

The e-MANTIS emulator: fast predictions of the non-linear matter power spectrum in $f(R)$ CDM cosmology

Iñigo Sáez-Casares

Collaborators: Yann Rasera, Baojiu Li

June 5, 2023

Testing the laws of gravity at cosmological scales

Why modified gravity?

- probe gravity at **cosmological scales** (less constrained than at local scales)
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The formation of the Large-Scale Structure (LSS) of the Universe is mainly driven by gravity:

- small initial fluctuations (inflation) grow by **gravitational collapse**
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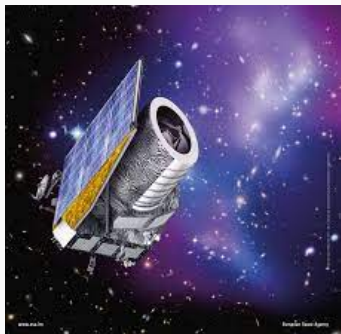
- small initial fluctuations (inflation) grow by **gravitational collapse**
- formation of galaxy clusters and groups

+ **accelerated** expansion driven by an *unknown* **dark energy** component

⇒ interesting playground to probe simultaneously the laws of gravity and the nature of dark energy

Measuring the structure of the Universe

→ One of the objectives of the ongoing (DESI) and future (Euclid, LSST) next generation large surveys.



EUCLID satellite.



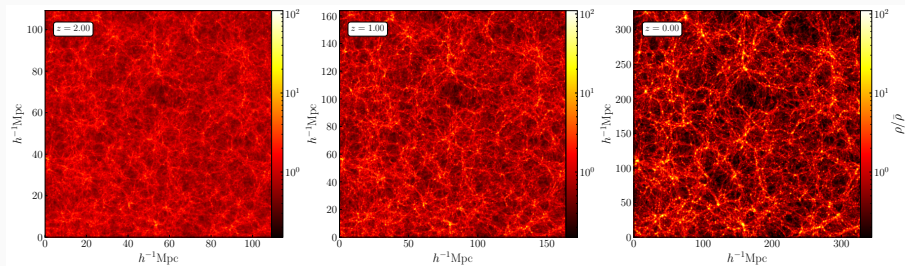
LSST telescope.

→ **Theoretical predictions** will be required in order to interpret the observations.

Cosmological N-body simulations

→ In the **non-linear regime** ($k \gtrsim 0.1 \text{ hMpc}^{-1}$) it is easier to discriminate between different **dark energy** and **modified gravity** models.

→ **N-body simulations** are required to obtain accurate theoretical predictions (but are very time consuming...).



Faster predictions: emulator approach

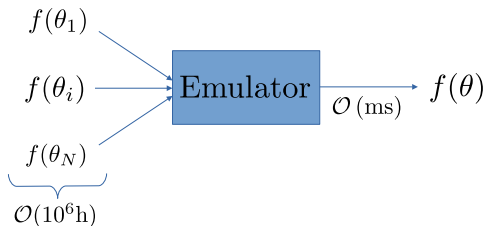
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Solution : Build an *emulator*.

→ Interpolate between the results of a set of simulations, run with different cosmological parameters.



The $f(R)$ gravity model

Simple MG model, with an extra term in the action:

$$S_{\text{EH}} = \frac{c^4}{16\pi G} \int d^4x \sqrt{-g} [R - 2\Lambda] \longrightarrow \frac{c^4}{16\pi G} \int d^4x \sqrt{-g} [R + f(R)]$$

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→ Depending on the form of $f(R)$ this model can:

- produce **cosmic acceleration**,
- exhibit a **screening** mechanism \Rightarrow GR recovered in **high density** environments (where gravity is well constrained).

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In the high density limit, fixing $c_1/c_2 = 6\Omega_\Lambda/\Omega_m$:

$$f(R) \simeq -2\Lambda + \frac{c_1}{c_2} m^2 \left(\frac{m^2}{R} \right)^n \xrightarrow{R \gg m^2} -2\Lambda$$

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→ Closely reproduces the expansion history of Λ CDM.

