The Cosmological Flow

of Primordial Correlators

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IAP

erc — European Research Council
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GEODESI

Cosmology: A History of Time

Cosmological fluctuations are **correlated** on large scales

Time

LSS

CMB

Reheating surface

Transfer function

$$\langle X^2(\boldsymbol{k}, t)\rangle = \mathbf{T}^2(\boldsymbol{k}; t, t_0)\langle X^2(\boldsymbol{k}, t_0)\rangle$$

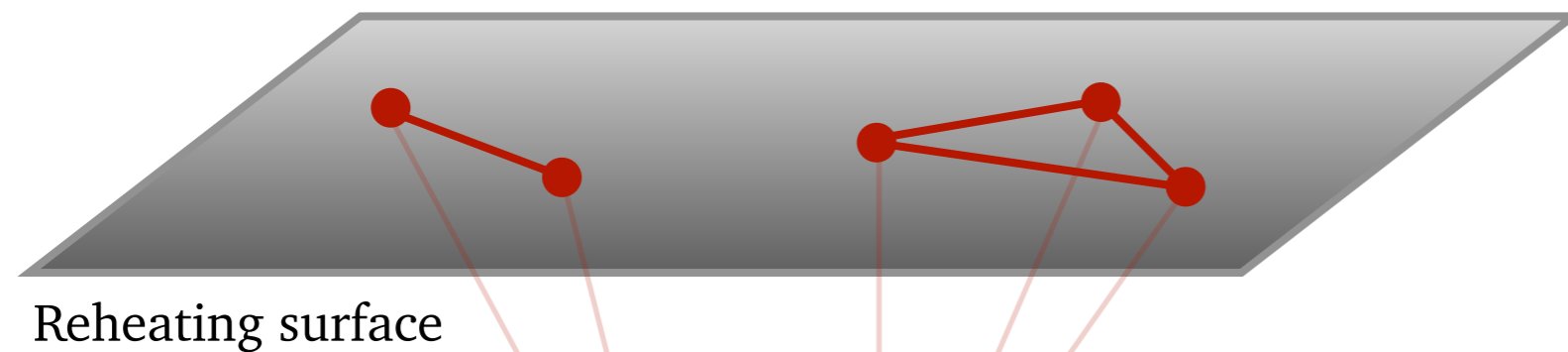- Linear clustering
- Dark matter, photons, baryons
- …
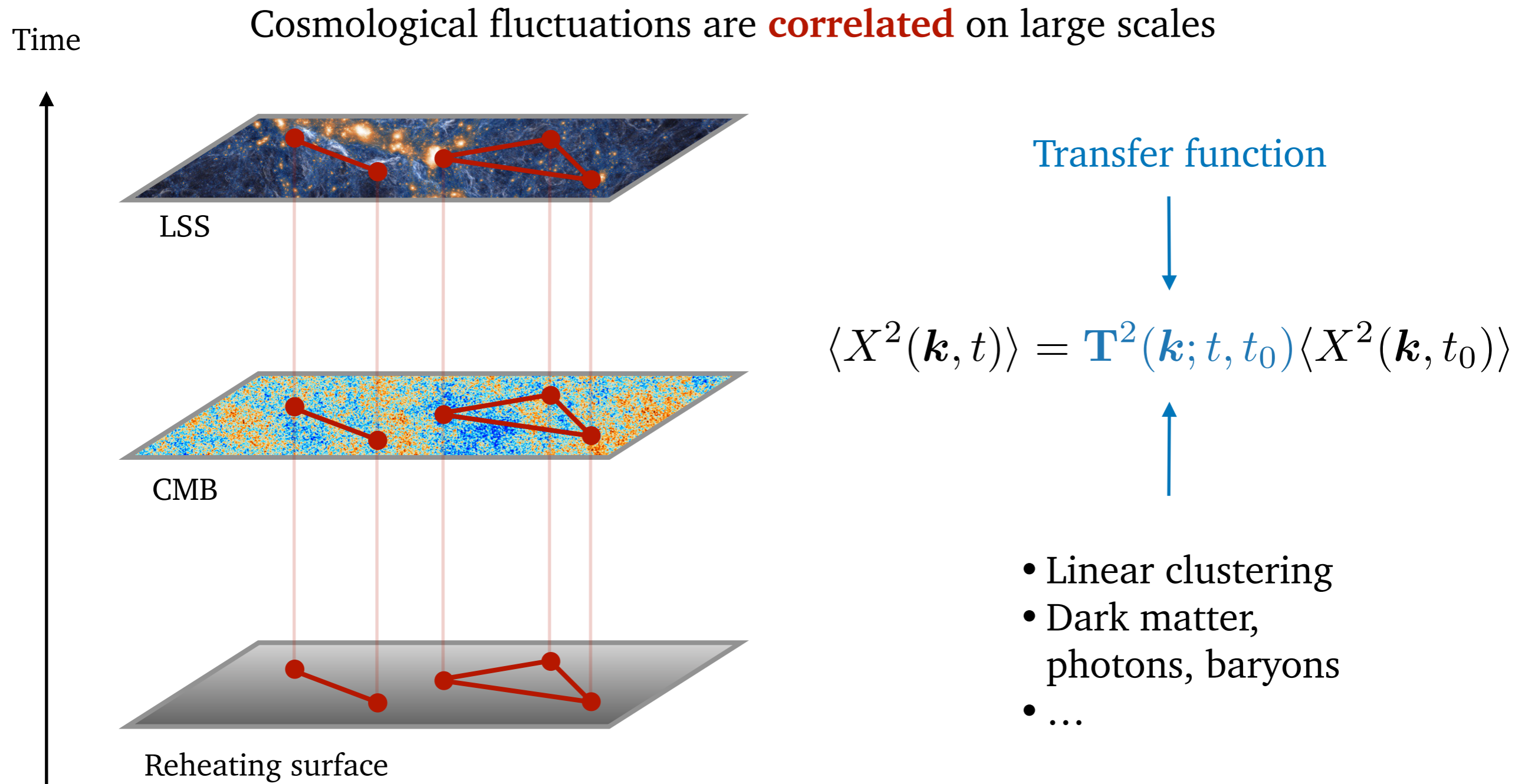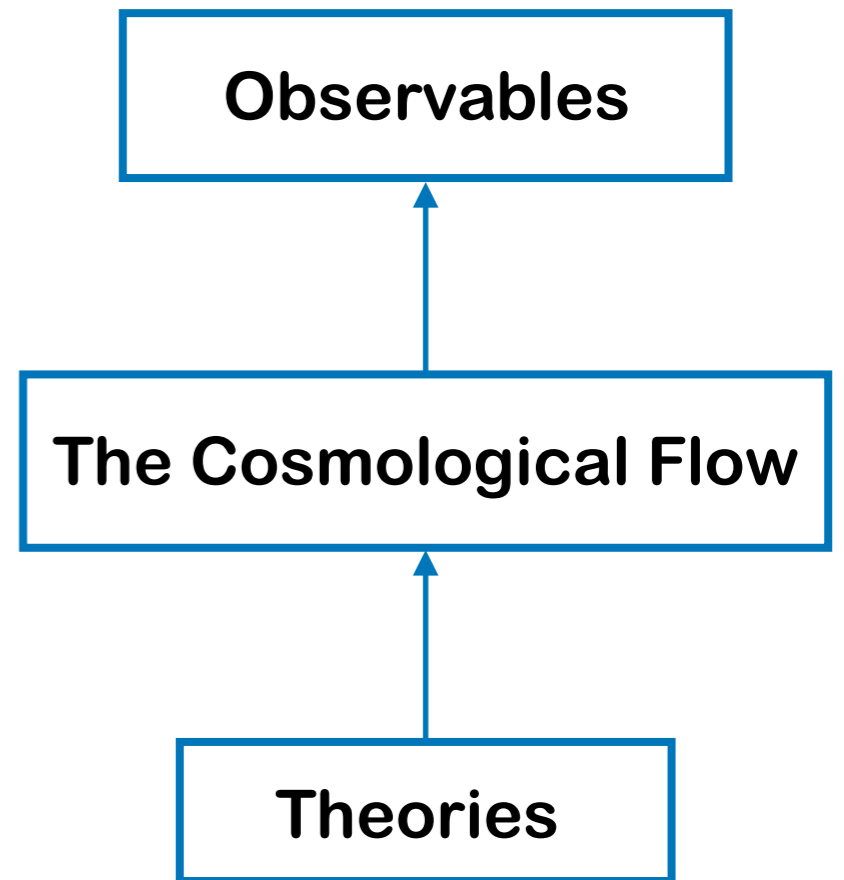
The physics is encoded in the **time evolution** of these fluctuations

