Electroweak monopoles and their black hole counterparts

Gervalle Romain

Journees Relativistes, Tours

June 1st, 2023

Reference papers:

R. G. and M. S. Volkov, *"Electroweak monopoles and their stability"*, Nucl. Phys. B 984 (2022) 115937

R. G. and M. S. Volkov, *"Electroweak multi-monopoles"*, Nucl. Phys. B 987 (2023) 116112

INSTITUT ORLÉANS • TOURS Mathématiques & Physique Théorique DENIS POISSON

université de TOURS

Electroweak theory of Weinberg and Salam

$$u_a v^a = u_1 v^1 + u_2 v^2 + u_3 v^3$$
$$\mu \in \{0, 1, 2, 3\}, \; a \in \{1, 2, 3\}$$

We conisder the *bosonic* sector of the **Weinberg-Salam theory**:

$$\mathcal{L}_{\text{WS}} = -\frac{1}{4g'^2} \overset{(Y)}{F}_{\mu\nu} \overset{(Y)}{F}{}^{\mu\nu} - \frac{1}{4g^2} \overset{(W)}{F}{}^a_{\mu\nu} \overset{(W)}{F}{}^{a\mu\nu} - (D_\mu \Phi)^\dagger D^\mu \Phi - \frac{\beta}{8} \left(\Phi^\dagger \Phi - 1 \right)^2,$$

where $\Phi = (\phi_1, \phi_2)^T$ is the Higgs *doublet*,

$$\overset{(Y)}{F}_{\mu\nu} = \partial_\mu Y_\nu - \partial_\nu Y_\mu,$$

$$\overset{(W)}{F}{}^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + \epsilon_{abc} W^b_\mu W^c_\nu,$$

$$D_\mu \Phi = \left(\partial_\mu - \frac{i}{2} Y_\mu - \frac{i}{2} \tau^a W^a_\mu \right) \Phi,$$

and g, g' and β are **experimentally known** parameters:

$$g' = \sqrt{0.23}, \; g^2 + g'^2 = 1, \; \beta = 1.88.$$

Boson masses (in units of 128.6 GeV):

$$m_Z = \frac{1}{\sqrt{2}}, \; m_W = \frac{g}{\sqrt{2}}, \; m_H = \sqrt{\frac{\beta}{2}}.$$

1. Electroweak monopoles
   - Spherically symmetric monopoles
   - Axially symmetric monopoles
2. Magnetic black holes
3. Conclusion

Dirac monopole embedded in the electroweak theory

$$W^1 = W^2 = 0, \quad W^3 = Y = \frac{n}{2}\cos\vartheta\, d\varphi, \quad \Phi = \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \tag{1}$$

→ Describes a **pointlike** magnetic charge located at $r = 0$:

$$\boxed{\vec{B} = \frac{P\vec{r}}{r^3} \quad \text{with} \quad P = \underbrace{P_{\mathrm{U}(1)}}_{\text{pointlike}} + \underbrace{P_{\mathrm{SU}(2)}}_{\text{pointlike}} = -\frac{n}{2e}, \quad e \equiv gg'.}$$

Dirac monopole embedded in the electroweak theory

$$W^1 = W^2 = 0, \quad W^3 = Y = \frac{n}{2} \cos\vartheta \, d\varphi, \quad \Phi = \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \tag{1}$$

→ Describes a **pointlike** magnetic charge located at $r = 0$:

$$\boxed{\vec{B} = \frac{P\vec{r}}{r^3} \quad \text{with} \quad P = \underbrace{P_{\mathrm{U}(1)}}_{\text{pointlike}} + \underbrace{P_{\mathrm{SU}(2)}}_{\text{pointlike}} = -\frac{n}{2e}, \quad e \equiv gg'.}$$

The fields (1) contains a **Dirac singular string** at $\vartheta = 0, \pi$ which can be removed by gauge transformations if $\boxed{n \in \mathbb{Z}}$.