→ Deviations from GR disappear in **high density** environments (CMB, solar system).

→ Two remaining free parameters: n and $c_1/c_2^2 \sim f_{R_0}$.

Cosmological simulations in $f(R)$ gravity

N-body cosmological simulations with ECOSMOG [Li et al. 2012, Bose et al. 2017], a modified version of RAMSES [Teyssier 2002].

→ optimized $f(R)$ solver limited to $n = 1$, remaining free parameter: f_{R_0} .

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Newtonian limit:

$$\frac{1}{a^2} \nabla^2 \phi = \frac{4}{3} \times 4\pi G \delta\rho - \frac{1}{6} \delta R(f_R) \quad \& \quad \frac{1}{a^2} \nabla^2 f_R = \frac{1}{3} [\delta R(f_R) - 8\pi G \delta\rho]$$

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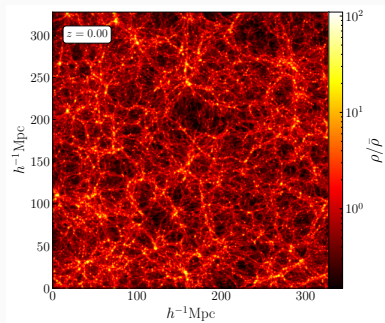
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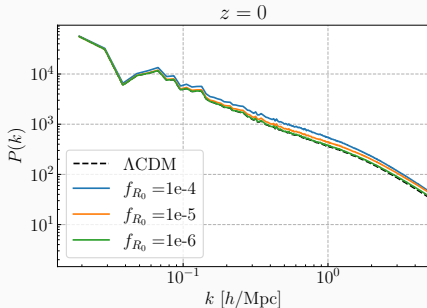
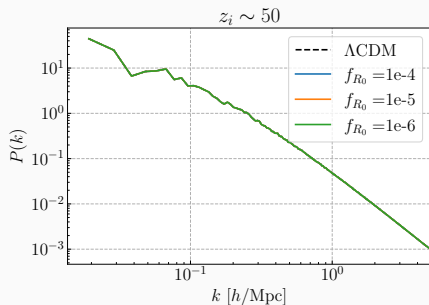
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Simulations:

- Volume: $(328.125 h^{-1} \text{Mpc})^3$
- 512^3 dark matter particles
- Mass resolution: $m_p \sim 2 \cdot 10^{10} h^{-1} M_\odot$
- ~ 2 – 10 slower than Λ CDM



Influence de $f(R)$ sur la distribution de matière



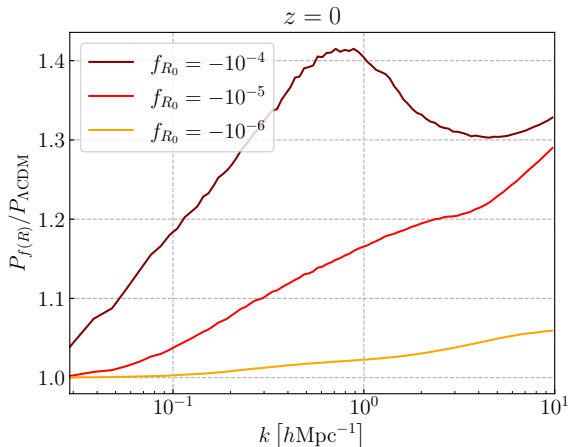
Left: Initial spectrum (start of the simulation).

Right: Final spectrum (end of the simulation).

→ The power spectrum is **amplified** by $f(R)$ gravity.

→ This amplification is stronger at **non-linear scales** ($k > 0.1 \text{ hMpc}^{-1}$).