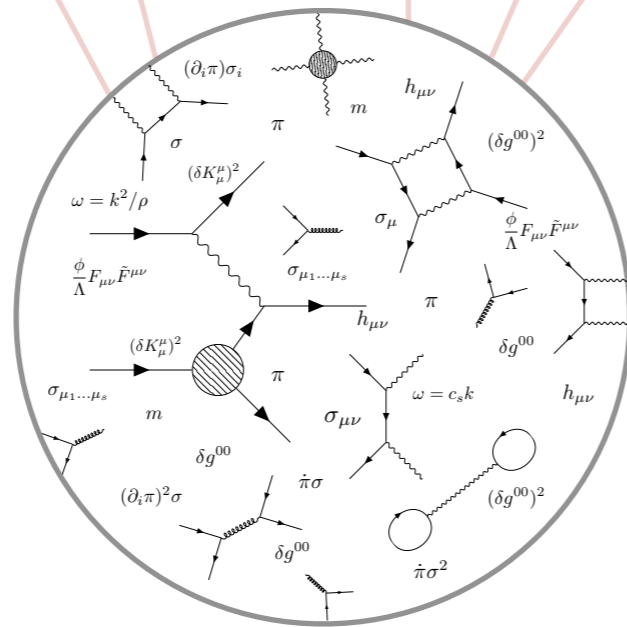# The Cosmological Flow Philosophy

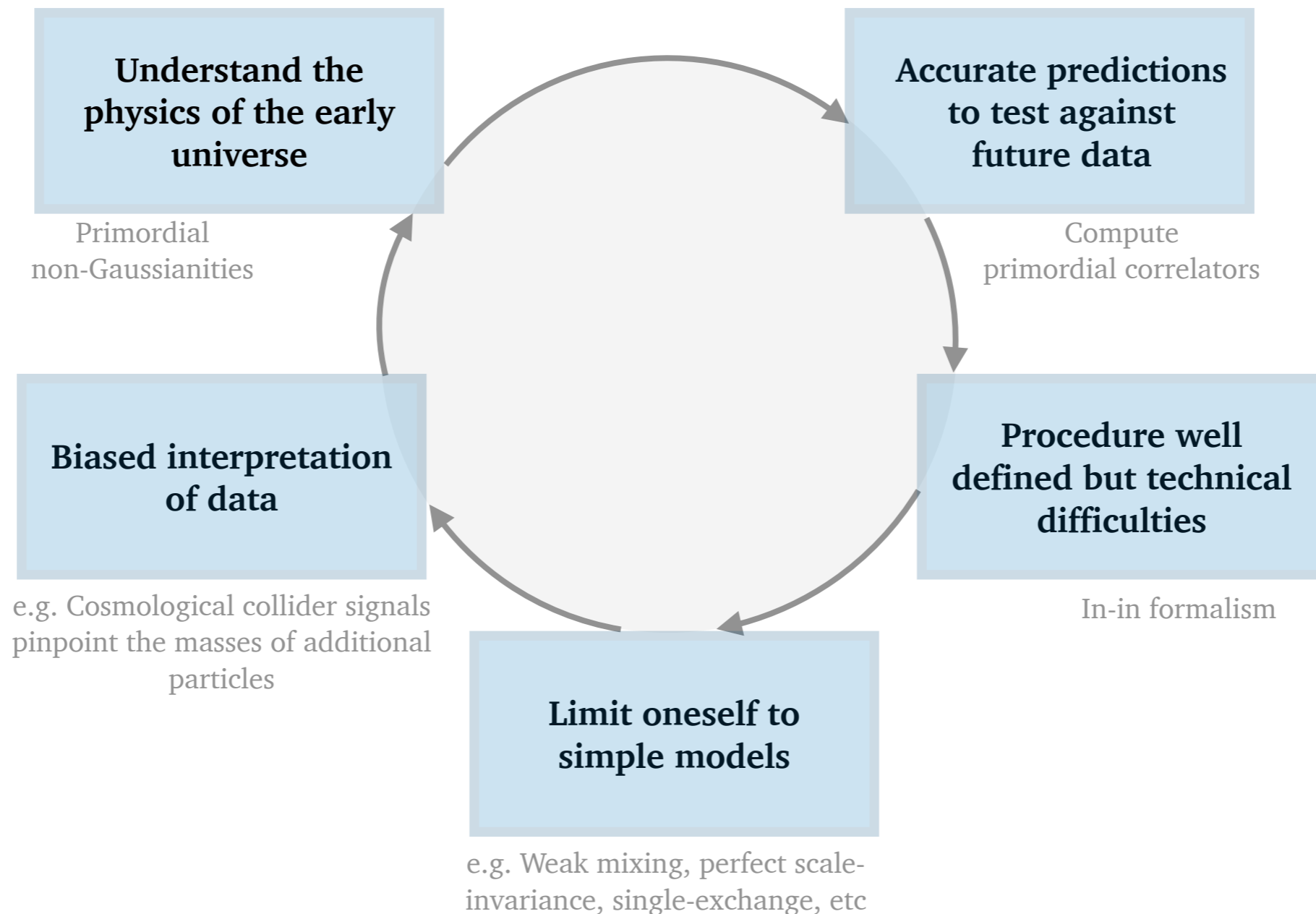Inflation is a unique probe of the physics at the highest reachable energies

Follow the **time evolution** of primordial fluctuations from their origin as **quantum vacuum fluctuations** to the reheating surface

Why the Cosmological Flow: Break the Vicious Circle

The **Cosmological Flow** is an **efficient** and **systematic** approach
to compute primordial correlators

