^i\beta^j\partial_{ij})\hat{\mathscr{C}} = \frac{1}{\sigma}\left(\mathscr{C} - \hat{\mathscr{C}} - \tau\partial_0\hat{\mathscr{C}}\right)$$

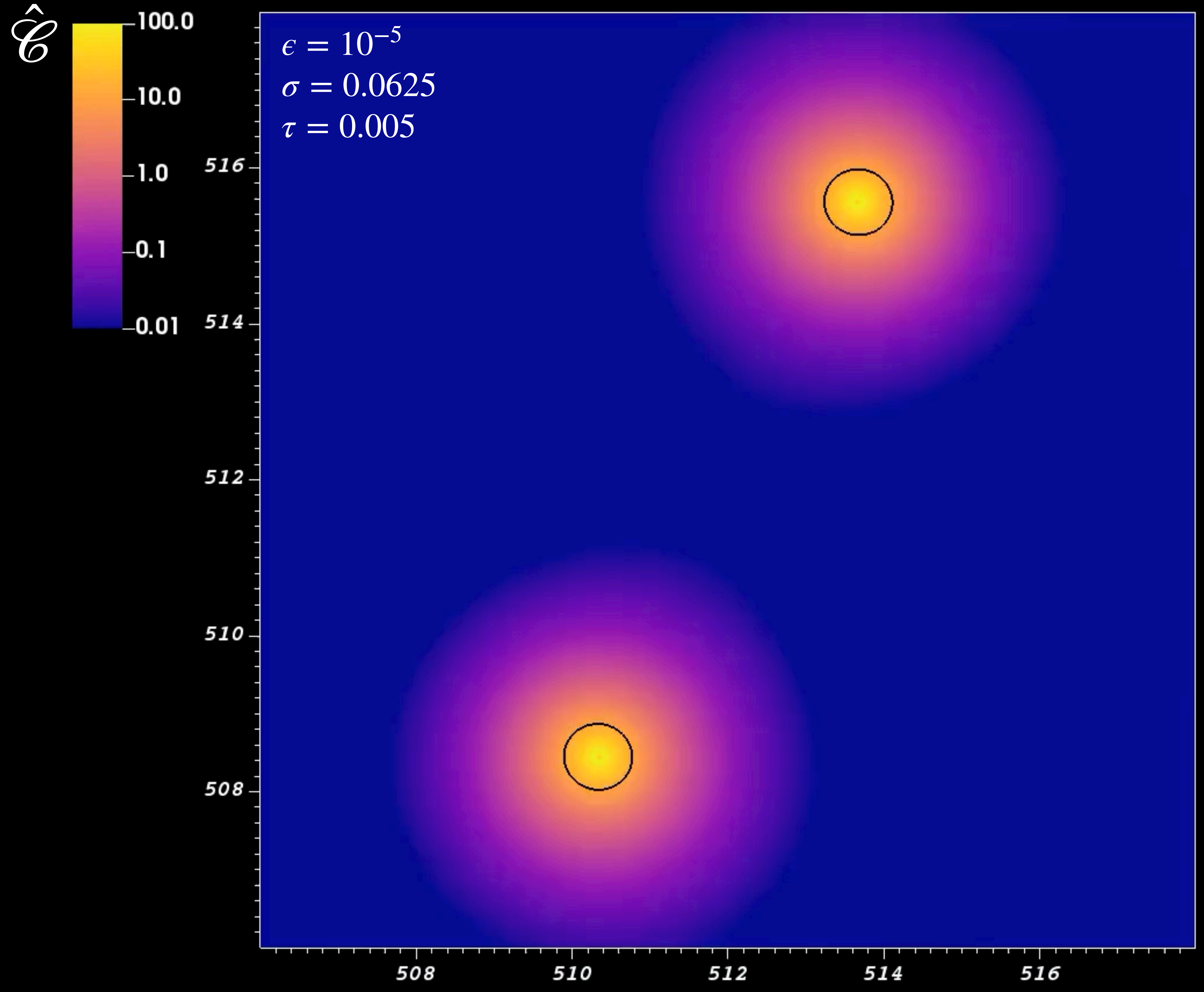
- Reduction of order to replace time derivatives on the RHS