Power spectrum boost due to $f(R)$ gravity

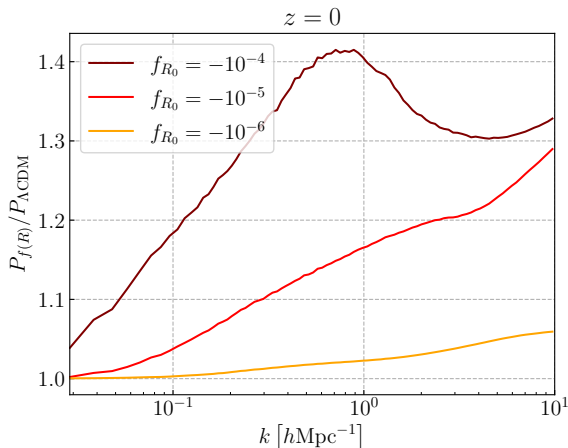


Power spectrum boost:

- cancellation of large scale variance
- cancellation of numerical resolution errors
- only three parameters: f_{R_0} , Ω_m and σ_8

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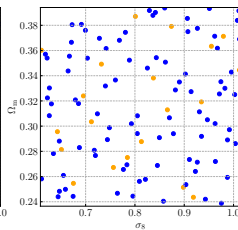
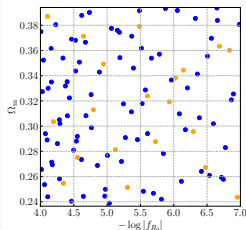
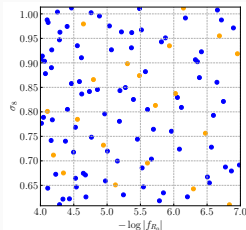
Strategy: emulate the boost $B(k)$, using pairs of $f(R)$ CDM and Λ CDM simulations.

Caveat: Λ CDM emulator required to get the full power spectrum.

Sampling the parameter space

Latin Hypercube Sampling (LHS):

- 90 **training** models (blue)
- 20 **validation** models (orange)



→ 5 pairs of Λ CDM & $f(R)$ CDM simulations per model

→ a total of 1100 simulations $\Rightarrow \sim 3\text{Mh}$

Data compression & emulation

We want to build an emulator for $B(k; \theta_i)$, where different k -bins ($N_k \simeq 100$) are highly correlated between each other.

→ We can compress the data with a Principal Component Analysis (PCA):

$$B(k; \theta_i) = \sum_{j=1}^{N_{\text{PCA}}} \alpha_j(\theta_i) \phi_j(k) + \epsilon$$

where the $\phi_j(k)$ are a set of (empirical) **orthogonal** basis functions.

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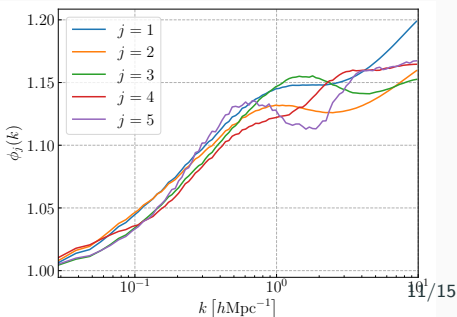
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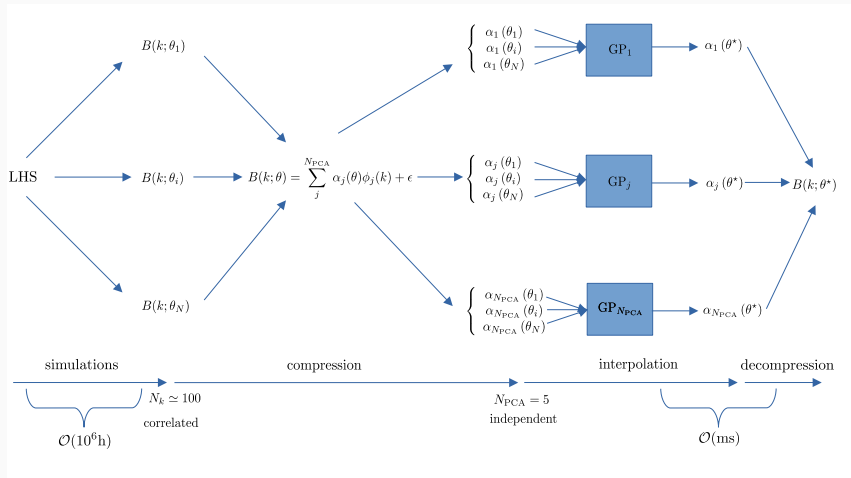
where the $\phi_j(k)$ are a set of (empirical) **orthogonal** basis functions.