Understand the physics of the early universe

Primordial non-Gaussianities

Accurate predictions to test against future data

Compute primordial correlators

Biased interpretation of data

e.g. Cosmological collider signals pinpoint the masses of additional particles

Procedure well defined but technical difficulties

In-in formalism

Limit oneself to simple models

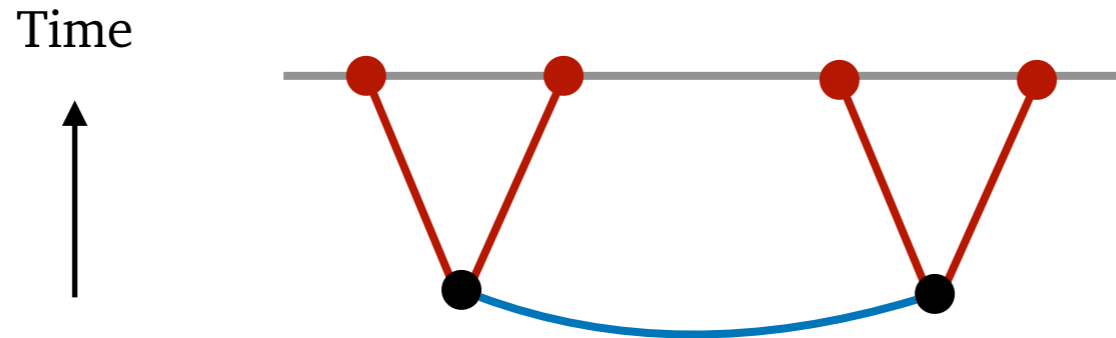e.g. Weak mixing, perfect scale-invariance, single-exchange, etc

Outline

I. The Cosmological Flow

II. CosmoFlow Starting Guide

The Cosmological Flow

In practice, we compute Feynman-Witten **diagrams**

Time

$$\langle \boldsymbol{X}^4 \rangle = \int_{-\infty}^{0} \mathrm{d}t \, \mathrm{d}t' V(t)V(t') \, K(k_1, t) \, K(k_2, t) \, \mathcal{G}(k_{12}, t, t') K(k_3, t') K(k_4, t')$$

Nested time integrals

Late-time correlators
receive contributions
from all times
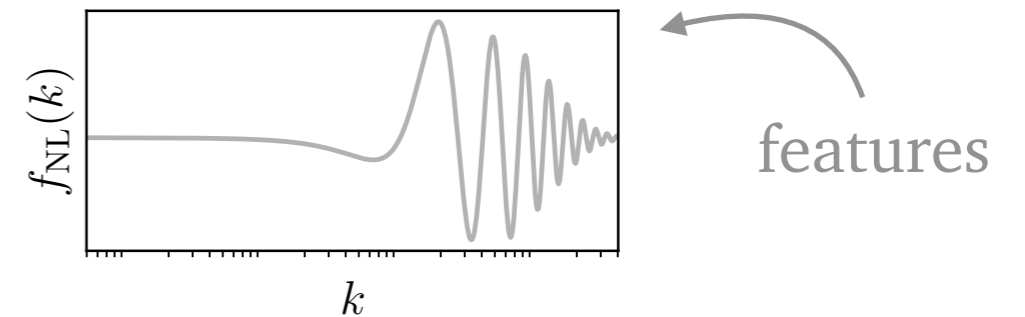
Complicated
hypergeometric functions

Limitations of Analytical Methods

Weak Quadratic Mixing

$$\mathcal{L}^{(2)} \supset \rho\dot{\phi}\sigma$$

treated perturbatively

(Near) Scale-Invariance

features

Only Single-Exchange Diagram
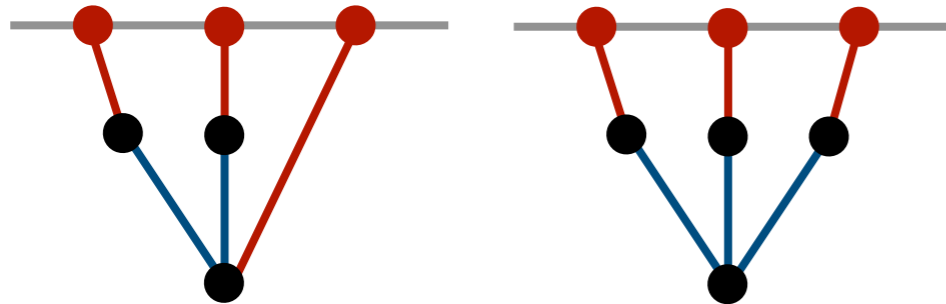
Large hierarchy of masses/couplings but not the intermediate regimes

$$\rho \quad \ll \quad H \quad \ll \quad m$$

energy

Often only 1 or 2 Fields

$$m_\phi \quad m_1 \quad m_2 \quad H \quad m_3$$

energy
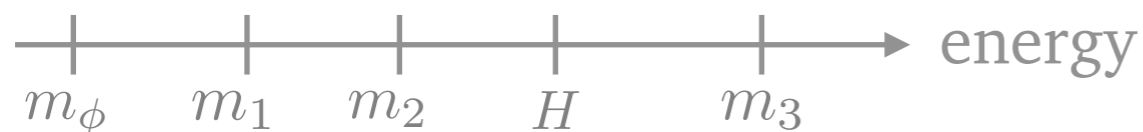
Treatment of Equilateral and Squeezed Configurations Separately