- $\hat{\mathscr{C}} \to \mathscr{C}$ on a time scale set by σ/τ

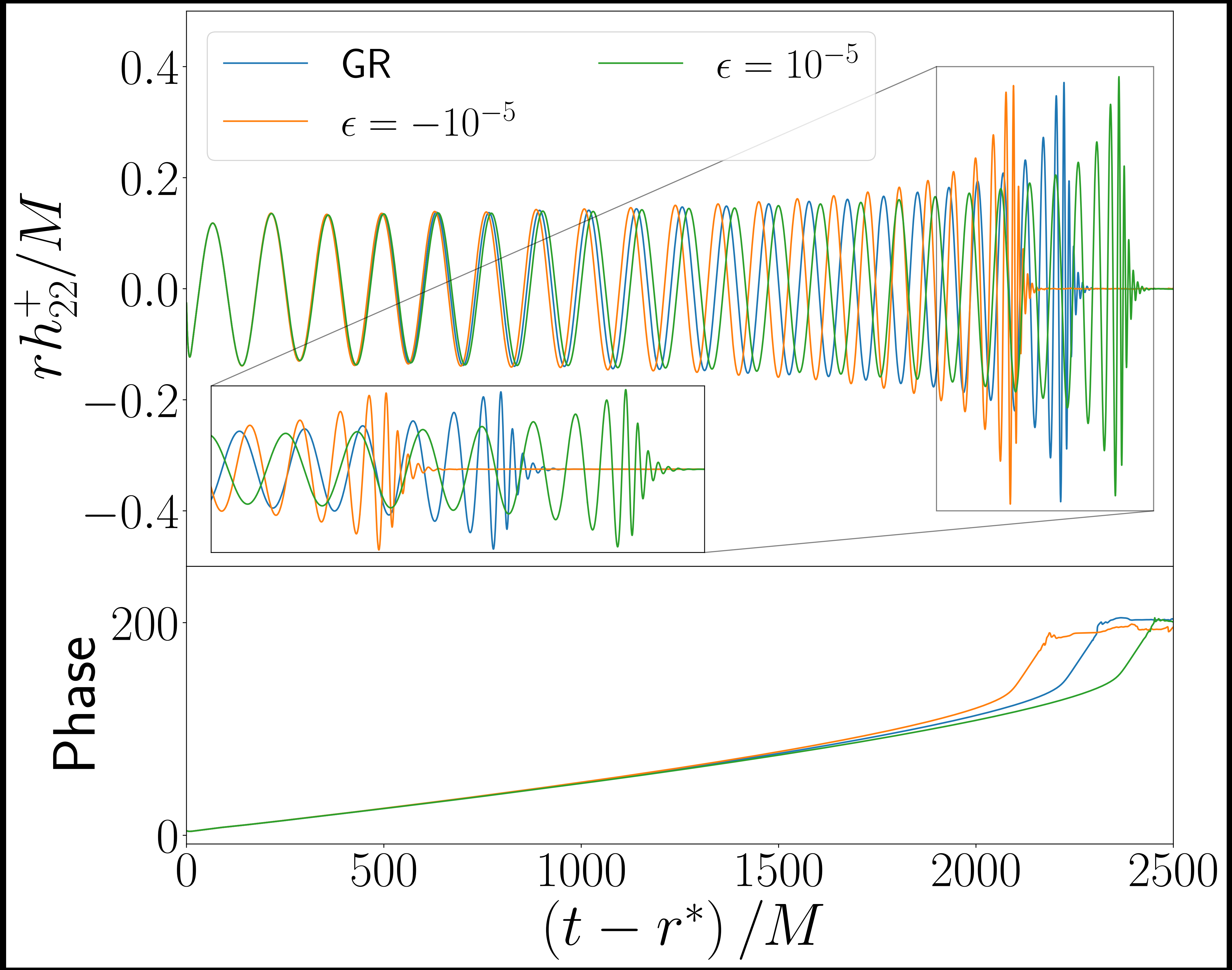
MIS for gravity: results



MIS for gravity: results



MIS for gravity: results



Conclusions and outlook

Conclusions

- We have performed simulations of black hole binary mergers in the 4∂ST and higher derivative theories of gravity treating them fully non-linearly
- Physically relevant initial data can be found such that the theory remains weakly coupled throughout the evolutions
- The waveforms exhibit a generic $O(1)$ de-phasing compared to the GR
- Applications: compact binary mergers, EFT of inflation, endpoint of the GL instability of black strings, finite coupling effects in holography

Thank your for your attention!