→ **5 independent** $\alpha_j(\theta_i)$ coefficients are enough to fully describe the power spectrum boost.

→ We build an emulator for each $\alpha_j(\theta_i)$ with a **Gaussian Processes Regression**.



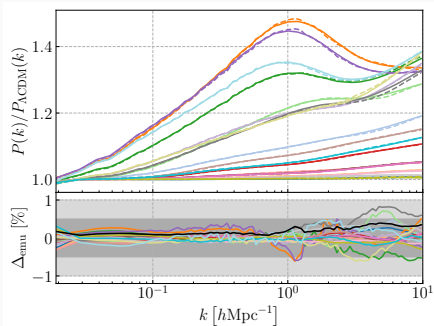
Emulator design



Standard procedure: [Habib et al. 2007], [Lawrence et al. 2010], [Lawrence et al. 2017], [Nishimichi et al. 2019], [Angulo et al. 2021], [Arnold et al. 2022] and others.

→ PCA + GP using SCIKIT-LEARN [Pedregosa et al. 2011].

Emulator validation



Emulation errors smaller than 1% for:

- $0.03h\text{Mpc}^{-1} < k < 10h\text{Mpc}^{-1}$,
- $0 < z < 2$.

Additional checks:

- large scale variance errors $< 1\%$
- small scale resolution errors $< 3\%$
for $k < 7h\text{Mpc}^{-1}$

→ **Accurate** and **fast** ($\sim 10\text{ms}$) emulator able to predict the matter power spectrum boost in $f(R)\text{CDM}$ cosmology.

→ Such an emulator could be used to constrain $f(R)$ gravity with weak lensing analyses.

[Submitted on 15 Mar 2023]

The e-MANTIS emulator: fast predictions of the non-linear matter power spectrum in $f(R)$ CDM cosmology

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In order to probe modifications of gravity at cosmological scales, one needs accurate theoretical predictions. N-body simulations are required to explore the non-linear regime of structure formation but are very time consuming. In this work, we build an emulator, dubbed e-MANTIS, that performs an accurate and fast interpolation between the predictions of a given set of cosmological simulations, in $f(R)$ modified gravity, run with ECOSMOG. We sample a wide 3D parameter space given by the current background scalar field value $10^{-7} < |f_{R_0}| < 10^{-4}$, matter density $0.24 < \Omega_m < 0.39$, and primordial power spectrum normalisation $0.6 < \sigma_8 < 1.0$, with 110 points sampled from a Latin Hypercube. For each model we perform pairs of $f(R)$ CDM and Λ CDM simulations covering an effective volume of $(560 h^{-1} \text{Mpc})^3$ with a mass resolution of $\sim 2 \times 10^{10} h^{-1} M_\odot$. We compute the matter power spectrum boost due to $f(R)$ gravity $B(k) = P_{f(R)}(k)/P_{\Lambda\text{CDM}}(k)$ and build an emulator using a Gaussian Process Regression method. The boost is mostly independent of h , n_s , and Ω_b , which reduces the dimensionality of the relevant cosmological parameter space. Additionally, it is much more robust against statistical and systematic errors than the raw power spectrum, thus strongly reducing our computational needs. The resulting emulator has a maximum error of 3% across the whole cosmological parameter space, for scales $0.03 h\text{Mpc}^{-1} < k < 7 h\text{Mpc}^{-1}$, and redshifts $0 < z < 2$, while in most cases the accuracy is better than 1%. Such an emulator could be used to constrain $f(R)$ gravity with weak lensing analyses.