$$\sim e^{-\pi\mu} \qquad \sim \frac{1}{\mu^2}$$

The Cosmological Flow

From first principles and at tree-level, the time evolution of primordial correlators is encoded in the **flow equations** (Ehrenfest theorem)
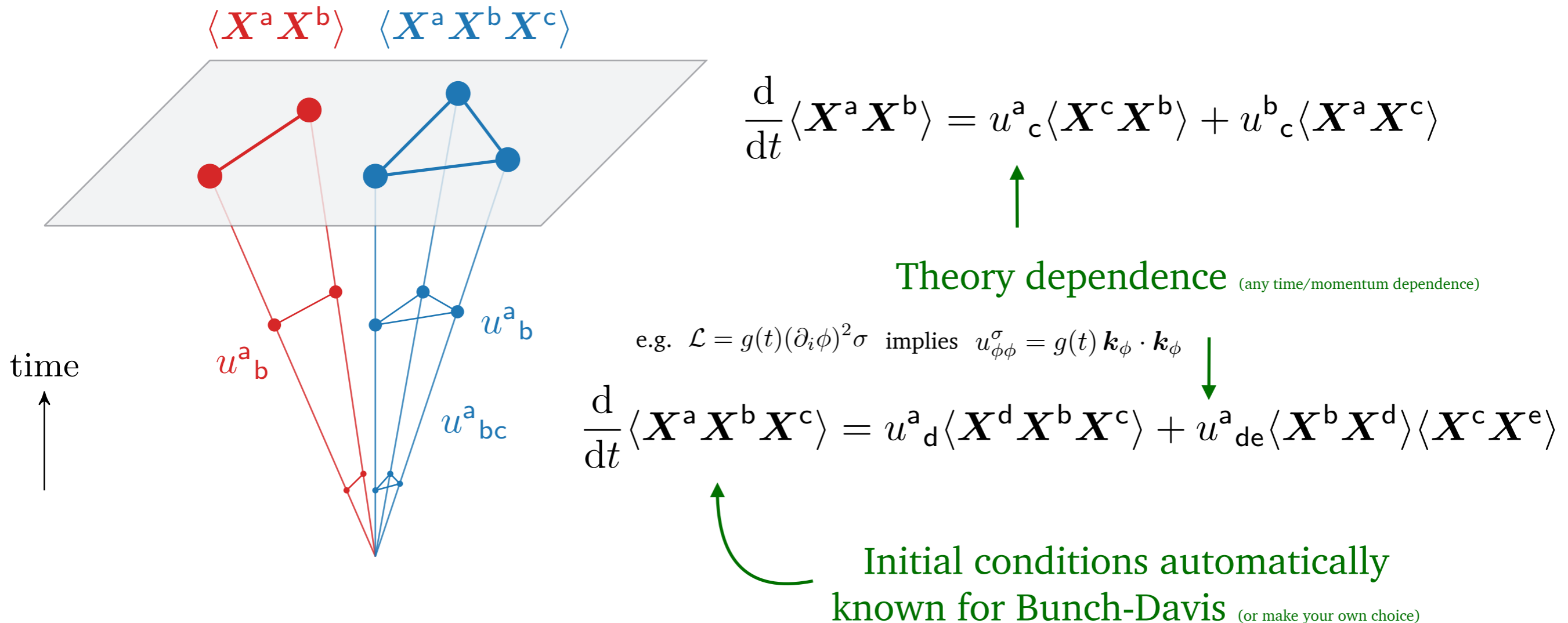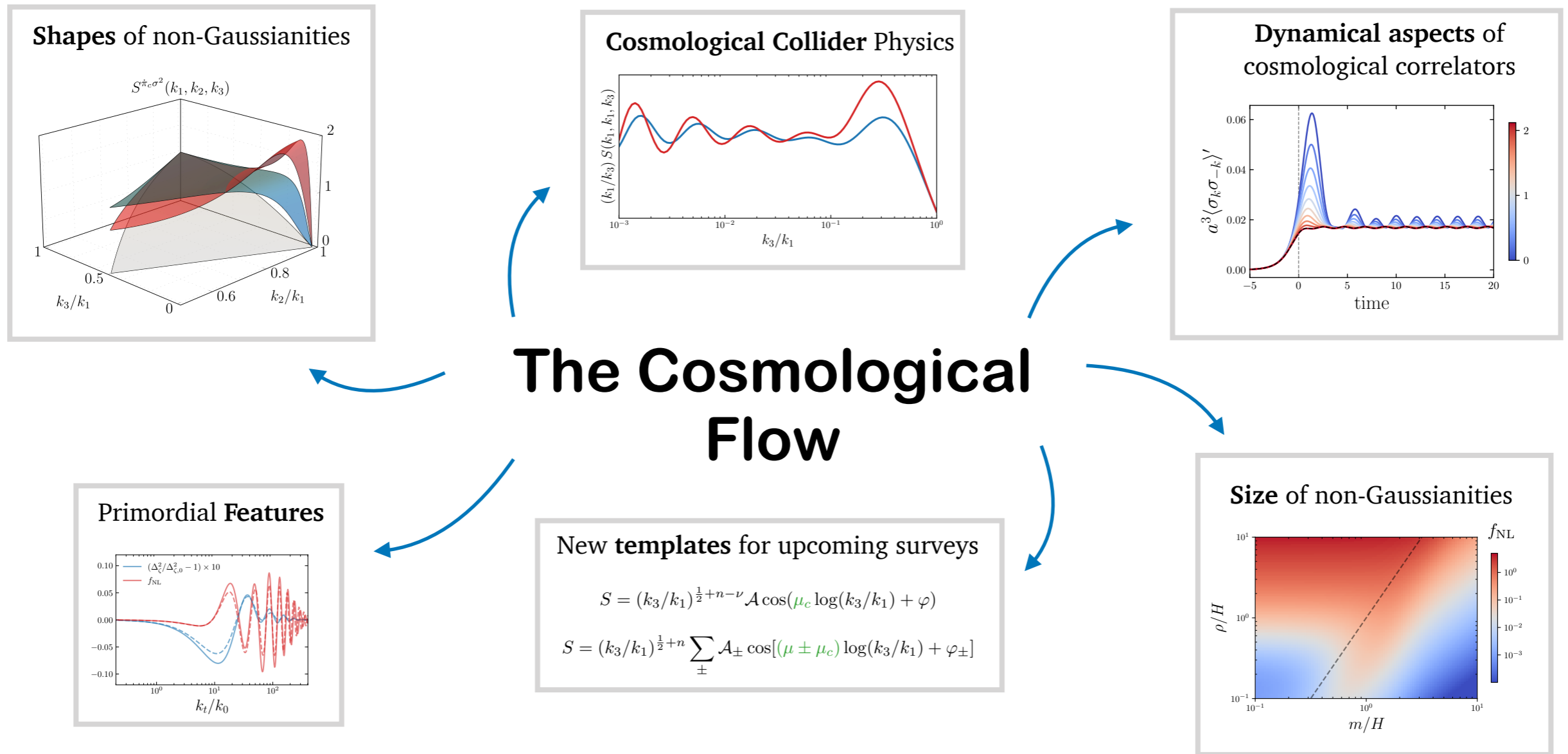