→ **Dirac charge quantization** (n is called *"magnetic charge number"*).

The Dirac monopole has **infinite** energy,

→ Topological arguments exclude the regularization of the Y-sector.

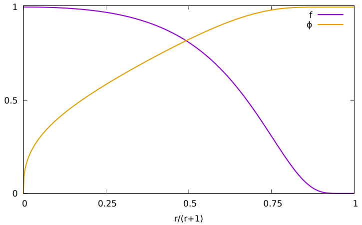
→ A regularization of the W-sector is possible thanks to the *non-abelian* degrees of freefom.

Cho-Maison monopole (1996)

$$W^1 = -f(r)\sin\vartheta\, d\varphi, \quad W^2 = f(r)\, d\vartheta, \quad W^3 = Y = \pm\cos\vartheta\, d\varphi, \quad \Phi = \begin{pmatrix} 0 \\ \phi(r) \end{pmatrix}.$$

Cho-Maison monopole (1996)

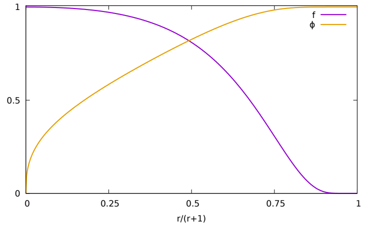
$$W^1 = -f(r)\sin\vartheta\, d\varphi, \quad W^2 = f(r)\, d\vartheta, \quad W^3 = Y = \pm\cos\vartheta\, d\varphi, \quad \Phi = \begin{pmatrix} 0 \\ \phi(r) \end{pmatrix}.$$



Profiles of the functions $f(r)$ and $\phi(r)$.
$\rightarrow$ At large r the Dirac monopole fields are recovered.

Cho-Maison monopole (1996)

$$W^1 = -f(r)\sin\vartheta\, d\varphi, \quad W^2 = f(r)\, d\vartheta, \quad W^3 = Y = \pm\cos\vartheta\, d\varphi, \quad \Phi = \begin{pmatrix} 0 \\ \phi(r) \end{pmatrix}.$$



Profiles of the functions $f(r)$ and $\phi(r)$.

$\rightarrow$ At large r the Dirac monopole fields are recovered.

Describes a Coulombian singularity at $r = 0$ surrounded by regular **non-Abelian** fields.

$$P = \underbrace{P_{\mathrm{U}(1)}}_{\text{pointlike}} + \underbrace{P_{\mathrm{SU}(2)}}_{\text{smooth}} = \pm\frac{1}{e}.$$

The energy consists of a **divergent** U(1) part + a **finite** SU(2) part:

$$E = \frac{2\pi}{g'^2}\int_0^\infty \frac{dr}{r^2} + E_{\mathrm{SU}(2)}.$$

Summary

Electroweak theory contains two types of static and **spherically symmetric** monopoles, both with **infinite** energy:

- Pointlike Dirac monopole for any value of the magnetic charge $n = \pm 1, \pm 2, \ldots$
- Cho-Maison monopole for $n = \pm 2$, can be viewed as a hybrid between a U(1) Dirac monopole and a SU(2) 't Hooft-Polyakov monopole.

Are they stable ? [R. G., Volkov, 2022]

Summary

Electroweak theory contains two types of static and **spherically symmetric** monopoles, both with **infinite** energy:

- Pointlike Dirac monopole for any value of the magnetic charge $n = \pm 1, \pm 2, \ldots$
- Cho-Maison monopole for $n = \pm 2$, can be viewed as a hybrid between a U(1) Dirac monopole and a SU(2) 't Hooft-Polyakov monopole.

Are they stable ? [R. G., Volkov, 2022]

- Cho-Maison is **stable** with respect to any (small) perturbations.
- All Dirac monopoles with $|n| > 1$ are **unstable**.
- The $|n| = 2$ Dirac monopole is unstable with respect to **spherically symmetric** perturbations.
  $\rightarrow$ The Cho-Maison monopole is spherically symmetric and has $|n| = 2$: it is the **stable remnant** of Dirac's monopole decay.