→ arXiv:2303.08899

→ Emulator available as a
python package:

`https://gitlab.obspm.fr/
e-mantis/e-mantis`

emantis 1.0.3

`pip install emantis`

Latest version

Released: Apr 19, 2023

A cosmological emulator for non-linear large-scale structure formation studies in extended dark energy and gravity theories.

Navigation

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Project description

python 3.8.3 DOI: 10.5281/zenodo.7790961

e-MANTIS: Emulator for Multiple observable ANalysis in extended cosmological TheorieS

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- Installation
- Documentation and usage

Future developments

→ **New observables:**

- DM halo power spectrum multipoles (RSD)
- DM halo mass function
- DM halo density profiles

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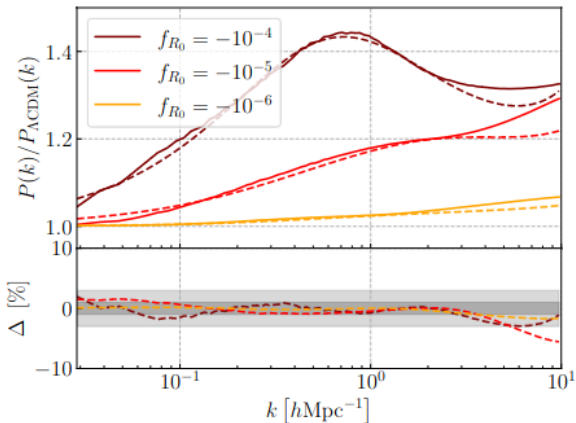
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→ Our simulation data are available upon request: don't hesitate to contact us.

Thank you!

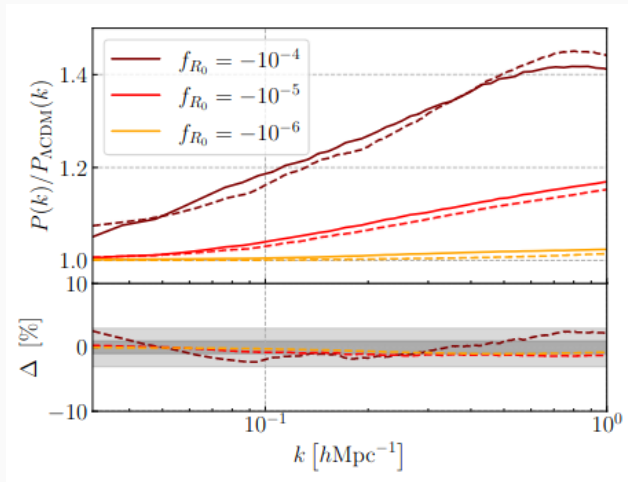
Comparison with other predictions



e-MANTIS vs Winther et al. (2019) fitting formula

→ Only depends on f_{R_0} .

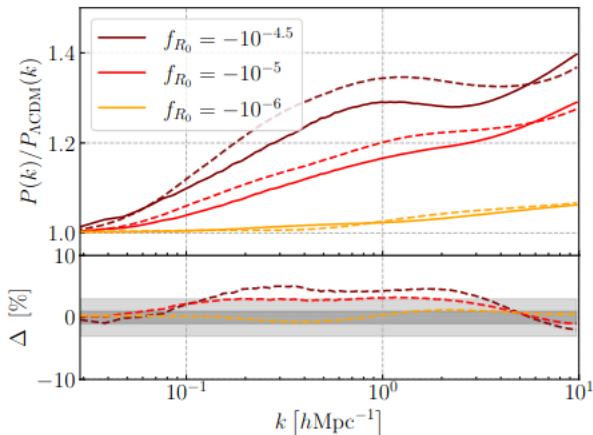
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e-MANTIS vs Ramachandra et al. (2021) emulator

→ Based on COLA simulations, less accurate than N-body at small scales.

Comparison with other predictions



e-MANTIS vs FORGE (Arnold et al. 2022) emulator

→ Based on N-body, using a different simulation code (AREPO vs RAMSES).