$\langle \boldsymbol{X}^{\mathsf{a}} \boldsymbol{X}^{\mathsf{b}} \rangle$ $\langle \boldsymbol{X}^{\mathsf{a}} \boldsymbol{X}^{\mathsf{b}} \boldsymbol{X}^{\mathsf{c}} \rangle$

$u^{\mathsf{a}}{}_{\mathsf{b}}$

$u^{\mathsf{a}}{}_{\mathsf{b}}$

$u^{\mathsf{a}}{}_{\mathsf{bc}}$

time

$$\frac{\mathrm{d}}{\mathrm{d}t} \langle \boldsymbol{X}^{\mathsf{a}} \boldsymbol{X}^{\mathsf{b}} \rangle = u^{\mathsf{a}}{}_{\mathsf{c}} \langle \boldsymbol{X}^{\mathsf{c}} \boldsymbol{X}^{\mathsf{b}} \rangle + u^{\mathsf{b}}{}_{\mathsf{c}} \langle \boldsymbol{X}^{\mathsf{a}} \boldsymbol{X}^{\mathsf{c}} \rangle$$

Theory dependence (any time/momentum dependence)

e.g. $\mathcal{L} = g(t)(\partial_i \phi)^2 \sigma$ implies $u^{\sigma}_{\phi\phi} = g(t)\, \boldsymbol{k}_\phi \cdot \boldsymbol{k}_\phi$

$$\frac{\mathrm{d}}{\mathrm{d}t} \langle \boldsymbol{X}^{\mathsf{a}} \boldsymbol{X}^{\mathsf{b}} \boldsymbol{X}^{\mathsf{c}} \rangle = u^{\mathsf{a}}{}_{\mathsf{d}} \langle \boldsymbol{X}^{\mathsf{d}} \boldsymbol{X}^{\mathsf{b}} \boldsymbol{X}^{\mathsf{c}} \rangle + u^{\mathsf{a}}{}_{\mathsf{de}} \langle \boldsymbol{X}^{\mathsf{b}} \boldsymbol{X}^{\mathsf{d}} \rangle \langle \boldsymbol{X}^{\mathsf{c}} \boldsymbol{X}^{\mathsf{e}} \rangle$$