Axially symmetric ansatz

$$W^a_\mu dx^\mu \frac{\tau^a}{2} = \frac{\tau_2}{2}\left(-\frac{1}{r}H_1\, dr + H_2\, d\vartheta\right) + \frac{n}{2}\left((\cos\vartheta + H_3\,\sin\vartheta)\frac{\tau_3}{2} - H_4\,\sin\vartheta\,\frac{\tau_1}{2}\right) d\varphi,$$

$$Y^a_\mu dx^\mu = \frac{n}{2}\left(\cos\vartheta + y\,\sin\vartheta\right) d\varphi, \qquad \Phi = (\phi_1, \phi_2)^T \text{ with } \phi_1, \phi_2 \in \mathbb{R},$$

where $H_1,\ H_2,\ H_3,\ H_4$, y, $\phi_1,\ \phi_2$ are **7** functions of (r, ϑ) and $\boxed{2n \in \mathbb{Z}}$.

Axially symmetric ansatz

$$W^a_\mu dx^\mu \frac{\tau^a}{2} = \frac{\tau_2}{2}\left(-\frac{1}{r}H_1\, dr + H_2\, d\vartheta\right) + \frac{n}{2}\left((\cos\vartheta + H_3\,\sin\vartheta)\frac{\tau_3}{2} - H_4\,\sin\vartheta\,\frac{\tau_1}{2}\right) d\varphi,$$

$$Y^a_\mu dx^\mu = \frac{n}{2}(\cos\vartheta + y\,\sin\vartheta)\, d\varphi, \qquad \Phi = (\phi_1, \phi_2)^T \text{ with } \phi_1, \phi_2 \in \mathbb{R},$$

where $H_1,\ H_2,\ H_3,\ H_4$, y, $\phi_1,\ \phi_2$ are **7** functions of (r, ϑ) and $\boxed{2n \in \mathbb{Z}}$.

Gauge condition: $\boxed{r\,\partial_r H_1 - \partial_\vartheta H_2 = 0}$.

Axially symmetric ansatz

$$W^a_\mu dx^\mu \frac{\tau^a}{2} = \frac{\tau_2}{2}\left(-\frac{1}{r} H_1\, dr + H_2\, d\vartheta\right) + \frac{n}{2}\left((\cos\vartheta + H_3\, \sin\vartheta)\frac{\tau_3}{2} - H_4\, \sin\vartheta\, \frac{\tau_1}{2}\right) d\varphi,$$

$$Y^a_\mu dx^\mu = \frac{n}{2}\,(\cos\vartheta + y\, \sin\vartheta)\, d\varphi, \qquad \Phi = (\phi_1, \phi_2)^T \text{ with } \phi_1, \phi_2 \in \mathbb{R},$$

where $H_1,\ H_2,\ H_3,\ H_4,\ y,\ \phi_1,\ \phi_2$ are **7** functions of (r, ϑ) and $\boxed{2n \in \mathbb{Z}}$.

Gauge condition: $\boxed{r\,\partial_r H_1 - \partial_\vartheta H_2 = 0}$.

→ The field equations reduce to **7 nonlinear coupled PDEs** which are manifestly **elliptic**.
→ The principal part of the differential operator is **diagonal** and contains the Laplacian

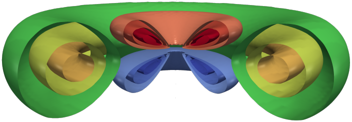
$$\boxed{\Delta = \frac{\partial^2}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2}{\partial \vartheta^2} + \dots}$$

→ We solve the PDEs with the **FreeFem finite element** solver, using a compactified radial coordinate $x = r/(r+1)$.

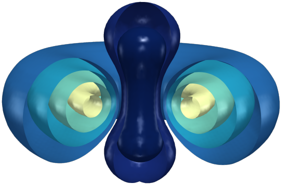
The solutions

We construct the axisymmetric monopoles up to $|n| = 200$ [R. G., Volkov, 2023].