Initial conditions automatically known for Bunch-Davis (or make your own choice)

The **Cosmological Flow** is an **efficient** and **systematic** approach to compute primordial correlators

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Probing High-Energy Aspects of Inflation

The Cosmological Flow offers new possibilities for **studying**, **exploring** and **understanding** inflationary physics

Shapes of non-Gaussianities

$S^{\dot\pi_c\sigma^2}(k_1, k_2, k_3)$

Cosmological Collider Physics

Dynamical aspects of cosmological correlators

The Cosmological Flow

Primordial **Features**

New **templates** for upcoming surveys

$$S = (k_3/k_1)^{\frac{1}{2}+n-\nu} \mathcal{A} \cos(\mu_c \log(k_3/k_1) + \varphi)$$

$$S = (k_3/k_1)^{\frac{1}{2}+n} \sum_{\pm} \mathcal{A}_\pm \cos[(\mu \pm \mu_c) \log(k_3/k_1) + \varphi_\pm]$$

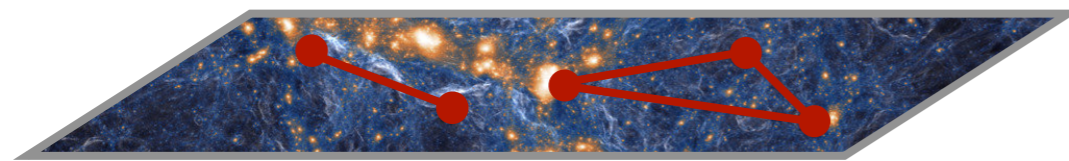**Size** of non-Gaussianities

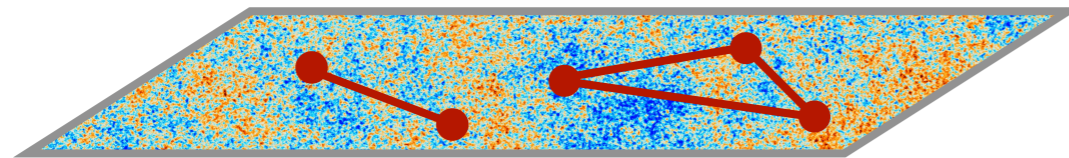f_{NL}

CosmoFlow Starting Guide

CosmoFlow for Initial Conditions of the Universe

We hope to include CosmoFlow in the already-existing
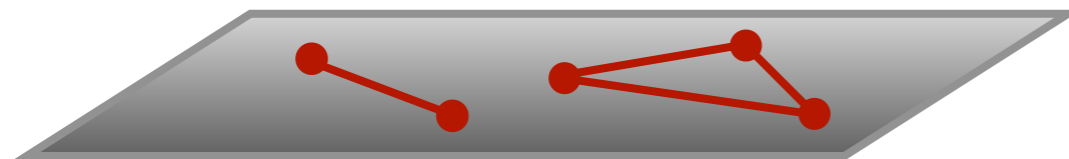chain of **late-time cosmological tools**

Time

LPT, 2LPT, Ramses, Gadget

CLASS

CosmoFlow

$$\langle X^a X^b \rangle \quad \langle X^a X^b X^c \rangle$$

Observables

CosmoFlow

Theories

$$\mathcal{L}(X^a)$$

Initial conditions for
subsequent evolution

CosmoFlow Architecture

background_inputs.py

• Parameters of the Lagrangian

model.py

• Define the theory
• Feed the $u^a{}_b, u^a{}_{bc}$ tensors

solver.py

• Initialise the correlators
• Numerical solver (solve_ivp)