$n = 4$



Level surfaces of SU(2) charge density (green $\to$ orange) and of current density (red and blue).



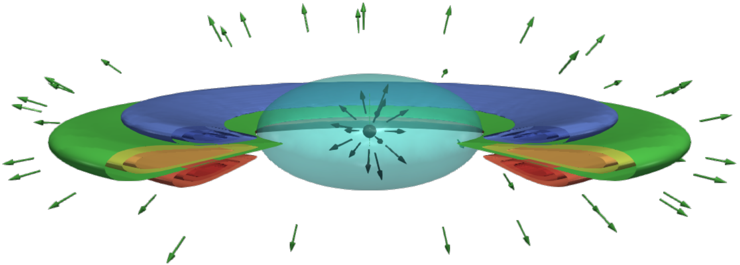
Level surfaces of regularized energy density (dark blue $\to$ white).

$$P = -\frac{n}{2e} = P_{\mathrm{U}(1)} + P_{\mathrm{SU}(2)}.$$

- $P_{\mathrm{U}(1)}$: pointlike.
  $P_{\mathrm{SU}(2)}$: **smoothly** distributed over tori centered on the equatorial plane.
- Monopoles contain two **oppositely directed** loops of **electric current**.
- **Repulsion** between the two current loops is balanced by the monopole magnetic field ($\sim$Laplace force).
- $|n| \to 2$: $\rho_{\mathrm{mag}}^{\mathrm{SU}(2)} \to$ **spherically symmetric** and the currents vanish (Cho-Maison monopole).

Large charge limit

$n = 40$



- The intense U(1) magnetic field in the central region produces a *bubble* of Higgs **false vacuum**: restoration of the full electroweak gauge symmetry.
- Inside the bubble: $|\Phi| = 0$, $W_\mu^a = 0$, $Y_\mu \sim$ Dirac monopole.
- Outside the bubble: $|\Phi| = 1$, W_μ^a, $Y_\mu \sim$ Dirac monopole.
- The non-linearly interacting fields produce the SU(2) magnetic ring and the superconducting current loops in a transition region.
  $\rightarrow$ **Electroweak corona** [Maldacena, 2020].

Coupling to gravity

The Weinberg-Salam equations are coupled to the **Einstein equations**,

$$G_{\mu\nu} = \kappa\, T_{\mu\nu}(Y_\mu, W^a_\mu, \Phi) \quad \text{with} \quad \kappa = 5.44 \times 10^{-33}.$$

Static and **axially symmetric** line element:

$$ds^2 = -e^{2U} N(r) dt^2 + e^{2K} \left(\frac{dr^2}{N(r)} + r^2 d\vartheta^2 \right) + e^{2W} r^2 \sin^2\vartheta\, d\varphi^2,$$

with $N(r) = 1 - r_H/r$ and U, K, W, **3** functions of $(r, \vartheta) \rightarrow$ **10 PDEs** to solve.

Coupling to gravity

The Weinberg-Salam equations are coupled to the **Einstein equations**,

$$G_{\mu\nu} = \kappa\, T_{\mu\nu}(Y_\mu, W^a_\mu, \Phi) \quad \text{with} \quad \kappa = 5.44 \times 10^{-33}.$$

Static and **axially symmetric** line element:

$$ds^2 = -e^{2U} N(r) dt^2 + e^{2K}\left(\frac{dr^2}{N(r)} + r^2 d\vartheta^2\right) + e^{2W} r^2 \sin^2\vartheta\, d\varphi^2,$$

with $N(r) = 1 - r_H/r$ and U, K, W, **3** functions of $(r, \vartheta) \rightarrow$ **10 PDEs** to solve.