CosmoFlow Starting Guide

https://github.com/deniswerth/CosmoFlow
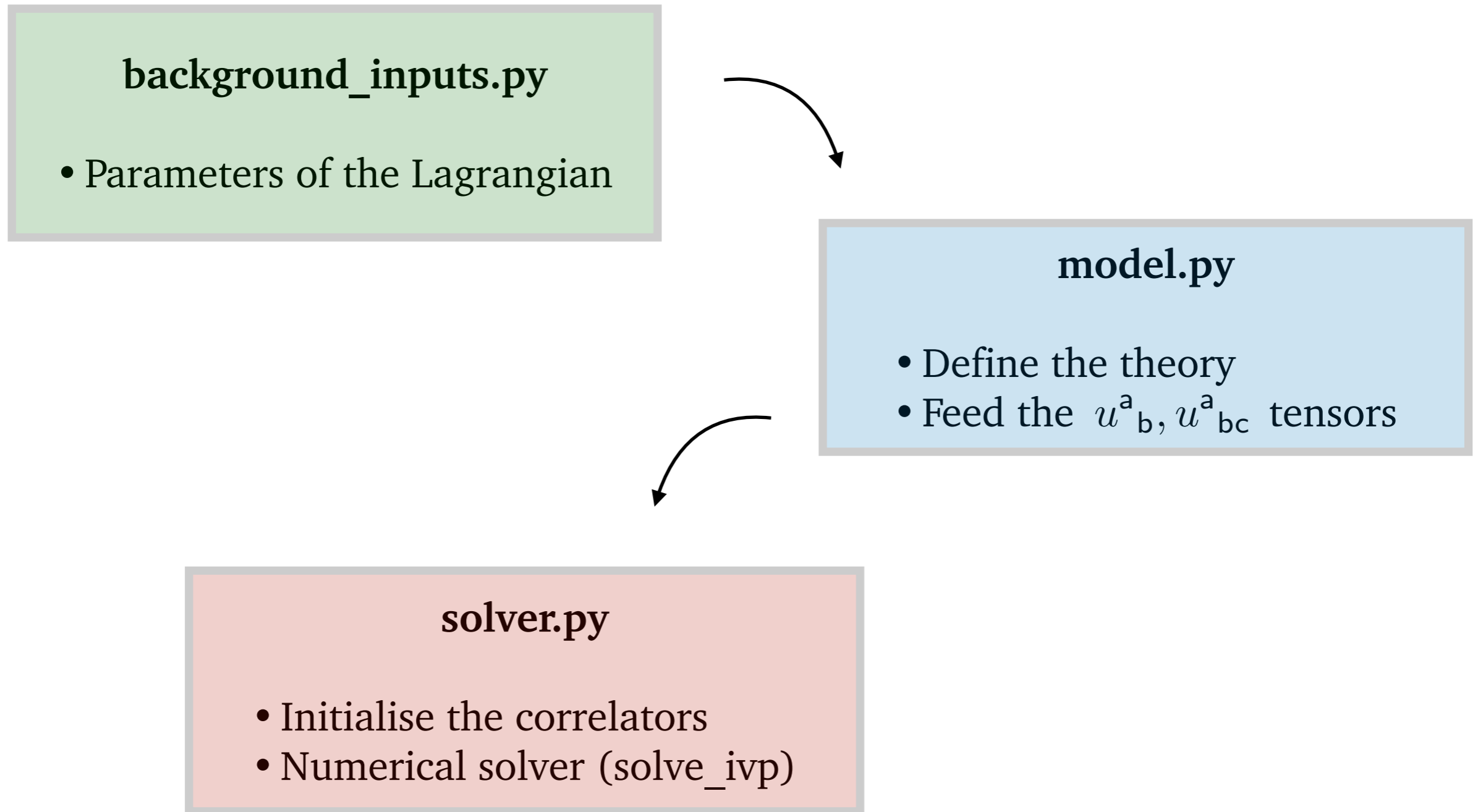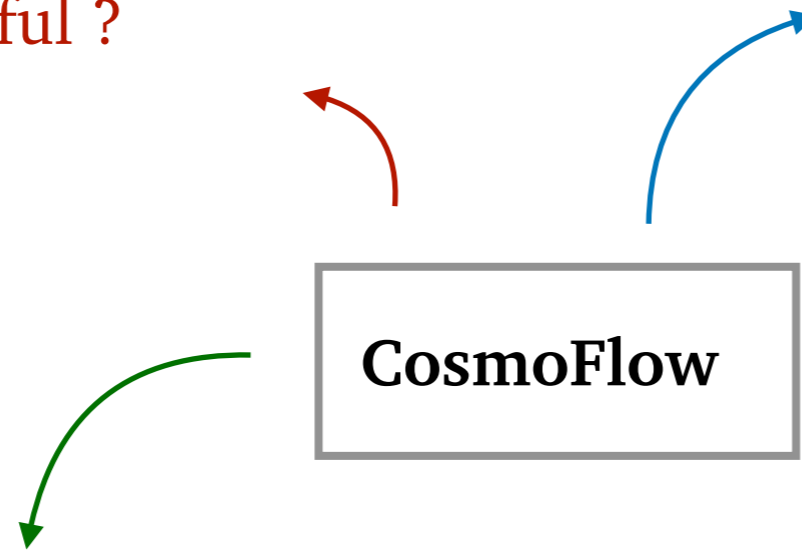
(Not yet released)

What's Next ?

Can **CosmoFlow** be the next **"primordial CLASS"** ?

CosmoFlow Package
- Any advice to write it as a package ?
- How to efficiently advertise CosmoFlow ?
- Is "code paper" useful ?

General Interest
- Would you use CosmoFlow ?
- How can it be used by late-time universe cosmologists ?

CosmoFlow

Numerical Improvements
- How to speed up numerical integration ? (Ideally run it on the laptop)
- How to render CosmoFlow user-friendly ? (For now everything must be done "by hand")
- Which programming language should it be written in ?
- Numerical instability on sub-horizon scales ? $(2N)^3$ ODEs to solve ... ?

Thank you