Solutions:

- Reissner-Nordström (RN) black hole with Dirac monopole at its center: only solution for $r_H \geq r_H^{\max}(n)$.
- Black hole surrounded by non-Abelian **hairs**: exists for $r_H^{\min} \leq r_H < r_H^{\max}$.

$$\boxed{r_H^{\min} \propto \sqrt{\kappa}|n|, \quad r_H^{\max} \propto \sqrt{|n|} \quad \Rightarrow \quad |n| \leq 1/\kappa \sim 10^{32}.}$$

Properties of black hole solutions

- U(1) singularity hidden inside the horizon $\rightarrow$ **finite** ADM mass.

Properties of black hole solutions

- U(1) singularity hidden inside the horizon $\rightarrow$ **finite** ADM mass.
- **Small charge** black holes are described by **RN geometry + corrections of order** κ. If $r_H \sim r_H^{\text{min}}(n)$: same matter fields as for flat space monopoles.

Properties of black hole solutions

- U(1) singularity hidden inside the horizon $\rightarrow$ **finite** ADM mass.
- **Small charge** black holes are described by **RN geometry + corrections of order** κ. If $r_H \sim r_H^{\text{min}}(n)$: same matter fields as for flat space monopoles.
- Deviations from RN becomes important for **very large charges**, $|n| \sim 10^{32}$, in which case $\boxed{r_H \sim 1/\sqrt{\kappa} \sim 1\,\text{cm}}$ and $\boxed{M \sim M_{\text{Pl}}/\kappa \sim 10^{25}\,\text{kg} \sim M_{\text{Earth}}}$.

Properties of black hole solutions

- U(1) singularity hidden inside the horizon $\rightarrow$ **finite** ADM mass.
- **Small charge** black holes are described by **RN geometry + corrections of order** κ. If $r_H \sim r_H^{\text{min}}(n)$: same matter fields as for flat space monopoles.
- Deviations from RN becomes important for **very large charges**, $|n| \sim 10^{32}$, in which case $\boxed{r_H \sim 1/\sqrt{\kappa} \sim 1\,\text{cm}}$ and $\boxed{M \sim M_{\text{Pl}}/\kappa \sim 10^{25}\,\text{kg} \sim M_{\text{Earth}}}$.
- Non-Abelian monopoles are associated with the unstable modes $Y_{jm}(r, \vartheta)$, $j = |n|/2 - 1$, of the Dirac monopole.

  $\rightarrow$ Axial symmetry is only a **special case**.

  $\rightarrow$ For a given charge n, there should be $|n| - 1$ different hairy black holes.

Final remarks

- Axially symmetric electroweak monopoles for arbitrary magnetic charge n are constructed.
- For $|n| \gg 1$, they are strongly squashed and present a central bubble of size $\propto \sqrt{|n|}$ containing the pointlike U(1) charge $P_{\mathrm{U}(1)} = ng/(2g')$. Outside the bubble, the **corona** made of massive fields carries the SU(2) magnetic charge $P_{\mathrm{SU}(2)} = ng'/(2g)$.
- Coupling to gravity $\rightarrow$ **hairy** black holes whose mass and size vary from **Planck's values** up to values of **planetary mass** black holes.
- These black holes may be formed from primordial fluctuations at the early times of the Universe.

Final remarks

- Axially symmetric electroweak monopoles for arbitrary magnetic charge n are constructed.
- For $|n| \gg 1$, they are strongly squashed and present a central bubble of size $\propto \sqrt{|n|}$ containing the pointlike U(1) charge $P_{\mathrm{U}(1)} = ng/(2g')$. Outside the bubble, the **corona** made of massive fields carries the SU(2) magnetic charge $P_{\mathrm{SU}(2)} = ng'/(2g)$.
- Coupling to gravity $\rightarrow$ **hairy** black holes whose mass and size vary from **Planck's values** up to values of **planetary mass** black holes.
- These black holes may be formed from primordial fluctuations at the early times of the Universe.

Other results with FreeFem:

R. G. *"Chains of rotating boson stars"*, Phys. Rev. D 105, 124052 (2022).

Thank you for your attention.